A new description of lattice Yang-Mills theory and non-Abelian magnetic monopoles as the quark confiner

Akihiro Shibata (KEK)

In collaboration with:

Seikou Kato (Takamatsu NCT)

Kei-Ichi Kondo (Chiba Univ.)

Toru Shinohara (Chiba Univ.)

Takeharu Murakami (Chiba Univ.)

Shoichi Ito (Nagano NCT)

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Overview

The purpose of this talk is to give a new description of the Yang-Mills theory on a lattice, which enable one to explain quark confinement based on the dual superconductivity.

CONTENTS:

1. Introduction
2. New descriptions of the Yang-Mills theory & non-Abelian Stoks’ theorem
3. Numerical simulation for new variables
4. The gauge invariant monopole
5. Numerical results
6. Conclusion and discussion

Dual superconductor picture from lattice studies

- Quark confinement follows from the area law of the Wilson loop average [Wilson,1974]

Non-Abelian Wilson loop $\left\langle \mathrm{tr}\left[\mathscr{P}\exp\left\{ig\oint_C dx^\mu \mathscr{A}_\mu(x)\right\}\right]\right\rangle_{\mathrm{YM}}^{\mathrm{no\ GF}} \sim e^{-\sigma_{NA}|S|}$

- Numerical simulations support this picture:
  - **Abelian dominance** ⇔ $\sigma_{Abel} \simeq \sigma_{NA}$ (92 ± 4)%
    - [Suzuki & Yotsuyanagi, PRD42,4257,1990]
  - **(Abelian) Monopole dominance** ⇔ $\sigma_{monopole} \simeq \sigma_{Abel}$ (95)%
    - [Stack, Neiman and Wensley, hep-lat/9404014],[Shiba & Suzuki, hep-lat/9404015]

SU(2) case

Abelian-projected Wilson loop $\left\langle \exp\left\{ig\oint_C dx^\mu A_\mu^3(x)\right\}\right\rangle_{\mathrm{YM}}^{\mathrm{MAG}} \sim e^{-\sigma_{Abel}|S|}$!?

Problems

- How can we establish the **gauge-invariant** "Abelian" dominance and magnetic monopole dominances?

  These result are obtained

  - Only for gauge fixings by the maximal Abelian (MA) gauge and the Laplacian Abelian gauge ,
  - however, these gauge fixing breaks color symmetry.

- For the $SU(3)$ case, is there any possibility other than projecting to the maximal torus group?

$$U_{x,\mu} = X_{x,\mu} V_{x,\mu} \in G = SU(3)$$

$$V_{x,\mu} = \mathrm{diag}(\exp(i\alpha), \exp(i\beta), \exp(i\gamma)) \in U(1) \times U(1) \qquad \alpha + \beta + \gamma = 0 \pmod{2\pi}$$

Possible sub groups for SU(3):

minimal case $U(2) \cong SU(2) \times U(1) \in SU(3)$

maximal case $U(1) \times U(1) \in SU(3)$

A new description of lattice Yang-Mills theory

Question:

- Can we obtain a gauge independent decomposition of the link variable U=XV, which reproduces the "Abelian" dominance for Wilson loop?
  - V corresponds to the conventional "Abelian" part.
  - V and X transform under the SU(N) gauge transformation

Yes, for the SU(2) YM theory.

- Compact representation of Cho-Faddeev-Niemi-Shabanov (CFNS) decomposition on lattice. PLB**632** 326(2006), PLB**645** 67(2007), PLB**653** 101(2007)

➔ Obtaining the decomposition of a link variable for the **fundamental rep. of Wilson loop** in SU(3) YM.

$$W_C[U] := \mathrm{Tr}\left[P \prod_{\langle x,x+\mu\rangle \in C} U_{x,\mu} \right] / \mathrm{Tr}(\mathbf{1})$$

$$U_{x,\mu} = X_{x,\mu} V_{x,\mu}$$

$$U_{x,\mu} \to U'_{x,\mu} = \Omega_x U_{x,\mu} \Omega^\dagger_{x+\mu}$$

$$V_{x,\mu} \to V'_{x,\mu} = \Omega_x V_{x,\mu} \Omega^\dagger_{x+\mu}$$

$$X_{x,\mu} \to X'_{x,\mu} = \Omega_x X_{x,\mu} \Omega^\dagger_{x}$$

$$\Omega_x \in G = SU(N)$$

$$W_C[V] := \mathrm{Tr}\left[P \prod_{\langle x,x+\mu\rangle \in C} V_{x,\mu} \right] / \mathrm{Tr}(\mathbf{1})$$

$$W_C[U] = \mathrm{const.} W_C[V] \quad !!$$

A new description of the SU(3) YM theory for Wilson loop of the fundamental rep.

