GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Hadronic equation of state and relativistic heavy-ion collisions

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Workshop on Excited Hadronic States and the Deconfinement Transition
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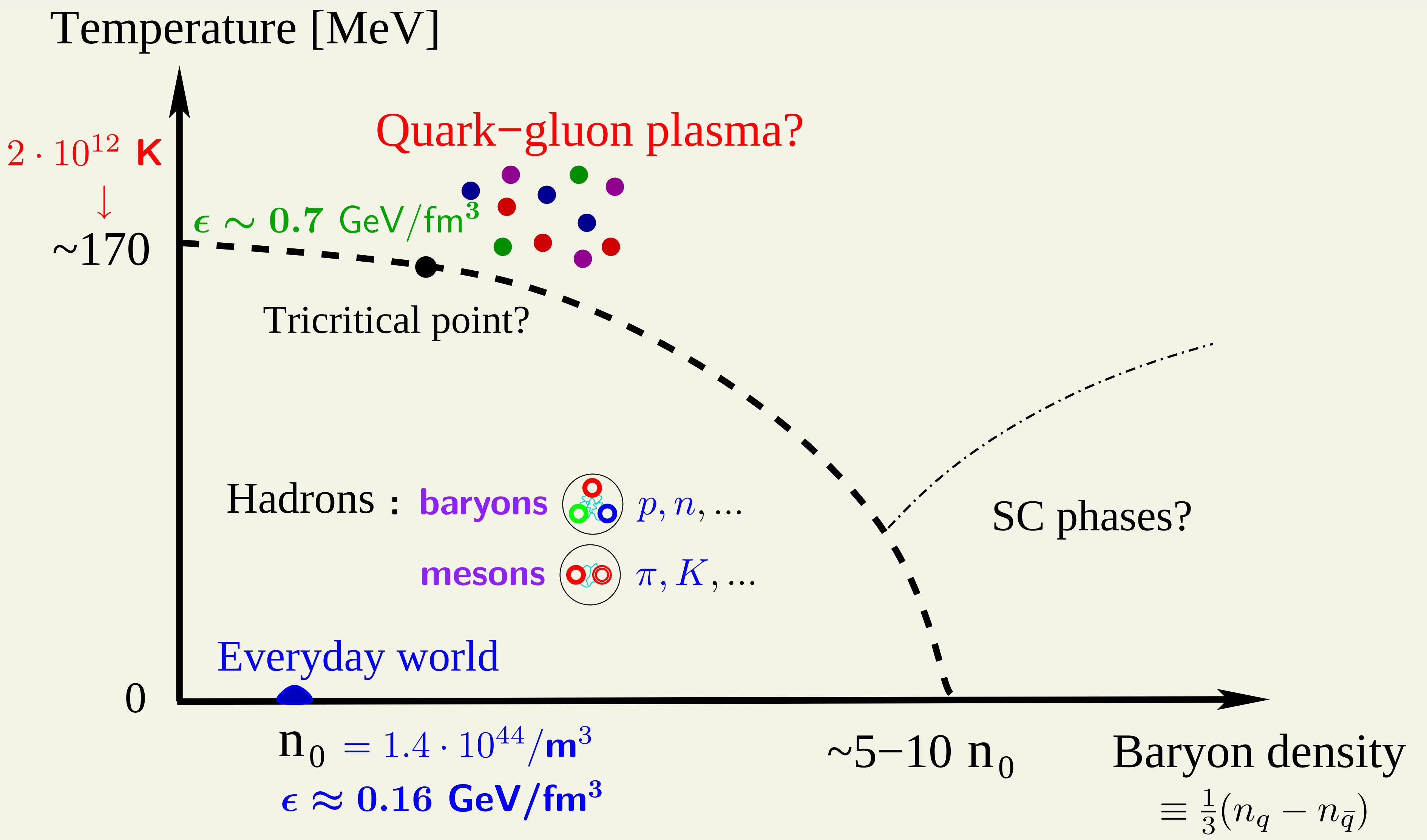
in collaboration with P. Petreczky, H. Niemi and G. Denicol

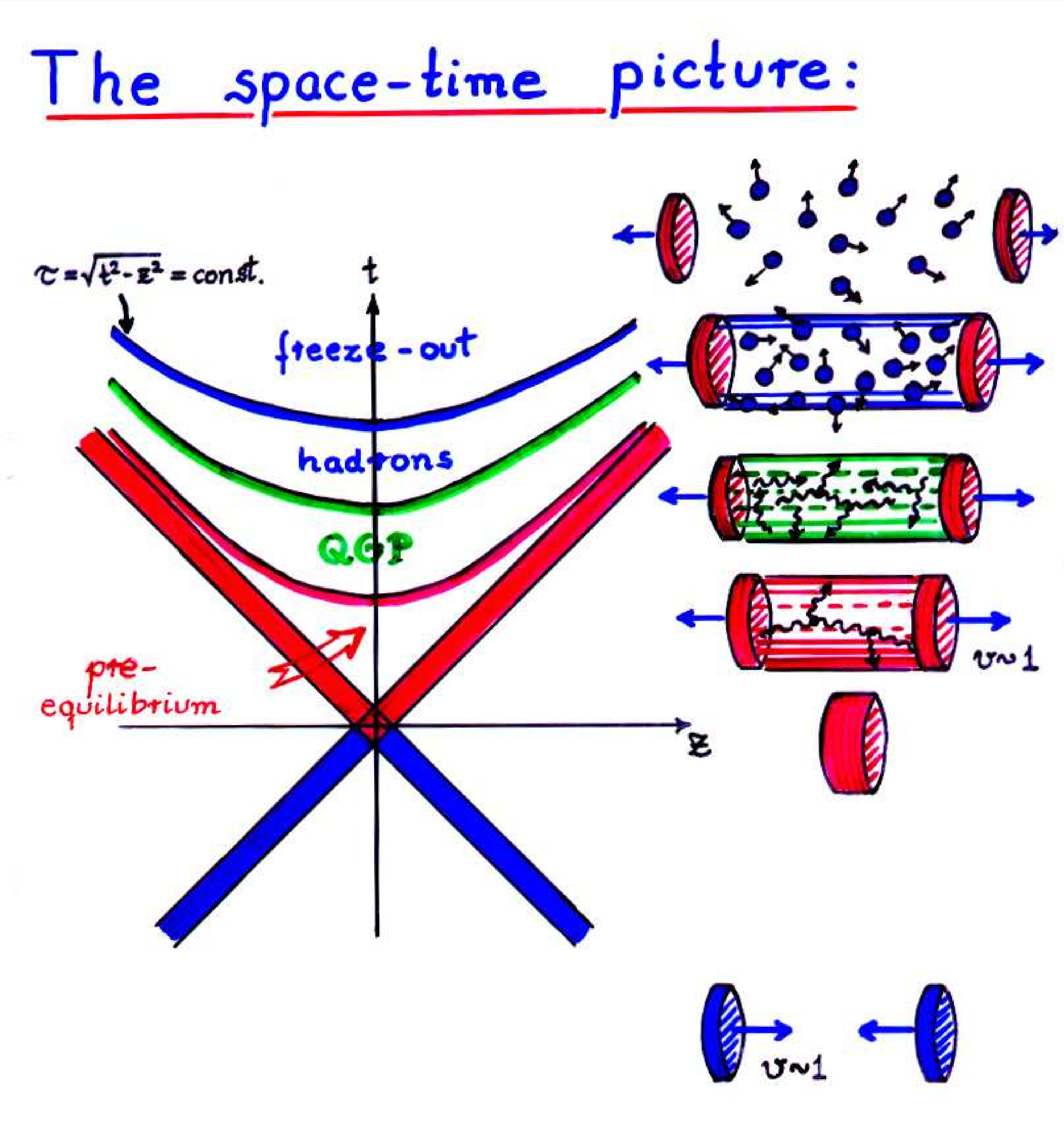
Hadron properties, hydrodynamics, and relativistic heavy-ion collisions

or

Why we need to know hadron properties to learn about QGP

Nuclear phase diagram





- free streaming

  ⇑
- freeze-out

  ⇑
- fluid dynamics

  ⇑
- thermalization

  ⇑
- initial collision

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Ideal hydrodynamics

local conservation of energy, momentum and baryon number:

$$\partial_\mu T^{\mu\nu}(x) = 0 \quad \textbf{and} \quad \partial_\mu N^\mu(x) = 0$$

local equilibrium, no dissipation: $T^{\mu\nu} = (e+p)u^\mu u^\nu - Pg^{\mu\nu}$
$N^\mu = nu^\mu$

