# HISQ action in dynamical simulations

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HISQ action

Fermion force

Exploratory study of the pion spectrum

Conclusions

# HISQ action

- HISQ = Highly Improved Staggered Quarks[1]
- $O(a^2)$ Symanzik-improved action with further suppressed taste-symmetry violations
- Additional suppression of taste-exchange interactions is achieved by replacing the original gauge links in the Dirac operator by

$$U \rightarrow \mathcal{F}_2 \mathcal{U} \mathcal{F}_1 \, U$$

  - $\mathcal{F}_1$ - smearing level 1 (Fat 7)
  - $\mathcal{U}$ - reunitarization
  - $\mathcal{F}_2$ - smearing level 2 (Asq)

[1] E. Follana, Q. Mason, C. Davies, K. Hornbostel, G.P. Lepage, J. Shigemitsu, H. Trottier, K. Wong, Phys. Rev. D 75 (2007) 054502

# Fermion force

- Calculated as the derivative of the action using the chain rule:
$$\frac{\partial S}{\partial U} = \frac{\partial S}{\partial X}\frac{\partial X}{\partial W}\frac{\partial W}{\partial V}\frac{\partial V}{\partial U},$$
- where
  - $U$ – fundamental gauge links
  - $V$ – fat links level 1
  - $W$ – reunitarized links
  - $X$ – fat links level 2
- The following parts are the same as for Asqtad:
$$\frac{\partial S}{\partial X},\ \frac{\partial X}{\partial W},\ \frac{\partial V}{\partial U}$$

# Reunitarization

- For projecting to U(3) we have chosen

$$W = V(V^\dagger V)^{-1/2}$$

- The derivative

$$\frac{\partial W}{\partial V}$$

can be calculated analytically by applying Cayley-Hamilton theorem[2]

$$Q^{-1/2} = f_0 + f_1 Q + f_2 Q^2, \quad Q = V^\dagger V$$

- May give large contribution to the force

[2] C. Morningstar, M.J. Peardon, Phys. Rev. D 69 (2004) 054501, Hasenfratz, Hoffmann, Schaefer, JHEP 05 (2007) 029

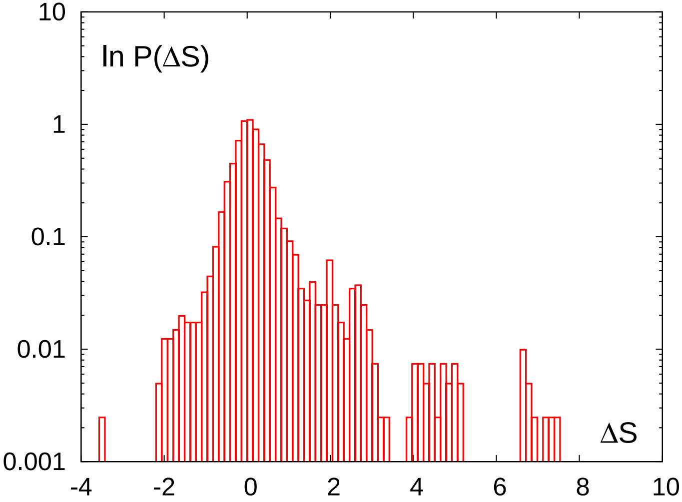
# Example of reunitarization for U(1)

- Let $V = r\, e^{i\theta}$, then $W = e^{i\theta}$
- The derivative

$$\frac{\partial W}{\partial V} = \left(\frac{\partial W}{\partial V}\right)_{V^\dagger} = \frac{\partial(W, V^\dagger)}{\partial(V, V^\dagger)}$$
$$= \frac{\partial(W, V^\dagger)}{\partial(r, \theta)} \cdot \frac{\partial(r, \theta)}{\partial(V, V^\dagger)} = \frac{1}{2r}$$

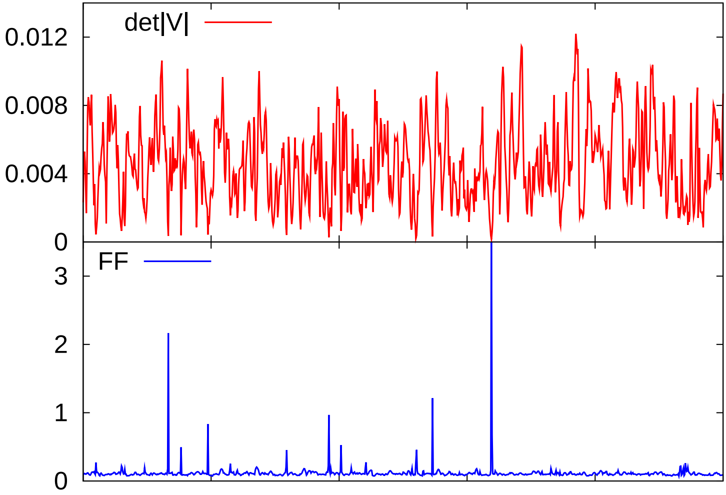
- For matrix case the derivative is dominated by the smallest eigenvalue of $V$