YM	equipollent	YM'
$U_{x,\mu}$	$U_{x,\mu} = X_{x,\mu} V_{x,\mu}$	$V_{x,\mu}, X_{x,\mu}$
$SU(3)^{local}_{\Omega}$	⟺	$SU(3)^{local}_{\Omega'=\Theta'}$

Extending our SU(2) formulation

- To obtain the equipollent theory by new variables ***V*** and ***X*** ,
- Extend the local gauge symmetry by introducing the color field ***h*(x)**

$$U_{x,\mu} \to U'_{x,\mu} = \Omega_x U_{x,\mu} \Omega^\dagger_{x+\mu}$$
$$V_{x,\mu} \to V'_{x,\mu} = \Omega_x V_{x,\mu} \Omega^\dagger_{x+\mu}$$
$$X_{x,\mu} \to X'_{x,\mu} = \Omega_x X_{x,\mu} \Omega^\dagger_{x}$$

Fundamental rep. of Wilson loop

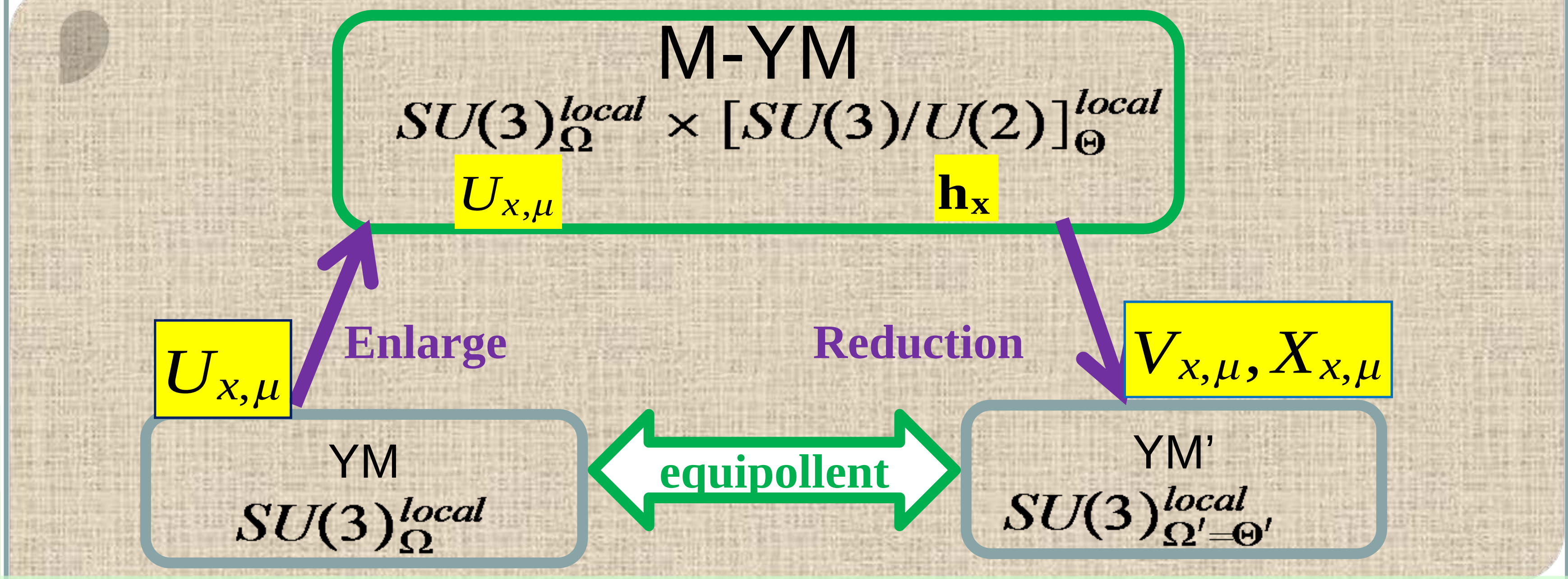
= Minimal case

$$\mathbf{h}_x \in SU(3)/U(2)$$
$$h_x \to h'_x = \Theta_x h_x \Theta^\dagger_x$$

➔Defining the M-YM (master Yang-Mills)

See arXiv:0803.2451v1 [hep-lat]

Symmetry of M-YM and new variables



A necessary and sufficient condition for

arXiv0801.4203[hep-th]

$$W_C[U] = \text{const.} W_C[V] \quad !!$$

which is derived from non-Abelian Stokes' theorem (NAST) on a lattice.

$$V_{x,\mu} h_{x+\mu} = h_x V_{x,\mu}$$
$$Tr((X_{x,\mu} - X^{\dagger}_{x,\mu}) h_{x,\mu}) = 0$$

- Extending the SU(2) case
- Defining equation from fundamental rep. of Wilson loop

Solving the defining equations

$$V_{x,\mu} h_{x+\mu} = h_x V_{x,\mu}$$
$$Tr((X_{x,\mu} - X^{\dagger}_{x,\mu}) h_{x,\mu}) = 0$$

arXiv:0803.2451[hep-lat]

Adopting an ansatz for V ;

$$\tilde{V}_{s,\mu} = U_{s,\mu} + \alpha(\mathbf{h}_s U_{s,\mu} + U_{s,\mu}\mathbf{h}_{s+\hat{\mu}}) + \beta \mathbf{h}_s U_{s,\mu} \mathbf{h}_{s+\hat{\mu}}$$

