11th Workshop on Hadron physics in China and Opportunities Worldwide

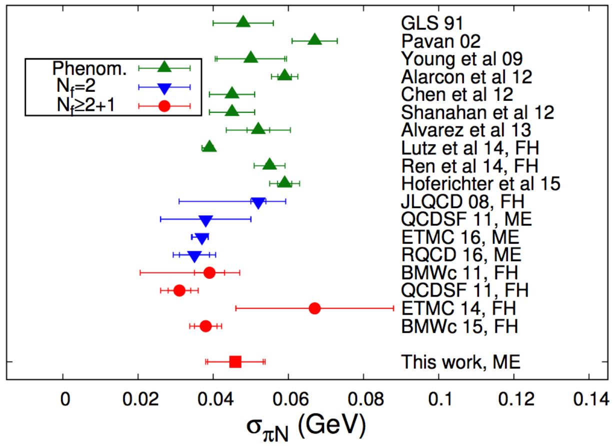
Proton mass from Lattice QCD

Yi-Bo Yang

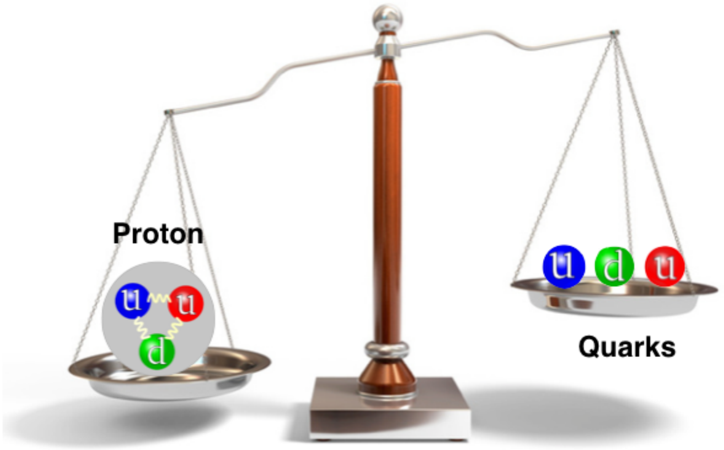
Tianjin, Nankai

Aug. 25th, 2019

Outline

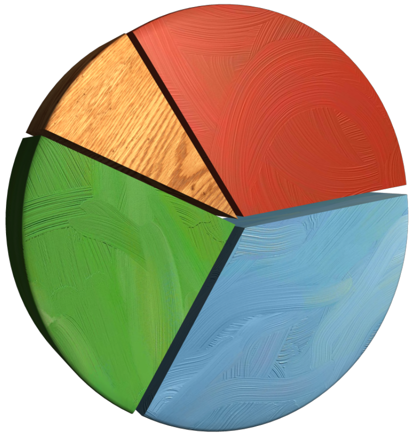


Quark mass and proton mass

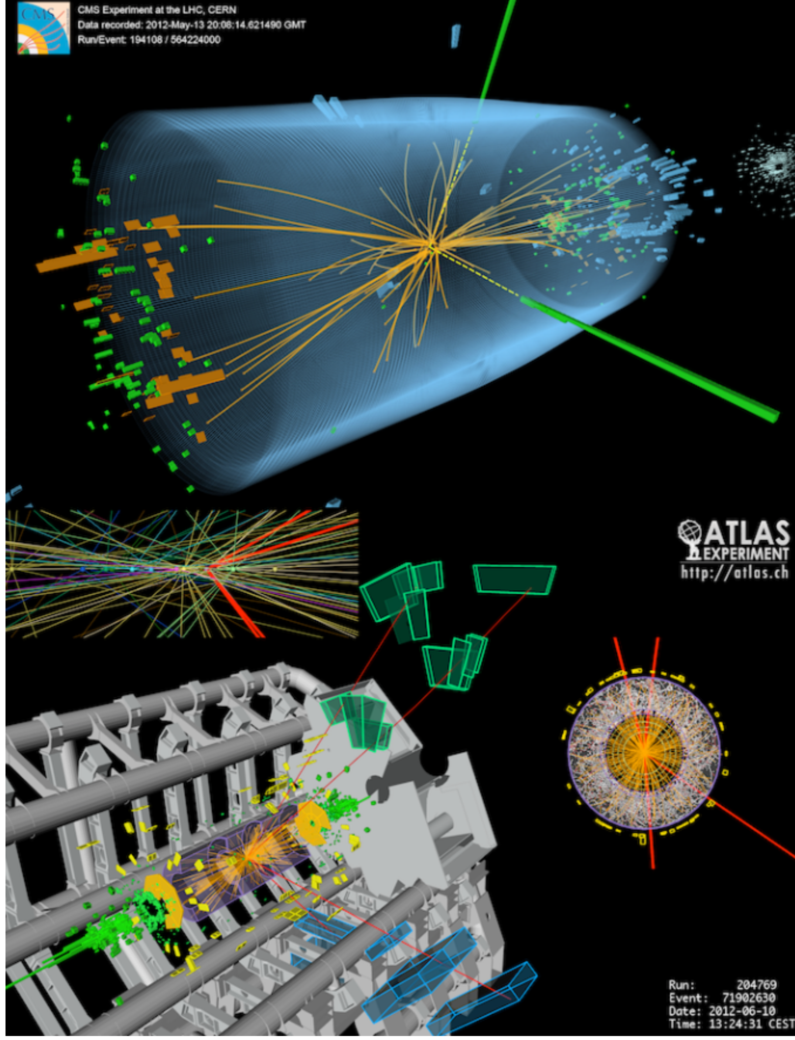


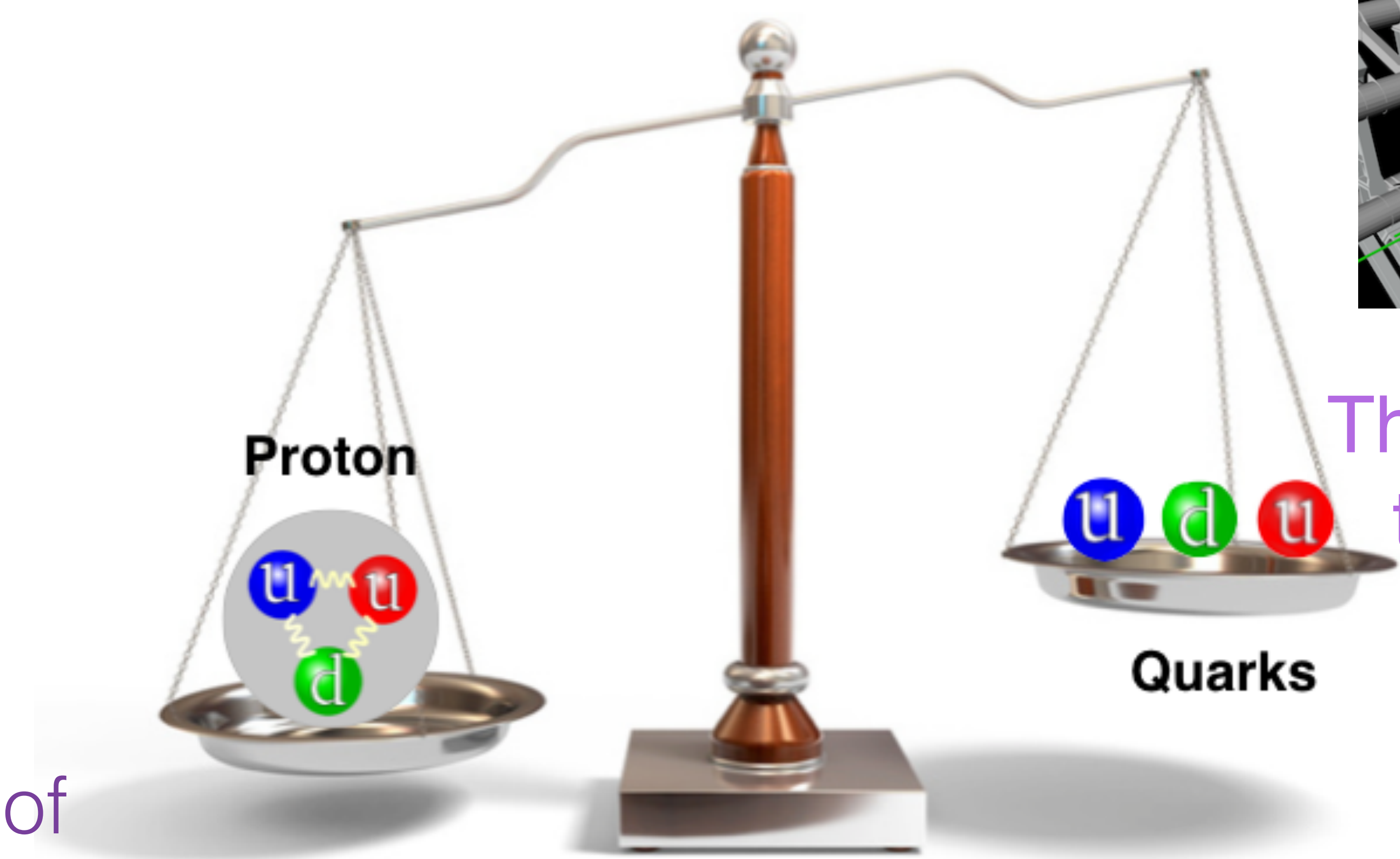
The decompositions of proton mass

Results and further challenges



How does the mass of nucleon arise?





The Higgs boson make the u/d quark having masses (2GeV MS-bar):

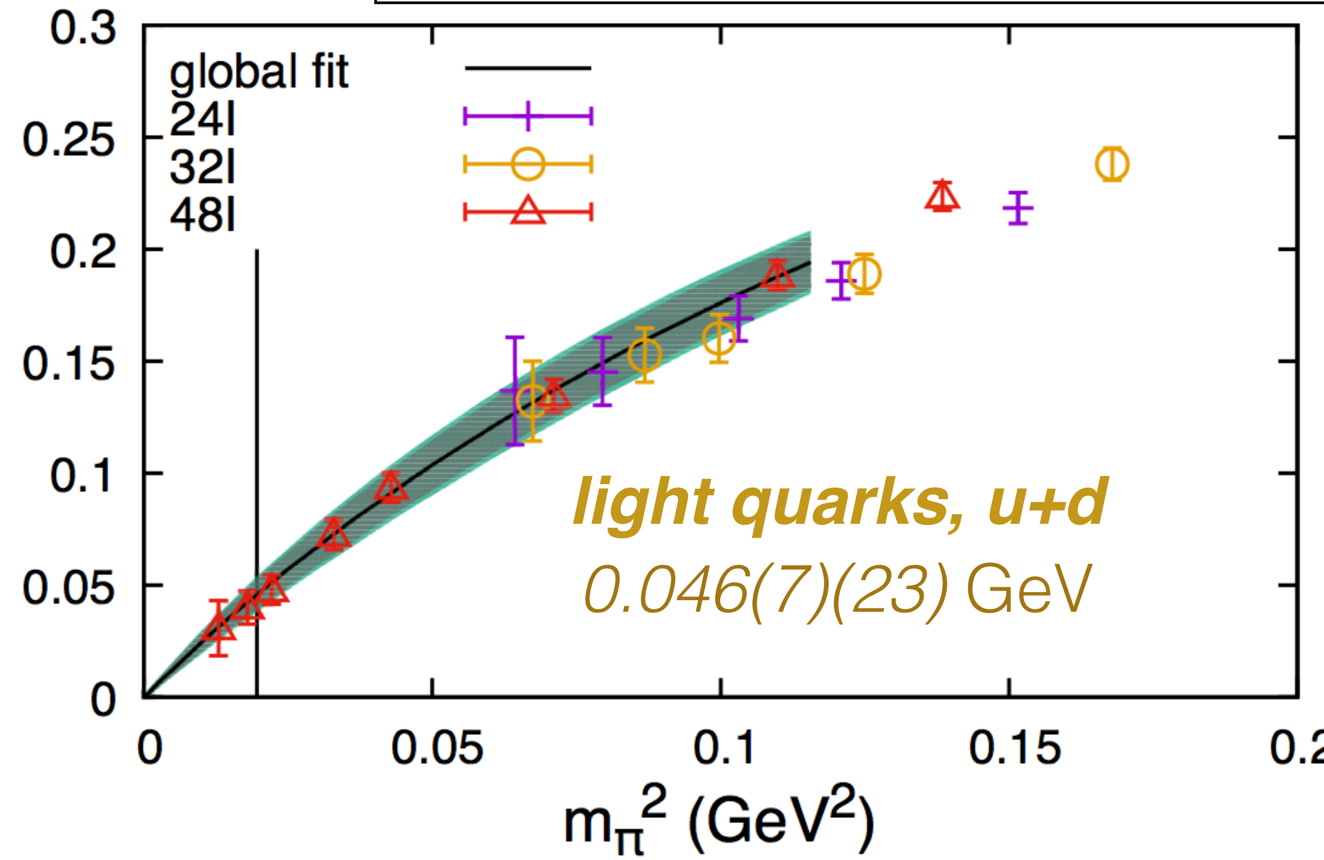
m_u = 2.08(09) MeV
m_d = 4.73(12) MeV

http://flag.unibe.ch/2019/Quark%20masses

But the mass of the proton is ***938.272046(21) MeV.***
~**100 times of the sum of the quark masses!**