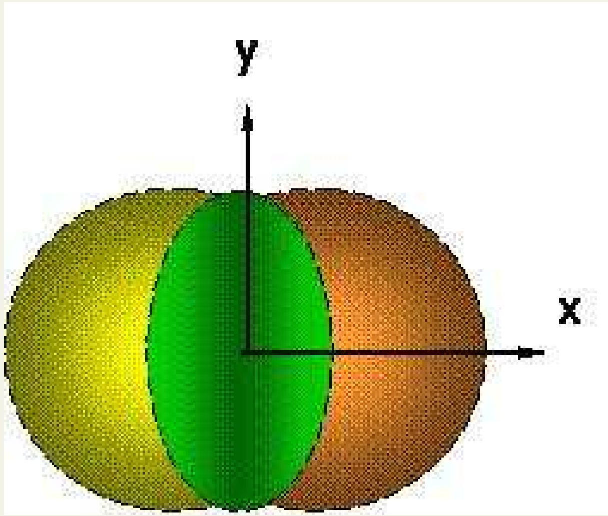
local, macroscopic variables: **energy density** $e(x)$
pressure $P(x)$
flow velocity $u^\mu(x)$

matter characterized by: **equation of state** $P = P(e, n)$

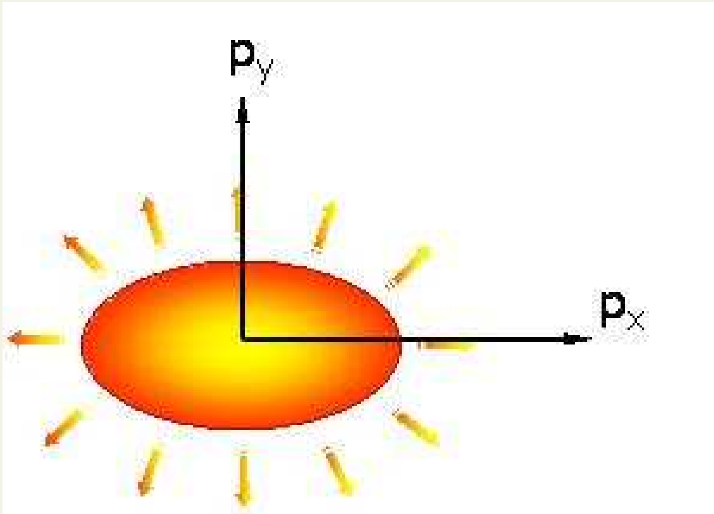
Unknowns: **initial state, final state, equation of state**

Elliptic flow

spatial anisotropy $\rightarrow$ final azimuthal momentum anisotropy



$\longrightarrow$



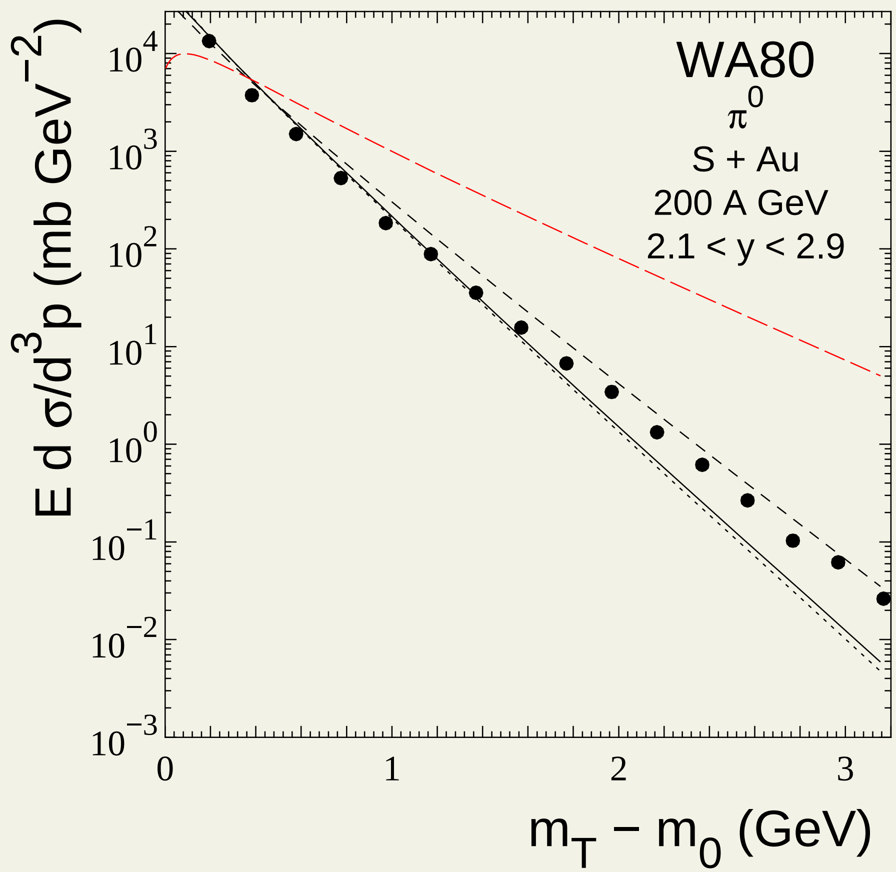
$$\varepsilon \equiv \frac{\langle x^2 - y^2 \rangle}{\langle x^2 + y^2 \rangle}$$

$$v_2 \equiv \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle}$$

sensitive to speed of sound $c_s^2 = \partial p / \partial e$ **and shear viscosity** η

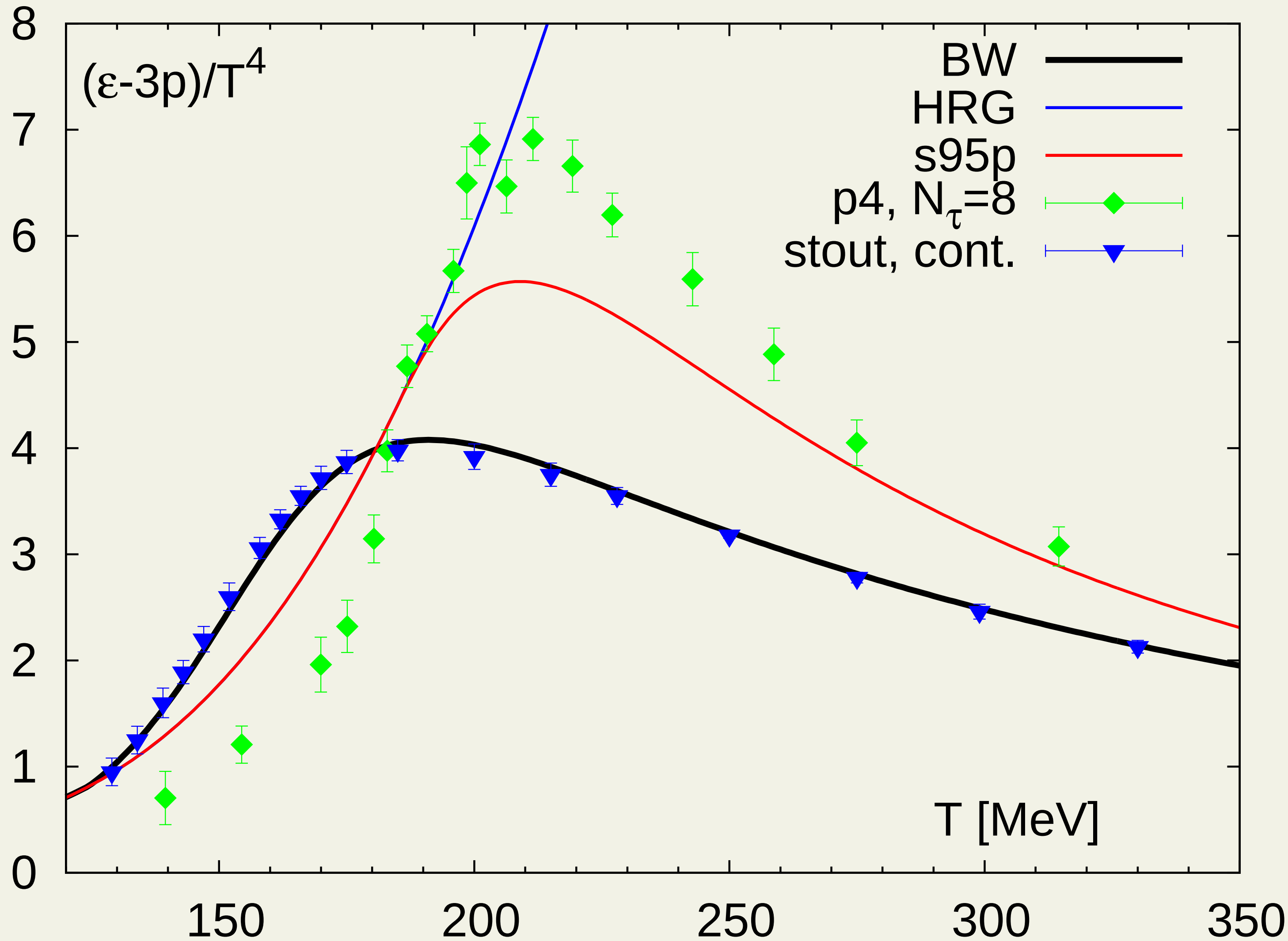
Pion gas EoS

Sollfrank *et al*, PRC55, 392 (1997)



- **ideal pion gas EoS**
- **too stiff**
- **must have "many" hadronic d.o.f's in the EoS**

