Hagedorn States in Relativistic Heavy Ion Collisions

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Excited Hadrons : February 25^{th}, 2011 : Jefferson Lab
Newport News, VA USA

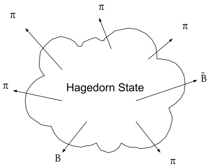
Outline

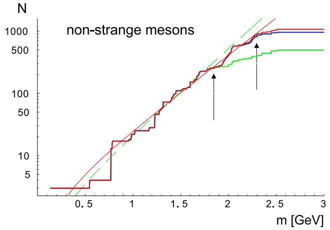
Hagedorn's Original Idea

Hagedorn States

"fireballs consist of fireballs, which consist of fireballs..."

- Proposed an exponentially increasing mass to explain spectra in $p-p$ and $\pi-p$ scattering
- Original model included hadronic states up to $\Delta(1232)$





Broniowski,Florkowski,Glozman,PRD70,117503(2004)

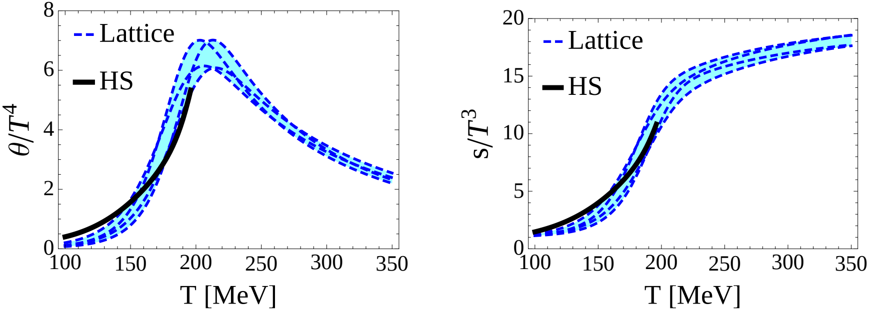
- Exponential mass spectrum
- Constant T_H: $\uparrow$ energy of system, $\uparrow$ new particles, NOT T_H
- Lead to Statistical Bootstrap Model:

$$\rho(M) = \int_{M_0}^{M} \frac{A}{\left[m^2 + (m_0)^2\right]^{\frac{5}{4}}} e^{\frac{m}{T_H}} dm$$

Comparison to Lattice Results

$T_H = 196$ MeV, $M = 15$ GeV, $M_0 = 2$ GeV, $A = 0.5\ \mathrm{GeV}^{\frac{3}{2}}$, $B = 250\ \mathrm{GeV}^4$, and $m_0 = 500$ MeV

Bielefeld-BNL-Columbia Collaboration (BBC)



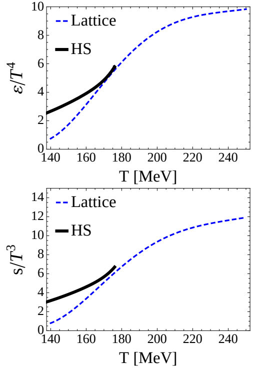
BNL et al,PRD77(2008)014511; PRD80(2009)014504

Comparison to Lattice Results

Budapest-Marseille-Wuppertal Collaboration (BMW)

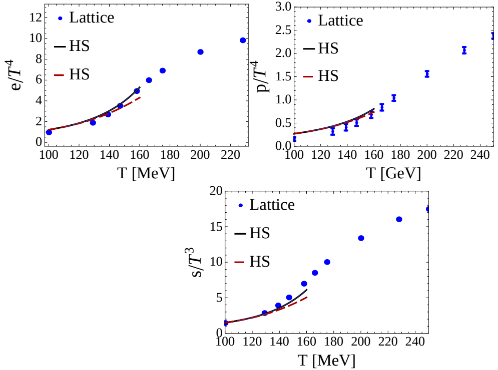
$$\rho(M) = \int_{2GeV}^{15GeV} \frac{0.5\ \mathrm{GeV}^{\frac{3}{2}}}{\left[m^2 + (0.5GeV)^2\right]^{\frac{5}{4}}} e^{\frac{m}{176MeV}}\, dm$$

$$\rho(M) = \int_{1.7GeV}^{2.5GeV} \frac{0.715}{252MeV} e^{m/252MeV}\, dm$$



-no volume corrections

Majumder and Muller,PRL105(2010)252002



Fodor et al, JHEP 0601, 089 (2006); JNH et al, PLB 643, 46 (2006)

