A Fitting Robot for Variational Analysis

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CSSM

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Introduction

- All-to-all propagators
- The folly of effective mass plots
- Variational analysis
- Need to minimise work *and uncertainty*

All-to-all Propagators - Hybrid Method

The quark propagator is broken up into two subspaces:
$Q^{-1} = \bar{Q}_0 + \bar{Q}_1$,

- $\bar{Q}_0$ is given by truncated spectral decomposition.
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Construct the *hybrid list*

$$w^{(i)} = \left\{ \frac{v^{(1)}}{\lambda_1}, \cdots, \frac{v^{(N_{ev})}}{\lambda_{N_{ev}}}, \eta^{(1)}, \cdots, \eta^{(N_d)} \right\}$$

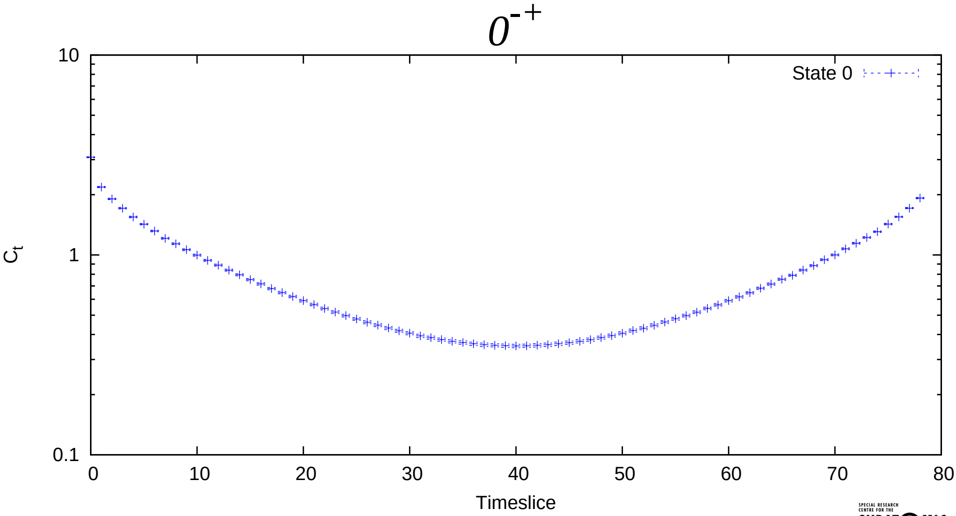
$$u^{(i)} = \left\{ v^{(1)}, \cdots, v^{(N_{ev})}, \psi^{(1)}, \cdots, \psi^{(N_d)} \right\}$$

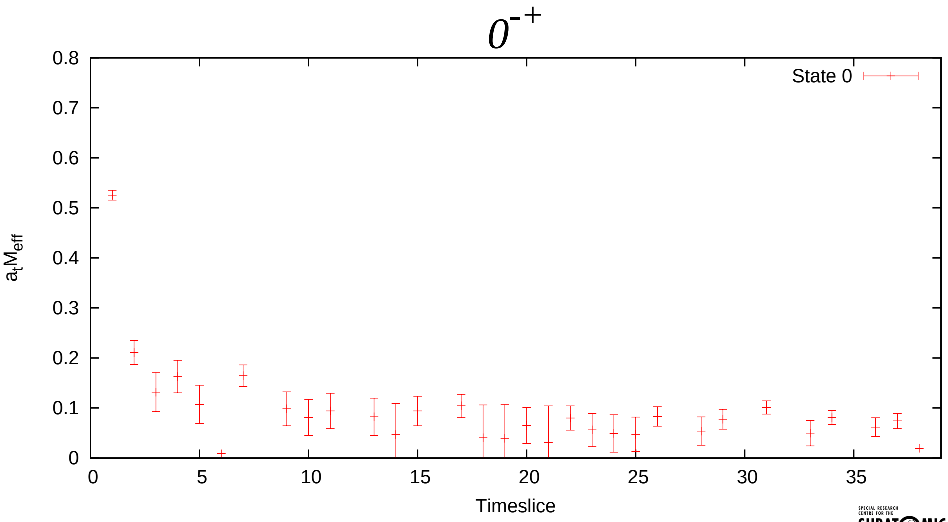
The hybrid formula for the all-to-all quark propagator (where $Q = \gamma_5 M$) is given by

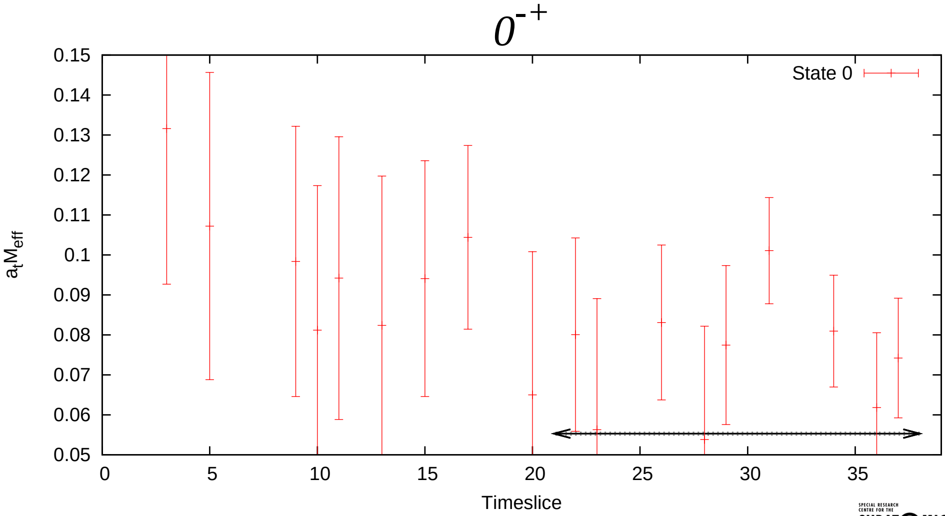
$$M^{-1} = \sum_{i=1}^{N_{HL}} u^{(i)}(\vec{x}, x_4) \otimes w^{(i)}(\vec{y}, y_4)^{\dagger} \gamma_5$$

Lattice Parameters

- $N_f = 2$ dynamical background
- $12^3 \times 80$ anisotropic lattice with $\xi = 6$ and $a_s = 0.2fm$.
- 96 gauge configurations
- Operators - quark bilinears, extended, smeared.
- Light quark mass comparable to strange.
- 20 eigenvectors, time and colour dilution


0-+
State 0
10
1
0.1
C_t
0
10
20
30
40
50
60
70
80
Timeslice


0^{-+}
State 0
$a_t M_{eff}$
Timeslice
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0
5
10
15
20
25
30
35


0^{-+}
State 0
$a_t M_{eff}$
Timeslice
0.15
0.14
0.13
0.12
0.11
0.1
0.09
0.08
0.07
0.06
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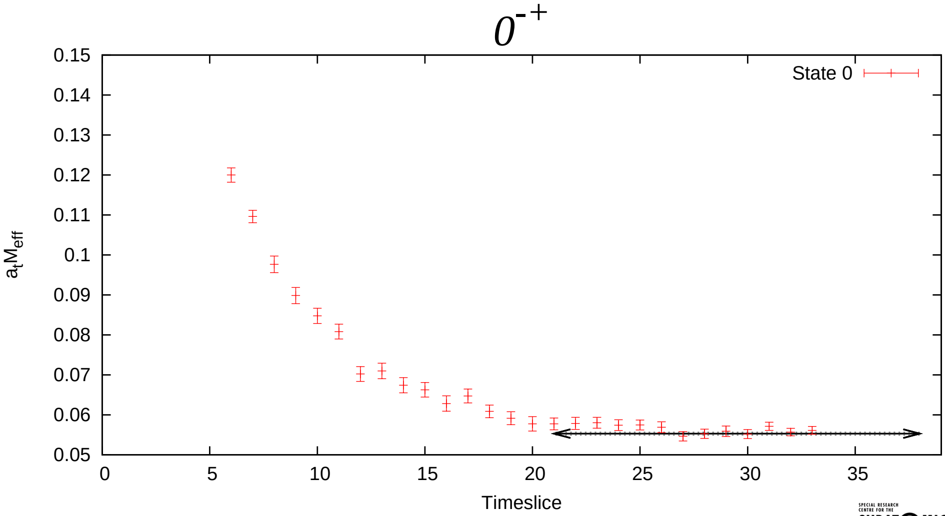
35


0-+
State 0
a_tM_eff
0.15
0.14
0.13
0.12
0.11
0.1
0.09
0.08
0.07
0.06
0.05
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5
10
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30
35
Timeslice

A Fitting Robot

- Identify data limitations

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- χ^2_{PDOF} - measure of fit