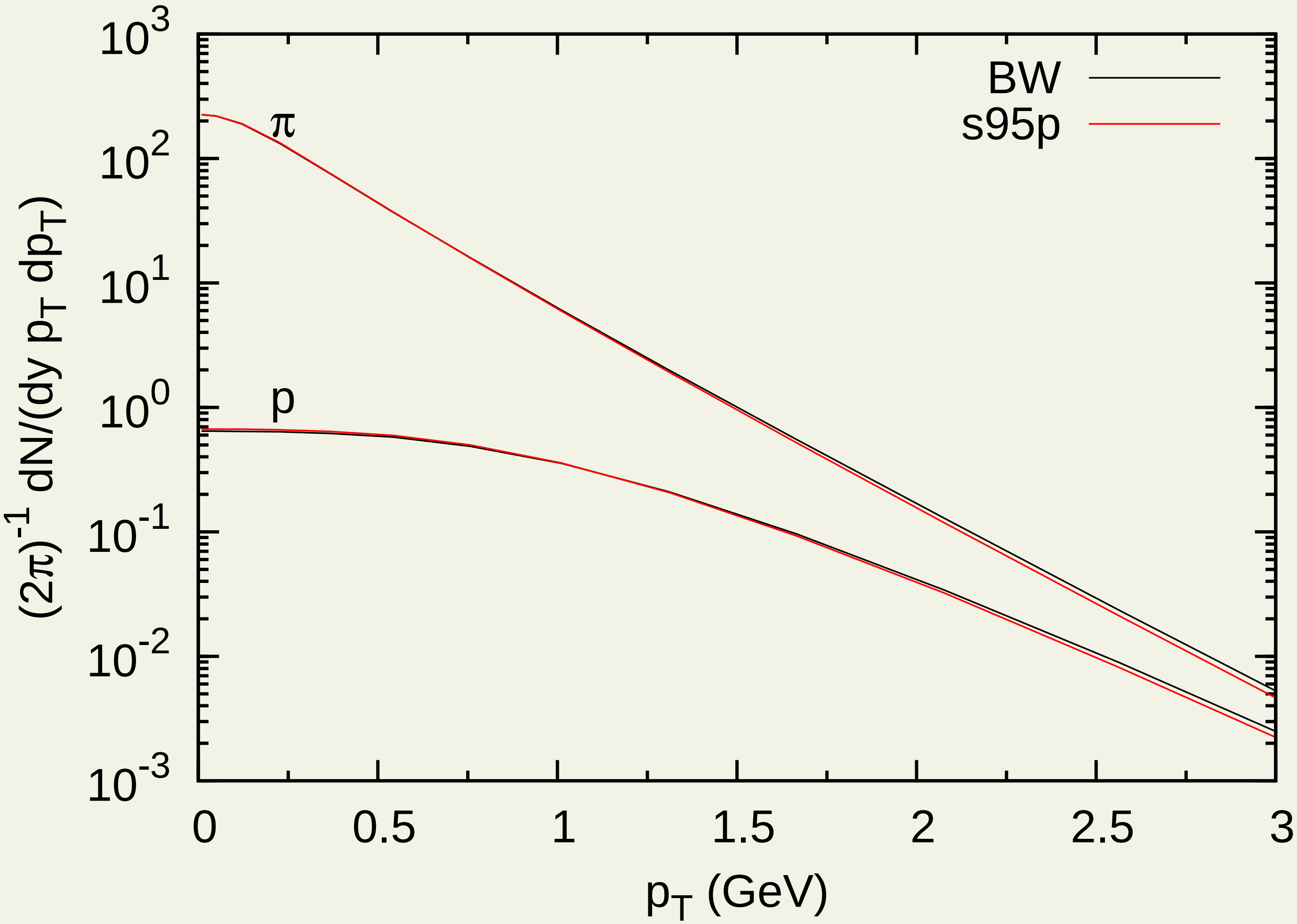
Hagedorn states?



- **BW: fit to Wuppertal results, Hagedorn states?**
- **s95p: hadrons up to 2 GeV mass below T_c**

have tiny effect

- **ideal fluid**
- **Au+Au collision at RHIC,** $\sqrt{s} = 200$ **GeV, b=7 fm**
- $T_{\rm dec} = 124$ **MeV; all EoSs!**



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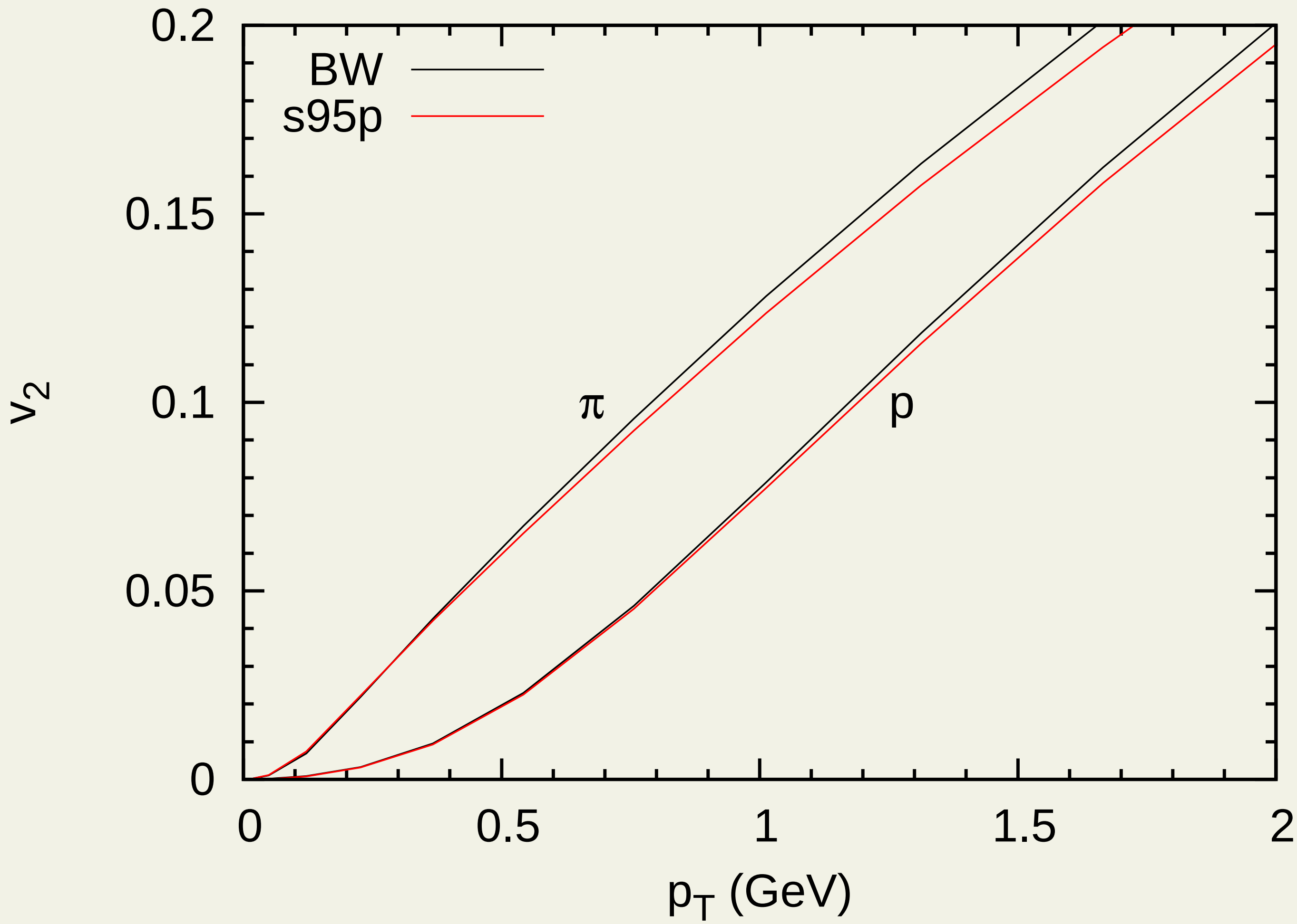
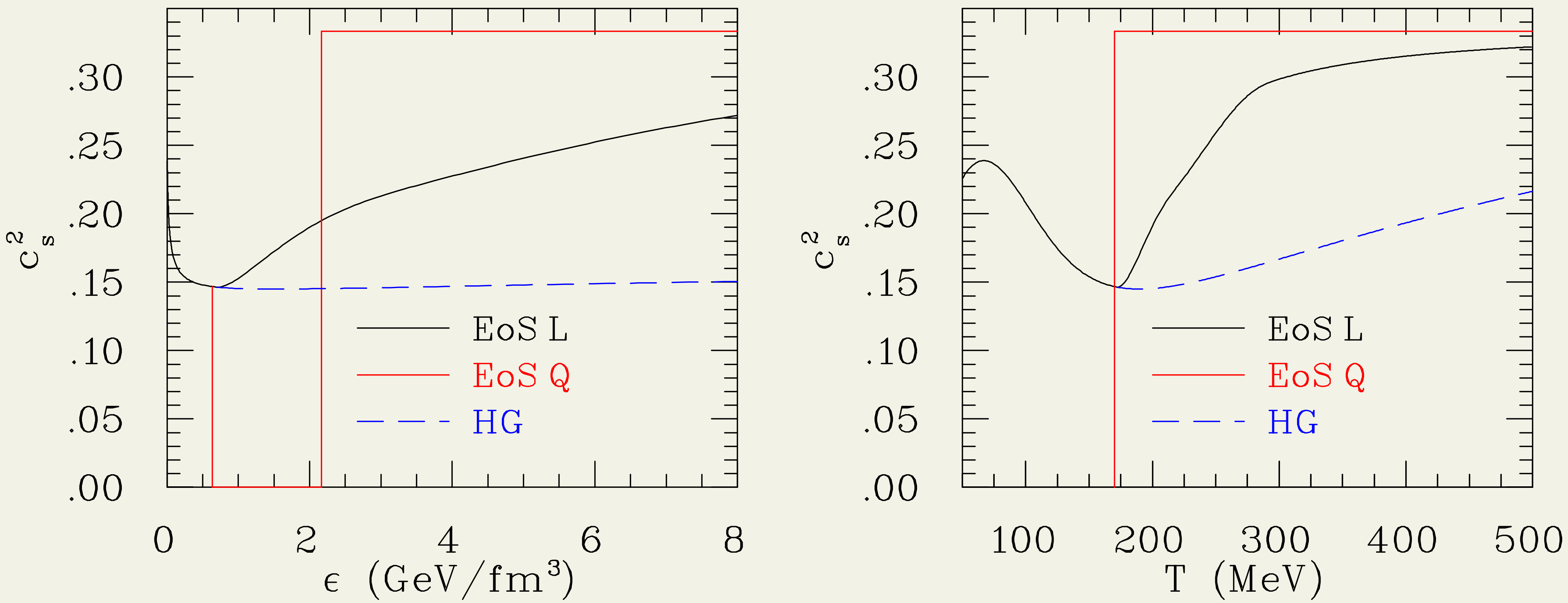


