Transverse Hadron Structures from Lattice QCD with LaMET

Yong Zhao
Massachusetts Institute of Technology
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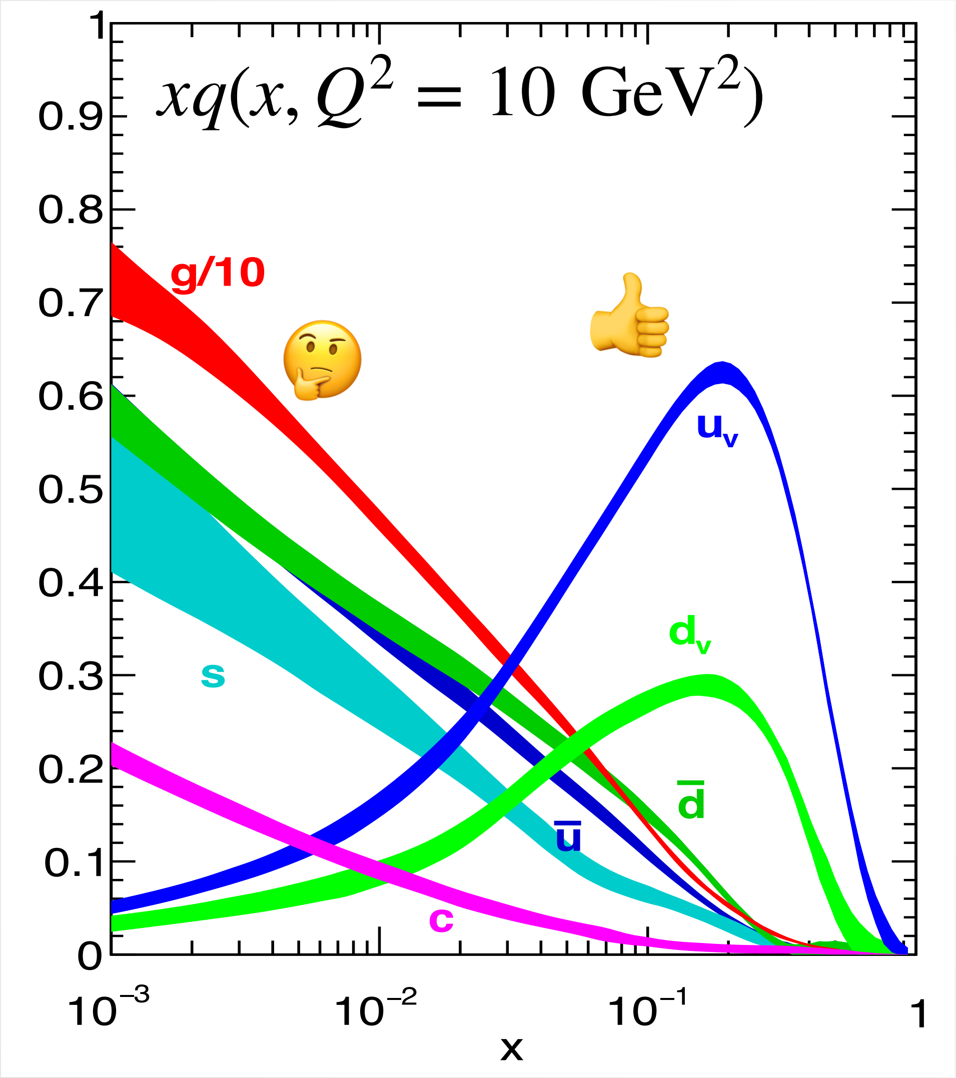
Outline

- Large-momentum effective theory
  - Physical picture and factorization formula
  - Systematic approach to extract PDFs from lattice QCD
- Transverse hadron structures from lattice QCD
  - Generalized parton distributions
  - Transverse momentum dependent PDFs
  - Collins-Soper kernel from lattice QCD

So far our knowledge of the PDFs mostly comes from the analysis of high-energy scattering data

Unpolarized PDF



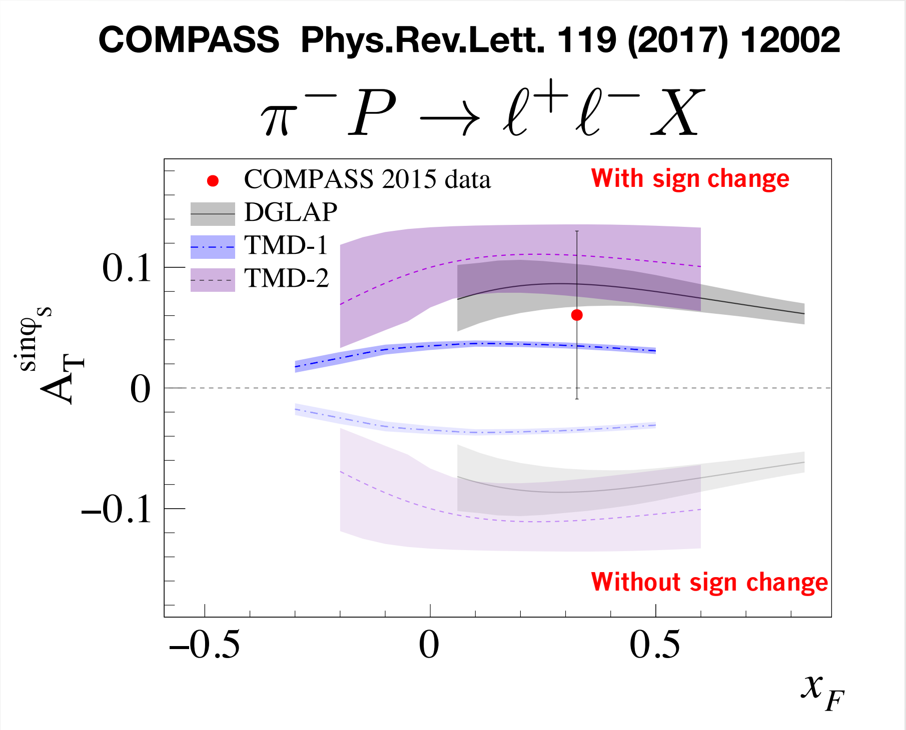
NNPDF 3.1, EPJ C77 (2017)

TMD PDF

Existing global analyses of TMDPDFs or TMD fragmentation functions rely on the modeling of their nonperturbative evolution.

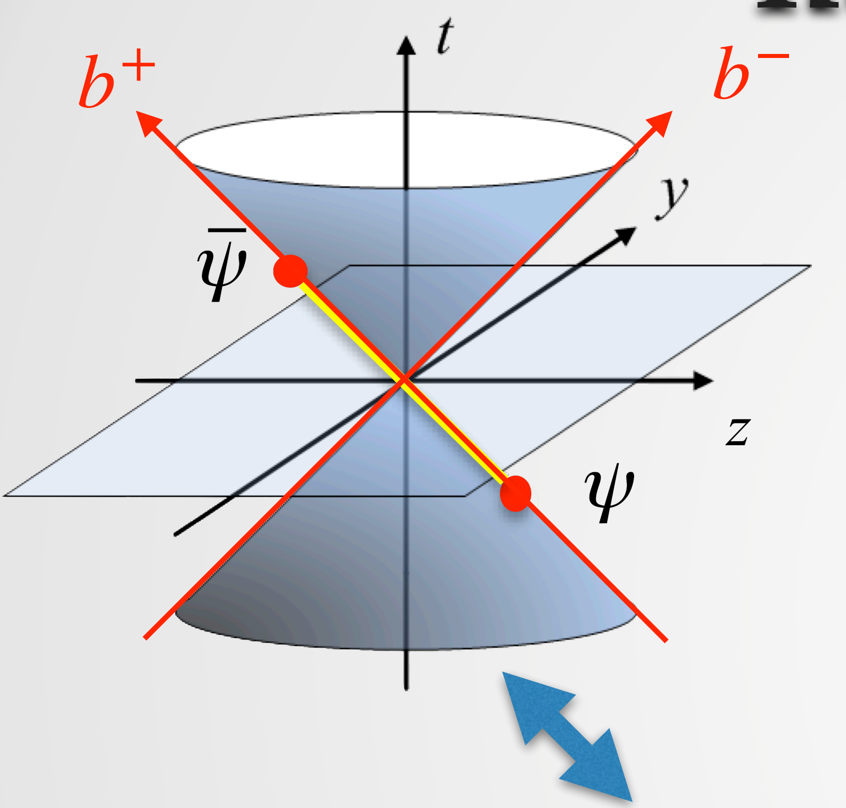
- Kang, Prokudin, Sun and Yuan, PRD93 (2016);
- Bacchetta et al., JHEP1706 (2017);
- Eur.Phys.J. C78 (2018) no.2, 89;
- Bertone, Scimemi and Vladimirov, arXiv:1902.08474.

The most definite experimental finding so far is the sign change of the Sivers function in SIDIS and Drell-Yan processes.



See also STAR Collaboration, PRL116 (2016).

Lattice QCD calculation of partonic hadron structures?

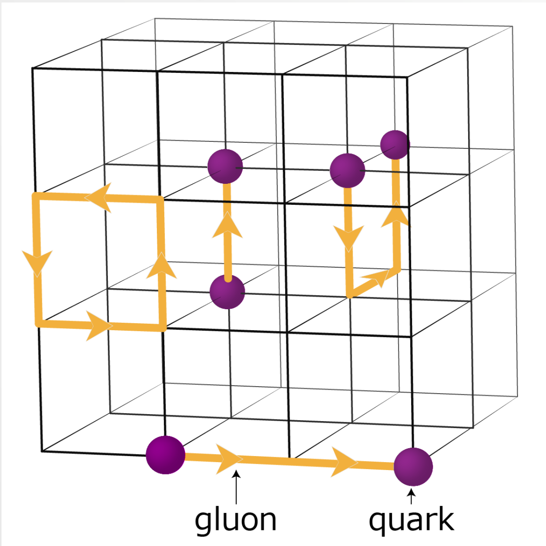


PDF:

$$q(x,\mu) = \int \frac{db^-}{2\pi} e^{-ib^-(xP^+)} \langle P \,|\, \bar\psi(b^-) \frac{\gamma^+}{2} W[b^-,0] \psi(0) \,|\, P \rangle$$

$$b^\pm = \frac{t \pm z}{\sqrt{2}}$$

- Minkowski space, real time;
- Defined on the light-cone which depends on the real time.