# Change in the action during MD evolution

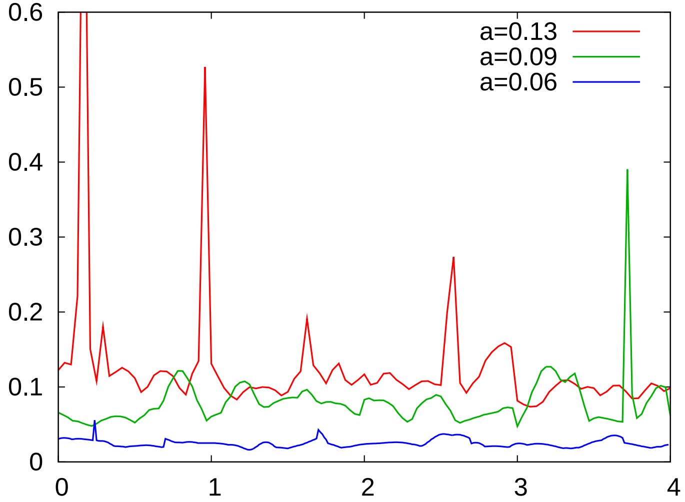



Outliers in the histogram are caused by exceptionally large forces

# Spikes in the force and $\det|V|$



# Fermion force for different ensembles



# Exploratory study

Two 2+1+1 flavor ensembles (50-60 configurations):

- `b675m010m050m600`, $20^3 \times 64$
  - time step 0.0417 (70% acceptance) (0.049 for Asqtad)
  - $a = 0.127$ fm ($r_1/a = 2.50$ from the static potential)
- `b707m007m035m420`, $28^3 \times 96$
  - time step 0.03125 (70% acceptance) (0.038 for Asqtad)
  - $a = 0.093$ fm ($r_1/a = 3.43$ from the static potential)

Recently started 2+1+1 flavor ensemble ($\sim 6$ configurations):

- `b747m004m020m240`, $48^3 \times 144$
  - time step 0.0125 ($\sim 60 - 70\%$ acceptance)
  - $a \sim 0.06$ fm ($r_1/a \sim 5.3$ from the static potential)

# Pion splittings

Ensemble: `b675m010m050m600`

$$\Delta \equiv m^2 - m^2_{\texttt{PION\_5}} \ , \qquad R \equiv \frac{\Delta_{ASQ}}{\Delta_{HISQ}}$$

| | $m_{ASQ}(658)$ | $m_{HISQ}(40)$ | $\Delta_{ASQ}$ | $\Delta_{HISQ}$ | $R$ |
|---|---|---|---|---|---|
| P_5 | 0.2244(02) | 0.1889(07) | | | |
| P_05 | 0.2815(11) | 0.2071(27) | 0.029(1) | 0.0072(11) | 4.0(6) |
| P_i5 | 0.2822(05) | 0.2058(10) | 0.029(0) | 0.0067(05) | 4.4(3) |
| P_ij | 0.3134(20) | 0.2224(33) | 0.048(1) | 0.0138(15) | 3.5(4) |
| P_i0 | 0.3126(11) | 0.2188(19) | 0.047(1) | 0.0122(08) | 3.9(3) |
| P_i | 0.3347(28) | 0.2306(56) | 0.048(1) | 0.0175(26) | 3.5(5) |
| P_0 | 0.3373(15) | 0.2311(22) | 0.063(1) | 0.0178(10) | 3.6(2) |
| P_s | 0.359(5) | 0.252(12) | 0.048(1) | 0.0280(61) | 2.8(6) |

