Center-symmetric dimensional reduction of hot Yang-Mills theory

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arXiv:0704.1416,
arXiv:0801.1566 with Philippe de Forcrand and Aleksi Vuorinen

Dimensional reduction

- At high T: For long distance properties ($\Delta x \gg 1/T$), the system looks 3d.

- Degrees of freedom are static modes $\phi_0(\mathbf{x})$
  $\phi(\mathbf{x},\tau) = T\sum_{n=-\infty}^{\infty} \exp(i\omega_n\tau)\phi_n(\mathbf{x})$
- Effective action: Integrate out non-static modes

$$Z = \int \mathcal{D}\phi_0 \mathcal{D}\phi_n \exp(-S_0(\phi_0) - S_n(\phi_0, \phi_n))$$
$$= \int \mathcal{D}\phi_0 \exp(-S_0(\phi_0) - S_{\text{eff}}(\phi_0))$$

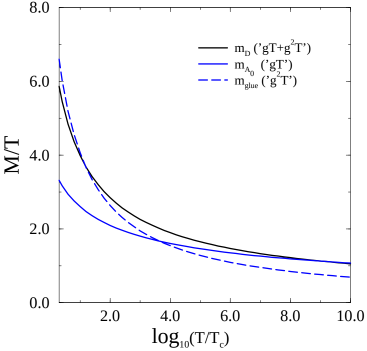
- In practice: Need scale separation between static and non-static modes

Where Dimensional Reduction works?

Scales in hot Yang-Mills:

- Perturbatively ($g \sim 1/\log(T)$):
  - Hard scale: $2\pi T$ Typical thermal momentum, non-static modes
  - Soft scale: $m_D \sim gT$ Debye screening, static modes

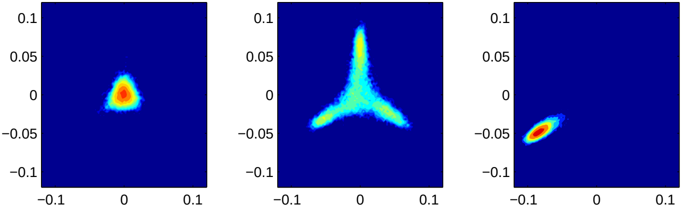
    $\Rightarrow$ Asymptotic dimensional reduction
- Non-perturbatively: $m(T_c) \sim 3T_c \overset{?}{\ll} 2\pi T_c$



Laermann, Philipsen

Perturbative dimensional reduction

Polyakov loop $\Omega = \mathrm{Tr}\left[P\exp\left(ig\int \mathrm{d}\tau A_0\right)\right]$, has N_c minima in the deconfined phase $\Rightarrow Z_N$ center symmetry.



$T < T_c$ $\quad$ $T \approx T_c$ $\quad$ $T > T_c$

Deep in deconfined phase: Expand fields around one minimum to get EQCD (= 3D Yang-Mills + adjoint Higgs):

$$S_{\mathrm{EQCD}} = \int \mathrm{d}^3x \Big[\underbrace{\frac{1}{2}\mathrm{Tr}F_{ij}}_{\text{spatial gluons}} + \underbrace{\mathrm{Tr}(D_i A_0)^2}_{\text{adjoint kinetic}} + \underbrace{\frac{1}{2}m_{\mathrm{E}}^2 \mathrm{Tr}A_0^2 + \frac{1}{4}\lambda_E \mathrm{Tr}A_0^4}_{\text{interactions from integration out}} + \ldots \Big]$$

Center-symmetric effective theories

- Goal: Want to construct an effective theory that
  - Preserves the Z_N center symmetry
  - Reduces to EQCD at high T
  - Is superrenormalizable
- Effective theory of Wilson lines not (super)renormalizable

  (Pisarski hep-ph/0608242)

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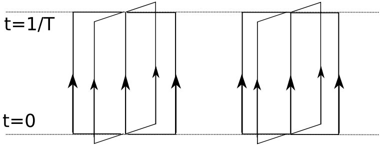
Idea: Construct effective theory for *coarse grained* Wilson loop

(Yaffe+Vuorinen hep-ph/0604100)

$$\mathcal{Z}(\mathbf{x}) = \frac{T}{V_{\text{Block}}} \int_V \mathrm{d}^3 y U(\mathbf{x},\mathbf{y}) W(\mathbf{y}) U(\mathbf{y},\mathbf{x}), \notin SU(N_\mathrm{c})$$



Center-symmetric theory for SU(2)