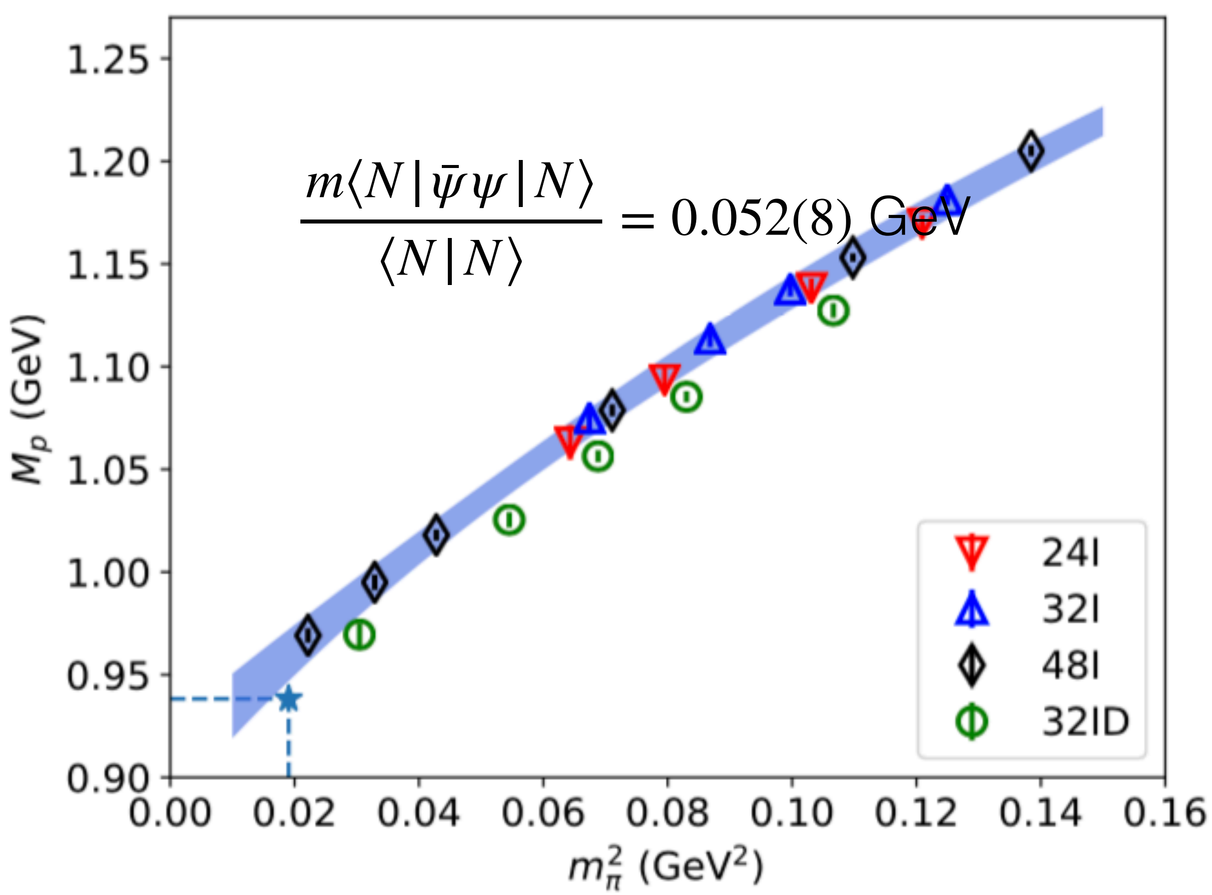
Scale independent quark mass contribution

YBY, et.al. χQCD Collaboration, PRD94(2016)054503



$$m \rightarrow \frac{m\langle N|\bar{\psi}\psi|N\rangle}{\langle N|N\rangle}$$

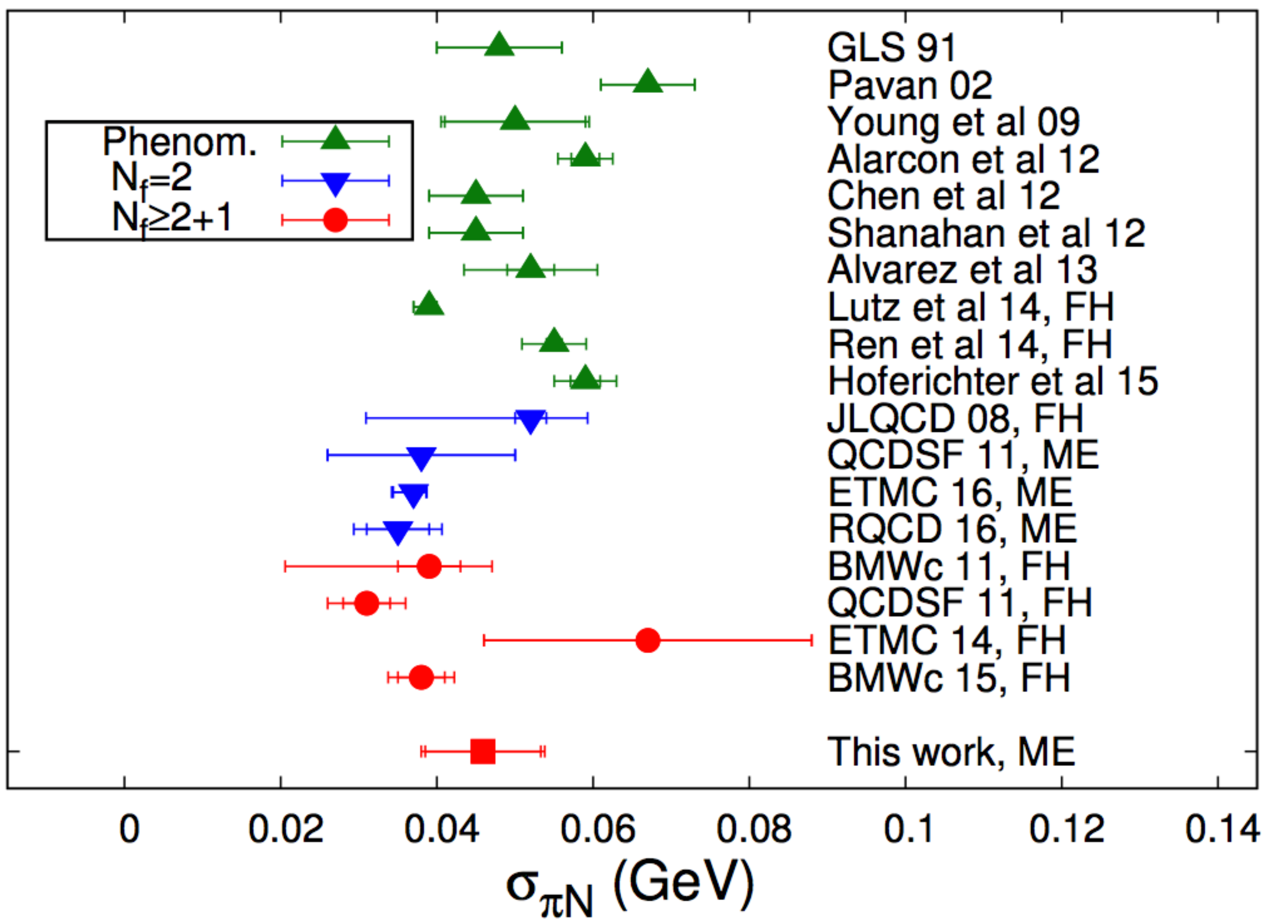
$$= m\frac{\partial m_N}{\partial m} \simeq \frac{m_\pi}{2}\frac{\partial m_N}{\partial m_\pi}$$

