# Meson-Baryon Scattering in Lattice QCD

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# Overview

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# Motivation

- Meson interaction studies have reached a high level of precision (see papers by the NPLQCD collaboration: Precise Determination of the I=2 $\pi\pi$ Scattering Length..., The $K^+K^+$ Scattering Length..., Multi-Pion States in Lattice QCD... )
- Baryon correlators have a decreasing signal to noise ratio as time increases, while the meson S/N ratio remains constant.
- The meson baryon scattering signal should be better than the baryon baryon
- Determination of the low energy constants in $HB\chi PT$ contribute to the understanding of the nuclear force.
- Meson baryon interactions are a fundamental aspect of nuclear physics.

# Energy Eigenvalues in a Box

The exact energy eigenvalue equation for $E_n$:

$$\Delta E_n \equiv E_n - m_1 - m_2 = \sqrt{p_n^2 + m_1^2} + \sqrt{p_n^2 + m_2^2} - m_1 - m_2$$

Energy levels occur at momenta $\mathbf{p} = 2\pi\mathbf{j}/L$; The Lüscher formula relates the phase shift to the momenta:

$$p\cot\delta(p) = \frac{1}{\pi L}\,\mathbf{S}\left(\frac{pL}{2\pi}\right), \qquad \mathbf{S}\left(\frac{pL}{2\pi}\right) \equiv \sum_{\mathbf{j}}^{\Lambda_j} \frac{1}{|\mathbf{j}|^2 - \left(\frac{pL}{2\pi}\right)^2} - 4\pi\Lambda_j$$

the effective range expansion for $pcot\delta(p) \to 1/a$, as $p \to 0$.

# Meson-Baryon Correlation Functions

The correlation functions are computed as follows:

$$C_{\phi}(t) = \sum_{\mathbf{x}} \langle \phi^{\dagger}(t,\mathbf{x})\ \phi(0,\mathbf{0})\rangle, \qquad C_B(t) = \sum_{\mathbf{x}} \langle \bar{B}(t,\mathbf{x})\ B(0,\mathbf{0})\rangle$$

$$C_{\phi B}(p,t) \quad = \quad \sum_{|\mathbf{p}|=p} \sum_{\mathbf{x},\mathbf{y}} e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})} \langle \phi^{\dagger}(t,\mathbf{x})\ \bar{B}(t,\mathbf{y})\ \phi(0,\mathbf{0})\ B(0,\mathbf{0})\rangle$$

And the following ratio yields the energy:

$$G_{\phi B}(p,t) \quad \equiv \quad \frac{C_{\phi B}(p,t)}{C_{\phi}(t) C_B(t)} \rightarrow \sum_{n=0}^{\infty} \mathcal{A}_n\ e^{-\Delta E_n\ t}$$

# Meson-Baryon Scattering Processes

There are six elastic MB scattering processes that we calculated on the Lattice:

$$
\begin{aligned}
K^+ + n &\longrightarrow K^+ + n \\
K^+ + p &\longrightarrow K^+ + p \\
\bar{K}^0 + \Sigma^+ &\longrightarrow \bar{K}^0 + \Sigma^+ \\
\bar{K}^0 + \Xi^0 &\longrightarrow \bar{K}^0 + \Xi^0 \\
\pi^+ + \Sigma^+ &\longrightarrow \bar{K}^0 + \Sigma^+ \\
\pi^+ + \Xi^0 &\longrightarrow \bar{K}^0 + \Xi^0
\end{aligned}
$$

these processes do not have disconnected diagrams.