Then we obtain parameters $\alpha = \frac{2\sqrt{3}}{5}, \quad \beta = \frac{24}{5}$

To obtain the variable $V_{x,\mu} \in SU(3)$,

(1) Adopting the polar decomposition:

$$\underline{V}_{x,\mu} = \tilde{V}_{x,\mu} H^{-1}_{x,\mu}, \qquad H^2_{x,\mu} \equiv \tilde{V}^{\dagger}_{x,\mu} \tilde{V}_{x,\mu},$$

(2) Applying to special unitarity condition:

$$V_{x,\mu} = \underline{V}_{x,\mu} (\det \underline{V}_{x,\mu})^{-1/3}$$

(3) so $\quad X_{x,\mu} = U_{x,\mu} V^{\dagger}_{x,\mu}$

Reduction condition

- Imposing a constraint called the reduction condition to reduce enlarged symmetry SU(3) × [SU(3)/U(2)] to SU(3)

➔ **Equipollent theory to YM**

- To obtain the gauge independent decomposition

the functional F_{rc} should be invariant under the SU(3) gauge transformation

$$h_x \to h'_x = \Theta_x h_x \Theta_x^\dagger$$
$$U_{x,\mu} \to U'_{x,\mu} = \Omega_x U_{x,\mu} \Theta_{x+\mu}^\dagger$$

$$\Omega_x = \Theta_x$$

- **Reduction condition is given by minimizing the functional:**

$$F_{rc}[\Omega_x, \Theta_x; h_x, U_{x,\mu}] = \sum_{x,\mu} Tr\Big[(D_\mu^\epsilon[{}^\Omega U_{x,\mu}]^\Theta \mathbf{h}_x)(D_\mu^\epsilon[{}^\Omega U_{x,\mu}]^\Theta \mathbf{h}_x)^\dagger \Big]$$

Numerical Simulation for new variables

- YM filed **U can be generated by the STANDARD method**.
- Color field **h** can be determined by the reduction condition by using the gauge fixing technique:
  - **Gauge invariance of reduction condition**

$$\mathbf{h}^0 = \Theta_x^\dagger \mathbf{h}_x \Theta_x = \lambda^8$$
$${}^G U_{x,\mu} = \Theta_x^{\dagger\Omega} U_{x,\mu} \Theta_{x+\mu}$$

➔
$$F_{rc}[\Omega_x, \Theta_x; \mathbf{h}_\mathbf{x}, U_{x,\mu}]$$
$$\to \sum_{x,\mu} Tr[{}^\Omega U_{x,\mu} {}^\Theta \mathbf{h}_{x+\mu} {}^\Omega U_{x,\mu}^\dagger {}^\Theta \mathbf{h}_x] = \sum_{x,\mu} Tr[{}^G U_{x,\mu} \mathbf{h}^0 {}^G U_{x,\mu}^\dagger \mathbf{h}^0]$$

