Resonances in Hadronic Transport

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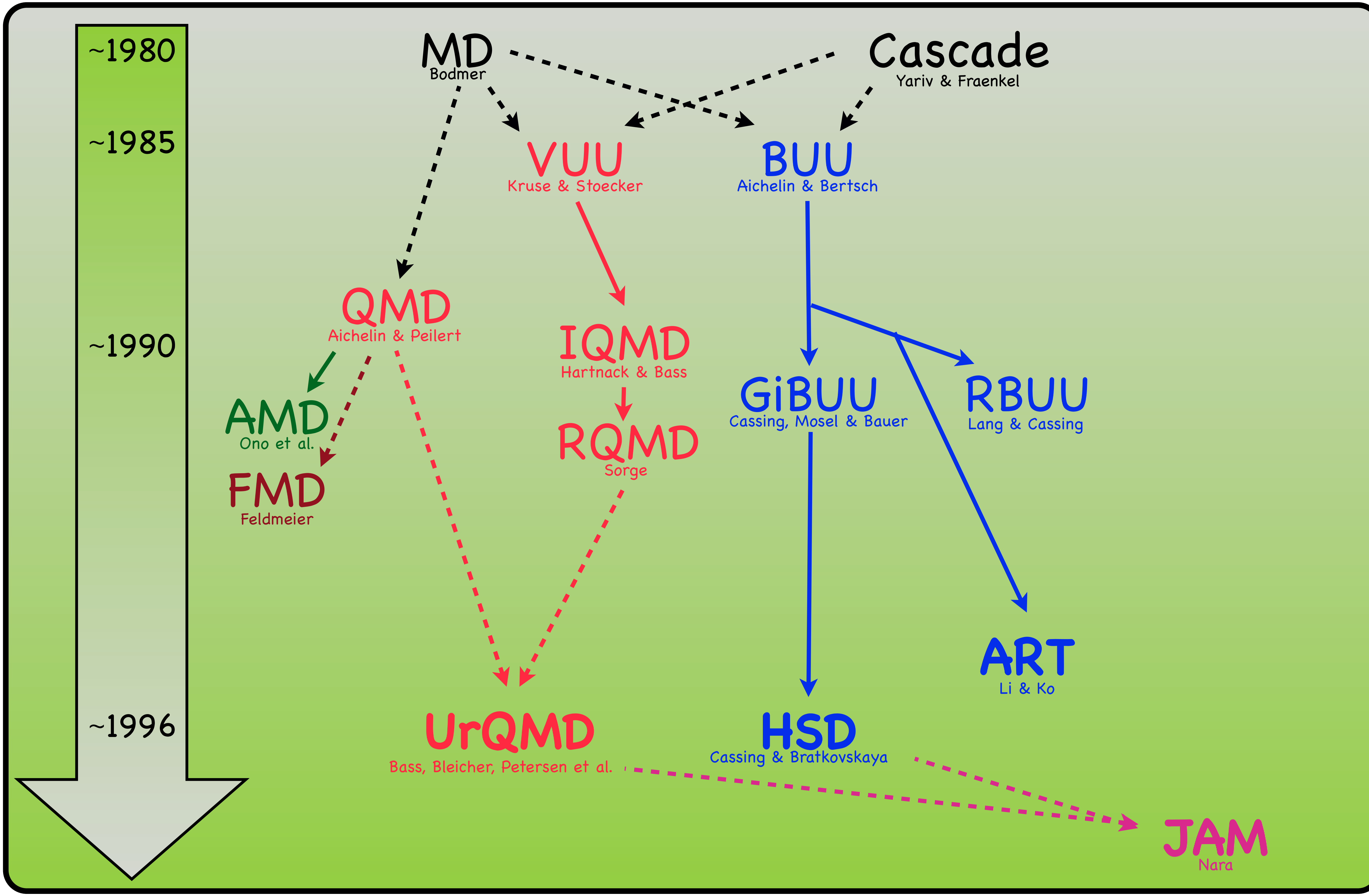
- The UrQMD Transport Model
- Infinite Matter
- Resonances out of Equilibrium
- Transport Coefficients: η/s

work supported through grants by

The UrQMD Transport Model

S.A. Bass et al.: Prog. Part. Nucl. Phys. **41**, 225 (1998)
M. Bleicher et al.: J. Phys. **G25**, 1895 (1999)

A Brief History of Hadronic Transport



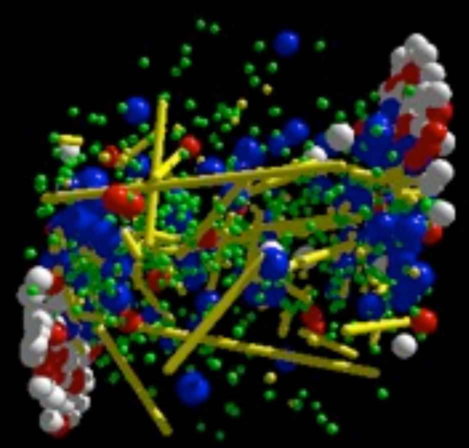
Describing Hadronic Matter: UrQMD

- elementary degrees of freedom: **hadrons**, const. (di)quarks
- classical trajectories in phase-space (relativistic kinematics): evolution of phase-space distribution via Boltzmann Equation:

$$\left[\frac{\partial}{\partial t} + \frac{\vec{p}}{m}\nabla_r\right] f^1 = \mathcal{C}_{\rm coll}$$

with $\mathcal{C}_{\rm coll} = N \int \sigma \mathrm{d}\Omega \int \mathrm{d}\vec{p}_2 \, |\vec{v}_1 - \vec{v}_2| \, [f_1(\vec{p}_1')f_1(\vec{p}_2') - f_1(\vec{p}_1)f_1(\vec{p}_2)]$

- initial high energy phase of the reaction is modeled via the excitation and fragmentation of strings
- 55 baryon- and 32 meson species, among those 25 N*, Δ* resonances and 29 hyperon/hyperon resonance species
- full baryon-antibaryon and isospin symmetry



main physics input and parameters:
- cross sections: total and partial cross sections, angular distributions
- resonance parameters: total and partial decay widths
- string fragmentation scheme: fragmentation functions, formation time

The UrQMD Hadron Gas: Constituents

- 55 baryon- and 32 meson species, among those 25 N*, Δ* resonances and 29 hyperon/hyperon resonance species
- magenta states are not contained in RQMD

Mesons:

0^{-+}	1^{--}	0^{++}	1^{++}
π	ρ	a_0	a_1
K	K^*	K_0^*	K_1^*
η	ω	f_0	f_1
η'	ϕ	f_0^*	f_1'
1^{+-}	2^{++}	$(1^{--})^*$	$(1^{--})^{**}$
b_1	a_2	ρ_{1450}	ρ_{1700}
K_1	K_2^*	K_{1410}^*	K_{1680}^*
h_1	f_2	ω_{1420}	ω_{1662}
h_1'	f_2'	ϕ_{1680}	ϕ_{1900}

Baryons:

N	Δ	Λ	Σ	Ξ	Ω
938	1232	1116	1192	1317	1672
1440	1600	1405	1385	1530	
1520	1620	1520	1660	1690	
1535	1700	1600	1670	1820	
1650	1900	1670	1790	1950	
1675	1905	1690	1775	2025	
1680	1910	1800	1915		
1700	1920	1810	1940		
1710	1930	1820	2030		
1720	1950	1830			
$1990^{\dagger}$		2100			
2080		2110			
2190					
2200					
2250					

The UrQMD Hadron Gas: Cross Sections

- the general form of the cross-section is given by:

$$\sigma_{1,2\to 3,4}(\sqrt{s}) \;=\; (2S_3+1)(2S_4+1)\,\frac{\langle p_{3,4}\rangle}{\langle p_{1,2}\rangle}\,\frac{1}{(\sqrt{s})^2}\,|\mathcal{M}(m_3,m_4)|^2$$

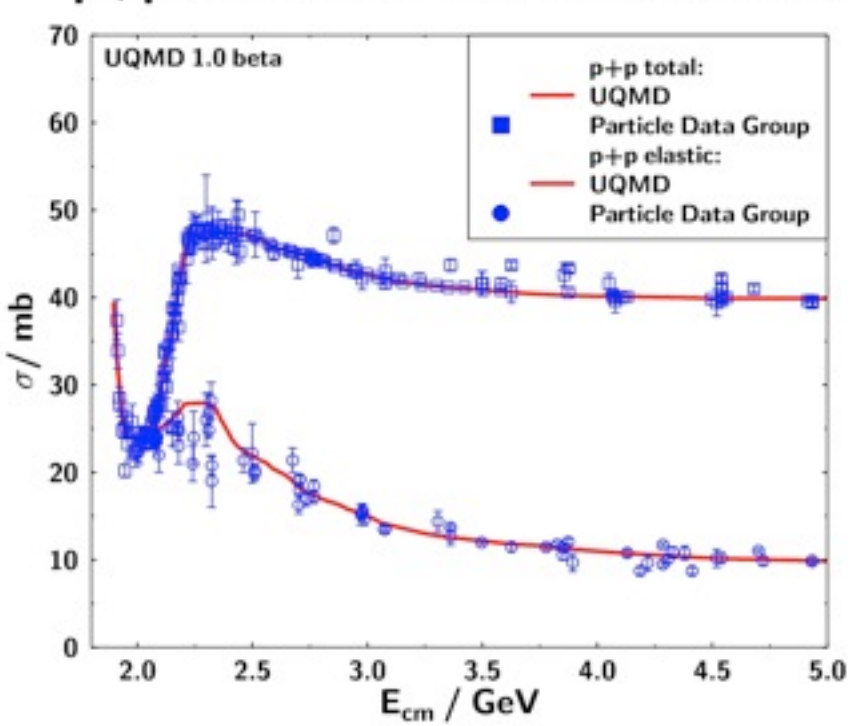
- spectral functions of resonances in the in/out channels are taken into account:

$$\langle p_{i,j}(M)\rangle \;=\; \int\!\!\int p_{CMS}(M,m_i,m_j)\,A_i(m_i)\,A_j(m_j)\,\mathrm{d}m_i\,\mathrm{d}m_j$$

