

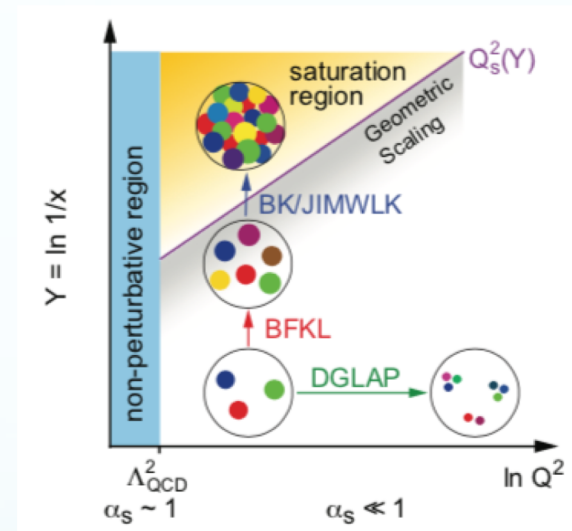
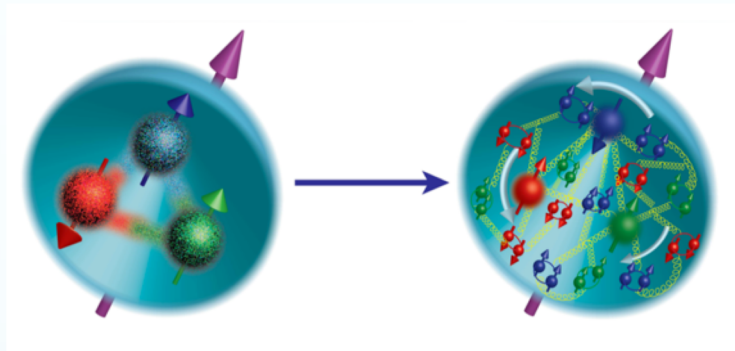
# Overview of TMDs

Zhongbo Kang  
UCLA

11<sup>th</sup> Workshop on Hadron Physics in China and Opportunities Worldwide  
August 23 - 28, 2019

# The proton in QCD

- Proton is made of
  - 2 up quarks + 1 down quarks → valence quarks
  - + any number of quark-antiquark pairs → sea quarks
  - + any number of gluons



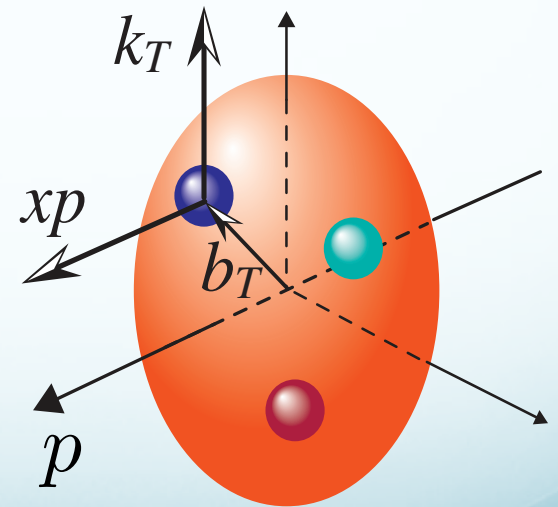
- ✓ Infinite many body dynamic system of quarks and gluons
- ✓ By changing  $x$  and  $Q$ , we probe different aspects of the proton wave function

# Quark and gluon structure of the nucleon

- Goal: quantum tomography in terms of quarks and gluons
  - **Momentum**: how do the quarks, antiquarks, gluons move inside?
  - **Position**: where are they located?
  - **Orbit**: do they orbit, carry orbital angular momentum?
  - **Correlation**: quantum correlations between motion and overall nucleon properties, e.g., spin? How do they respond to the external probes?

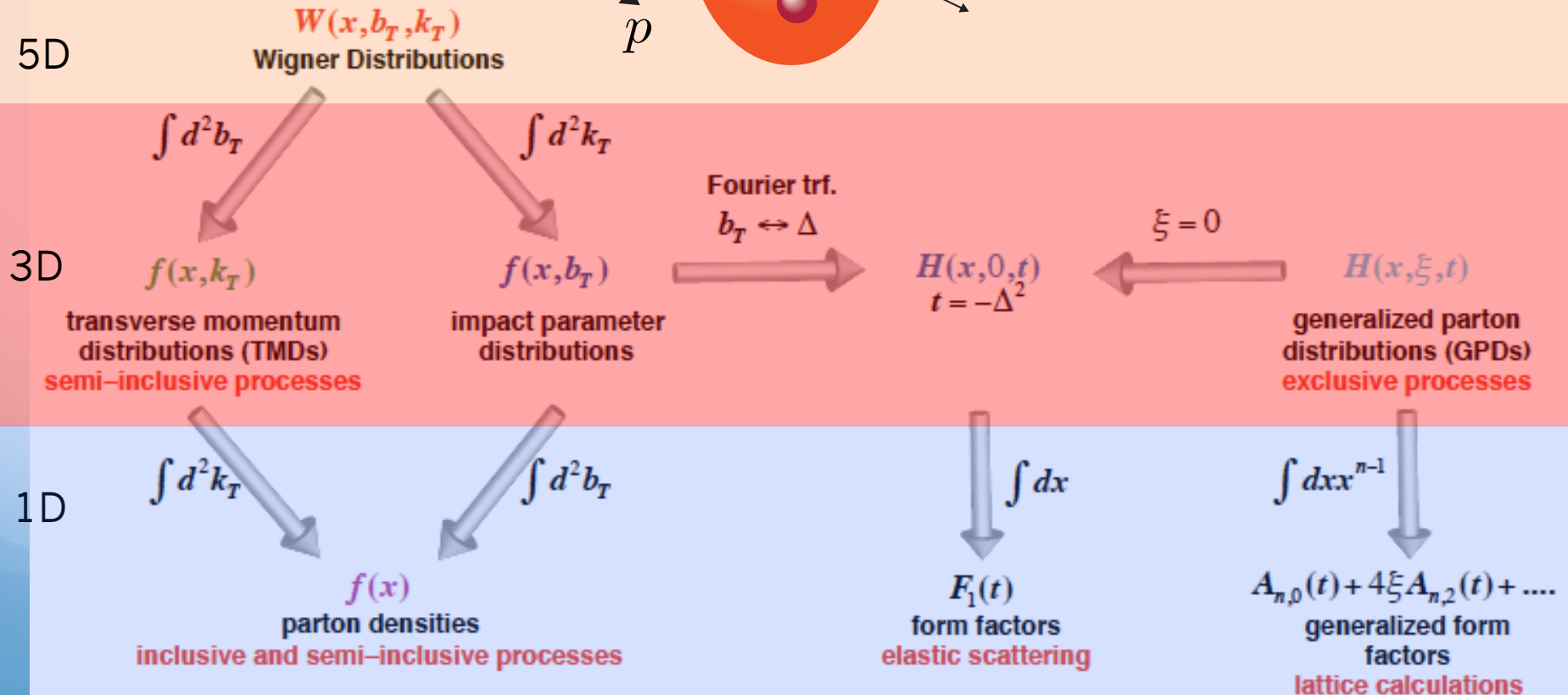
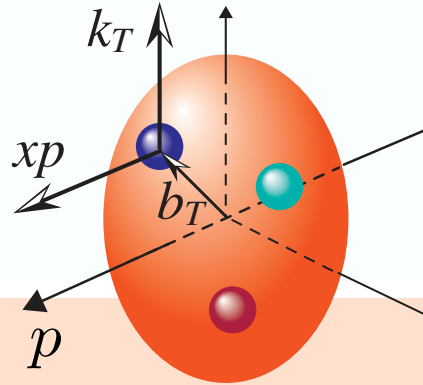
## Internal landscape of the nucleon

Such information are defined as a set of **parton distribution functions**



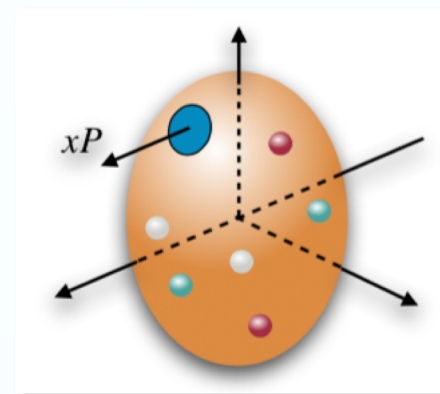
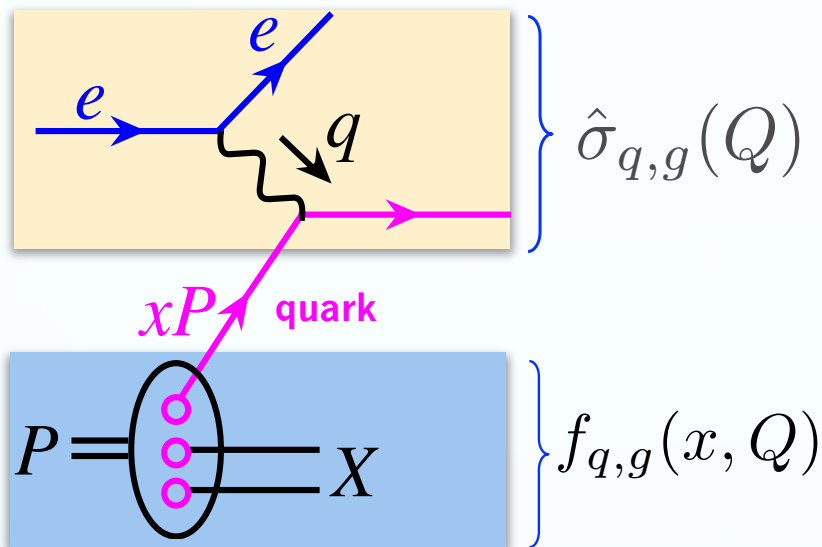
# Unified view: internal landscape