# Isospin Channels

The scattering amplitudes from HB $\chi PT$ are [Liu and Zhu hep-ph/0607100v3; N. Kaiser nucl-th/0107006v2]:

| Particles | Isospin content | $\chi PT$ L.O. | $\chi PT$ N.L.O. |
|---|---|---|---|
| $\pi^+\Sigma^+$ | $T^{(2)}_{\pi\Sigma}$ | $\frac{-m_\pi}{f_\pi^2}$ | $\frac{m_\pi^2}{f_\pi^2}C_1$ |
| $\pi^+\Xi^0$ | $T^{(3/2)}_{\pi\Xi}$ | $\frac{-m_\pi}{2f_\pi^2}$ | $\frac{m_\pi^2}{f_\pi^2}(C_1+C_0)$ |
| $K^+p$ | $T^{(1)}_{KN}$ | $\frac{-m_k}{f_k^2}$ | $\frac{m_k^2}{f_k^2}C_1$ |
| $K^+n$ | $\frac{1}{2}(T^{(1)}_{KN}+T^{(0)}_{KN})$ | $\frac{-m_k}{2f_k^2}$ | $\frac{m_k^2}{2f_k^2}(C_1+C_0)$ |
| $\bar{K}^0\Xi^0$ | $T^{(1)}_{K\Xi}$ | $\frac{-m_k}{f_k^2}$ | $\frac{m_k^2}{f_k^2}C_1$ |
| $\bar{K}^0\Sigma^+$ | $T^{(3/2)}_{K\Sigma}$ | $\frac{-m_k}{2f_k^2}$ | $\frac{m_k^2}{2f_k^2}(C_1+C_0)$ |

# Scattering Lengths

The threshold T matrix is related to the scattering length by:

$$T_{\phi B}^{(I)} = 4\pi \left(1 + \frac{m_\phi}{M_B}\right) a_{\phi B}^{(I)}$$

So at tree-level $a$ is:

$$a_{\phi B} = -\frac{\mu_{\phi B}}{4\pi f_\phi^2}, \qquad or \qquad a_{\phi B} = -\frac{\mu_{\phi B}}{8\pi f_\phi^2},$$

with $\mu$ being the reduced mass of the meson and baryon:

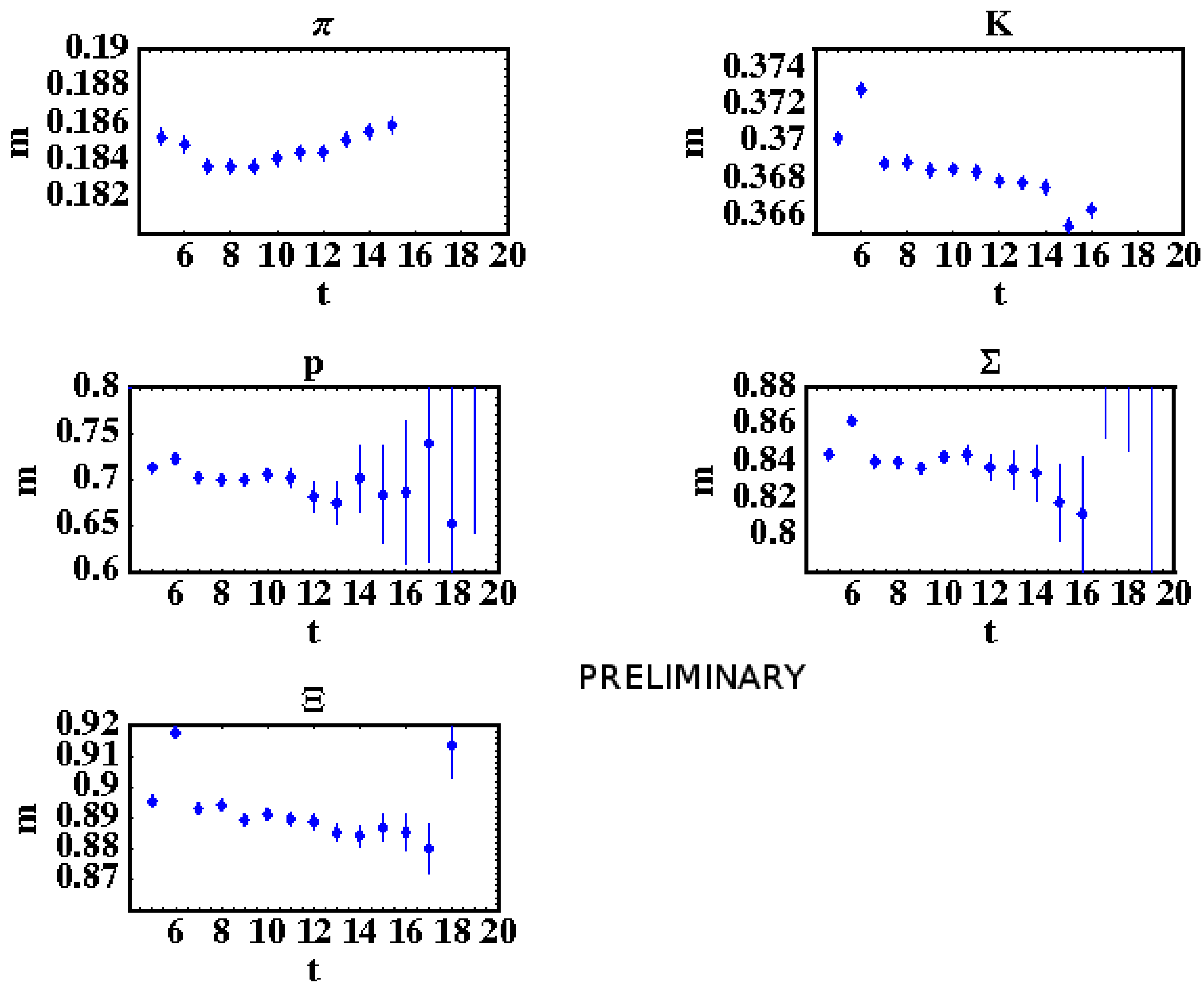
$$\mu_{\phi B} = \frac{m_\phi M_B}{m_\phi + M_B}$$