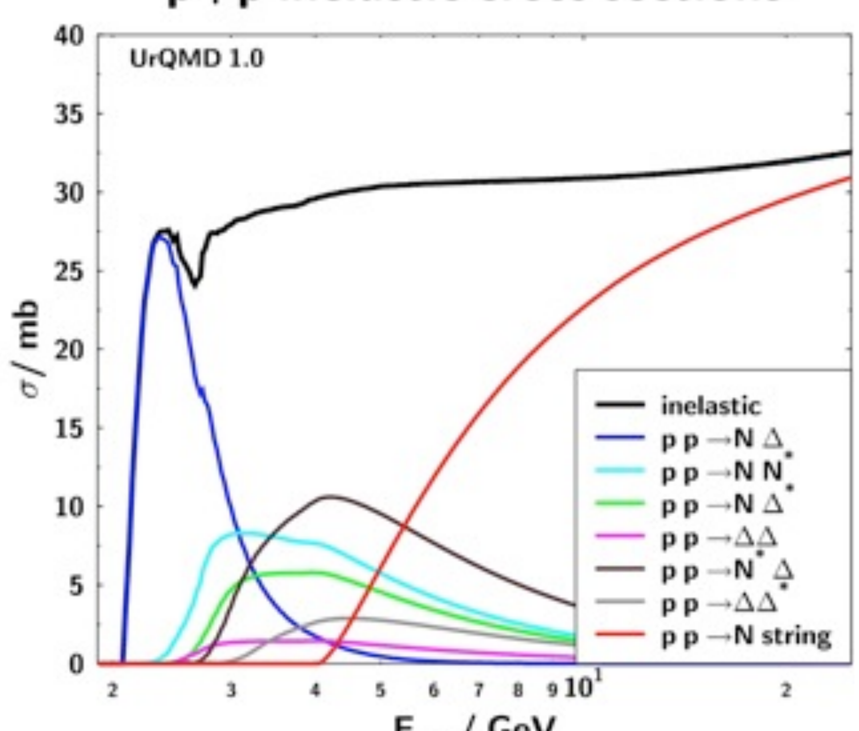
with $A_r(m) = \frac{1}{N}\frac{\Gamma(m)}{(m_r-m)^2+\Gamma(m)^2/4}$ and $\lim_{\Gamma\to 0} A_r(m) = \delta(m_r - m)$

- phase-space is treated correctly
- assumptions/fits have to be made for the matrix element $|\mathcal{M}(m_3,m_4)|^2$

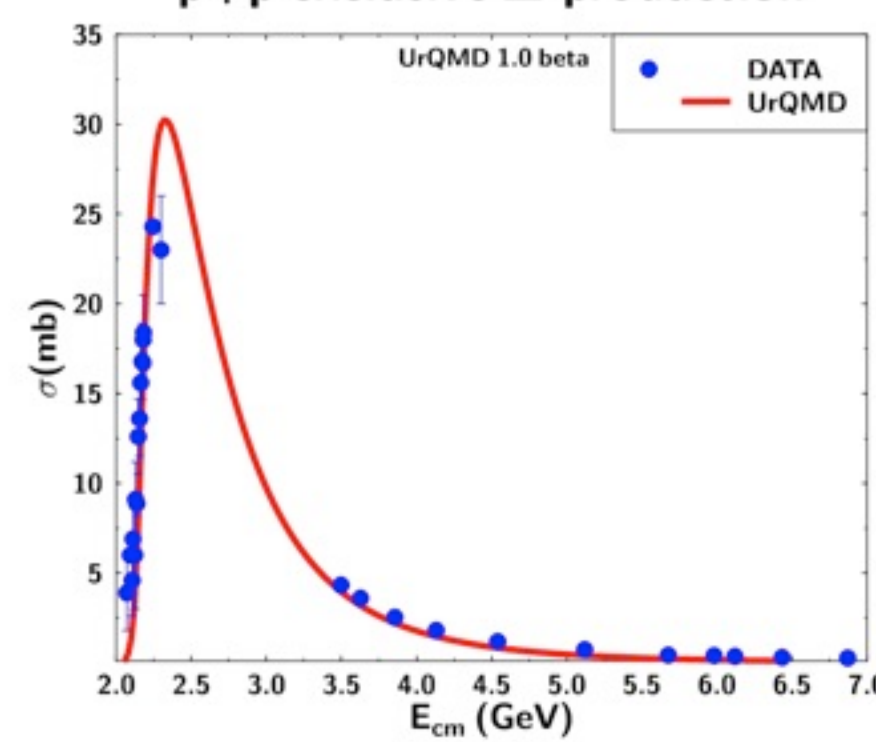
p+p total and elastic cross sections



p+p inelastic cross sections



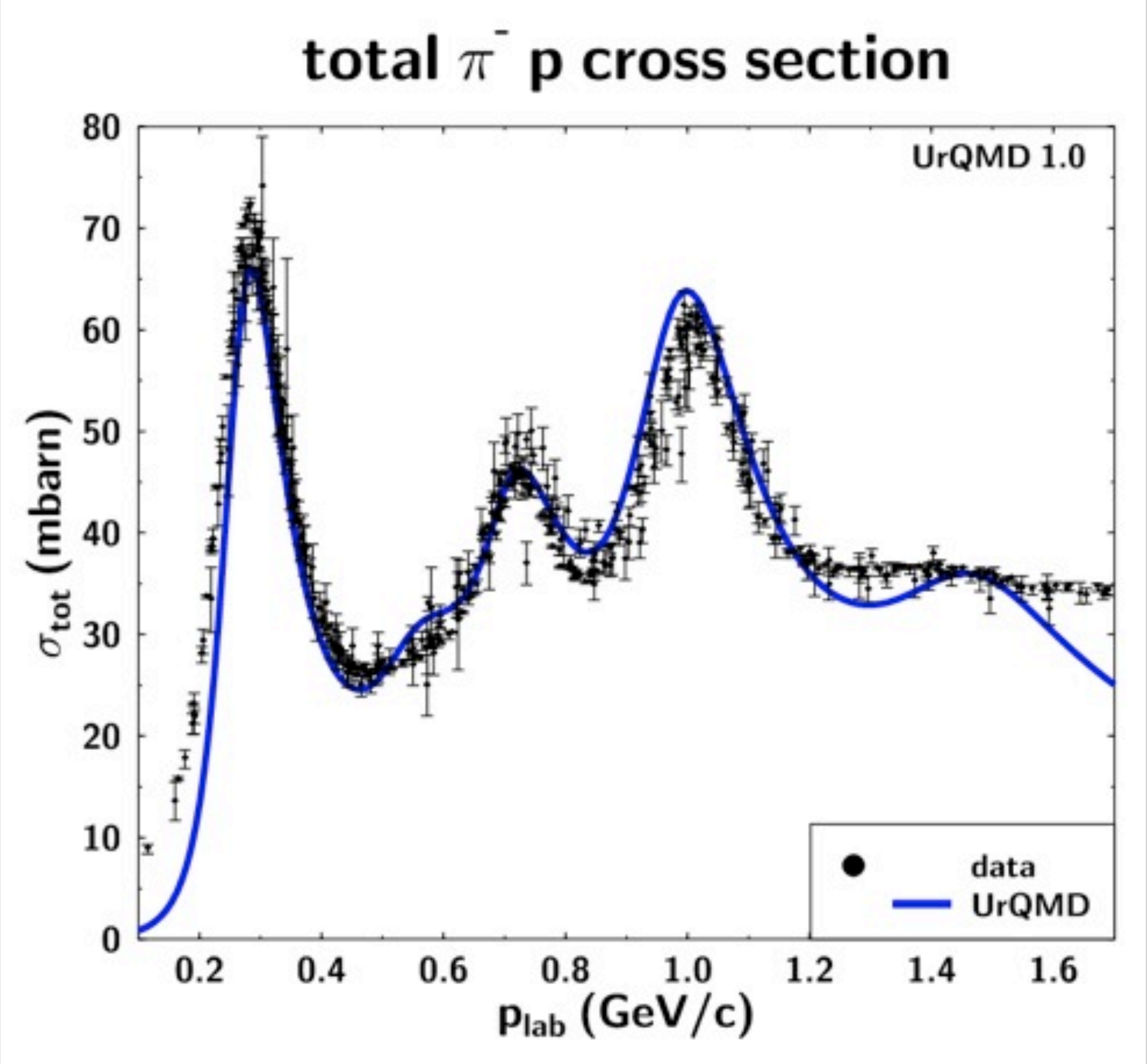
p+p exclusive Δ-production



The UrQMD Hadron Gas: MB Cross Sections

- all resonant cross sections are determined by the included resonances and their properties:

Δ^*	width	N^*	width
Δ_{1232}	120 MeV	N^*_{1440}	200 MeV
Δ_{1600}	350 MeV	N^*_{1520}	125 MeV
Δ_{1620}	150 MeV	N^*_{1535}	150 MeV
Δ_{1700}	300 MeV	N^*_{1650}	150 MeV
Δ_{1900}	200 MeV	N^*_{1675}	150 MeV
Δ_{1905}	350 MeV	N^*_{1680}	130 MeV
Δ_{1910}	250 MeV	N^*_{1700}	100 MeV
Δ_{1920}	200 MeV	N^*_{1710}	110 MeV
Δ_{1930}	350 MeV	N^*_{1720}	200 MeV
Δ_{1950}	300 MeV	N^*_{1990}	300 MeV