Lattice QCD:

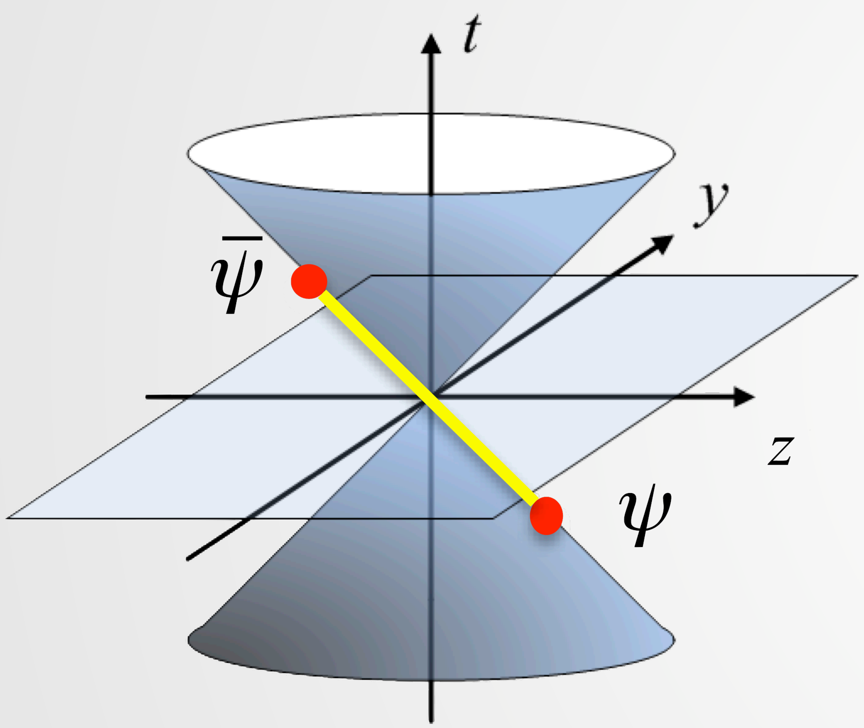
$$t = i\tau, \quad e^{iS} \to e^{-S}, \quad \langle O \rangle = \int D\psi D\bar\psi DA\ O(x) e^{-S}$$

- Euclidean space, imaginary time;
- Difficult to analytically continue lattice results back to Minkowski space.

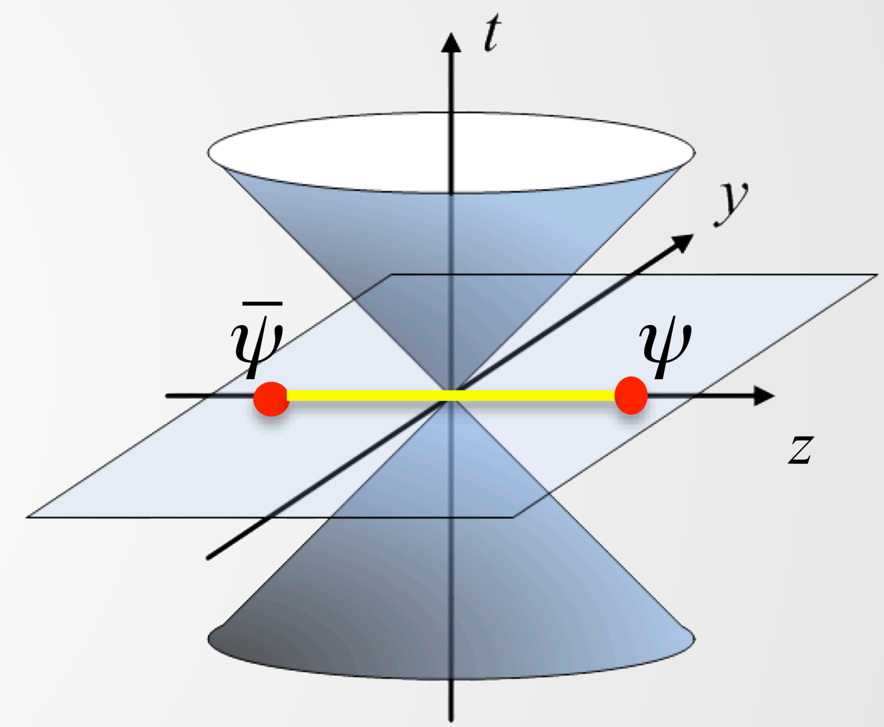
Light-cone PDFs not directly accessible from the lattice!

A novel approach to calculate light-cone PDFs

- Large-Momentum Effective Theory:
  - Ji, PRL110 (2013);
  - Ji, SCPMA57 (2014).



PDF $q(x)$:
Cannot be calculated on the lattice



Quasi-PDF $\tilde{q}(x, P^z)$:
Directly calculable on the lattice

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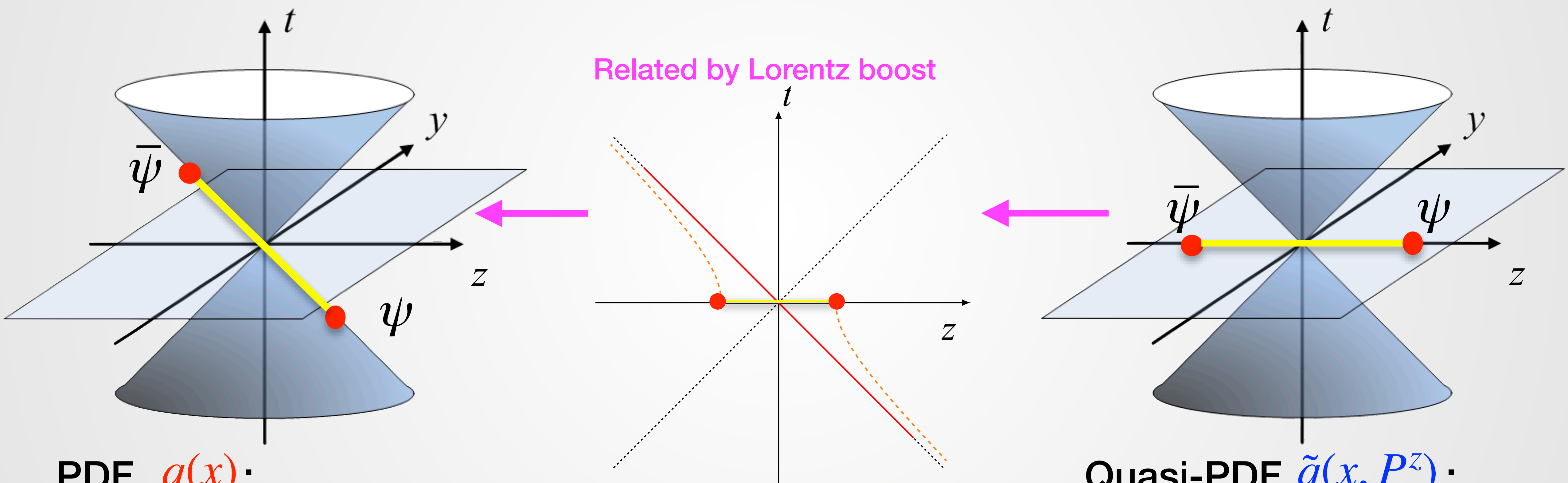

PDF $q(x)$:
Cannot be calculated on the lattice

Quasi-PDF $\tilde{q}(x, P^z)$:
Directly calculable on the lattice

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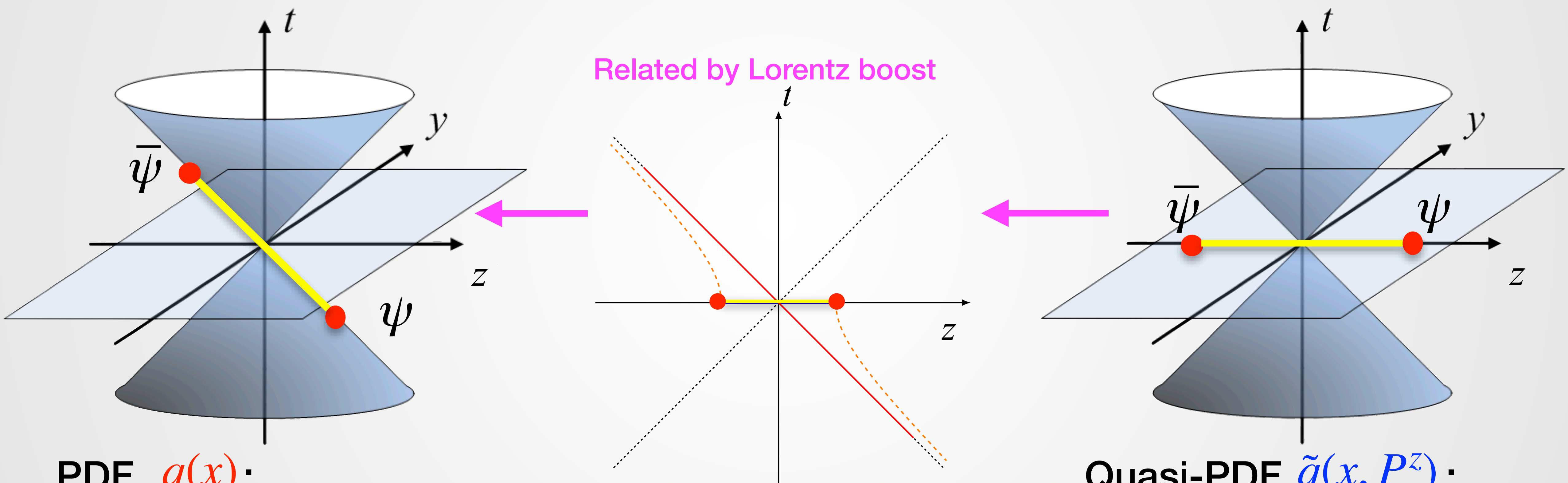