# MILC Configurations Used in the Calculation

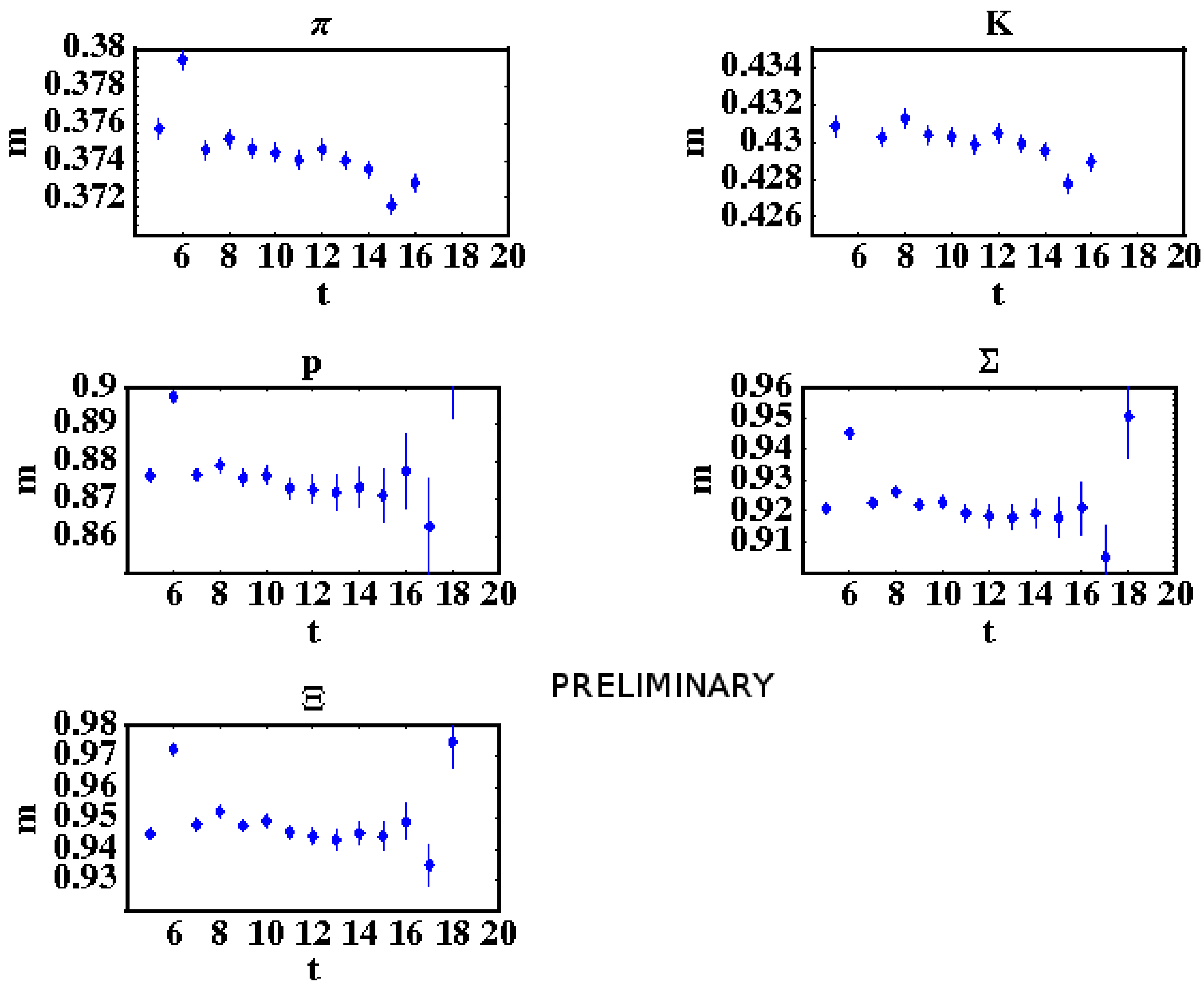
| Config Set | Dimensions | $bm_l$ | $bm_s$ | $m_\pi$ | # configs | # sources |
|---|---|---|---|---|---|---|
| 2896f21b709m0062m031 | $28^3 \times 96$ | 0.0062 | 0.05 | 317 MeV | 1001 | 7 |
| 2064f21b676m007m050 | $20^3 \times 64$ | 0.007 | 0.05 | 294 MeV | 1039 | 24 |
| 2064f21b676m010m050 | $20^3 \times 64$ | 0.010 | 0.05 | 348 MeV | 769 | 24 |
| 2064f21b679m020m050 | $20^3 \times 64$ | 0.020 | 0.05 | 484 MeV | 486 | 24 |
| 2064f21b681m030m050 | $20^3 \times 64$ | 0.030 | 0.05 | 565 MeV | 564 | 16 |