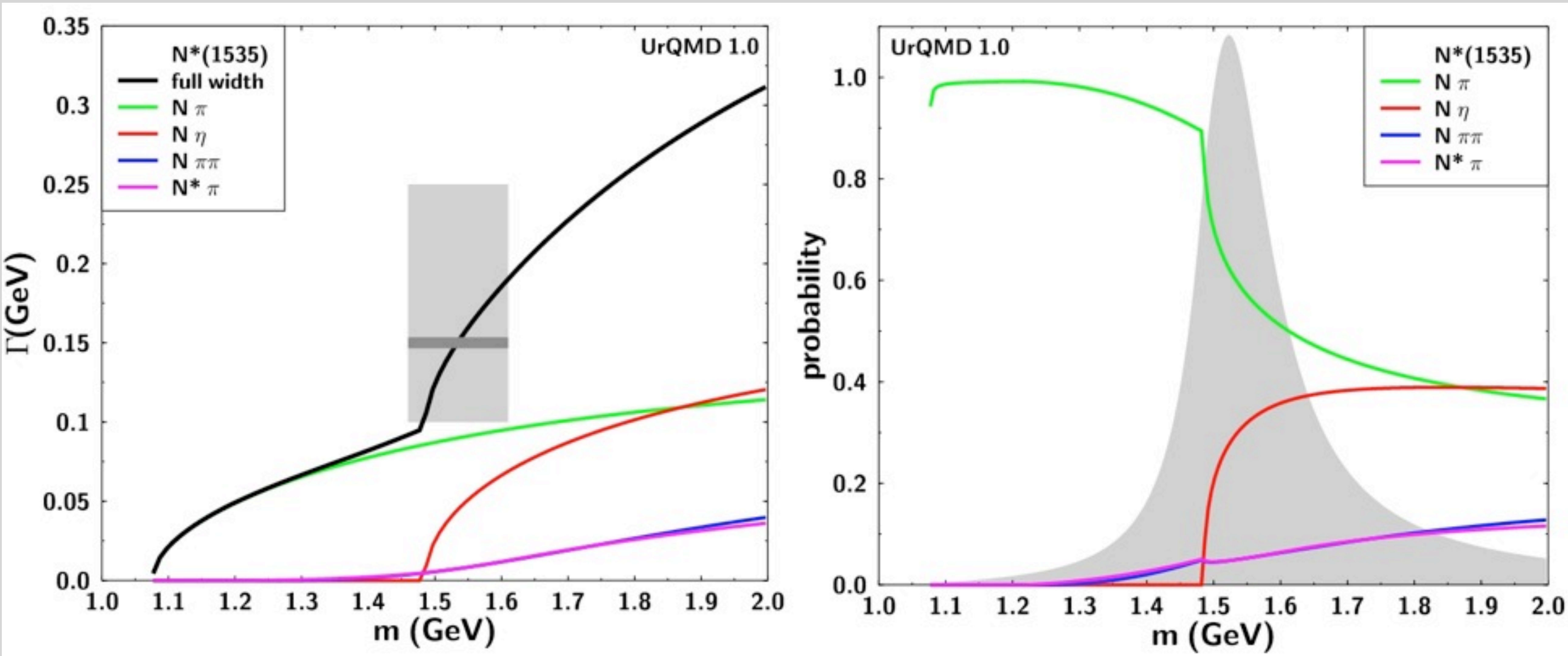


- calculate cross sections according to:

$$\sigma^{MB}_{tot} = \sum_{R=\Delta,N^*} \langle j_B, m_B, j_M, m_M \| J_R, M_R \rangle \frac{2I_R+1}{(2I_B+1)(2I_M+1)} \frac{\pi}{p^2_{CMS}} \frac{\Gamma_{R\to MB}\Gamma_{tot}}{(M_R-\sqrt{s})^2+\frac{\Gamma^2_{tot}}{4}}$$

The UrQMD Hadron Gas: Resonance-Widths

- all resonances have mass-dependent partial decay widths:



- the total decay width $\Gamma_{tot}(M)$ is defined as the sum over the partial widths $\Gamma_i(M)$:

$$\Gamma_{tot}(M) = \sum_{br=\{i.j\}}^{N_{br}} \left(\Gamma_R^{i,j} \frac{M_R}{M} \left(\frac{\langle p_{i,j}(M) \rangle}{\langle p_{i,j}(M_R) \rangle} \right)^{2l+1} \frac{1.2}{1 + 0.2 \left(\frac{\langle p_{i,j}(M) \rangle}{\langle p_{i,j}(M_R) \rangle} \right)^{2l}} \right)$$

Detailed Balance & Anti-Baryon Production

Detailed Balance:

for a given cross section dσ/dΩ(1,2→3,4) and entry channel (3,4) the principle of detailed balance can be used to calculate dσ/dΩ(3,4→1,2):

$$\frac{d\sigma}{d\Omega}(3,4 \to 1,2) = \langle j_3 m_3 j_4 m_4 \| JM \rangle^2 \cdot \lambda_{db} \cdot \frac{d\sigma}{d\Omega}(1,2 \to 3,4)$$

1) standard form (valid for stable particles):

$$\lambda_{db} = \frac{(2S_3+1)(2S_4+1)}{(2S_1+1)(2S_2+1)} \sum_{J=J_-}^{J_+} \langle j_1, j_2, m_1, m_2 \| J, M \rangle \frac{p_{1,2}^2}{p_{3,4}^2}$$

2) modified form for resonances:

$$p_{3,4}^2 \Rightarrow \langle p_{3,4}^2 \rangle = \int_{m_N+m_\pi}^{\sqrt{s}-(m_N+m_\pi)} \int_{m_N+m_\pi}^{\sqrt{s}-m_4} p_{CMS}^2(\sqrt{s}, m_3, m_4) A_3(m_3) A_4(m_4)\, dm_3\, dm_4$$

with $A_r(m) = \frac{1}{2\pi} \frac{\Gamma}{(m_r - m)^2 + \Gamma^2/4}$

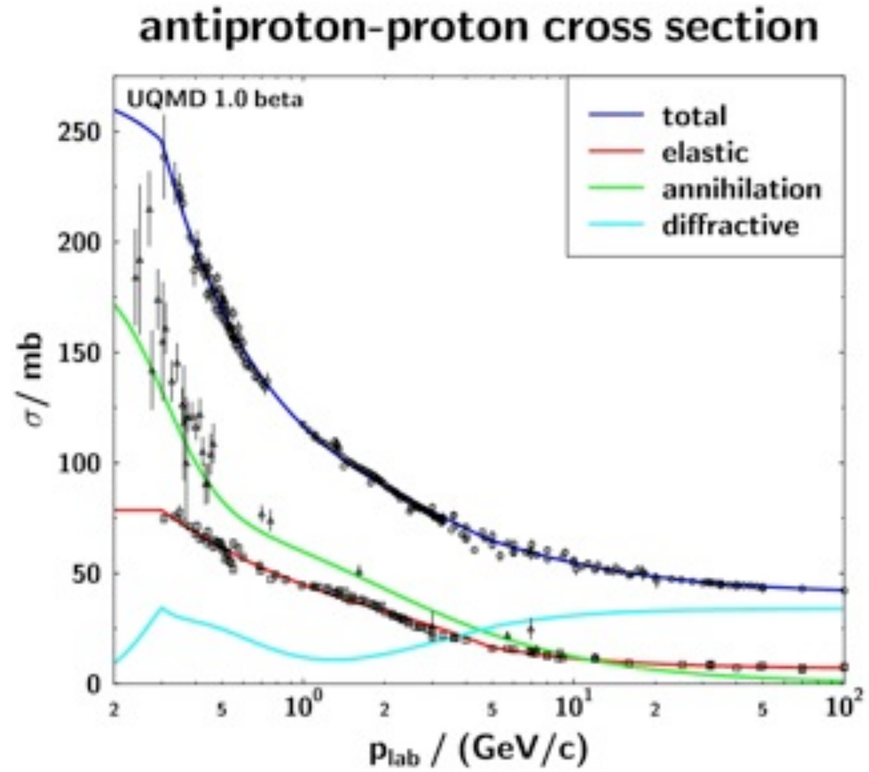
- to ensure detailed balance, all processes with multi-particle exit channels must be disabled (e.g. 2→3 scattering)

Anti-Baryons in Infinite Matter:

- produced via string fragmentation, violates detailed balance & is disabled
- annihilation produces a multi-particle final state & violates detailed balance
- solution: produce & annihilate anti-baryons via an effective 2↔2 process:

$$p + \bar{p} \leftrightarrow \rho + \omega(1420)$$

- subsequent decay of ρ & ω(1420) produce desired multi-particle state



UrQMD: Infinite Matter

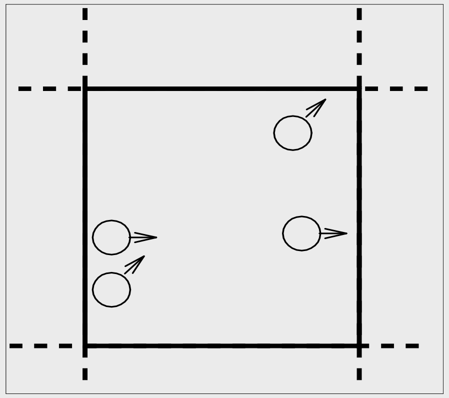
S.A. Bass et al.: Prog. Part. Nucl. Phys. **41**, 225 (1998)