- Wigner distributions: a quantum version of phase-space distribution



# QCD factorization

- Take deep inelastic scattering as an example

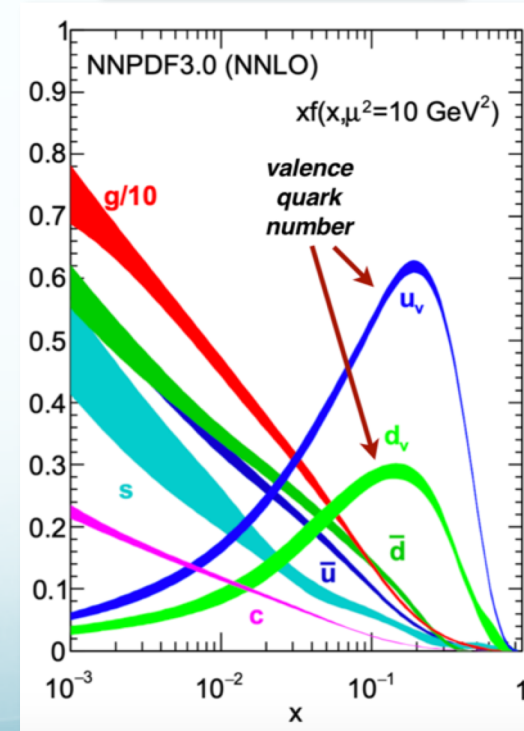
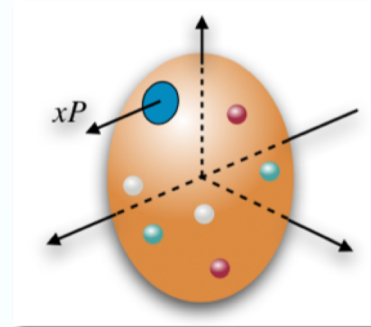
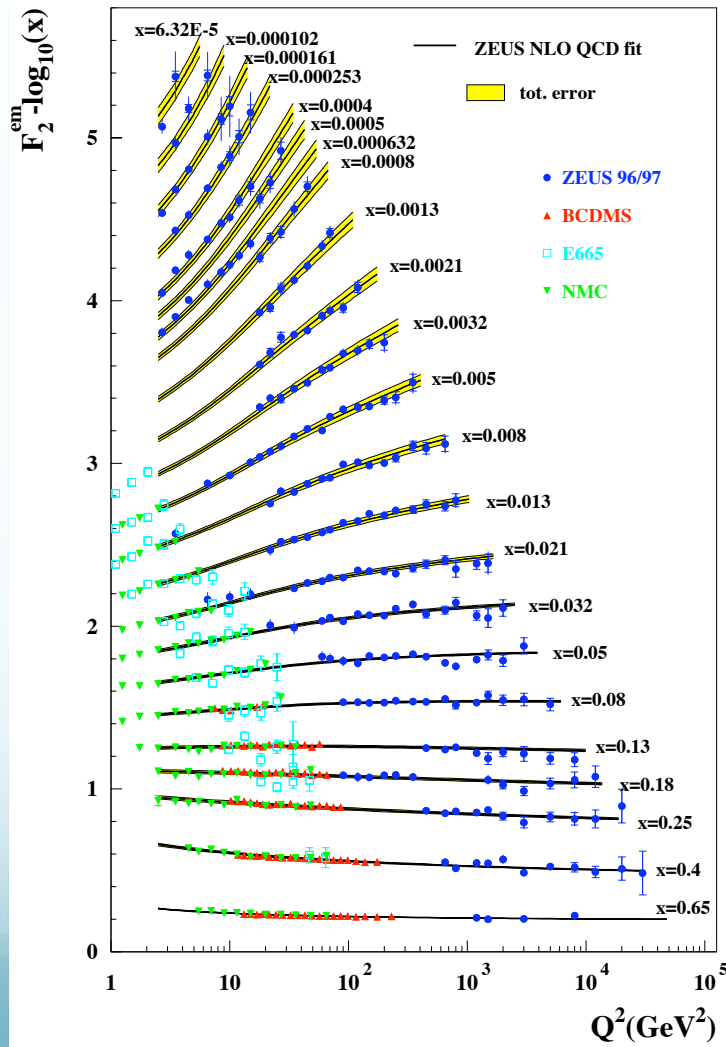


$$\underbrace{\sigma_{\text{proton}}(Q)}_{\text{measured}} = \underbrace{\hat{\sigma}_{q,g}(Q)}_{\text{calculable}} \otimes \underbrace{f_{q,g}(x, Q)}_{\text{extracted}}$$

- Proton structure: encoded in PDFs
- QCD dynamics at high-energy scale  $Q$

# Collinear PDFs

- One dimensional structure of the proton: longitudinal motion

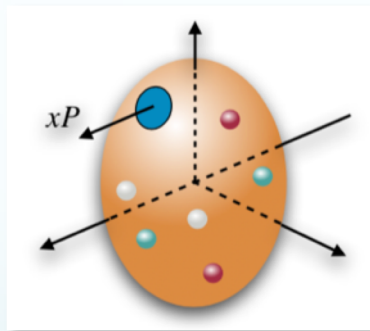


See E. R. Nocera talk

# Moving forward

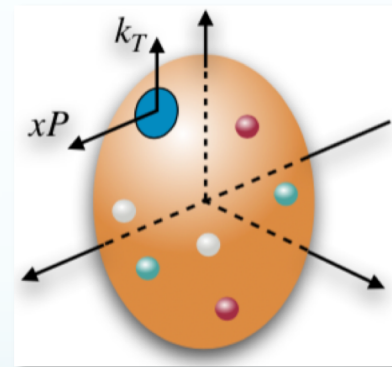
- 30+ years' study, good knowledge about parton's longitudinal motion: 1D
- Nucleon 3D structure: both longitudinal + transverse momentum dependent structure

## Transverse Momentum Dependent parton distributions (TMDs)



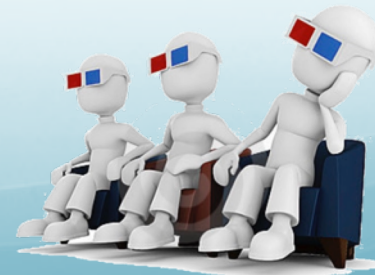
$$f(x)$$

Longitudinal motion only



$$f(x, k_T)$$

Longitudinal + transverse motion



# TMDs: rich quantum correlations

## Leading Twist TMDs



Nucleon Spin



Quark Spin

TMD parton distribution

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 =$		$h_1^\perp =$ - Boer-Mulders
	L		$g_{1L} =$ - Helicity	$h_{1L}^\perp =$ -
	T	$f_{1T}^\perp =$ - Sivers	$g_{1T} =$ - Transversal Helicity	$h_1 =$ - Transversity $h_{1T}^\perp =$ -

TMD fragmentation function

Quark Polarization			
	U	L	T
Pion	$D_1$		$H_1^\perp$ Collins



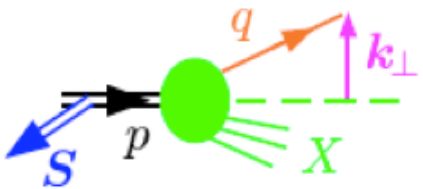
# Novel insights from TMDs

- Quantum correlation: spin-spin, spin-momentum (orbit) correlations
  - Akin to those in hydrogen atoms and topological insulators
- 3D imagining
  - Both longitudinal and transverse motion
- Orbital motion
  - Most TMDs would vanish in the absence of parton orbital angular momentum
- Color gauge invariance at a very deep level
  - Akin to Aharonov-Bohm Effect

**Using the nucleon as  
a QCD “laboratory”**

# Sivers function: non-universal

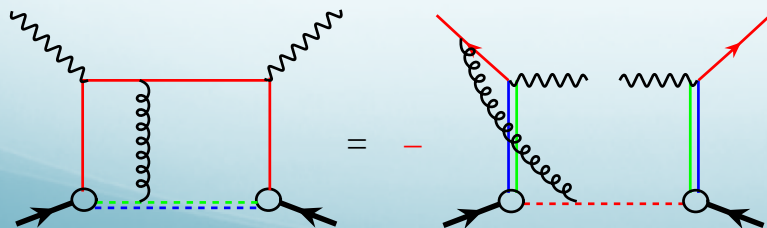
- Sivers function: unpolarized quark distribution inside a transversely polarized proton



$$f_{q/h^\uparrow}(x, \mathbf{k}_\perp, \vec{S}) \equiv f_{q/h}(x, k_\perp) - \frac{1}{M} f_{1T}^{\perp q}(x, k_\perp) \vec{S} \cdot (\hat{p} \times \mathbf{k}_\perp)$$

Spin-independent                      Spin-dependent

- ✓ 1990: introduced by D. Sivers, to describe the large single spin asymmetry measured in inclusive hadron production in p+p collisions at Fermilab
- ✓ 1993: J. Collins shows Sivers function has to vanish due to time-reversal invariance
- ✓ 2002: Brodsky, Hwang, Schmidt performed an explicit model calculation, showed the existence of the Sivers function
- ✓ 2002: Original proof missed the gauge link (needed to properly define gauge invariant distribution), once added, found Sivers function in SIDIS is **opposite** to that in Drell-Yan



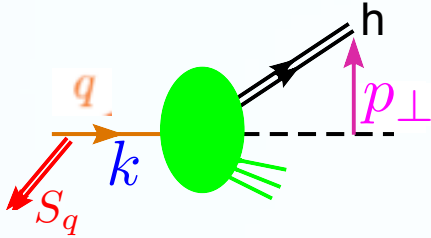
$$\text{SIDIS} = - \text{DY}$$

$$f_{1T}^{\perp \text{DIS}}(x, k_\perp) = - f_{1T}^{\perp \text{DY}}(x, k_\perp)$$

Collins 02, Boer-Mulders-Pijlman 03, Kang-Qiu, 09 ...

# Collins function: universal

- Collins function: unpolarized hadron from a transversely polarized quark

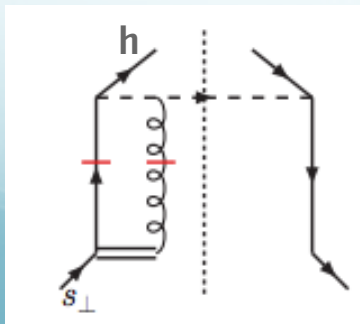


$$D_{h/q}(z, p_{\perp}) = D_1^q(z, p_{\perp}^2) + \frac{1}{zM_h} H_1^{\perp q}(z, p_{\perp}^2) \vec{S}_q \cdot (\hat{k} \times p_{\perp})$$

Spin-independent

Spin-dependent

- ✓ 2002: Metz studied the universality property of Collins function in a model-dependent way – very subtle – finally found it is universal between SIDIS and e+e-
- ✓ 2004: Collins and Metz have general arguments
- ✓ 2008: Yuan generalizes to pp
- ✓ 2010: perturbative tail calculation, demonstrate the gauge link does not contribute

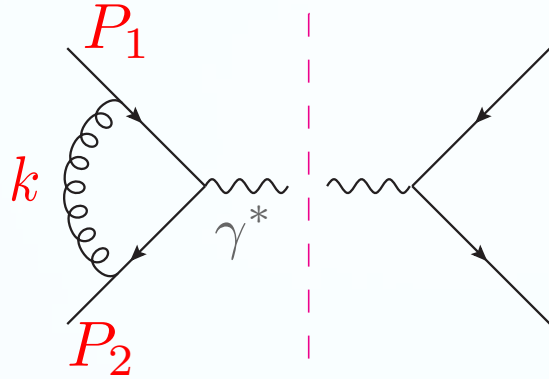


$$H_1^{\perp \text{SIDIS}}(z, p_{\perp}^2) = H_1^{\perp e^+e^-}(z, p_{\perp}^2) = H_1^{\perp \text{pp}}(z, p_{\perp}^2)$$

Metz 02, Collins, Metz 04, Yuan 08, Yuan, Zhou 09, Boer, Kang, Vogelsang, Yuan, PRL 10, ...