- For SU(2), sum of matrices proportional to SU(2)

$$
\begin{aligned}
\mathcal{Z} &= \lambda\Omega, \quad \Omega \in \mathrm{SU}(2), \quad \lambda > 0 \\
\mathcal{Z} &= \frac{1}{2}\Big\{ \underbrace{\Sigma \mathbb{1}}_{\text{Singlet}} + i \underbrace{\Pi_a \sigma_a}_{\text{Adjoint scalar}} \Big\} = \begin{pmatrix} \frac{1}{2}\Sigma + i\Pi_1 & i\Pi_2 - \Pi_3 \\ i\Pi_2 + \Pi_3 & \frac{1}{2}\Sigma - i\Pi_1 \end{pmatrix}
\end{aligned}
$$

- Transforms exactly like Wilson line

$$
\begin{aligned}
\mathcal{Z} &\longrightarrow \lambda^{-1}(\mathbf{x})\mathcal{Z}\lambda(\mathbf{x}) && \text{gauge} \\
\mathcal{Z} &\longrightarrow -\mathcal{Z} && \text{center } Z_2
\end{aligned}
$$

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- Most general superrenormalizable Lagrangian with A_i and $\mathcal{Z}$:

$$\mathcal{L}_{Z(2)} = \underbrace{\frac{1}{2}\mathrm{Tr}\, F_{ij}^2}_{\text{spatial gluons}} + \underbrace{\mathrm{Tr}\left(D_i\mathcal{Z}^\dagger D_i\mathcal{Z}\right)}_{\text{Adjoint Kinetic}} + V(\mathcal{Z})$$

$$V(\mathcal{Z}) = \underbrace{b_1\Sigma^2 + b_2\Pi_a^2 + c_1\Sigma^4 + c_2(\Pi_a^2)^2 + c_3\Sigma^2\Pi_a^2}_{\text{interaction from integration out}}$$

- Higher order terms suppressed by scale difference m_D/T.

Matching at $T \to \infty$

Parameters can be (almost) mached in perturbation theory (series in $\frac{g^2}{16\pi^2}$!):

$$
\begin{aligned}
b_1 &= -\frac{1}{4}r^2T^2, \\
b_2 &= -\frac{1}{4}r^2T^2 + 0.441841g^2T^2, \\
c_1/g_3^2 &= 0.0311994r^2 + 0.0135415g^2, \\
c_2/g_3^2 &= 0.0311994r^2 + 0.008443432g^2, \\
c_3/g_3^2 &= 0.0623987r^2, \\
g_3^2 &= g^2T
\end{aligned}
$$

- Parameters functions of full theory parameters (g, T) and r
  - rT: mass of fluctuation from SU(2) manifold
- r needs to be matched non-perturbatively

On the lattice:

$$S_a = S_W + S_{\mathcal{Z}} + V(\hat{\Sigma}, \hat{\Pi}),$$

$$\boxed{\beta = \frac{4}{a g_3^2}}$$

$$S_W = \beta \sum_{x,i<j} \left[1 - \frac{1}{2} \mathrm{Tr}\left[U_{ij}\right] \right],$$

$$S_{\mathcal{Z}} = 2\left(\frac{4}{\beta}\right) \sum_{x,i} \mathrm{Tr}\, \left[\hat{\Pi}^2 - \hat{\Pi}(x) U_i(x) \hat{\Pi}(x+\hat{i}) U_i^\dagger(x)\right]$$

$$+ \left(\frac{4}{\beta}\right) \sum_{x,i} \left(\hat{\Sigma}^2(x) - \hat{\Sigma}(x)\hat{\Sigma}(x+\hat{i})\right),$$

$$V = \left(\frac{4}{\beta}\right)^3 \sum_x \left[\hat{b}_1 \hat{\Sigma}^2 + \hat{b}_2 \hat{\Pi}_a^2 + \hat{c}_1 \hat{\Sigma}^4 + \hat{c}_2 \left(\hat{\Pi}_a^2\right)^2 + \hat{c}_3 \hat{\Sigma}^2 \hat{\Pi}_a^2\right],$$

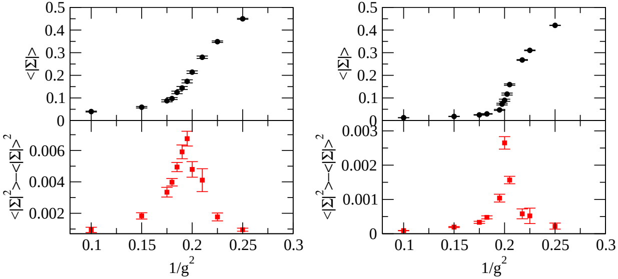
Exact renormalization (2-loop lat-PT) AK [0711.1796/hep-lat]