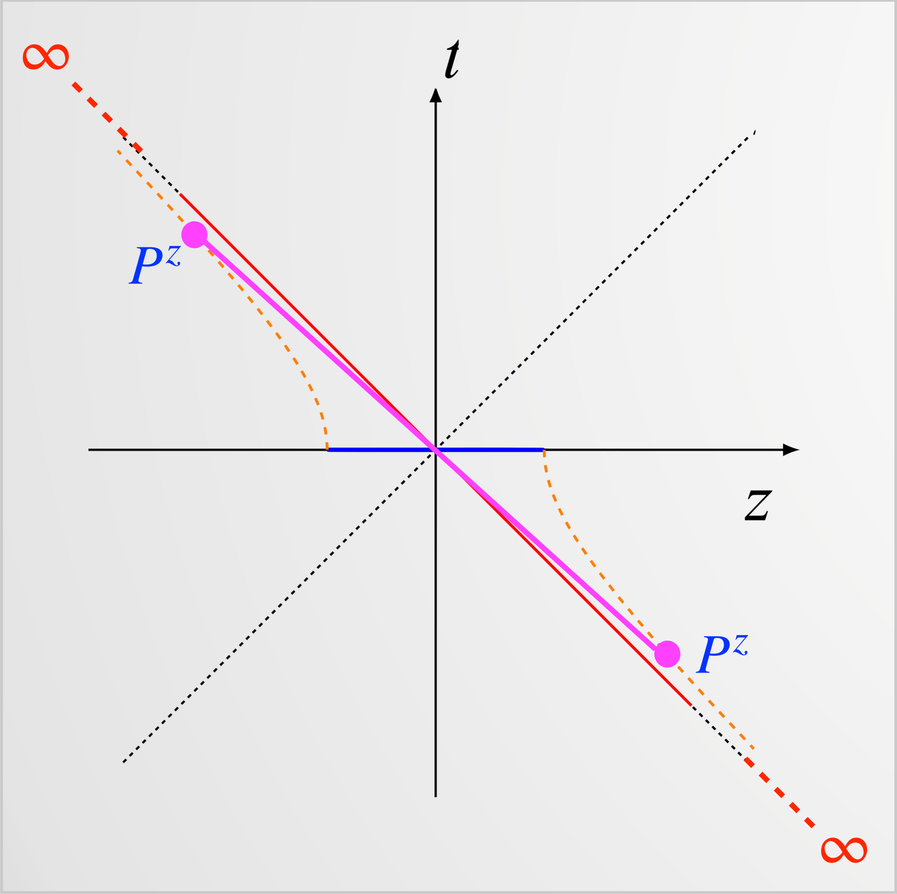
PDF $q(x)$:
Cannot be calculated on the lattice

Calculating the quasi-PDF at **hadron momentum** P^z is equivalent to boosting it.

Quasi-PDF $\tilde{q}(x, P^z)$:
Directly calculable on the lattice

A novel approach to calculate light-cone PDFs

$$\lim_{P^z\to\infty} \tilde{q}(x, P^z) = ?$$ ✗



Instead of taking $P^z\to\infty$ limit, one can perform an expansion for large but finite P^z:

$$\tilde{q}(x, P^z) = C\left(x, P^z\right) \otimes q(x) + O\left(1/(P^z)^2\right)$$

- $\tilde{q}(x, P^z)$ and $q(x)$ have the same infrared physics (nonperturbative), but different ultraviolet (UV) physics (perturbative);
- Therefore, the matching coefficient $C(x, P^z)$ is perturbative, which controls the logarithmic dependences on P^z.

Systematic procedure of calculating the PDFs

$$\tilde{q}(x,P^z,\mu)=\int\frac{dy}{|y|}C\left(\frac{x}{y},\frac{\mu}{yP^z}\right)q(y,\mu)+O\left(\frac{M^2}{P_z^2},\frac{\Lambda_{\rm QCD}^2}{x^2P_z^2}\right)$$

- X. Xiong, X. Ji, J.-H. Zhang and Y.Z., PRD90 (2014);
- Y.-Q. Ma and J. Qiu, PRD98 (2018), PRL 120 (2018);
- T. Izubuchi, X. Ji, L. Jin, I. Stewart, and Y.Z., PRD98 (2018).

1. Lattice simulation of the quasi-PDF;
2. Lattice renormalization and the physical limits (continuum, infinite volume, physical pion mass);
3. Power corrections;
4. Perturbative matching.

For complete review of LaMET, see:

- Cichy and Constantinou, Adv.High Energy Phys. 2019 (2019) 3036904;
- Y.Z., Int.J.Mod.Phys. A33 (2019);
- C. Alexandrou et al. (ETMC), Phys.Rev. D99 (2019) no.11, 114504;

Also see Y.-Z. Liu's talk on Sunday for more detailed introduction.

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  - Transverse momentum dependent PDFs
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Three-dimensional partonic hadron structures

- Longitudinal Parton Distribution Functions (PDFs): $q_{i=q,\bar{q},g}(x)$

- Generalized Parton Distributions (GPDs):

  $$F_i(x, \xi = 0, \vec{b}_T)$$

  - $\vec{b}_T$: transverse position of the parton.

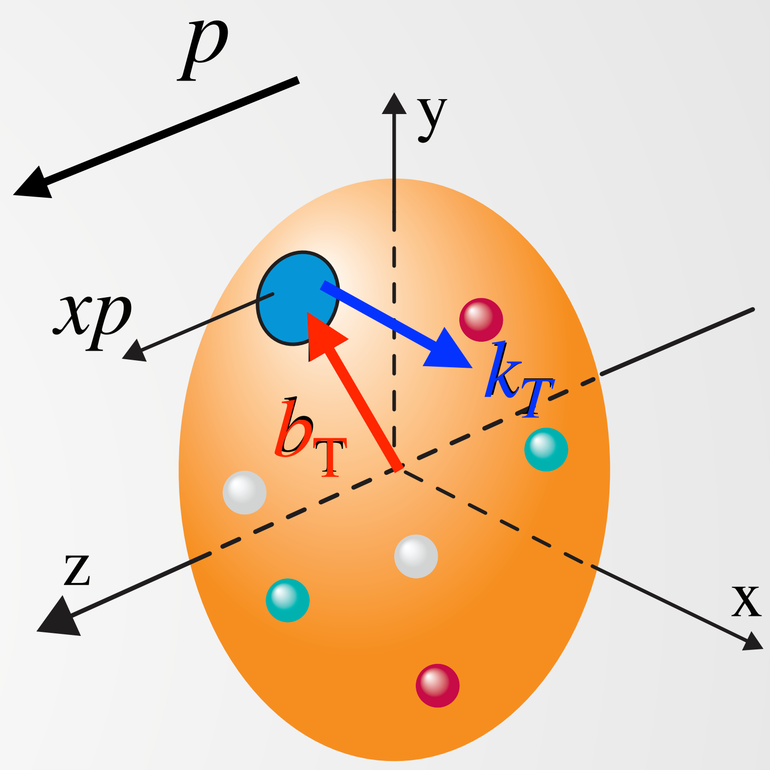
- Transverse momentum dependent (TMD) PDFs

  $$q_i(x, \vec{k}_T)$$

  - $\vec{k}_T$: transverse momentum of the parton.

- Wigner distributions or generalized transverse momentum dependent distributions

  $$W_i(x, \xi = 0, \vec{k}_T, \vec{b}_T)$$



The longitudinal and transverse PDFs provide complete 3D structural information of the proton.

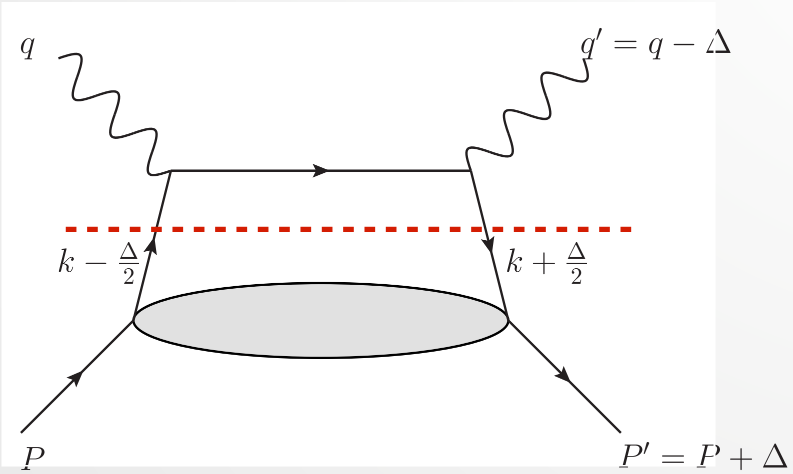
GPD

- Light-cone GPD:

$$\xi = \frac{P^+ - P'^+}{P^+ + P'^+}, \quad t = (P' - P)^2 \equiv \Delta^2$$

$$F_\Gamma(x, \xi, t, \mu) = \int \frac{d\zeta^-}{4\pi} e^{-ix\bar{P}^+\zeta^-} \langle P', S' | \bar{\psi}(\frac{\zeta^-}{2}) \Gamma U(\frac{\zeta^-}{2}, -\frac{\zeta^-}{2}) \psi(-\frac{\zeta^-}{2}) | P, S \rangle$$

- Measurable in hard exclusive processes such as deeply virtual Compton scattering:



$$\sim \int dx \; C(x, \xi) F(x, \xi, t)$$

Quasi-GPD

$$\tilde{\xi} = \frac{P^z - P'^z}{P^z + P'^z} = \xi + O\left(\frac{M^2}{P_z^2}\right)$$

- **Definition:**

$$\tilde{F}_{\tilde{\Gamma}}(x, \tilde{\xi}, t, \mu) = \int \frac{dz}{4\pi} e^{-ixP^z z} \langle P', S' | \bar{\psi}\left(\frac{z}{2}\right) \tilde{\Gamma} U\left(\frac{z}{2}, -\frac{z}{2}\right) \psi\left(-\frac{z}{2}\right) | P, S \rangle$$

- **Renormalization:**
  - Same operator as the quasi-PDF, can be renormalized the same way!