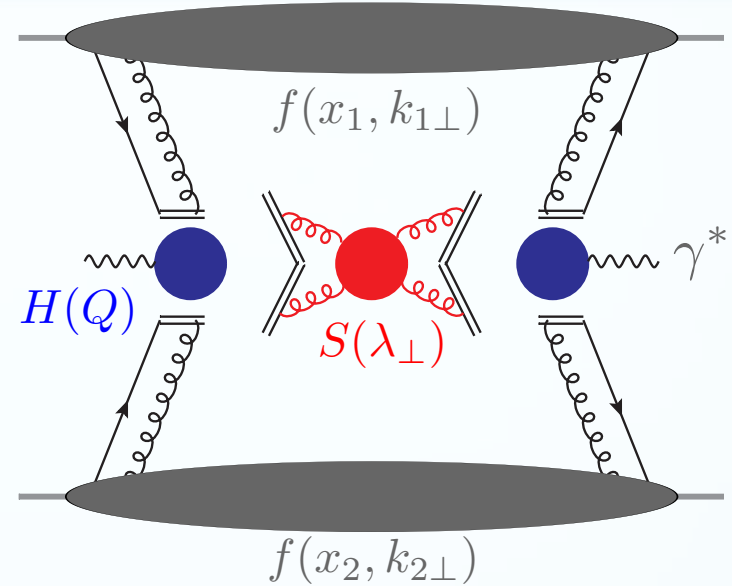
# TMD factorization in a nut-shell

- Drell-Yan:  $p + p \rightarrow [\gamma^* \rightarrow l^+ l^-] + X$



Factorization of regions:

(1)  $k \ll P_1$ , (2)  $k \ll P_2$ , (3)  $k$  soft, (4)  $k$  hard



- Factorized form and mimic “parton model”

$$\begin{aligned} \frac{d\sigma}{dQ^2 dy d^2q_\perp} &\propto \int d^2k_{1\perp} d^2k_{2\perp} d^2\lambda_\perp H(Q) f(x_1, k_{1\perp}) f(x_2, k_{2\perp}) S(\lambda_\perp) \delta^2(k_{1\perp} + k_{2\perp} + \lambda_\perp - q_\perp) \\ &= \int \frac{d^2b}{(2\pi)^2} e^{iq_\perp \cdot b} H(Q) f(x_1, b) f(x_2, b) S(b) \end{aligned}$$

$$F(x, b) = f(x, b) \sqrt{S(b)}$$

$$= \int \frac{d^2b}{(2\pi)^2} e^{iq_\perp \cdot b} H(Q) F(x_1, b) F(x_2, b)$$

mimic “parton model”

# TMD evolves

- Just like collinear PDFs, TMDs also depend on the scale of the probe = evolution

Collinear PDFs

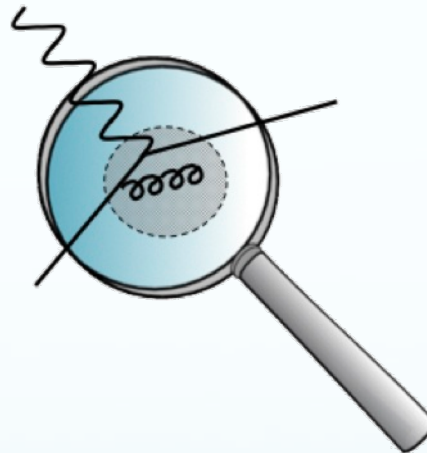
$$F(x, Q)$$

- ✓ DGLAP evolution
- ✓ Resum  $[\alpha_s \ln(Q^2/\mu^2)]^n$
- ✓ Kernel: purely **perturbative**

TMDs

$$F(x, k_\perp; Q)$$

- ✓ Collins-Soper/rapidity evolution equation
- ✓ Resum  $[\alpha_s \ln^2(Q^2/k_\perp^2)]^n$
- ✓ Kernel: can be **non-perturbative** when  $k_\perp \sim \Lambda_{\text{QCD}}$



$$F(x, Q_i)$$

$$R^{\text{coll}}(x, Q_i, Q_f)$$

$$F(x, Q_f)$$

$$F(x, k_\perp, Q_i)$$

$$R^{\text{TMD}}(x, k_\perp, Q_i, Q_f)$$

$$F(x, k_\perp, Q_f)$$

# TMD evolution in a nutshell

$$F(x, k_{\perp}; Q) = \frac{1}{(2\pi)^2} \int d^2 b e^{i k_{\perp} \cdot b} F(x, b; Q) = \frac{1}{2\pi} \int_0^{\infty} db b J_0(k_{\perp} b) F(x, b; Q)$$

$$F(x, b; Q) \approx C \otimes F(x, c/b^*) \times \exp \left\{ - \int_{c/b^*}^Q \frac{d\mu}{\mu} \left( A \ln \frac{Q^2}{\mu^2} + B \right) \right\} \times \exp \left( -S_{\text{non-pert}}(b, Q) \right)$$

longitudinal/collinear part

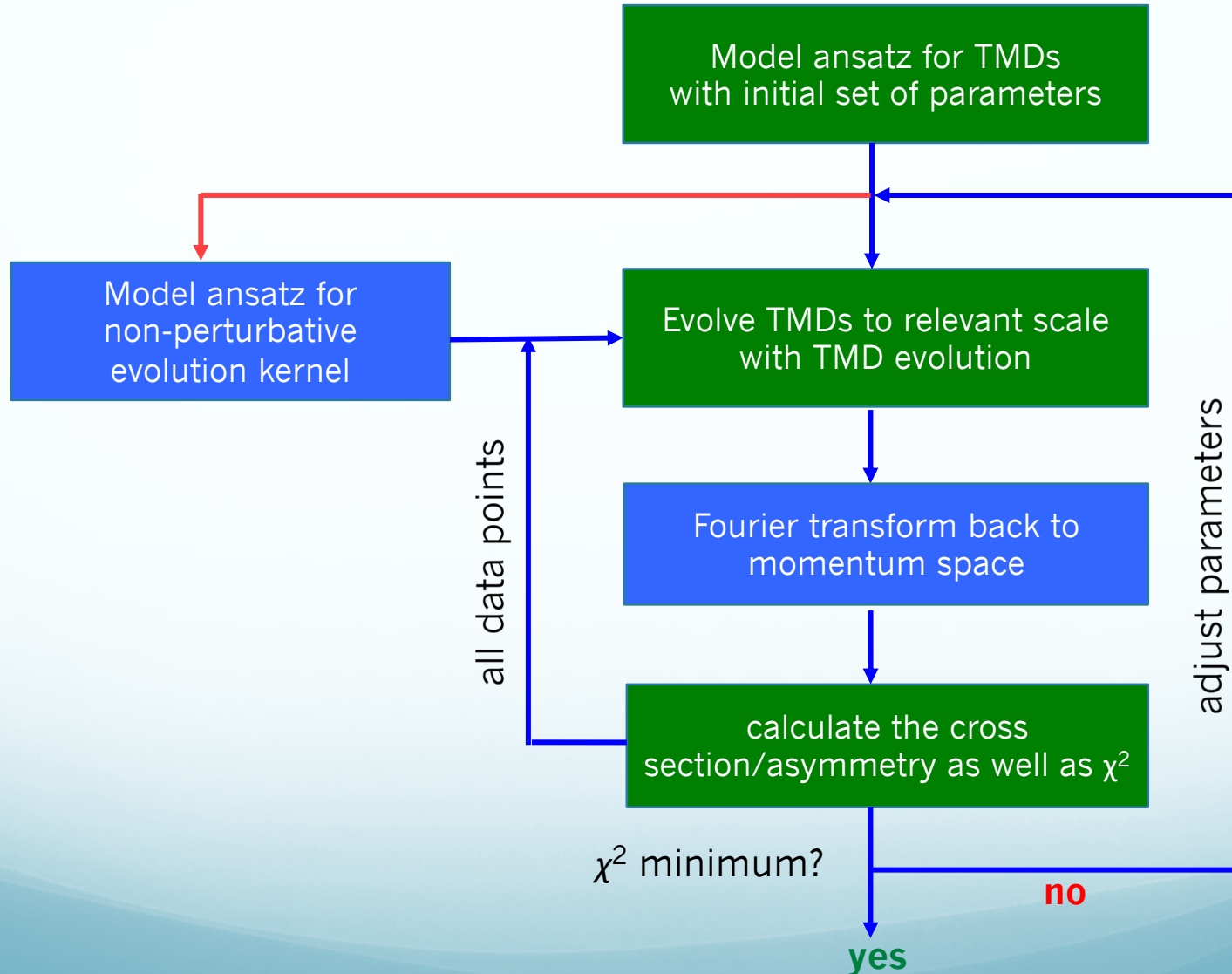
transverse part

- ✓ Non-perturbative: fitted from data
- ✓ The key ingredient –  $\ln(Q)$  piece is spin-independent