M. Bleicher et al.: J. Phys. **G25**, 1895 (1999)

The UrQMD Hadron Gas: Infinite Matter

Strategy: confine UrQMD to box with periodic boundary conditions

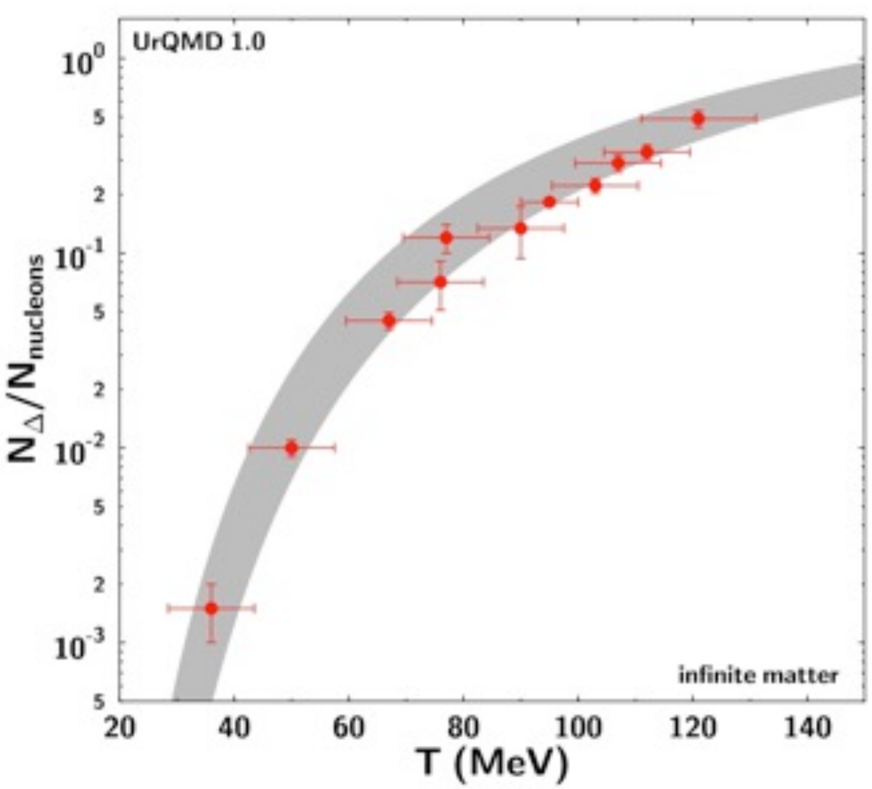
- system will evolve into equilibrium state (no freeze-out occurs)
- need to disable multi-body processes to maintain detailed balance
- example: π-N-Δ(1232) system



chemical equilibrium:

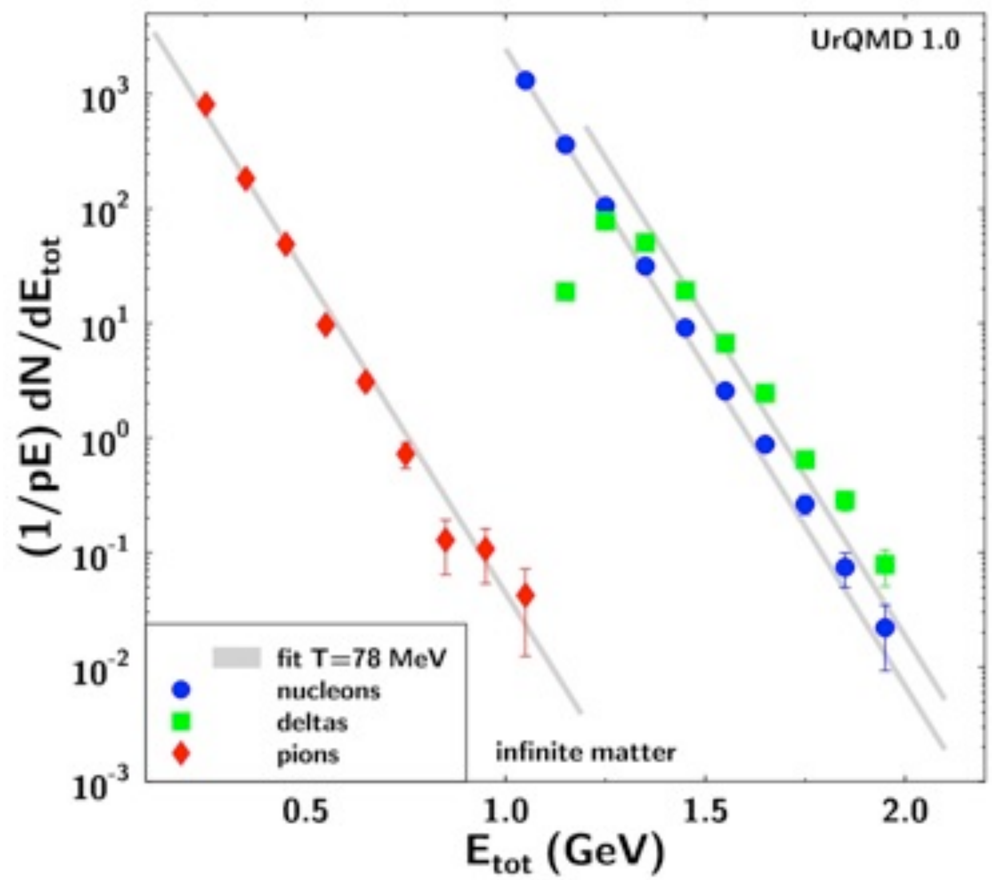
- fit to Statistical Model can be used to extract μ_i
- in non-relat. Boltzmann approx., Δ/N ratio is:

$$\frac{N_\Delta}{N_N} = \frac{g_\Delta}{g_N}\left(\frac{m_\Delta}{m_N}\right)^{\frac{3}{2}} \exp\left(\frac{m_N - m_\Delta}{T}\right)$$

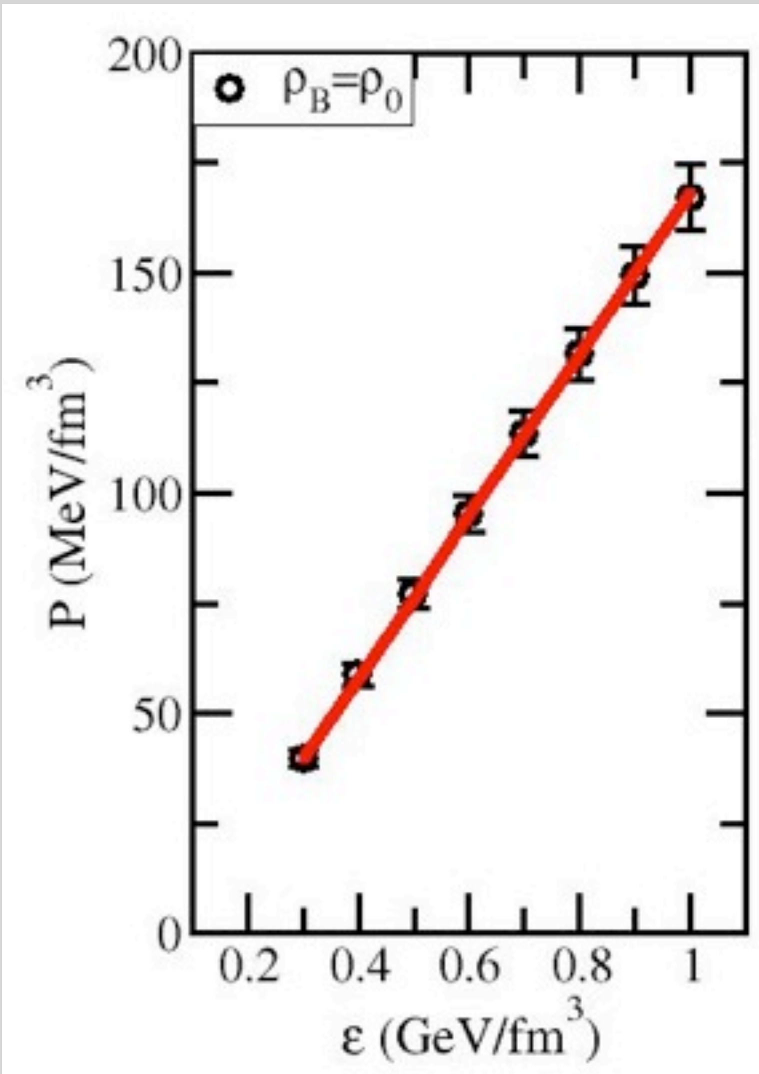


kinetic equilibrium:

