Fermions in higher representations. Some results about SU(2) with adjoint fermions.

Agostino Patella

Swansea University

with L. Del Debbio, C. Pica – arXiv:0805.2058 [hep-lat]

Lattice08, Williamsburg 18/7/2008

Outline

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Motivations: Why higher representations?

- Technicolor models.
  $SU(2)$ gauge theories + fermions in the symmetric two-index representation
- Orientifold planar equivalence.
  $SU(3)$ + fund. fermions $\longrightarrow$ $SU(N)$ + 2AS fermions $\longrightarrow$ $SU(\infty)$ + Adj fermions
- Softly-broken SYM.
  $SU(N)$ gauge theories + one Majorana fermion in the adjoint representation

Outline

The HiRep code

- Wilson action + Wilson fermions.
- Standard HMC/RHMC algorithm.
- Second order Omelyan integrator for the molecular dynamics evolution, with different time steps for the gauge and fermion actions.
- Link update implemeted by left multiplication of a unitary matrix that is a second-order approximation for $\exp(i\pi\Delta t)$.
- Even/odd preconditiong for the Dirac operator.
- Fermions in the representation R (fund, 2AS, 2S, Adj).

$$D\psi(x) = \psi(x) - \frac{1}{\kappa}\sum_{\mu}\left\{(1-\gamma_\mu)U^R(x,\mu)\psi(x+\mu) + (1+\gamma_\mu)U^R(x-\mu,\mu)^\dagger\psi(x-\mu)\right\}$$

$$\frac{d}{d\tau}U(x,\mu) = i\pi^a(x,\mu)T^a_F U(x,\mu)$$

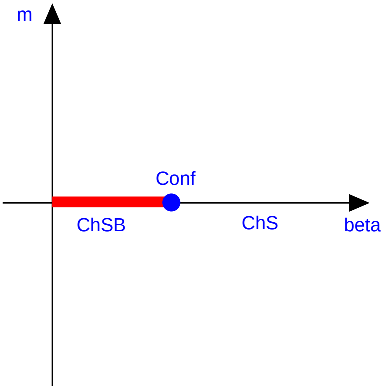
$$H = \frac{1}{4}\sum_{x,\mu}\pi^a(x,\mu)^2 - \frac{\beta}{N}\sum_{x,\mu<\nu}\mathcal{P}_{\mu\nu}(x) + \sum_x \phi^\dagger(x)[D_R^\dagger D_R - s]^{-1}\phi(x)$$

$$\frac{dH_f}{d\tau} = i\sum_{a,x,\mu}\pi^a(x,\mu)\mathrm{tr}_R\{T^a_R F_f[U_R](x,\mu)\}$$

$$\frac{1}{2}\frac{d}{d\tau}\pi^a(x,\mu) + \mathrm{tr}_F[iT^a_F F_g(x,\mu)] + \mathrm{tr}_R[iT^a_R F_f(x,\mu)] = 0$$

Outline

Conformal point?



- Catteral, Giedt, Sannino, Schneible, hep-lat/0807.0792
- next talk by Hietanen

The parameters

$\beta = 2.0$

lattice	V	κ	$-am_0$	N_{traj}	$\langle P \rangle$	τ
T2-A1	8×4^3	0.12500	0.0	28800	0.5093(14)	2.9(0.4)
T2-A2	8×4^3	0.14286	0.5	28800	0.5163(16)	3.1(0.5)
T2-A3	8×4^3	0.15385	0.75	28800	0.5235(18)	3.1(0.5)
T2-A4	8×4^3	0.16667	1.0	28800	0.5373(20)	6.0(1.2)
T2-A5	8×4^3	0.18182	1.25	27200	0.5742(37)	12.0(3.6)
T2-A6	8×4^3	0.18382	1.28	25600	0.5850(50)	22.3(9.3)
T2-A7	8×4^3	0.18587	1.31	41600	0.6013(55)	48.3(23.3)
T2-A8	8×4^3	0.18657	1.32	51200	0.6159(58)	40.7(16.3)
T2-A1′	8×4^3	0.12500	0.0	3000	0.5094(45)	2.7(1.2)
T2-B7	16×8^3	0.18587	1.31	3200	0.5951(42)	5.8(3.6)
T2-B8	16×8^3	0.18657	1.32	1600	0.6040(56)	9.0(9.6)
T2-B9	16×8^3	0.18692	1.325	2240	0.6107(53)	4.2(2.6)
T2-B10	16×8^3	0.18727	1.33	1100	0.6168(73)	2.6(1.8)
T2-B11	16×8^3	0.18797	1.34	3840	0.6347(58)	13.6(11.5)

