

Extended schematic model for hadrons
and
what happens to the radius
of an excited hadron

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- classification of hadrons: traditional quark model
 - mesons: $q\bar{q}$
 - baryons: qqq
- not everything fits \implies "exotic mesons"
 - light scalar nonet
 - some charmed mesons
- use diquarks \mathcal{Q} as building blocks
- but diquarks were never used *systematically* as a classification tool for *all known mesons*...

Idea: construct a [schematic model](#) with quarks and diquarks as building blocks to reclassify mesons, and see what we might learn about QCD in the process.

arXiv:0910.2229

... .. I: Diquarks and Classification of Mesons

Which diquarks?

- interquark forces not known at low energies
- derive diquark building blocks from meson phenomenology:

$$Q\bar{Q} = \text{flavor nonets} \implies Q \text{ is flavor-antisymmetric}$$

Table: [Diquark configurations](#)

	<i>Flavor</i>		<i>Spin</i>	<i>Color</i>	\mathcal{H}_{CM}	\mathcal{H}_{CE}
	$SU(3)_f$	$SU(4)_f$	$SU(2)_s$	$SU(3)_c$		
Q_1	$\bar{\mathbf{3}}_f(A)$	$\bar{\mathbf{6}}_f(A)$	$\mathbf{1}_s(A)$	$\bar{\mathbf{3}}_c(A)$	-8	-8/3
Q_2	$\bar{\mathbf{3}}_f(A)$	$\bar{\mathbf{6}}_f(A)$	$\mathbf{3}_s(S)$	$\mathbf{6}_c(S)$	-4/3	4/3
Q_3	$\mathbf{6}_f(S)$	$\mathbf{10}_f(S)$	$\mathbf{3}_s(S)$	$\bar{\mathbf{3}}_c(A)$	8/3	-8/3
Q_4	$\mathbf{6}_f(S)$	$\mathbf{10}_f(S)$	$\mathbf{1}_s(A)$	$\mathbf{6}_c(S)$	4	4/3

Meson quantum numbers

- Color

$$\begin{aligned} q\bar{q} &: \mathbf{3}_c \otimes \bar{\mathbf{3}}_c = \mathbf{8}_c \oplus \mathbf{1}_c & SU(3)_c, \\ Q_1\bar{Q}_1 &: \bar{\mathbf{3}}_c \otimes \mathbf{3}_c = \mathbf{8}_c \oplus \mathbf{1}_c & SU(3)_c, \\ Q_2\bar{Q}_2 &: \mathbf{6}_c \otimes \bar{\mathbf{6}}_c = \mathbf{27}_c \oplus \mathbf{8}_c \oplus \mathbf{1}_c & SU(3)_c, \end{aligned}$$

- Flavor

$$\begin{aligned} q\bar{q} &: \mathbf{3}_f \otimes \bar{\mathbf{3}}_f = \mathbf{8}_f \oplus \mathbf{1}_f & SU(3)_f, \\ Q_i\bar{Q}_i &: \bar{\mathbf{3}}_f \otimes \mathbf{3}_f = \mathbf{8}_f \oplus \mathbf{1}_f & SU(3)_f, \end{aligned}$$

- J^{PC}

$$J = L \otimes S .$$

$$\begin{aligned} q\bar{q} &: P = (-1)^{L+1}, \quad C = (-1)^{L+S} \\ Q_i\bar{Q}_i &: P = (-1)^L, \quad C = (-1)^{L+S} . \end{aligned}$$

Tables 2a, 2b, 2c: Meson quantum numbers for $q\bar{q}$, $Q_1\bar{Q}_1$, and $Q_2\bar{Q}_2$ up to $L = 3$.

Table 2a: $q\bar{q}$				
L	S	J^{PC}	$^{2S+1}L_J$	
0	0	0^{-+}	1S_0	$\sqrt{\bullet}$
0	1	1^{--}	3S_1	$\sqrt{\bullet}$
1	0	1^{+-}	1P_1	$\sqrt{\bullet}$
1	1	2^{++}	3P_2	$\sqrt{\bullet}$
		1^{++}	3P_1	$\sqrt{\bullet}$
		0^{++}	3P_0	$\sqrt{\bullet}$
2	0	2^{-+}	1D_2	$\sqrt{\bullet}$
2	1	3^{--}	3D_3	$\sqrt{\bullet}$
		2^{--}	3D_2	$\sqrt{}$
		1^{--}	3D_1	$\sqrt{\bullet}$
3	0	3^{+-}	1F_3	$\sqrt{}$
3	1	4^{++}	3F_4	$\sqrt{\bullet}$
		3^{++}	3F_3	
		2^{++}	3F_2	$\sqrt{\bullet}$