- isotropy of momentum distributions
- use energy spectrum to extract



Infinite Matter: Equation of State

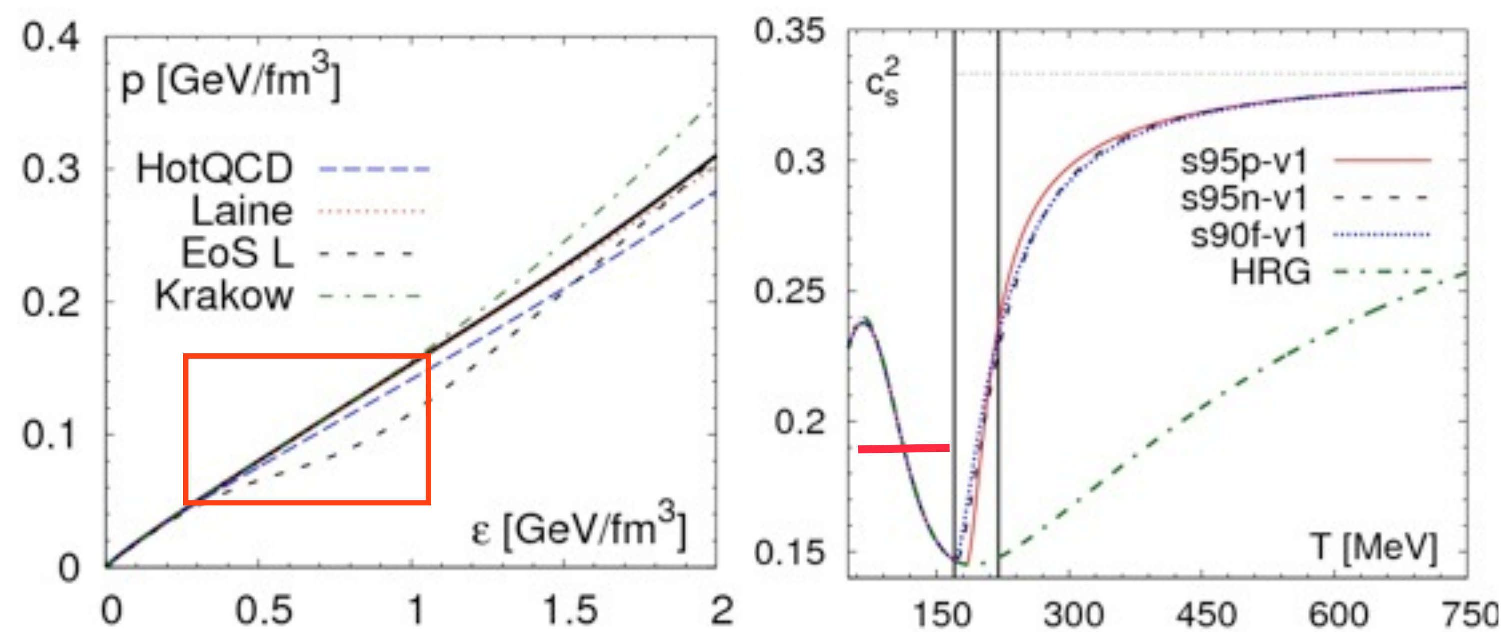


determine speed of sound c_s, using pressure and energy-density:

$$c_s^2 = \left(\frac{\partial P}{\partial \epsilon}\right)$$

- analysis yields $c_s^2=0.18$

Hadron Resonance Gas Reference:

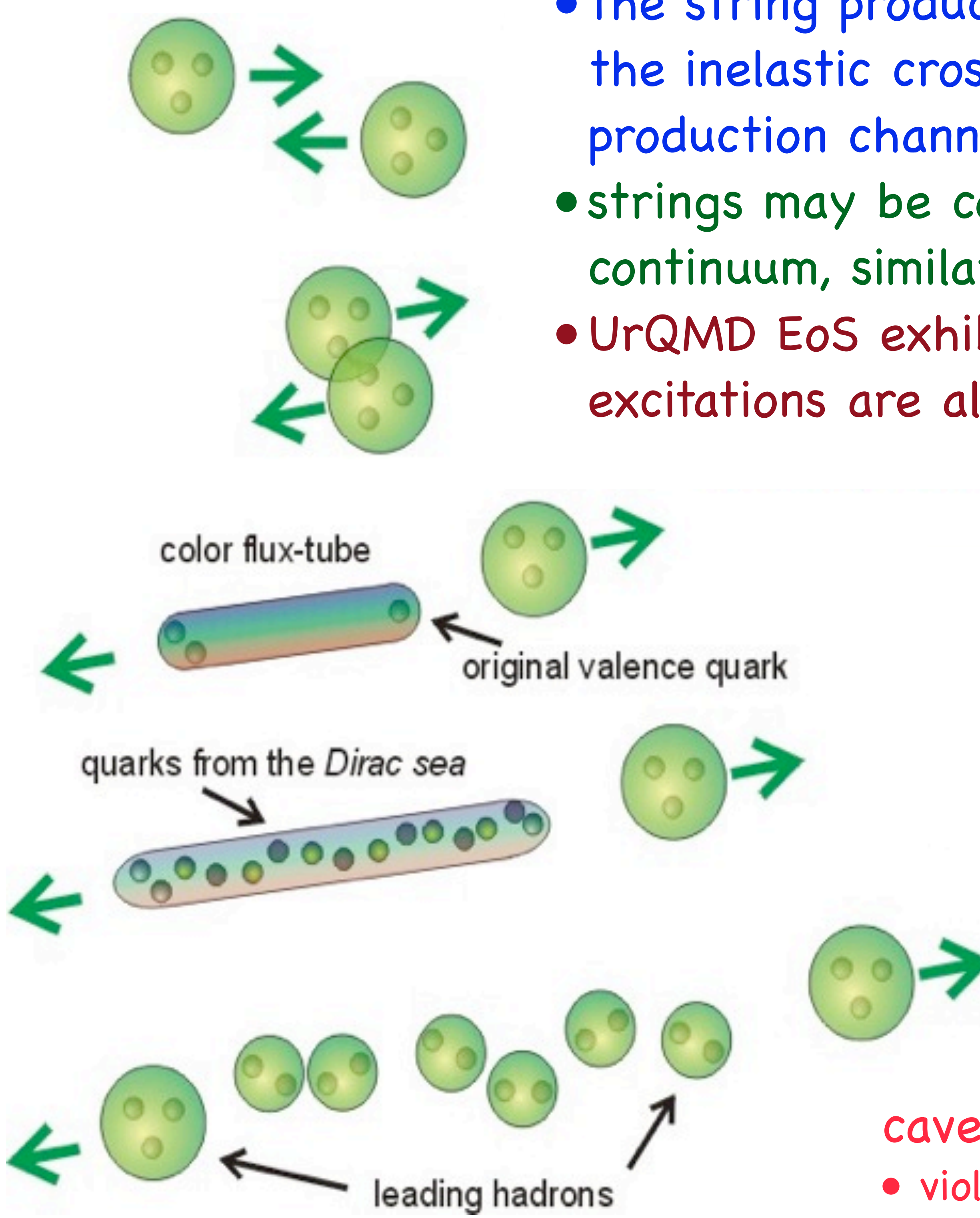


- UrQMD EoS exhibits same bulk behavior as HRG & Lattice EoS
- speed of sound extraction in UrQMD currently too coarse-grained to resolve variation in c_s vs. temperature

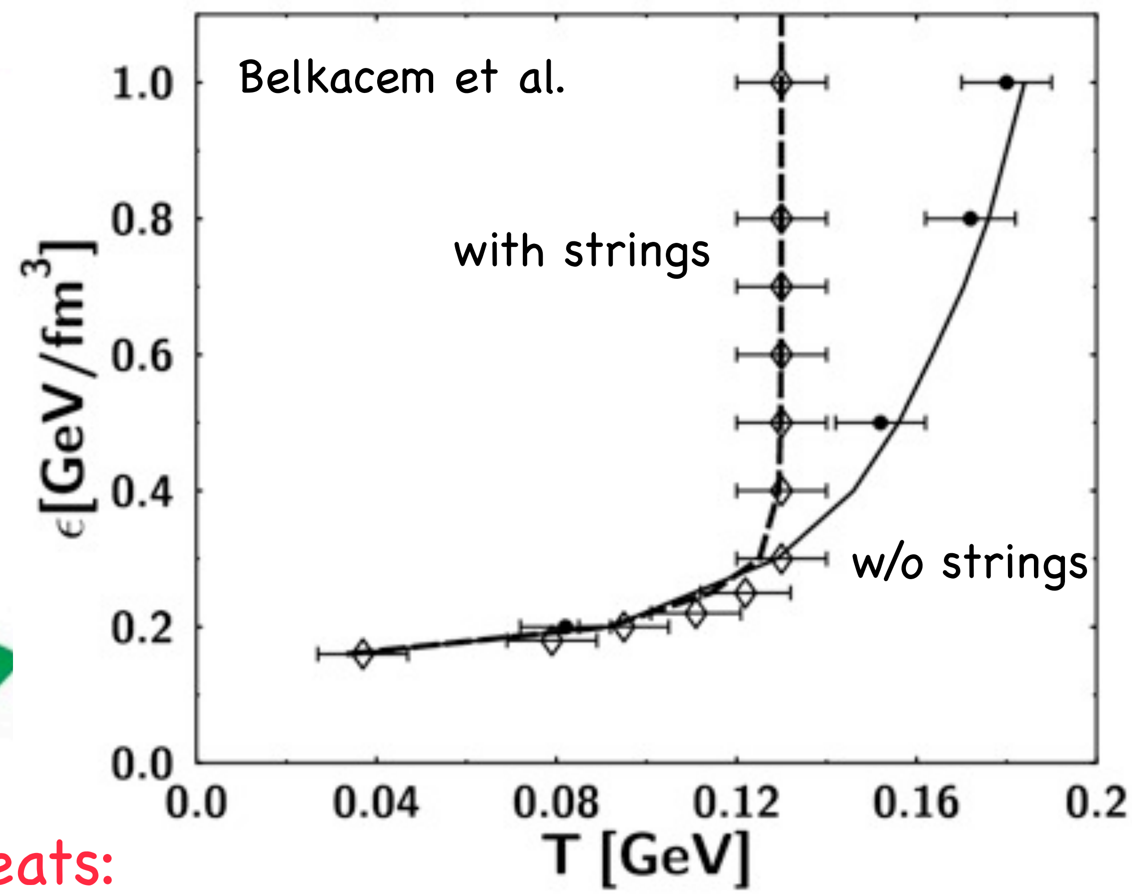
P. Huovinen & P. Petreczky: Nucl. Phys. **A837** (2010) 26

UrQMD can be used as an effective model of a HRG, but is not constrained by equilibrium physics