The chiral limit

Wilson fermions explictly break the chiral symmetry for each value of κ. The chiral point is fine-tuned by requiring that the Ward identities for the chiral symmetry are recovered.

$$\langle\bar{\psi}\gamma_5\psi(x)\ \partial_\mu\bar{\psi}\gamma_\mu\gamma_5\psi(y)\rangle = 2m\langle\bar{\psi}\gamma_5\psi(x)\ \bar{\psi}\gamma_5\psi(y)\rangle$$

$$am \simeq A\left(\frac{1}{\kappa} - \frac{1}{\kappa_c}\right)$$

In the chiral point $m = 0$ and in the continuum limit, we assume that the chiral symmetry is spontaneously broken. Then the lightest PS meson is massless and for small values of m_{PS}, the χPT is valid.

$$\frac{M_{PS}}{4\pi F_{PS}} \ll 1$$

$$aM_{PS} \simeq B\sqrt{am}$$

$$F_{PS} \simeq F_{PS}(0) + Cm$$

$$M_V \simeq M_V(0) + Dm$$

We want to check this assumption. We need to go to small PCAC masses but we need to be careful in this region.

Troubles at small m at fixed lattice spacing

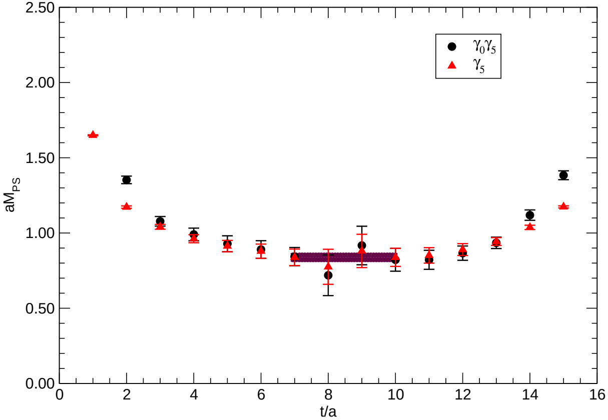
$$SU(4) \xrightarrow{\text{broken by } m,\, a} SO(4) \xrightarrow{\text{sp. broken in Aoki phase}} U(1) \times U(1)$$

- In the Aoki phase, flavour is spontaneously broken. Four Goldstone bosons are expected in this case.
- The transition to the Aoki phase is expected around the chiral limit in a width $a\Delta m \sim (a\Lambda)^3$.
- In the Aoki phase, exact zero modes of the Dirac operator (and instability of the algorithm) are expected.
- At finite volume, the phase transition becomes a wide cross-over. Thus, we can have a region of stability of the algorithm in which the measured observables are highly sensitive to lattice artifacts.

Safe chiral limit

We need to check the stability of our results close to the chiral point, by increasing the volume and reducing the lattice spacing (the width of the distribution of the lowest Dirac eigenvalue shrinks as $a/\sqrt{V}$ and the Aoki phase width shrinks as a^3).