• Y.-S. Liu, Y.Z. et al., PRD100 (2019) no.3, 034006

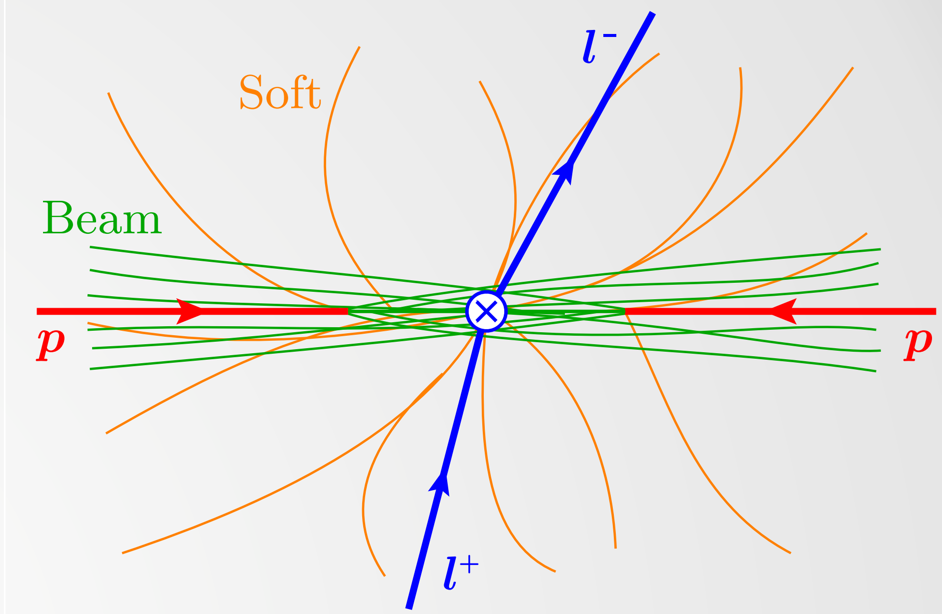
- **Factorization formula:**

$$\tilde{F}_{\tilde{\gamma}^z}(x, \xi, t, \mu) = \int_{-1}^{1} \frac{dy}{|\xi|} C\left(\frac{x}{\xi}, \frac{y}{\xi}, \frac{\mu}{\xi P^z}\right) F_{\gamma^+}(y, \xi, t, \mu) + O\left(\frac{M^2}{P_z^2}, \frac{t}{P_z^2}, \frac{\Lambda_{QCD}^2}{x^2 P_z^2}\right)$$

$$= \int_{-1}^{1} \frac{dy}{|y|} \bar{C}\left(\frac{x}{y}, \frac{\xi}{y}, \frac{\mu}{y P^z}\right) F_{\gamma^+}(y, \xi, t, \mu) + O\left(\frac{M^2}{P_z^2}, \frac{t}{P_z^2}, \frac{\Lambda_{QCD}^2}{x^2 P_z^2}\right)$$

- First lattice calculation of pion GPD, Chen, Lin and Zhang, arXiv: 1904.12376.
- Preliminary results for quasi-GPDs (ETMC), see M. Constantinou's talk at QCD Evolution 2019.

TMDPDF



- Collinear factorization (e.g., for Drell-Yan):

$$\frac{d\sigma}{dQdY} = \sum_{a,b} \sigma_{ab}(Q, \mu, Y)\ f_a(x_1, \mu)\ f_b(x_2, \mu)$$

- TMDPDF factorization:

$$\frac{d\sigma}{dQdYd^2q_T} = \sum_{i,j} H_{ij}(Q, \mu) \int d^2b_T\, e^{i\vec{b}_T\cdot\vec{q}_T} f_i^{\mathrm{TMD}}(x_a, \vec{b}_T, \mu, \zeta_a) f_j^{\mathrm{TMD}}(x_b, \vec{b}_T, \mu, \zeta_b)$$

q_T: Net transverse momentum of the color-singlet final state, and $q_T << Q$;

ζ: Collins-Soper Scale. $\zeta_a\zeta_b = Q^4$

- The definition of TMDPDF involves a collinear beam function (or un-subtracted TMD) and soft function:

Rapidity divergence regulator

$$f_i^{\mathrm{TMD}}(x, \vec{b}_T, \mu, \zeta) = \lim_{\epsilon\to 0, \tau\to 0} Z_{\mathrm{UV}}(\epsilon, \mu, xP^+) B_i(x, \vec{b}_T, \epsilon, \tau, xP^+) \Delta S^i(b_T, \epsilon, \tau)$$

UV divergence regulator

Evolution of TMDPDF

- Evolution of TMDPDF:

$$f_i^{\mathrm{TMD}}(x, \overrightarrow{b}_T, \mu, \zeta) = f_i^{\mathrm{TMD}}(x, \overrightarrow{b}_T, \mu_0, \zeta_0) \exp\left[\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_\mu^i(\mu', \zeta_0)\right] \exp\left[\frac{1}{2}\gamma_\zeta^i(\mu, b_T) \ln\frac{\zeta}{\zeta_0}\right]$$

- $\mu \sim Q, \zeta \sim Q^2 >> \Lambda_{\mathrm{QCD}}^2$;
- μ_0, ζ_0: initial or reference scales, measured in experiments or determined from lattice (~2 GeV).

$\gamma_\mu^i(\mu', \zeta_0)$ Anomalous dimension for μ evolution, perturbatively calculable;

$\gamma_\zeta^i(\mu, b_T)$ Collins-Soper kernel, becomes nonperturbative when $b_T \sim 1/\Lambda_{\mathrm{QCD}}$.

Both Initial-scale TMDPDF and the Collins-Soper kernel must be modeled in global fits of TMDPDF from experimental data.

$$\gamma_\zeta^i(\mu, b_T) = -2\int_{\mu(b_T)}^{\mu} \frac{df\mu'}{\mu'} \Gamma_{\mathrm{cusp}}^i[\alpha_s(\mu')] + \gamma_\zeta^i[\alpha_s(\mu(b_T))]$$

$$+ g_K(b_T)$$

$\mu(b_T) \gg \Lambda_{\mathrm{QCD}}$

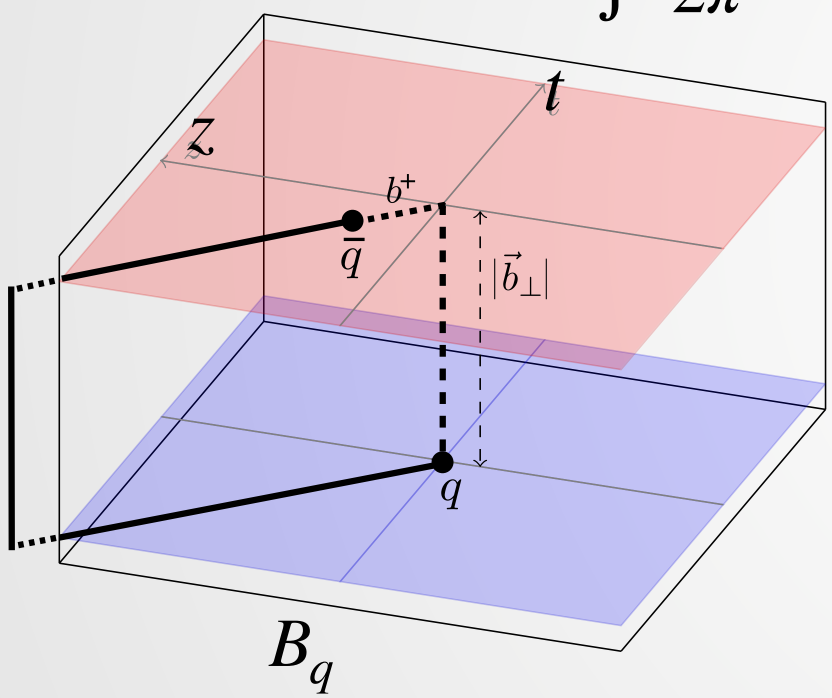
- Bachetta et al., JHEP 1706 (2017);
- Scimemi and Vladimirov, EPJC78 (2018);
- Bertone, Scimemi and Vladimirov, JHEP 1906 (2019).

Towards a lattice calculation of the TMDPDF

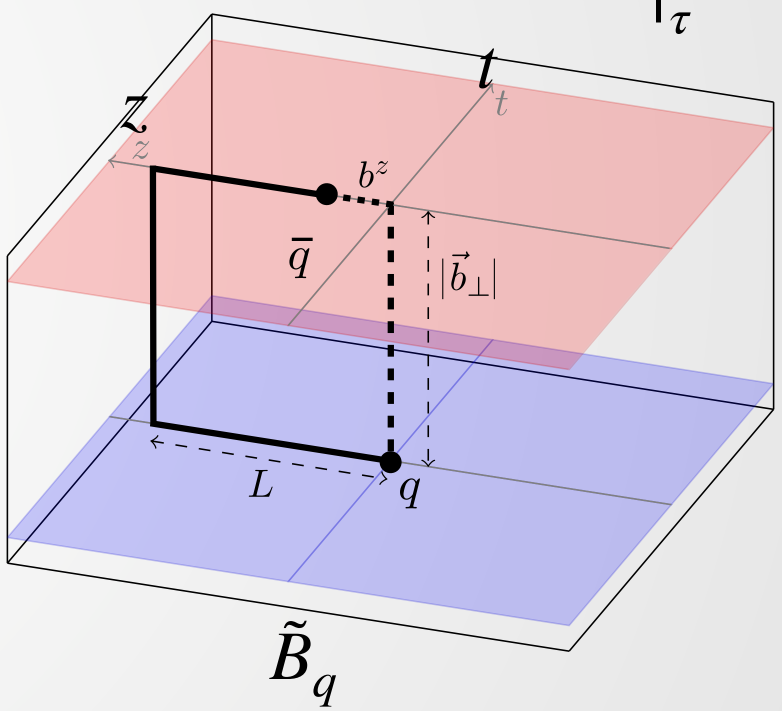
- Ji, Sun, Xiong and Yuan, PRD91 (2015);
- Ji, Jin, Yuan, Zhang and Y.Z., PRD99 (2019);
- M. Ebert, I. Stewart, Y.Z., PRD99 (2019);
- M. Ebert, I. Stewart, Y.Z., arXiv:1901.03685.

- Beam function:

$$B_q(x, \vec{b}_T, \epsilon, \tau) = \int \frac{db^+}{2\pi} e^{-i(xP^+)b^-} \langle P | \bar{q}(b^\mu) W(b^\mu) \frac{\gamma^+}{2} W_T(-\infty\bar{n}; \vec{b}_T, \vec{0}_T) W^\dagger(0) q(0) \Big|_\tau | P \rangle$$



Lorentz boost and $L \to \infty$



- Quasi-beam function on lattice:

$$\tilde{B}_q(x, \vec{b}_T, a, L, P^z) = \int \frac{db^z}{2\pi} e^{ib^z(xP^z)} \tilde{B}_q(b^z, \vec{b}_T, a, L, P^z)$$

$$= \int \frac{db^z}{2\pi} e^{ib^z(xP^z)} \langle P | \bar{q}(b^\mu) W_{\hat{z}}(b^\mu; L - b^z) \frac{\Gamma}{2} W_T(L\hat{z}; \vec{b}_T, \vec{0}_T) W_{\hat{z}}^\dagger(0) q(0) | P \rangle$$