Strings: Hagedorn-like behavior



- the string production cross section in UrQMD fills up the inelastic cross section above the discrete resonance production channels
- strings may be considered to be populating a resonance continuum, similar to Hagedorn States...
- UrQMD EoS exhibits a limiting temperature if string excitations are allowed to take place



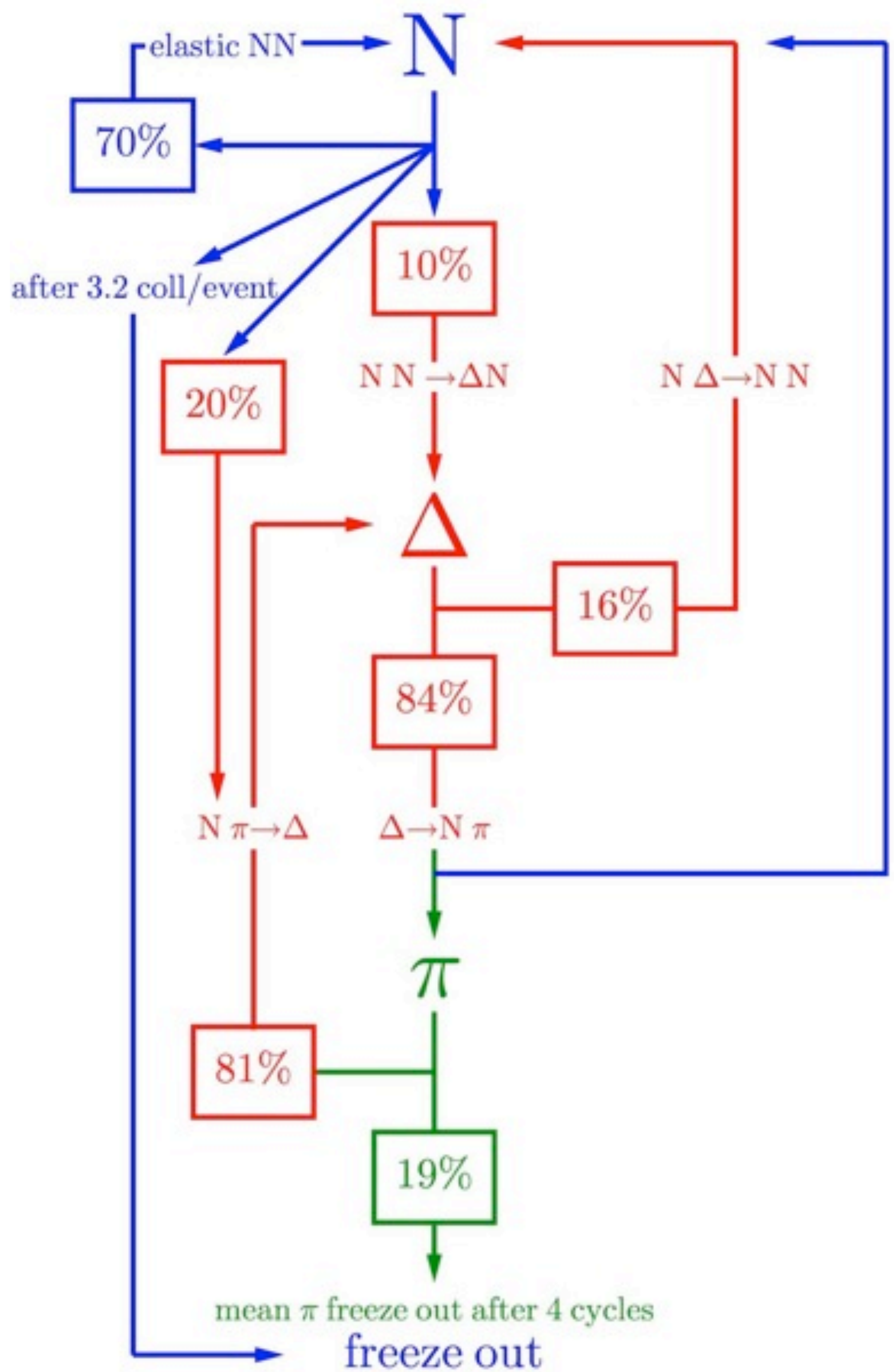
caveats:

- violation of detailed balance in calculation
- "wrong" limiting temperature: signature of missing discrete resonance states?

Resonances out of Equilibrium: Hadro-Chemistry in Heavy-Ions

S.A. Bass et al. Phys. Lett. **B335** (1994) 289
S.A. Bass et al.: Prog. Part. Nucl. Phys. **41** (1998) 225
S.A. Bass et al. Prog. Part. Nucl. Phys. **42** (1999) 313

Example: the N-Δ-π Cycle



Au+Au Collisions @ 1 GeV/nucleon:

- relevant hadronic degrees of freedom: nucleon, Δ(1232) & pion

Particle Production/Absorption:

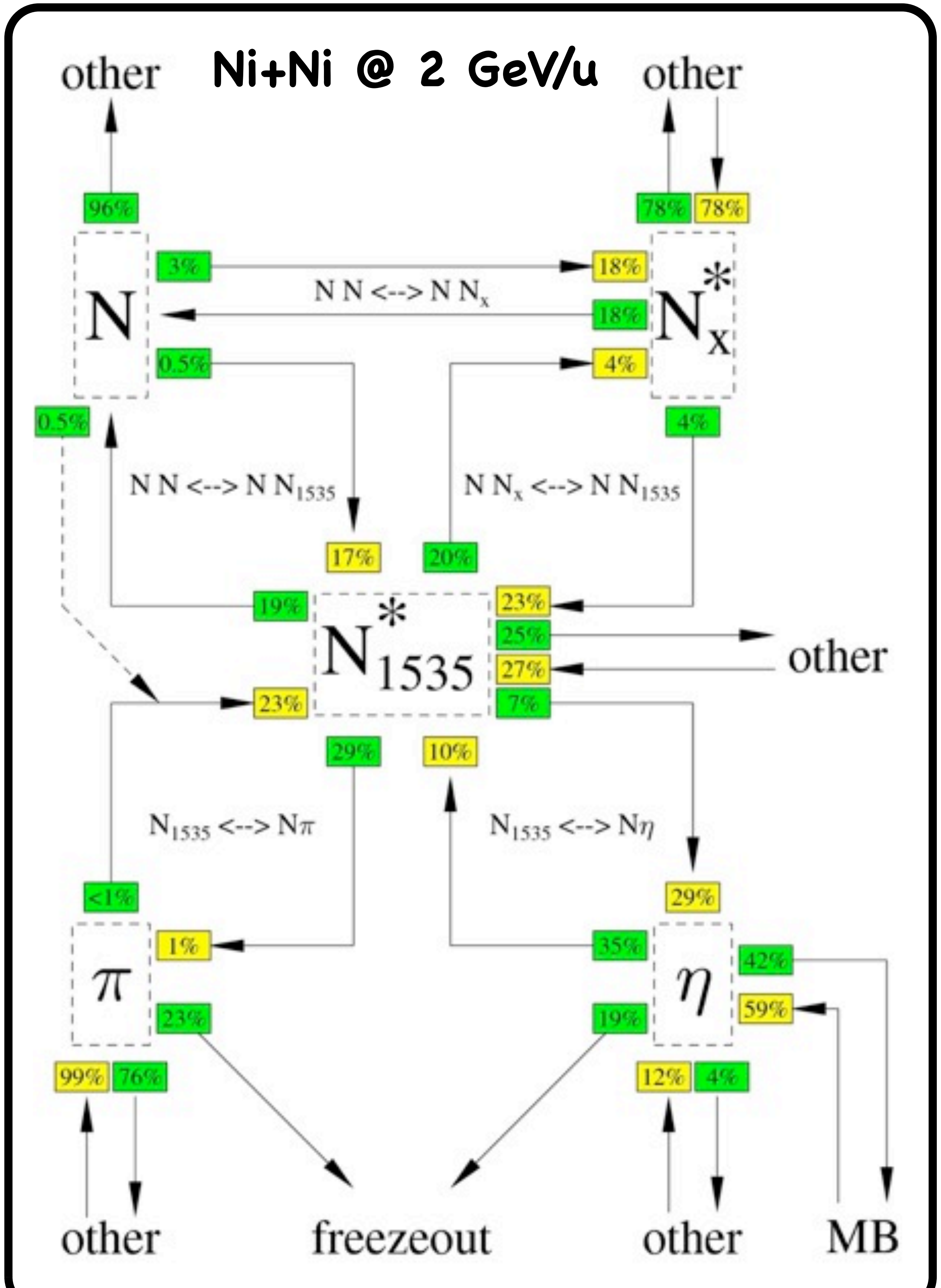
- pion production via Δ(1232)→N+π
- pion absorption requires two steps: π+N→Δ(1232) & Δ(1232)+N→N+N

- Δ(1232) plays a crucial role in the particle production dynamics
- the average pion goes through approx. 4 Δ-cycles before freeze-out!