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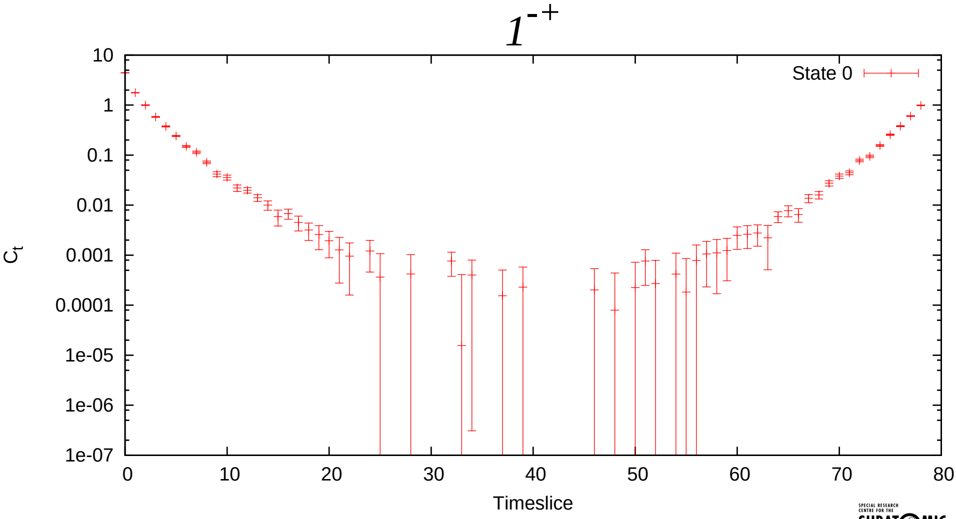
- χ^2_{PDOF} - measure of fit
- Search for maximum sized fit window
- t_{min} constrains fit and χ^2_{PDOF}, search in t_{max} direction first
- Resulting bias - largest fit window beginning at lowest t_{min} with acceptable χ^2_{PDOF} value