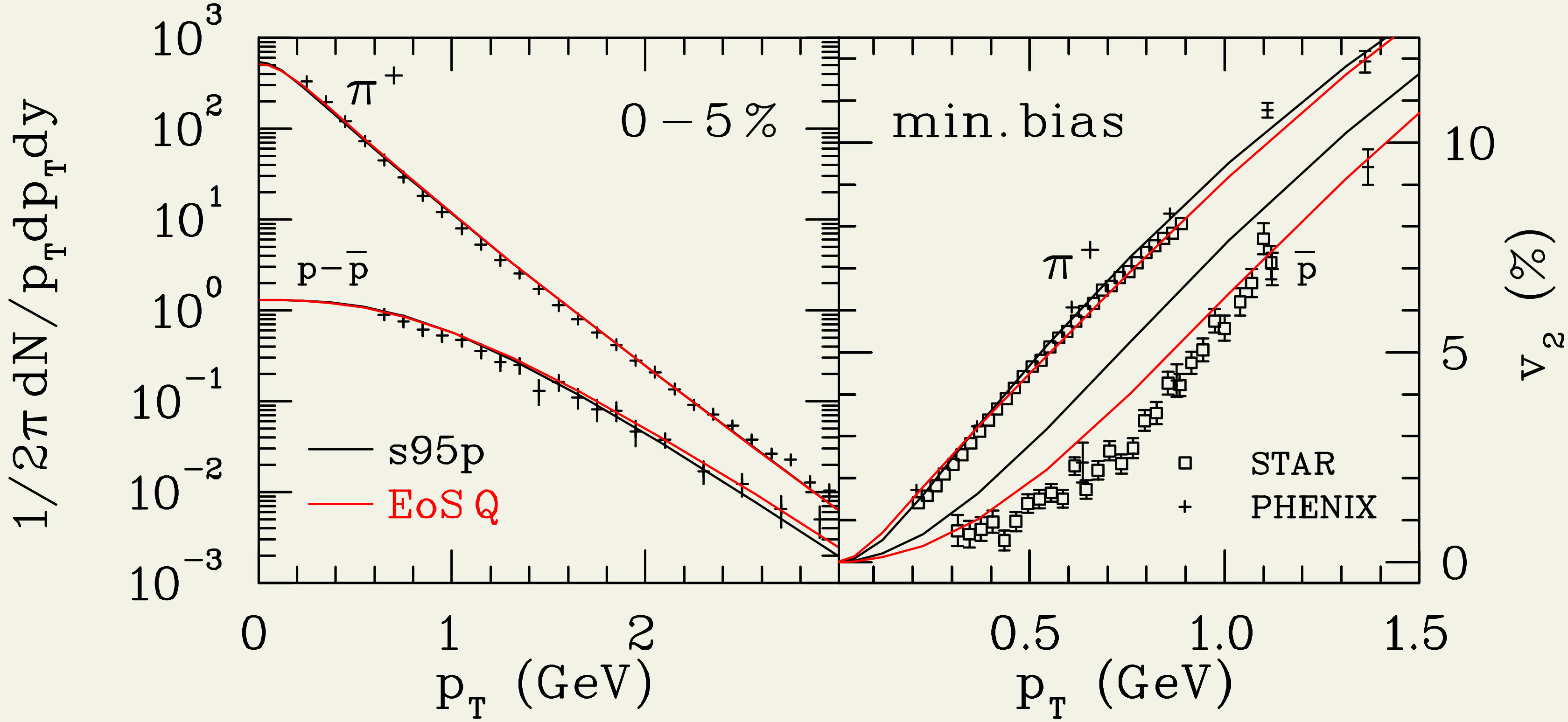
First order phase transition



- **first order phase transition** ⇔ **region with zero speed of sound**

has an observable effect!

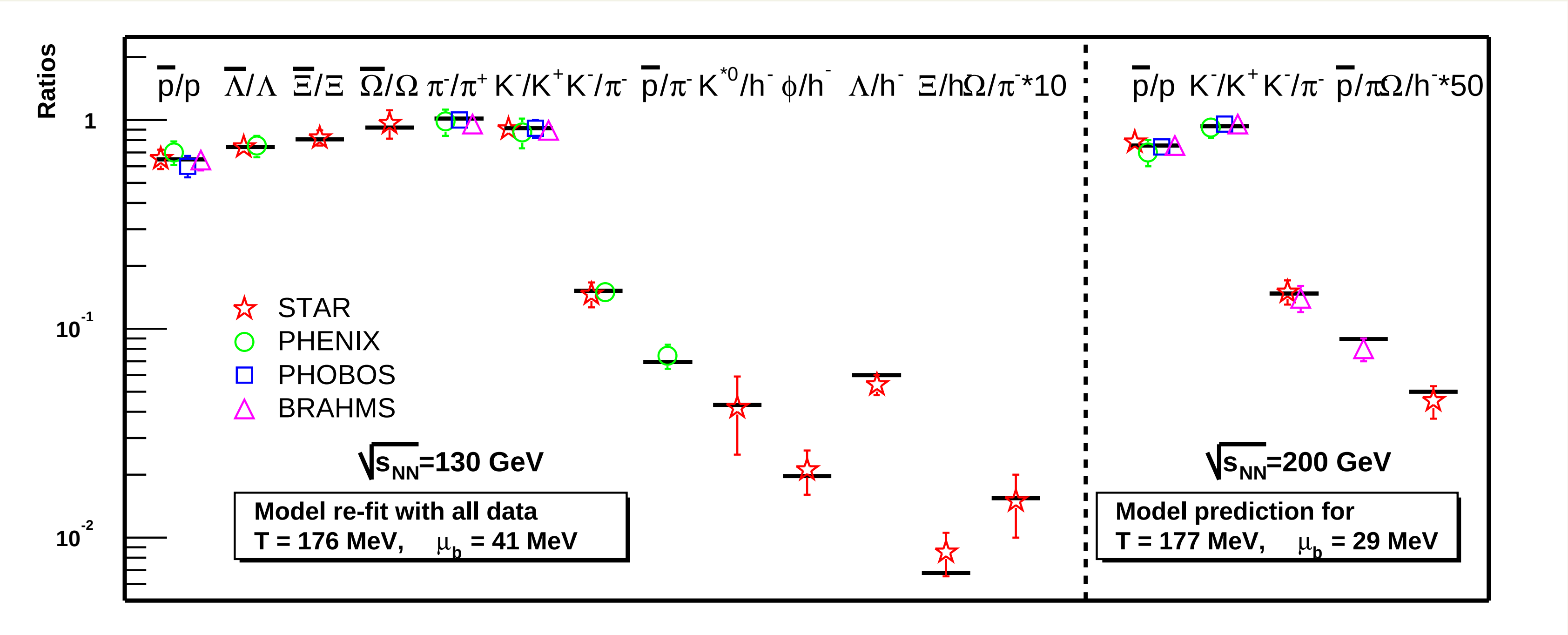
- **ideal hydro, Au+Au at $\sqrt{s_{NN}} = 200$ GeV**



- **s95p:** $T_{dec} = 140$ **MeV**
- **EoS Q: first order phase transition at** $T_c = 170$ **MeV,** $T_{dec} = 125$ **MeV**

Thermal models

- Hadronic phase: ideal gas of massive hadrons and resonances
- in chemical equilibrium

