# *the "quark model" and beyond - informed by lattice QCD*

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*&*

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based upon Hadron Spectrum Collaboration results in

Phys. Rev. Lett.103 262001 (2009)
Phys. Rev. D 82 034508 (2010)

JJD, Robert Edwards, Mike Peardon,
David Richards, Christopher Thomas

# the "constituent quark model"

the simplest quantum-mechanical description of meson flavor & $J^{PC}$ systematics

a statement of the apparent degrees-of-freedom in the spectrum

isospin ≤ 1
|strangeness| ≤ 1

⇒

$q(I_z,S) = \{u(½,0), d(-½,0), s(0,-1)\}$

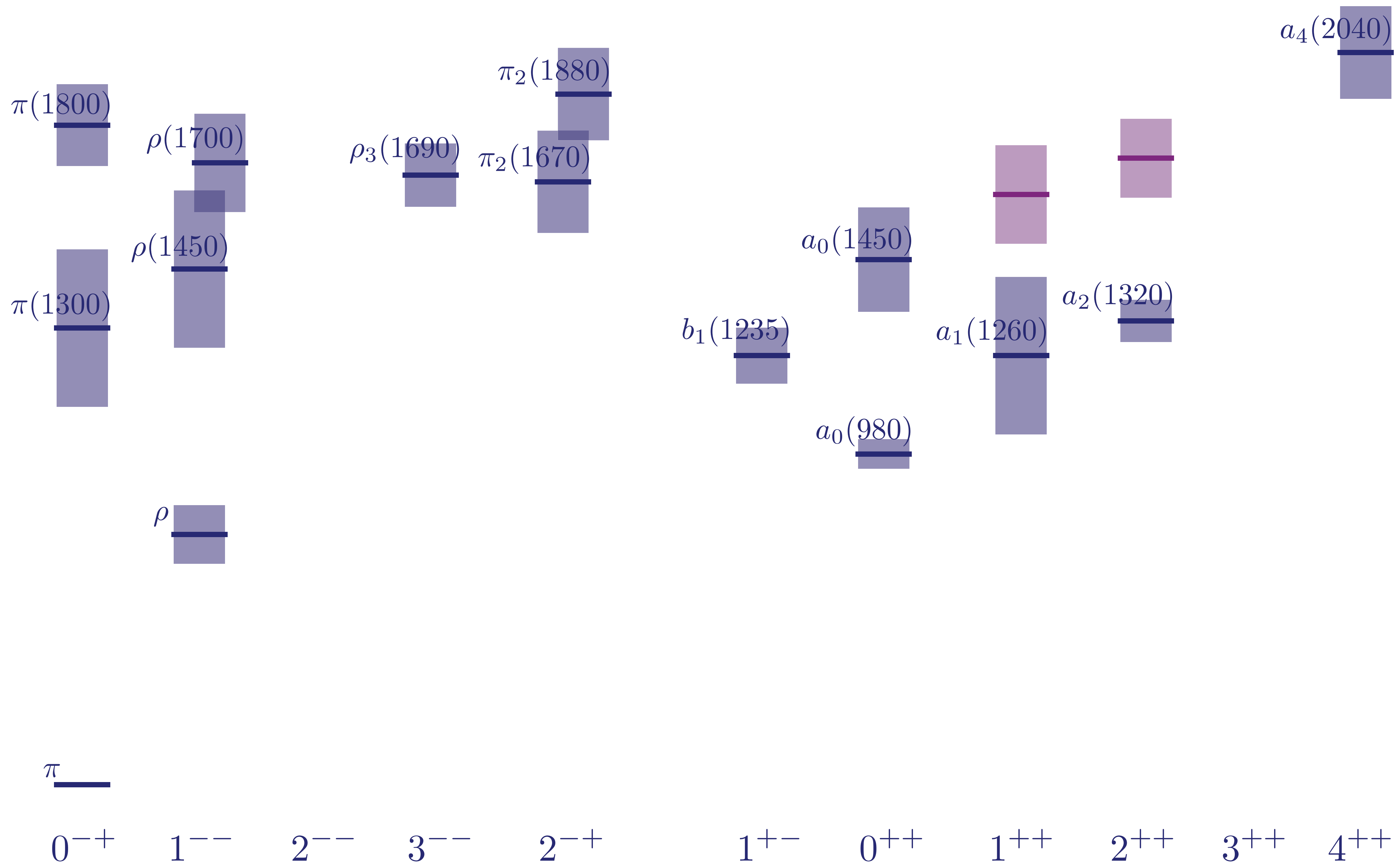
$M = q\bar{q}$

& generically $M \neq qq\bar{q}\bar{q}$

# isovector meson spectrum

"naive" reading of PDG summary table

not always
a good idea



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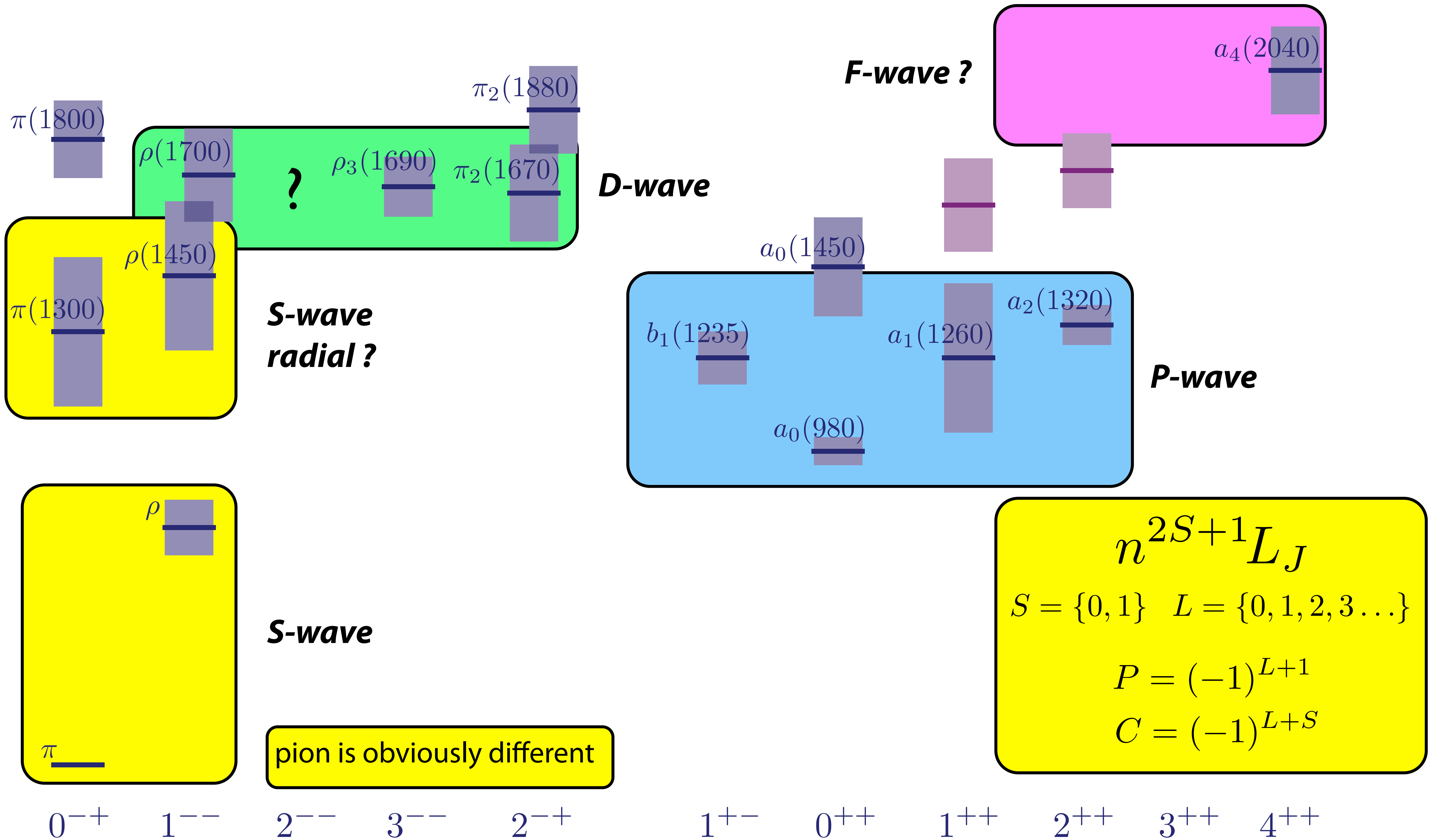

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$$[0^{-+}, 1^{--}] < [1^{+-}, (0,1,2)^{++}] < [2^{-+}, (1,2,3)^{--}]$$

&

absence of $0^{--}, 0^{+-}, 1^{-+}, 2^{+-}$

"exotic quantum numbers"

⇒

$q$ as spin-½ "heavy" d.o.f.
& orbital angular momentum

$$n^{2S+1}L_J$$

$S = \{0, 1\}$ $L = \{0, 1, 2, 3 \ldots\}$

$$P = (-1)^{L+1}$$
$$C = (-1)^{L+S}$$

will not consider details of the dynamics of these d.o.f.

# "predicting" kaons in the quark model

kaons as $\ell\bar{s}$

charge conjugation no longer good quantum number

mixtures of opposite *C* :

$$J^P = \begin{cases} J^{P+} \\ J^{P-} \end{cases}$$

$$0^- = \begin{cases} 0^{-+} \\ 0^{--} \end{cases}$$

$^1S_0$

exotic

$$1^+ = \begin{cases} 1^{++} \\ 1^{+-} \end{cases}$$

$^3P_1$

$^1P_1$

★ "doubled state"

$$1^- = \begin{cases} 1^{-+} \\ 1^{--} \end{cases}$$

exotic

$^3S_1$

$$2^+ = \begin{cases} 2^{++} \\ 2^{+-} \end{cases}$$

$^3P_2$

exotic

$$0^+ = \begin{cases} 0^{++} \\ 0^{+-} \end{cases}$$

$^3P_0$

exotic

$$2^- = \begin{cases} 2^{-+} \\ 2^{--} \end{cases}$$

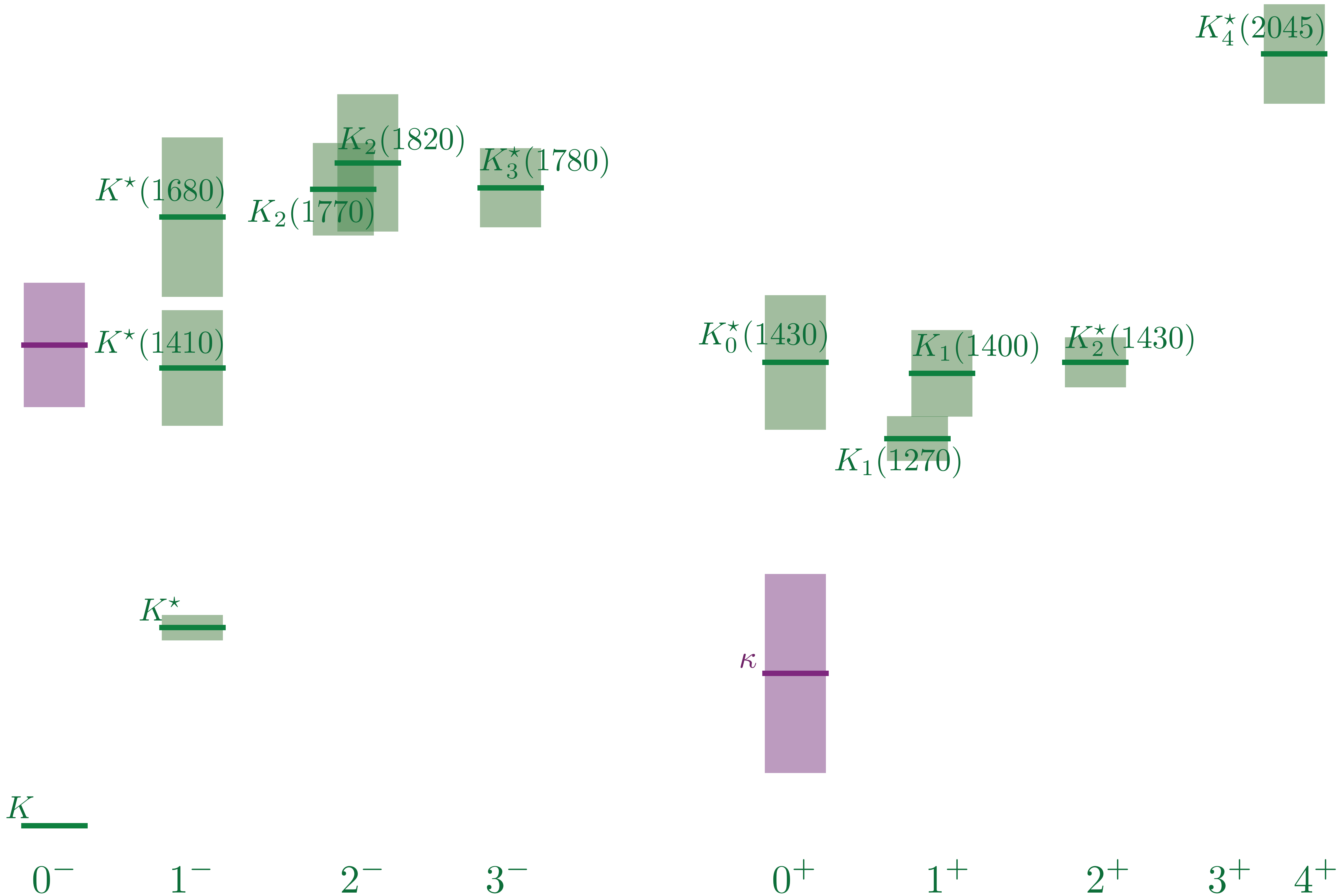
$^1D_2$

$^3D_2$

★ "doubled state"