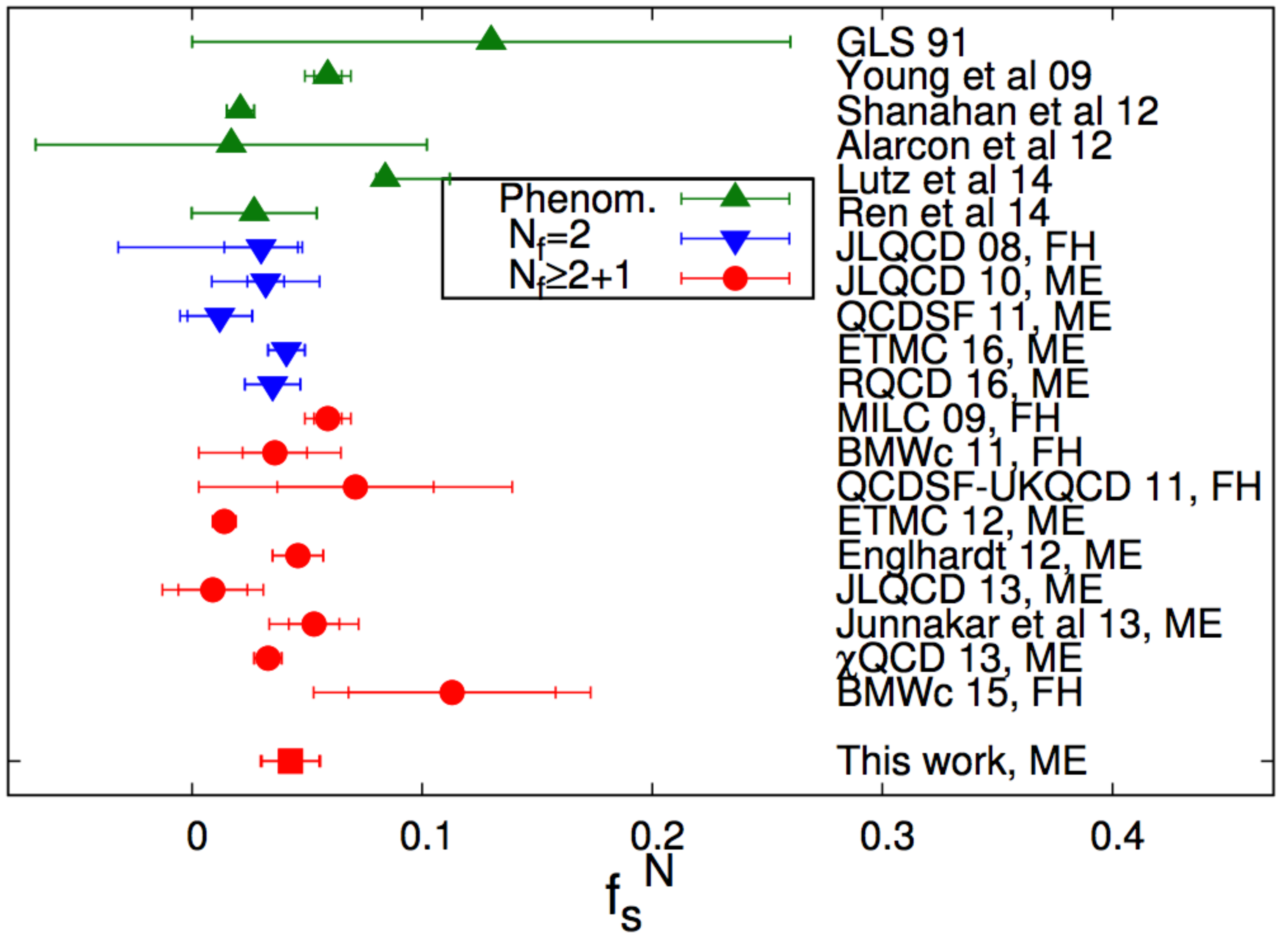


YBY, J. Liang, et. al., χQCD Collaboration, PRL121(2018)212001

- **Such a quantity can be obtain by either the direct matrix element calculation, or the derivative of the nucleon mass on the quark mass;**
- **Or extracted from the πN scattering experiments with χPT.**
- **But it is just ~50 MeV for three light quarks.**

Scale independent quark mass contribution





$\sigma_{\pi N} = \langle H_m(u)+H_m(d)\rangle = 45.9(7.4)(2.8)$ MeV

$f_s^N M_N = \langle H_m(s)\rangle = 40.2(11.7)(3.5)$ MeV

YBY, et.al. χQCD Collaboration, PRD94(2016)054503

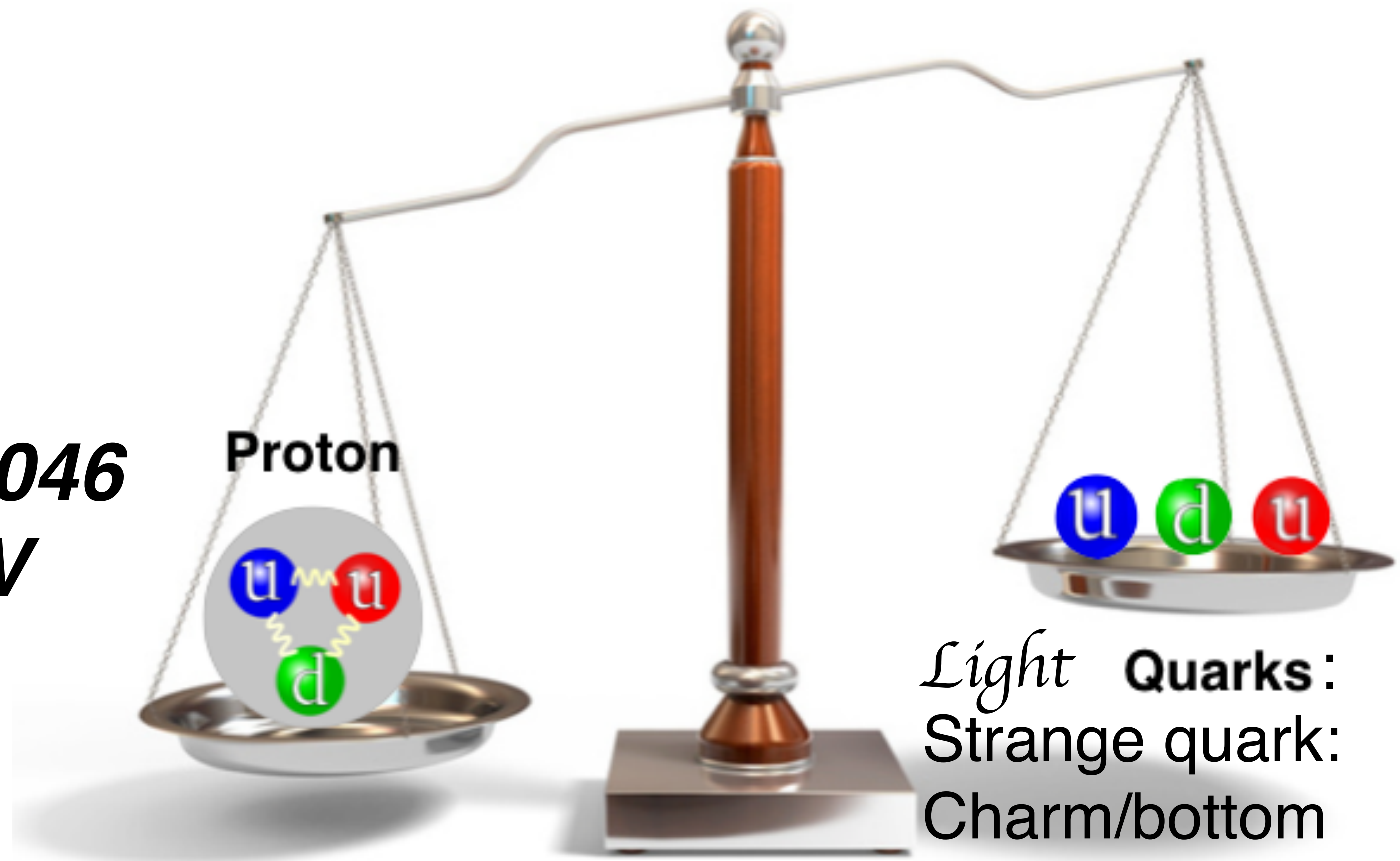
$m_{u/d}$ (2 GeV) = 3.36(4) MeV

m_s (2 GeV) = 92(1) MeV

http://flag.unibe.ch/2019/Quark%20masses

Where does the rest part come from?

938.272046 (21) MeV



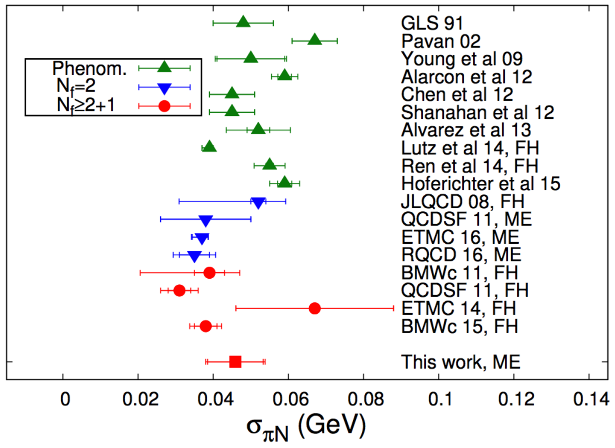
Light **Quarks**:	~50 MeV
Strange quark:	~40 MeV
Charm/bottom /top quarks:	~60 MeV each

} ~1/3 m_N

M. Gong, et.al., χQCD Collaboration , PRD88(2013)014503
ETM Collaboration, PRL116(2016)252001

$$\longrightarrow \quad m_N = \langle -T_{\mu\mu} \rangle = \langle m\bar{\psi}\psi + \boxed{\gamma_m m\bar{\psi}\psi + \frac{\beta(g)}{2g}F^2} \rangle$$

Outline



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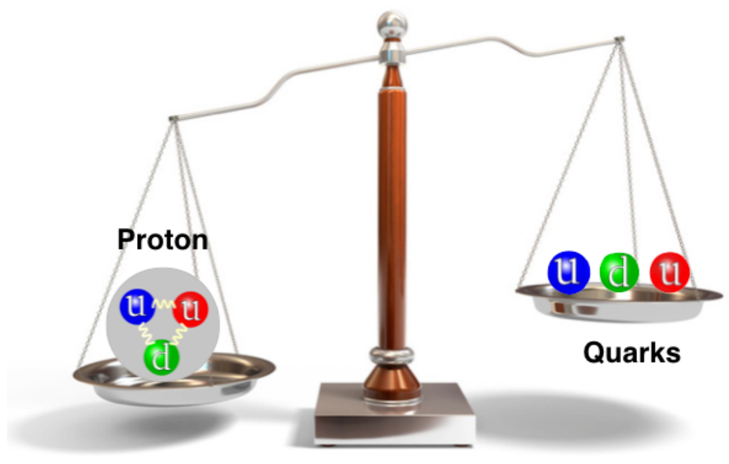

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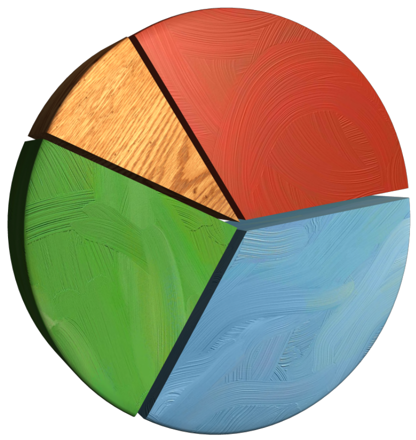


QCD Energy momentum Tensor

The gauge-invariant, symmetric QCD energy momentum tensor (EMT) in the Euclidean space is given by:

$$T_{\mu\nu} = \frac{1}{4}\bar{\psi}\gamma_{(\mu}\overleftrightarrow{D}_{\nu)}\psi + F_{\mu\rho}F^{\rho}{}_{\nu} - \frac{1}{4}g_{\mu\nu}F^2$$

$D_\mu = \partial_\mu + igA_\mu$, $A_{(\mu}B_{\nu)} = A_\mu B_\nu + A_\nu B_\mu$, $\overleftrightarrow{D}_\mu = D_\mu - \overleftarrow{D}_\mu$

For the nucleon with momentum p, its matrix element of QCD EMT satisfies:

$$\langle T_{\mu\nu}\rangle = \begin{pmatrix} -E & ip_i \\ ip_i & \frac{p_i p_j}{E} \end{pmatrix} \xrightarrow[p_i \to 0]{} \begin{pmatrix} -m_N & 0 \\ 0 & 0 \end{pmatrix}$$

The rest frame

$$\xrightarrow[p_z \to \infty]{} \begin{pmatrix} -p_z & ip_z \\ ip_z & p_z \end{pmatrix}$$

The infinite momentum frame

Trace anomaly in Dim. Reg.

Under the dimensional regularization, the QCD EMT can be decomposed into the trace part and the traceless part:

$$T_{\mu\nu} = \frac{1}{4}\bar{\psi}\gamma_{(\mu}\overleftrightarrow{D}_{\nu)}\psi + F_{\mu\rho}F_{\nu}{}^{\rho} - \frac{1}{4}g_{\mu\nu}F^2$$

$$= \left(T_{\mu\nu} - \frac{g_{\mu\nu}}{d}T^\alpha_\alpha\right) + \frac{g_{\mu\nu}}{d}T^\alpha_\alpha \equiv \bar{T}_{\mu\nu} + \hat{T}_{\mu\nu},$$

where $T^\alpha_\alpha = m\bar{\psi}\psi - 2\epsilon\frac{F^2}{4} + \mathcal{O}(\epsilon^2)\,.$ See Y. Hatta, et.al., JHEP12(2018)008 as an example

After the renormalization,

$$m\bar{\psi}\psi = \left(m\bar{\psi}\psi\right)_R,\ F^2 = -\frac{1}{\epsilon}\left(\frac{\beta_R}{g_R}F_R^2 + 2\gamma_m^R\left(m\bar{\psi}\psi\right)_R\right) + \mathcal{O}(\epsilon^0),$$

And then,

$$T^\alpha_\alpha = (1+\gamma_m^R)\left(m\bar{\psi}\psi\right)_R + \frac{\beta_R}{2g_R}F_R^2\,.$$

Trace anomaly in Dim. Reg.