**The presence of non-perturbative evolution kernel makes TMD global analysis much more involved**

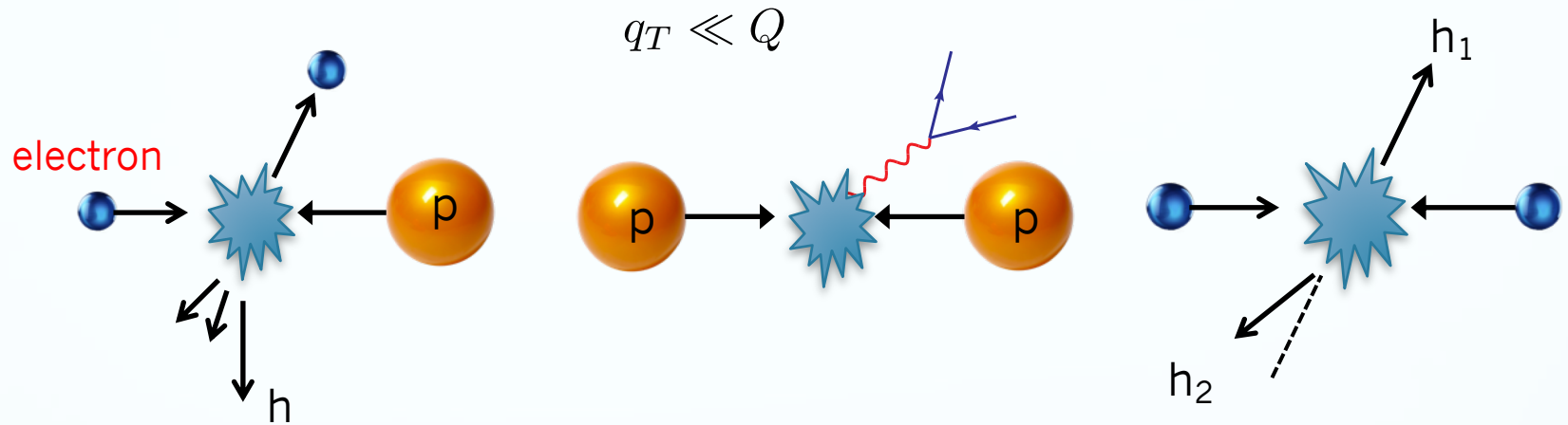
# TMD global analysis

- Outline of a TMD global analysis: numerically more heavy



# Standard processes to extract TMDs

- SIDIS, Drell-Yan, dihadron in  $e^+e^-$



- They have a well-established TMD factorization formalism



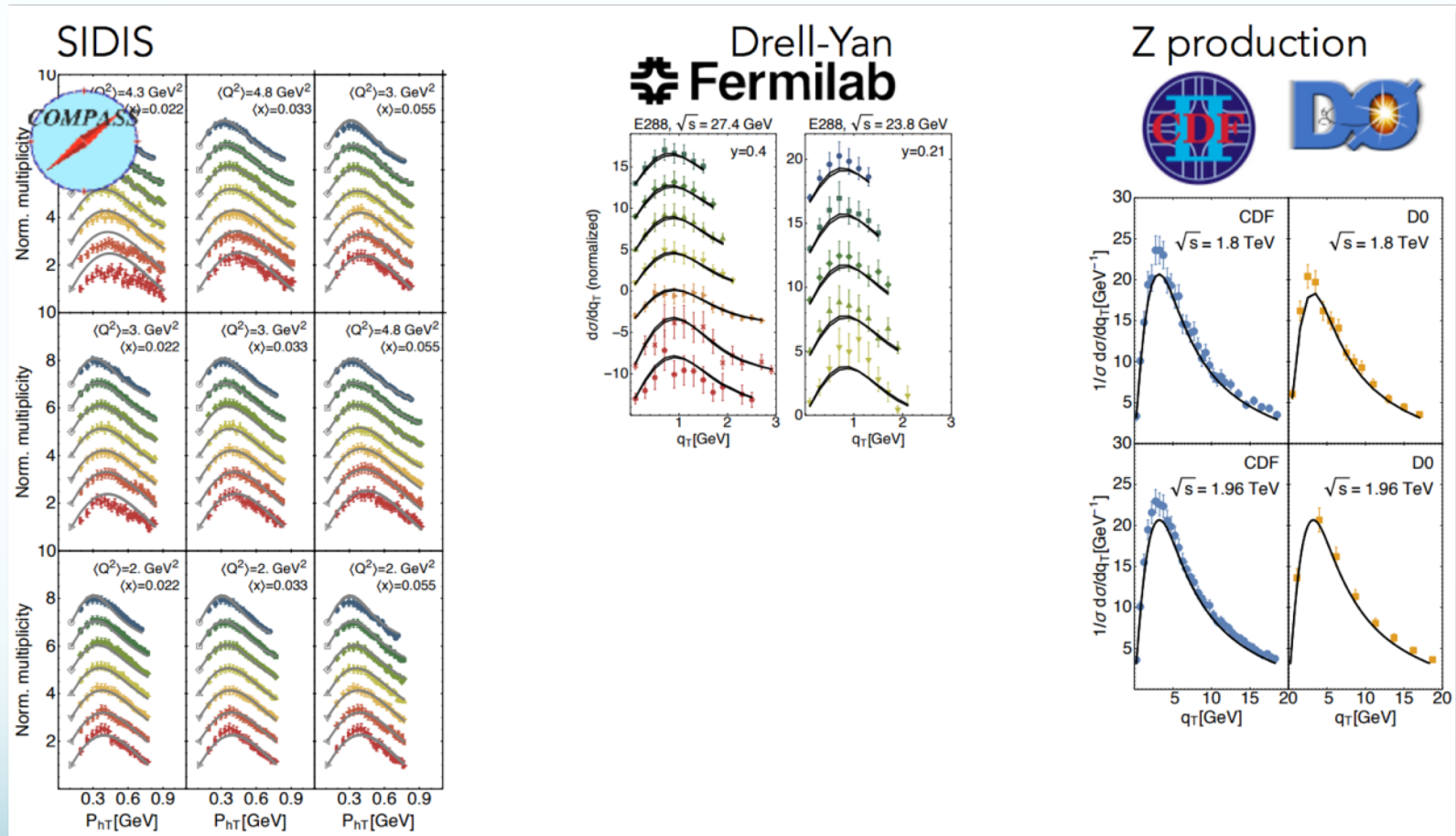
# Extremely active phenomenology - 1

- Examples: Pavia, Torino, EIKV, KSPY, DEMS, SV...

	Framework	W+Y	HERMES	COMPASS	DY	Z production	N of points
KN 2006 <a href="#">hep-ph/0506225</a>	LO-NLL	W	✗	✗	✓	✓	98
QZ 2001 <a href="#">hep-ph/0506225</a>	NLO-NLL	W+Y	✗	✗	✓	✓	28 (?)
RESBOS <a href="#">resbos@msu</a>	NLO-NNLL	W+Y	✗	✗	✓	✓	>100 (?)
Pavia 2013 <a href="#">arXiv:1309.3507</a>	LO	W	✓	✗	✗	✗	1538
Torino 2014 <a href="#">arXiv:1312.6261</a>	LO	W	✓ (separately)	✓ (separately)	✗	✗	576 (H) 6284 (C)
DEMS 2014 <a href="#">arXiv:1407.3311</a>	NLO-NNLL	W	✗	✗	✓	✓	223
EIKV 2014 <a href="#">arXiv:1401.5078</a>	LO-NLL	W	1 (x,Q <sup>2</sup> ) bin	1 (x,Q <sup>2</sup> ) bin	✓	✓	500 (?)
SIYY 2014 <a href="#">arXiv:1406.3073</a>	NLO-NLL	W+Y	✗	✓	✓	✓	200 (?)
Pavia 2017 <a href="#">arXiv:1703.10157</a>	LO-NLL	W	✓	✓	✓	✓	8059
SV 2017 <a href="#">arXiv:1706.01473</a>	NNLO-NNLL	W	✗	✗	✓	✓	309
BSV 2019 <a href="#">arXiv:1902.08474</a>	NNLO-NNLL	W	✗	✗	✓	✓	457

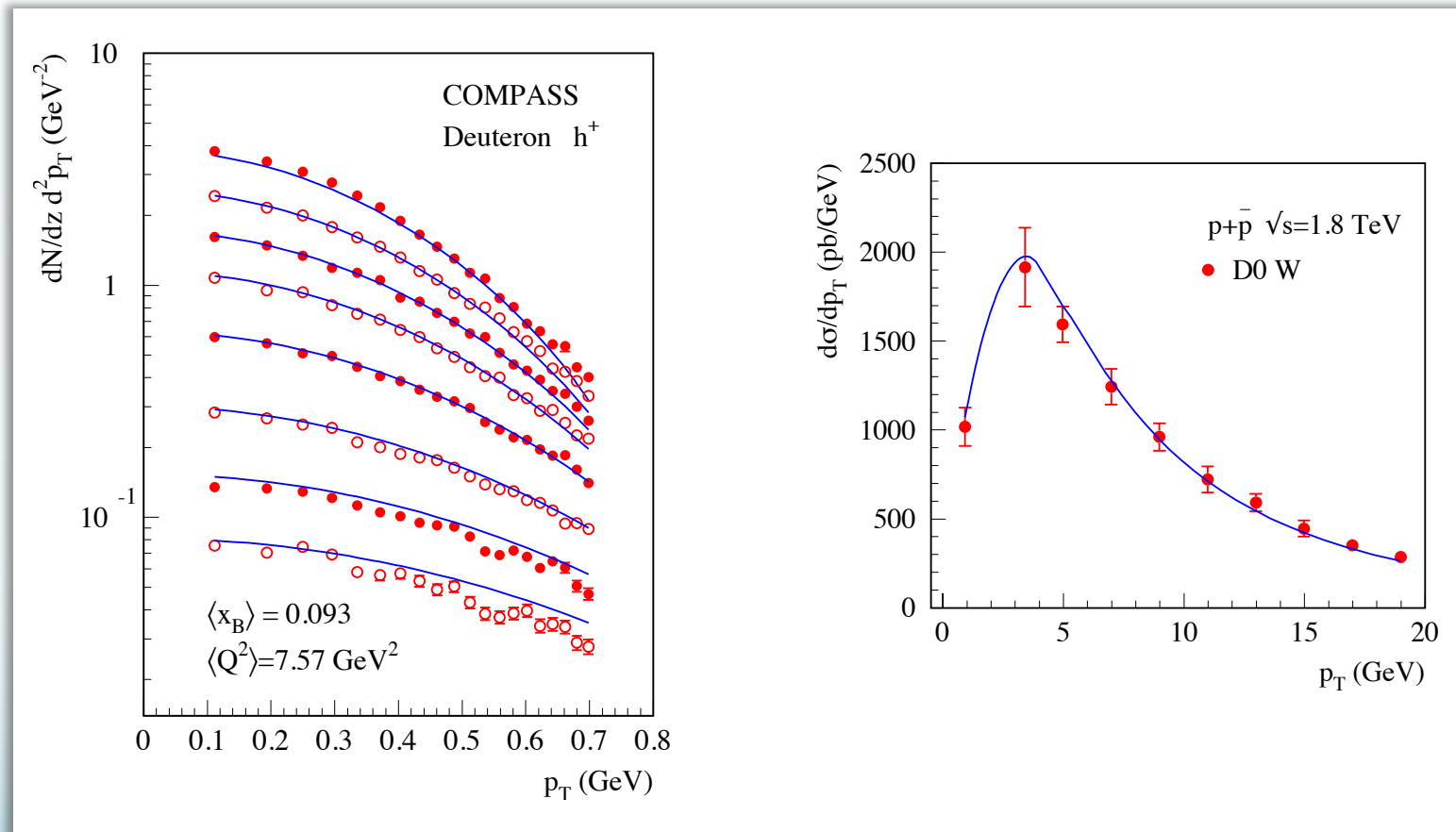
# Extremely active phenomenology - 2

- Example: Pavia group



# Extremely active phenomenology - 3

- Example: our group

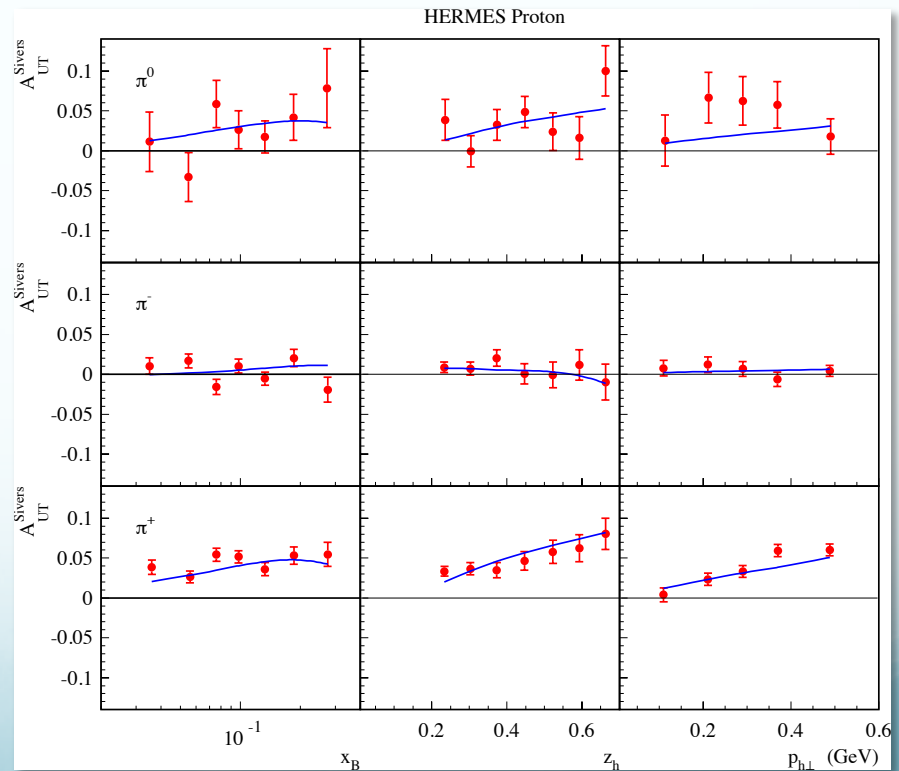
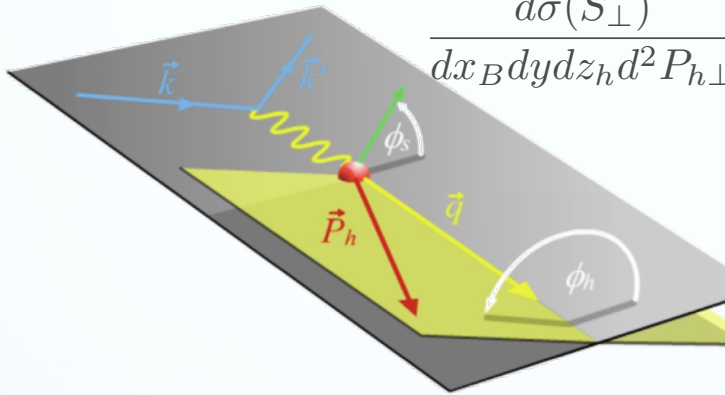