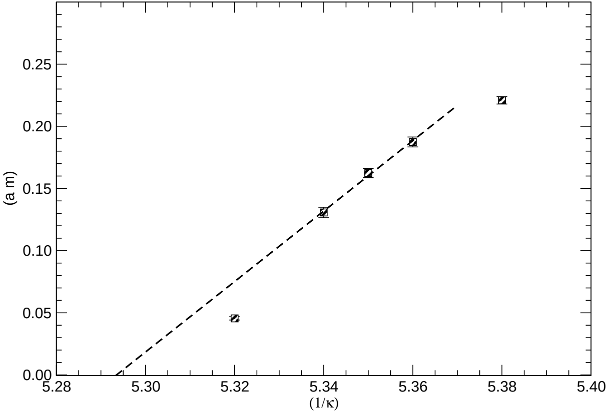
Extracting the masses from correlators



$$\frac{h(t+1,M)}{h(t,M)} = \frac{C_{\gamma_5,\gamma_5}(t+1)}{C_{\gamma_5,\gamma_5}(t)}$$

$$h(t,M) = e^{-Mt} + e^{-M(T-t)}$$

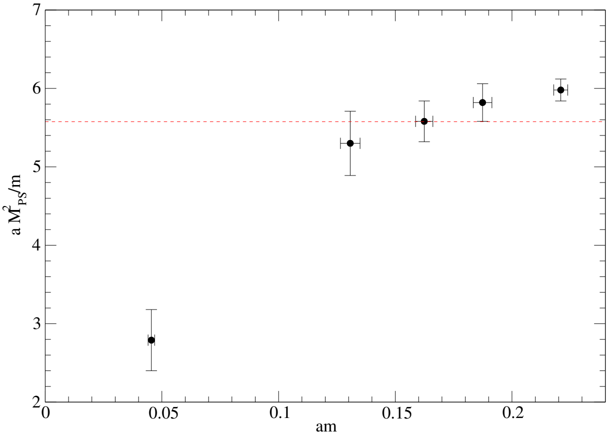
The PCAC mass and the chiral limit



$$m_{eff}(t) = \frac{C_{\gamma_0\gamma_5,\gamma_5}(t+1) - C_{\gamma_0\gamma_5,\gamma_5}(t-1)}{C_{\gamma_5,\gamma_5}(t)}$$

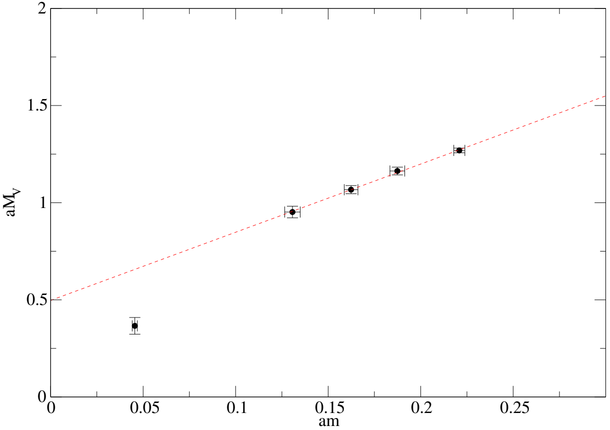
$$am = A\left(\frac{1}{\kappa} - \frac{1}{\kappa_c}\right) \quad \longrightarrow \quad \kappa_c = 0.18679(7)$$