At 1-loop level, the explicit form of the EMT trace part is,

$$T^{\alpha}_{\alpha} = (1+\gamma_m)m\bar{\psi}\psi + \frac{\beta}{2g}F^2 = (1+\frac{2}{\pi}\alpha_s)m\bar{\psi}\psi + (-\frac{11}{8\pi}+\frac{n_f}{12\pi})\alpha_s F^2 + \mathcal{O}(\alpha_s^2)\,.$$

Since we also have the following relation for the heavy quarks:

$$m_Q\bar{\psi}_Q\psi_Q = -\frac{1}{12\pi}\alpha_s F^2 + \mathcal{O}(\alpha_s^2) + \mathcal{O}(\frac{1}{m_Q}),$$

the ME of F^2 is independent to the number of heavy flavors, and the mass contribution from a heavy quark mass term can be directly estimated by:

$$m_Q\bar{\psi}_Q\psi_Q = \frac{1}{12\pi}\frac{m_N - \langle H_m^{u,d,s}\rangle}{\frac{11}{8\pi}-\frac{3}{12\pi}} + \mathcal{O}(\alpha_s^2) + \mathcal{O}(\frac{1}{m_Q}) \simeq 0.063 \text{ GeV}$$

The trace less part of EMT

Let us go back to the ME of the traceless EMT:

$$\frac{\langle P|\bar{T}^{q,g}_{\mu\nu}|P\rangle}{\langle P|P\rangle} = A^{q,g}\frac{P_\mu P_\nu + \frac{1}{d}g_{\mu\nu}m_N^2}{P_0},$$

where $\bar{T}^q_{\mu\nu} = \frac{1}{4}\bar{\psi}\gamma_{(\mu}\overleftrightarrow{D}_{\nu)}\psi - \frac{1}{d}g_{\mu\nu}m\bar{\psi}\psi,\ \bar{T}^g_{\mu\nu} = F_{\mu\rho}F_\nu{}^\rho - \frac{1}{d}g_{\mu\nu}F^2.$

The Lorentz quark/gluon momentum fraction *A* can be obtained in any frame, likes the rest frame:

$$\left.\frac{\langle P|\bar{T}^{q,g}_{\mu\nu}|P\rangle}{\langle P|P\rangle}\right|_{P_{x,y,z}=0,} = \frac{d-1}{d}A^{q,g}m_N,$$

or on the light-cone as:

$$\left.\frac{\langle P|\bar{T}^{q,g}_{++}|P\rangle}{\langle P|P\rangle}\right|_{P_{x,y,z}=0,} = A^{q,g}P_+,$$ where $\bar{T}^q_{++} = \frac{1}{2}\bar{\psi}\gamma_+\overleftrightarrow{D}_+\psi,\ \bar{T}^g_{++} = F_{+\rho}F_+{}^\rho.$

The trace terms are omitted as $P_+ >> m_N$

Momentum fractions

as the moments of PDF

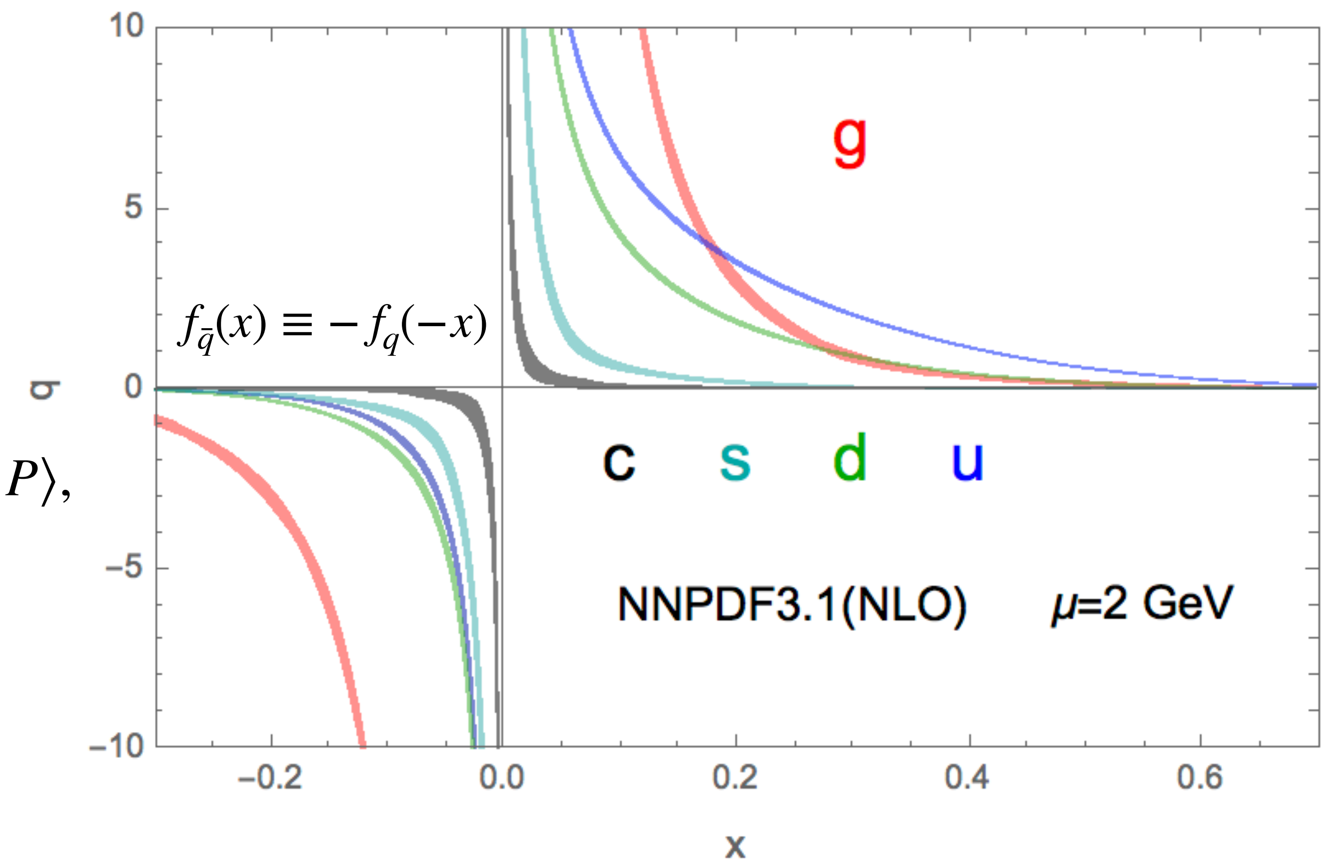
On the light-cone, the quark and gluon unpolarized parton distribution function (PDF) are defined by:

$$q(x) = \int \frac{\mathrm{d}\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P | \bar{\psi}(\xi^-)\gamma_+ U(\xi^-,0)\psi(0) | P \rangle,$$

$$g(x) = \int \frac{\mathrm{d}\xi^-}{2x\pi} e^{-ix\xi^- P^+} \langle P | \mathrm{Tr}\left[F_{+\rho}(\xi^-)U(\xi^-,0)F_+^{\rho}(0)U(0,\xi^-)\right] | P \rangle,$$

and it is easy to obtain that,

NNPDF Collaboration, NPB887(2014)276

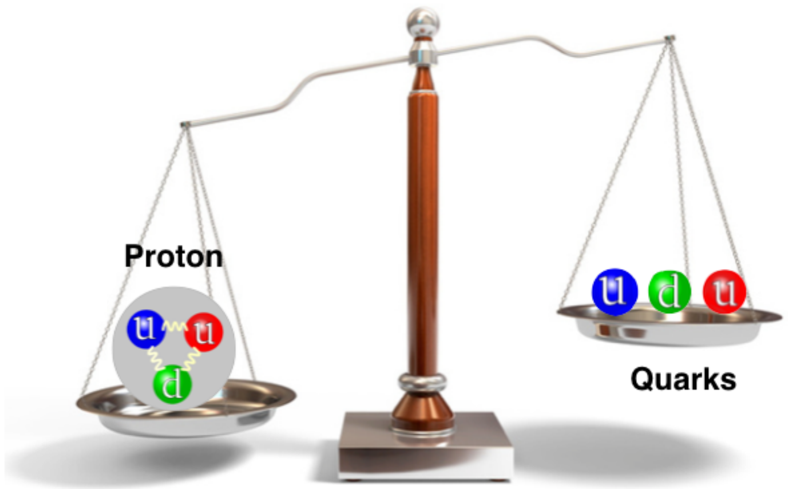


$$\int_{-1}^{1} xq(x)\mathrm{d}x = \frac{\langle P | \bar{T}^q_{++} | P \rangle}{P_+ \langle P | P \rangle} = A^q, \quad \int_{-1}^{1} xg(x)\mathrm{d}x = \frac{\langle P | \bar{T}^g_{++} | P \rangle}{P_+ \langle P | P \rangle} = A^g,$$

with $\bar{T}^q_{++} = \frac{1}{2}\bar{\psi}\gamma_+ \overleftrightarrow{D}_+ \psi,\ \bar{T}^g_{++} = F_{+\rho}F_+{}^{\rho}.$

Thus the momentum fractions we obtained in the rest frame is directly the moments of the unpolarized PDF.

The decompositions of the QCD EMT



Thus one can have the following Ji's decomposition of the nucleon mass (the energy in the rest frame) :

$$m_N = \langle T_{44} \rangle_{P_{x,y,z}=0, d\to 4} = \langle \bar{T}^q_{44} \rangle + \langle \bar{T}^g_{44} \rangle + \frac{1}{4}(1+\gamma_m)\langle H_m \rangle + \frac{\beta}{8g}\langle F^2 \rangle$$

$$= \langle \bar{\psi}\gamma_4 \overrightarrow{D}_4 \psi \rangle + \langle \bar{T}^g_{44} \rangle + \frac{1}{4}\gamma_m \langle H_m \rangle + \frac{\beta}{8g}\langle F^2 \rangle$$

$$= \langle \sum_i \bar{\psi}\gamma_i \overrightarrow{D}_i \psi \rangle + \langle \bar{T}^g_{44} \rangle + \langle H_m \rangle + \frac{1}{4}\gamma_m \langle H_m \rangle + \frac{\beta}{8g}\langle F^2 \rangle$$

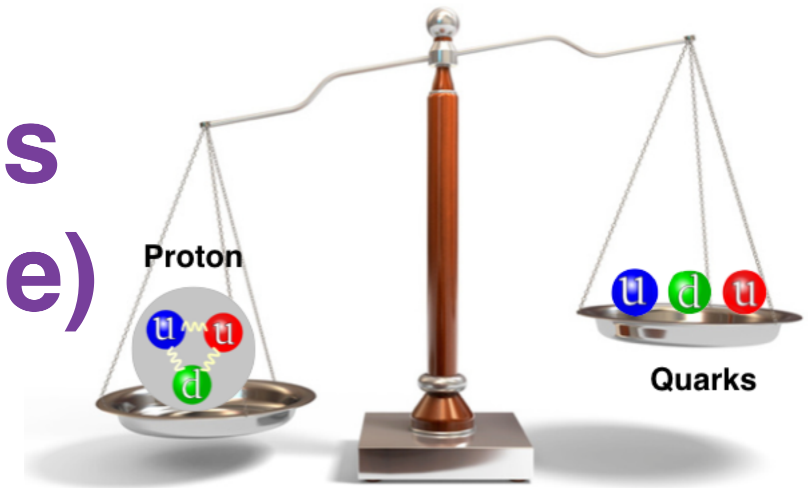
Xiangdong Ji, PRL 74(1995)1071

Or the following decomposition of EMT following the structure of perfect fluid,

$$\langle P | T_{i,\mu,\nu} | P \rangle = \frac{\langle P | P \rangle}{2E}\left(2P_\mu P_\nu \langle x \rangle_i - 2m_N g_{\mu\nu}\bar{p}_i\right), \ \bar{p}_i = (-\langle x \rangle_i + \langle H_{m,i} \rangle)/4.$$

C. Lorce, EPJC78(2018)120

Ji's decomposition of proton mass (the proton energy in the rest frame)



$$M = -\langle T_{44}\rangle = \langle H_E\rangle + \langle H_m\rangle + \langle H_g\rangle + \frac{1}{4}\langle H_a\rangle,$$

$$M = -\langle \hat{T}_{44}\rangle = \langle H_m\rangle + \langle H_a\rangle$$

Xiangdong Ji, PRL 74(1995)1071

With

$$H_m = \sum_{u,d,s\cdots}\int d^3x\, m\bar{\psi}\psi,$$

The quark mass

$\langle H_m(u,d,s)\rangle / M_N = 9(2)\%$

YBY, et.al. χQCD Collaboration, PRD94(2016)054503

Gauge Invariant and scale independent combinations.