$b_c = 0.125$ fm, $b_f = 0.09$ fm, $L = 2.5$ fm

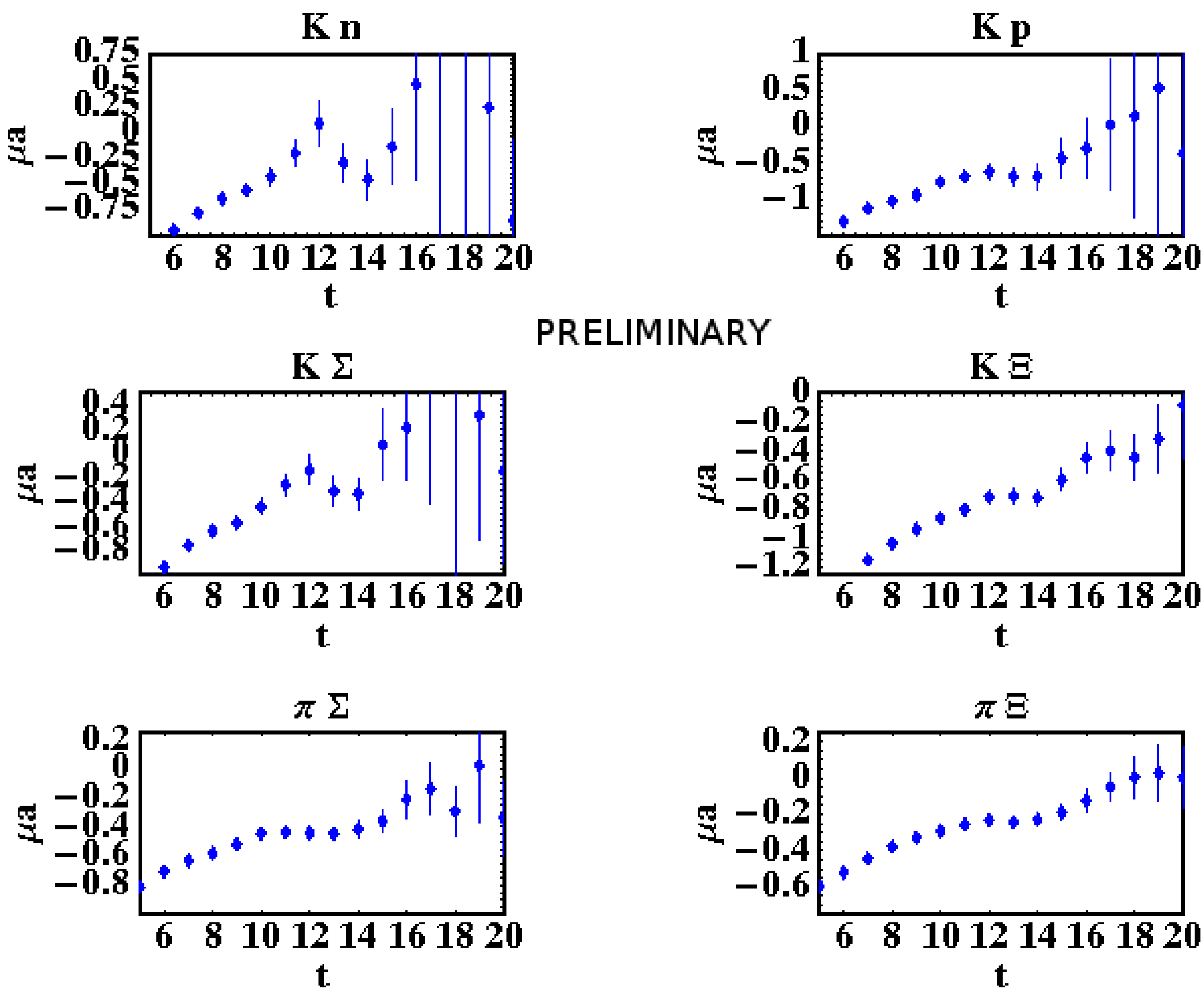
# m007 Effective Mass Plots



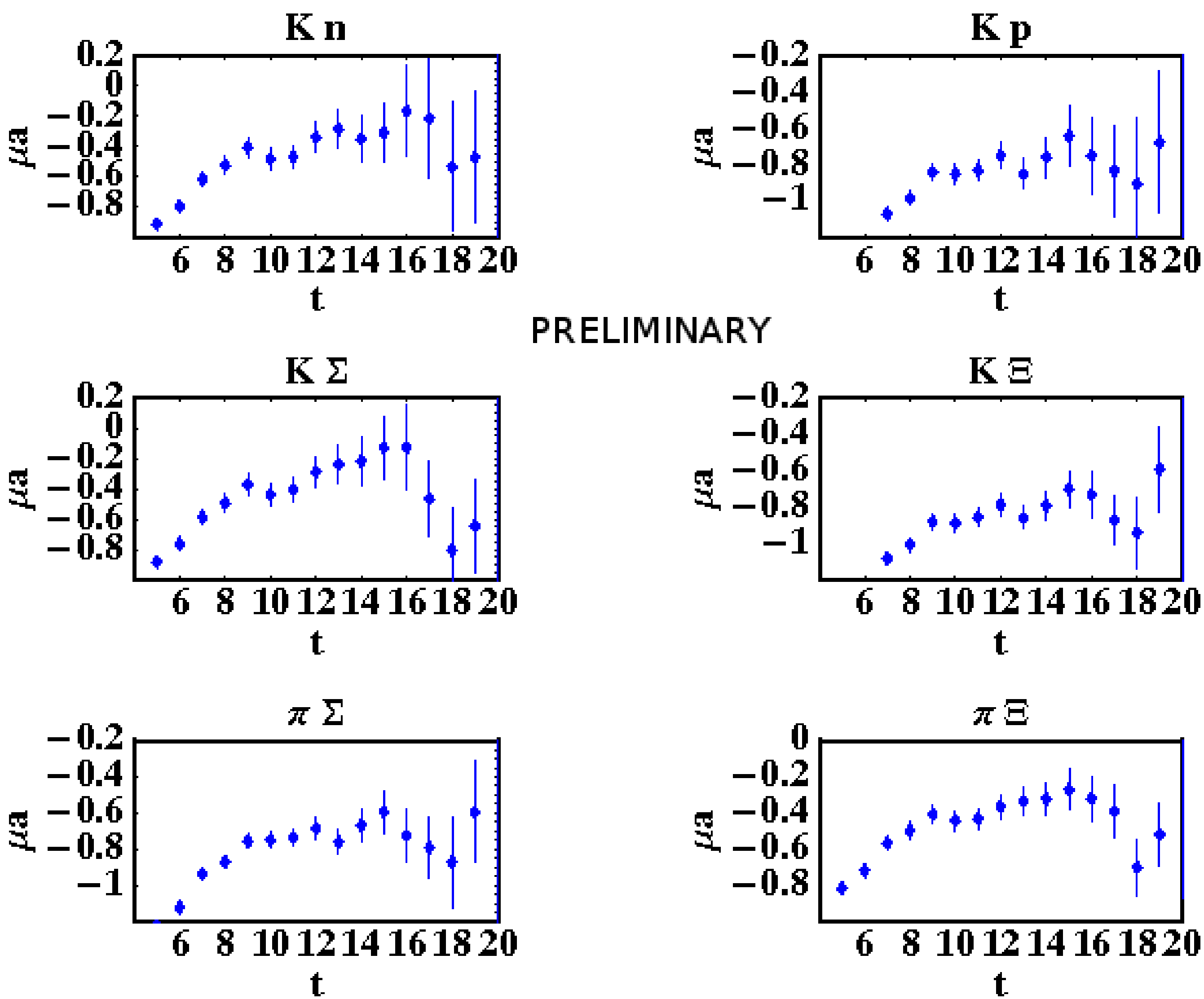
# m030 Effective Mass Plots



# m0062 $\mu a$ Effective Plots

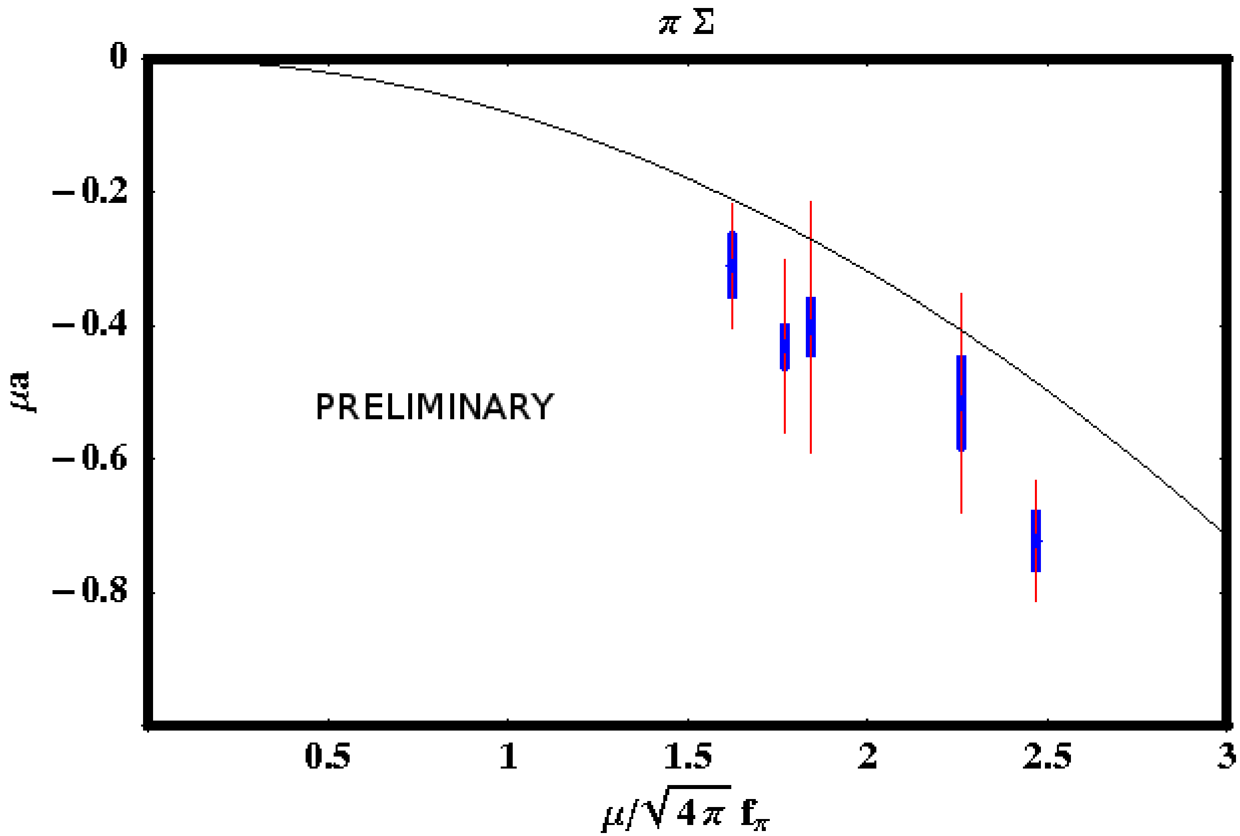


# m030 $\mu a$ Effective Plots



# $\pi^+\Sigma^+$ Scattering Length vs. Reduced Mass

From left to right, the mass points are ordered: m007, m0062, m010, m020, m030.




the blue error bar is statistical, and the red error bar is an estimate of the systematic error due to fitting.