Table 2b: $Q_1\bar{Q}_1$				
L	S	J^{PC}	$^{2S+1}L_J$	
0	0	0^{++}	1S_0	$\sqrt{\bullet}$
1	0	1^{--}	1P_1	$\sqrt{\bullet}$
2	0	2^{++}	1D_2	$\sqrt{}$
3	0	3^{--}	1F_3	$\sqrt{}$

Table 2c: $Q_2\bar{Q}_2$				
L	S	J^{PC}	$^{2S+1}L_J$	
0	0	0^{++}	1S_0	
0	1	1^{+-}	3S_1	
0	2	2^{++}	5S_2	
1	0	1^{--}	1P_1	
1	1	2^{-+}	3P_2	$\sqrt{\bullet}$
		1^{-+}	3P_1	$\sqrt{}$
		0^{-+}	3P_0	$\sqrt{\bullet}$
1	2	3^{--}	5P_3	
		2^{--}	5P_2	$\sqrt{\bullet}$
		1^{--}	5P_1	$\sqrt{}$
		0^{--}	5P_0	
2	0	2^{++}	1D_2	$\sqrt{}$
2	1	3^{+-}	3D_3	
		2^{+-}	3D_2	
		1^{+-}	3D_1	$\sqrt{}$
2	2	4^{++}	5D_4	
		3^{++}	5D_3	
		2^{++}	5D_2	$\sqrt{}$
		1^{++}	5D_1	$\sqrt{}$
		0^{++}	5D_0	$\sqrt{}$

Table 2c, cont'd				
L	S	J^{PC}	$^{2S+1}L_J$	
3	0	3^{--}	1F_3	$\sqrt{}$
3	1	4^{-+}	3F_4	$\sqrt{}$
		3^{-+}	3F_3	
		2^{-+}	3F_2	$\sqrt{}$
3	2	5^{--}	5F_5	
		4^{--}	5F_4	
		3^{--}	5F_3	
		2^{--}	5F_2	
		1^{--}	5F_1	

Classification of mesons:

- arrange mesons from PDG into flavor multiplets
- to each multiplet assign
 - meson type: $q\bar{q}$, $Q_1\bar{Q}_1$, or $Q_2\bar{Q}_2$
 - L and S numbers

Table 3a: Our suggested assignments for observed light mesons. Compare with Table 14.2 in the PDG.

J^{PC}	constituents	$^{2S+1}L_J$	$I = 1$	$I = \frac{1}{2}$	$I = 0$		$n^{2S+1}L_J(PDG)$
0^{-+}	$q\bar{q}$	1S_0	$\bullet\pi$	$\bullet K$	$\bullet\eta$	$\bullet\eta'(958)$	1^1S_0
0^{-+}	$Q_2\bar{Q}_2$	3P_0	$\bullet\pi(1300)$	$K(1460)$	$\bullet\eta(1475)$	$\bullet\eta(1295)$	2^1S_0
0^{++}	$Q_1\bar{Q}_1$	1S_0	$\bullet a_0(980)$	$\kappa(800)$	$\bullet f_0(980)$	$\bullet f_0(600)$	—
0^{++}	$q\bar{q}$	3P_0	$\bullet a_0(1450)$	$\bullet K_0^*(1430)$	$\bullet f_0(1710)$	$\bullet f_0(1370)$	1^3P_0
0^{++}	$Q_2\bar{Q}_2$	5D_0		$K_0^*(1950)$	$f_0(2100)$	$\bullet f_0(2020)$	—
1^{--}	$q\bar{q}$	3S_1	$\bullet\rho(770)$	$\bullet K^*(892)$	$\bullet\phi(1020)$	$\bullet\omega(782)$	1^3S_1
1^{--}	$Q_1\bar{Q}_1$	1P_1	$\bullet\rho(1450)$	$\bullet K^*(1410)$	$\bullet\phi(1680)$	$\bullet\omega(1420)$	2^3S_1
1^{--}	$Q_2\bar{Q}_2$	5P_1	$\rho(1570)$				—
1^{--}	$q\bar{q}$	3D_1	$\bullet\rho(1700)$	$\bullet K^*(1680)$	$\bullet\omega(1650)$		1^3D_1
1^{--}	$Q_2\bar{Q}_2$	5F_1	$\rho(2150)$				—
1^{-+}	$Q_2\bar{Q}_2$	3P_1	$\bullet\pi_1(1600)$	$K(1630)$			—
1^{++}	$q\bar{q}$	3P_1	$\bullet a_1(1260)$	$\bullet K_1(1400)$	$\bullet f_1(1420)$	$\bullet f_1(1285)$	1^3P_1
1^{++}	$Q_2\bar{Q}_2$	5D_1	$a_1(1640)$	$K_1(1650)$	$f_1(1510)$		—
1^{+-}	$q\bar{q}$	1P_1	$\bullet b_1(1235)$	$\bullet K_1(1270)$	$h_1(1380)$	$\bullet h_1(1170)$	1^1P_1
1^{+-}	$Q_2\bar{Q}_2$	3D_1			$h_1(1595)$		—
2^{-+}	$Q_2\bar{Q}_2$	3P_2	$\bullet\pi_2(1670)$	$K_2(1580)$	$\eta_2(1870)$	$\bullet\eta_2(1645)$	1^1D_2
2^{-+}	$q\bar{q}$	1D_2	$\bullet\pi_2(1880)$				—
2^{-+}	$Q_2\bar{Q}_2$	3F_2	$\pi_2(2100)$	$K_2(2250)$			—
2^{--}	$Q_2\bar{Q}_2$	5P_2		$\bullet K_2(1770)$			—
2^{--}	$q\bar{q}$	3D_2		$\bullet K_2(1820)$			1^3D_2