The QCD anomaly

$$H_a = H_g^a + H_m^\gamma,$$

The glue anomaly

$$H_g^a = \int d^3x\, \frac{-\beta(g)}{g}(E^2+B^2),$$

$$H_m^\gamma = \sum_{u,d,s\cdots}\int d^3x\, \gamma_m m\bar{\psi}\psi.$$

The quark mass anomaly

The total energy

$$H_E = \sum_{u,d,s\ldots}\int d^3x\, \bar{\psi}(\vec{D}\cdot\vec{\gamma})\psi,$$

The quark energy

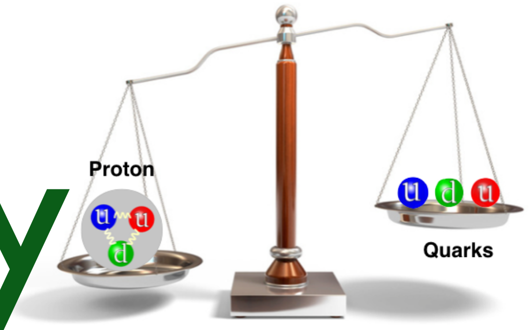
$$H_g = \int d^3x\, \frac{1}{2}(B^2 - E^2),$$

The glue field energy

YBY, et. al., χQCD collaboration**,** Phys. Rev. D 91(2015)074516

Proton mass decomposition

The QCD anomaly



Then we have

$$M = -\langle T_{44}\rangle = \langle H_q\rangle + \langle H_g\rangle + \langle H_a^a\rangle + \langle H_m^\gamma\rangle$$
$$= \langle H_E\rangle + \langle H_m\rangle + \langle H_g\rangle + \frac{1}{4}\langle H_a\rangle,$$

$$M = -\langle \hat{T}_{44}\rangle = \langle H_m\rangle + \langle H_a\rangle,$$

- **The joint contribution of the QCD anomaly can be deduced from the quark mass term, with the sum rule above.**

The QCD anomaly

$$H_a = H_g^a + H_m^\gamma,$$

The glue anomaly

$$H_g^a = \int d^3x \, \frac{-\beta(g)}{g}(E^2 + B^2),$$

$$H_m^\gamma = \sum_{u,d,s\cdots} \int d^3x \, \gamma_m m \overline{\psi}\psi.$$

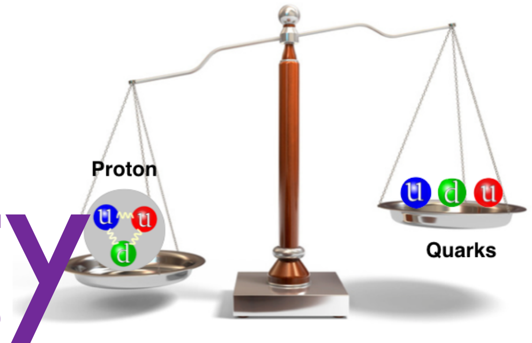
The quark mass anomaly

- The total QCD anomaly is renormalization scheme/scale independent.

- $H_a / 4M_N = 23(1)\%$

Proton mass decomposition

The quark/gluon energy



Then we have

$$M = -\langle T_{44}\rangle = \langle H_q\rangle + \langle H_g\rangle + \langle H_a^a\rangle + \langle H_m^\gamma\rangle$$
$$= \langle H_E\rangle + \langle H_m\rangle + \langle H_g\rangle + \frac{1}{4}\langle H_a\rangle,$$

$$M = -\langle \hat{T}_{44}\rangle = \langle H_m\rangle + \langle H_a\rangle,$$

- The quark/glue energy can be deduced from the momentum fraction,

$$\langle H_E\rangle = \frac{3}{4}\langle x\rangle_q M - \frac{3}{4}\langle H_m\rangle$$
$$\langle H_q\rangle = \frac{3}{4}\langle x\rangle_q M + \frac{1}{4}\langle H_m\rangle$$

$$\langle H_g\rangle = \frac{3}{4}\langle x\rangle_g M.$$

- The renormalization of the quark momentum fraction is much more trivial, which is just mixed with the glue one.

- **It is more straightforward to obtain the quark/glue momentum fraction first, and convert it to the quark/glue energy.**

The total energy

$$H_E = \sum_{u,d,s\ldots} \int d^3x\, \overline{\psi}(\vec{D}\cdot\vec{\gamma})\psi,$$

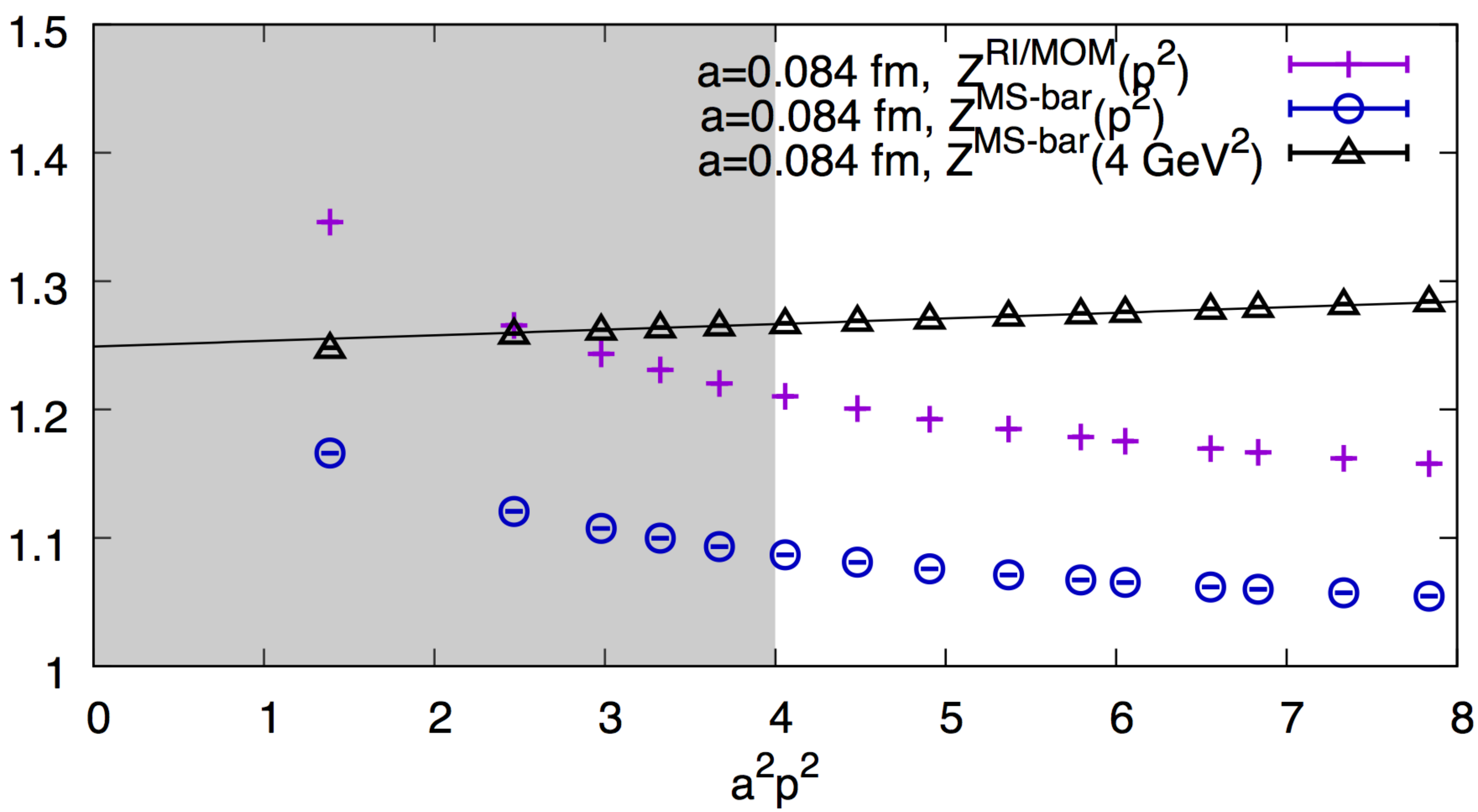
The quark energy

$$H_g = \int d^3x\, \frac{1}{2}(B^2 - E^2),$$

The glue field energy

Renormalization

of the **quark** momentum fractions



$$Z_{QQ}^{\overline{\rm MS}}(\mu) = \left[(Z_{QQ} R_{QQ}) \left(\mu_R, \frac{\mu}{\mu_R}\right)|_{a^2\mu_R^2 \to 0} \right]^{-1}$$

- Strong scale $\mu_R{}^2 = p^2$ dependence in the RI/MOM renormalization constant Z_{QQ} and converting ratio R_{QQ}
- But only the discretization error a^2p^2 left in final MS-bar renormalization constant at a fixed scale.