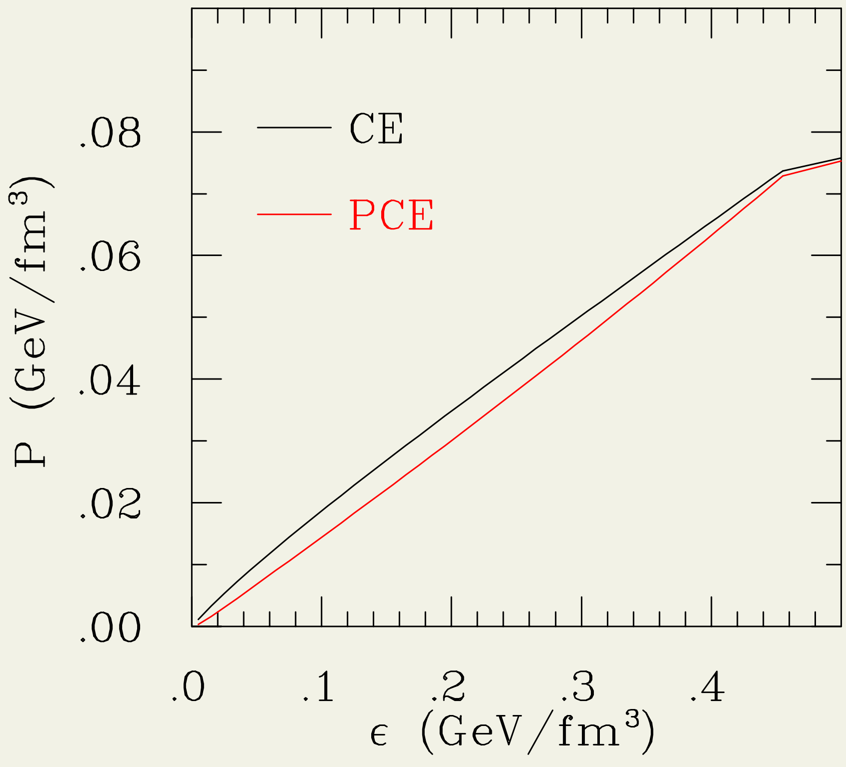


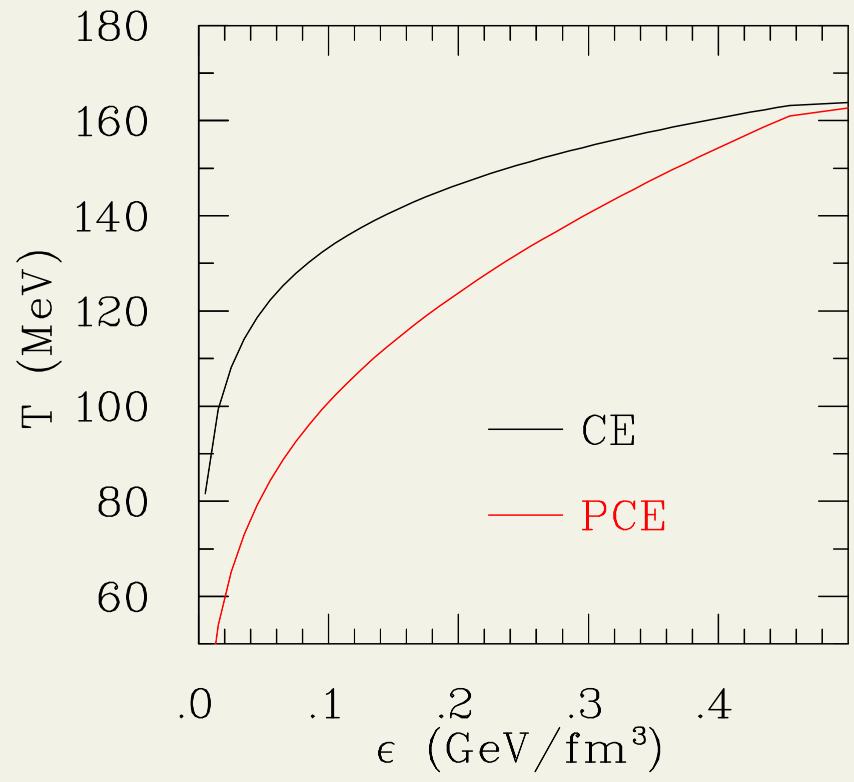
- **Particle ratios (approximately) correspond to a system in $T \approx 160$–170 MeV temperature**
- **Evolution to $T \approx 100$–120 MeV temperature**

$\Rightarrow$ **In hydro particle ratios become wrong**

Chemical non-equilibrium

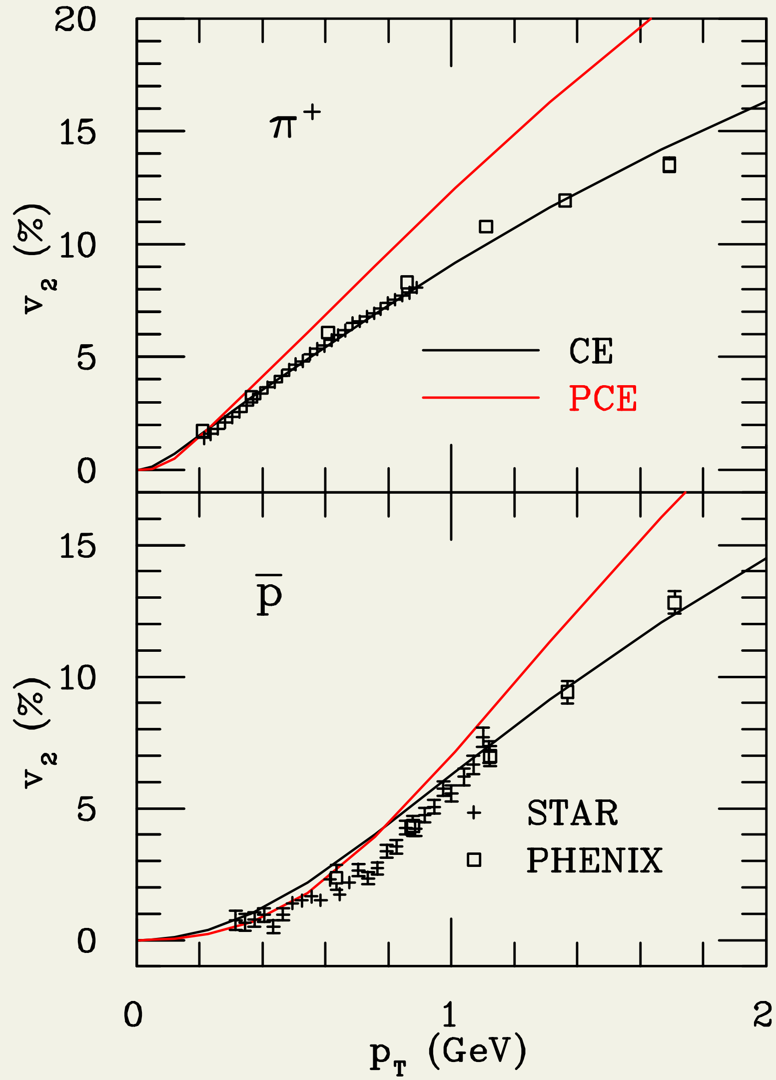
- **Treat number of pions, kaons etc. as conserved quantum numbers below T_{ch} (Bebie et al, Nucl.Phys.B378:95-130,1992)**
- $P = P(\epsilon, n_b)$ **changes very little, but** $T = T(\epsilon, n_b)$ **changes. . .**





Effect on flow

- ideal hydro, Au+Au at $\sqrt{s_{NN}} = 200$ GeV
- $T_{\rm chem} = 150$ MeV



Viscous hydrodynamics

relativistic Navier-Stokes hydro: small corrections linear in gradients

$$T^{\mu\nu}_{NS} = T^{\mu\nu}_{ideal} + \eta(\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}\partial^\alpha u_\alpha) + \zeta\Delta^{\mu\nu}\partial^\alpha u_\alpha$$

$$N^\mu_{NS} = N^\mu_{ideal} - \kappa\left(\frac{nT}{e+p}\right)^2 \nabla^\mu \frac{\mu}{T}$$

where $\Delta^{\mu\nu} \equiv u^\mu u^\nu - g^{\mu\nu}$, $\nabla^\mu = \Delta^{\mu\nu}\partial_\nu$

η, ζ **shear and bulk viscosities,** κ **heat conductivity**

two problems:

parabolic equations $\rightarrow$ **acausal** Müller ('76), Israel & Stewart ('79) ...

instabilities Hiscock & Lindblom, PRD31, 725 (1985) ...

Causal viscous hydro

Müller, Israel & Stewart...

$$\Delta T^{\mu\nu} \equiv \pi^{\mu\nu} + \Pi\Delta^{\mu\nu} \quad , \qquad \Delta N^{\mu} = -\frac{n}{e+p} q^{\mu}$$

bulk pressure Π, shear stress $\pi^{\mu\nu}$ heat flow q^{μ} treated as independent dynamical quantities that relax to their Navier-Stokes value on time scales $\tau_{\Pi}(e,n)$, $\tau_{\pi}(e,n)$, $\tau_{q}(e,n)$