J^{PC}	constituents	$^{2S+1}L_J$	$I = 1$	$I = \frac{1}{2}$	$I = 0$	$n^{2S+1}L_J(PDG)$
2^{++}	$q\bar{q}$	3P_2	$\bullet a_2(1320)$	$\bullet K_2^*(1430)$	$f_2(1430)$ $\bullet f_2(1270)$	1^3P_2
2^{++}	$Q_2\bar{Q}_2$	1D_2			$\bullet f_2'(1525)$	—
2^{++}	$Q_1\bar{Q}_1$	1D_2	$a_2(1700)$		$f_2(1640)$ $f_2(1565)$	—
2^{++}	$Q_2\bar{Q}_2$	5D_2			$f_2(1810)$	—
2^{++}	$q\bar{q}$	3F_2		$K_2^*(1980)$	$\bullet f_2(2010)$ $\bullet f_2(1950)$	—*
3^{--}	$q\bar{q}$	3D_3	$\bullet \rho_3(1690)$	$\bullet K_3(1780)$	$\bullet \phi_3(1850)$ $\bullet \omega_3(1670)$	1^3D_3
3^{--}	$Q_1\bar{Q}_1$	1F_3	$\rho_3(1990)$			—
3^{--}	$Q_2\bar{Q}_2$	1F_3	$\rho_3(2250)$			—
3^{+-}	$q\bar{q}$	1F_3		$K_3(2320)$		—
4^{-+}	$Q_2\bar{Q}_2$	3F_4		$K_4(2500)$		—
4^{++}	$q\bar{q}$	3F_4	$\bullet a_4(2040)$	$\bullet K_4^*(2045)$	$f_4(2220)$ $\bullet f_4(2050)$	1^3F_4
5^{--}	$Q_2\bar{Q}_2$	5F_5	$\rho_5(2350)$	$K_5^*(2380)$		1^3G_5
6^{++}	$Q_2\bar{Q}_2$	5G_6	$a_6(2450)$		$f_6(2510)$	1^3H_6

* This nonet was classified as 2^3P_2 , a radial excitation, between 1992 and 2002.

Table 3b: Our suggested assignments for observed heavy (charm and bottom) mesons. Compare with Table 14.3 in the PDG.