YBY, J. Liang, et. al., χQCD Collaboration, PRL121(2018)212001, **ViewPoint and Editor's suggestion**

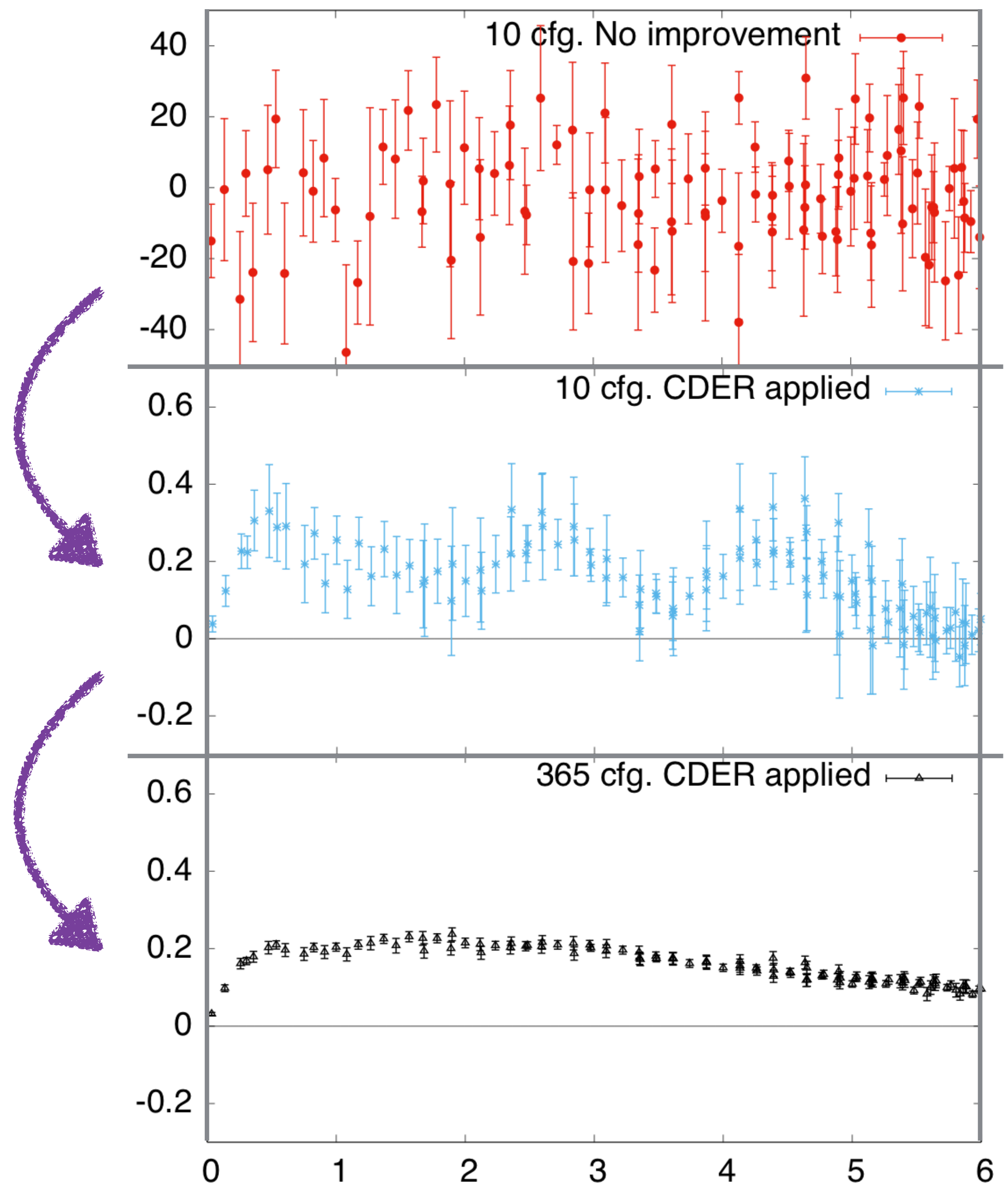
Gluon renormalization with CDER

W. Sun, et.al, χQCD collaboration, CPC42, 063102(2018), 1507.02541
K. Liu, J. Liang, **YBY**, PRD96, 114504(2017), 1805.00531

- Calculate the renormalization factor of the glue EMT non-perturbatively on a ~5 fm box will require ~30,000,000 configurations to make the uncertainty to be ~0.01;

- Taking the localization of the correlations between the glue fields/ operators into account, the uncertainty can be reduced by a factor ~200;

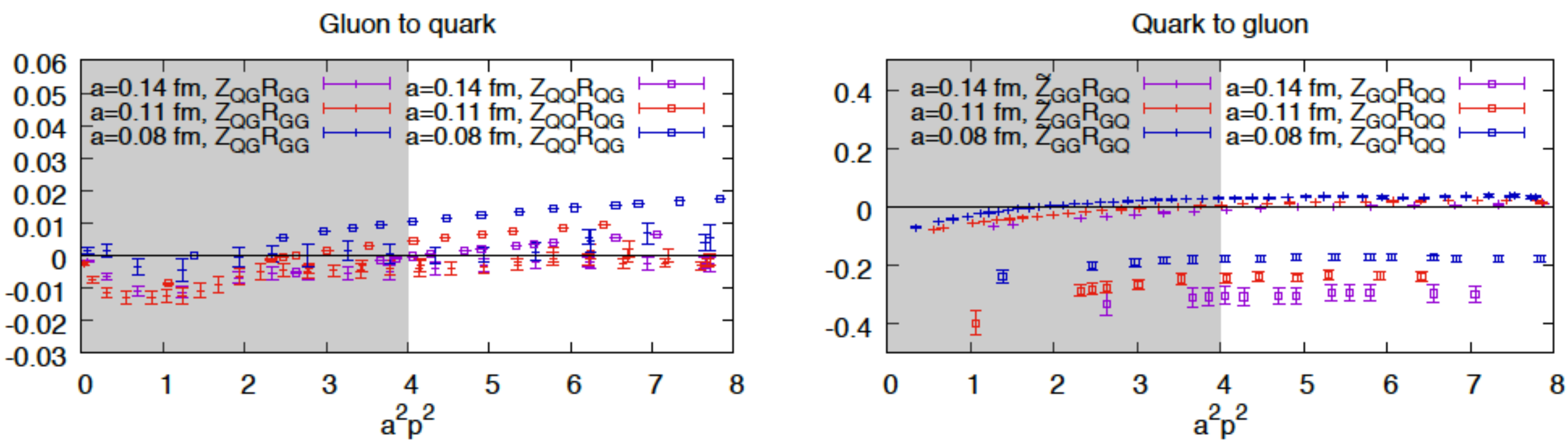
- Use reasonable computer resource (~1M CPU hours) to increase the statistics, the ~0.01 uncertainty goal can be obtained with 365 configurations.



YBY, et. al., χQCD collaboration, PRD98(2018) 074506

Mixing

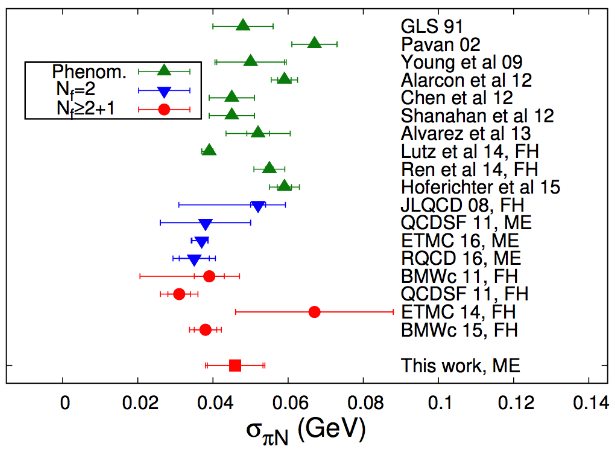
between the quark and glue momentum fractions



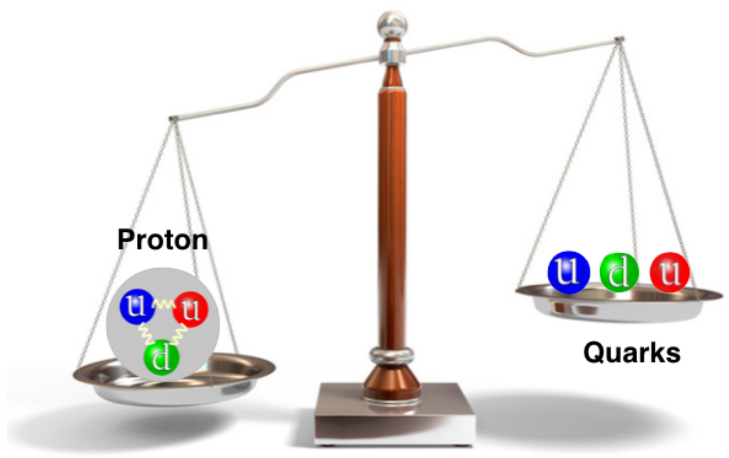
- The mixing from **glue** to **quark** is at 1% level;
- But that from **quark** to **glue** is significant.

YBY, J. Liang, et. al., χQCD Collaboration, PRL121(2018)212001,
ViewPoint and Editor's suggestion