# Sivers asymmetry from SIDIS

- Sivers asymmetry has been measured in SIDIS process: HERMES, COMPASS, JLab

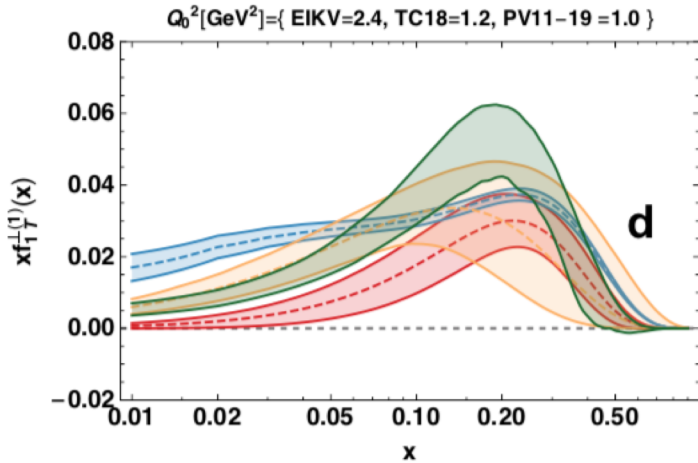
$$\ell + p^\uparrow \rightarrow \ell' + \pi(p_T) + X$$

$$\frac{d\sigma(S_\perp)}{dx_B dy dz_h d^2 P_{h\perp}} = \sigma_0(x_B, y, Q^2) \left[ F_{UU} + \sin(\phi_h - \phi_s) F_{UT}^{\sin(\phi_h - \phi_s)} + \dots \right]$$

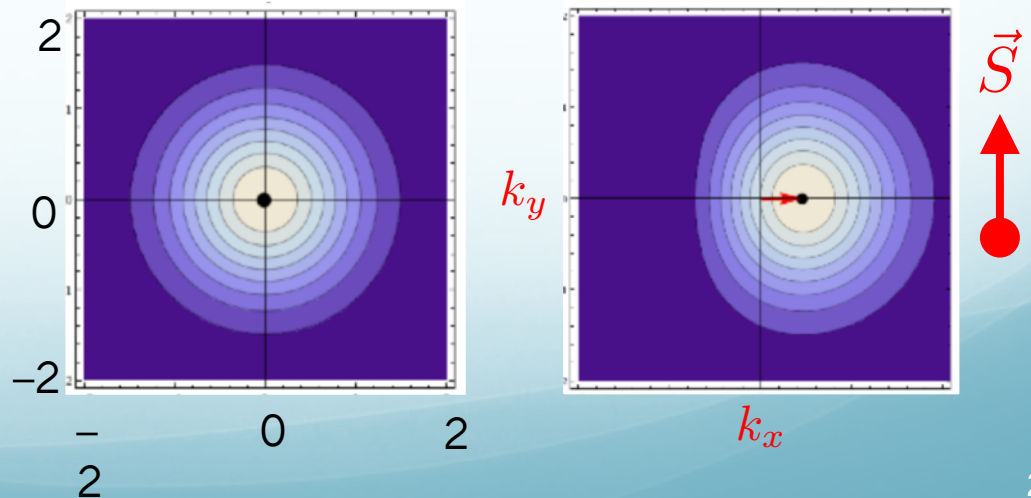
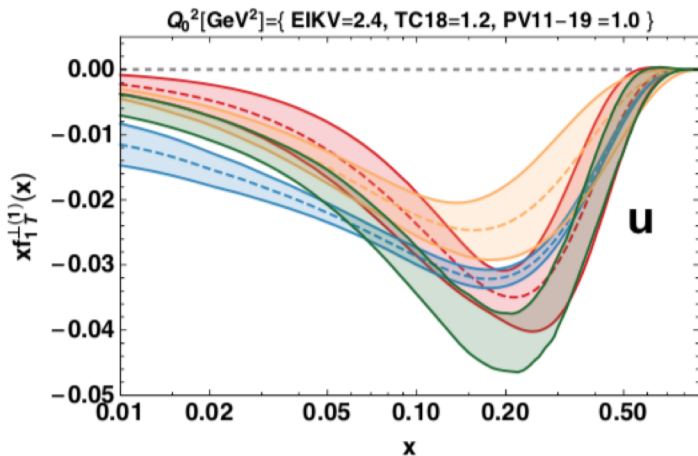


# Current status of Sivers extraction

- Large uncertainties

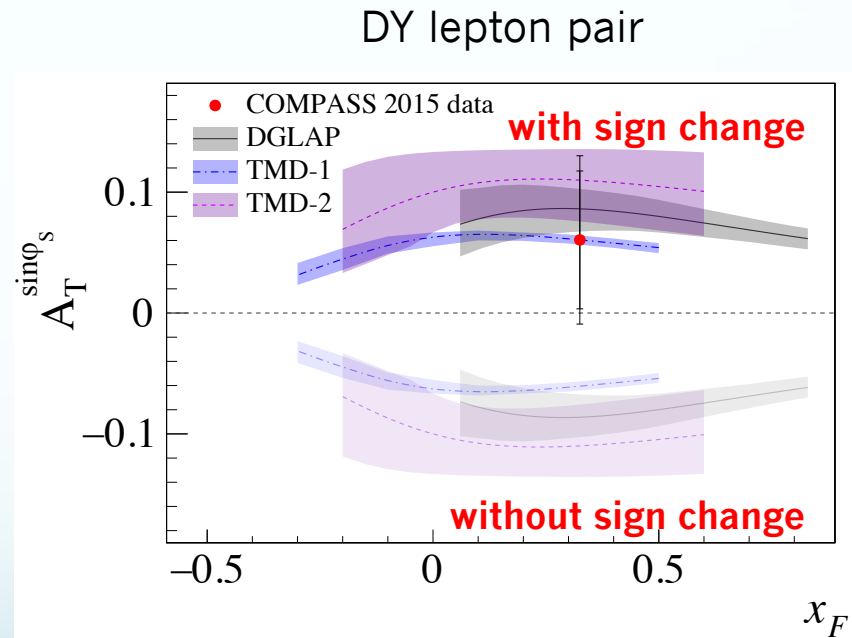
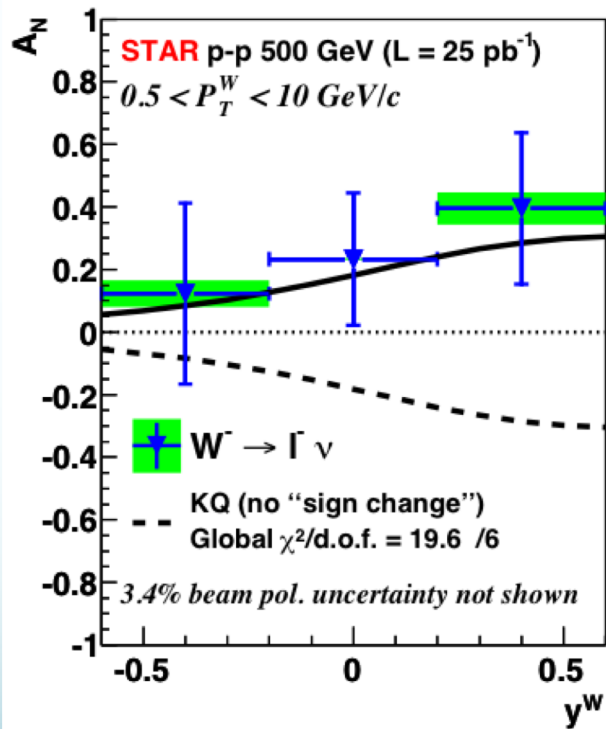


— EIKV [1401.5078]  
— PV11 [1107.5755]  
— TC18 [1806.10645]  
— PV19 **preliminary**  $\approx$  UCLA preliminary



# Experimental evidence of sign change

- STAR and COMPASS: the data seem to favor sign change
- Both theory and experiment has large uncertainty: will be improved in the future runs



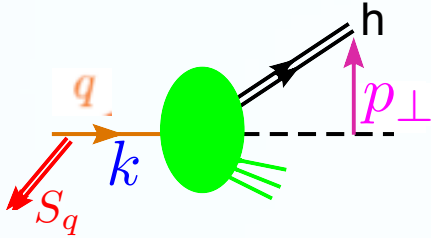
KQ = Kang, Qiu

STAR, arXiv:1511.06003, PRL

COMPASS, 1704.00488

# Collins function: universal

- Collins function: unpolarized hadron from a transversely polarized quark

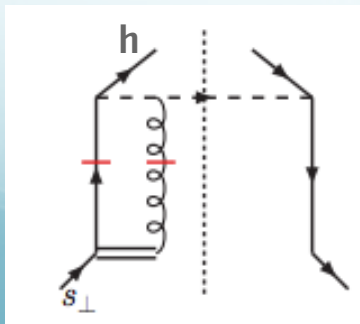


$$D_{h/q}(z, p_{\perp}) = D_1^q(z, p_{\perp}^2) + \frac{1}{zM_h} H_1^{\perp q}(z, p_{\perp}^2) \vec{S}_q \cdot (\hat{k} \times p_{\perp})$$

Spin-independent

Spin-dependent

- ✓ 2002: A. Metz studied the universality property of Collins function in a model-dependent way – very subtle – finally found it is universal between SIDIS and e+e-
- ✓ 2004: Collins and Metz have general arguments
- ✓ 2008: Yuan generalizes to pp
- ✓ 2010: Boer, Kang, Vogelsang, and Yuan perform perturbative tail calculation, demonstrate the gauge link does not contribute

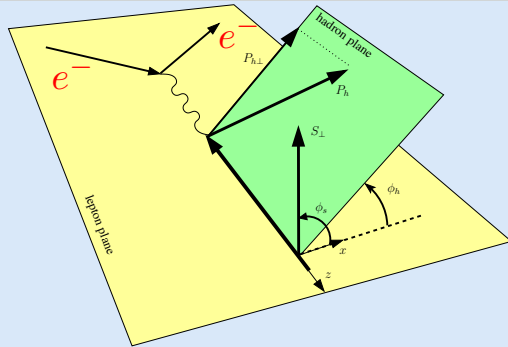


$$H_1^{\perp \text{SIDIS}}(z, p_{\perp}^2) = H_1^{\perp e^+e^-}(z, p_{\perp}^2) = H_1^{\perp \text{pp}}(z, p_{\perp}^2)$$

Metz 02, Collins, Metz 04, Yuan 08,  
Boer, Kang, Vogelsang, Yuan, PRL 10, ...