$$\hat{b}_1 = b_1/g_3^4 - \frac{2.38193365}{4\pi}(2\hat{c}_1 + \hat{c}_3)\beta + \frac{1}{16\pi^2}\left\{(48\hat{c}_1^2 + 12\hat{c}_3^2 - 12\hat{c}_3)\left[\log 1.5\beta + 0.08849\right] - 6.9537\,\hat{c}_3\right\} + \mathcal{O}(a),$$

$$\hat{b}_2 = b_2/g_3^4 - \frac{0.7939779}{4\pi}(10\hat{c}_2 + \hat{c}_3 + 2)\beta + \frac{1}{16\pi^2}\Big\{(80\hat{c}_2^2 + 4\hat{c}_3^2 - 40\hat{c}_2)\left[\log 1.5\beta + 0.08849\right] - 23.17895\,\hat{c}_2 - 8.66687\Big\} + \mathcal{O}(a).$$

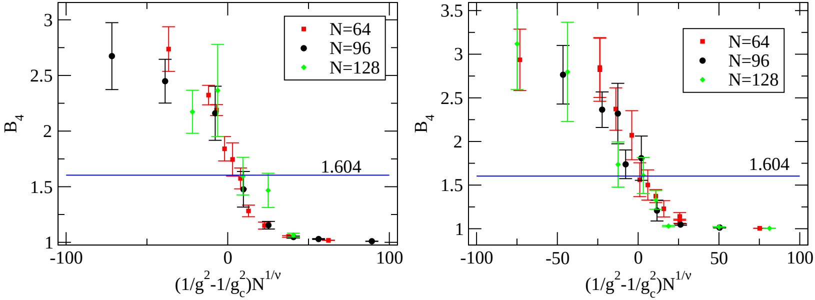
Results from simulations:

Z_2-restoring phase transition



- left ($\beta = 12, n = 64, r^2 = 5$)
- right ($\beta = 6, n = 64, r^2 = 5$)

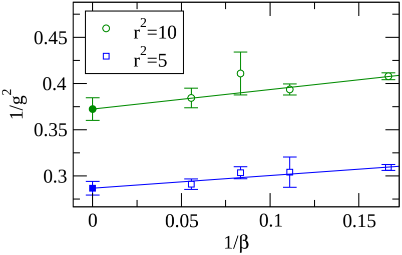
Results from simulations:



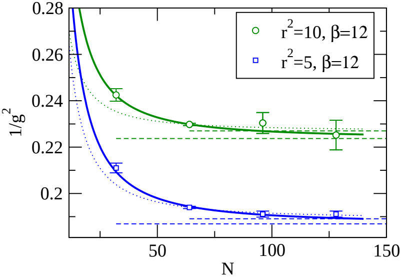
- left $r^2 = 5$, right $r^2 = 10$.
- 3d Ising universality class.

$$B_4 = \langle \bar{\Sigma}^4 \rangle / \langle \bar{\Sigma}^2 \rangle^2 = 1.604 \ldots \text{at criticality}$$
$$\nu = 0.63$$

Results from simulations:



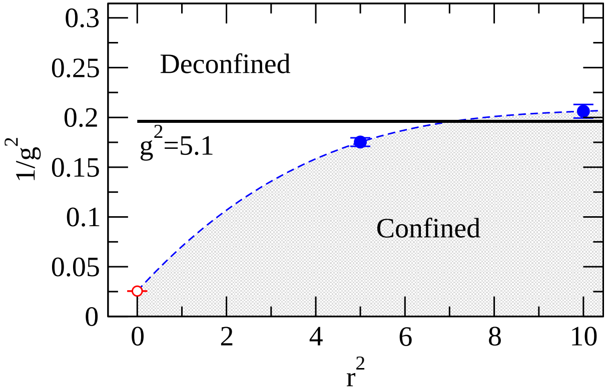
$a \to 0$



$V \to \infty$ $(A + BL^{-1/\nu})$

- large r → short correlation length → fine lattice
- small r → long correlation length → large volume
- $r = 0$: Σ decouples → $\lambda\phi^4$ already done, X.P. Sun hep-lat/0209144

Results from simulations:



- Phase diagram resembles the full theory (unlike in EQCD).
- Insensitive to $r > 1$
- Phase transition at correct g!

Outlook

Implementing center-symmetry to the effective theory gives correct phase transition in SU(2)

Lots of simulations to do:

- Check accuracy near T_c:
  - Domain wall tension
  - Spatial string tension
  - Screening masses
- Make predictions:
  - Heavy quarks: Z_N breaking terms
  - Finite chemical potential (Correct phase transitions?)
  - Extension to large N_c apparent:
    - At $N_c > 4$ no more N_c-dependent (super-renormalizable) operators.
- Can the theory accommodate "fuzzy bag"?
  - $p(T) = B_{\text{MIT}} + B_{\text{fuzzy}} T^2 + f_{\text{pert}} T^4$