Noteworthy:

- resonances allow for particle production below the N+N threshold for the respective hadron species
- the regions of the Δ(1232) spectral function probed during the reaction may change as a function of time

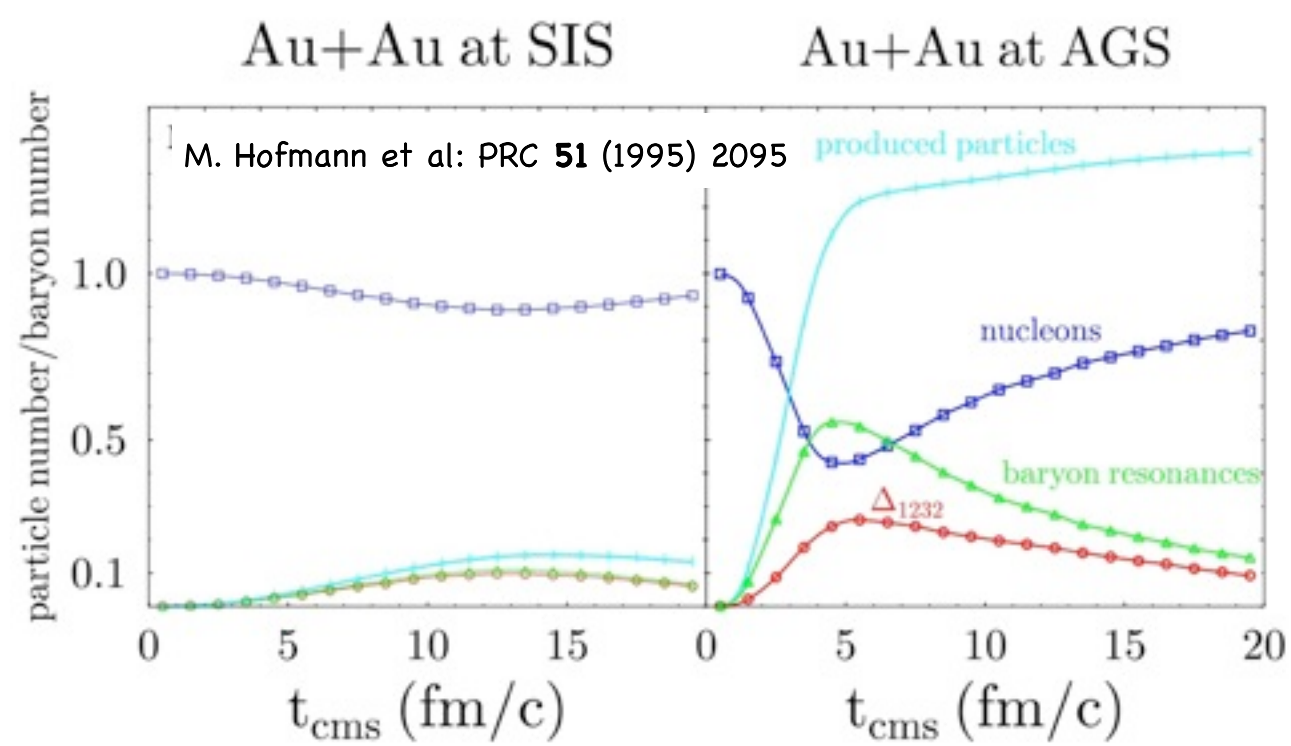
Resonance Matter



higher beam-energy leads to the excitation of more resonance degrees of freedom:

- example: π-Δ(1232)-N-N*(1535)-η system in Ni+Ni collisions at 2 GeV/nucleon
- complex dynamics with multiple gain/loss reactions

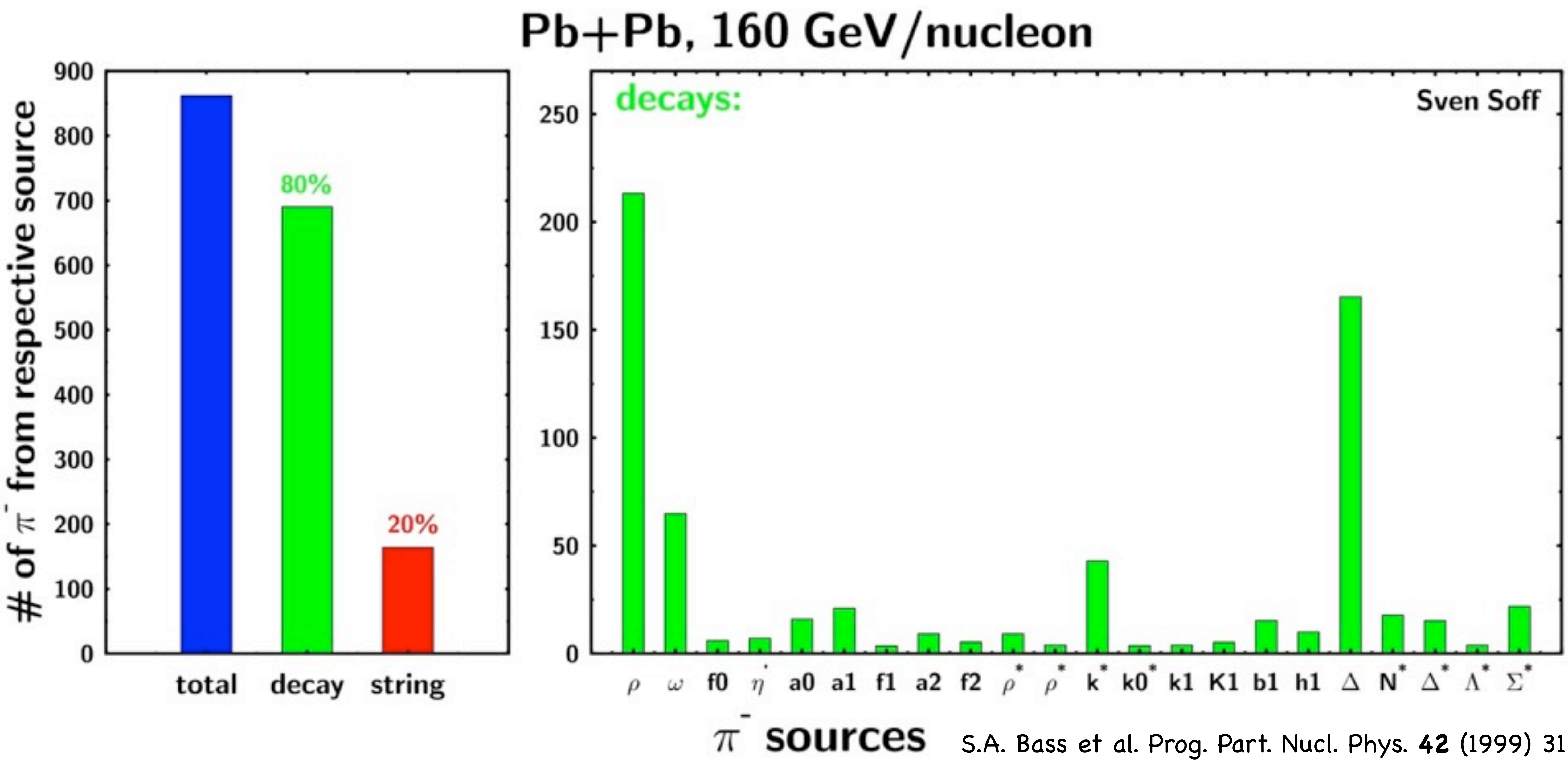
- in the high-density phase of Au+Au collisions at AGS energies, more baryons are in resonance states than in the ground state: **Resonance Matter**



From Baryon- to Meson-Dominated Matter

at higher beam energies the meson multiplicity overtakes the baryon multiplicity:

- meson absorption via baryon resonances becomes ineffective
- dominant resonances in the system e.g. at RHIC are: ρ(770) and K*, however, many other resonances will contribute to the pion-source
- ‣ need exclusive decay channels to probe specific resonances...



S.A. Bass et al. Prog. Part. Nucl. Phys. **42** (1999) 313

η/s of a Hadron Gas

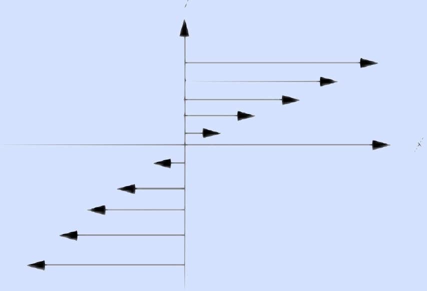
N. Demir & S.A. Bass: Phys. Rev. Lett. **102**, 172302 (2009)

Shear Viscosity: Linear Transport Equation & Green - Kubo Formalism

Mechanical definition of shear viscosity:

- application of a shear force to a system gives rise to a non-zero value of the xy-component of the pressure tensor P_{xy}. P_{xy} is then related to the velocity flow field via the shear viscosity coefficient η:

$$P_{xy} = -\eta \frac{\partial v_x}{\partial y}$$



- a similar linear transport equation can be defined for other transport coefficients: thermal conductivity, diffusion ...