JNH, Jorge Noronha, Carsten Greiner

Volume Corrections

$$p_{xv} = \frac{p_{pt}\left(T^*\right)}{1 - \frac{p_{pt}(T^*)}{4B}}$$

$$\varepsilon_{xv} = \frac{\varepsilon_{pt}\left(T^*\right)}{1 + \frac{\varepsilon_{pt}(T^*)}{4B}}$$

$$T = \frac{T^*}{1 - \frac{p_{pt}(T^*)}{4B}}$$

$$s_{xv} = \frac{s_{pt}\left(T^*\right)}{1 + \frac{\varepsilon_{pt}(T^*)}{4B}}$$

$$n_{xv} = \frac{n_{pt}\left(T^*\right)}{1 + \frac{\varepsilon_{pt}(T^*)}{4B}}$$

η/s in a Hadronic gas near T_c

JNH, Jorge Noronha, and Carsten Greiner, PRL103(2009)172302

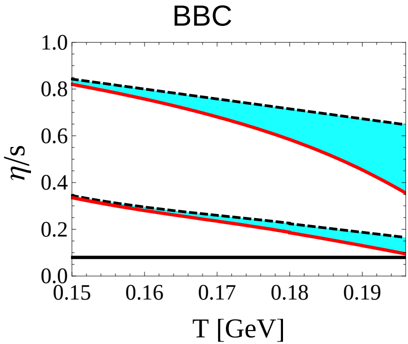
- η/s can be rewritten:

$$\left(\frac{\eta}{s}\right)_{tot} = \frac{\eta_{HG}+\eta_{HS}}{s_{HG}+s_{HS}} = \frac{s_{HG}}{s_{HG}+s_{HS}}\left[\left(\frac{\eta}{s}\right)_{HG}+\frac{\eta_{HS}}{s_{HG}}\right]$$

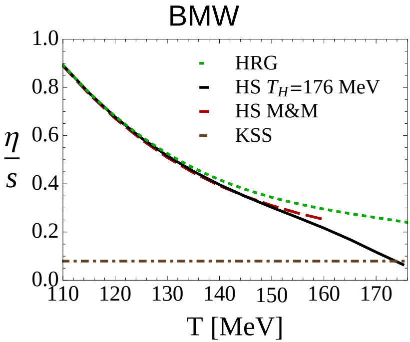
- From kinetic theory arguments: $\eta_{NR}=\frac{1}{3}\sum_i n_i\langle p\rangle_i\,\lambda_i$
- $\langle p_i\rangle = m_i\langle v_i\rangle = \sqrt{3T\,m_i}$
- $\lambda_i=\tau_i\,\langle v_i\rangle$
- $\langle v_i\rangle=\sqrt{\frac{3T}{m_i}}$
- $\tau_i=\frac{1}{\Gamma_i}\Rightarrow$ most conservative estimate!

$$\left(\frac{\eta}{s}\right)_{HS} = \frac{\sum_i T\,n_i\tau_i}{s_{HS}}. \tag{1}$$

Result: η/s



JNH, Jorge Noronha, and Carsten Greiner, PRL103(2009)172302



JNH, Jorge Noronha, Carsten Greiner

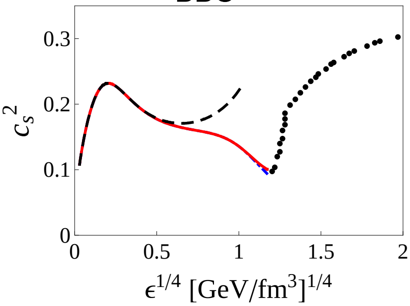
Because HS allow for η/s to drop to the KSS limit, it provides a smooth transition for hydro

Sufficiently near T_c, η/s can be close to the viscosity bound already in the hadronic phase!!!!