Towards a lattice calculation of the TMDPDF

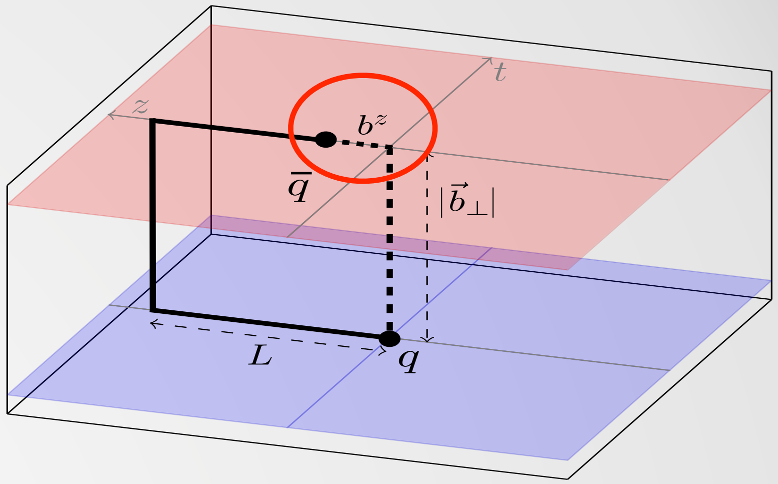
- Soft function:

$$S_q(b_T,\epsilon,\tau)=\frac{1}{N_c}\langle 0|\,\mathrm{Tr}\left[S_n^\dagger(\vec{b}_T)S_{\bar n}(\vec{b}_T)S_T(-\infty\bar n;\vec{b}_T,\vec{0}_T)S_{\bar n}^\dagger(\vec{0}_T)S_n(\vec{0}_T)S_T^\dagger(-\infty n;\vec{b}_T,\vec{0}_T)\right]\Big|_\tau|0\rangle$$

- Quasi-soft function on lattice (naive definition):

$$\tilde S_q(b_T,a,L)=\frac{1}{N_c}\langle 0|\,\mathrm{Tr}\left[S_{\hat z}^\dagger(\vec{b}_T;L)S_{-\hat z}(\vec{b}_T;L)S_T(L\hat z;\vec{b}_T,\vec{0}_T)S_{-\hat z}^\dagger(\vec{0}_T;L)S_n(\vec{0}_T;L)S_T^\dagger(-L\hat z;\vec{b}_T,\vec{0}_T)\right]|0\rangle$$

Quasi-TMDPDF and its relation to TMDPDF



- Quasi-TMDPDF (in the MSbar scheme):

$$\tilde{f}_q^{\rm TMD}(x, \vec{b}_T, \mu, P^z) = \lim_{L\to\infty} \int \frac{db^z}{2\pi} e^{ib^z(xP^z)} \tilde{Z}'(b^z, \mu, \tilde{\mu}) \tilde{Z}_{\rm UV}(b^z, \mu, a) \frac{\tilde{B}_q(b^z, \vec{b}_T, a, L, P^z)}{\sqrt{\tilde{S}_q(b_T, a, L)}}$$

- Relation to TMDPDF:

- M. Ebert, I. Stewart, Y.Z., arXiv:1901.03685.

$$\tilde{f}_{\rm ns}^{\rm TMD}(x, \vec{b}_T, \mu, P^z) = C_{\rm ns}^{\rm TMD}\left(\mu, xP^z\right) g_q^S(b_T, \mu) \exp\left[\frac{1}{2}\gamma_\zeta^q(\mu, b_T) \ln \frac{(2xP^z)^2}{\zeta}\right]$$

$$\times f_{\rm ns}^{\rm TMD}(x, \vec{b}_T, \mu, \zeta) + \mathcal{O}\left(\frac{b_T}{L}, \frac{1}{b_T P^z}, \frac{1}{P^z L}\right)$$

Hierarchy of scales: $b^z \sim \frac{1}{P^z} \ll b_T \ll L$

For $b_T \ll \Lambda_{\rm QCD}^{-1}$ $\quad g_q^{S_{\rm naive}}(b_T, \mu) = 1 + \frac{\alpha_s C_F}{2\pi} \ln \frac{b_T^2 \mu^2}{4e^{-2\gamma_E}} + O(\alpha_s^2)$

- Ji, Jin, Yuan, Zhang and Y.Z., PRD99 (2019);
- M. Ebert, I. Stewart, Y.Z., arXiv:1901.03685.

Collins-Soper kernel of TMDPDF from lattice QCD

- M. Ebert, I. Stewart, Y.Z., PRD99 (2019);
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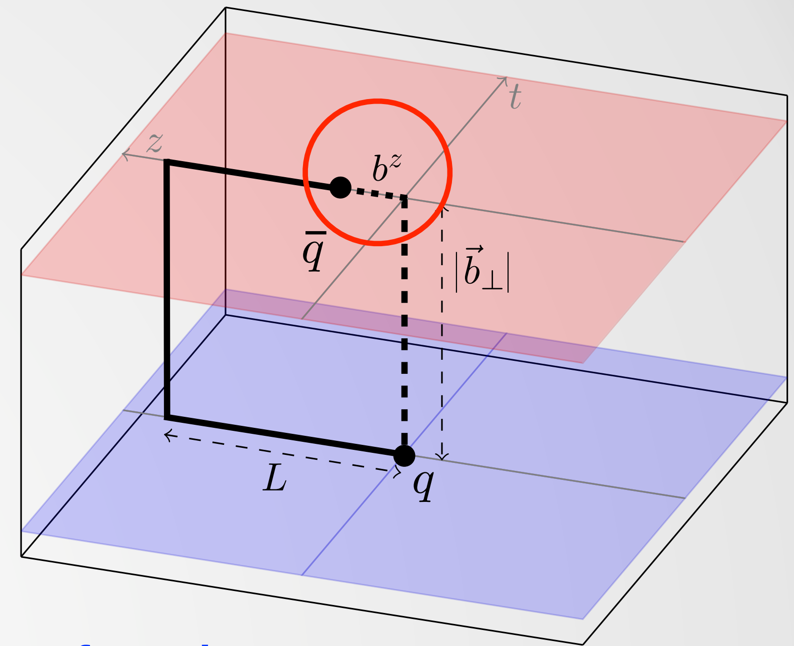

Physical limit:

$$b^z \sim \frac{1}{P^z} \ll b_T \ll L$$

Quasi-beam function
(or un-subtracted quasi-TMD)

$$\gamma_\zeta^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)}$$

$$\times \ln \frac{C_{\rm ns}^{\rm TMD}(\mu, xP_2^z) \int db^z \, e^{ib^z x P_1^z} \, \tilde{Z}'(b^z, \mu, \tilde{\mu}) \tilde{Z}_{\rm UV}(b^z, \tilde{\mu}, a) \tilde{B}_{\rm ns}(b^z, \vec{b}_T, a, L, P_1^z)}{C_{\rm ns}^{\rm TMD}(\mu, xP_1^z) \int db^z \, e^{ib^z x P_2^z} \, \tilde{Z}'(b^z, \mu, \tilde{\mu}) \tilde{Z}_{\rm UV}(b^z, \tilde{\mu}, a) \tilde{B}_{\rm ns}(b^z, \vec{b}_T, a, L, P_2^z)}$$

The idea of forming ratios to cancel the soft function has been used in the calculation of *x*-moments of TMDPDFs by
Hagler, Musch, Engelhardt, Yoon, et al., EPL88 (2009), PRD83 (2011), PRD85 (2012), PRD93 (2016), arXiv:1601.05717, PRD96 (2017)

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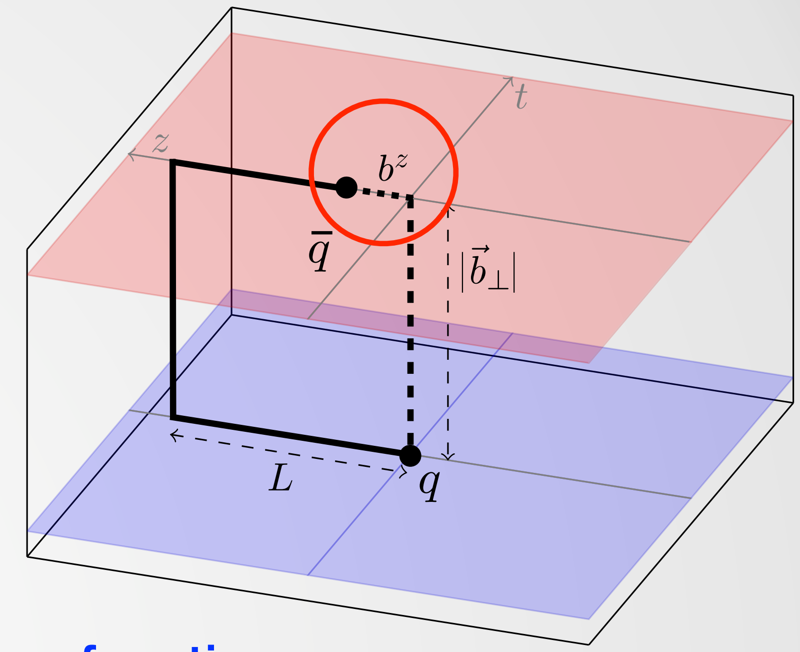

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$$\times \ln \frac{C_{\rm ns}^{\rm TMD}(\mu, xP_2^z) \int db^z\, e^{ib^z xP_1^z}\, \tilde{Z}'(b^z,\mu,\tilde{\mu})\tilde{Z}_{\rm UV}(b^z,\tilde{\mu},a)\tilde{B}_{\rm ns}(b^z,\vec{b}_T,a,L,P_1^z)}{C_{\rm ns}^{\rm TMD}(\mu, xP_1^z) \int db^z\, e^{ib^z xP_2^z}\, \tilde{Z}'(b^z,\mu,\tilde{\mu})\tilde{Z}_{\rm UV}(b^z,\tilde{\mu},a)\tilde{B}_{\rm ns}(b^z,\vec{b}_T,a,L,P_2^z)}$$

Physical Collins-Soper (CS) kernel does not depend on the external hadron state, which means that one can just calculate it with a pion state including heavy valence quarks.

work in progress with

Phiala Shanahan and Michael Wagman.