J^{PC}	constituents	$2^{S+1}L_J$	Charmed mesons				Bottom mesons				$n^{2S+1}L_J$ (PDG)
			$I = 1^\circ$	$I = \frac{1}{2}$	$I = 0$	$I = 0$	$I = 1^\circ$	$I = \frac{1}{2}$	$I = 0$	$I = 0$	
0^{-+}	$q\bar{q}$	1S_0	$\bullet D$	$\bullet D_s^\#$	$\bullet \eta_c(1S)$		$\bullet B^\dagger$	$\bullet B_s^\dagger, B_c^\dagger$	$\eta_b(1S)^\dagger$	1^3S_0	
0^{-+}	$Q_2\bar{Q}_2$	3P_0			$\bullet \eta_c(2S)^\dagger$					2^1S_0	
0^{++}	$Q_1\bar{Q}_1$	1S_0	$D_0^*(2400)^\#$	$\bullet D_{s0}^*(2317)$	$\bullet \chi_{c0}(1P)$				$\bullet \chi_{b0}(1P)$	1^3P_0	
0^{++}	$q\bar{q}$	3P_0							$\chi_{b0}(2P)^{\dagger\dagger}$	2^3P_0	
1^{--}	$q\bar{q}$	3S_1	$\bullet D^*$	$\bullet D_s^{*\#}$	$\bullet J/\psi(1S)$		$\bullet B^{*\dagger}$	$B_s^{*\dagger}$	$\bullet \Upsilon(1S)$	1^3S_1	
1^{--}	$Q_1\bar{Q}_1$	1P_1			$\bullet \psi(2S)$				$\bullet \Upsilon(2S)$	2^3S_1	
1^{--}	$q\bar{q}$	3D_1			$\bullet \psi(3770)$				$\bullet \Upsilon(3S)$	—*	
1^{--}	$Q_2\bar{Q}_2$	5F_1			$\bullet \psi(4040)$				$\bullet \Upsilon(4S)$	—*	
1^{++}	$q\bar{q}$	3P_1	$D_1(2420)$	$\bullet D_{s1}(2536)^\#$	$\bullet \chi_{c1}(1P)$		$\bullet B_1(5721)^{0\dagger}$	$\bullet B_{s1}(5830)^{0\dagger}$	$\bullet \chi_{b1}(1P)^{\dagger\dagger}$	—**	
1^{++}	$Q_2\bar{Q}_2$	5D_1		$\bullet D_{s1}(2460)$	$\bullet X(3872)^\#\#$				$\bullet \chi_{b1}(2P)^{\dagger\dagger}$	—**	
2^{++}	$q\bar{q}$	3P_2	$\bullet D_2^*(2460)$	$\bullet D_{s2}(2573)^\#$	$\bullet \chi_{c2}(1P)$		$\bullet B_2^*(5747)^{0\dagger}$	$\bullet B_{s2}^*(5840)^\dagger$	$\bullet \chi_{b2}(1P)^{\dagger\dagger}$	$1^3P_2^{**}$	
2^{++}	$Q_1\bar{Q}_1$	1D_2			$\chi_{c2}(2P)$				$\bullet \chi_{b2}(2P)^{\dagger\dagger}$	2^3P_2	

Table 3c: Isorons and magic J^{PC} .

Isorons						
	0^{-+}	0^{++}	1^{--}	1^{-+}	2^{++}	4^{++}
Light	<ul style="list-style-type: none"> •$\eta(1405)$ $\eta(1760)$ •$\pi(1800)$ $K(1830)$ $\eta(2225)$ 	<ul style="list-style-type: none"> •$f_0(1500)$ $f_0(2200)$ $f_0(2330)$ 	<ul style="list-style-type: none"> $\rho(1900)$ 	<ul style="list-style-type: none"> •$\pi_1(1400)$ 	<ul style="list-style-type: none"> $f_2(1910)$ $f_2(2150)$ •$f_2(2300)$ •$f_2(2340)$ 	<ul style="list-style-type: none"> $f_4(2300)$
Heavy			<ul style="list-style-type: none"> •$\psi(4160)$ •$X(4260)$ $X(4360)$ •$\psi(4415)$ $\Upsilon(10860)$ $\Upsilon(11020)$ 			
Glueballs						
	0^{-+}	0^{++}	1^{--}	1^{-+}	2^{++}	4^{++}
	X	X			X	

- isorons
- magic numbers and glueballs
- no "exotics"

former exotic mesons:

- the "cryptoexotic" light scalar nonet with $J^{PC} = 0^{++}$ is a $Q_1\bar{Q}_1, {}^1S_0$
- a manifestly "exotic" meson with $J^{PC} = 1^{-+}$ is a $Q_2\bar{Q}_2, {}^3P_1$
- some newly discovered charmed mesons including:
 - * the $D_{sJ}^*(2317)$ with $J^{PC} = 0^{++}$ is a $Q_1\bar{Q}_1, {}^1S_0$;
 - * the $D_{sJ}(2460)$ with $J^{PC} = 1^{++}$ is a $Q_2\bar{Q}_2, {}^5D_1$;
 - * the $X(3872)$ with $J^{PC} = 1^{++}$ is a $Q_2\bar{Q}_2, {}^5D_1$.

former outcasts:

- some heavier scalar mesons with $J^{PC} = 0^{++}$ now form a nonet which is classified as $Q_2\bar{Q}_2, {}^5D_0$;
- some vector mesons with $J^{PC} = 1^{++}$ are now $Q_2\bar{Q}_2, {}^5D_1$;
- some 2^{++} mesons which are now $Q_1\bar{Q}_1, {}^1D_2$.