Outline

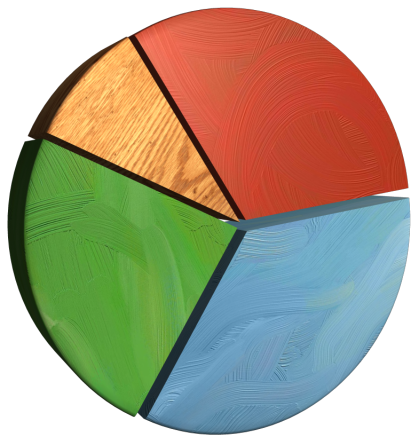


Quark mass and proton mass



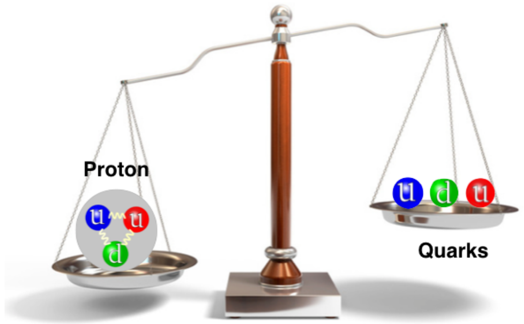
The decompositions of proton mass

Results and further challenges

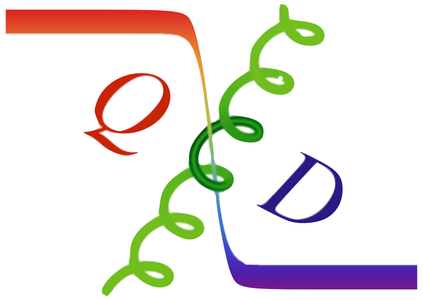


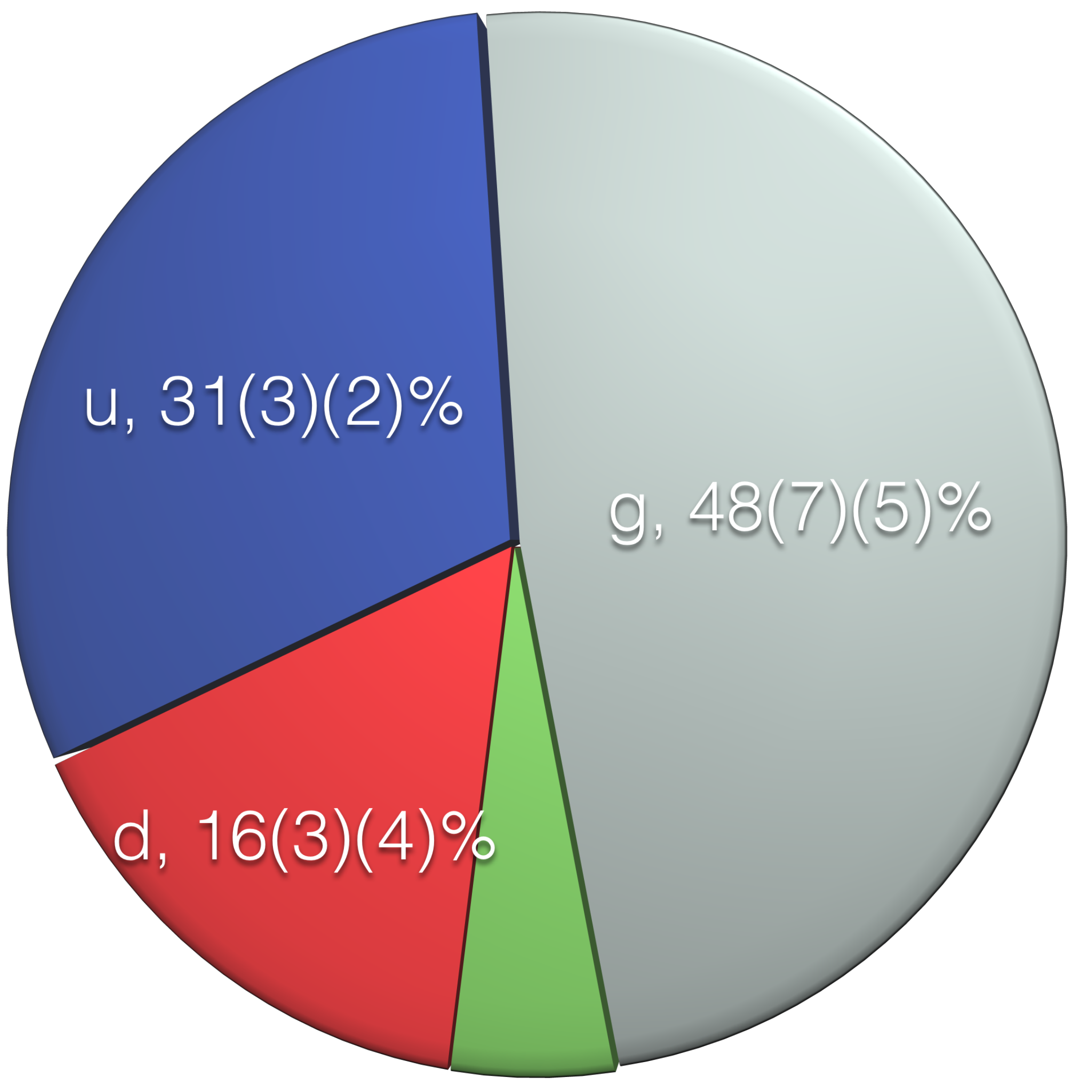
Proton mass decomposition

Comparing the momentum fractions



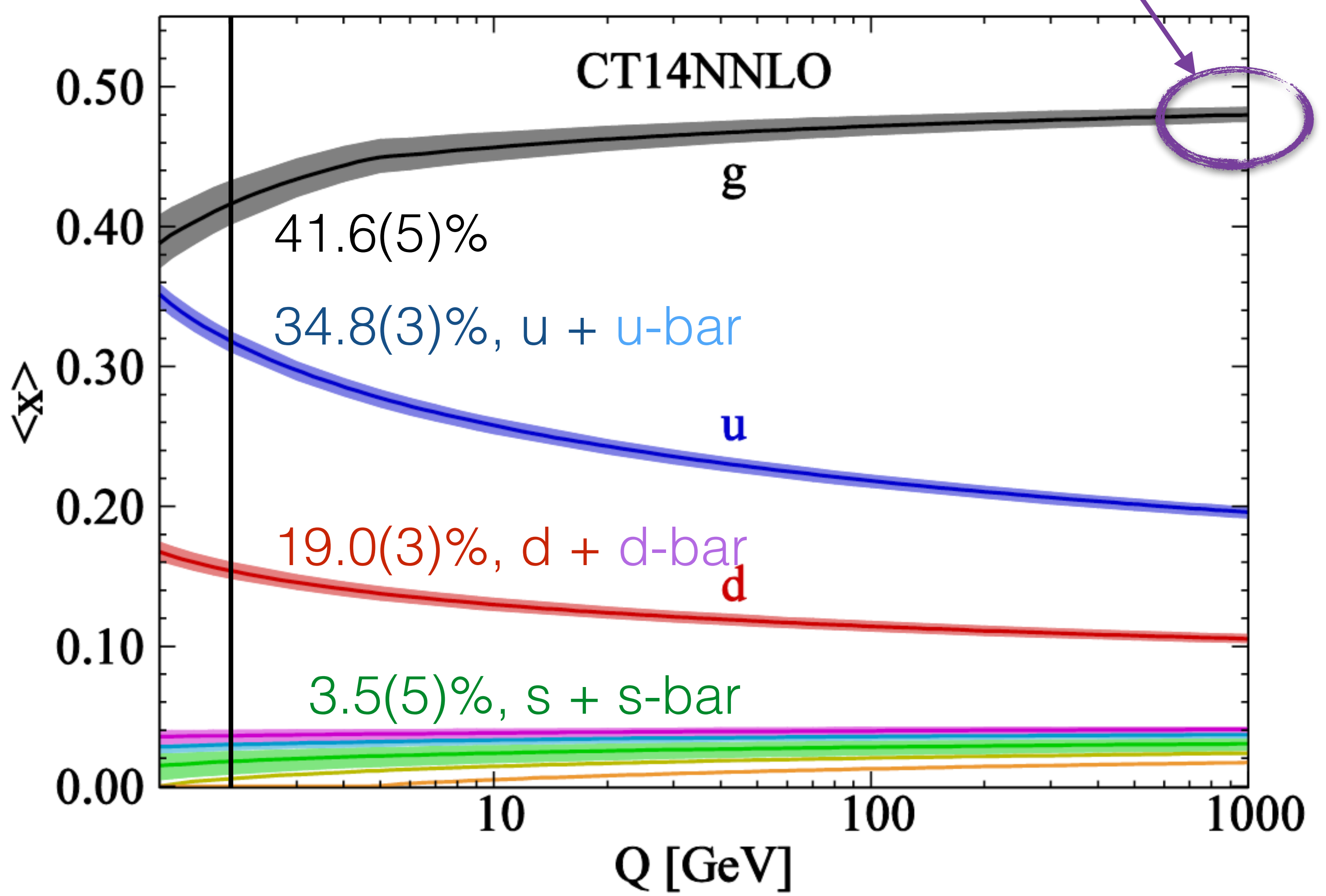
YBY, J. Liang, et. al., χQCD Collaboration, PRL121(2018)212001,
ViewPoint and Editor's suggestion





from the experiment

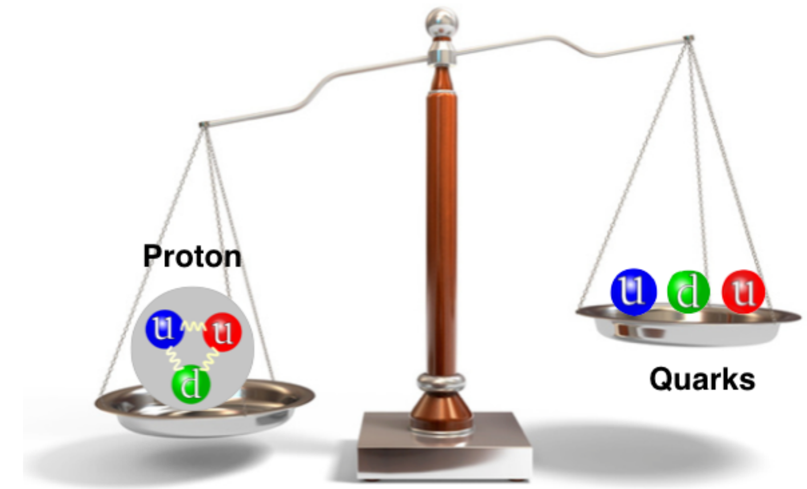
$$\langle x\rangle_g(\mu=\infty)=\frac{\frac{8}{3}C_F}{\frac{8}{3}C_F+\frac{2}{3}N_f}=0.471$$