# kaon meson spectrum

"naive" reading of PDG summary table



# kaon meson spectrum

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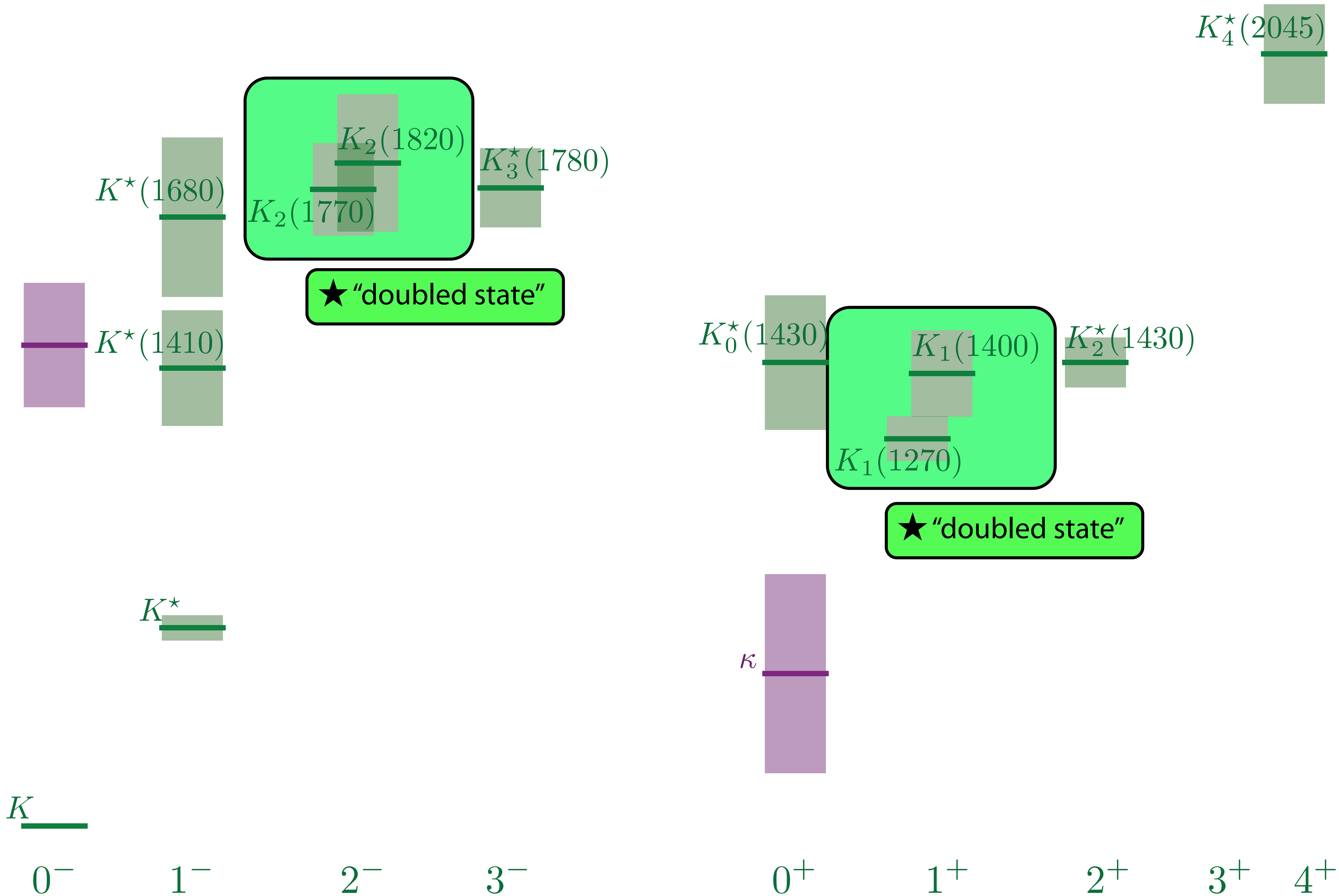


# quark model and QCD

so the quark model roughly describes the spectrum

is it compatible with QCD ?

current quarks in the lagrangian ($m_q$ ~ 1 MeV)

↓ chiral symmetry breaking ?

constituent quarks in the spectrum ($m_q$ ~ 350 MeV)

---

prosaic role of glue in the model

provides binding but not quantum numbers

excitations of the gluonic field ?

present but heavy ?

# quark model and QCD

excited gluonic field could provide quantum numbers $\Rightarrow$ hybrids can have exotic $J^{PC}$

(and non-exotic)

might be heavy (mass gap) ?

heard from Curtis that experimental signals are unclear

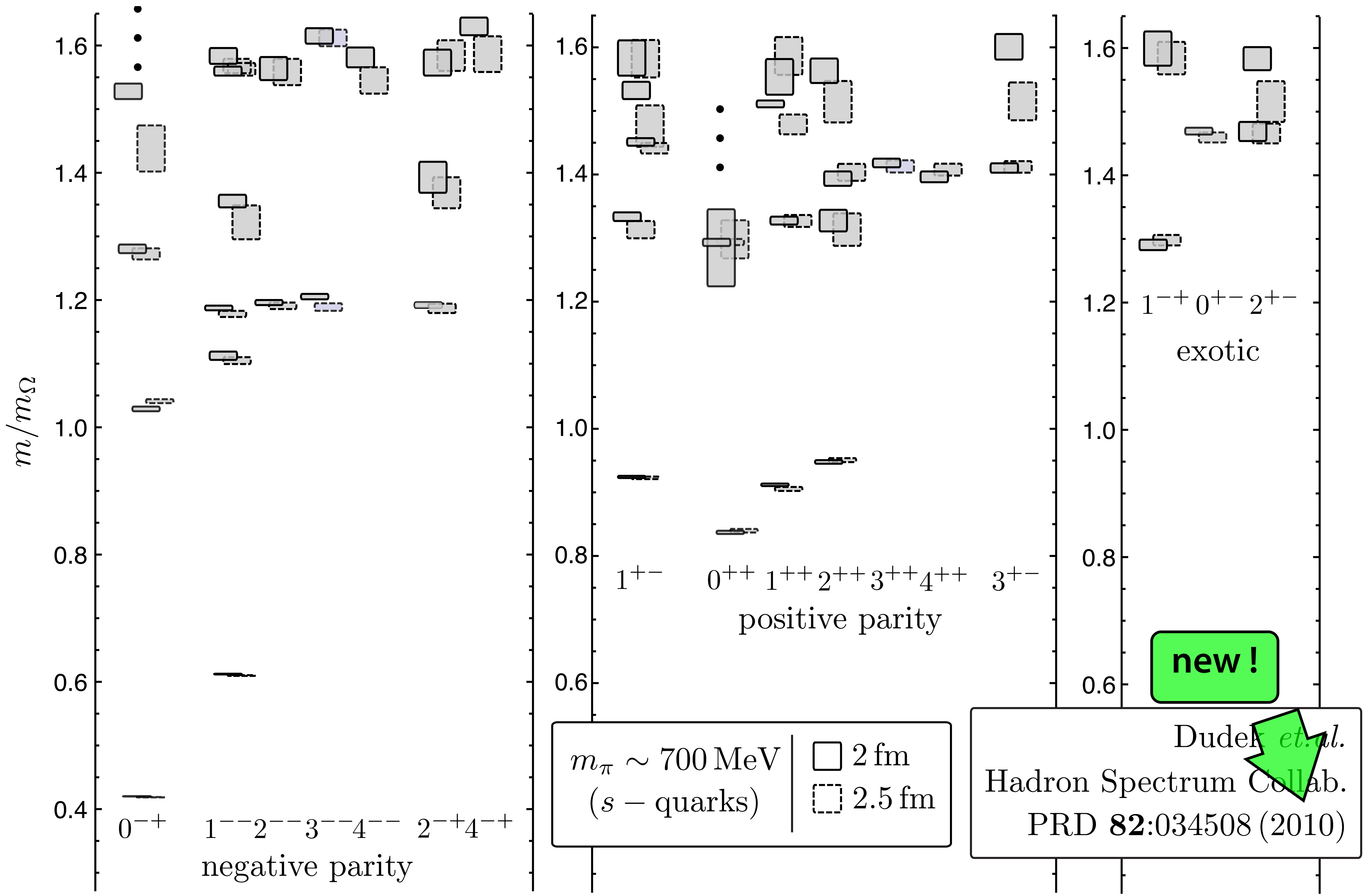
can we get anything from QCD directly ?

heard from Mike that excited meson spectrum is directly calculable in lattice QCD

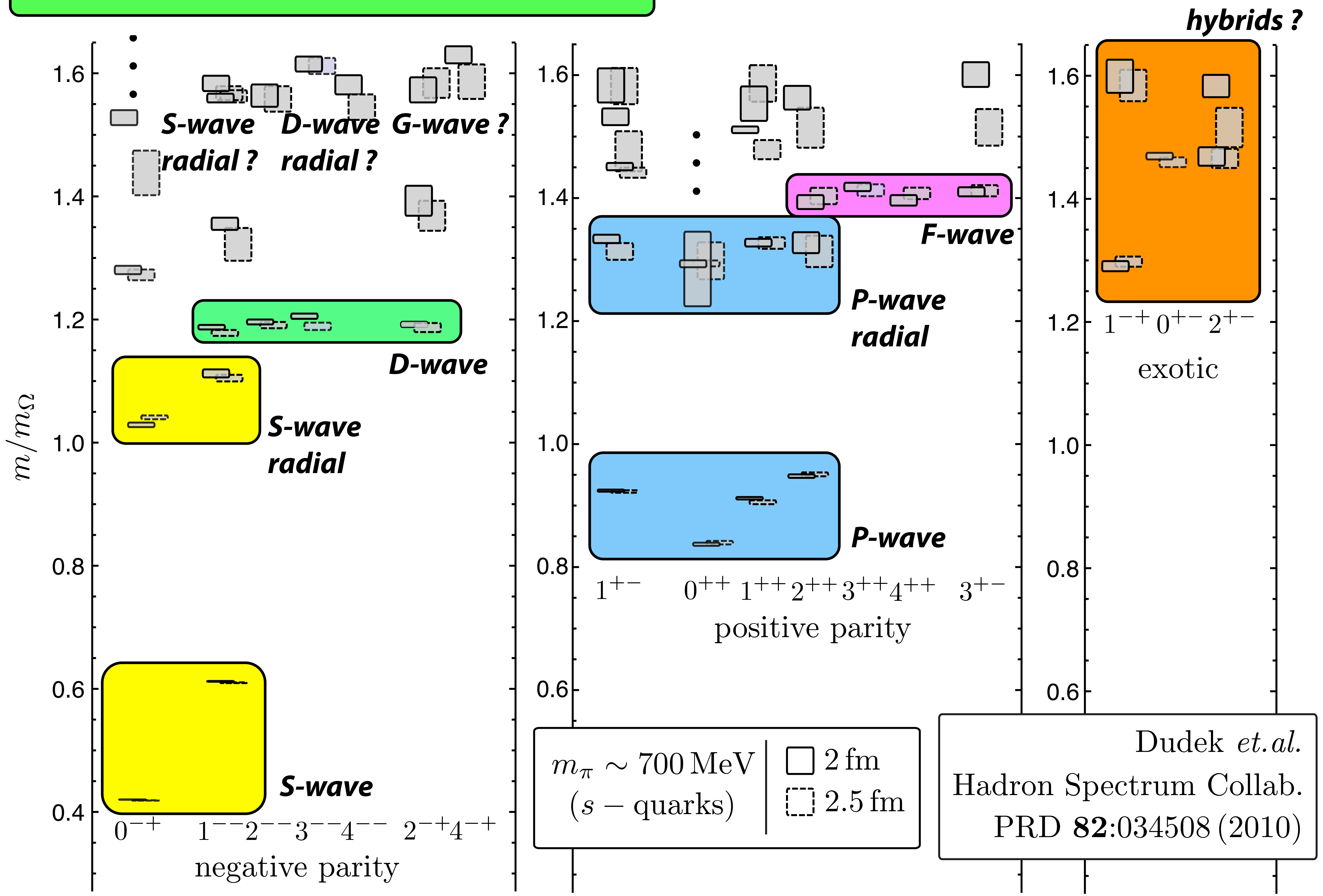
can we build a phenomenology from the results ? → quark model ?

→ gluonic excitations ?