The PS mass



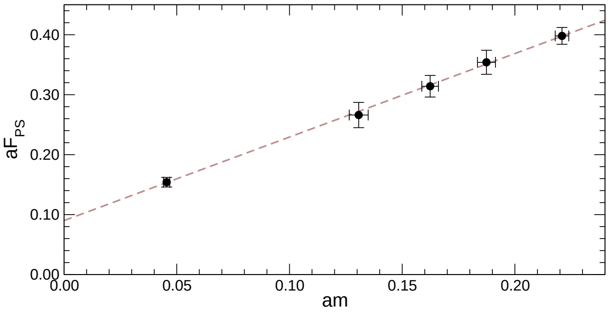
$$a^2 M_{PS}^2 = Bam$$

The V mass



$$M_V = M_V(0) + Dm$$

The PS decay constant

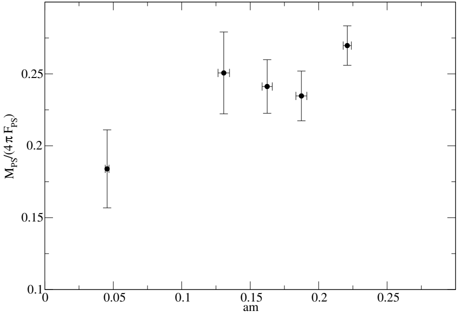


$$C_{\gamma_5,\gamma_5}(t) \simeq \frac{G_{PS}^2}{M_{PS}} \exp\{-M_{PS} t\}$$

$$F_{PS} = \frac{m}{M_{PS}^2} G_{PS}$$

$$F_{PS} = F_{PS}(0) + Cm$$

Validity region of χPT



$$\frac{M_{PS}}{4\pi F_{PS}} \simeq 0.2$$

We are at the superior corner of the region of applicability of χPT. So far data are compatible with the standard scenario of chiral symmetry breaking at the chiral point. Anyway more exotic scenarios (like the presence of a conformal chiral point) cannot be excluded. We need to go closer to the chiral point in a safe way (increasing the volume and reducing the lattice spacing).

Outline

Conclusions

- $SU(N)$ gauge theories with fermions in two-index representations are relevant for the physics beyond SM.
- We have implemented and tested the HMC/RHMC algorithm for fermions in the generic representation of $SU(N)$.
- We have produced some preliminary phenomenological results for $SU(2)$ with $n_f = 2$ adjoint fermions at fixed lattice spacing.
- Our results are affected by systematic errors, due to both finite lattice spacing and finite volume. In particular the chiral limit and the scaling region require deeper investigation.
- Our results are compatible with the standard scenario of chiral symmetry breaking in the chiral point and χPT. However more exotic scenarios cannot be excluded. Lighest quarks are necessary.

Behaviour of the HMC/RHMC algorithm

- If ΔH is the difference of the value of the Hamiltonian at the beginning and at the end of the MD evolution, we expect

$$\langle \exp(-\Delta H) \rangle = 1$$

- If Δt is the MD step size, we expect

$$\langle \Delta H \rangle \sim \Delta t^4$$

- If P_{acc} is the acceptance probability, we expect

$$P_{acc} = \text{erfc}(\sqrt{\langle \Delta H \rangle}/2)$$

- The average of the plaquette is independent of Δt.
- Violation of reversibility. Fix a starting configuration, evolve for $\tau = 1$, flip the momenta and evolve back for $\tau = 1$. The starting and ending configurations should be the same. We get

$$|\delta H| \simeq 10^{-7}$$

Test of the group structure

- $SU(3) + n_f = 2$ in the fundamental representation, checked against:

  M. Luscher, Comput. Phys. Commun. 165, 199 (2005) [arXiv:hep-lat/0409106]

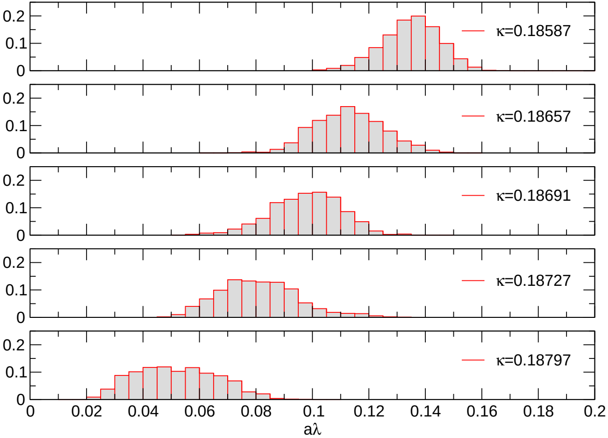
- $SU(3) + n_f = 2$ in the fundamental representation =
  = $SU(3) + n_f = 2$ in the antisymmetric two-index representation