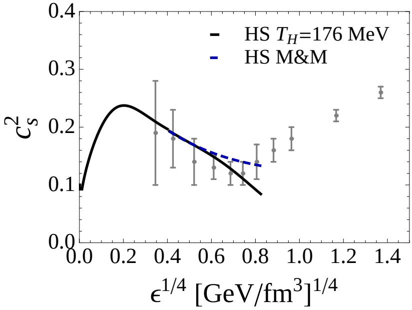
Theory: c_s^2

$c_s^2 = dp/d\varepsilon$

BBC



BMW



Note that c_s^2 does not go to zero.

Strangeness Enhancement

SPS

- SPS observed enhancement of anti-hyperons, multi-strange baryons, and kaons compared to pp-data
- Used binary collisions

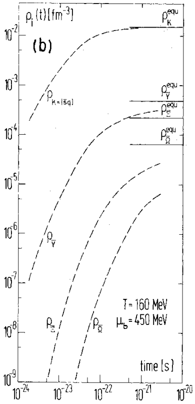
Binary strangeness production reactions

$$\pi + \bar{p} \leftrightarrow \bar{K} + \bar{\Lambda} \tag{2}$$

Binary strangeness exchange reactions

$$K + \bar{p} \leftrightarrow \pi + \bar{\Lambda} \tag{3}$$

- Gave small cross-sections $\rightarrow$ QGP! Because strange quarks produced more efficiently by gluon fusion.
P. Koch, B. Muller, and J. Rafelski



Strangeness enhancement was considered a signal for QGP!

Strangeness Enhancement
SPS

- Used multi-mesonic reactions

For anti-protons

$$\bar{p} + N \leftrightarrow n\pi \qquad (4)$$

R. Rapp and E. Shuryak

For anti-hyperons

$$\begin{aligned} \bar{\Sigma}, \bar{\Lambda} + N &\leftrightarrow n\pi + K \\ \bar{\Xi} + N &\leftrightarrow n\pi + 2K \\ \bar{\Omega} + N &\leftrightarrow n\pi + 3K \\ \bar{Y} + N &\leftrightarrow n\pi + n_{\bar{Y}} K \end{aligned} \qquad (5)$$

C. Greiner and S. Leupold.

- Giving the time scale

$$\tau_{\bar{Y}} := \frac{1}{\Gamma_{\bar{Y}}} = \frac{1}{\langle\langle \sigma_{N\bar{Y} \to n\pi + n_{\bar{Y}} K} v_{\bar{Y}N} \rangle\rangle \rho_B} \qquad (6)$$

assuming $\sigma_{\rho\bar{Y}} \approx \sigma_{\rho\bar{p}} \approx 50\ \mathrm{mb}$,
$\rho_B \approx 0.16 - 0.32 \frac{1}{\mathrm{fm}^3}$, and
$v \approx 0.5 - 0.6\ c$ (typical for SPS)

Time Scale

$$\tau_{\bar{Y}} \approx 1 - 3\ \frac{\mathrm{fm}}{c} \qquad (7)$$

- Fits within typical lifetime of fireball of 5-10 $\frac{\mathrm{fm}}{c}$!

Strangeness Enhancement

RHIC

At $T = 170$ MeV

- $\rho_B^{eq} = \rho_{\bar{B}}^{eq} \approx 0.04\ \mathrm{fm}^{-3}$
  $\langle \sigma v \rangle \approx 30\ \mathrm{mb\ c}$

Time Scale

$$\tau_{\bar{B}} \approx 10\ \frac{\mathrm{fm}}{c}. \qquad (8)$$

- Too large!!! In fireball $\tau \leq 4\ \frac{\mathrm{fm}}{c}$.