# lattice QCD meson spectrum



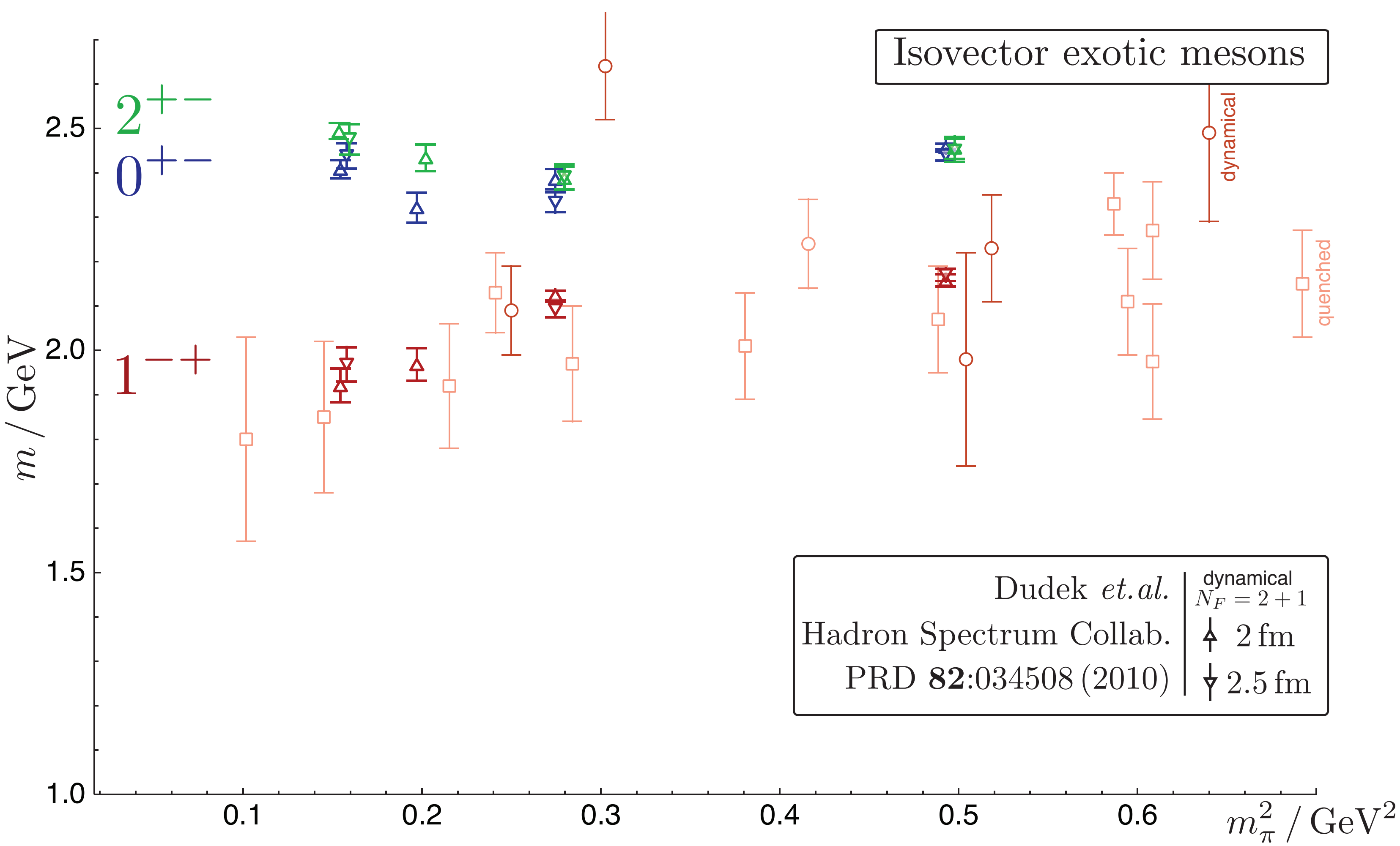
# degeneracy pattern

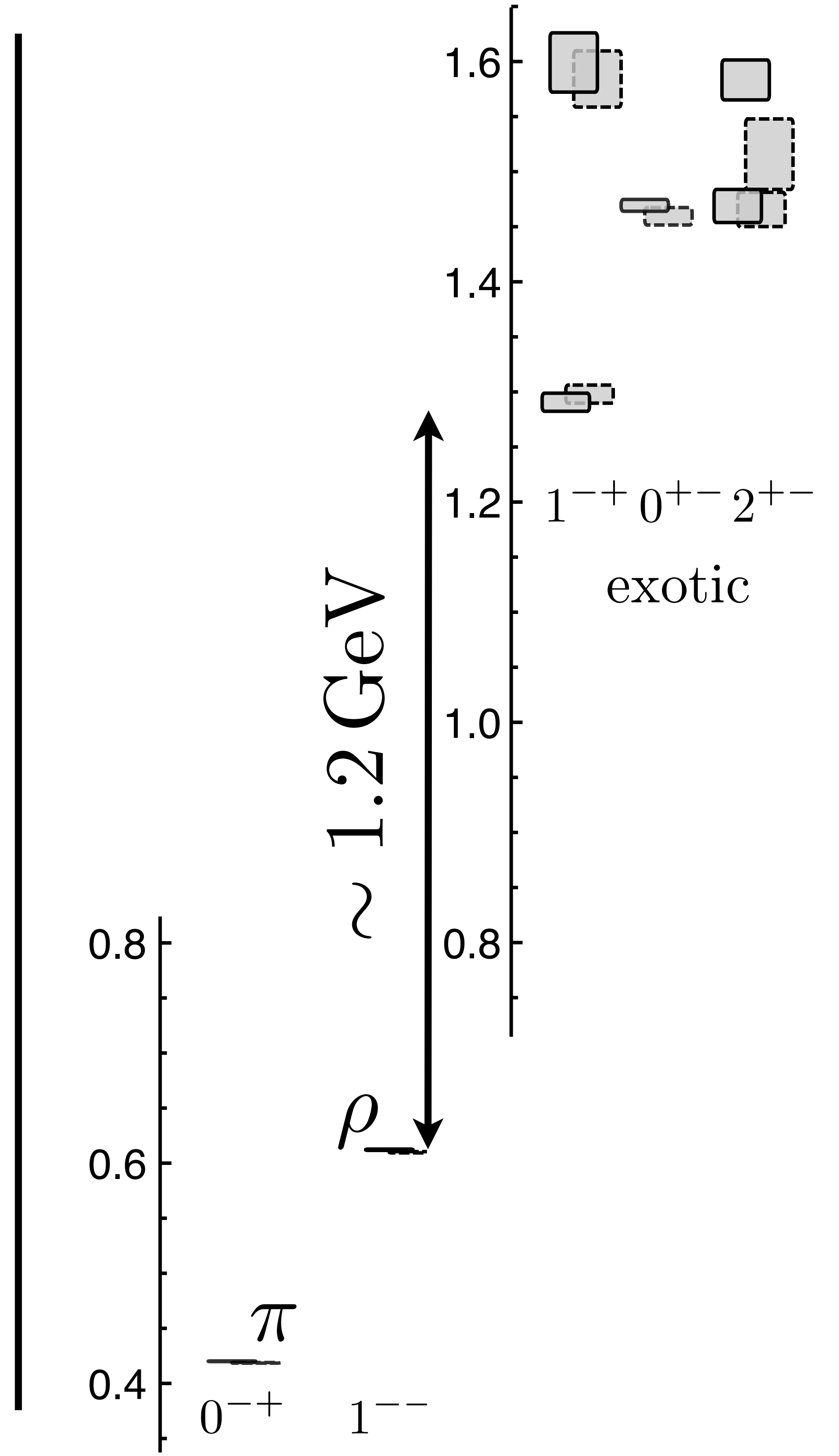


# quark model and QCD

spectrum degeneracy pattern shows quark model systematics

but also exotic $J^{PC}$ states - significant mass gap



# quark model and QCD

additional qualitative information in the matrix elements

$$\left\langle M \middle| \bar{\psi}\boldsymbol{\Gamma}\psi \middle| 0 \right\rangle$$

e.g. selection of our vector operators :

$\rho$ $\quad\gamma_i\quad$ $\xrightarrow{\text{non. rel.}}$ ${}^3S_1$

$\left(\rho \times D^{[2]}_{J=2}\right)^{J=1}$ $\quad D^{[2]}_{J=2} \equiv \langle 1, m_1; 1, m_2 | 2, m \rangle \overleftrightarrow{D}_{m_1} \overleftrightarrow{D}_{m_2}$ $\xrightarrow[D \to \partial]{\text{non. rel.}}$ ${}^3D_1$

$\left(\pi \times D^{[2]}_{J=1}\right)^{J=1}$ $\quad D^{[2]}_{J=1} \equiv \langle 1, m_1; 1, m_2 | 1, m \rangle \overleftrightarrow{D}_{m_1} \overleftrightarrow{D}_{m_2}$ $\xrightarrow{\text{non. rel.}}$ hybrid?

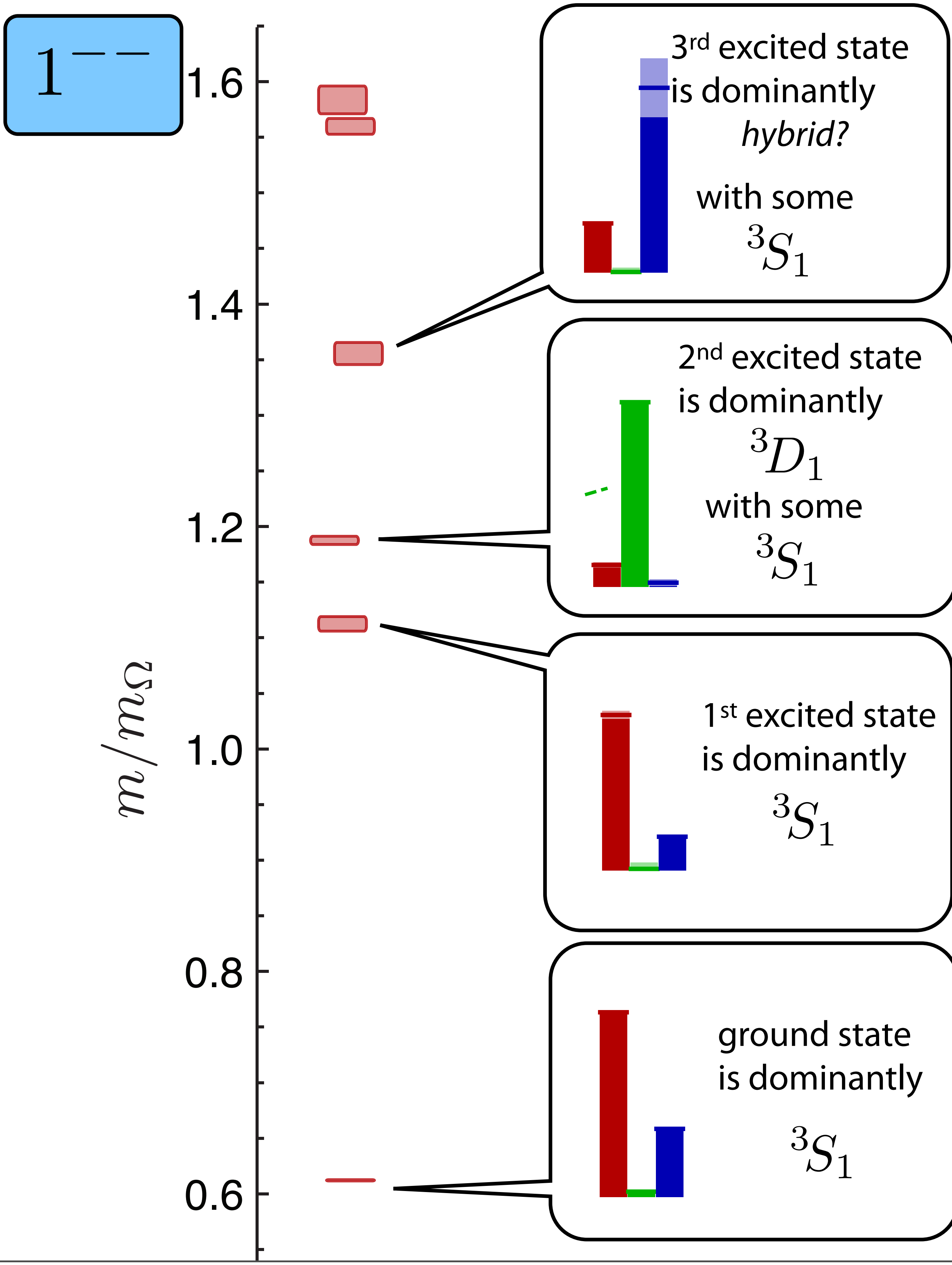
$\sim [D, D] \sim F$

(spin-singlet)

# matrix elements

***$N_F = 3$ (s,s,s)*** $m_\pi \sim 700$ MeV

look at the 'overlaps' $Z_n^{\mathbf{\Gamma}} = \langle n | \bar{\psi} \mathbf{\Gamma} \psi | 0 \rangle$