# Collins asymmetry from SIDIS and e+e-

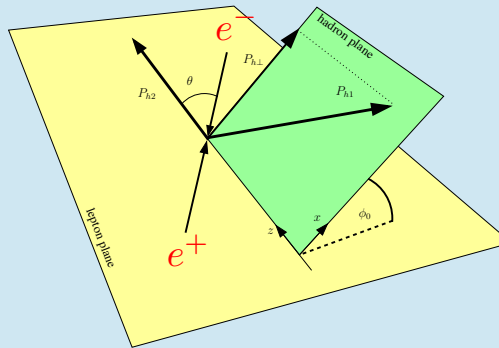
- SIDIS and e+e-: combined global analysis



$$F_{UT}^{\sin(\phi_h + \phi_s)} \sim h_1(x_B, k_\perp) H_1^\perp(z_h, p_\perp)$$

transversity          Collins function

$$\frac{d\sigma(S_\perp)}{dx_B dy dz_h d^2 P_{h\perp}} = \sigma_0(x_B, y, Q^2) \left[ F_{UU} + \sin(\phi_h + \phi_s) \frac{2(1-y)}{1+(1-y)^2} F_{UT}^{\sin(\phi_h + \phi_s)} + \dots \right]$$



$$Z_{\text{collins}}^{h_1 h_2} \sim H_1^\perp(z_1, p_{1\perp}) H_1^\perp(z_2, p_{2\perp})$$

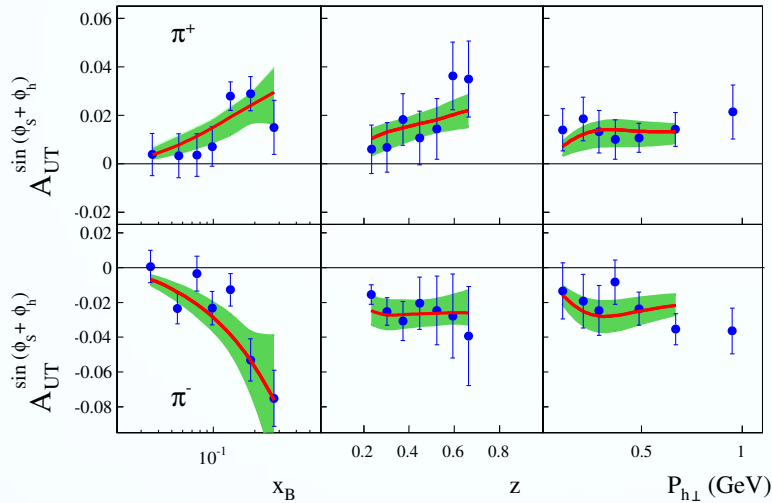
Collins function

$$\frac{d\sigma^{e^+ e^- \rightarrow h_1 h_2 + X}}{dz_{h1} dz_{h2} d^2 P_{h\perp} d \cos \theta} = \frac{N_c \pi \alpha_{\text{em}}^2}{2Q^2} \left[ (1 + \cos^2 \theta) Z_{uu}^{h_1 h_2} + \sin^2 \theta \cos(2\phi_0) Z_{\text{collins}}^{h_1 h_2} \right]$$

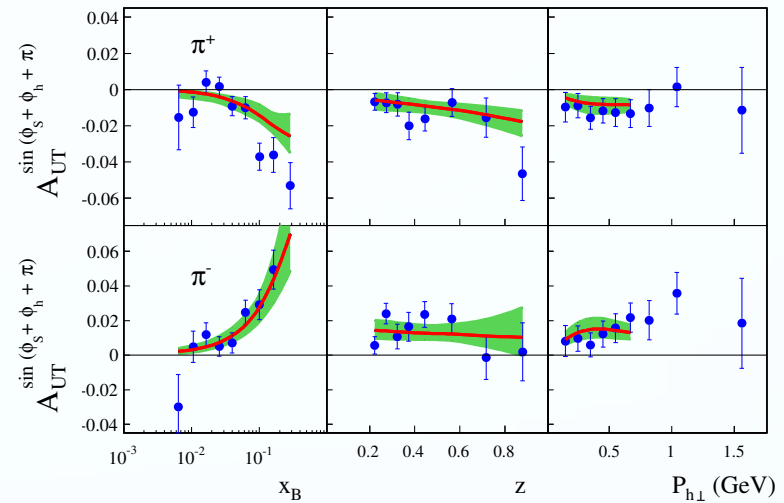


# Global fitting of Collins asymmetry

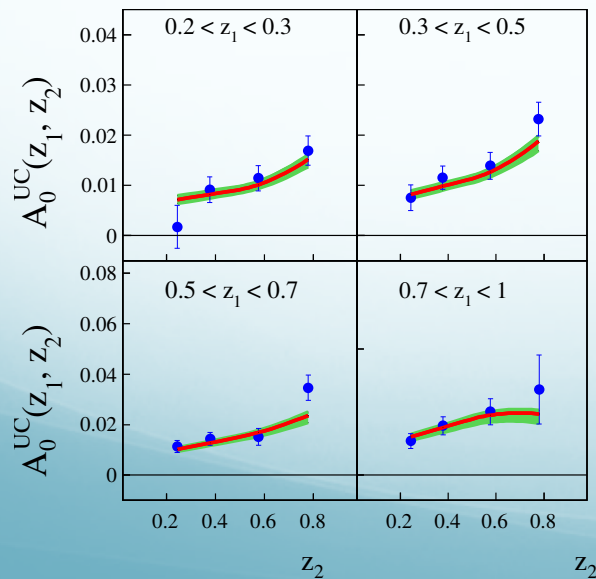
HERMES



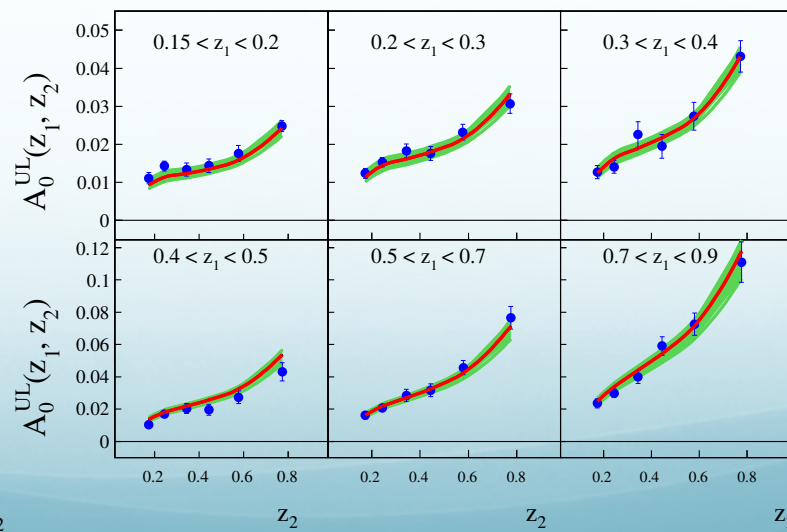
COMPASS



BELLE



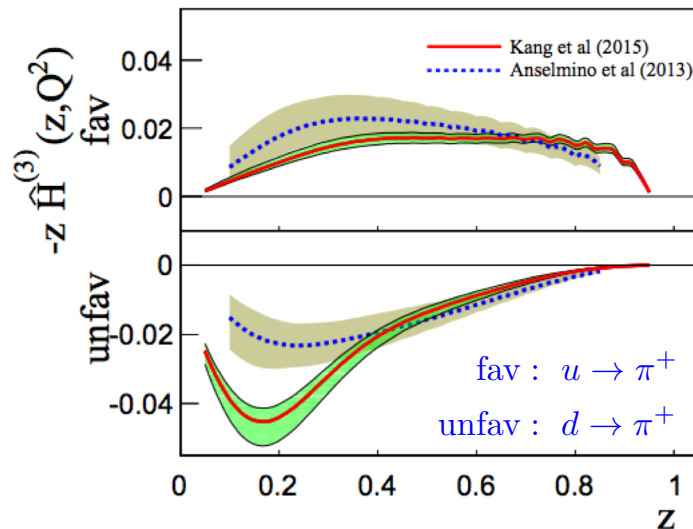
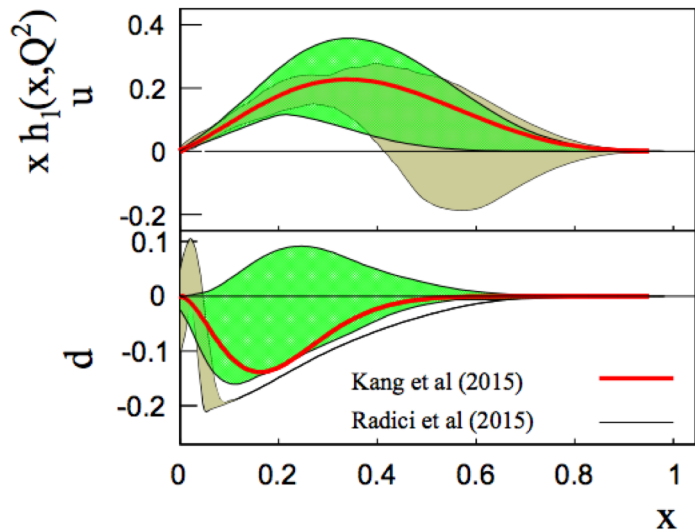
BaBar



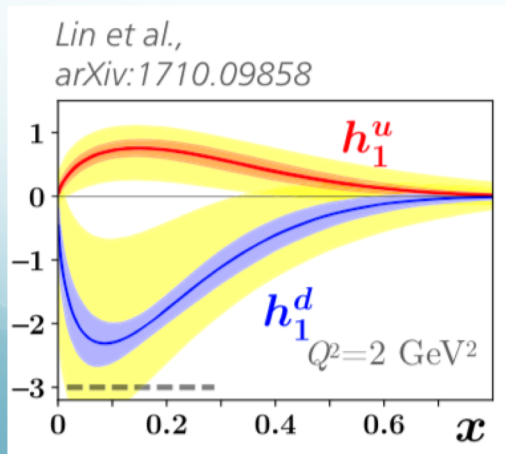
Kang, Prokudin,  
Sun, Yuan, PRD 15 & 16

# Fitted TMDs

- Fitted quark transversity and Collins function



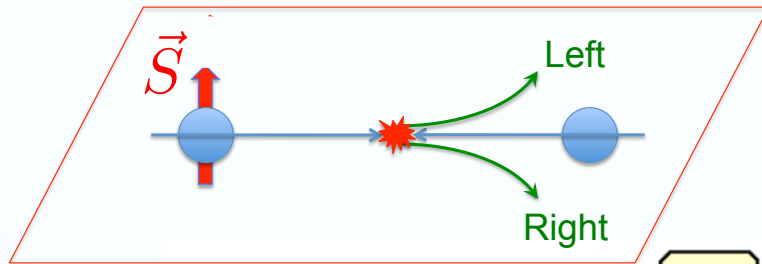
- Using lattice data (tensor charge) to constrain transversity