➔The color field is given by the solution of F_{RC}

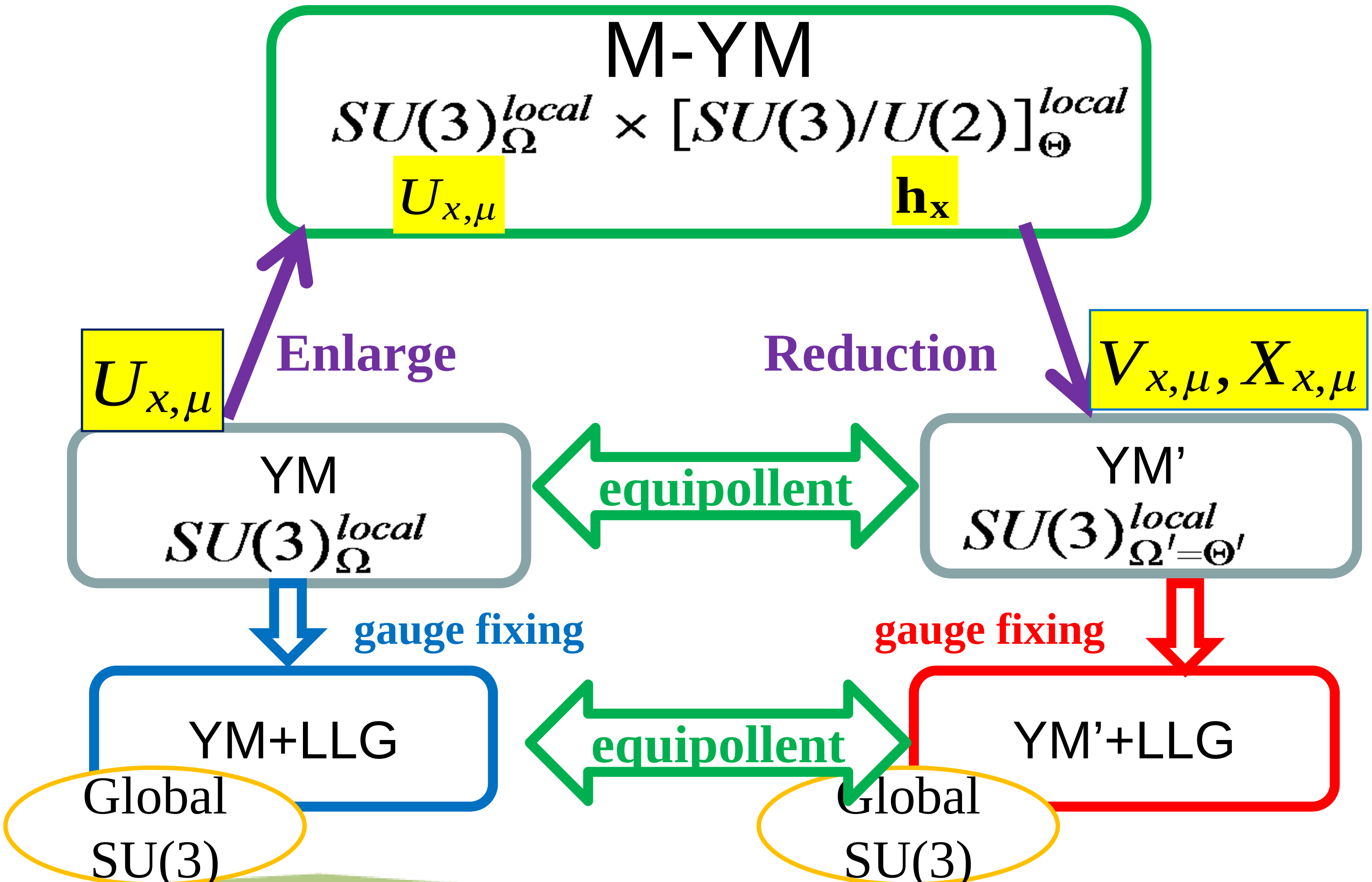
$${}^G U = {}^{sol} U = (\Theta_x^\dagger \Omega)^{LLG} U (\Theta_x^\dagger \Omega)^\dagger$$
$$\mathbf{h}_x = (\Theta_x^\dagger \Omega) \mathbf{h}^0 (\Theta_x^\dagger \Omega)^\dagger$$

gauge independent

- New variables are given by decomposing an arbitrary link variable ***U*** , by **using the solution of defining equation** for a given color field **h** from $\mathbf{F_{RC}}$

$$\{U_{x,\mu}, \mathbf{h}_x\} \to \tilde{V}_{x,\mu} \to \underline{V}_{x,\mu} \to V_{x,\mu} \to X_{x,\mu} = U_{x,\mu} V_{x,\mu}^\dagger$$

A new description of the YM theory for SU(3) minimal case



Defining gauge invariant non-Abelian monopole

- The gauge invariant field strength for V

$$V_{x,\mu}V_{x+\mu,\nu}V^{\dagger}_{x+\nu,\mu}V^{\dagger}{}_{x,\nu} \simeq \exp(-ig\epsilon F_{\mu\nu}[\mathbf{V}])$$

- The magnetic monopole current can by define by F[V]

$$k_{x,\mu} = -\frac{1}{4\pi}\epsilon_{\mu\nu\rho\sigma}\partial_\nu \Theta^8_{x,\rho\sigma}$$

Gauge invariant under G=SU(3)

$$\Theta^8_{x,\mu\nu} \equiv -argTr[(\frac{1}{3}\mathbf{1} - \frac{2}{\sqrt{3}}\mathbf{h}_x)V_{x,\mu}V_{x+\hat{\mu},\nu}V^{\dagger}_{x+\hat{\nu},\mu}V^{\dagger}_{x,\nu}]$$

Naïve conti.lim.

$$\longrightarrow \frac{g}{\sqrt{3}}tr(2\mathbf{h}_x\mathcal{F}_{x,\mu\nu}[V]) \equiv \frac{g}{\sqrt{3}}G_{x,\mu\nu}$$

$$Q_m = \int d^3x k_{x,0} \simeq \frac{g}{\sqrt{3}}\int d^3x \frac{1}{2}\epsilon^{jkl}\partial_l tr(2\mathbf{h}(x)\mathcal{F}_{jk}[V](x)) = n \in \mathcal{Z}$$

$$\Longleftrightarrow \pi_2(SU(3)/U(2)) = \mathcal{Z}$$ (Second homotopy group)

NAST for Wilson operator & magnetic monopole

» (e.g. K.-I. Kondo PRD77 085929(2008))