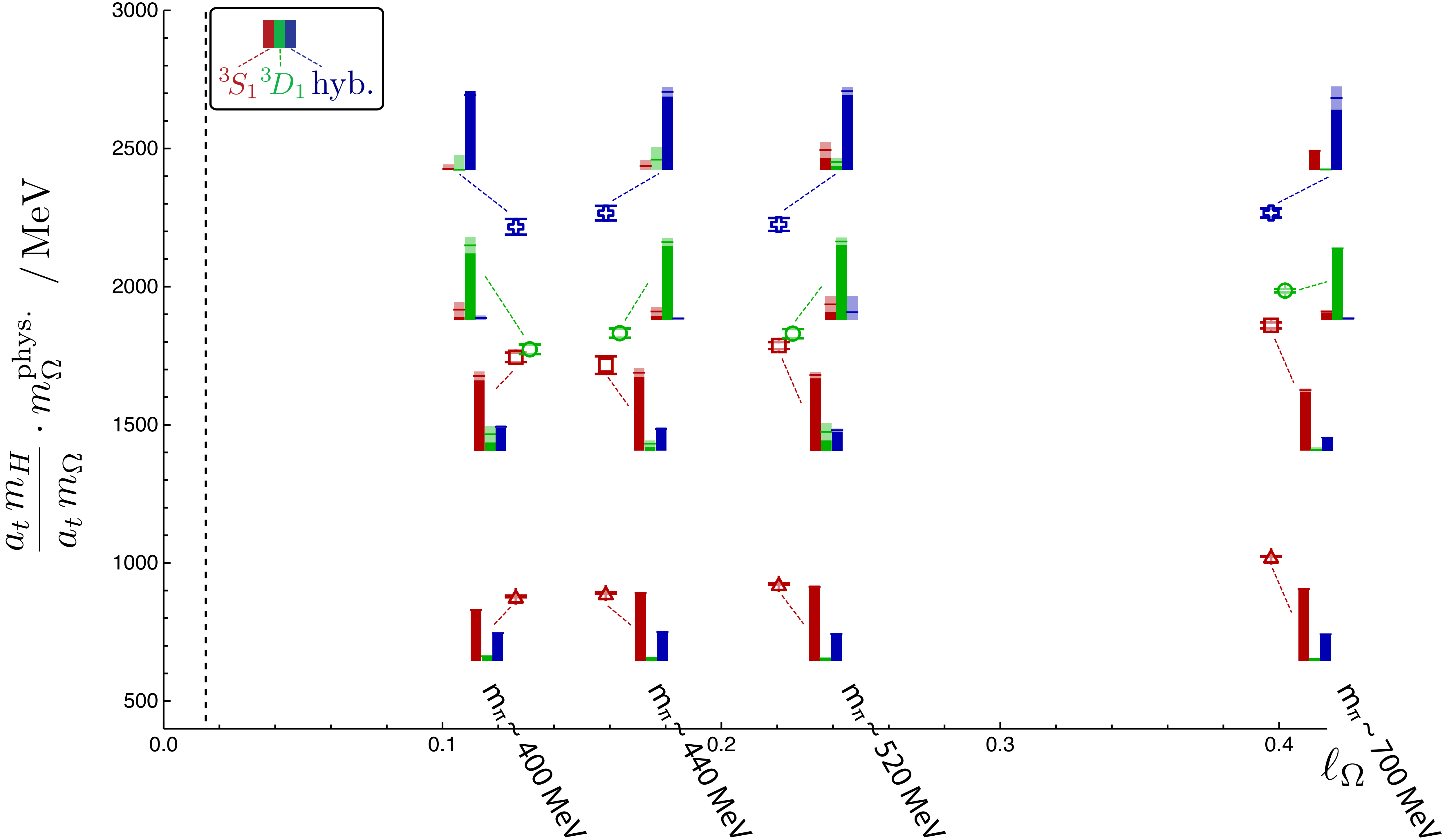



$\rho$ $^3S_1$

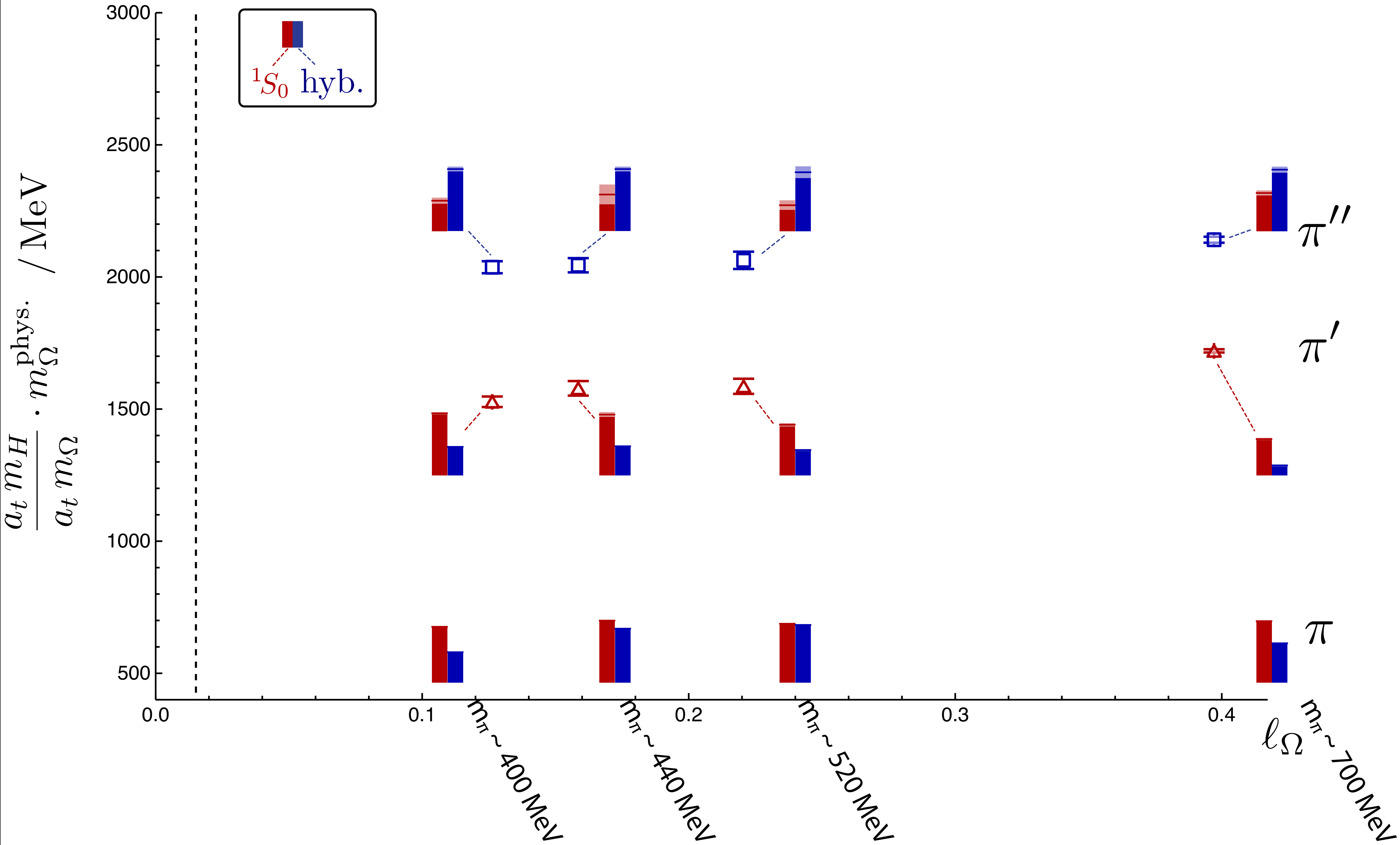
$\left(\rho \times D^{[2]}_{J=2}\right)^{J=1}$ $^3D_1$

$\left(\pi \times D^{[2]}_{J=1}\right)^{J=1}$ hybrid?

# vector mesons

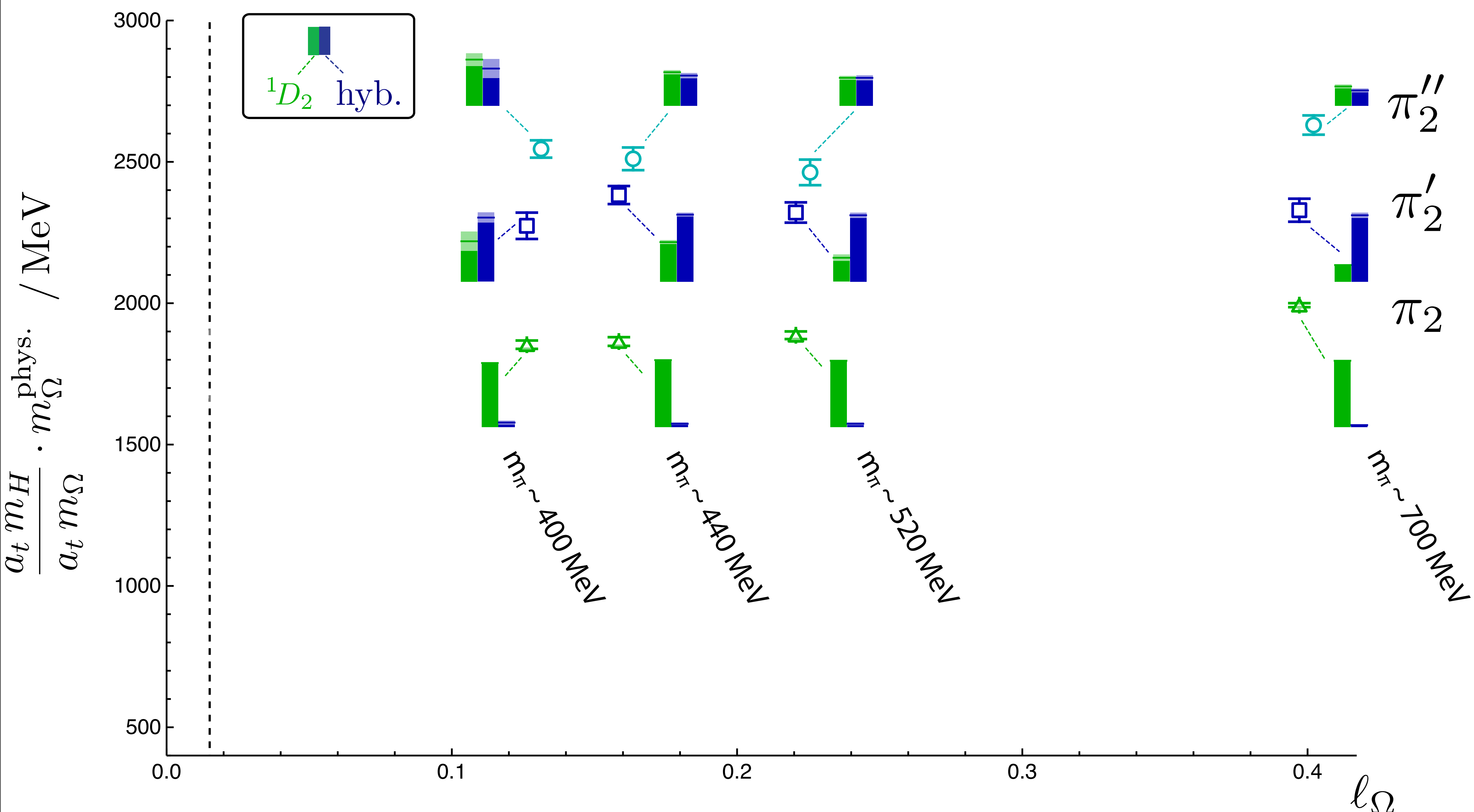


# pseudoscalar mesons



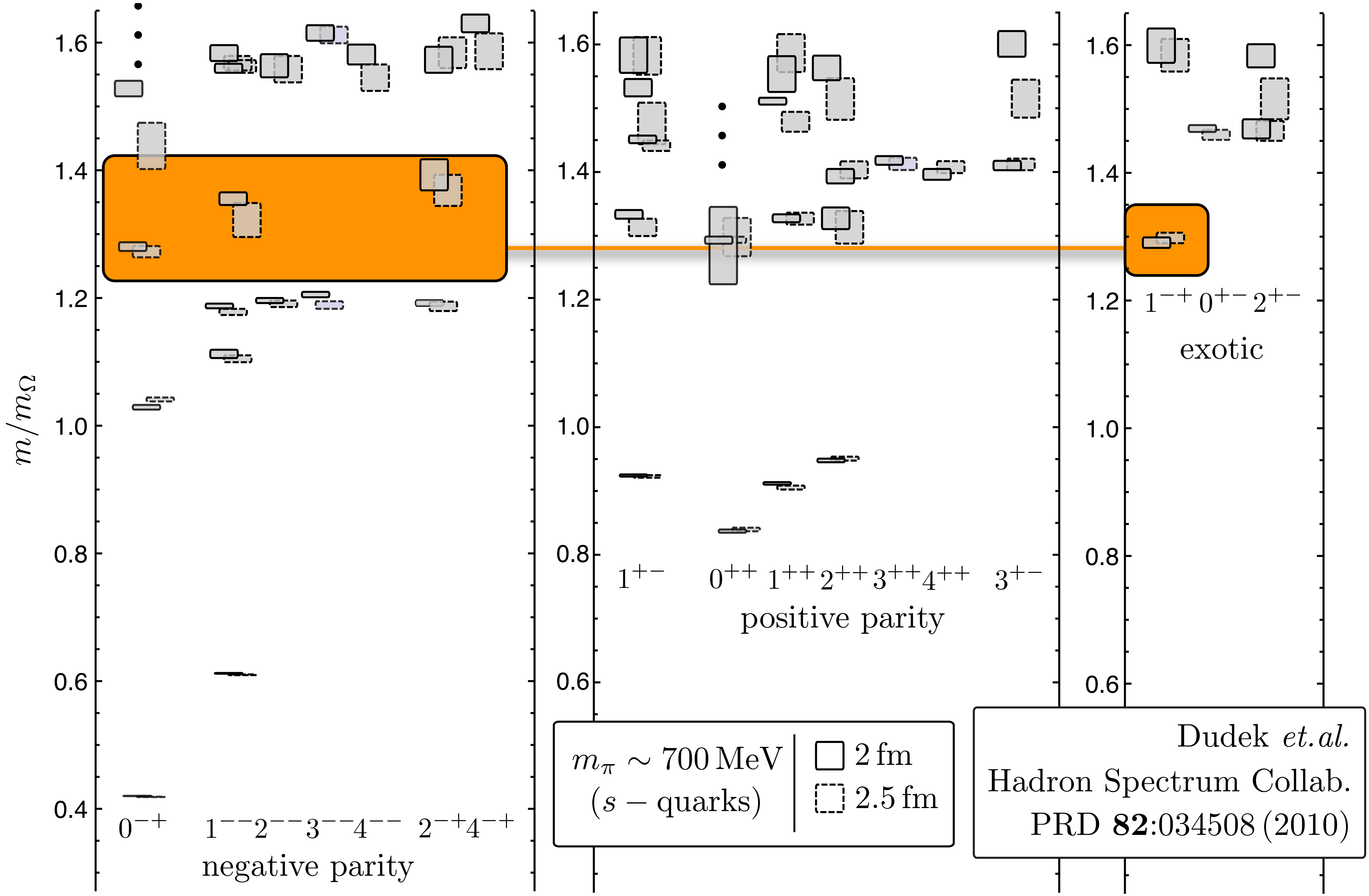

separation of "quark model"-like and hybrid not so clear

# $2^{-+}$ mesons




separation of “quark model”-like and hybrid not so clear

# excited glue phenomenology



# excited glue phenomenology

lightest hybrids : $0^{-+}, 1^{-+}, 2^{-+}, 1^{--}$

overlap with ops : $\underbrace{\left(\rho \times D^{[2]}_{J=1}\right)^{J=0,1,2}}$ $\hookrightarrow \left(\pi \times D^{[2]}_{J=1}\right)^{J=1}$

$m/m_\Omega \sim 1.3$

& other operators ...

$D^{[2]}_{J=1} \sim \epsilon_{ijk} F^a_{jk} \sim B^a_i$ chromomagnetic field

$\{\pi, \rho\} \sim L_{q\bar{q}} = 0$

quark *S*-wave

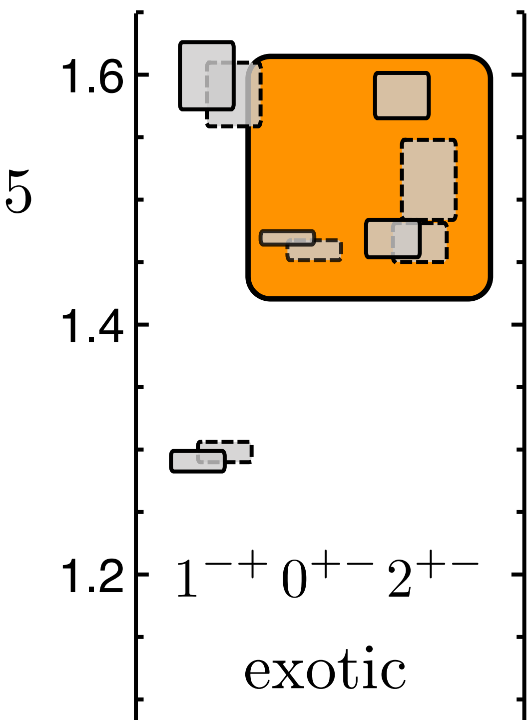
## excited glue phenomenology

heavier hybrids : $0^{+-}, 2^{+-}, 2^{+-}, \ldots$

overlap with ops : e.g. $\left(a_1 \times D^{[2]}_{J=1}\right)^{J=0,1,2}$

& other operators ...

$m/m_\Omega \sim 1.5$




$D^{[2]}_{J=1} \sim \epsilon_{ijk} F^a_{jk} \sim B^a_i$ chromomagnetic field

$\{a_1\} \sim L_{q\bar{q}} = 1$ quark *P*-wave

# excited glue phenomenology

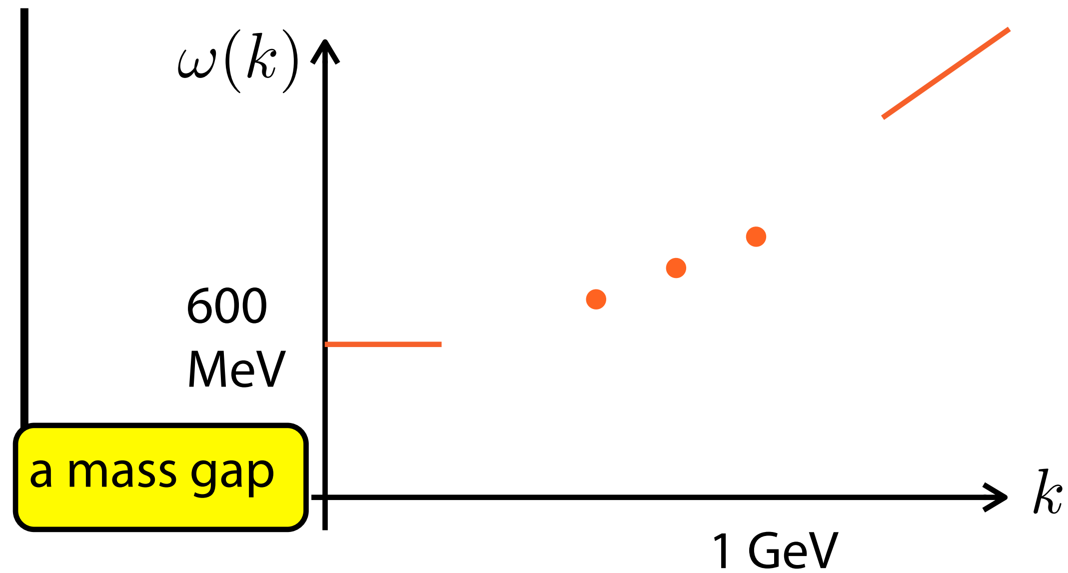
e.g. Guo, Szczepaniak ...
*"Heavy quarkonium hybrids from Coulomb gauge QCD"*
Phys.Rev.D78:056003,2008