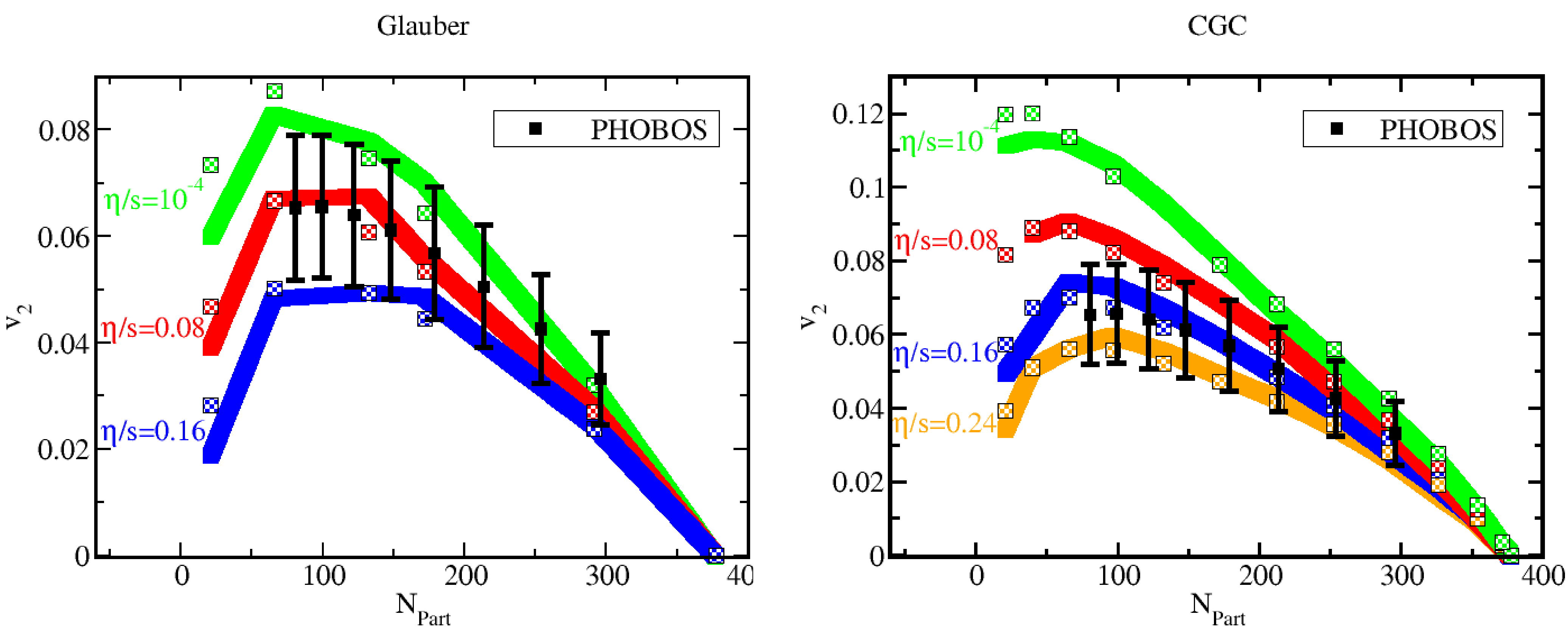
schematically

$$\dot{X} = -\frac{X - X_{NS}}{\tau_X} + \mathcal{F}(X)$$

restores causality (for not too small τ_X)

estimate for η/s

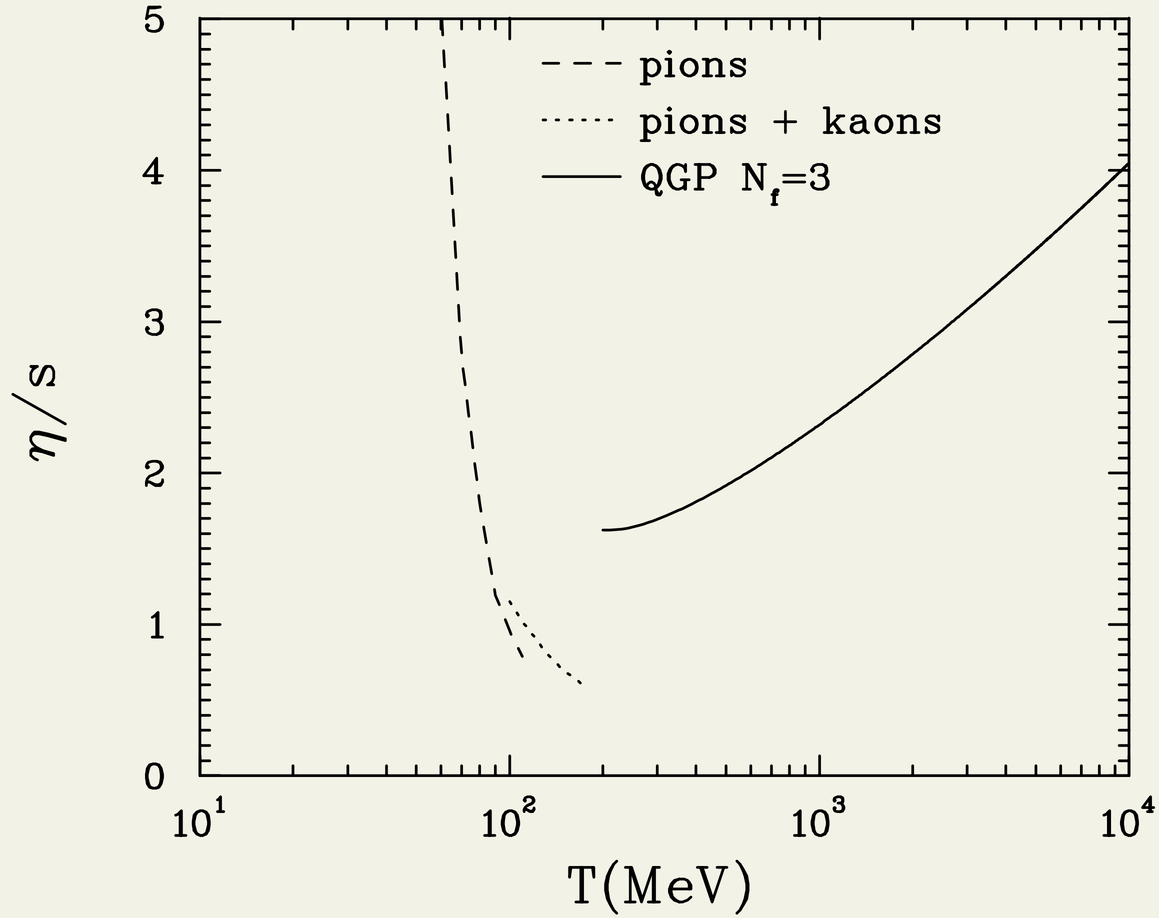
Luzum & Romatschke, PRC78, 034915 (2008)



- $\eta/s = 0.08$ or $\eta/s = 0.16$ depending on initialization

$\eta/s(T)$

Kapusta, McLerran and Csernai, nucl-th/0604032:

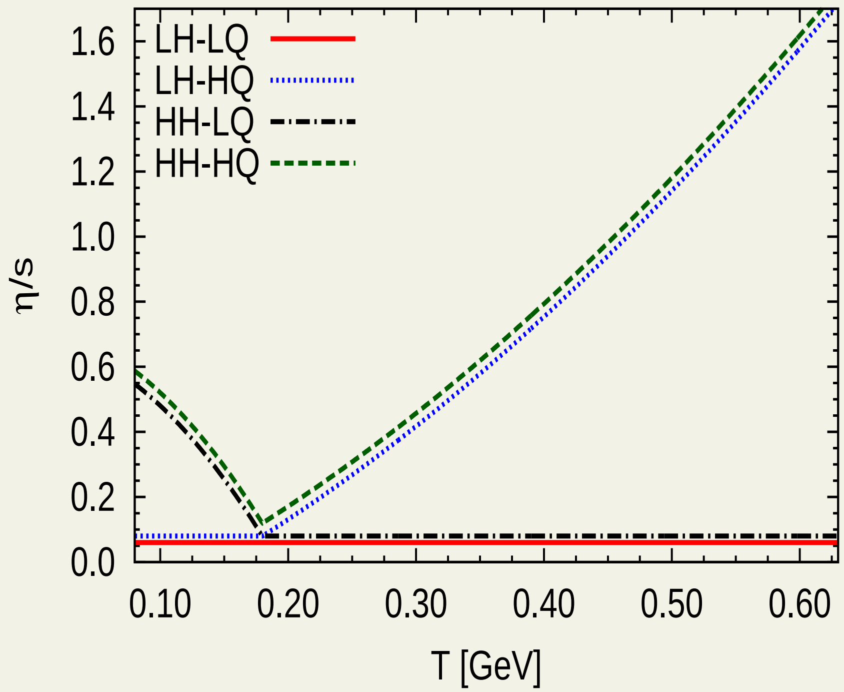


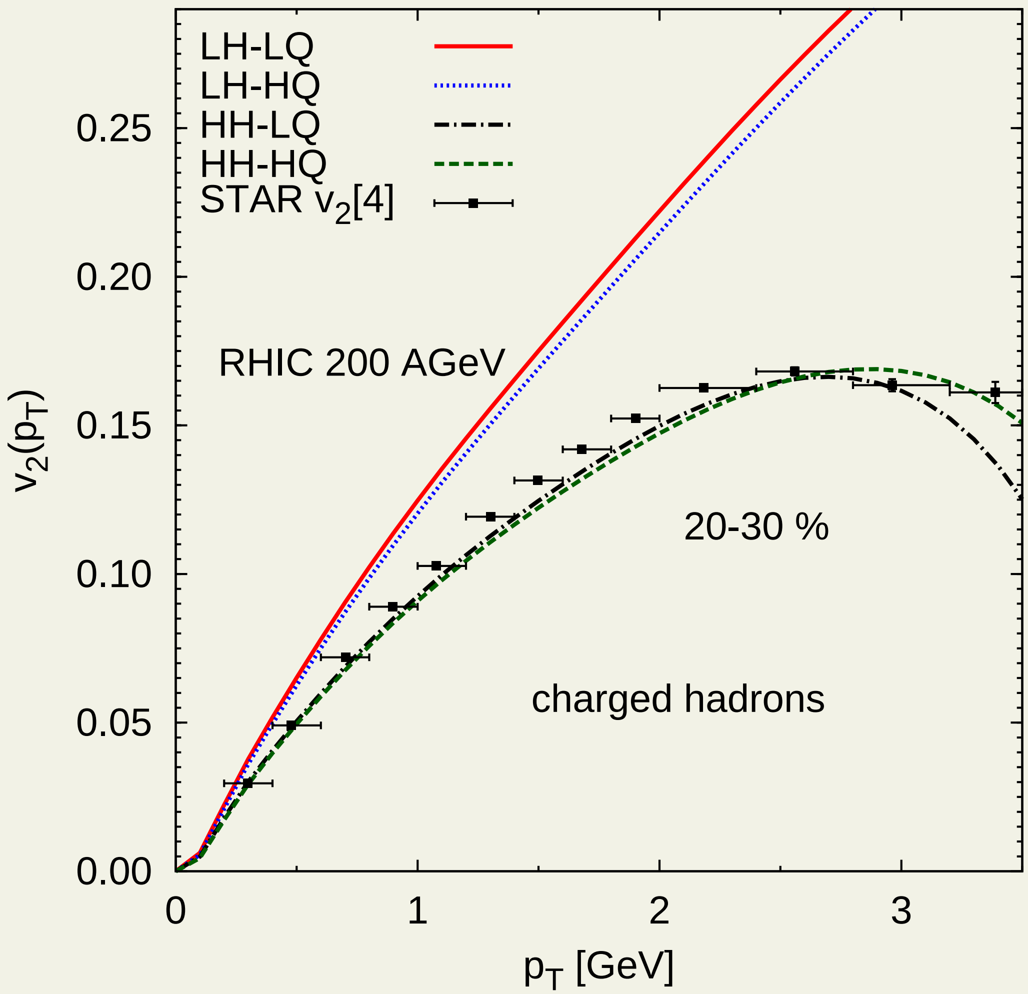
Low T (*Prakash et al.*) **using experimental data for 2-body interactions**