# Conclusion

- we will be able to extract the scattering lengths for some of these processes, but not with high precision.
- we can also fit the low energy constants $C_1$ and $C_0$ that appear at NNLO in the $HB\chi PT$ Lagrangian

To Do:

- Use Mixed Action $\chi PT$ for the chiral extrapolations

Thanks to NPLQCD: Silas Beane, William Detmold, Tom Luu, Kostas Orginos, Assumpta Parreño, Martin Savage, and André Walker-Loud

Thanks to The College of William and Mary, and the organizers

# Jackknife

Once we have the numbers from the correlators, we average over the number of gauge configurations using the jackknife method

$$\alpha_i = [\alpha_1, \alpha_2, \cdots, \alpha_N]$$

$$\alpha_i^{jackknife} = \frac{1}{N-1}\left[\sum_{i=1}^{N}\alpha_i - \alpha_1, \sum_{i=1}^{N}\alpha_i - \alpha_2, \ldots, \sum_{i=1}^{N}\alpha_i - \alpha_N\right]$$

| time | config 1 | config 2 | ... | config N |
|---|---|---|---|---|
| 0 | 0.00012 | 0.00013 | ... | 0.00012 |
| 1 | 0.00007 | 0.00006 | ... | 0.00004 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| 31 | 0.00009 | 0.00008 | ... | 0.00007 |

The reason for jackknife is that the measurements at different coordinates $(x, y)$ are not statistically independent (see DeGrand & DeTar, "Lattice Methods for Quantum Chromodynamics").