Suggestions

- Born in Equilibrium?
- Near T_c, extra large particle density overpopulated with pions and kaons?
  - Overpopulation of (anti-)baryons, which cannot be killed off
- Hagedorn resonances?

Contribution of HS to Chemical Equilibrium Values

Effective $X = p, K$, or Λ

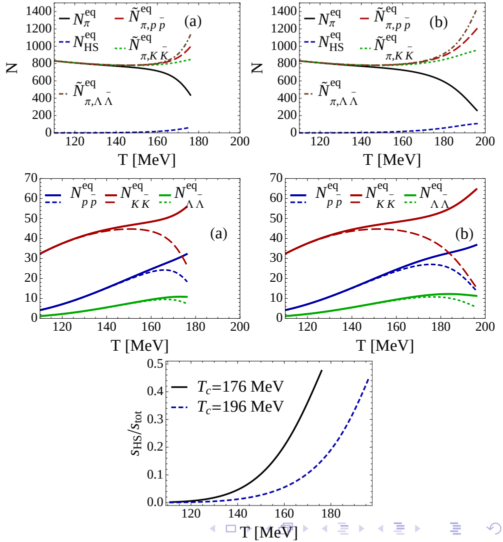
$$\tilde{N}_X = N_X + \sum_i N_i \langle X_i \rangle$$

Effective π's

$$\tilde{N}_\pi = N_\pi + \sum_i N_i \langle n_i \rangle$$

$\langle X_i \rangle$ and $\langle n_i \rangle$ are calculated within a microcanonical model

Liu, et.al. PRC68(2003)024905, JPG30(2004)S589, PRC69(2004)054002



Rate Equations for the Chem. Eq. Time of Hadrons

$n\pi \leftrightarrow HS \leftrightarrow n'\pi + X\bar{X}$

$$\frac{d\lambda_i}{dt} = \Gamma_{i,\pi}\left(\sum_n^{\infty} B_{i,n}\lambda_\pi^n - \lambda_i\right) + \Gamma_{i,X\bar{X}}\left(\lambda_\pi^{\langle n_{i,x}\rangle}\lambda_{X\bar{X}}^2 - \lambda_i\right),$$

$$\frac{d\lambda_\pi}{dt} = \sum_i \Gamma_{i,\pi}\frac{N_i^{eq}}{N_\pi^{eq}}\left(\lambda_i\langle n_i\rangle - \sum_n^{\infty} B_{i,n} n \lambda_\pi^n\right) + \sum_i \Gamma_{i,X\bar{X}}\langle n_{i,x}\rangle\frac{N_i^{eq}}{N_\pi^{eq}}\left(\lambda_i - \lambda_\pi^{\langle n_{i,x}\rangle}\lambda_{X\bar{X}}^2\right),$$

$$\frac{d\lambda_{X\bar{X}}}{dt} = \sum_i \Gamma_{i,X\bar{X}}\frac{N_i^{eq}}{N_{X\bar{X}}^{eq}}\left(\lambda_i - \lambda_\pi^{\langle n_{i,x}\rangle}\lambda_{X\bar{X}}^2\right)$$

$\lambda = \frac{N}{N^{eq}}$, N is the total number of each particle, its equilibrium value is N^{eq}.