High T (*Yaffe et al.*) **using perturbative QCD**

$\eta/s(T)$

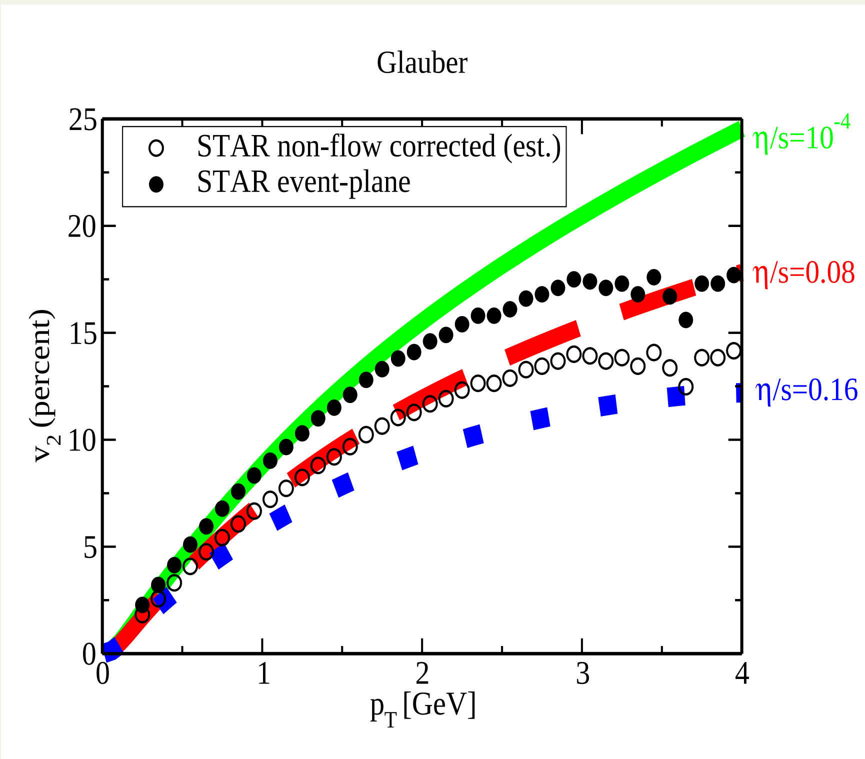
Niemi *et al.*, arXiv:1101.2442

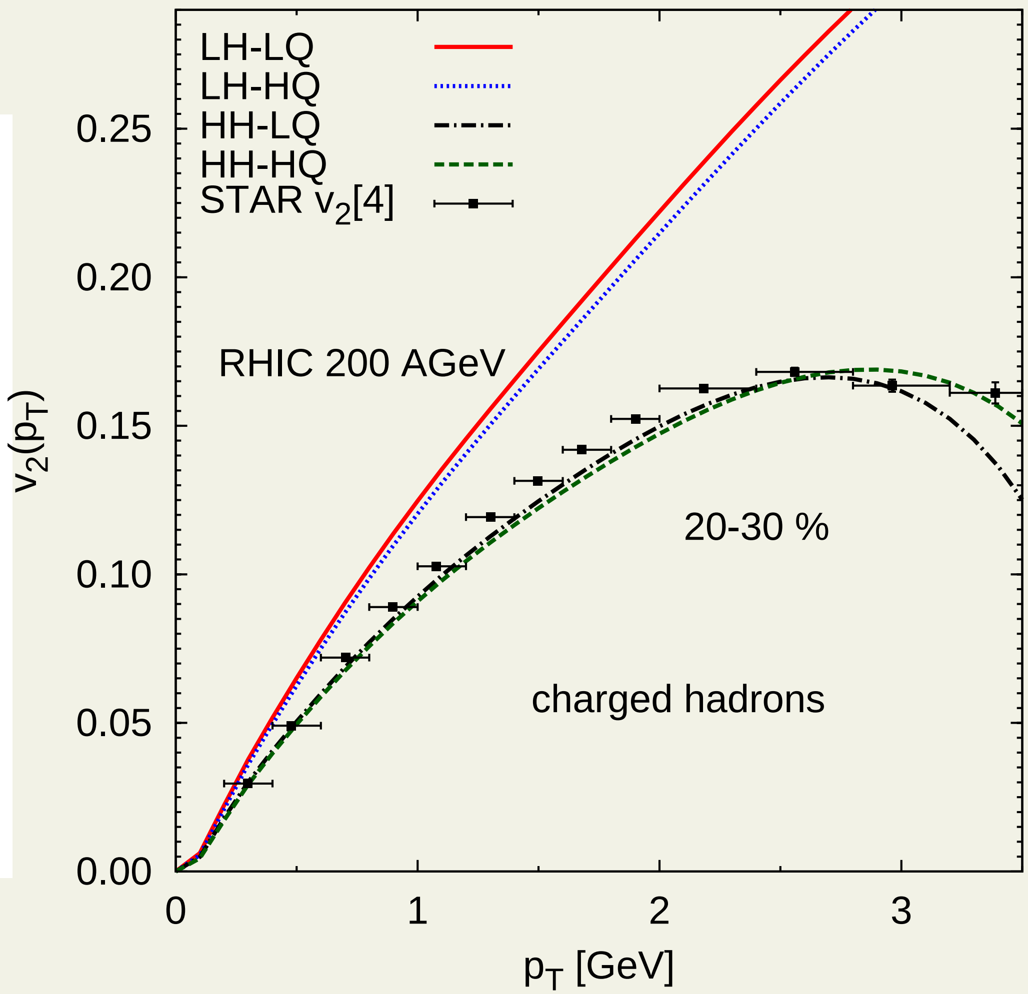




- $v_2(p_T)$ **mostly sensitive to hadronic** $\eta/s(T)$ **at RHIC!**

η/s?