Procedure of lattice calculation

- 1. Lattice simulation of the bare quasi-beam function

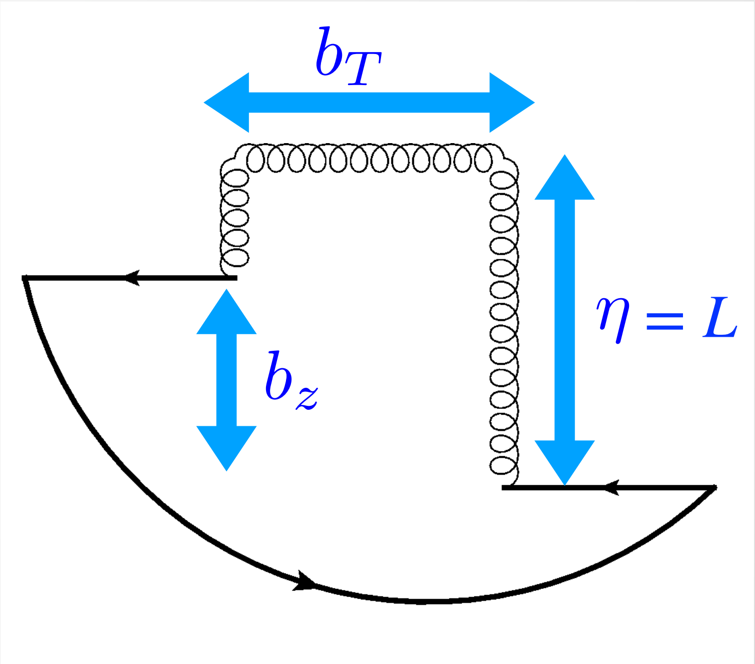
$$\gamma_\zeta^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)}$$

$$\times \ln \frac{C_{\text{ns}}^{\text{TMD}}(\mu, xP_2^z) \int db^z\, e^{ib^z xP_1^z} Z'(b^z, \mu, \tilde{\mu}) \tilde{Z}_{\text{UV}}(b^z, \tilde{\mu}, a) \tilde{B}_{\text{ns}}(b^z, \vec{b}_T, a, L, P_1^z)}{C_{\text{ns}}^{\text{TMD}}(\mu, xP_1^z) \int db^z\, e^{ib^z xP_2^z} Z'(b^z, \mu, \tilde{\mu}) \tilde{Z}_{\text{UV}}(b^z, \tilde{\mu}, a) \tilde{B}_{\text{ns}}(b^z, \vec{b}_T, a, L, P_2^z)}$$

$$b^z \sim \frac{1}{P^z} \ll b_T \ll \eta < \frac{L_{\text{Lat}}}{2}$$

Choice of γ matrix: to choose γ^t or γ^z depending on operator mixing.

- M. Constantinou et al., PRD99 (2019)



Procedure of lattice calculation

- 2. Renormalization and matching to the MSbar scheme

$$\gamma_\zeta^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)} \times \ln \frac{C^{\rm TMD}_{\rm ns}(\mu, xP_2^z)\int db^z\, e^{ib^z xP_1^z}\, \tilde Z'(b^z,\mu,\tilde\mu)\tilde Z_{\rm UV}(b^z,\tilde\mu,a)\tilde B_{\rm ns}(b^z,\vec b_T,a,L,P_1^z)}{C^{\rm TMD}_{\rm ns}(\mu, xP_1^z)\int db^z\, e^{ib^z xP_2^z}\, \tilde Z'(b^z,\mu,\tilde\mu)\tilde Z_{\rm UV}(b^z,\tilde\mu,a)\tilde B_{\rm ns}(b^z,\vec b_T,a,L,P_2^z)}$$

Multiplicative renormalizability of the Wilson line operator assumed to be provable using the auxiliary field formalism.

- X. Ji, J.-H. Zhang, and Y.Z., PRL120 (2018);
- J. Green et al., PRL121 (2018);

Linear power divergence: $\sim \frac{|L-b_z|}{a} + \frac{b_T}{a} + \frac{L}{a}$

Nonperturbative Renormalization: $\tilde Z_{\rm UV}(b^z,\tilde\mu,a)$

Perturbative matching to MSbar scheme: $\tilde Z'(b^z,\mu,\tilde\mu)$

Procedure of lattice calculation

- 3. Fourier transform and calculate the ratio at different P^z

$$\gamma_\zeta^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)}$$

$$\times \ln \frac{C_{\rm ns}^{\rm TMD}(\mu, xP_2^z) \int db^z\ e^{ib^z xP_1^z}\ \tilde{Z}'(b^z, \mu, \tilde{\mu})\tilde{Z}_{\rm UV}(b^z, \tilde{\mu}, a)\tilde{B}_{\rm ns}(b^z, \overrightarrow{b}_T, a, L, P_1^z)}{C_{\rm ns}^{\rm TMD}(\mu, xP_1^z) \int db^z\ e^{ib^z xP_2^z}\ \tilde{Z}'(b^z, \mu, \tilde{\mu})\tilde{Z}_{\rm UV}(b^z, \tilde{\mu}, a)\tilde{B}_{\rm ns}(b^z, \overrightarrow{b}_T, a, L, P_2^z)}$$

  - Independent of the choice of *x*!
  - Independent of P^z!
  - Fourier transform has truncation errors, but for given P^z there is always a region of *x* that is insensitive to such truncation effects;
  - One may still seek alternatives to Fourier transforms that can be done directly in coordinate space.

Lattice calculation

- Lattice setup:

generated by Michael Endres

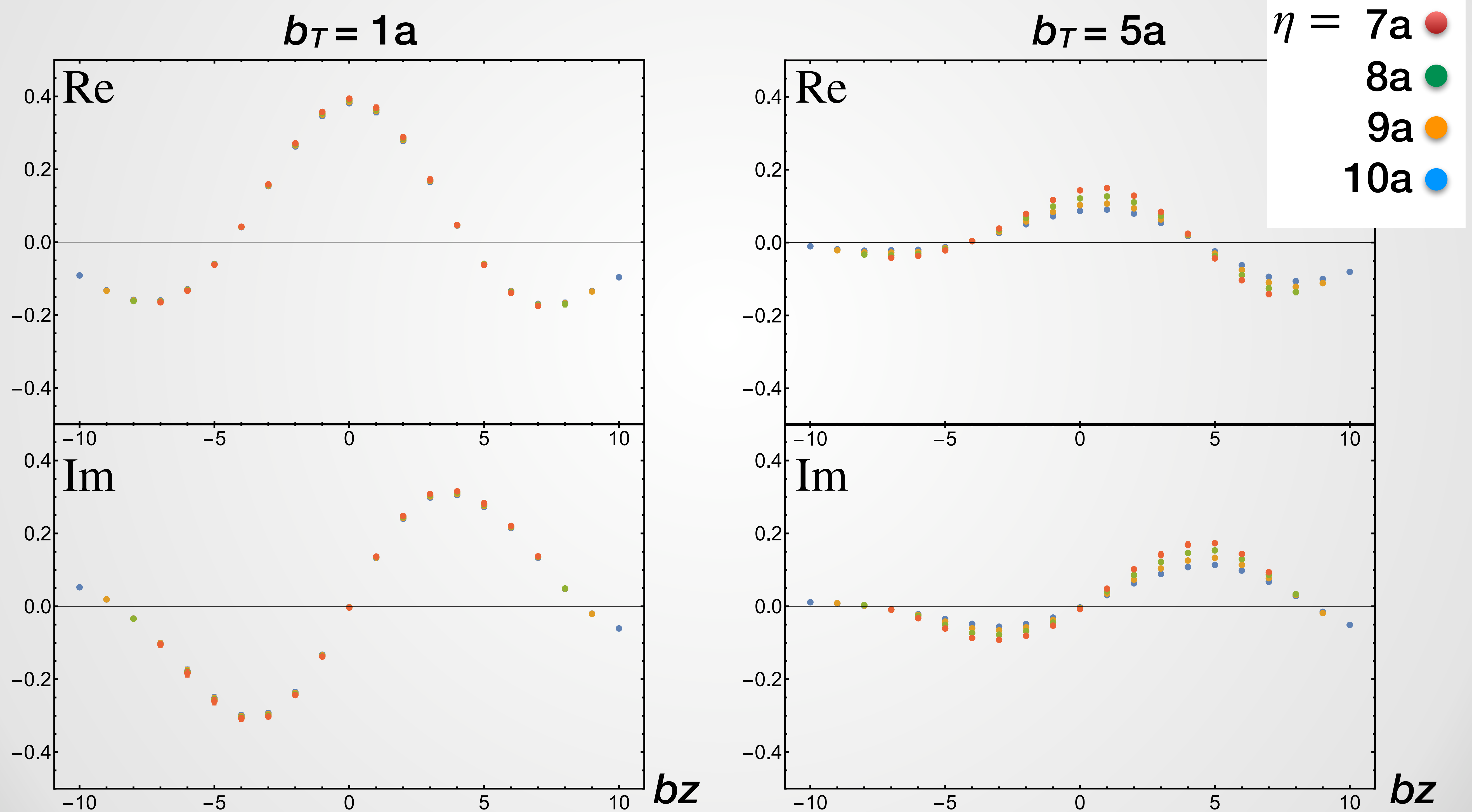
  - Quenched Wilson gauge configurations;
  - β=6.30168, a=0.06(1) fm, $32^3\times64$;
  - Probe valence pion with $m_\pi \sim 1.2$ GeV
  - Each momentum uses 2 gauge fixed plane wave ("wall") quark sources;
  - **A first look at N_{cfg}=7.**

$$0 \le b^z, b_T \le \eta, \quad \eta = \{7,8,9,10\}a \quad p^z = \{2,\ 3,\ 4\}\frac{2\pi}{L}, p^z_{\max} = 2.6\ \text{GeV}$$

Bare matrix elements

Caveat N_{cfg}=7

P^z=2.6 GeV



- Error bars smaller than point size.
- Asymmetric real and imaginary parts in b_z due to stapled shape.

Lattice renormalization in the RI′/MOM Scheme

Green's function:

$$G(b,p) = \sum_x \left\langle \gamma_5 S^\dagger(p, b+x)\gamma_5 U(b+x, x)\frac{\Gamma}{2}S(p,x)\right\rangle$$

Amputated Green's function (or vertex function):

$$\Lambda(b,p) = \left(\gamma_5 \left[S^{-1}(p)\right]^\dagger\right) G(b,p)S^{-1}(p)$$

Momentum subtraction condition:

$$Z_{\mathcal{O}}^{-1}(b, p_\mu^R, \mu_R) Z_q(\mu_R)\ G(b,p)\Big|_{p_\mu = p_\mu^R} = G^{\text{tree}}(b, p_R)\,,$$

- I. Stewart and Y.Z., PRD97 (2018);
- Constantinou and Panagopoulos, PRD96 (2017);
- M. Constantinou et al., PRD99 (2019).

$$Z_q(\mu_R) = \frac{1}{12}\text{Tr}\left[S^{-1}(p)S^{\text{tree}}(p)\right]\Big|_{p^2=\mu_R^2}$$

Mixing

- Tracing with a projection operator to define the renormalization factors.