- Wilson loop for the **fundamental representation**

$$W_C[\mathbf{A}] = \mathrm{tr}\left[P\exp ig\oint_C dx^\mu A_\mu(x)\right]/\mathrm{tr}(\mathbf{1}) = \int [d\mu(\xi)]_\Sigma \exp\left\{\int_{S:C=\partial S} dS^{\mu\nu}\mathcal{F}_{\mu\nu}[V]\right\}$$

$$= \int[d\mu(\xi)]_\Sigma \exp\left\{ig\sqrt{\frac{N-1}{2N}}(k,\Xi_\Sigma) + ig\sqrt{\frac{N-1}{2N}}(j,N_\Sigma)\right\}$$

$$\Xi_\Sigma := *d\Theta_\Sigma\Delta^{-1} = \delta * \Theta_\Sigma\Delta^{-1}, N_\Sigma := \delta\Theta_\Sigma\Delta^{-1}$$

D−dimensional Laplacian $\Delta = d\delta + \delta d$

Θ_Σ : the vorticity tensor with support on the surface Σ_C sppaned by Willson loop C

lattice version

$$\langle W_C[U]\rangle \approx \langle W_C[\mathrm{Mag}]\rangle = \left\langle \exp\left\{2\pi i\sum_{s,\mu} k_{x,\mu}\Xi_{x,\mu}\right\}\right\rangle$$

$$\Xi_{x,\mu} := \sum_s \Delta_L^{-1}(s-s')\frac{1}{2}\epsilon_{\mu\alpha\beta\gamma}\partial_\alpha S^J_{\beta\gamma}(s'+\mu), \qquad \partial'_\beta S^J_{\beta\gamma}(s) = J_\gamma(s)$$

Numerical simulation

Parameters:

- Wilson action
  - Configurations are generated by using pseudo head bath algorithm (Cabibbo-Marinari) for the Wilson action
- Lattics size , 16^4, beta=5.7

- Gauge fixing technique to calculate the reduction condition
- Study in the case of the lattice Landau gauge for original YM theory(Propagators /correlations)
  - For gauge fixing of YM theory over-relaxation algorithm is used.

Preliminary Result

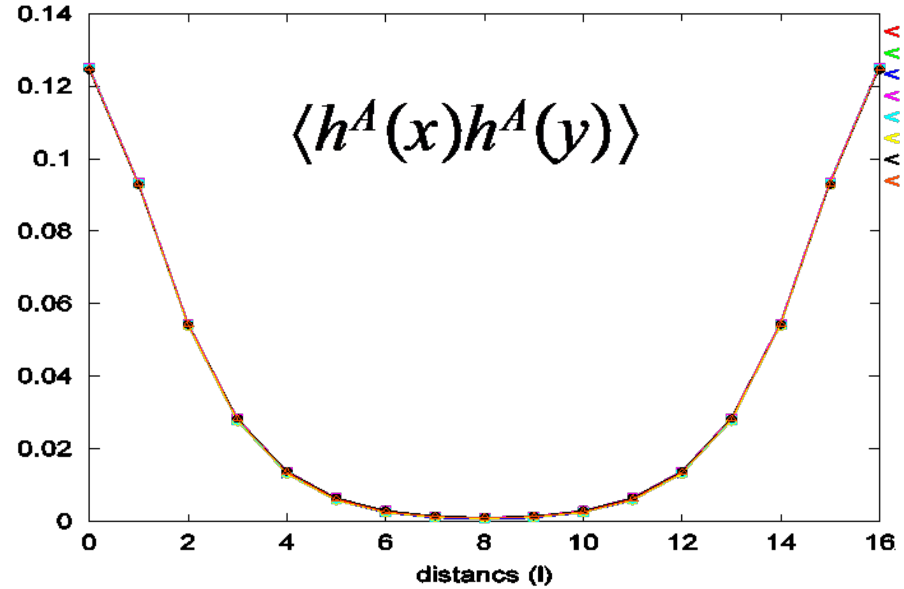
Color symmetry
(correlation of color vector fields)

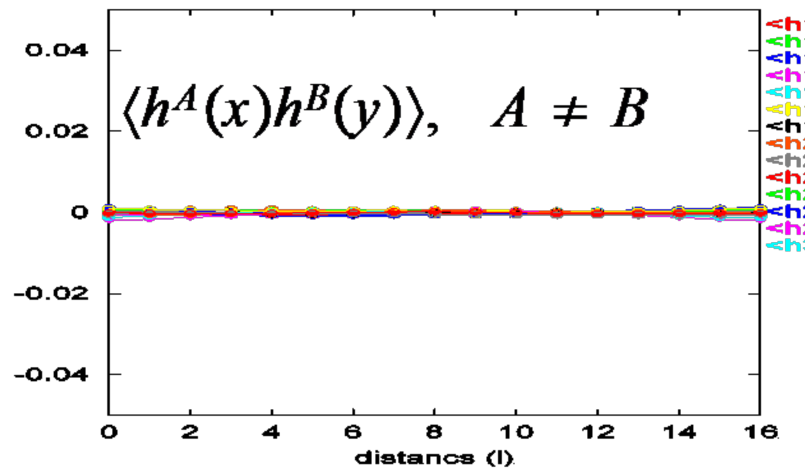
- The Landau gauge preserve global SU(3) symmetry, color symmetry, of YM theory.
- The color fields ***h*** mast have color symmetry.