- no radials

- the second 0^{-+} nonet, previously a radial excitation with $n^{2S+1}L_J = 2^1S_0$, is a $Q_2\bar{Q}_2$ with $^{2S+1}L_J = {}^3P_0$;
- the second 1^{--} nonet, previously a radial excitation with $n^{2S+1}L_J = 2^3S_1$, is a $Q_1\bar{Q}_1$ with $^{2S+1}L_J = {}^1P_1$.

Baryon sector: which diquarks for baryons?

color singlets \implies color antisymmetric

Table: [Diquark configurations](#)

	<i>Flavor</i>		<i>Spin</i>	<i>Color</i>	\mathcal{H}_{CM}	\mathcal{H}_{CE}
	$SU(3)_f$	$SU(4)_f$	$SU(2)_s$	$SU(3)_c$		
Q_1	$\bar{\mathbf{3}}_f(A)$	$\bar{\mathbf{6}}_f(A)$	$\mathbf{1}_s(A)$	$\bar{\mathbf{3}}_c(A)$	-8	$-8/3$
Q_2	$\bar{\mathbf{3}}_f(A)$	$\bar{\mathbf{6}}_f(A)$	$\mathbf{3}_s(S)$	$\mathbf{6}_c(S)$	$-4/3$	$4/3$
Q_3	$\mathbf{6}_f(S)$	$\mathbf{10}_f(S)$	$\mathbf{3}_s(S)$	$\bar{\mathbf{3}}_c(A)$	$8/3$	$-8/3$
Q_4	$\mathbf{6}_f(S)$	$\mathbf{10}_f(S)$	$\mathbf{1}_s(A)$	$\mathbf{6}_c(S)$	4	$4/3$

– Ropers

$1/2^+$: $N(1440)$, $\Lambda(1600)$, $\Sigma(1660)$;

$1/2^+$: $N(1710)$, $\Lambda(1810)$, $\Sigma(1880)$;

$3/2^+$: $\Delta(1600)$.

Ropers as pentaquarks, $Q_1 Q_1 \bar{q}$ P-waves

\implies There are no radial excitations in the hadron spectrum

[The title of the paper was:

No Radial Excitations in Low-Energy QCD I: Diquarks and
Classification of Mesons (arXiv: 0910.2229)]

● PDG - 1960's: $2S+1L_J$

● PDG - 1980's: $n^{2S+1}L_J$

● PDG 2004

● experiment

What happens to the radius of hadrons
when they are excited?

What happens to the radius of hadrons
when they are excited?

- The radius of a hadron is largest when the hadron is in its ground state.

Table: Measured sizes of ground state ($L = 0$) hadrons

Mesons				
	Mass (MeV)	Radius (fm)	Density (g/cm ³)	Source
π^\pm	140	.672	$.20 \times 10^{15}$	PDG
K^\pm	494	.560	1.2×10^{15}	PDG

Baryons				
	Mass (MeV)	Radius (fm)	Density (g/cm ³)	Source
p	938	.87	$.61 \times 10^{15}$	PDG
Σ^-	1197	.78	1.1×10^{15}	PDG
Δ	1382	.650	2.1×10^{15}	Lattice
	1425	.632	2.4×10^{15}	Lattice
	1470	.614	2.7×10^{15}	Lattice

What happens to the radius of hadrons when they are excited?

- The radius of a hadron is largest when the hadron is in its ground state.
- The radius of a hadron decreases when the hadron's orbital excitation increases.

What happens to the radius of hadrons
when they are excited?

- The radius of a hadron is largest when the hadron is in its ground state.
- The radius of a hadron decreases when the hadron's orbital excitation increases.

Opposite of atoms:

The radius of an atom is **smallest** in the ground state

The radius of an atom **increases** with L

- Corollary: the path from confinement to asymptotic freedom is a Regge trajectory

– simple testable prediction about a fundamental property of hadrons: their size

– Request to lattice QCD people: please compute more radii!

– Request to experimentalists: please measure more radii!

The title was:

No Radial Excitations in Low-Energy QCD II:

[The Shrinking Radius of Hadrons](#) (arXiv: 0910.2231)

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”[Shrinking the proton](#)”

Physics Today on size of protons (August 2010):

”... the proton is a quark composite whose [size and shape](#)
[are quantum-chromodynamic manifestations](#) beyond the purview
of QED.”

Summary:

Diquarks and quarks as building blocks

\implies reclassification of hadron spectrum

\implies no radial quantum number

\implies shrinking radius of hadrons