# Pion splittings

Ensemble: `b707m007m035m420`

$$\Delta \equiv m^2 - m^2_{\texttt{PION\_5}} \;, \qquad R \equiv \frac{\Delta_{ASQ}}{\Delta_{HISQ}}$$

| | $m_{ASQ}(572)$ | $m_{HISQ}(50)$ | $\Delta_{ASQ}$ | $\Delta_{HISQ}$ | $R$ |
|---|---|---|---|---|---|
| P_5 | 0.2069(05) | 0.1378(08) | | | |
| P_05 | 0.2177(10) | 0.1420(08) | 0.0046(5) | 0.0012(4) | 4(1) |
| P_i5 | 0.2187(07) | 0.1428(08) | 0.0050(4) | 0.0014(3) | 3.6(8) |
| P_ij | 0.2256(11) | 0.1467(21) | 0.0081(5) | 0.0025(7) | 3.2(8) |
| P_i0 | 0.2259(07) | 0.1475(11) | 0.0082(4) | 0.0028(4) | 3.0(4) |
| P_i | 0.2311(15) | 0.1485(16) | 0.0106(7) | 0.0031(5) | 3.5(6) |
| P_0 | 0.2318(10) | 0.1509(11) | 0.0109(5) | 0.0038(4) | 2.9(3) |
| P_s | 0.2398(25) | 0.1522(27) | 0.015(1) | 0.0042(9) | 3.5(8) |

# Conclusions

- The reunitarization step introduces spikes in the fermion force whenever a matrix with small determinant is created during the smearing level 1
- Spikes lead to large fluctuations in the action increasing the number of rejected trajectories
- This problem becomes less severe closer to the continuum limit where gauge configurations are smoother
- Comparing to Asqtad two levels of smearing in HISQ action tend to create smoother configurations that require less iterations of the conjugate gradient
- Including the charm quark is cheap for CG, but more expensive for building fat links due to the correction to the Naik term

# Valence: HISQ/Sea: Asqtad

Ensemble: `b676m010m050`, tuned $m_{val} = 0.01365$

$$\Delta \equiv m^2 - m^2_{\mathtt{PION\_5}} \;, \qquad R \equiv \frac{\Delta_{ASQ}}{\Delta_{HISQ}}$$

| | $m_{ASQ}(658)$ | $m_{HISQ}(128)$ | $\Delta_{ASQ}$ | $\Delta_{HISQ}$ | $R$ |
|---|---|---|---|---|---|
| PION_5 | 0.2244(02) | 0.2224(04) | | | |
| PION_05 | 0.2815(11) | 0.2439(12) | 0.029(1) | 0.0100(6) | 2.9 |
| PION_i5 | 0.2822(05) | 0.2433(07) | 0.029(0) | 0.0097(4) | 3.0 |
| PION_ij | 0.3134(20) | 0.2630(20) | 0.048(1) | 0.020(1) | 2.4 |
| PION_i0 | 0.3126(11) | 0.2634(12) | 0.047(1) | 0.0199(7) | 2.4 |
| PION_i | 0.3347(28) | 0.2815(64) | 0.062(2) | 0.030(4) | 2.1 |
| PION_0 | 0.3373(15) | 0.2813(17) | 0.063(1) | 0.0297(1) | 2.1 |
| PION_s | 0.359(5) | 0.2882(49) | 0.079(4) | 0.034(4) | 2.4 |

# Valence: HISQ-Stout/Sea: Asqtad

Ensemble: `b676m010m050`, tuned $m_{val} = 0.01365$

$$\Delta \equiv m^2 - m^2_{\mathtt{PION\_5}} \ , \qquad R \equiv \frac{\Delta_{ASQ}}{\Delta_{HISQ-S}}$$

| | $m_{ASQ}(658)$ | $m_{H-S}(124)$ | $\Delta_{ASQ}$ | $\Delta_{HISQ-S}$ | $R$ |
|---|---|---|---|---|---|
| PION_5 | 0.2244(02) | 0.2358(04) | | | |
| PION_05 | 0.2815(11) | 0.2686(15) | 0.029(1) | 0.0165(8) | 1.8 |
| PION_i5 | 0.2822(05) | 0.2690(08) | 0.029(0) | 0.0168(4) | 1.7 |
| PION_ij | 0.3134(20) | 0.2955(34) | 0.048(1) | 0.0317(2) | 1.5 |
| PION_i0 | 0.3126(11) | 0.2963(13) | 0.047(1) | 0.0322(8) | 1.5 |
| PION_i | 0.3347(28) | 0.3119(35) | 0.062(2) | 0.042(2) | 1.5 |
| PION_0 | 0.3373(15) | 0.3196(23) | 0.063(1) | 0.047(2) | 1.4 |
| PION_s | 0.359(5) | 0.3301(66) | 0.079(4) | 0.053(4) | 1.5 |