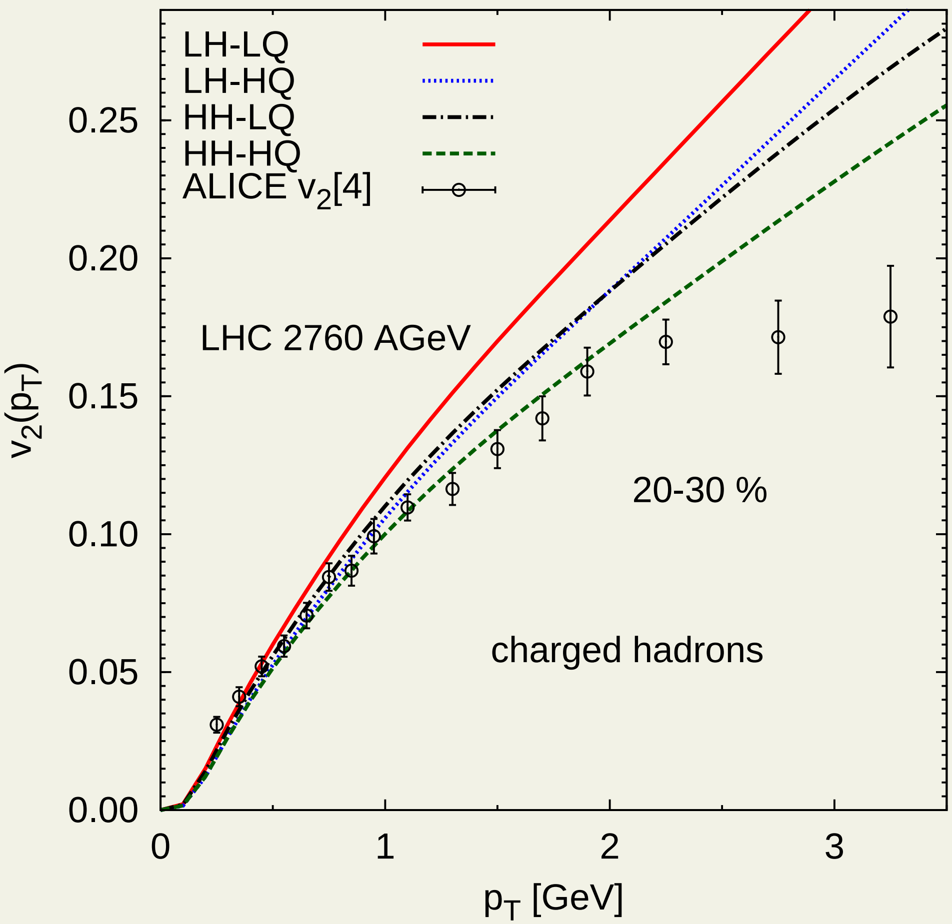


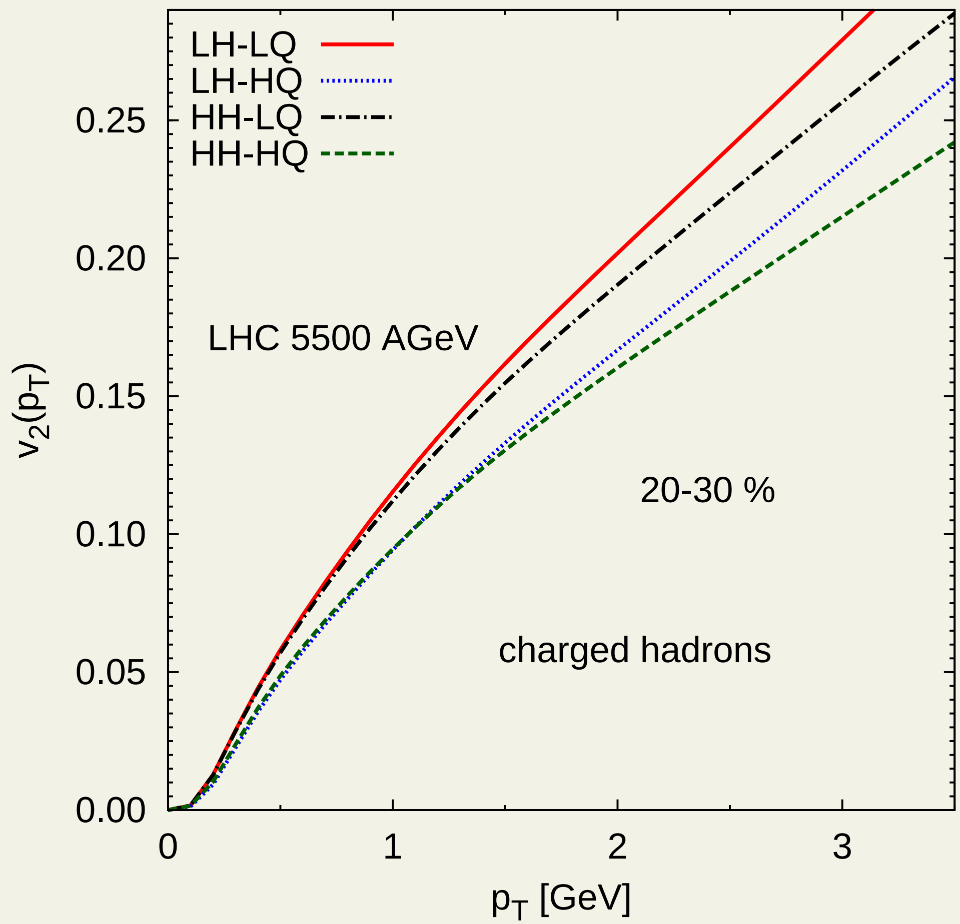


- **small η/s at minimum, large in HRG — or vice versa?**

$\eta/s(T)$ at LHC

Niemi *et al.*, arXiv:1101.2442





- **at the top LHC energy η/s of plasma dominates**

Fluid to particles

Cooper-Frye description:

$$E\frac{\mathrm{d}N}{\mathrm{d}\mathbf{p}^3} = \int_{\Sigma_{\mathrm{fo}}} \mathrm{d}\Sigma_\mu \, p^\mu f(x, p\cdot u)$$

- $f(x, p\cdot u)$ **thermal** $\Leftrightarrow$ **ideal fluid**
- **dissipation requires** $f_{\mathrm{th}} \to f_{\mathrm{th}}(1 + \delta f)$

Grad 14-moment approximation (Boltzmann distribution)

$$\delta f = \epsilon + \epsilon_\mu p^\mu + \epsilon_{\mu\nu} p^\mu p^\nu$$

Shear only, Landau matching gives

$$\delta f = \epsilon_{\mu\nu} p^\mu p^\nu = \frac{1}{2T^2(\epsilon + P)} \pi^{\mu\nu} p_\mu p_\nu$$

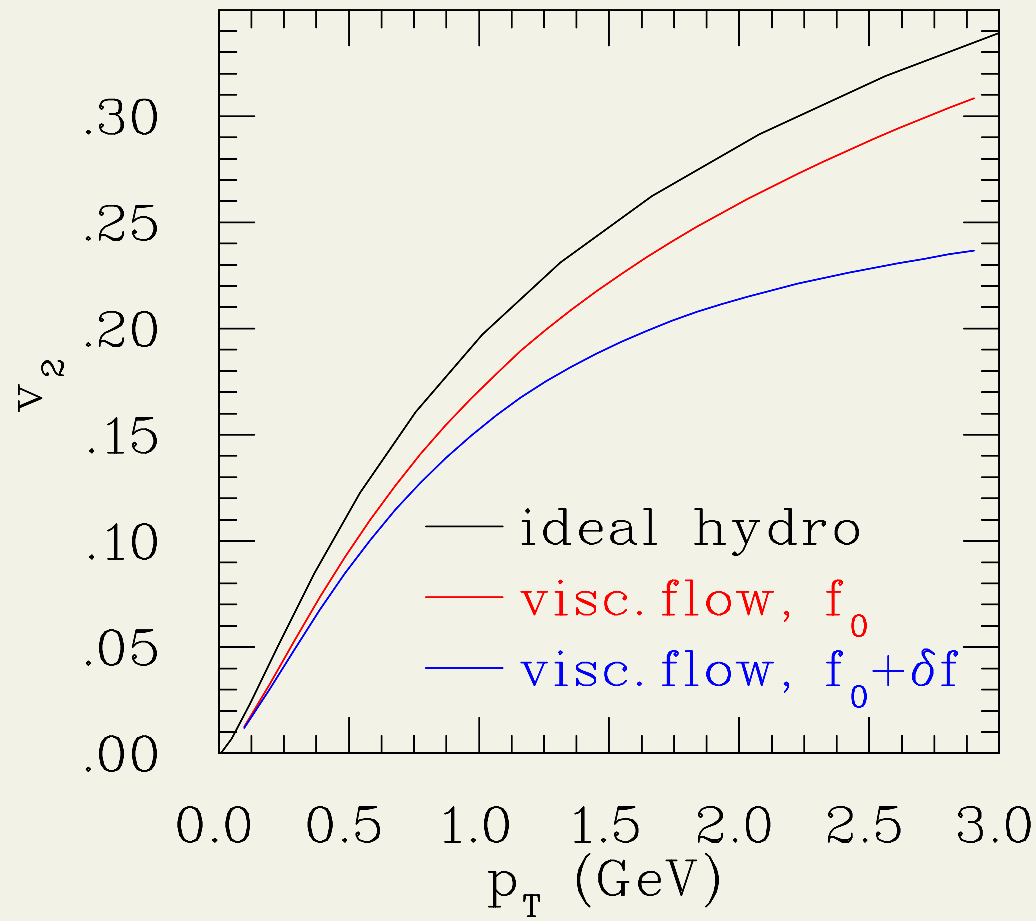
How to share $\pi^{\mu\nu}$ for each particle species?

δf

TWO effects: - **dissipative corrections to hydro fields** u^μ, T, n
- **dissipative corrections to thermal distributions** $f \to f_0 + \delta f$

$\eta/s \approx 1/(4\pi)$ $(\sigma_{\rm tr} \propto \tau^{2/3})$

$$\delta f = f_0 \frac{p^\mu p^\nu \pi_{\mu\nu}}{8nT^3}$$



δf for mixtures

Denicol *et al.*, work in progress:

Analogous to single component system:

$$\delta f_i = \frac{p_\mu p_\nu \, \pi_i^{\mu\nu}}{2(\varepsilon_i + P_i)T^2}$$

At Navier-Stokes limit:

$$\pi^{\mu\nu} = \eta\sigma^{\mu\nu} \Longrightarrow \pi_i^{\mu\nu} = \eta_i \sigma^{\mu\nu}$$

Thus

$$\delta f_i \approx \frac{p_\mu p_\nu}{2(\varepsilon_i + P_i)T^2} \frac{\eta_i}{\eta} \pi^{\mu\nu}$$

Using kinetic theory:

$$\eta_i = \mathcal{F}(T, \{\mu_i\}, \{m_i\}, \sigma_{ij},)$$

is a complicated integral over thermal distributions

Conclusions

- hadronic EoS has only a small observable effect
  - once the hadronic # of d.o.f is large enough
- chemical non-equilibrium of hadron phase has large effect!
- hadronic viscosity dominates at RHIC
  - less so at LHC
  - $\eta/s(T)$ of QGP cannot be constrained using RHIC v_2 data only
- δf a big uncertainty
  - hadron cross sections required to evaluate it!