- To check this, **VEV** and **correlation functions** of color vector fields are calculated.

$$\langle h^A(x) \rangle = 0 \pm 0.002$$
$$\langle h^A_x h^B_y \rangle = \delta^{AB} D(x-y)$$

➔ Color symmetry is preserved.





Magnetic monopole and charge quantization

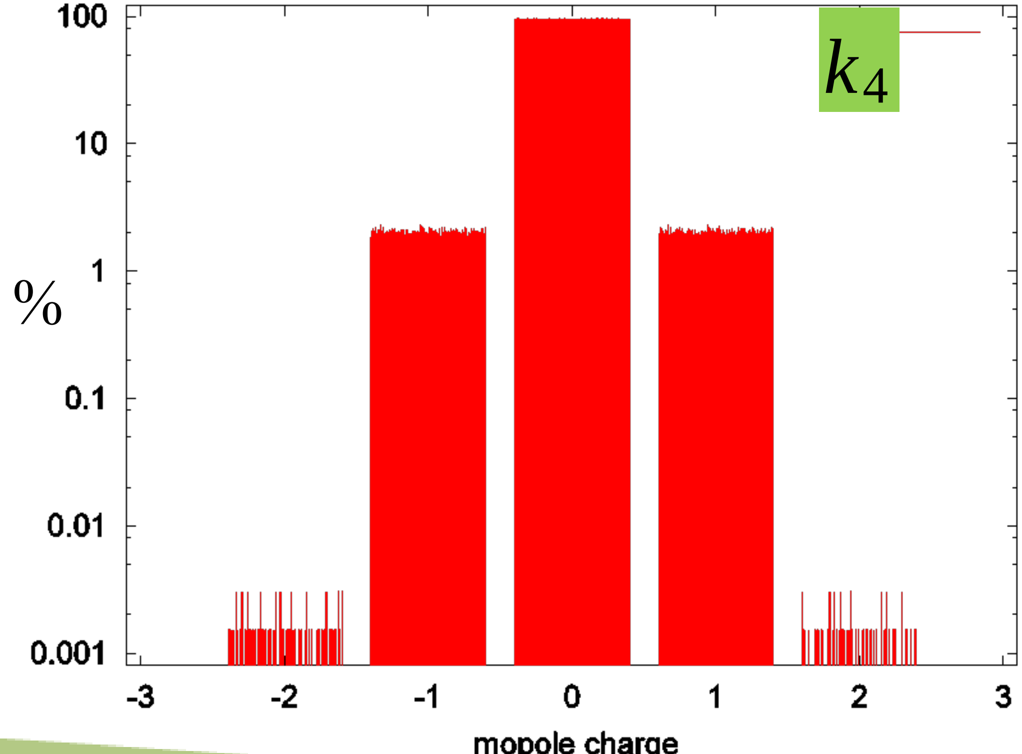
$$Q_m = \int d^3x k_{x,0} \simeq \frac{g}{\sqrt{3}} \int d^3x \frac{1}{2} \epsilon^{jkl} \partial_l tr(2\mathbf{h}(x)\mathcal{F}_{jk}[V](x)) = n \in \mathcal{Z}$$

$\Longleftrightarrow$ $\pi_2(SU(3)/U(2)) = \mathcal{Z}$ (Second homotopy group)

$$K_{x,\mu} \equiv -\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\partial_\nu \Theta^8_{x,\rho\sigma} = 2\pi k_{x,\mu}$$