1^{-+}
State 0
C_t
Timeslice
10
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
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40
50
60
70
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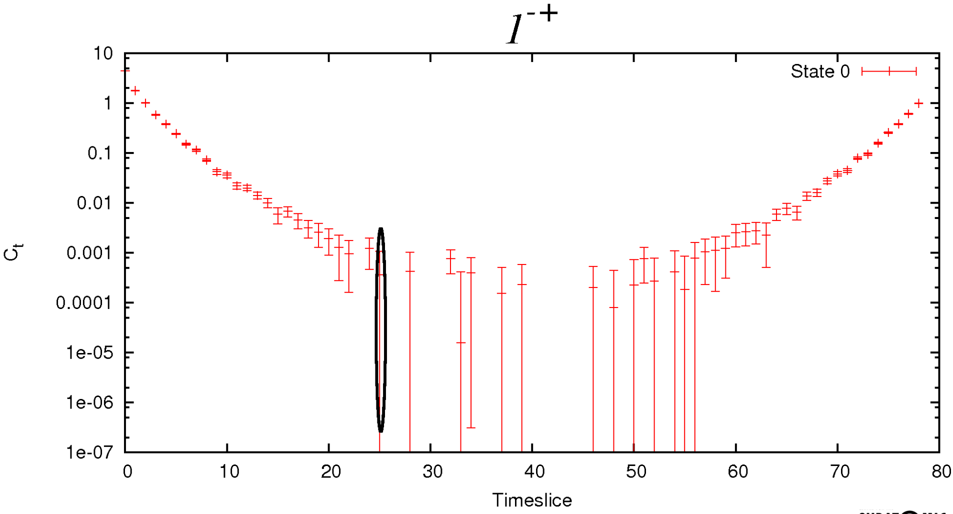
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C_t
10
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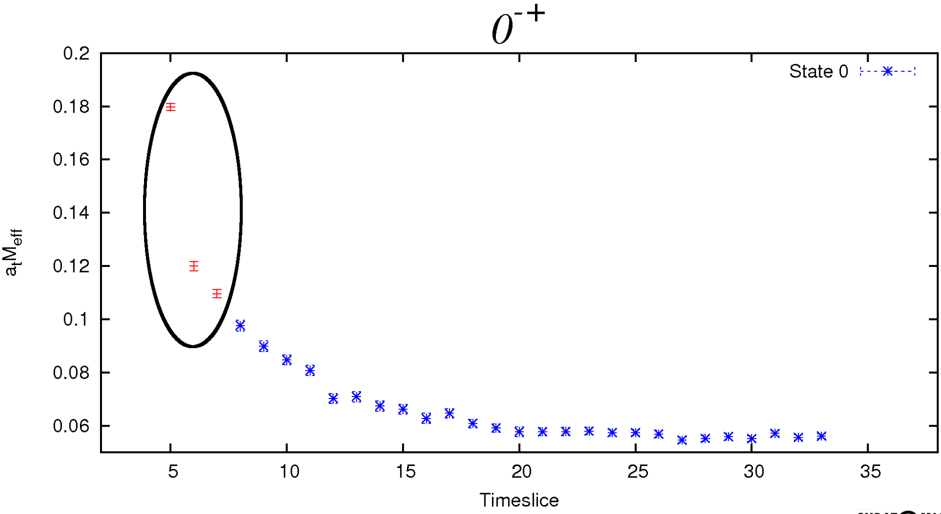
0^{-+}
State 0
$a_t M_{eff}$
Timeslice
0.2
0.18
0.16
0.14
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0.1
0.08
0.06
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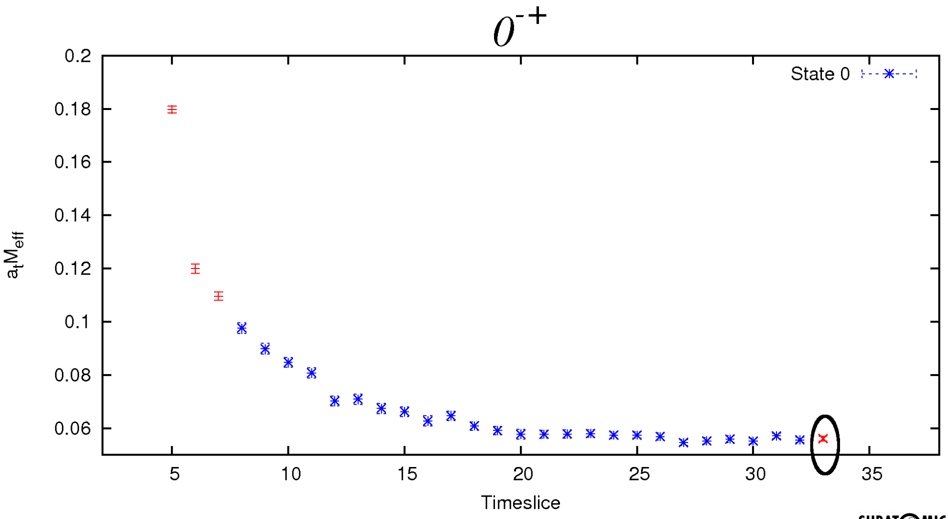
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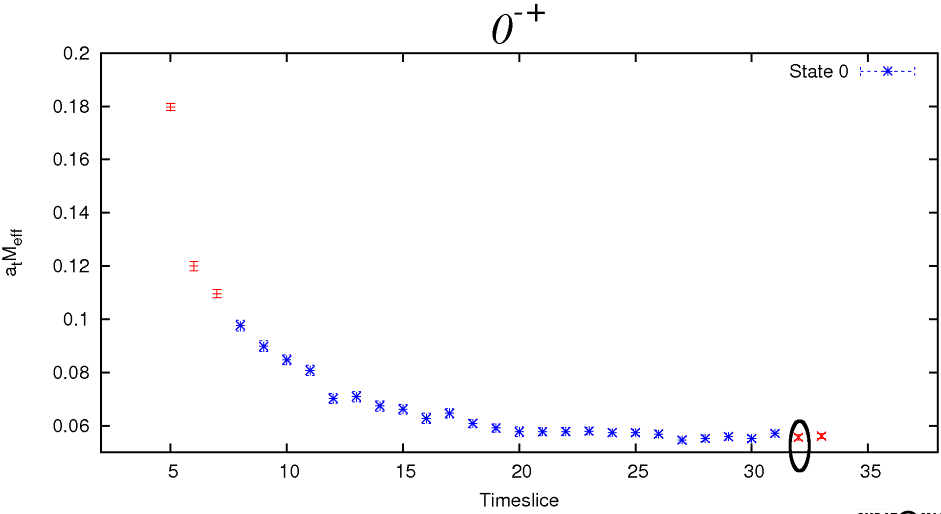
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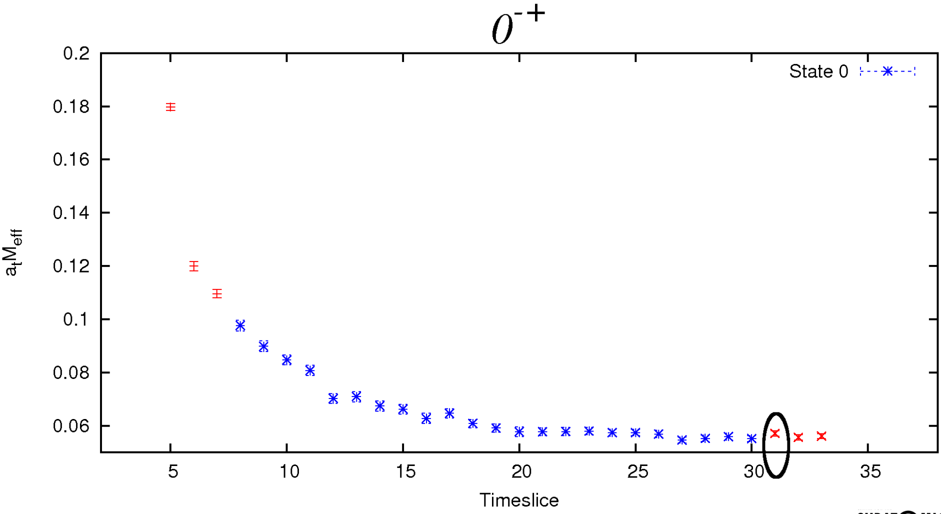
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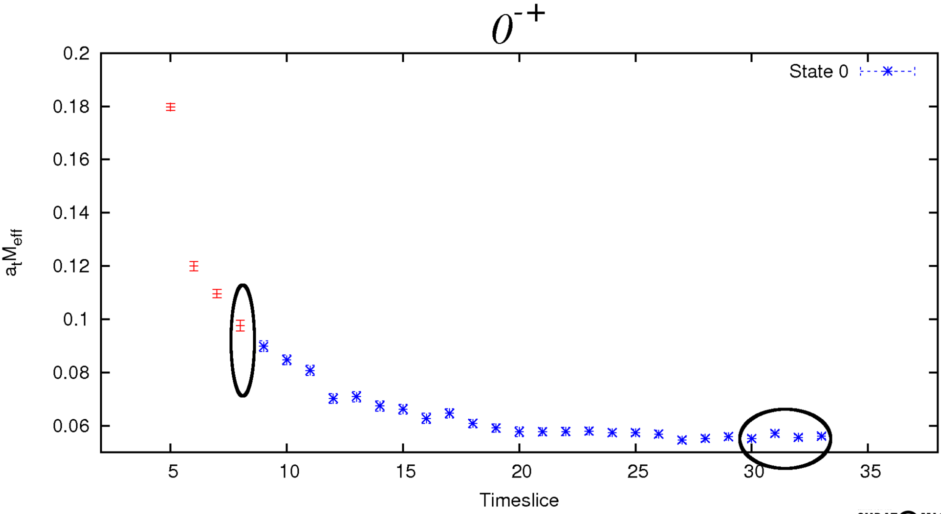
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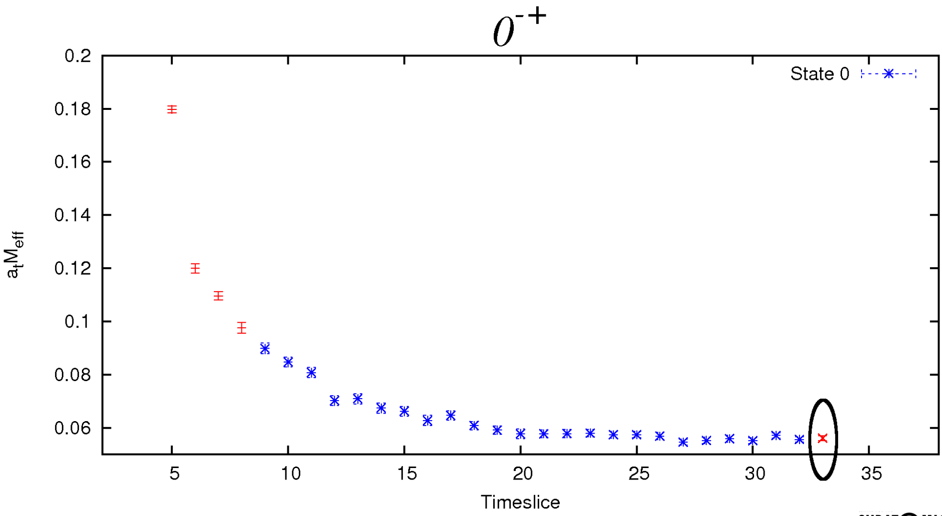
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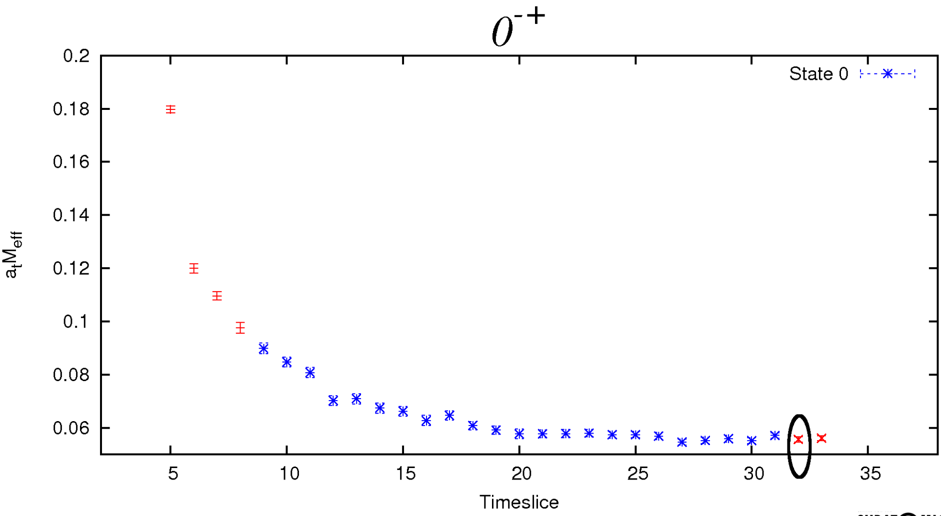
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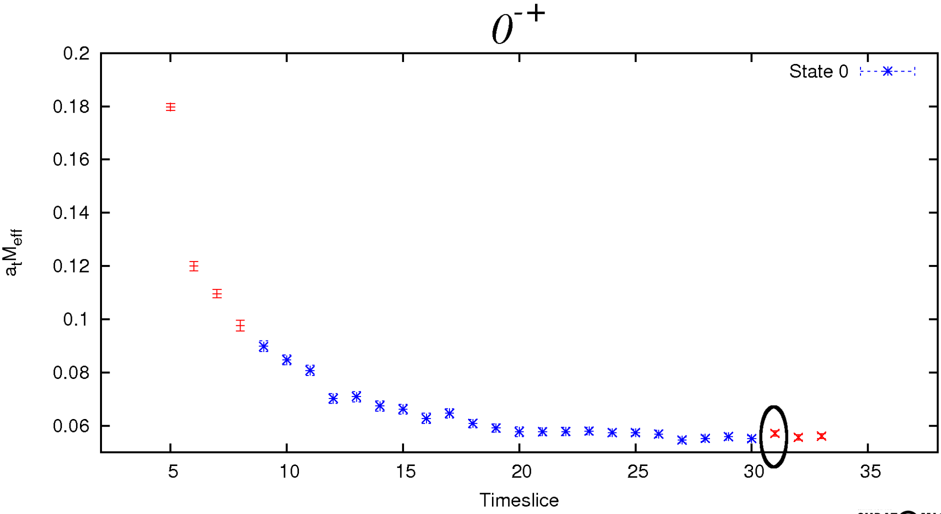
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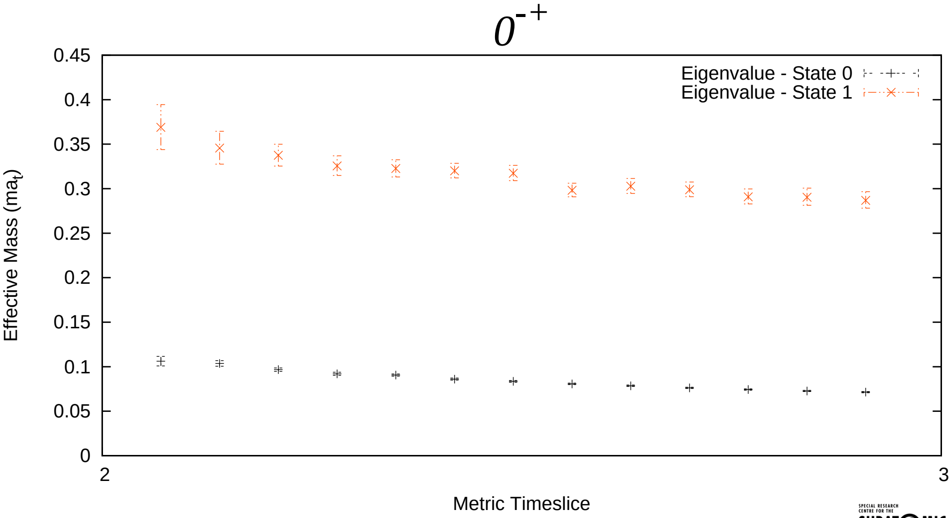
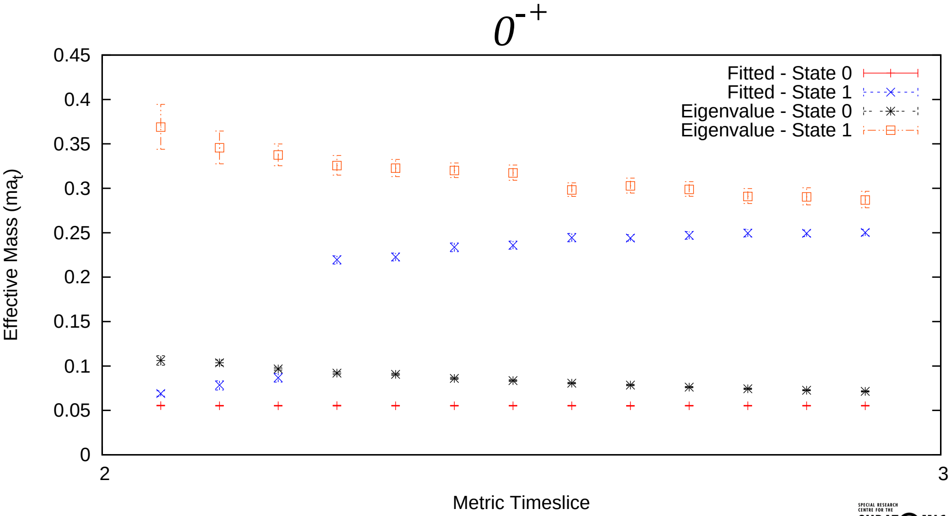
Variational Analysis