# Energy Eigenvalues and Scattering Length

The scattering length can be expressed in terms of known constants, and quantities we can measure on the lattice;

$$\Delta E_0 \quad = \quad -\frac{4\pi a}{M L^3}\left[ 1 \, + \, c_1 \frac{a}{L} \, + \, c_2 \left(\frac{a}{L}\right)^2 \right] + \mathcal{O}\left(\frac{1}{L^6}\right)$$

where the constants, $c_1, c_2$ contain infinite sums, and a regulator $\Lambda$, which have to be evaluated numerically (see S.R. Beane, P.F.Bedaque, A. Parreno, M.J. Savage, hep-lat/0312004)

Using the above expression, we can solve for the scattering length since we can fit both masses and $\Delta E$, from our lattice data.

This expression is obtained from the exact equation for $\mathbf{S}\left(\frac{pL}{2\pi}\right)$.

# Heavy Baryon $\chi$PT

invariant Lagrangian of HB$\chi$PT reads [a]

$$\mathcal{L} = \mathcal{L}_{\phi\phi} + \mathcal{L}_{\phi B},$$

$\mathcal{L}_{\phi\phi}$ incorporates even chiral order terms while the terms in $\mathcal{L}_{\phi B}$ start from $\mathcal{O}(p)$.

$$\mathcal{L}^{(2)}_{\phi\phi} = f^2 \mathrm{tr}(u_\mu u^\mu + \frac{\chi_+}{4}),$$

$$\mathcal{L}^{(1)}_{\phi B} = \mathrm{tr}(\overline{B}(i\partial_0 B + [\Gamma_0, B])) - D\mathrm{tr}(\overline{B}\{\vec{\sigma}\cdot\vec{u}, B\}) - F\mathrm{tr}(\overline{B}[\vec{\sigma}\cdot\vec{u}, B]),$$

$$\begin{aligned}
\mathcal{L}^{(2)}_{\phi B} \quad = \quad & b_D \mathrm{tr}(\overline{B}\{\chi_+, B\}) + b_F \mathrm{tr}(\overline{B}[\chi_+, B]) + b_0 \mathrm{tr}(\overline{B}B)\mathrm{tr}(\chi_+) \\
& + \Big(2d_D + \frac{D^2 - 3F^2}{2M_0}\Big)\mathrm{tr}(\overline{B}\{u_0^2, B\}) + \Big(2d_F - \frac{DF}{M_0}\Big)\mathrm{tr}(\overline{B}[u_0^2, B]) \\
& + \Big(2d_0 + \frac{F^2 - D^2}{2M_0}\Big)\mathrm{tr}(\overline{B}B)\mathrm{tr}(u_0^2) \\
& + \Big(2d_1 + \frac{3F^2 - D^2}{3M_0}\Big)\mathrm{tr}(\overline{B}u_0)\mathrm{tr}(u_0 B)
\end{aligned}$$

# HB$\chi$PT

$$\Gamma_\mu = \frac{i}{2}[\xi^\dagger, \partial_\mu \xi], \qquad u_\mu = \frac{i}{2}\{\xi^\dagger, \partial_\mu \xi\}, \qquad \xi = \exp(i\phi/2f),$$

$$\chi_+ = \xi^\dagger \chi \xi^\dagger + \xi \chi \xi, \qquad \chi = \mathrm{diag}(m_\pi^2,\, m_\pi^2,\, 2m_K^2 - m_\pi^2),$$

$$\phi = \sqrt{2}\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \overline{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}, B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}$$

where $C_{1,0}$ are defined in Ref. (N. Kaiser, Chiral Corrections to Kaon Nucleon Scattering lengths)

$$\begin{aligned} C_1 &= 2(d_0 - 2b_0) + 2(d_D - 2b_D) + d_1 - \frac{D^2 + 3F^2}{6M_0}, \\ C_0 &= 2(d_0 - 2b_0) - 2(d_F - 2b_F) - d_1 - \frac{D(D - 3F)}{3M_0}. \end{aligned}$$

Meson-Baryon Scattering Lengths in HB$\chi$PT, Liu and Zhu, 2007