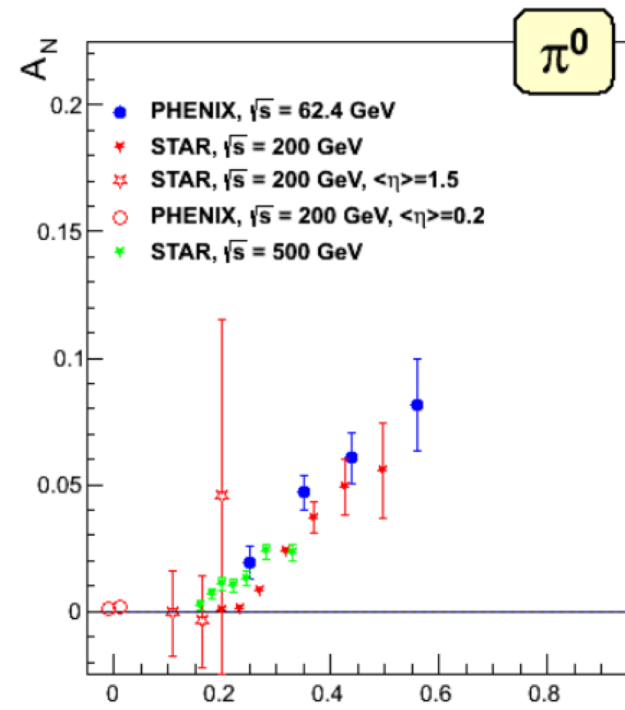
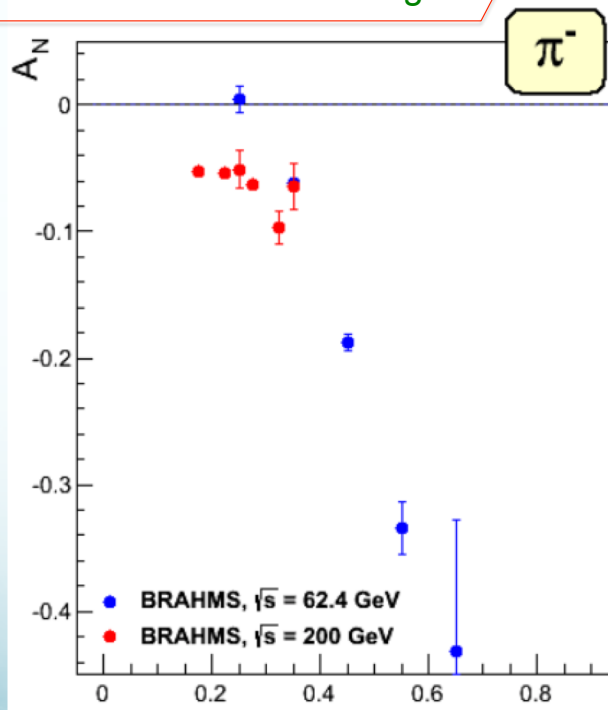
See more from M. Radici in the morning

# Single transverse spin asymmetry (SSA)

- For a long time, the single spin asymmetry for hadron production is due to **Qiu-Sterman** mechanism



$$A_N \equiv \frac{L - R}{L + R} = \frac{\sigma^\uparrow - \sigma^\downarrow}{\sigma^\uparrow + \sigma^\downarrow}$$



$$x_F \sim 2P_z / \sqrt{s}$$

# Sign mismatch problem

- In 2006, Ji-Qiu-Vogelsang-Yuan discovered Qiu-Sterman mechanism is closely related to Sivers functions

- Equation of motion

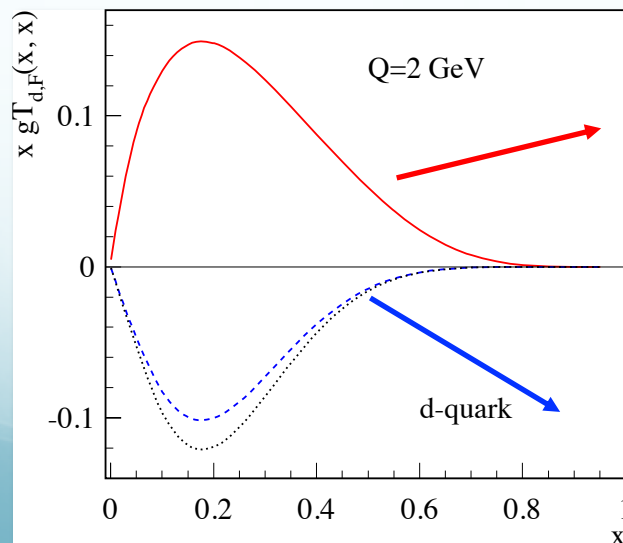
Ji-Qiu-Vogelsang-Yuan, hep-ph/0602239

$$T_{q,F}(x, x) \propto \int d^2 k_T \frac{k_T^2}{M} f_{1T}^{\perp,q}(x, k_T^2)$$

- Operator product expansion

$$f_{1T}^{\perp,q}(x, k_T^2) \propto C(k_T^2) \otimes T_{q,F}(x, x)$$

- This gives a way to relate the single spin asymmetry of hadrons in p+p to the Sivers asymmetry in SIDIS



Kang-Qiu-Vogelsang-Yuan, arXiv:1103.1591

Directly extracted from p+p

Computed based on SIDIS Sivers asymmetry

# Investigation on fragmentation function contributions

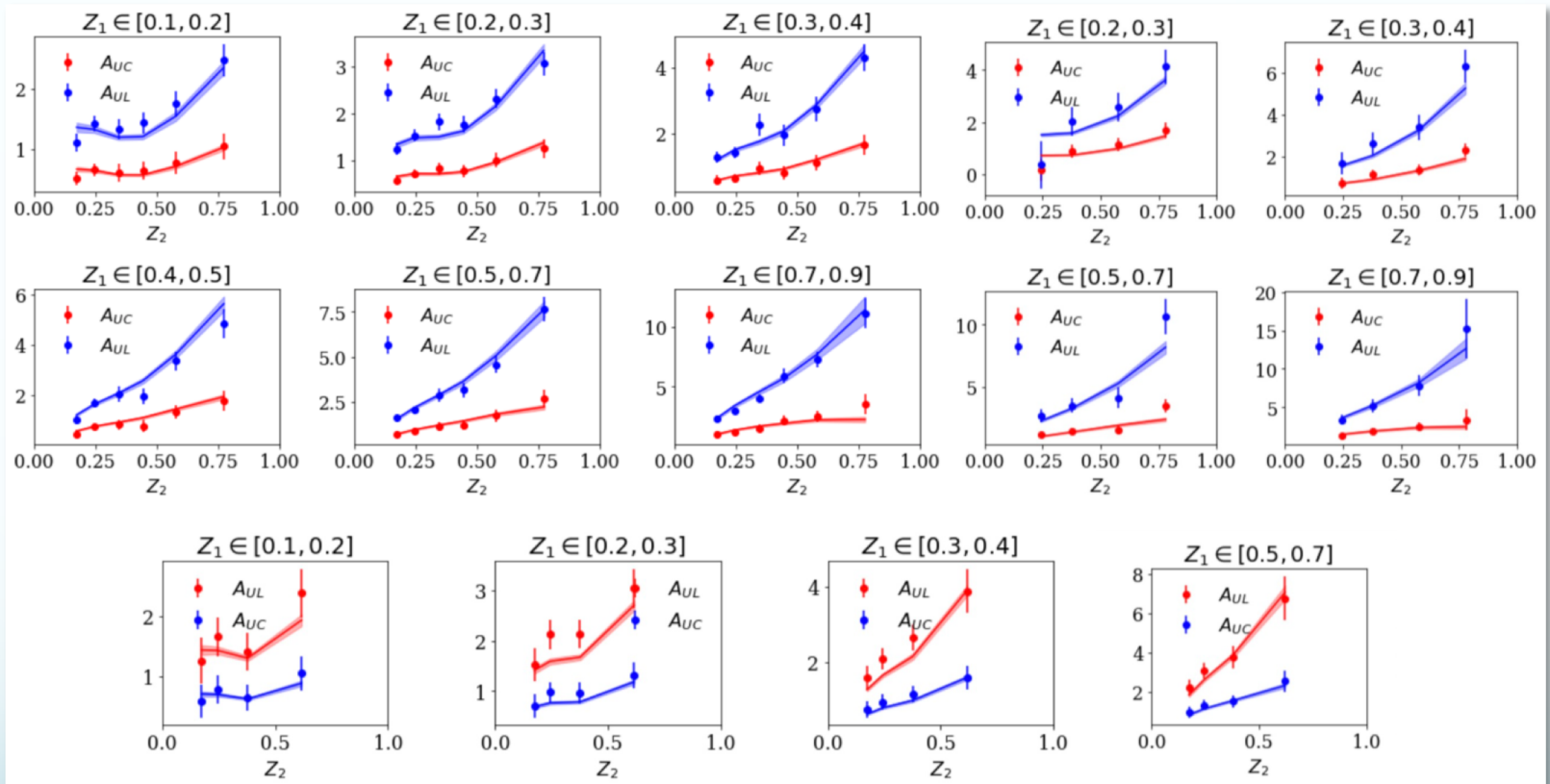
- This pinpoints to the importance of fragmentation functions
  - It is important to figure out if this will work
  
- Based on all possible sources, depending on different combinations of correlations in PDFs (Sivers) and FFs (Collins and more)
  - Collins effect in e+e-: product of Collins
  - Collins asymmetry in SIDIS: transversity × Collins
  - Integrated  $A_{UT}^{\sin(\phi_s)}$  in SIDIS: transversity × twist-3 FFs

$$F_{UT}^{\sin\phi_s} = -x \sum_a e_a^2 \frac{2M_h}{Q} h_1^a(x) \frac{\tilde{H}^a(z)}{z}$$