- Extracting excited-state energies requires matrix of correlators
- For a given $N \times N$ correlator matrix $C_{\alpha\beta}(t) = \langle 0|\mathcal{O}_\alpha(t)\mathcal{O}_\beta(0)|0\rangle$ one defines the N principal correlators $\lambda_\alpha(t, t_0)$ as the eigenvalues of

$$C(t_0)^{1/2}C(t)C(t_0)^{1/2}$$

  where t_0 (the time defining the metric) is small
- Can show that $lim_{t\to\inf}\lambda_\alpha(t, t_0) = e^{-(t-t_0)E_\alpha}(1 + e^{-t\Delta E_\alpha})$
- N principal effective masses defined by $m_\alpha^{eff}(t) = ln(\frac{\lambda_\alpha(t,t_0)}{\lambda_\alpha(t+1,t_0)})$ now tend (plateau) to the N lowest-lying stationary-state energies, as do the projected correlation functions


0-+
Eigenvalue - State 0
Eigenvalue - State 1
Effective Mass (ma_t)
Metric Timeslice
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
2
3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Effective Mass (ma_t)
Metric Timeslice
Fitted - State 0
Fitted - State 1
Eigenvalue - State 0
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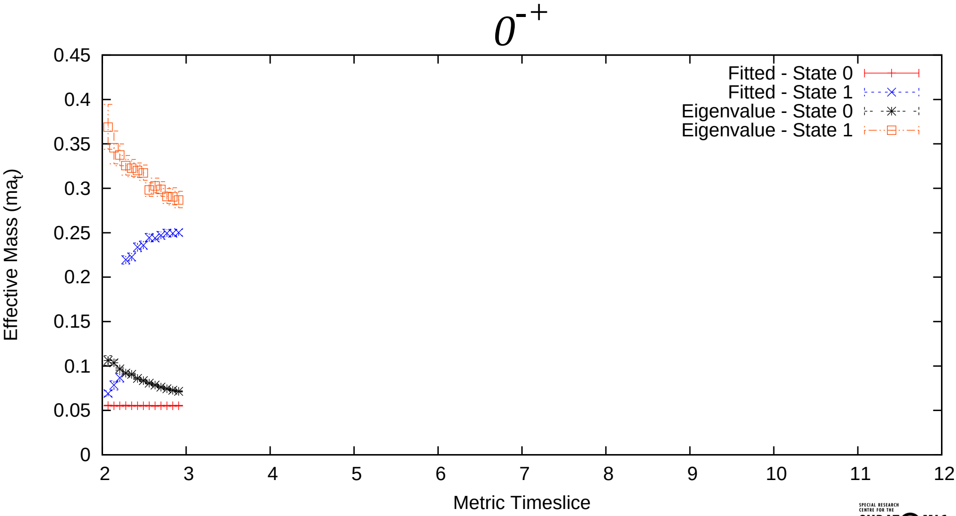
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Eigenvalue - State 0
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Effective Mass (ma_t)
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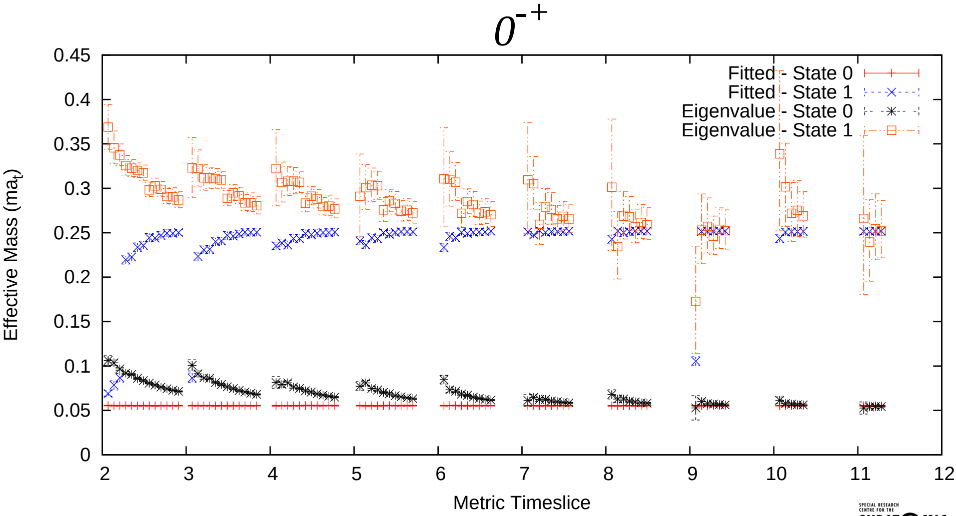
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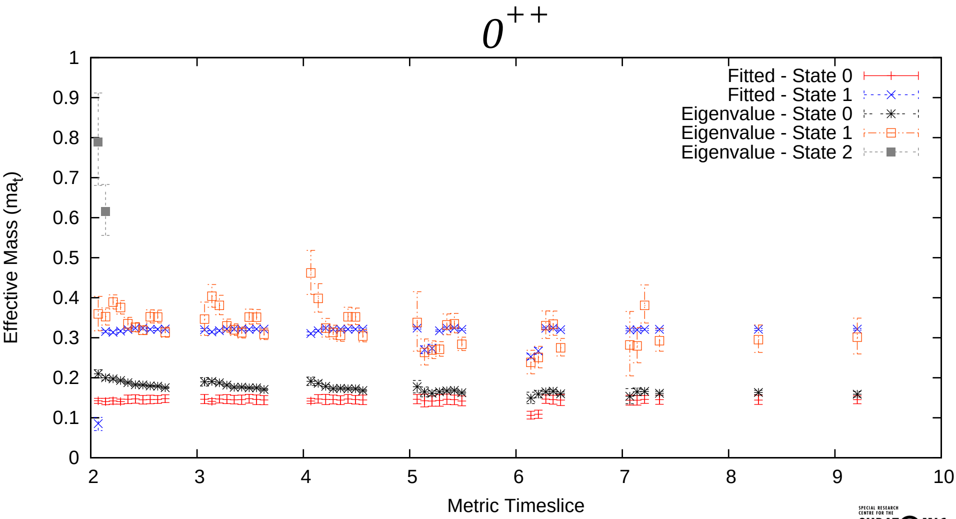
0^{++}
Fitted - State 0
Fitted - State 1
Eigenvalue - State 0
Eigenvalue - State 1
Eigenvalue - State 2
Effective Mass (ma$_t$)
Metric Timeslice
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
2
3
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7
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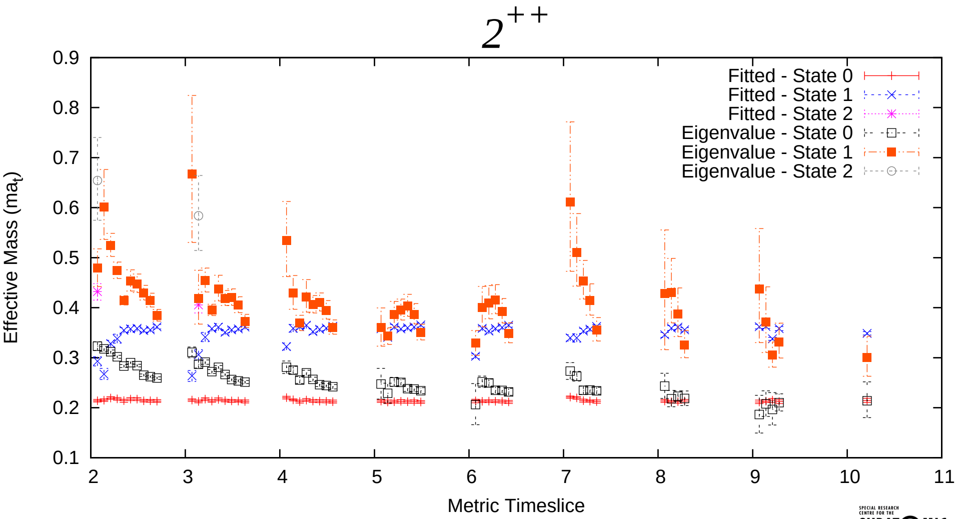
9
10