- $SU(2) + n_f = 2$ in the adjoint representation, checked against:

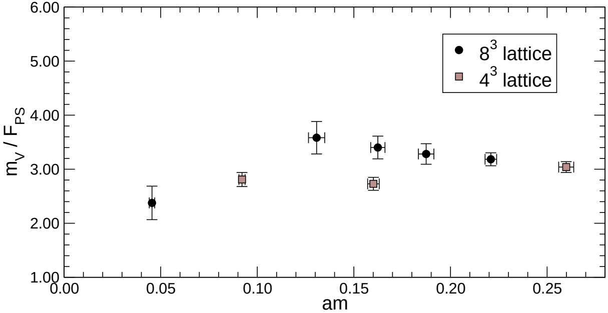
  S. Catteral and F. Sannino, Phys. Rev. D 76, 034504 (2007) [arXiv:hep-lat/0705.1664]

- $SU(2) + n_f = 2$ in the adjoint representation =
  = $SU(2) + n_f = 2$ in the symmetric two-index representation

The lowest eigenvalue of D



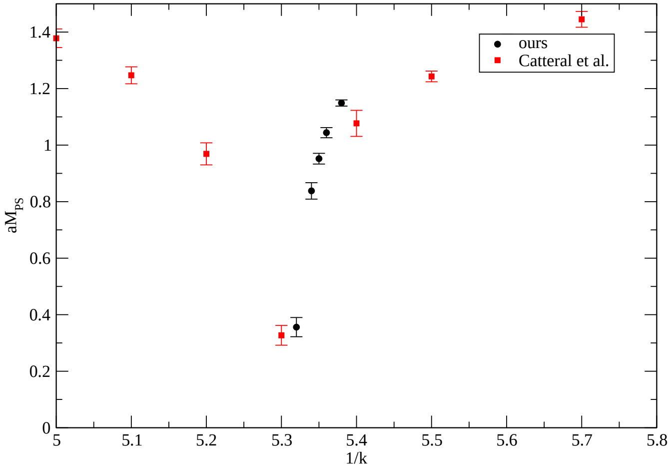
Failure of the naive TC scaling



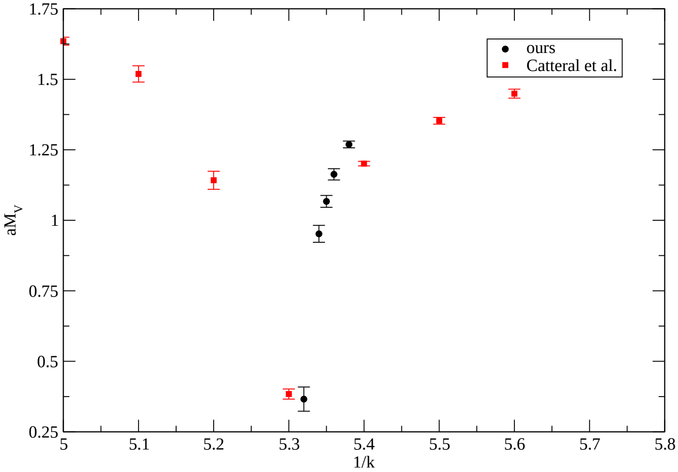
From the naive TC scaling:

$$\frac{M_V}{F_{PS}} = \frac{M_\rho}{F_\pi} \simeq 8.4$$

The PS mass


ours
Catteral et al.
aM_{PS}
1/k
0
0.2
0.4
0.6
0.8
1
1.2
1.4
5
5.1
5.2
5.3
5.4
5.5
5.6
5.7
5.8

The V mass


ours
Catteral et al.
aM_V
1/k
1.75
1.5
1.25
1
0.75
0.5
0.25
5
5.1
5.2
5.3
5.4
5.5
5.6
5.7
5.8

Compatibility with exotic scenarios