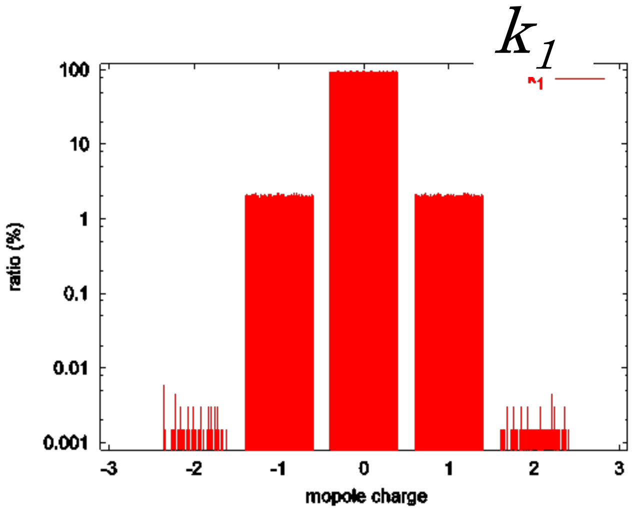
- 16^4 lattice β=5.7
#config. = 400

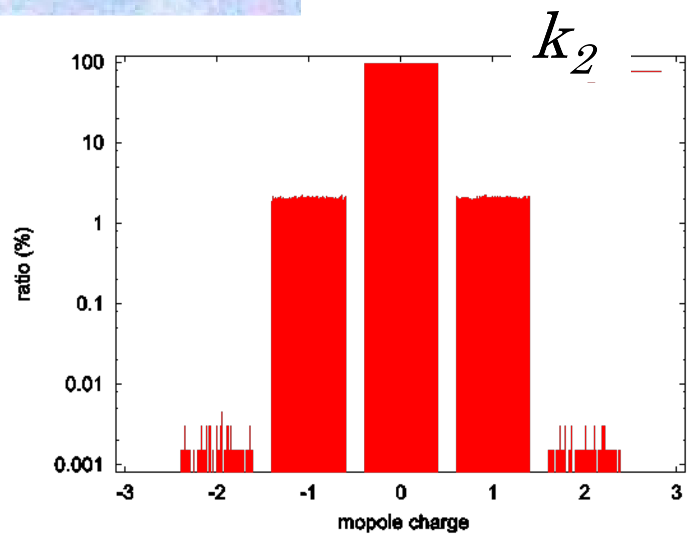
- Quantized magnetic monopole charge density

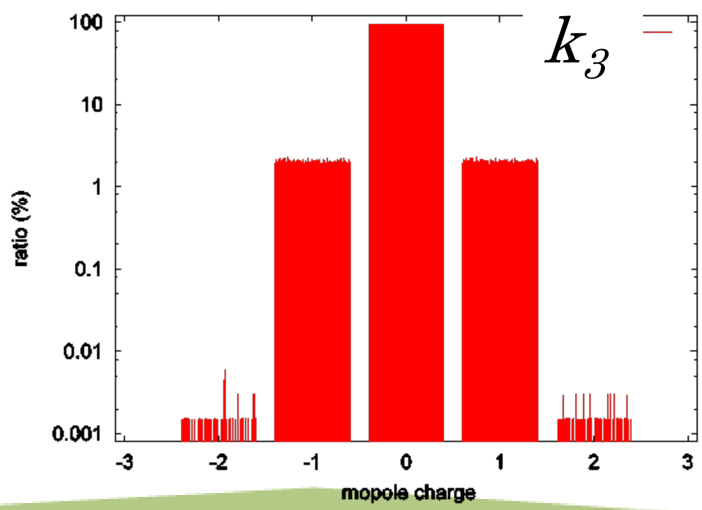


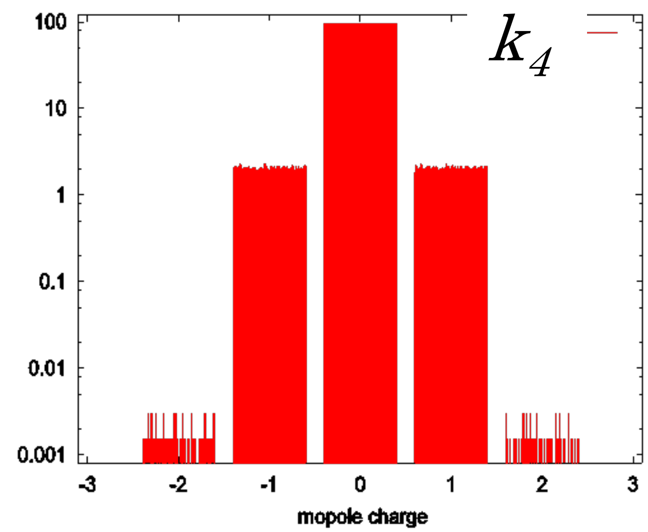
Monopole charge quantization and distributions

$$K_{x,\mu} \equiv -\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\partial_\nu \Theta^8_{x,\rho\sigma} = 2\pi k_{x,\mu}$$