Results


2^{++}
Effective Mass (ma_t)
Metric Timeslice
Fitted - State 0
Fitted - State 1
Fitted - State 2
Eigenvalue - State 0
Eigenvalue - State 1
Eigenvalue - State 2
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
2
3
4
5
6
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8
9
10
11

Conclusions and Further Work

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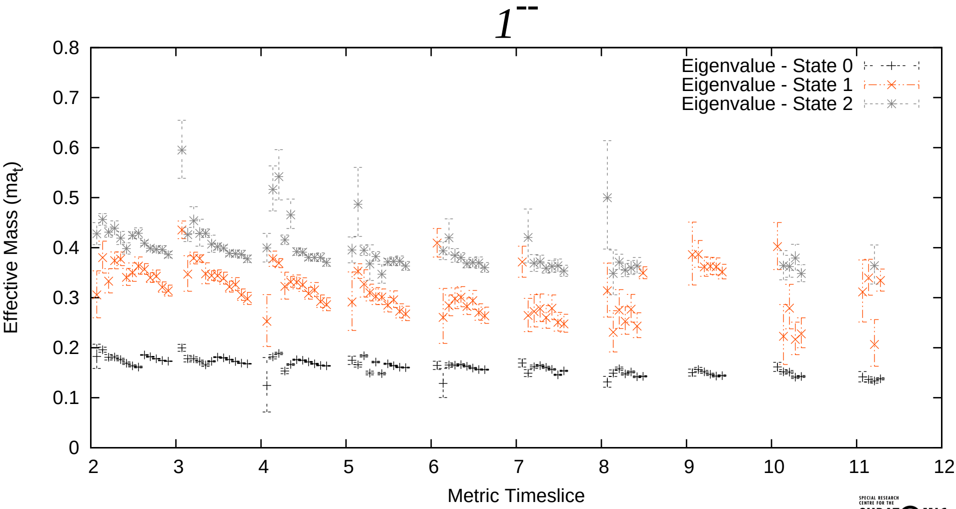
- Necessary to explore the variational analysis parameter space
- Need to replace effective mass plots with more informative visual aid