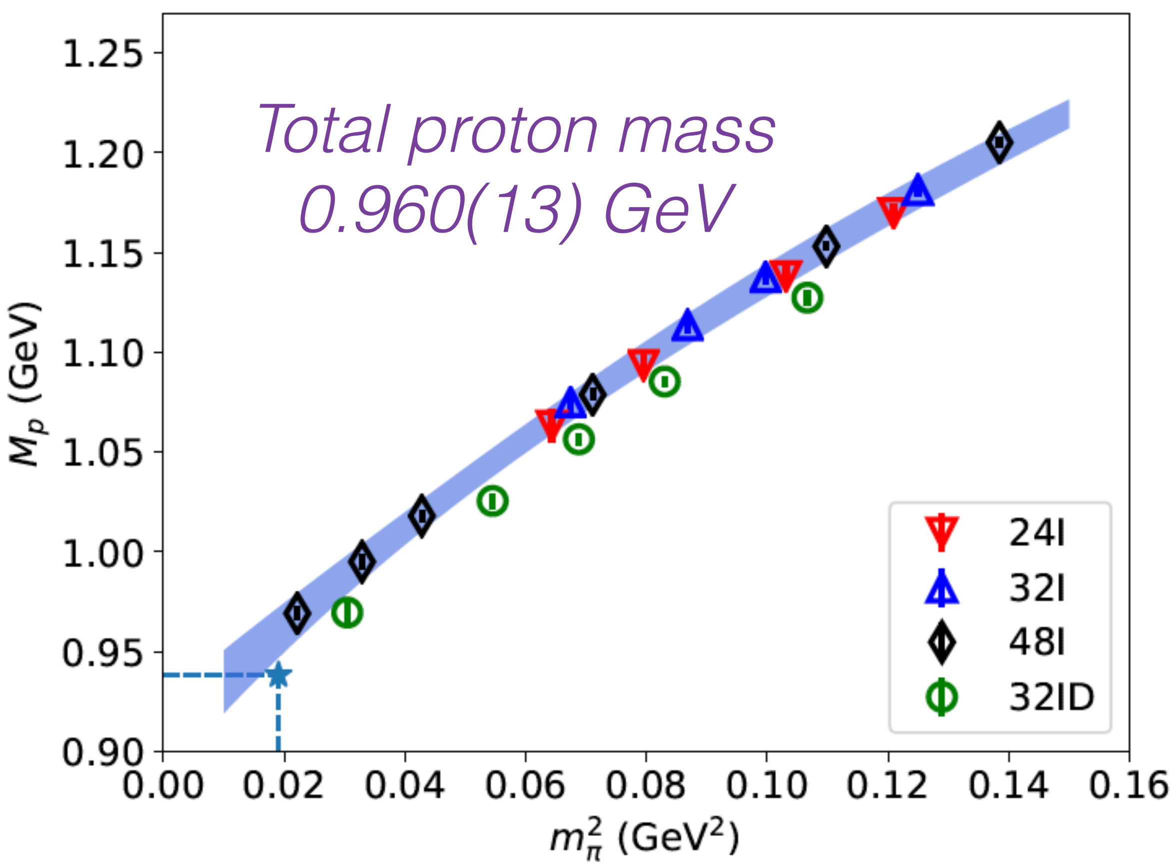
S. Dulat et al, PRD93(2016)033006

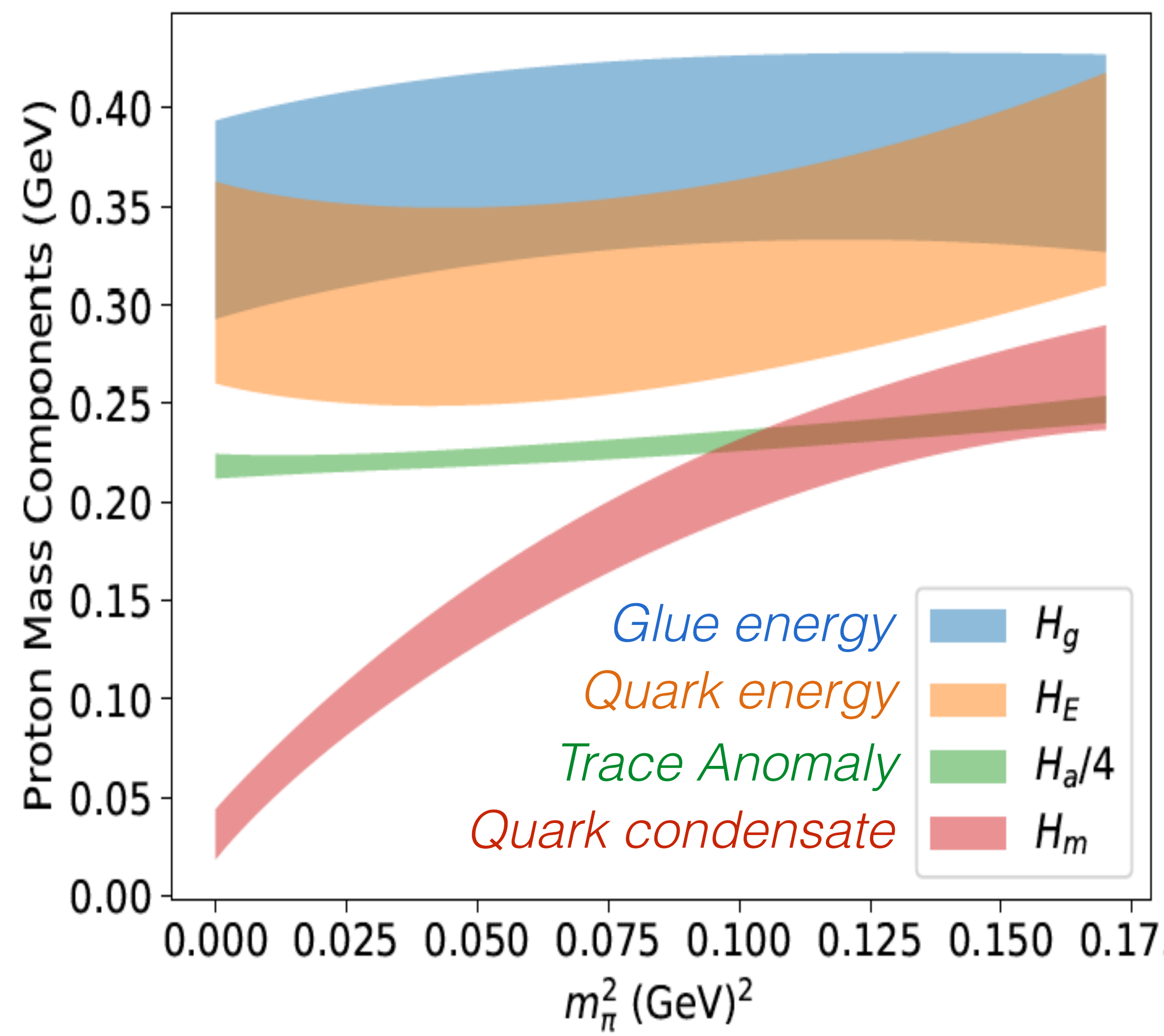
- Direct calculation of the quark/glue momentum fraction with non-perturbative renormalization and normalization.

- Trace anomaly contribution deduced by the direct calculation of the quark scalar condensate in nucleon, based on the sum rule



$$\frac{1}{4}M = -\langle \hat{T}_{44} \rangle = \frac{1}{4}\langle H_m \rangle + \frac{1}{4}\langle H_a \rangle$$





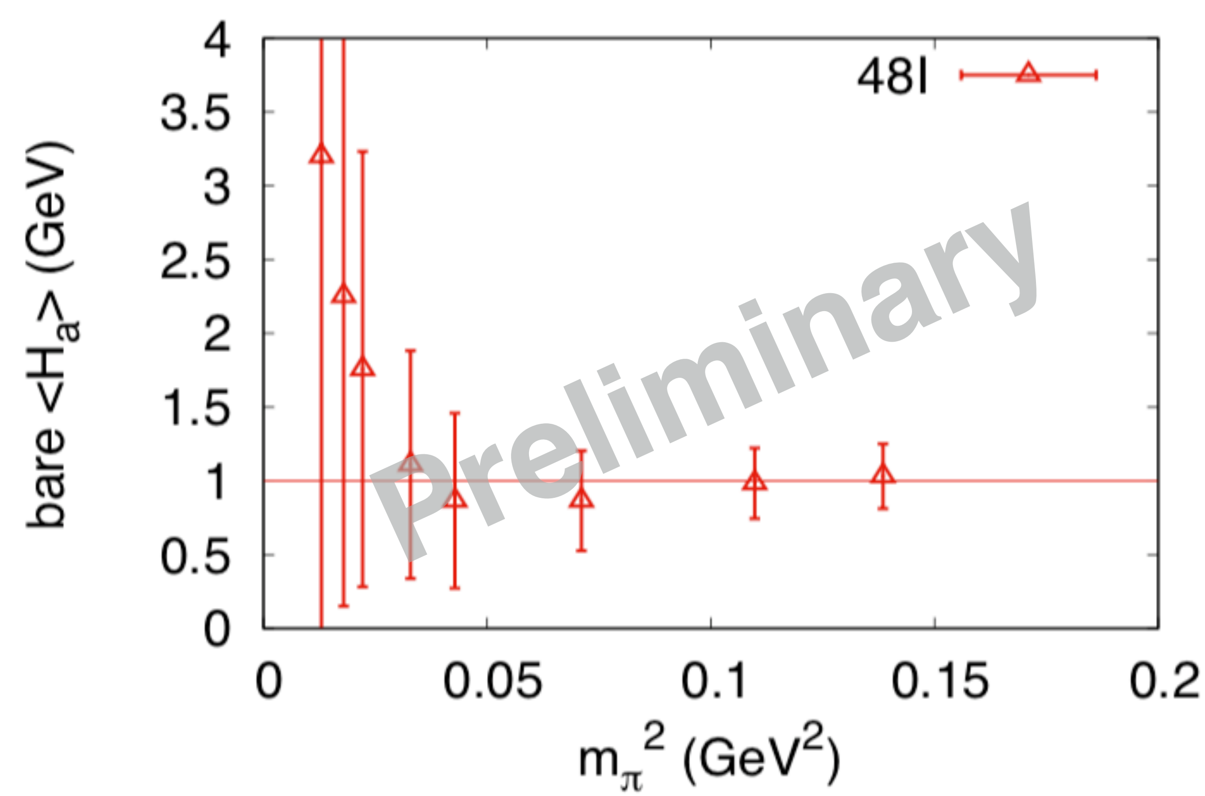
YBY, J. Liang, et. al., χQCD Collaboration, PRL121(2018)212001,
ViewPoint and Editor's suggestion

The next challenge: Trace anomaly under the Lattice Regularizations

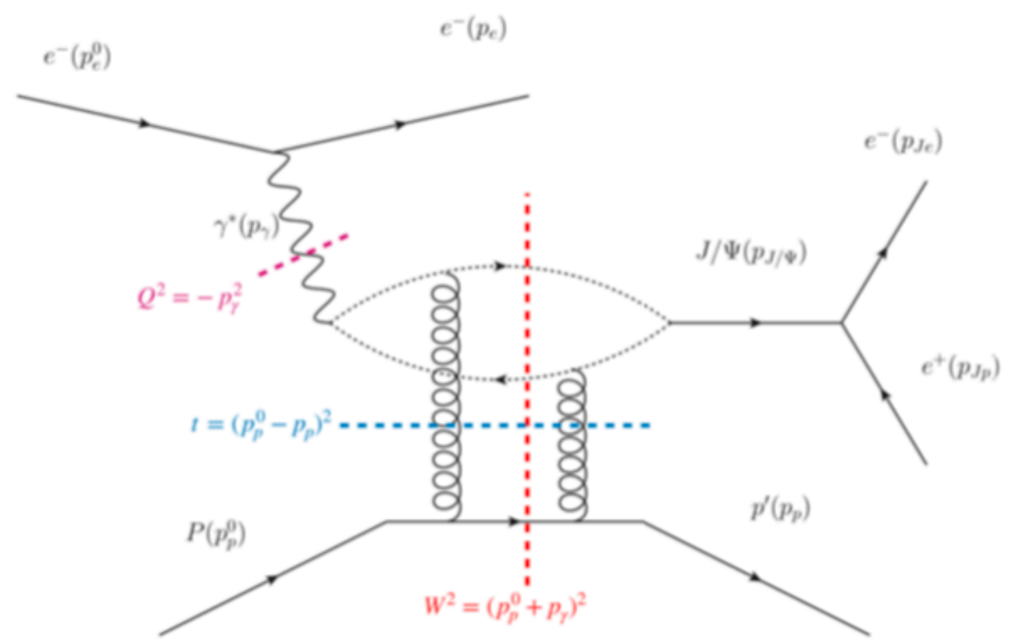
- Scheme 1: Define the exact EMT under the lattice regularization and then the trace anomaly can be obtained automatically

- Scheme 2: Renormalize the ME of F^2 in the RI/MOM scheme and convert to the MS-bar scheme, then one can use the MS-bar beta function.

- Scheme 3: Calculate the ME of F^2 of both the nucleon and pion, then normalize the nucleon case with the pion case



Possible experiments on the anomaly



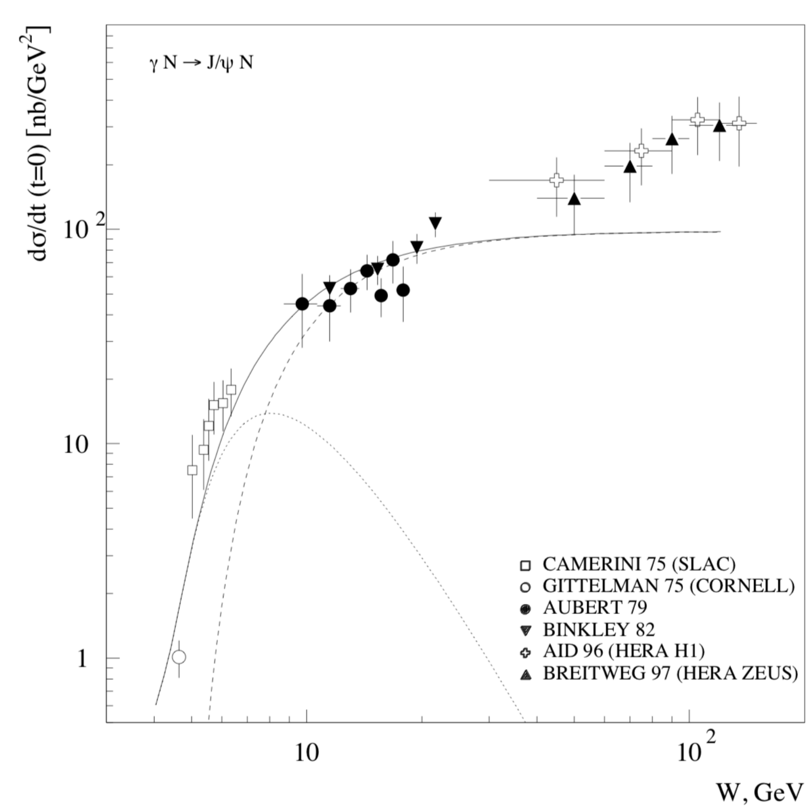
- The near-threshold γ+N->J/psi+N photo production cross section would be sensitive to the form factor of the trace anomaly…

D. Kharzeev, Proc. Int. Sch. Phys. Fermi 130 (1996), 105

- …if is Q2 dependence is similar to that of the traceless part of EMT.

Y. Hatta and D. L. Yang, PRD98(2018)074003

- Such an assumption can be checked with Lattice QCD.



Summary

- Lattice QCD provides a systematic way to investigate the decomposition of the nucleon mass and also QCD EMT;

- It is crucial to investigate the trace anomaly with the regularization other than the dim. reg., especially the Lat. reg.;

- The Lattice result of the trace anomaly will be available in the near future, for both the proton and pion.