'constituent' gluons

e.g. in Coulomb gauge many body approach

transverse "massive" $1^{--}$ degree-of-freedom

$$\vec{A}^a(\vec{x}) = \int \frac{d^3\vec{k}}{(2\pi)^3} \frac{1}{\sqrt{2\omega(k)}} \left[ \vec{\epsilon}_\lambda(\vec{k})\, a^a_\lambda(\vec{k})\, e^{i\vec{k}\cdot\vec{x}} + \vec{\epsilon}^{\,*}_\lambda(\vec{k})\, a^{a\dagger}_\lambda(\vec{k})\, e^{-i\vec{k}\cdot\vec{x}} \right]$$




1 constituent gluon state : $\left| g^a_\lambda(\vec{k}) \right\rangle = a^{a\dagger}_\lambda(\vec{k}) \left| \Psi_0 \right\rangle$

hybrid meson construction : $\left| \mathcal{H} \right\rangle = \left| g \right\rangle \otimes \left| q\bar{q} \right\rangle$

what quantum numbers ?

don't need the model details,
just the degrees-of-freedom

# excited glue phenomenology

$J_g^{P_g C_g} = 1^{--}$

gluon in an *S*-wave w.r.t $q\bar{q}$

| | $S=0$ | $S=1$ |
|---|---|---|
| $L=0$ | $1^{+-}$ | $(0,1,2)^{++}$ |
| $L=1$ | $(0,1,2)^{-+}$ | $(0,1^3,2^2,3)^{--}$ |

exactly resembles a $q\bar{q}$ *P*-wave

exotic $1^{-+},\ 0^{--}$

nothing like what we observe

$J_g^{P_g C_g} = 1^{+-}$

gluon in an *P*-wave w.r.t $q\bar{q}$

| | $S=0$ | $S=1$ |
|---|---|---|
| $L=0$ | $1^{--}$ | $(0,1,2)^{-+}$ |
| $L=1$ | $(0,1,2)^{++}$ | $(0,1^3,2^2,3)^{+-}$ |

exotic $0^{+-}, 2^{+-}, 2^{+-}$

exactly what we observe

# excited glue phenomenology

$J_g^{P_g C_g} = 1^{+-}$

gluon in an *P*-wave w.r.t $q\bar{q}$

| | $S=0$ | $S=1$ |
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exotic $0^{+-}, 2^{+-}, 2^{+-}$

exactly what we observe

operator overlap expectations :

$$\vec{A}^a(\vec{x}) = \int \frac{d^3\vec{k}}{(2\pi)^3} \frac{1}{\sqrt{2\omega(k)}} \left[ \vec{\epsilon}_\lambda(\vec{k})\, a^a_\lambda(\vec{k})\, e^{i\vec{k}\cdot\vec{x}} + \vec{\epsilon}^{\,*}_\lambda(\vec{k})\, a^{a\dagger}_\lambda(\vec{k})\, e^{-i\vec{k}\cdot\vec{x}} \right] \Longrightarrow D^{[2]}_{J=1} \sim \left(\vec{k}\times\vec{\epsilon}\right) a^\dagger$$

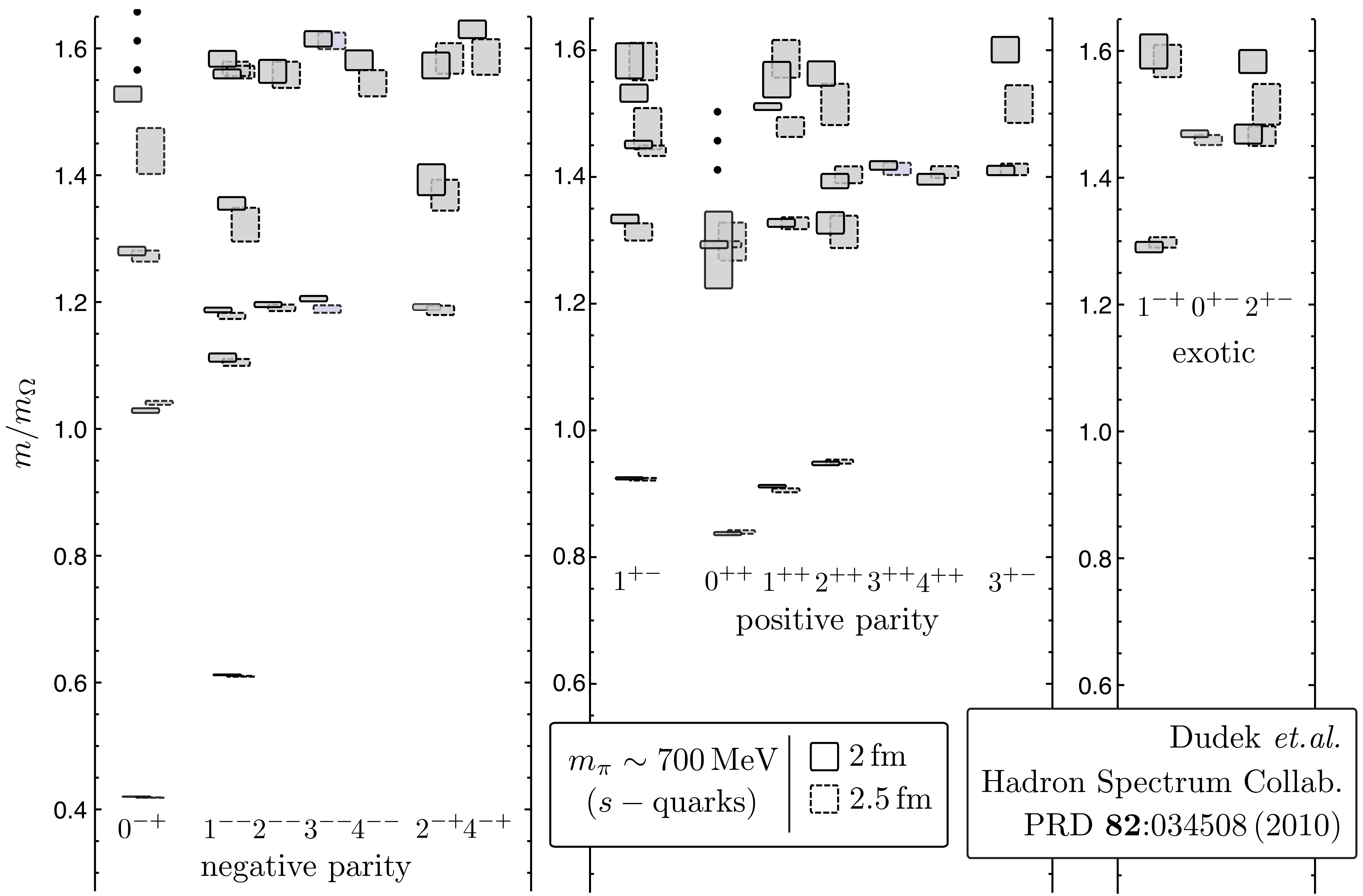
so e.g.

$$\left(\pi \times D^{[2]}_{J=1}\right)^{J=1} \Longrightarrow g[1^+]\, q\bar{q}({}^1S_0) \qquad J^{PC}_{\mathcal{H}} = 1^{--}$$

$$\left(\rho \times D^{[2]}_{J=1}\right)^{J=0,1,2} \Longrightarrow g[1^+]\, q\bar{q}({}^3S_1) \qquad J^{PC}_{\mathcal{H}} = (0,1,2)^{-+}$$

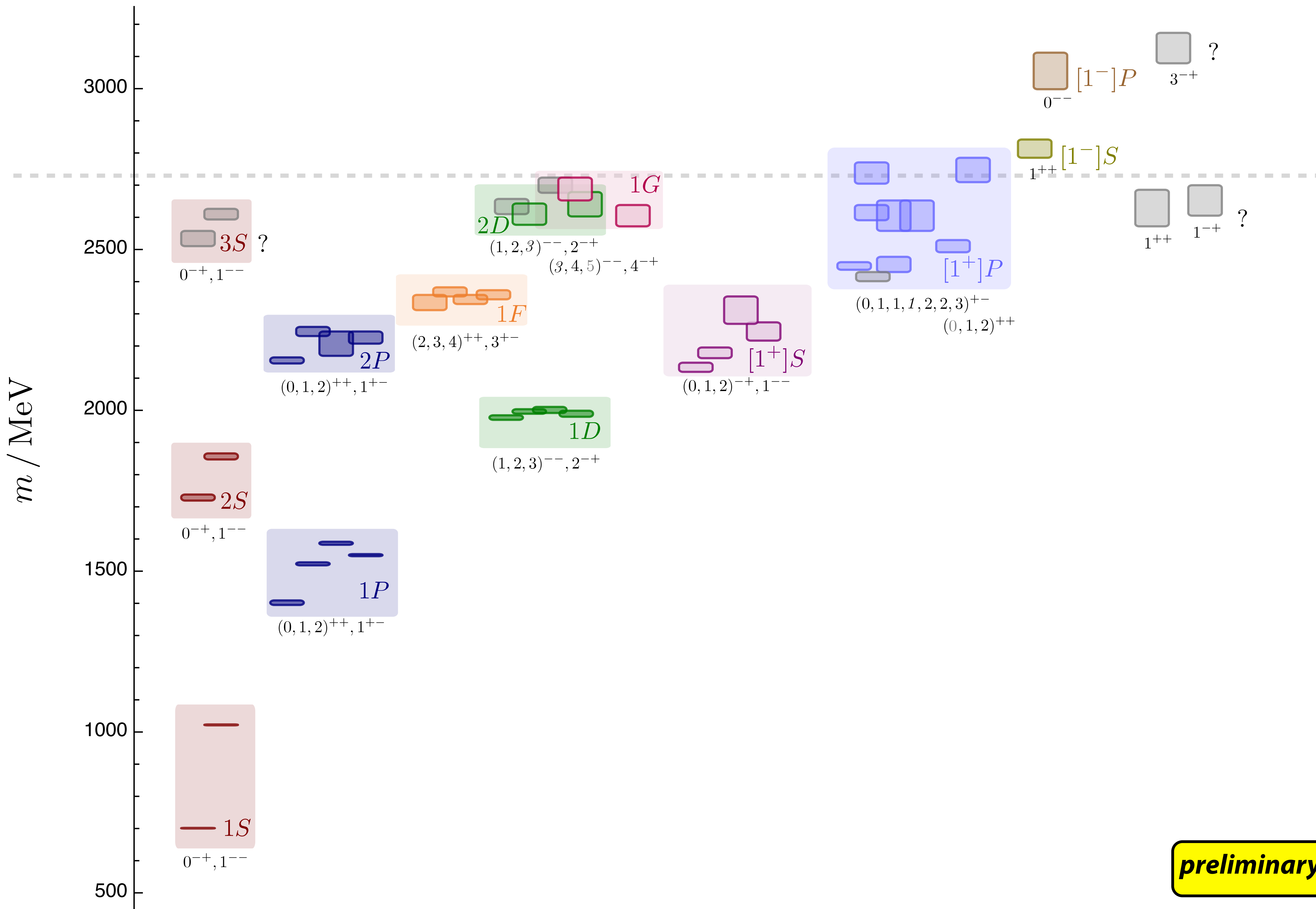
$$\left(a_1 \times D^{[2]}_{J=1}\right)^{J=0,1,2} \Longrightarrow g[1^+]\, q\bar{q}({}^3P_1) \qquad J^{PC}_{\mathcal{H}} = (0,1,2)^{+-}$$

# lattice QCD meson spectrum



# spectrum phenomenology

multiplets identified by overlap patterns




**preliminary**

# summary of observations

degeneracy pattern and operator overlaps have some quark model features

*S,P,D,F*... level ordering

spin singlets & triplets

radial excitations ?

clear need to supplement the constituent quark degrees-of-freedom with gluonic excitations

exotic $J^{PC}$

large overlaps with gluonic operators

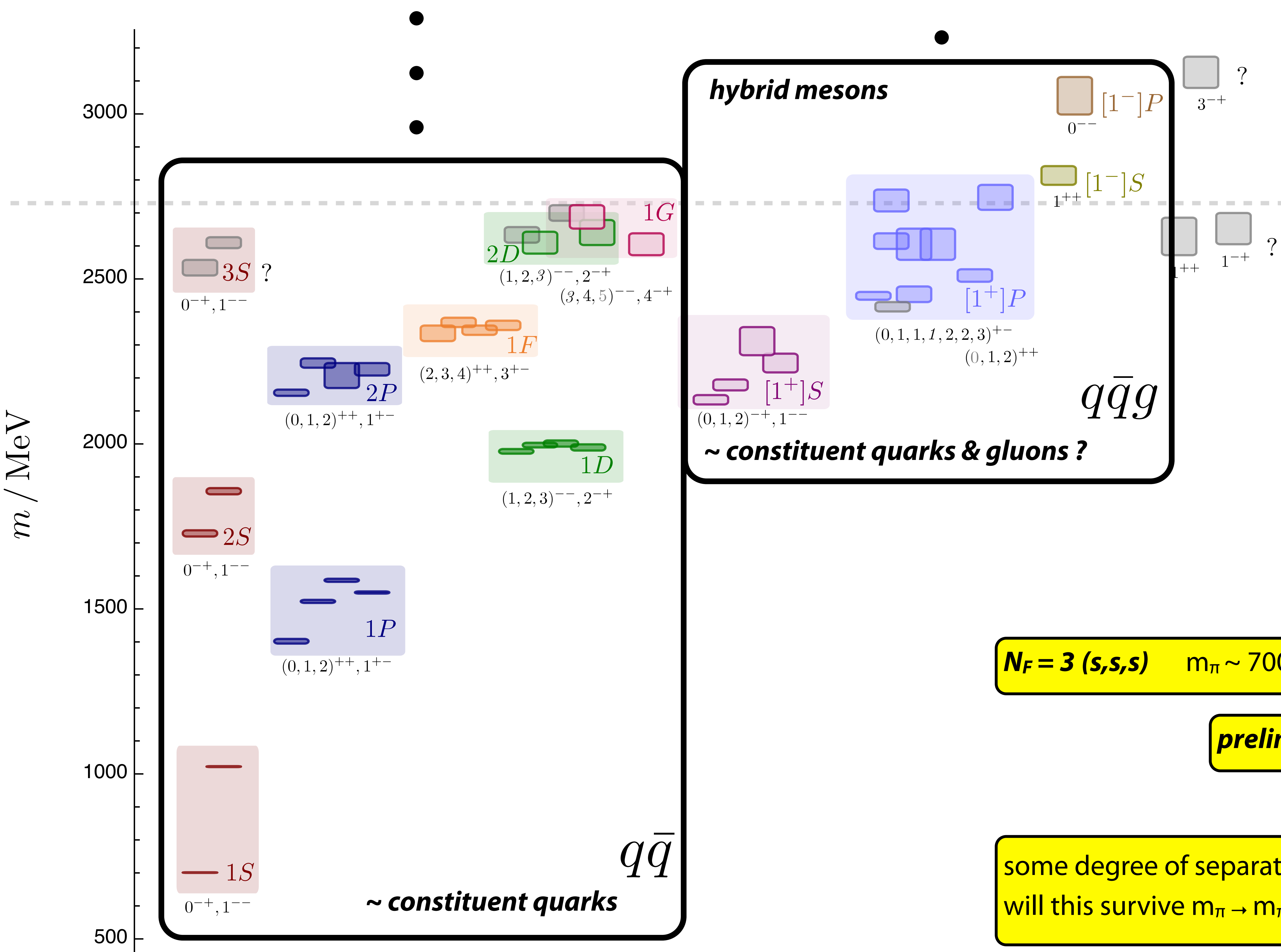
possibly described by a constituent gluon picture ?