1^{--}
Eigenvalue - State 0
Eigenvalue - State 1
Eigenvalue - State 2
Effective Mass (ma_t)
Metric Timeslice
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
2
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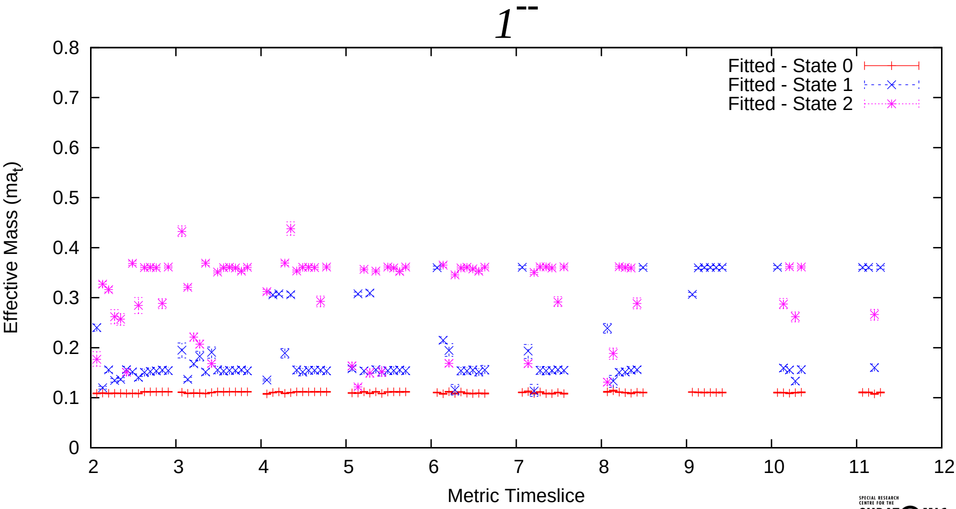
1^{--}
Effective Mass (ma_t)
Metric Timeslice
Fitted - State 0
Fitted - State 1
Fitted - State 2
0.8
0.7
0.6
0.5
0.4
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0.2
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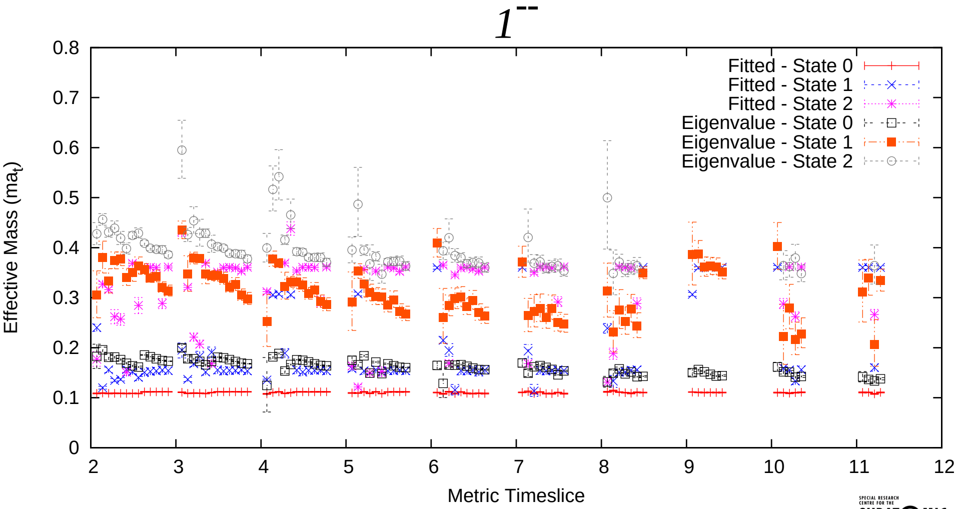
1--
Effective Mass (ma_t)
Metric Timeslice
Fitted - State 0
Fitted - State 1
Fitted - State 2
Eigenvalue - State 0
Eigenvalue - State 1
Eigenvalue - State 2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
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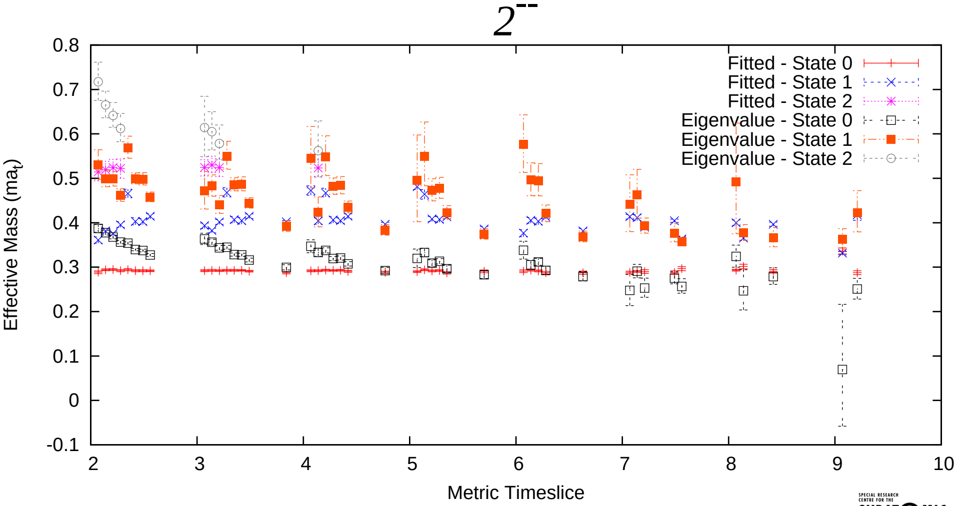
2^{--}
Fitted - State 0
Fitted - State 1
Fitted - State 2
Eigenvalue - State 0
Eigenvalue - State 1
Eigenvalue - State 2
Effective Mass (ma_t)
Metric Timeslice
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
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-0.1
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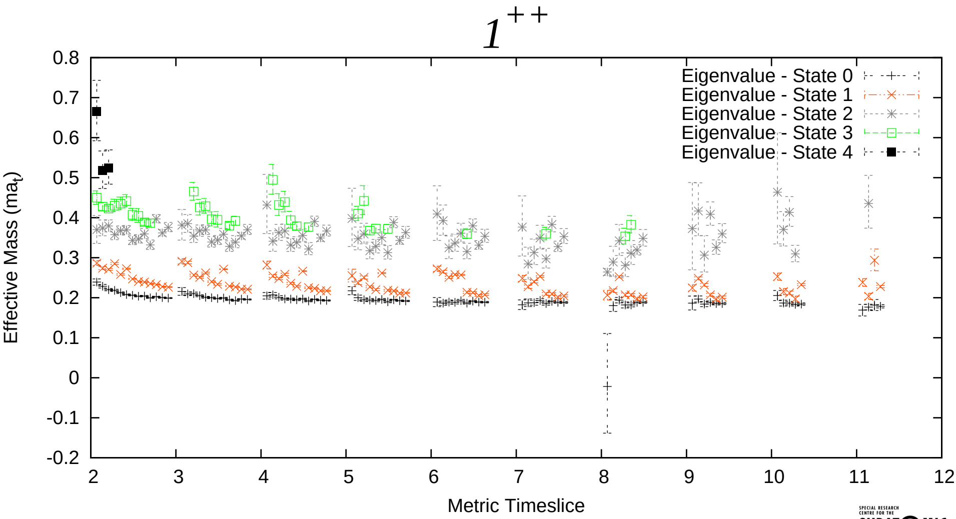
1^{++}
Eigenvalue - State 0
Eigenvalue - State 1
Eigenvalue - State 2
Eigenvalue - State 3
Eigenvalue - State 4
Effective Mass (ma_t)
Metric Timeslice
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
2