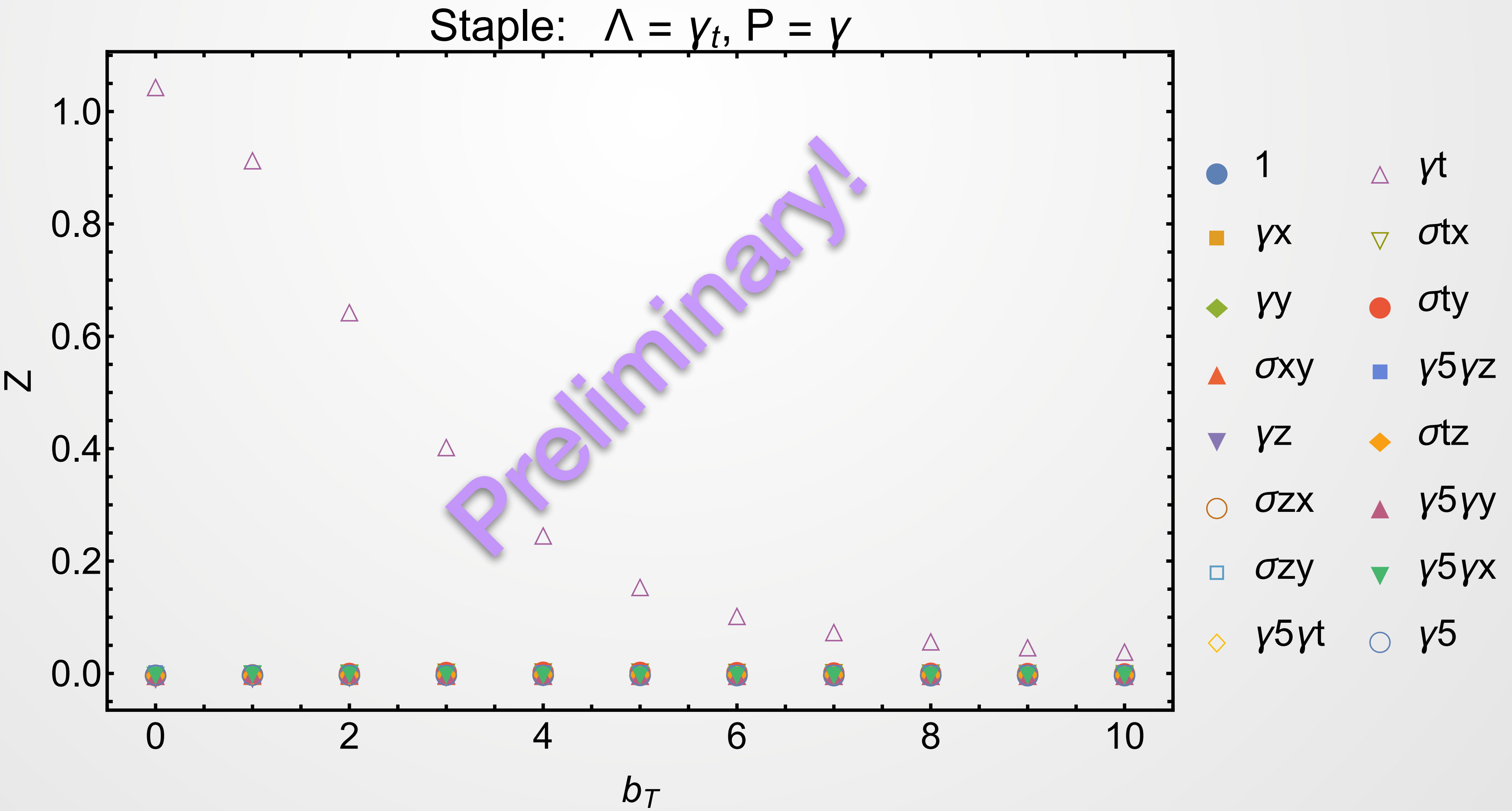
$$\mathrm{Tr}\left[\Lambda_{\gamma^t}(z,p)\mathscr{P}\right]$$

- For simplicity, we choose $\mathscr{P} = \gamma^t$

- To study the mixing effects, we also choose all the other 15 Gamma matrices.

$b_z = 0$,
***eta* = 10a.**

Real part



Mixing

- Tracing with a projection operator to define the renormalization factors.

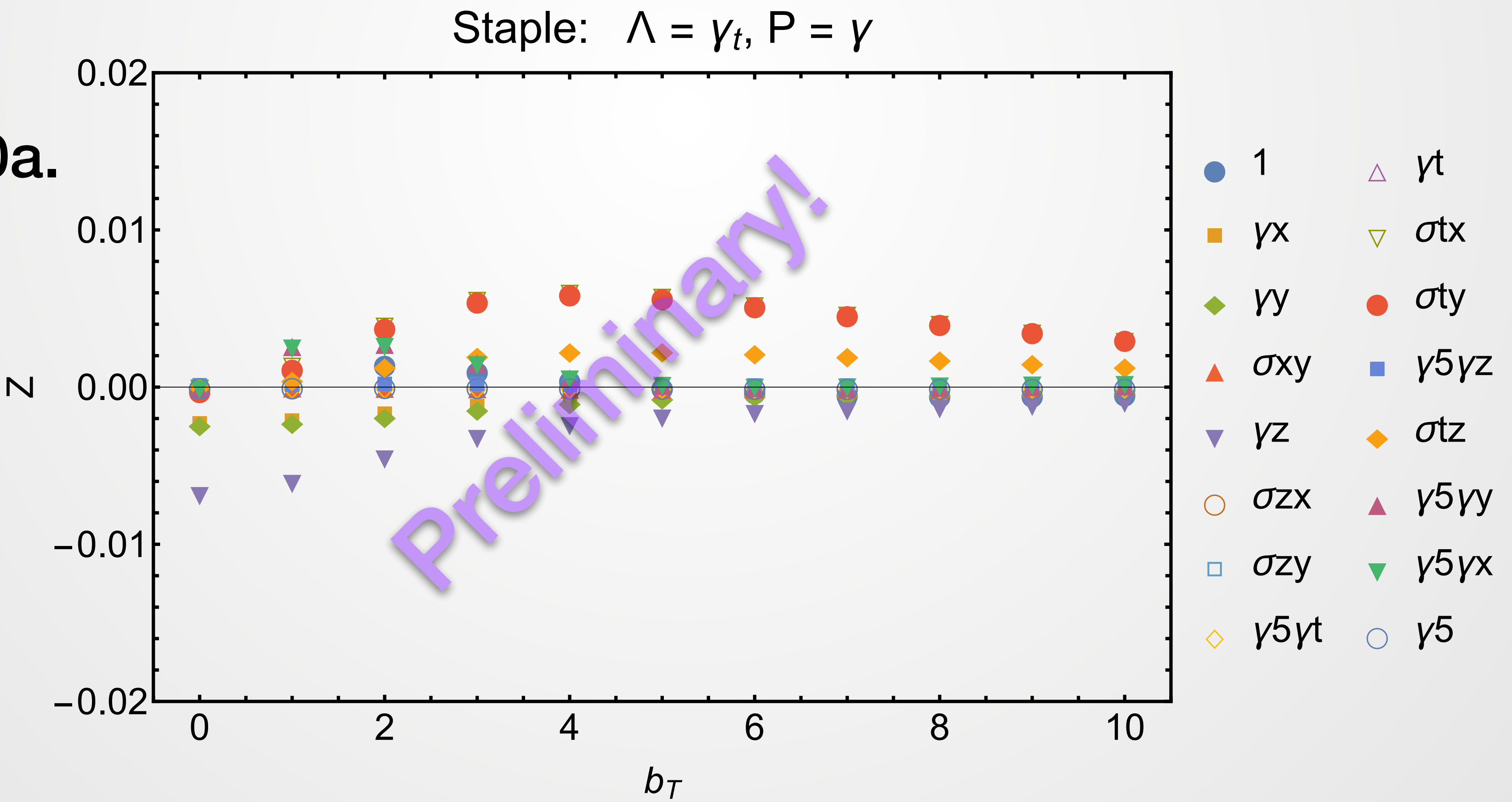
$$\mathrm{Tr}\left[\Lambda_{\gamma^t}(z,p)\mathscr{P}\right]$$

- For simplicity, we choose $\mathscr{P} = \gamma^t$

- To study the mixing effects, we also choose all the other 15 Gamma matrices.

$b_z = 0$,
$eta = 10a$.

Imaginary part

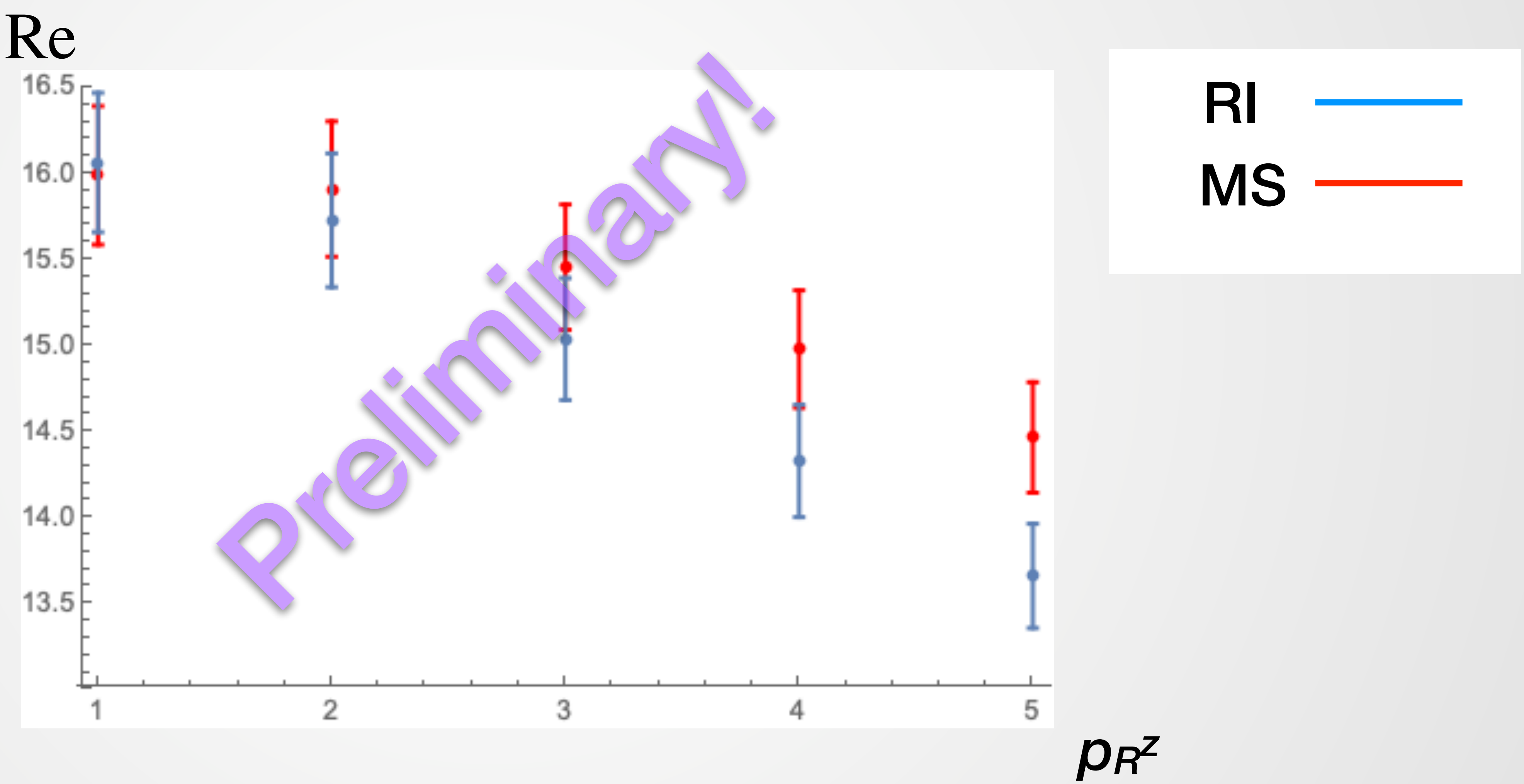


O(1%) effects, negligible for exploratory study.