- using linear-response theory, the **Green-Kubo relations** for the shear viscosity can be derived, expressing η as an integral of an near-equilibrium time correlation function of the stress-energy tensor:

$$\eta = \frac{1}{T} \int d^3r \int_0^\infty dt \left\langle \pi^{xy}(\vec{0},0)\, \pi^{xy}(\vec{r},t) \right\rangle_{\text{equil}}$$

with the stress-energy tensor: $\pi^{\mu\nu}(\vec{r},t) = \int d^3p \frac{p^\mu p^\nu}{p^0} f(x,p)$

Microscopic Transport: η/s of a Hadron Gas

- for particles in a fixed volume, the stress energy tensor discretizes

$$\pi^{xy} = \frac{1}{V} \sum_{j=1}^{N_{\rm part}} \frac{p^x(j) p^y(j)}{p^0(j)}$$

- and the Green-Kubo formula reads:

$$\eta = \frac{V}{T} \int_0^\infty dt \, \langle \pi^{xy}(0)\, \pi^{xy}(t) \rangle$$

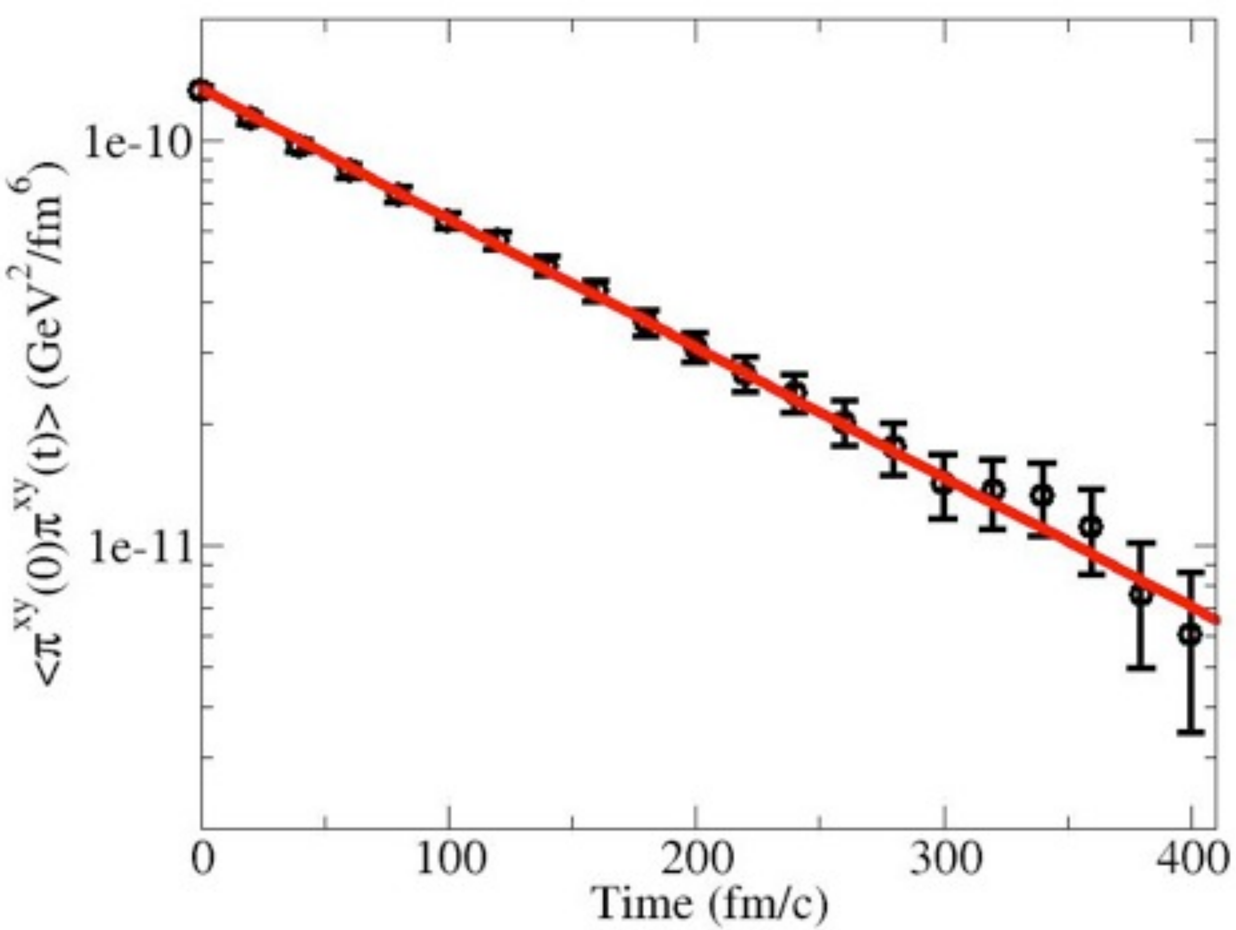
Entropy:

- extract thermodynamic quantities via:

$$P = \frac{1}{3V} \sum_{j=1}^{N_{\rm part}} \frac{|\vec{p}|^2(j)}{p^0(j)} \qquad \epsilon = \frac{1}{V} \sum_{j=1}^{N_{\rm part}} p^0(j)$$

- use Gibbs relation (with chem. pot. extratced via SM)

$$s_{\rm Gibbs} = \left(\frac{\epsilon + P - \mu_i \rho_i}{T} \right)$$



- evaluating the correlator numerically, e.g. in UrQMD, one empirically finds an exponential decay as function of time
- using the following ansatz, one can extract the relaxation time τ_π:

$$\langle \pi^{xy}(0)\, \pi^{xy}(t) \rangle \propto \exp\left(-\frac{t}{\tau_\pi} \right)$$

- the shear viscosity then can be calculated from known/extracted quantities:

$$\eta = \tau_\pi \frac{V}{T} \langle \pi^{xy}(0)^2 \rangle$$

A. Muronga: Phys. Rev. **C69**: 044901, 2004

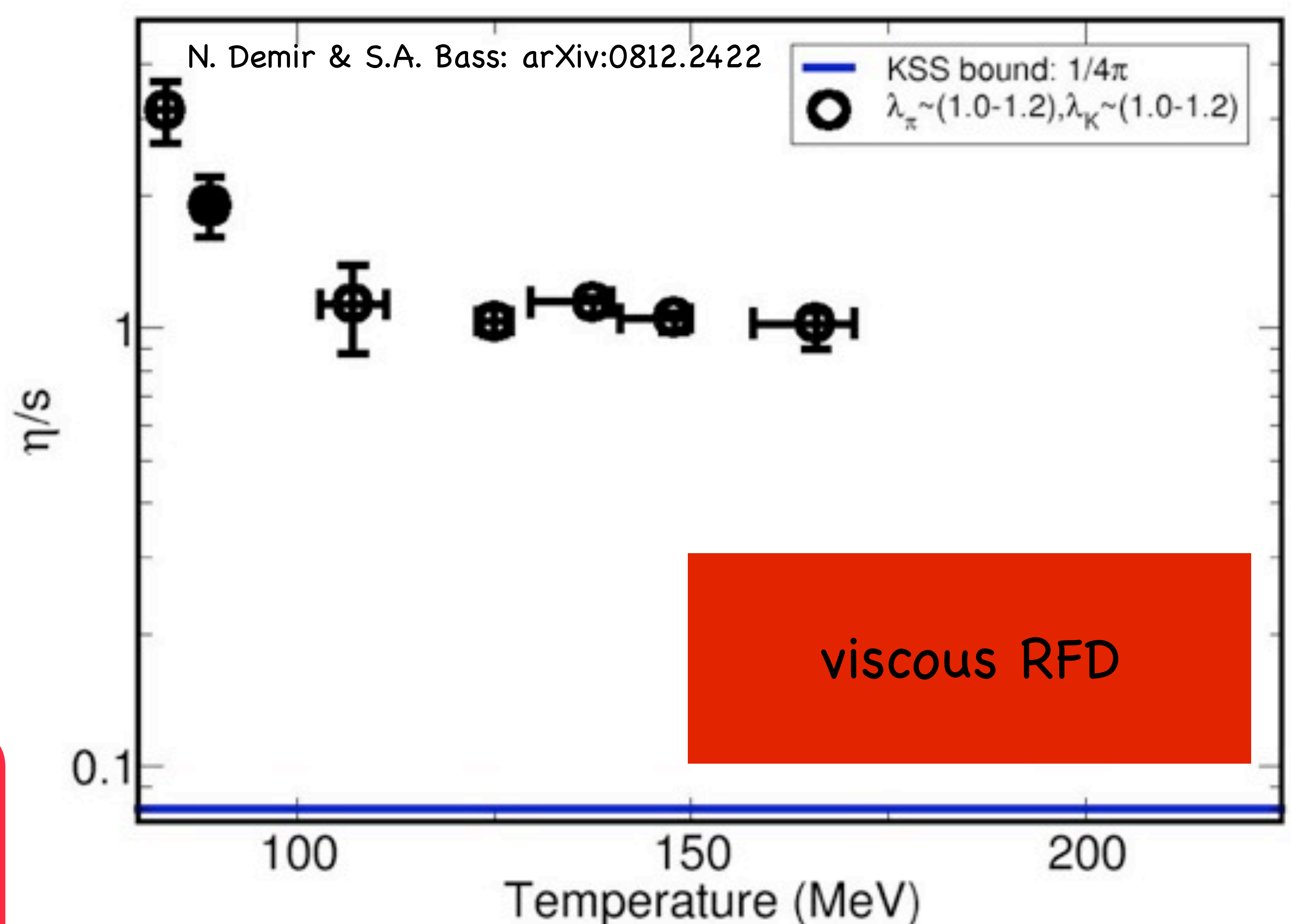
η/s of a Hadron Gas in & out of Equilibrium

first reliable calculation of of η/s for a full hadron gas including baryons and anti-baryons:

- breakdown of vRFD in the hadronic phase?
- what are the consequences for η/s in the deconfined phase?

- RFD freeze-out temperature to reproduce spectral shapes: ~110 MeV
- Statistical Model temperature fits to hadron yields/ratios: ~160 MeV
  - separation of chemical and kinetic freeze-out in the hadronic phase!
  - confirmed by hybrid models
  - implies non-unit species-dependent fugacities in RFD

- non-unit fugacities reduce η/s by a factor of two to η/s≈0.5
  - **improved constraint:** η/s needs to be significantly lower in deconfined phase for vRFD to reproduce elliptic flow!



T. Hirano & K. Tsuda: Nucl. Phys. **A715**, 821 (2003)
P.F. Kolb & R. Rapp: Phys. Rev. **C67**, 044903 (2003)

Summary

- The UrQMD Transport Model:
  - study of hadronic systems in/out of equilibrium
  - hadronic "afterburner" to QGP evolution models
- Infinite Matter:
  - microscopic model of a hadron gas
- Resonances out of Equilibrium:
  - catalysts for particle production
  - drivers for meson absorption in matter
- Transport Coefficients: η/s
  - improved constraints for QGP η/s
  - transport model allows for calculations not feasible in analytic approaches

work supported through grants by


That's it!
Any questions?