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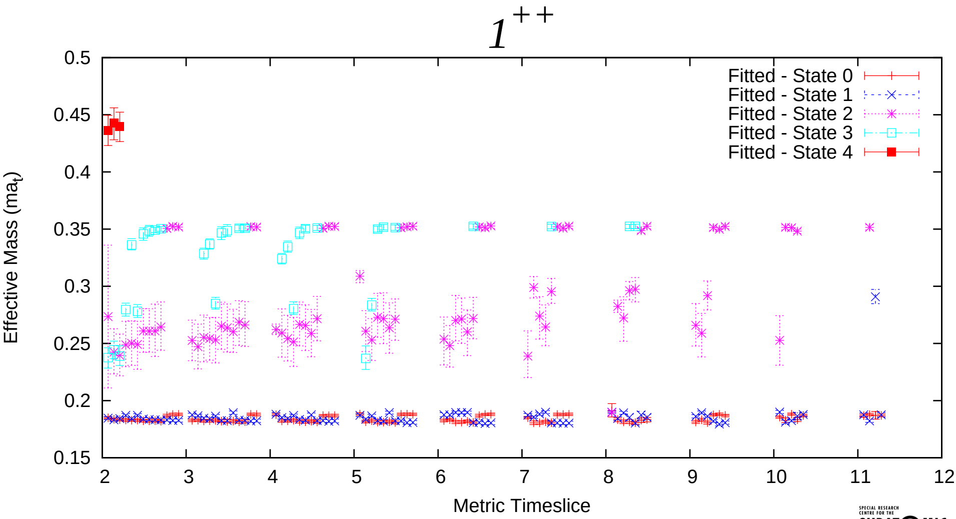
1^{++}
Fitted - State 0
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Effective Mass (ma_t)
0.5
0.45
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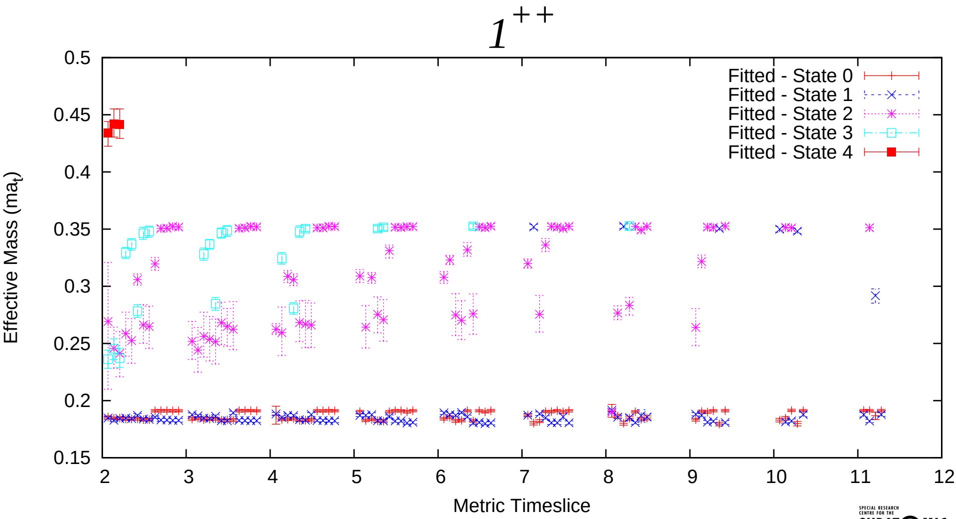
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Fitted - State 0
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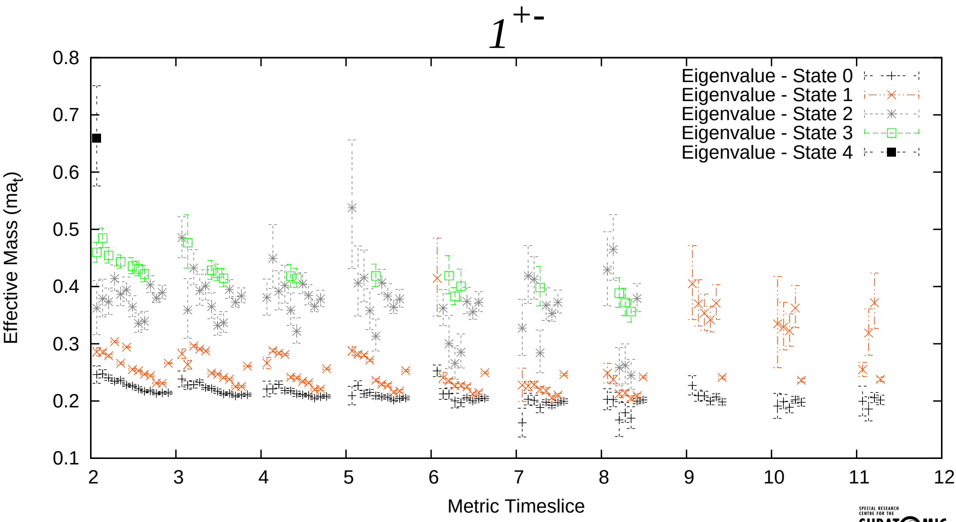
1^{+-}
Effective Mass (ma_t)
Metric Timeslice
Eigenvalue - State 0
Eigenvalue - State 1
Eigenvalue - State 2
Eigenvalue - State 3
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0.8
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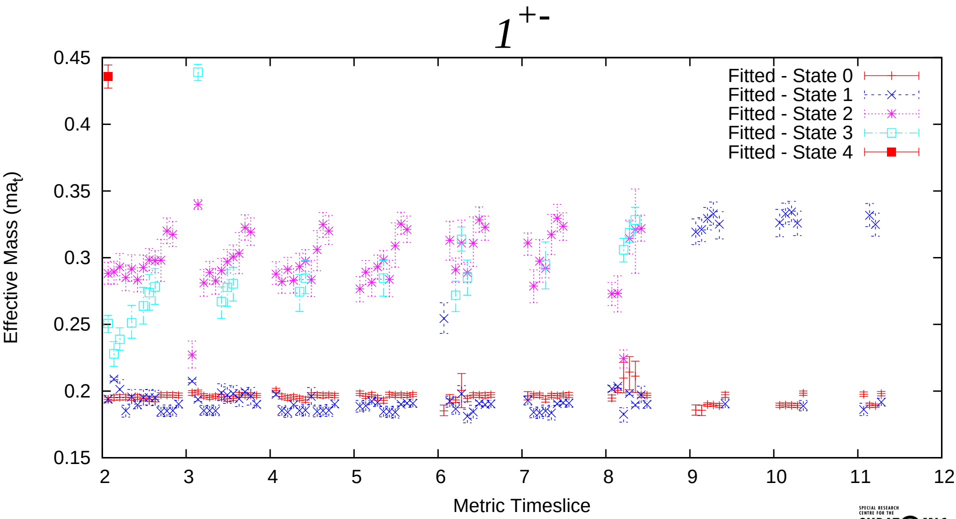
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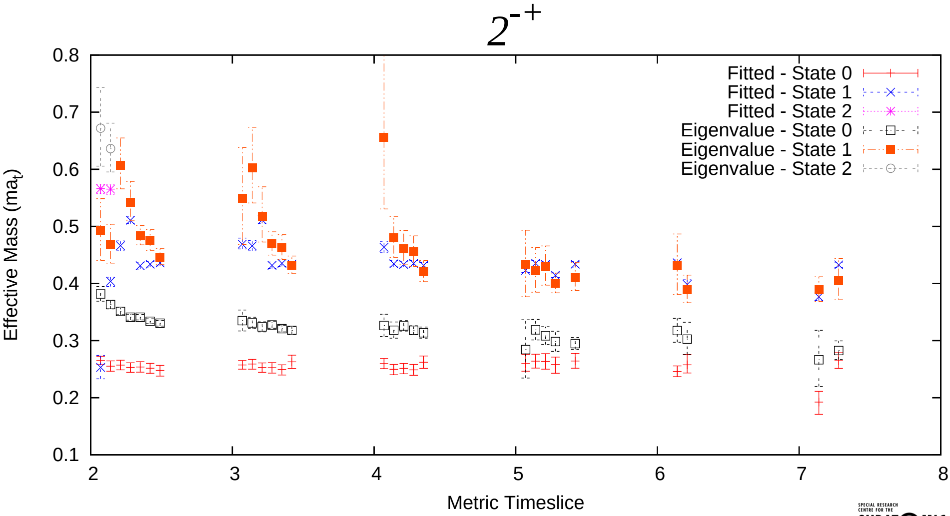
2-+
Effective Mass (ma_t)
Metric Timeslice
Fitted - State 0
Fitted - State 1
Fitted - State 2
Eigenvalue - State 0
Eigenvalue - State 1
Eigenvalue - State 2
0.8
0.7
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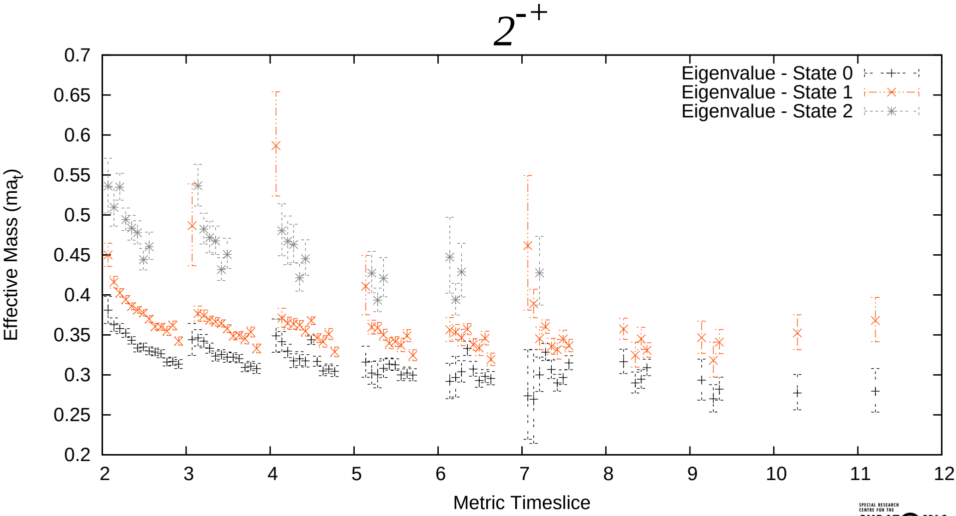
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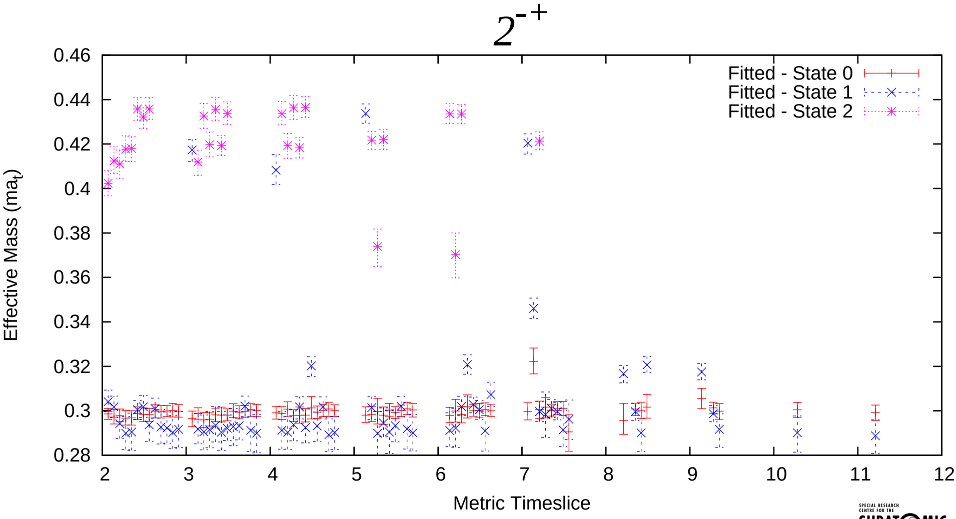
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Effective Mass (ma_t)
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Fitted - State 0
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0.46
0.44
0.42
0.4
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0.36