Time Scale Estimates

Naively, we would assume $N_\pi \approx N_\pi^{eq}$ and $N_i \approx N_i^{eq}$, then

$$\frac{d\lambda_{X\bar{X}}}{dt} = \sum_i \Gamma_{i,X\bar{X}} \frac{N_i^{eq}}{N_{X\bar{X}}^{eq}} \left(\lambda_i - \lambda_\pi^{\langle n_{i,x}\rangle} \lambda_{X\bar{X}}^2\right)$$

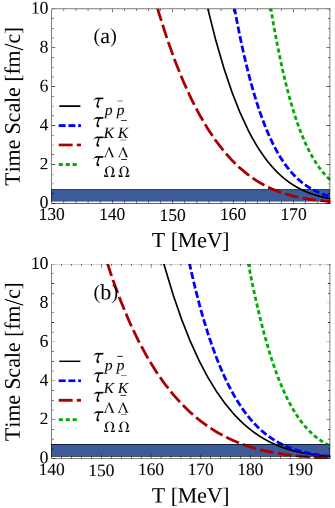
$$= \sum_i \Gamma_{i,X\bar{X}} \frac{N_i^{eq}}{N_{X\bar{X}}^{eq}} \left(1 - \lambda_{X\bar{X}}^2\right)$$

$$\lambda_{X\bar{X}} = \frac{\left(\frac{\phi+1}{\phi-1}\right)\exp\left(\frac{2t}{\tau_{X\bar{X}}}\right)+1}{\left(\frac{\phi+1}{\phi-1}\right)\exp\left(\frac{2t}{\tau_{X\bar{X}}}\right)-1}$$

where $\phi := \lambda_{X\bar{X}}(0)$ and

$$\tau_{X\bar{X}} := \frac{N_{X\bar{X}}^{eq}}{\sum_i \Gamma_{i,X\bar{X}} N_i^{eq}} < 1\frac{fm}{c}$$

Only true when the pions and the resonances are held in equilibrium!



Fireball Expansion

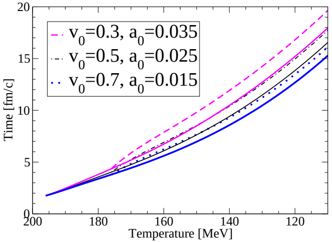
Use an isentropic expansion...

Find T(t) for the 5% most central collisions

$$\frac{S_\pi}{N_\pi}\int \frac{dN_\pi}{dy}dy = s(T)V(t) = const.$$

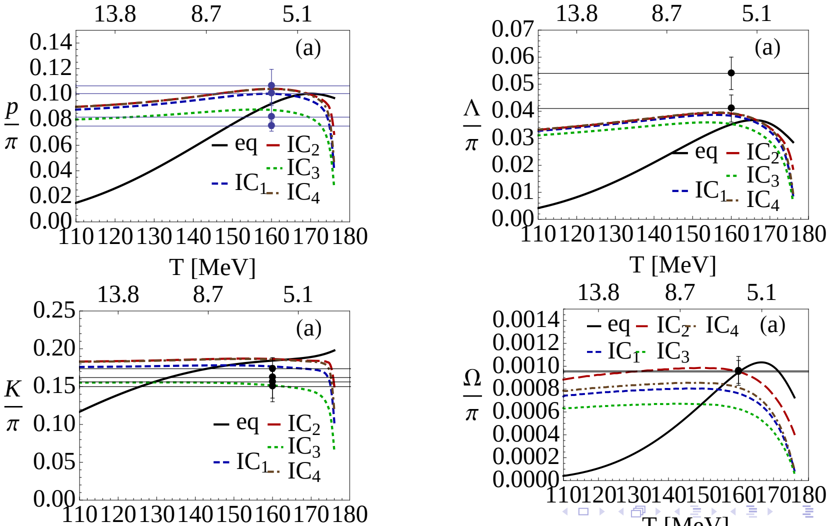
Volume

$$V_{eff}(t \geq t_0) = \pi\, ct\left(r_0 + v_0(t-t_0) + .5a_0(t-t_0)^2\right)^2$$