Matching to MSbar scheme @ 2GeV

$$Z_{\overline{\rm MS}}(\eta, b_z, b_T, \mu, a) = Z_{\mathcal{O}}^{-1}(\eta, b^z, b_T, p_\mu^R, \mu^R, a) \cdot C(\eta, b^z, b_T, p_\mu^R, \mu^R, \mu)$$

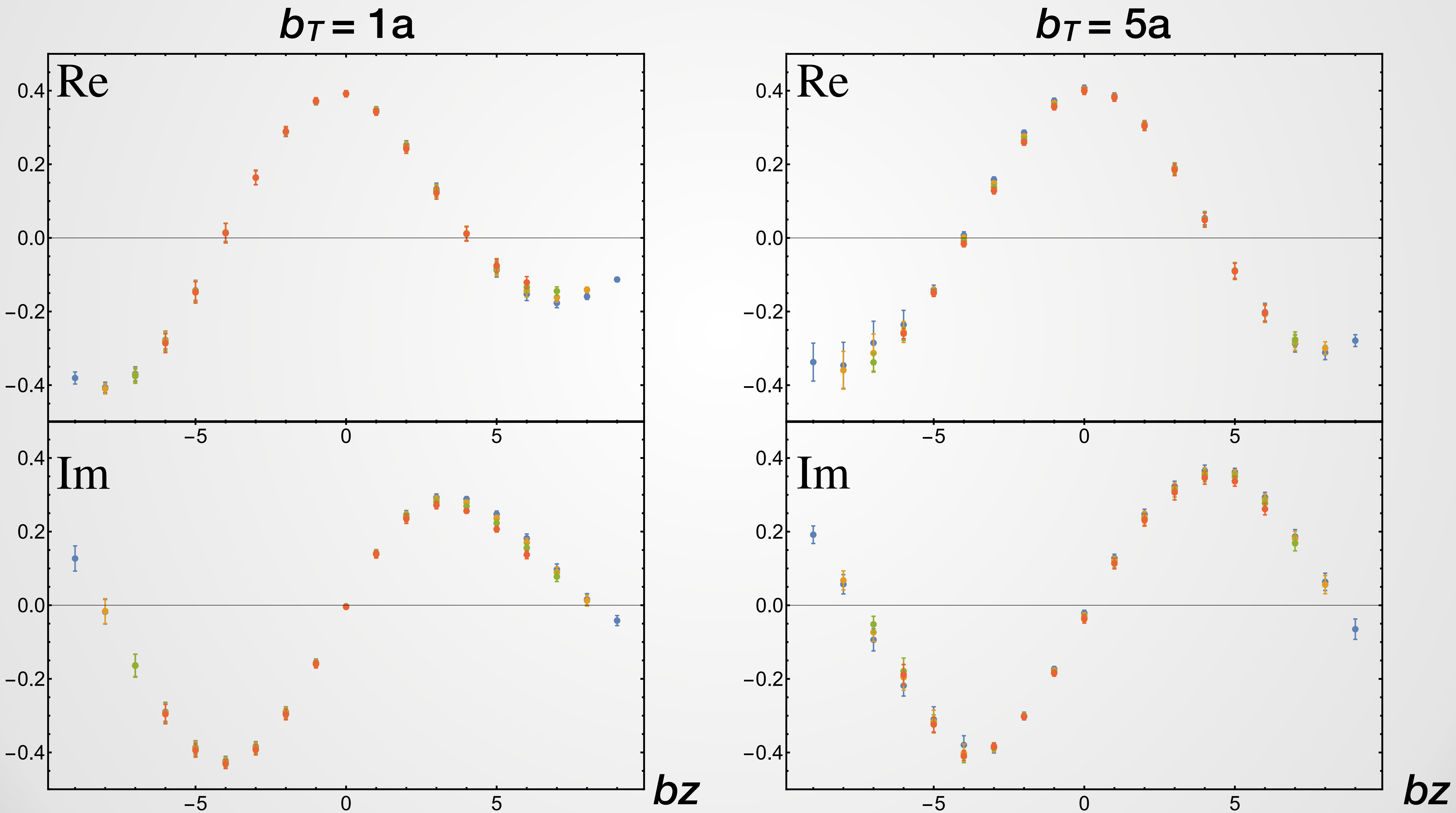
$b_z = 0$,
$b_T = 8a$,
$eta = 10a$,



- The RIMOM renormalization factor is most sensitive to p_R^z;
- Perturbative matching is a small correction, but it compensates for the p_R^z dependence in the matching factor;
- Matched result can be fitted with a constant within the uncertainties.

Renormalized matrix element in the MSbar scheme @ 2 GeV

Caveat N_{cfg}=7



Renormalization renders the real and imaginary parts more anti-symmetric in b_z

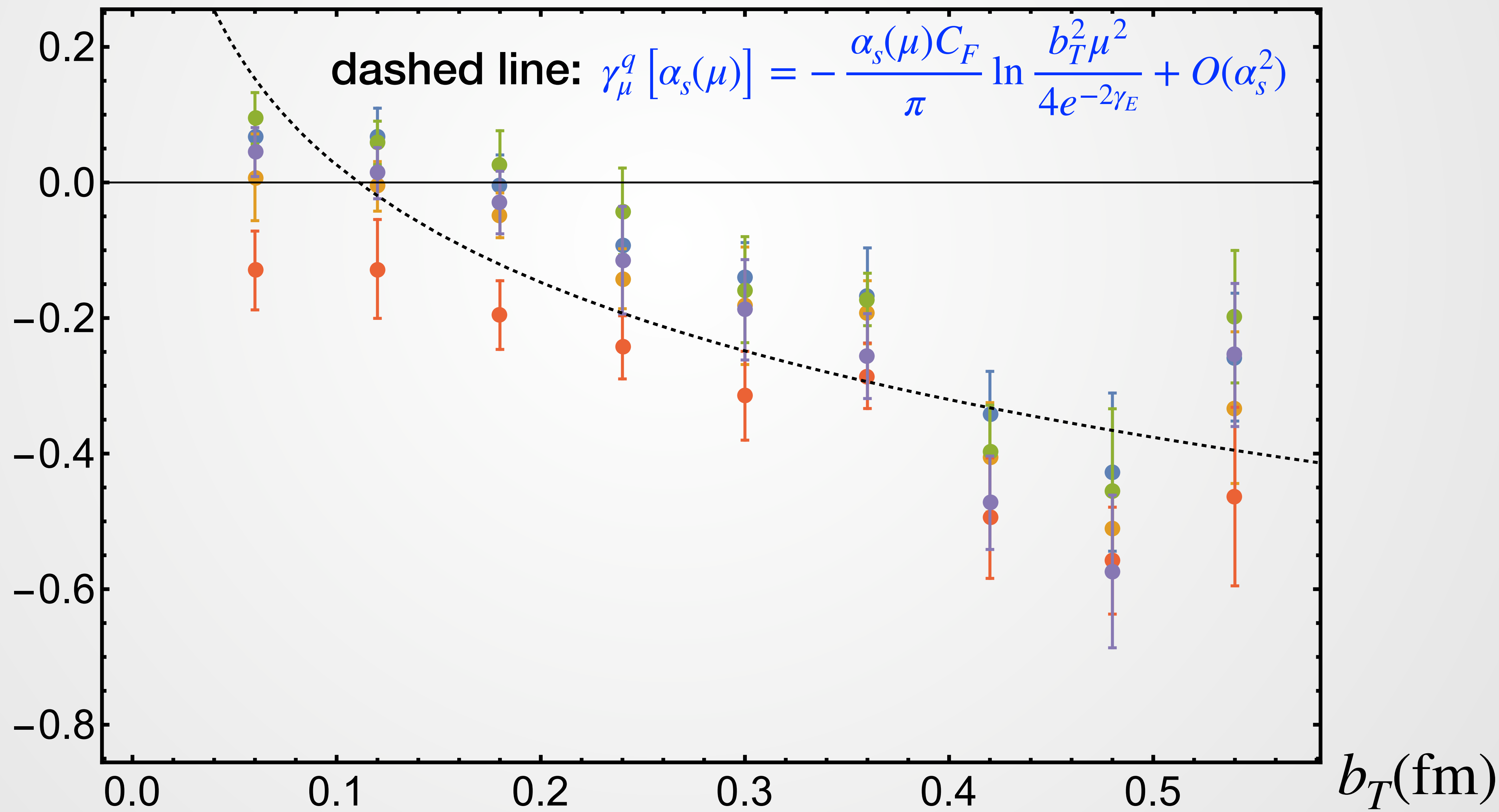
Extraction of the CS kernel with Naive Fourier transform and without matching

Comparison to perturbative results at one-loop (dashed line):

Caveat N_{cfg}=7

- M. Ebert, I. Stewart, Y.Z., PRD99 (2019).

$\gamma_\zeta(b_T, \mu = 2\ \text{GeV})$



Different colored points correspond to CS kernel calculated at *x* = 0.4, 0.45, 0.5, 0.55, 0.6.

Conclusion

- The LaMET approach can be readily applied to the lattice calculation of GPDs;
- Progress has been made in the application of LaMET to TMDPDFs;
- The Collins-Soper kernel can be calculated with LaMET by forming the ratios of quasi-beam functions;
- Encouraging results that LQCD calculations of the CS kernel might be achieved with present-day resources;
- Future work will include (much) larger statistics, different lattice spacings (for taking the continuum limit), and more systematic treatment than the naive Fourier transform.