but in a *P*-wave?

c.f. the bag model

build a QCD-motivated gluonic excitations phenomenology

# constituent quarks & gluons in the spectrum




**$N_F = 3$ (s,s,s)** $m_\pi \sim 700$ MeV

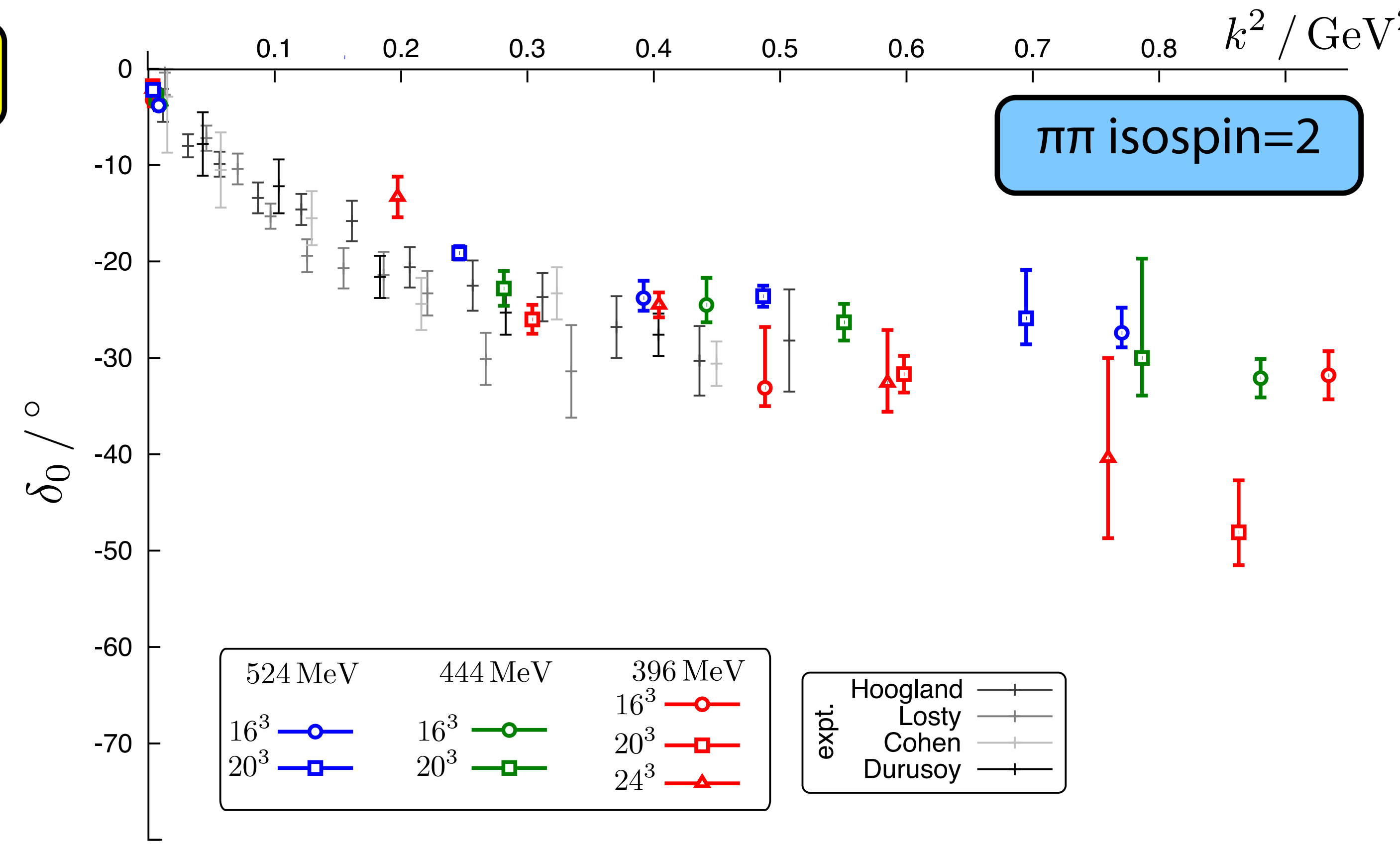
***preliminary***

some degree of separation - will this survive $m_\pi \rightarrow m_\pi^{phys}$ ?

# still to do ...

include explicit multiquark operators in our basis

check on our "prejudices"




study decay dynamics

quark model as presented ignores decays

treat mesons as bound states

lattice calc without meson-meson operators

don't resolve hadron widths

poor approximation for broad resonances like $a_1$

# still to do ...

reducing the quark mass

little qualitative spectrum change in region 400 MeV < $m_\pi$ < 700 MeV

still some mysteries :

lack of kaon mixing

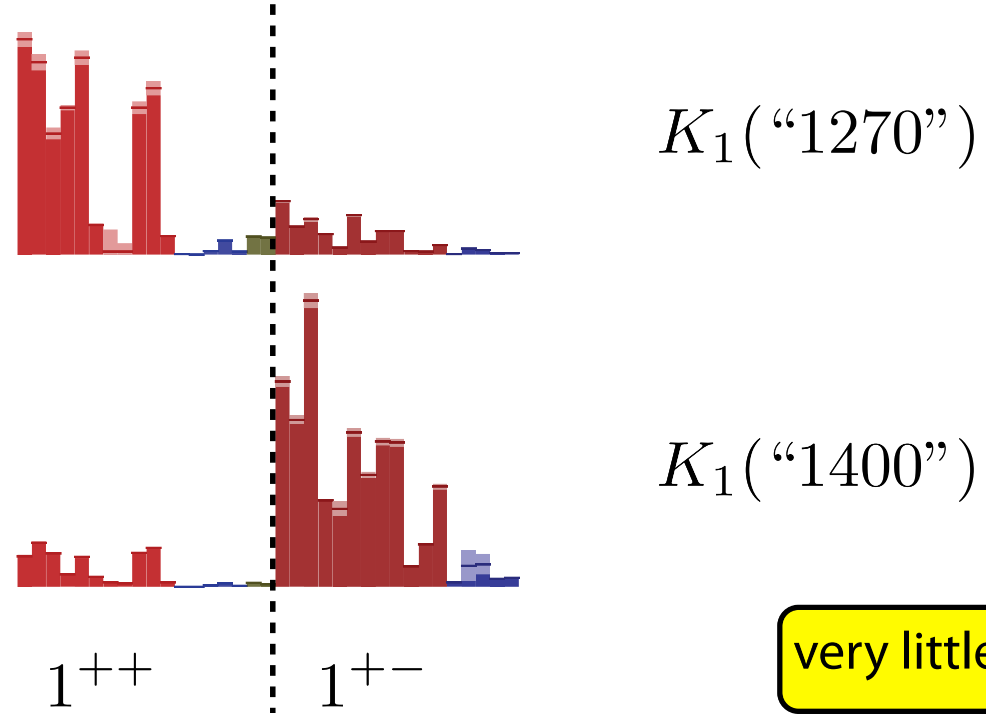
phenomenology :

$$\begin{pmatrix} K_1(1270) \\ K_1(1400) \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} K(1^{+-}) \\ K(1^{++}) \end{pmatrix}$$

$$\theta \sim 45^\circ$$

our calculation @ $m_\pi \sim$ 400 MeV :

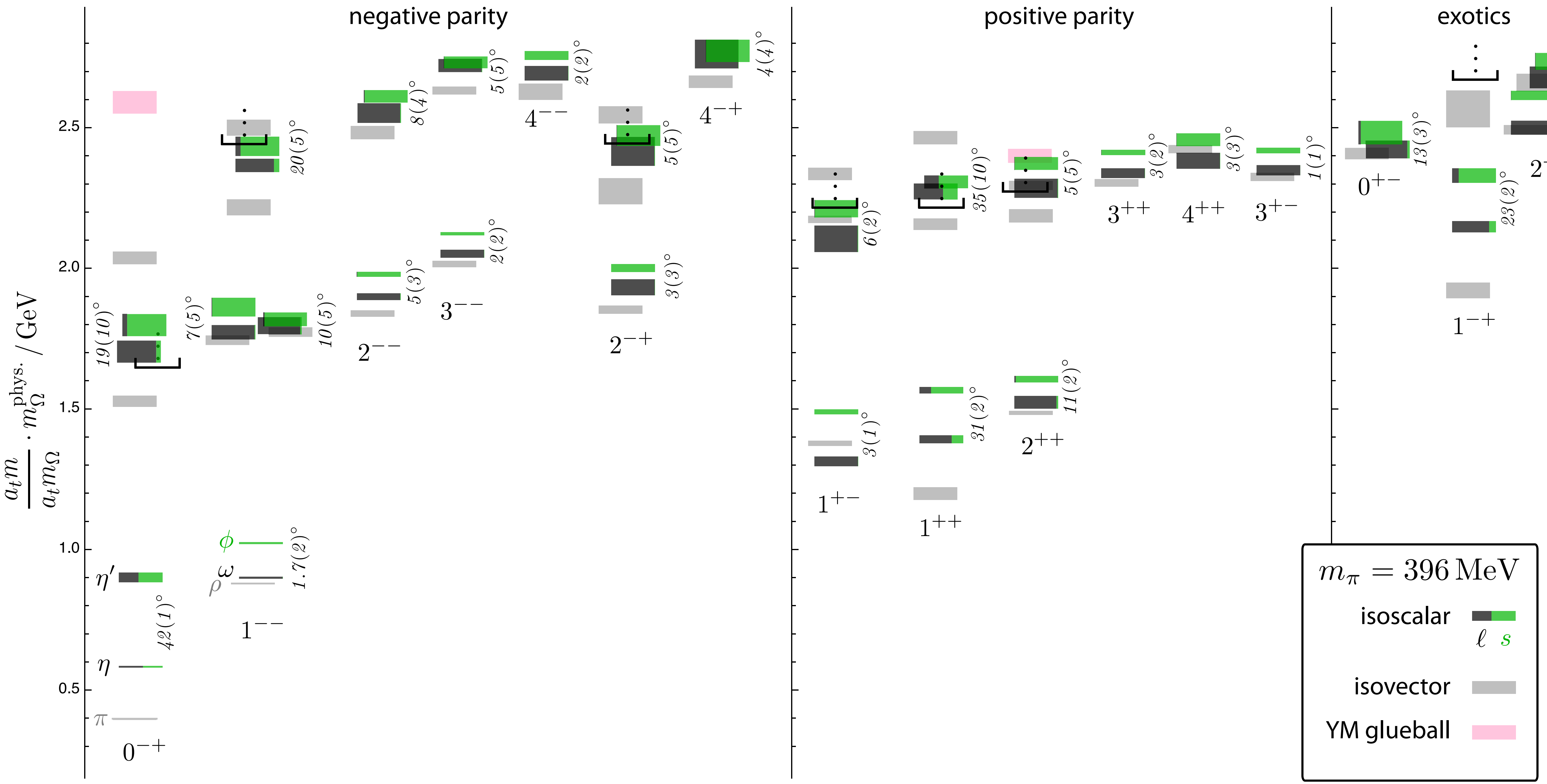
$$\frac{m_K}{m_\pi} = 1.39 \quad (840)$$




very little mixing

## & the rest

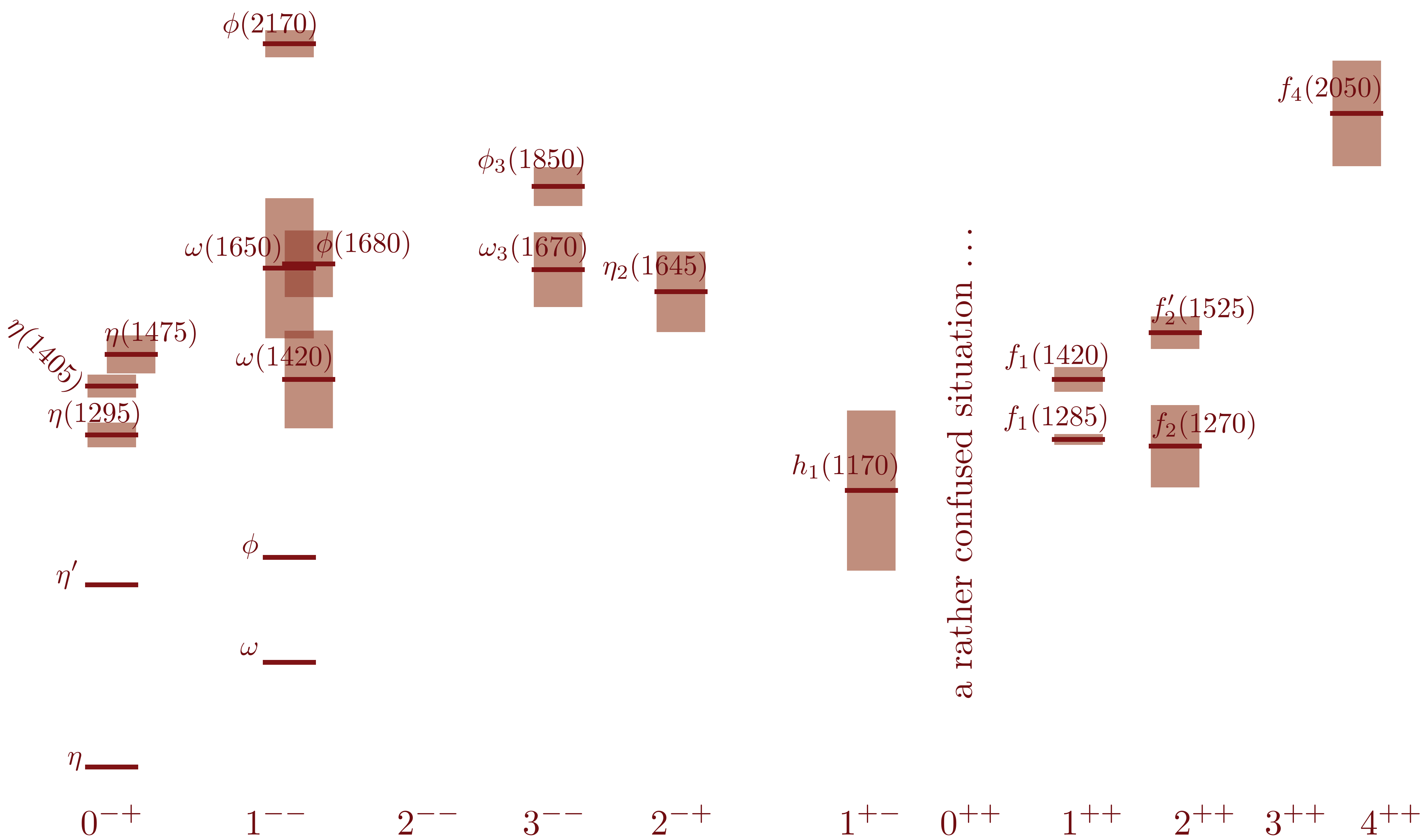
# isoscalars



negative parity
positive parity
exotics
$\frac{a_t m}{a_t m_\Omega} \cdot m_\Omega^{\text{phys.}}$ / GeV
2.5
2.0
1.5
1.0
0.5
$0^{-+}$
$1^{--}$
$2^{--}$
$3^{--}$
$4^{--}$
$2^{-+}$
$4^{-+}$
$1^{+-}$
$1^{++}$
$2^{++}$
$3^{++}$
$4^{++}$
$3^{+-}$
$0^{+-}$
$1^{-+}$
$2^{+-}$
$\pi$
$\eta$
$\eta'$
$\rho$
$\omega$
$\phi$
42(1)°
19(10)°
7(5)°
10(5)°
20(5)°
1.7(2)°
5(3)°
8(4)°
2(2)°
5(5)°
2(2)°
3(3)°
5(5)°
4(4)°
3(1)°
6(2)°
31(2)°
35(10)°
11(2)°
5(5)°
3(2)°
3(3)°
1(1)°
13(3)°
23(2)°
2(1)°
5(4)°
$m_\pi = 396\,\text{MeV}$
isoscalar
$\ell$ $s$
isovector
YM glueball