Particle Ratios

$T_H = 176$ MeV



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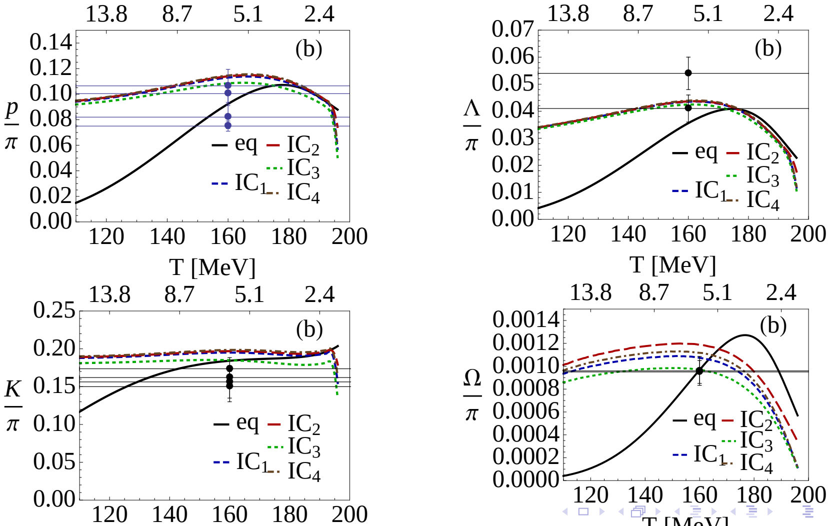
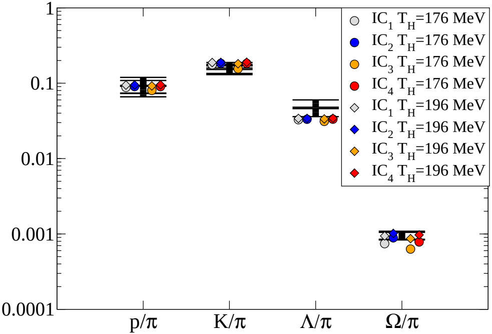
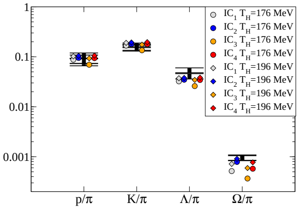


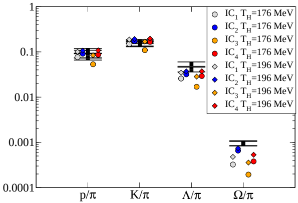
Summary Graph: Dynamic Chem. Eq. with HS



Dividing Γ_i by 2



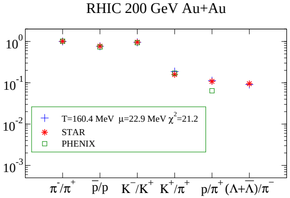
Dividing Γ_i by 4



Comparison of Thermal Fit with Hagedorn States

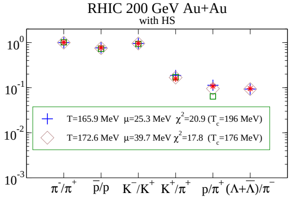
JNH, et al.,PRC82(2010)024913

No Hagedorn States



- $\chi^2 = 21.06$ Fit without Hagedorn States
- Matches other thermal fit models well: $T_{ch} = 155 - 169$ MeV and $\mu_b = 20 - 30$ MeV (PLB518,41(2001);PRC65,064905(2002); arXiv:nucl-th/0405068; Nucl.Phys.A757,102(2005); Nucl. Phys. A 772, 167 (2006) PRC78,054901(2008))

Hagedorn States



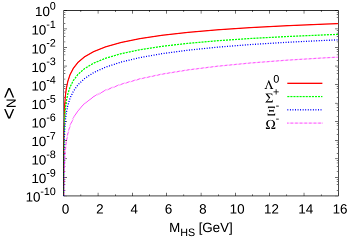
- $\chi^2 = 17.8$ Fit with $T_H = 176$ MeV
- $\chi^2 = 20.9$ Fit with $T_H = 196$ MeV