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0.32
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2
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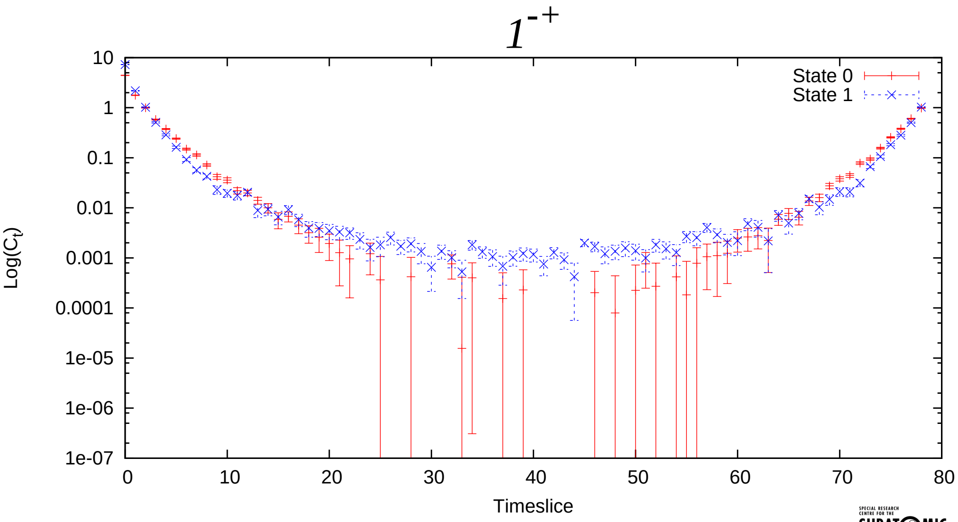
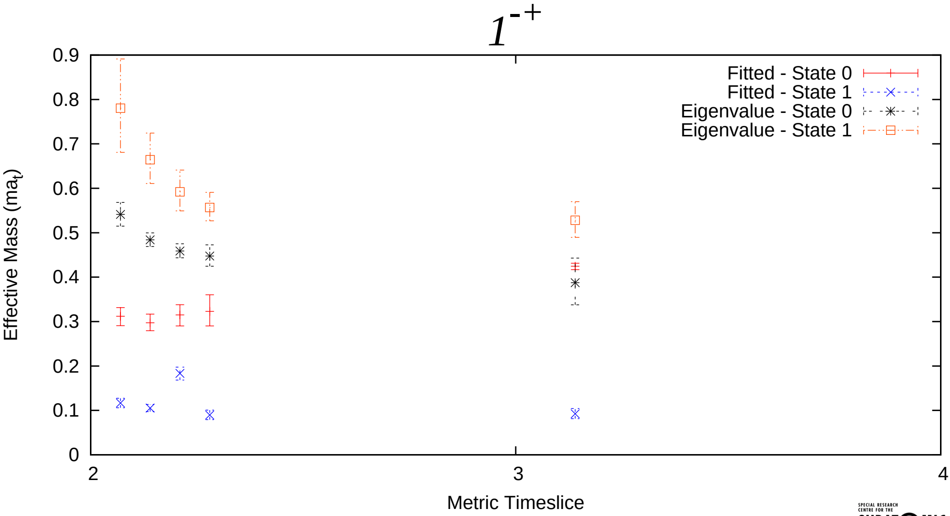
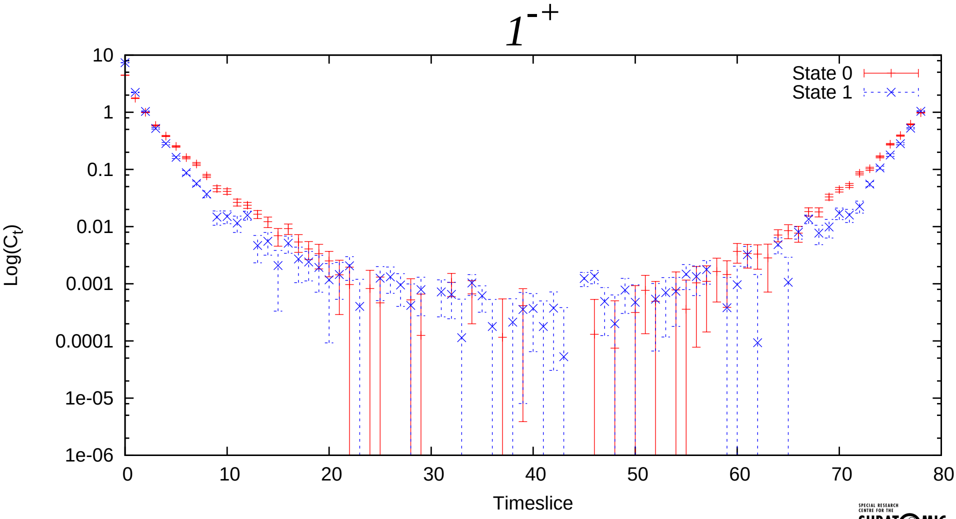
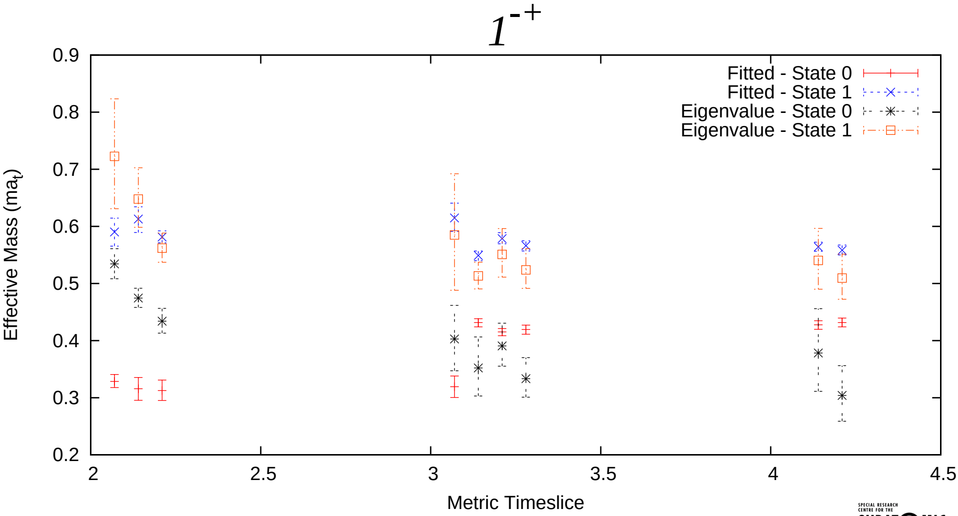
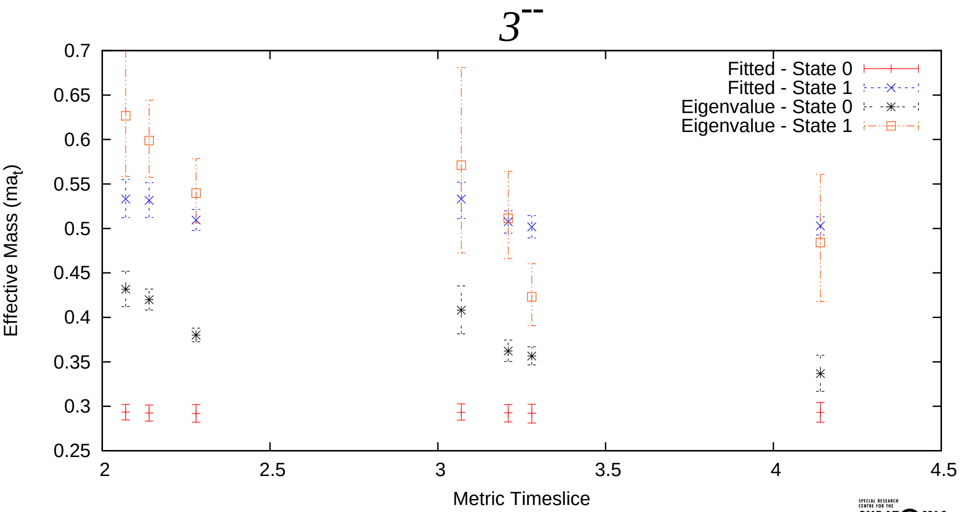
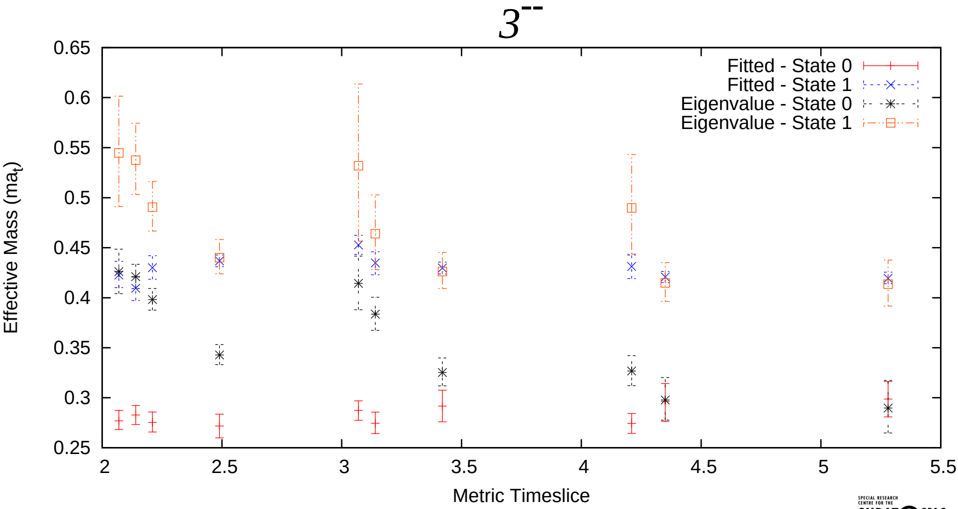

1^{-+}
State 0
State 1
10
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
1e-07
0
10
20
30
40
50
60
70
80
Log(C$_t$)
Timeslice


1^{-+}
Effective Mass (ma$_t$)
Metric Timeslice
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
2
3
4
Fitted - State 0
Fitted - State 1
Eigenvalue - State 0
Eigenvalue - State 1


1^{-+}
State 0
State 1
Log(C_t)
Timeslice
10
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
0
10
20
30
40
50
60
70
80


1^{-+}
Fitted - State 0
Fitted - State 1
Eigenvalue - State 0
Eigenvalue - State 1
Effective Mass (ma_t)
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
2
2.5
3
3.5
4
4.5
Metric Timeslice


3--
Fitted - State 0
Fitted - State 1
Eigenvalue - State 0
Eigenvalue - State 1
Effective Mass (ma_t)
Metric Timeslice
0.7
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
2
2.5
3
3.5
4
4.5


3^{--}
Effective Mass (ma_t)
Metric Timeslice
Fitted - State 0
Fitted - State 1
Eigenvalue - State 0
Eigenvalue - State 1
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
2
2.5
3
3.5
4
4.5
5
5.5