# isoscalar meson spectrum

"naive" reading of PDG summary table



# chiral restoration ?

suggestions that high in the spectrum the degeneracy pattern will be according to ***unbroken*** chiral symmetry

hence parity doubling, e.g. $\rho_\star(1^{--})$ degenerate with $a_{1\star}(1^{++})$ ...

and/or $\rho_\star(1^{--})$ degenerate with $h_{1\star}(1^{+-})$ ...

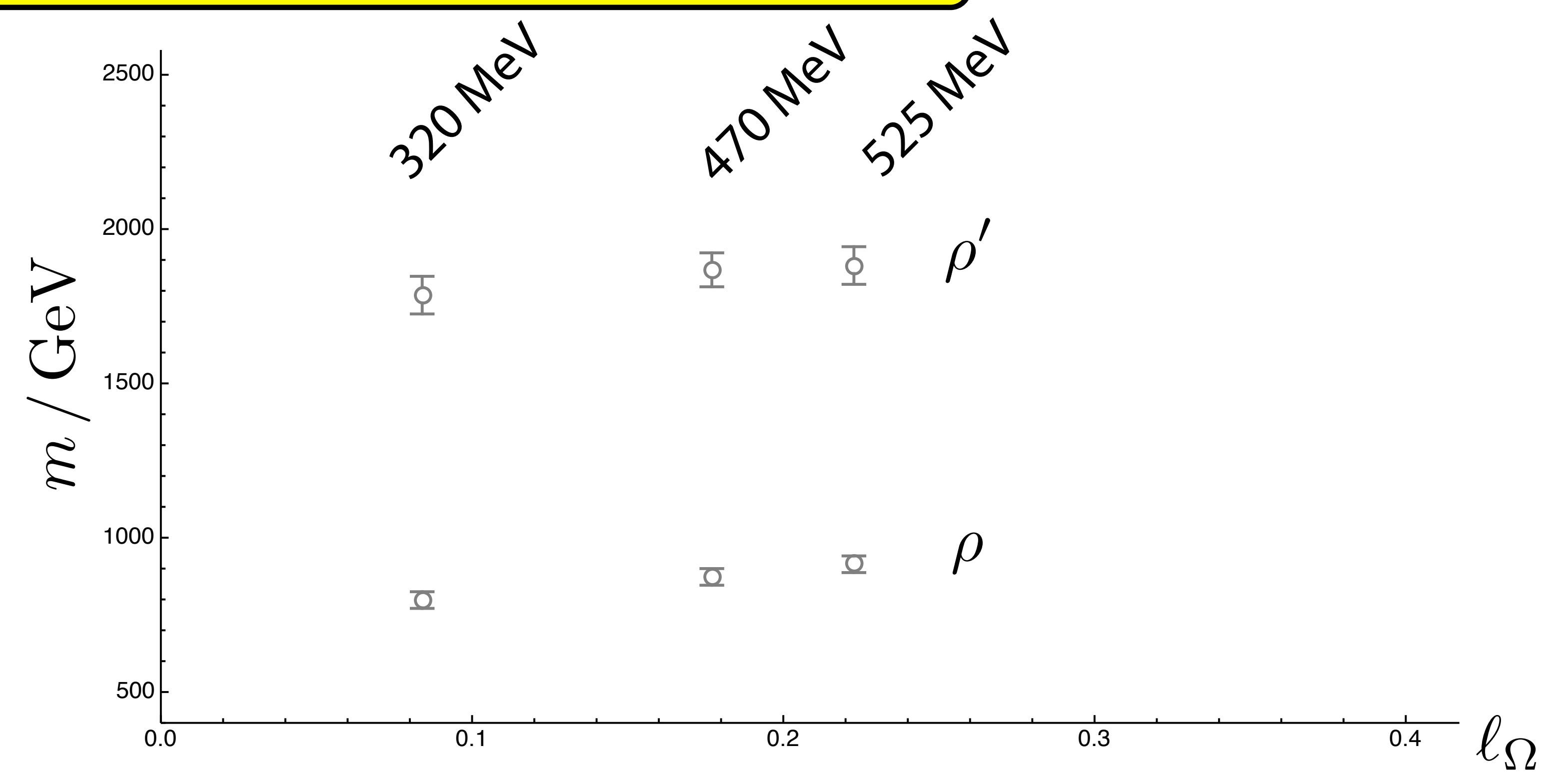
and breakdown of constituent quark-model pattern

but how high in the spectrum ?

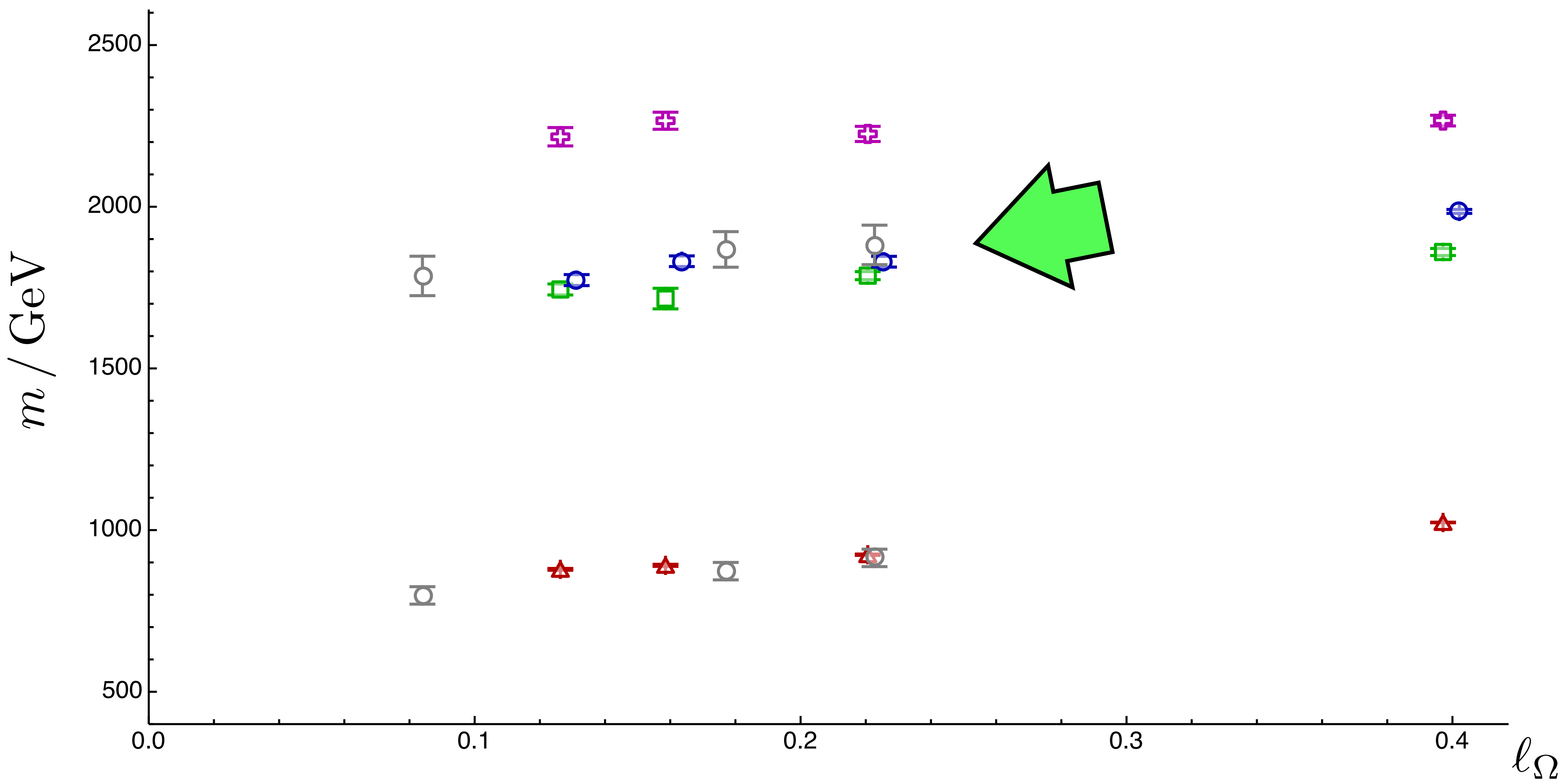
recent claims that lattice results show this even in the first excited rho !

Glozman, Lang, Limmer
PRD82 097501

$N_F = 2$



# chiral restoration ?




Graz use a very small operator basis :