Quest for Branching Ratios

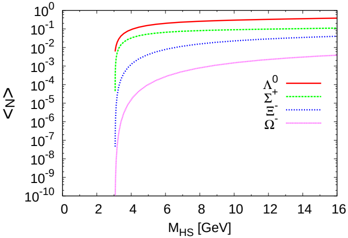
BSQ-Canonical Model: M. Beitel, JNH, C. Greiner

- Up until now we have been limited by the branching ratios of the Hagedorn states
- Using a canonical model that conserves baryon number (B), strangeness (S), and charge (Q), we are able to calculate the average number of X that a large resonance can decay into.

Mesonic, non-strange cluster at T=160 MeV



B=2, S=0, Q=2 cluster at T=160 MeV



Conclusions

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- We showed that the exponentially increasing Hagedorn spectrum (a property of QCD) may already account for the near perfect fluid behavior of hadronic matter close to T_c
- Hagedorn states are catalysts for quick dynamical reactions that can explain short chemical equilibrium times at RHIC, consistent with thermal fits. Thus, the hadrons do not need to be "born in equilibrium."
- As of yet they have shown little effects on the thermal fits.

Outlook

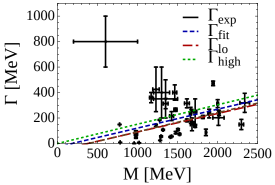
Outlook

- Include Hagedorn States in transport models (such as UrQMD)
- Consider effects of Hagedorn states at larger baryonic chemical potential (QCD critical point???)
- Consider strange and/or baryonic Hagedorn states (either for chemical equilibrium times or at large μ_b)
- Is strangeness enhancement really a signature of QGP or can it be described entirely by dynamical reactions within the hadronic phase?

Decay Width

- Linear fit (PDG)

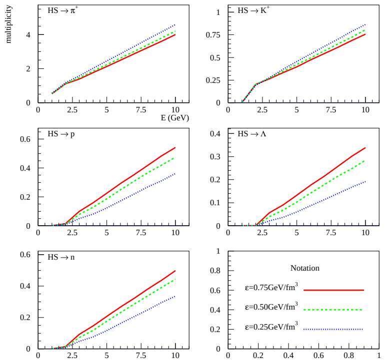
$$\Gamma_i = 0.15 m_i - 58$$
$$= 250 - 1000 \text{ MeV}$$



- $X\bar{X}$ (microcanonical)

$$\Gamma_{i,X\bar{X}} = \langle X \rangle \Gamma_i$$

$$\Gamma_{i,\pi} = \Gamma_i - \Gamma_{i,X\bar{X}}$$



C. Greiner et al., J.Phys.G31:S725-S732,2005.

$$\langle B \rangle \approx 0.06 \text{ to } 0.4$$
$$\langle K \rangle \approx 0.4 \text{ to } 0.5$$
$$\langle \Lambda \rangle \approx 0.01 \text{ to } 0.2$$

Branching Ratios

- Branching ratios for $n\pi \leftrightarrow HS$ are described by a Gaussian distribution

$$B_{i,n} \approx \frac{1}{\sigma_i\sqrt{2\pi}} e^{-\frac{(n-\langle n_i \rangle)^2}{2\sigma_i^2}}$$

- Average pion number (Liu, Werner, Aichelin, Phys. Rev. C 68, 024905 (2003).)

$$\langle n_i \rangle = 0.9 + 1.2\frac{m_i}{m_p}$$

- Standard deviation

$$\sigma_i^2 = (0.5\frac{m_i}{m_p})^2$$

- After cutoff $n \geq 2$, $\langle n_i \rangle \approx 3$ to 9 and $\sigma_i^2 \approx 0.8$ to 11
- For $HS \leftrightarrow n'\pi + X\bar{X}$, $\langle n_{i,x} \rangle = 2 - 4$