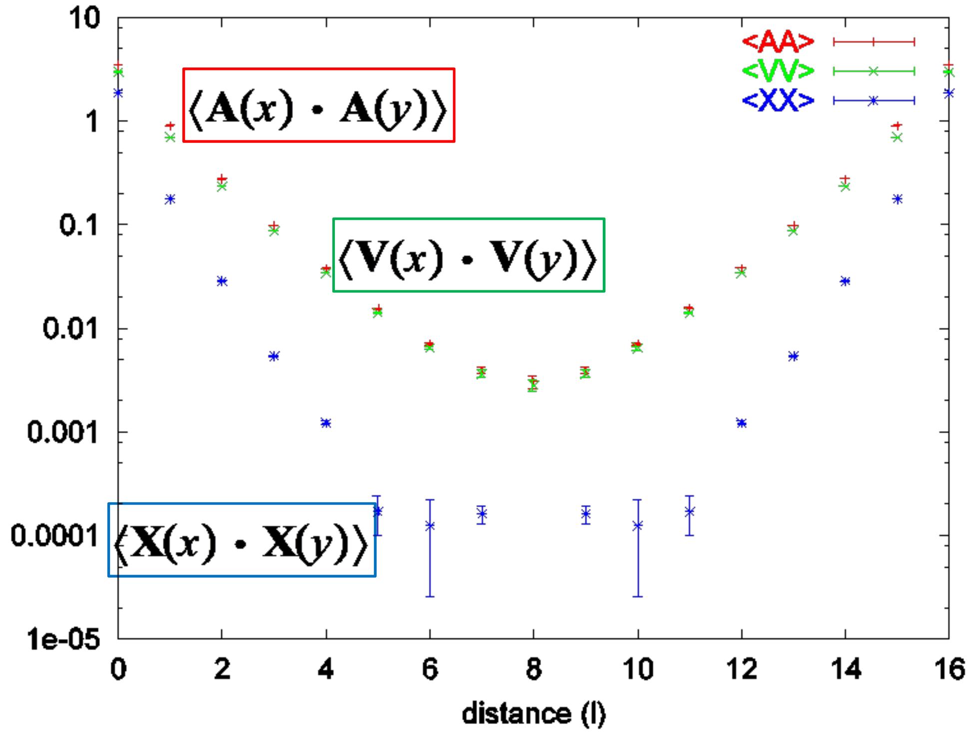






Infrared V dominance

- the **correlation function** for the original gauge field, A in the Landau gauge and new variables, V, X.
- Damping of <VV> is the almost same as that of <AA>
- Damping of <XX> is quickly and decoupled from V in IR region.



$$D_{AA}(x-y) \simeq D_{VV}(x-y) \gg D_{XX}(x-y),$$
$$|x-y| \gg 1$$

→infrared V dominance

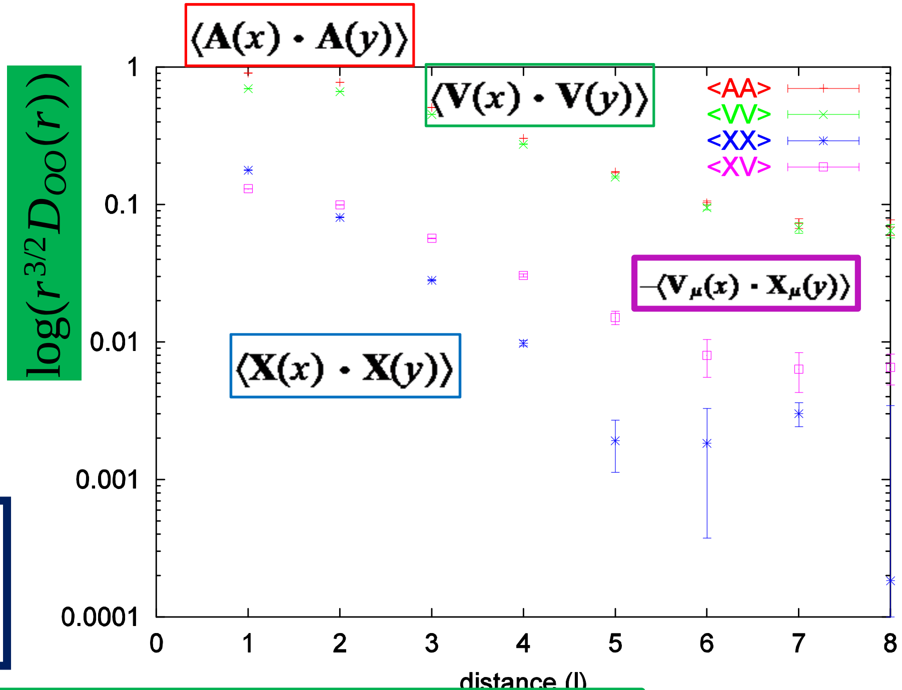
Mass Gap ?

$X_{x,\mu}$ is adjoint rep

$$X_{x,\mu} \rightarrow X'_{x,\mu} = \Omega_x X_{x,\mu} \Omega^\dagger_{x+\mu}$$

A gauge invariant mass term can be introduced

$$\mathcal{L}_{MX} = \frac{1}{2} M_x^2 (X_{x,\mu})^2$$



- Condensation of mass dimension two is possible?

Inverse Fourie transformation of the massive gauge boson propagator gives

$$G_{\mu\mu}(r;M) = \langle \mathbb{X}_\mu(x) \mathbb{X}_\nu(y) \rangle = \int \frac{d^4k}{(2\pi)^4} e^{ik(x-y)} \frac{1}{k^2+M^2} \left(4 + \frac{k^2}{M^2}\right)$$

$$\simeq \frac{3\sqrt{M}}{2(2\pi)^{3/2}} \frac{e^{-Mr}}{r^{3/2}}$$

Conclusion

- We have proposed a new description of the YM theory on a lattice and demonstrated the numerical simulation for **SU(3) minimal case:**
  - Gauge independent decomposition of link variable *U=XV* for the fundamental representation of Wilson loop.
  - **V** approve the conventional "Abelian" part
  - Gauge invariant non-Abelian magnetic monopole current is defined by V
  - Infrared "Abelian dominance"

- ***Outlook***

- **V dominance / monopole dominance for the Wilson loop (in progress)**
- N-ality for the string tension
- Relations between topological defects?
  - Center vortex or monopole loops ⇔ arXiv:0802.3829 [hep-th]
  - Magnetic Monopole Loops supported by meron pair? ⇔ arXiv:0806.3913 [hep-th]

THANK YOU FOR YOUR ATTENSION