$\bar{\psi}\gamma_i\psi$
$\bar{\psi}\gamma_0\gamma_i\psi$
& two smearing radii

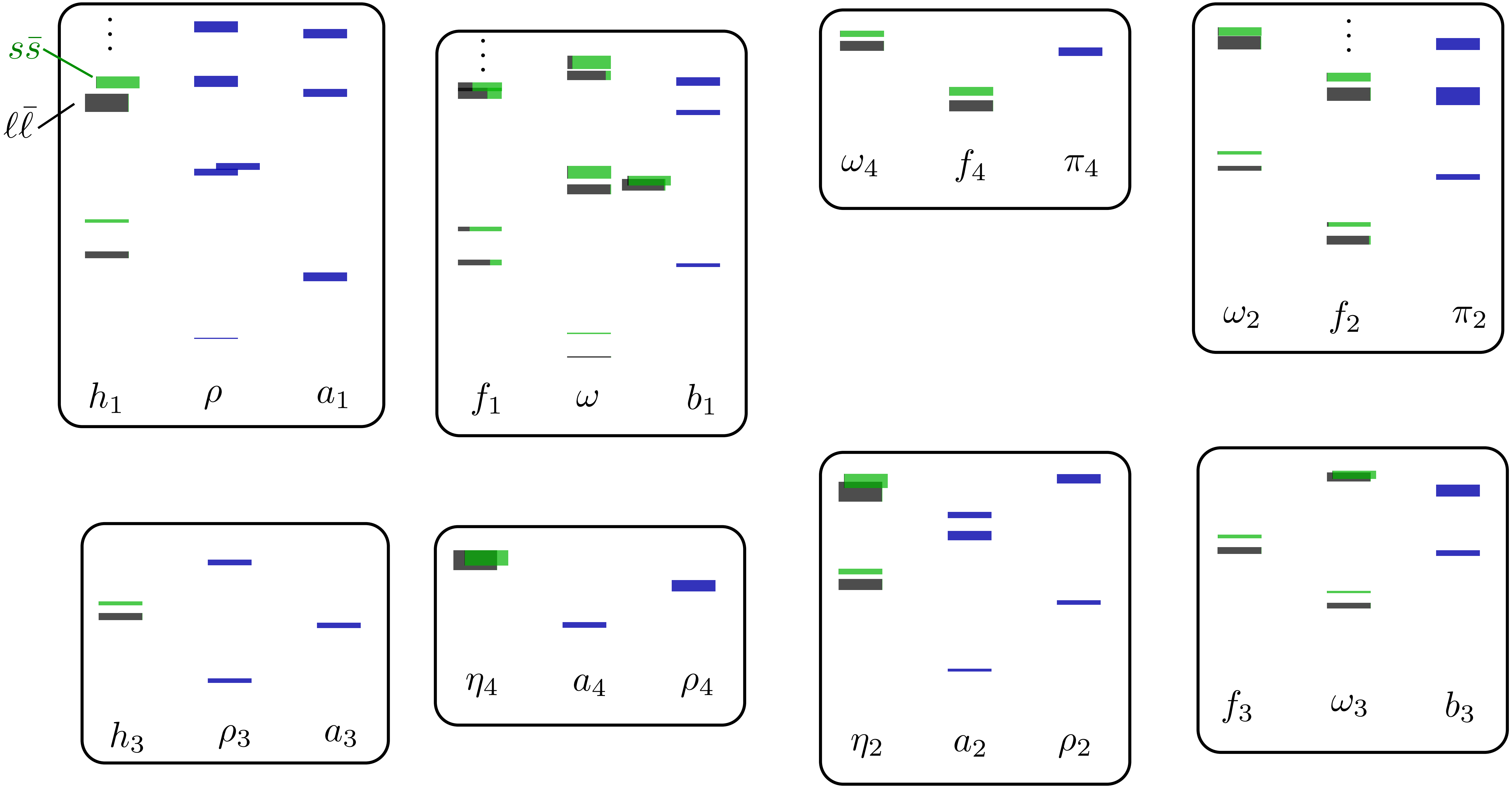
clearly not large enough

HSC have isoscalar spectra in hand - can look for degeneracy patterns

# chiral restoration ?

$m_\pi \sim 400$ MeV

large explicit χSB

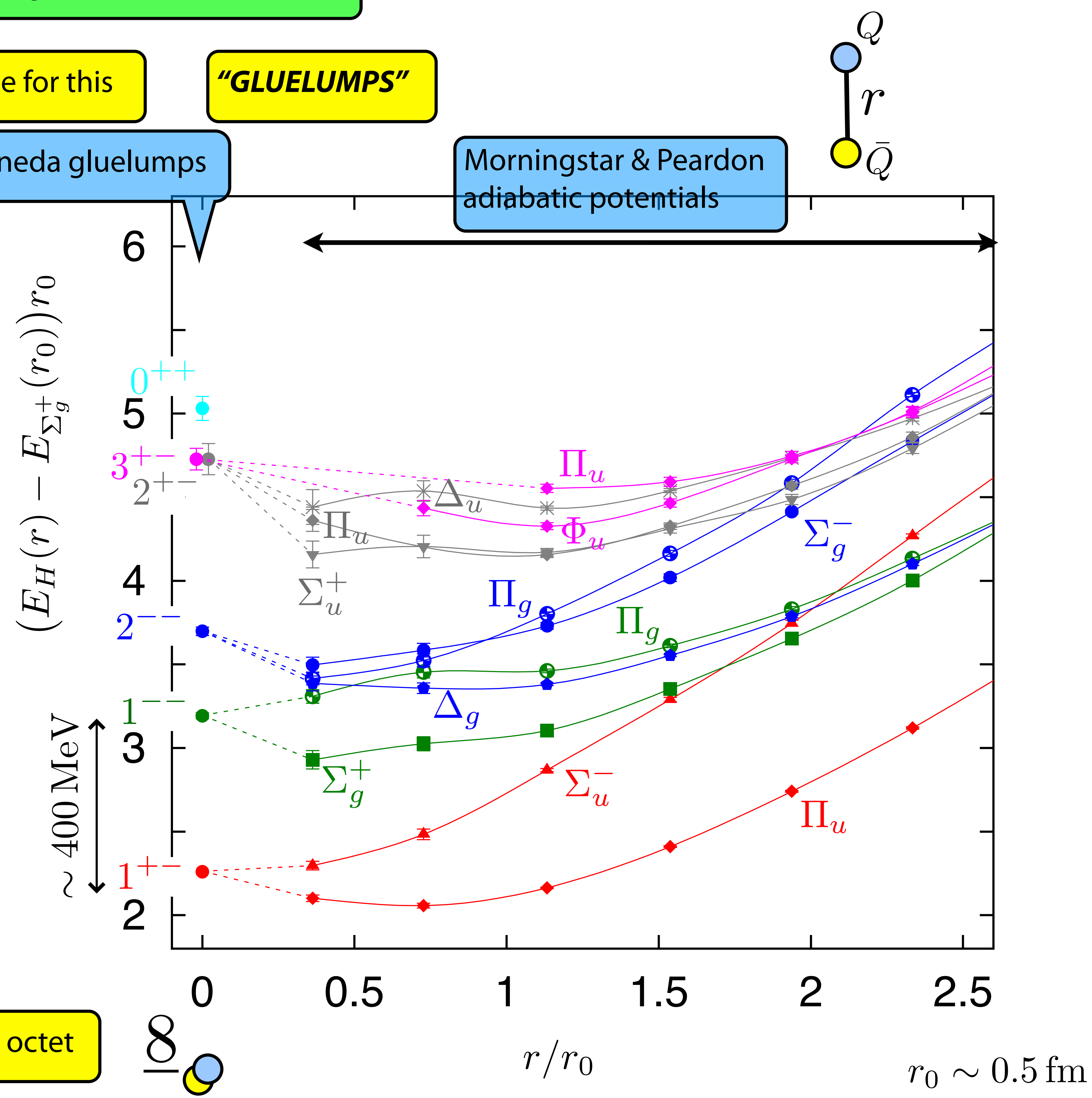



overlaps should indicate the chiral representation ... to come

# $J_g^{P_g} = 1^+$ lighter than $J_g^{P_g} = 1^-$ ?

this is not the first evidence for this

***"GLUELUMPS"***




energy of a static color octet

## $J_g^{P_g} = 1^+$ lighter than $J_g^{P_g} = 1^-$ ?

***"GLUELUMPS"***

in Coulomb gauge model a three-body force causes it - pushes the *S*-wave up

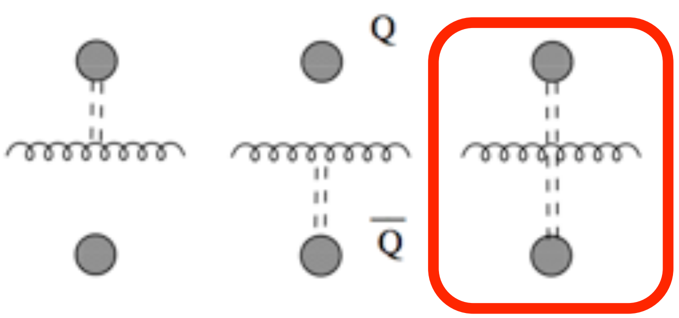
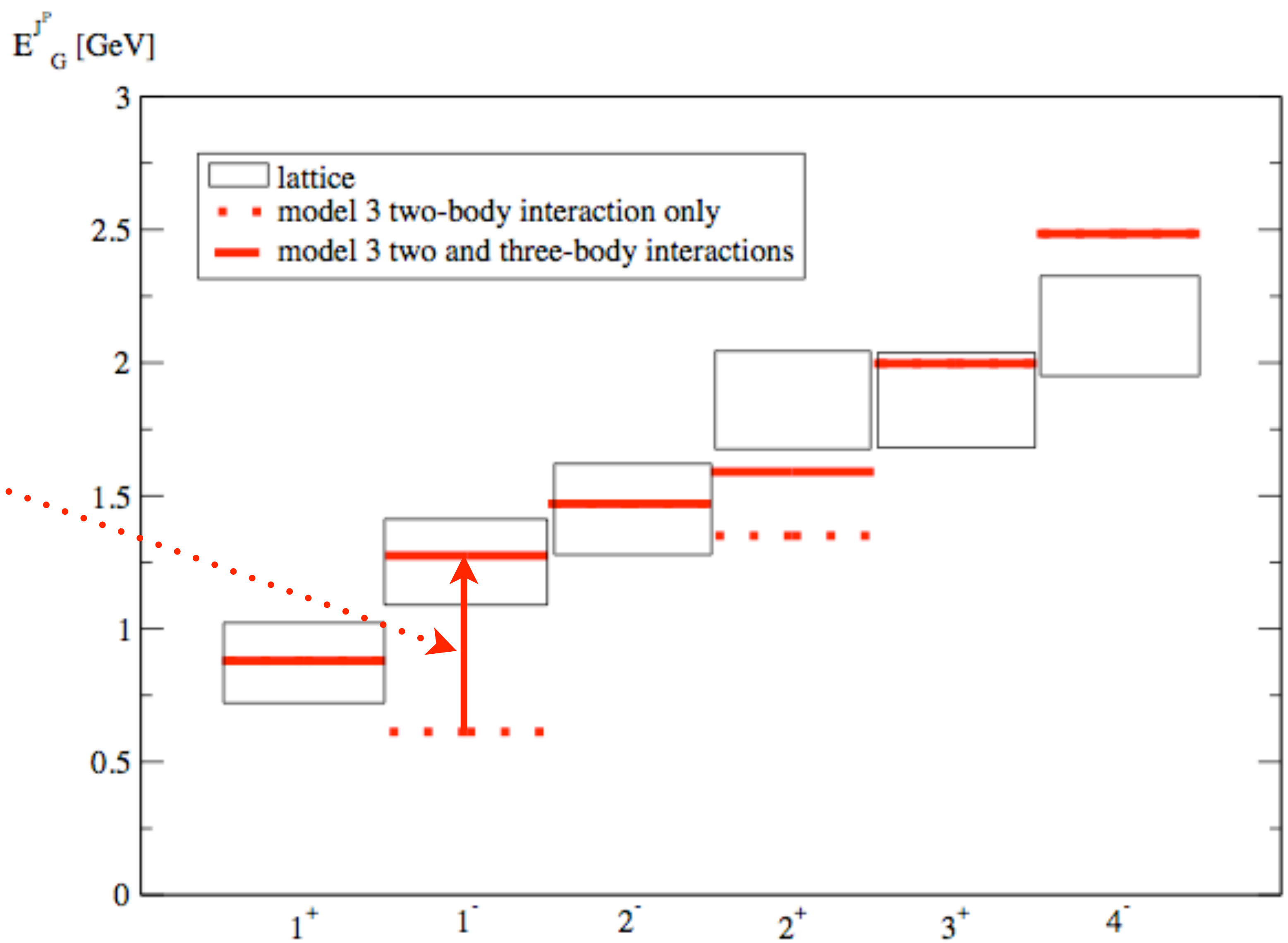
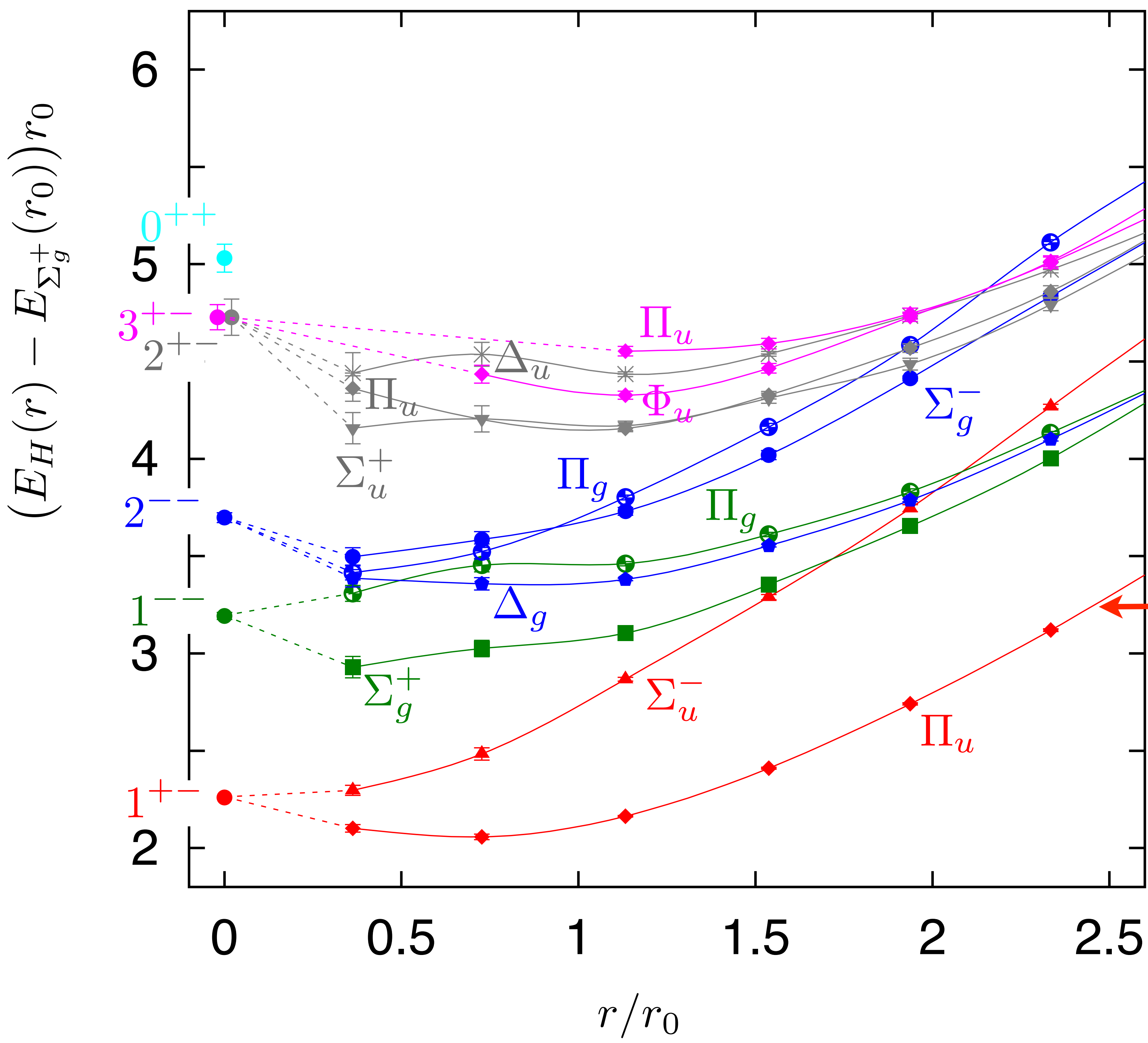
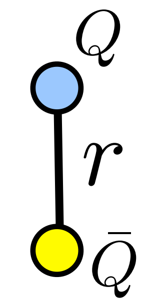



FIG. 2: Two -(left) and three- (right) body potential between the quasi-gluon and the $Q\bar{Q}$ pair. The static quark and the antiquark are represented by blobs. The dashed line represents the effective $V_{CL}$ potential (left) or the effective three-body interaction from Eq. (37)(right).



# adiabatic potentials




in Born-Oppenheimer approx. (slow quarks)

$(0,1,2)^{-+}$ $(0,1,2)^{+-}$
$1^{++}, 1^{--}$

"flux-tube" degeneracy pattern

? why does the short distance symmetry seem to dominate over the long ?

$r_0 \sim 0.5\,\text{fm}$

# bound state model constructions

fixed Fock state basis

$$\left|q\bar{q}(n^{2S+1}L_J)\right\rangle =$$

couple quark spins

couple spin and orb. ang. mom.

$$\sum_{s,t,m_S,m_L} \left\langle \tfrac{1}{2}s; \tfrac{1}{2}t \middle| S, m_S \right\rangle \left\langle S, m_S; L, m_L \middle| J, m_J \right\rangle$$

$$\int q^2 dq\, \varphi_{nL}(q) \int d\hat{q}\, Y_L^{m_L}(\hat{q})\, \left|q_s(\vec{q})\bar{q}_t(-\vec{q})\right\rangle$$

mom. space radial wavefunction

ang. behaviour fixed by ***L***-wave

---

$$\left|q\bar{q}\tilde{g}([J_g^{P_g}]^{2S+1}L_{J_q})_J\right\rangle \sim$$

couple quark ang. mom

couple quark to glue

$$\sum_{\text{spins}} \left\langle \tfrac{1}{2}s; \tfrac{1}{2}t \middle| S, m_S \right\rangle \left\langle S, m_S; L, m_L \middle| J_q, m_q \right\rangle \left\langle J_q, m_q; J_g, M_g \middle| J, M \right\rangle$$

gluon angular

$$\int k^2 dk\, q^2 dq\, \Psi(k,q) \int d\hat{q}\, d\hat{k}\, Y_L^{m_L}(\hat{q})\, \mathcal{D}^{(J_g)\star}_{M_g,-\sigma}(\hat{k})$$

$$\times \left[\delta_{\sigma,+} + P(-1)^{J_g+L+1}\delta_{\sigma,-}\right]$$

quark-gluon radial wavefunction

quark angular

$$\left|q_s(\tfrac{1}{2}\vec{k}+\vec{q})\, \bar{q}_t(\tfrac{1}{2}\vec{k}-\vec{q})\, \tilde{g}_\sigma(-\vec{k})\right\rangle$$