- $A_N$  in p+p collisions: all of them: all of them

# Global fit: e+e- data

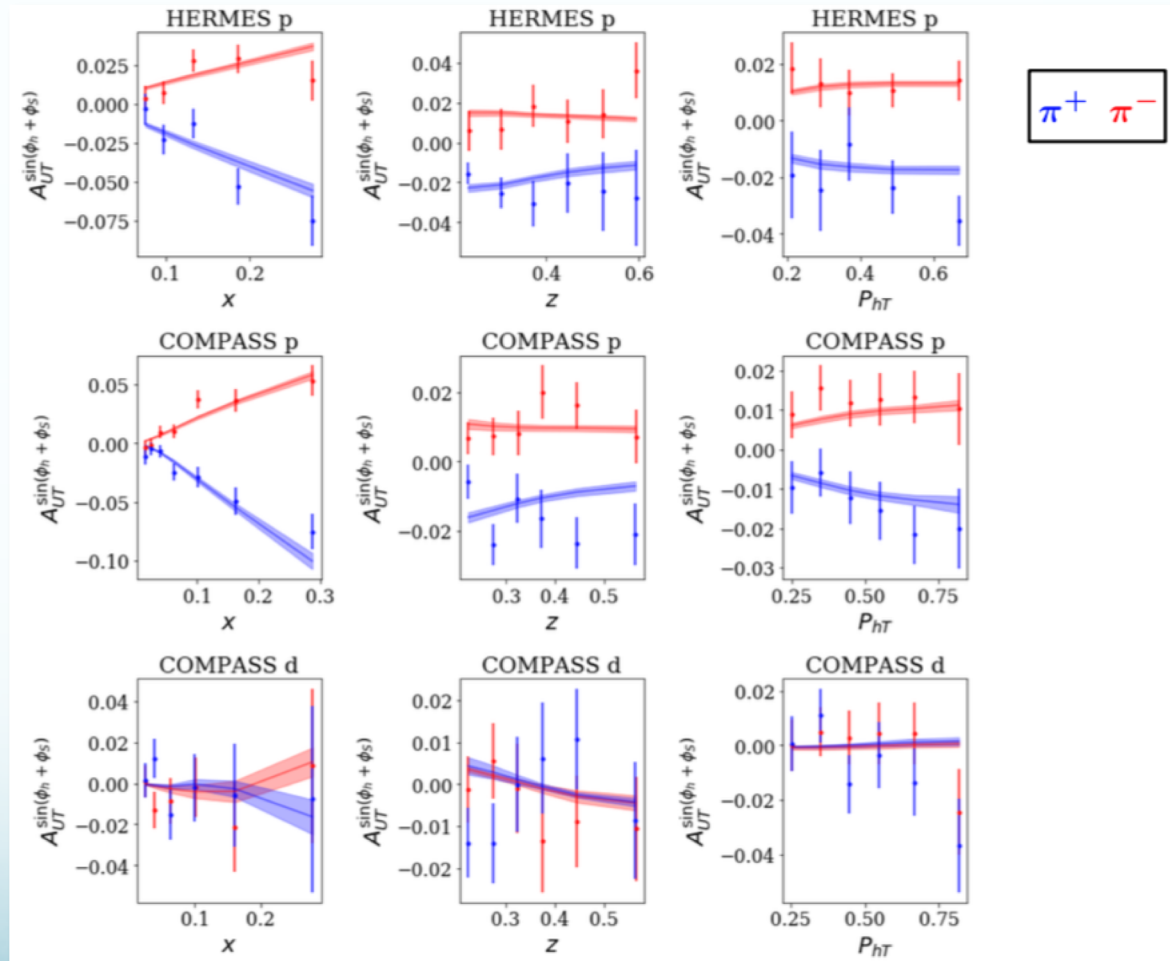
- Collins asymmetry in e+e-



Gamberg-Kang-Pitonyak-Prokudin-Sato, to appear, 2019

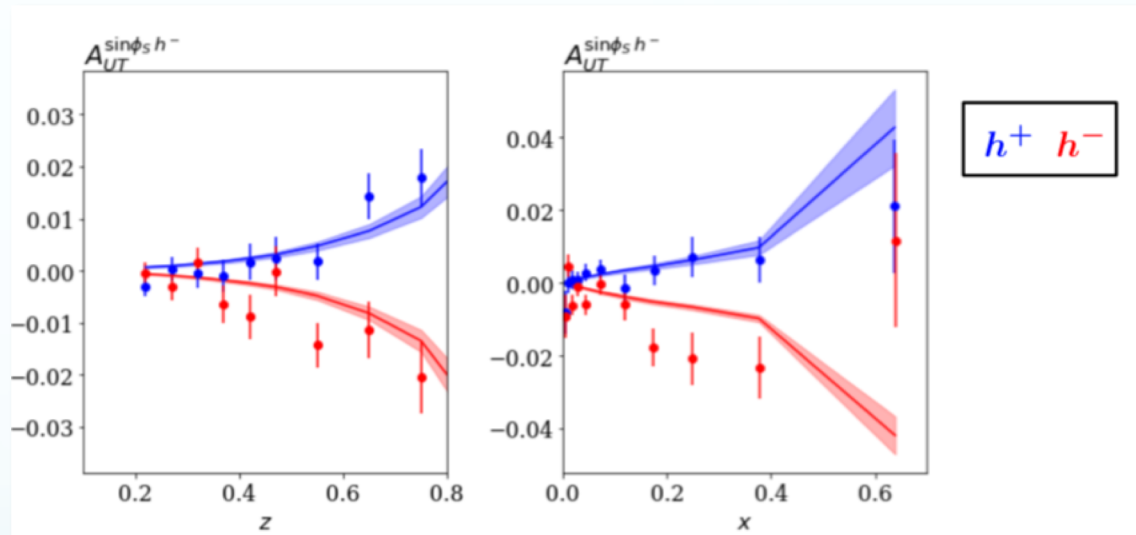
# Global fit: SIDIS data

- Collins asymmetry in SIDIS



# Global fit: SIDIS data

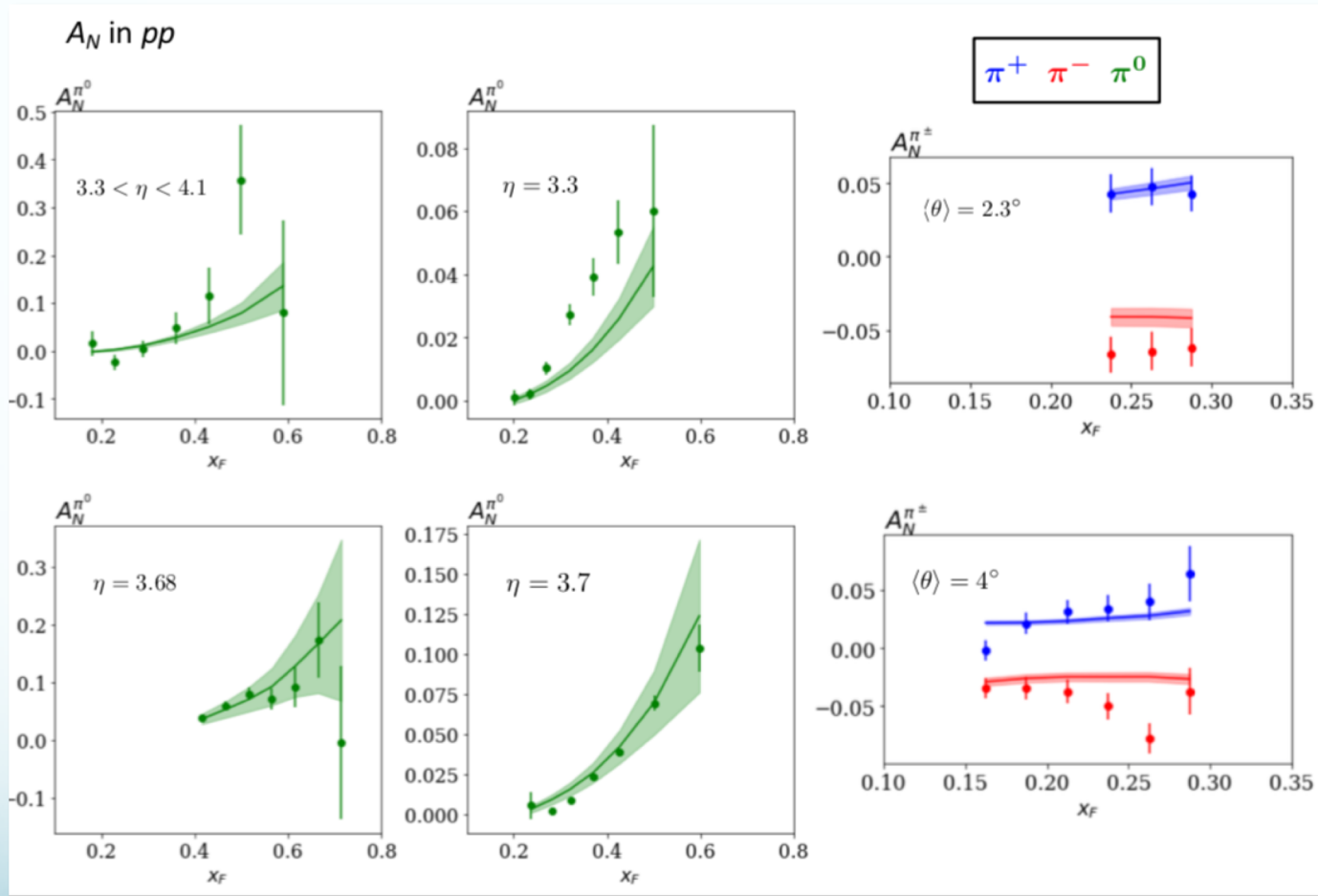
- Integrated  $A_{UT}^{\sin(\phi_s)}$  in SIDIS





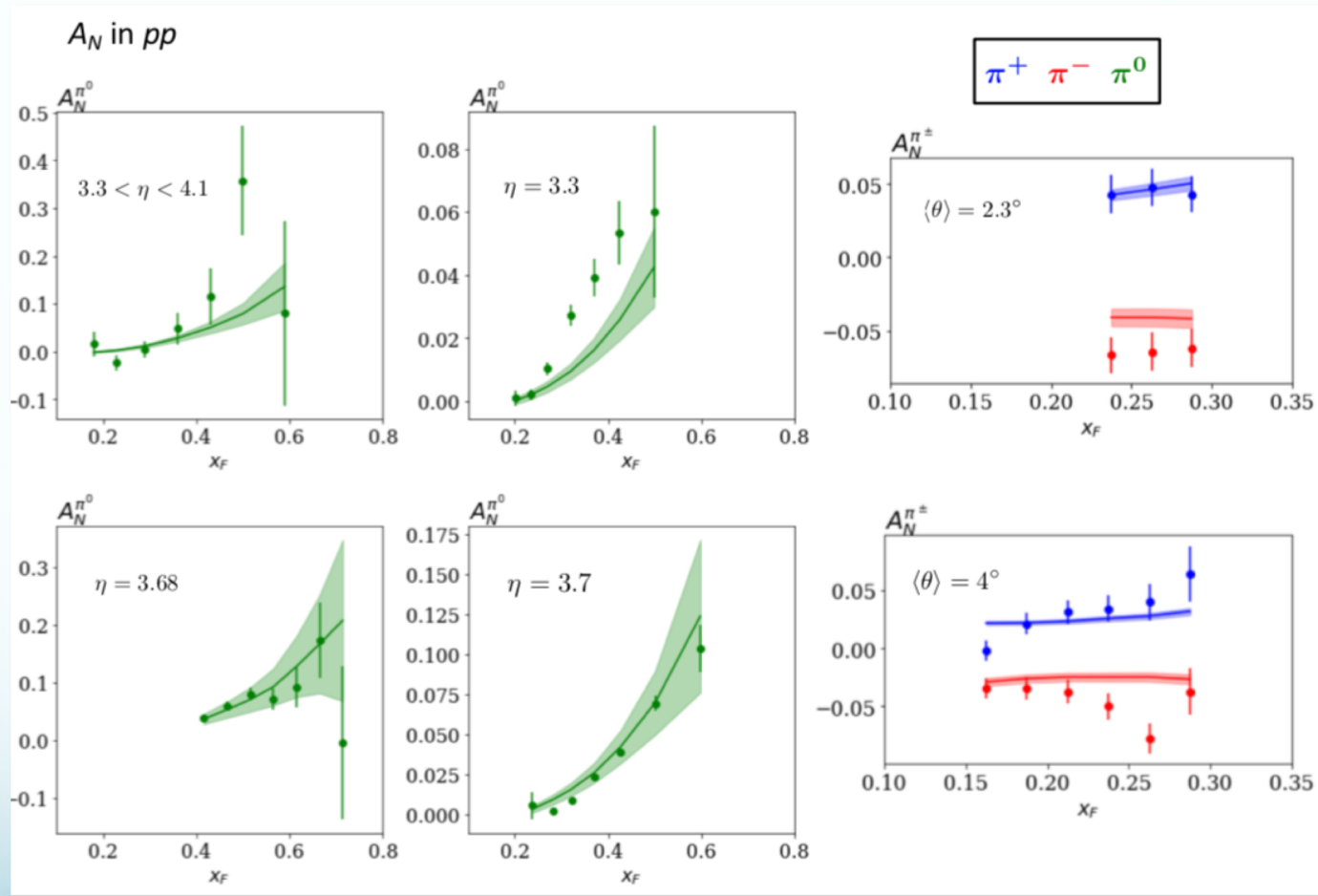
# Global fit: RHIC data

- Single spin asymmetry of hadron in p+p collisions



# Global fit: RHIC data

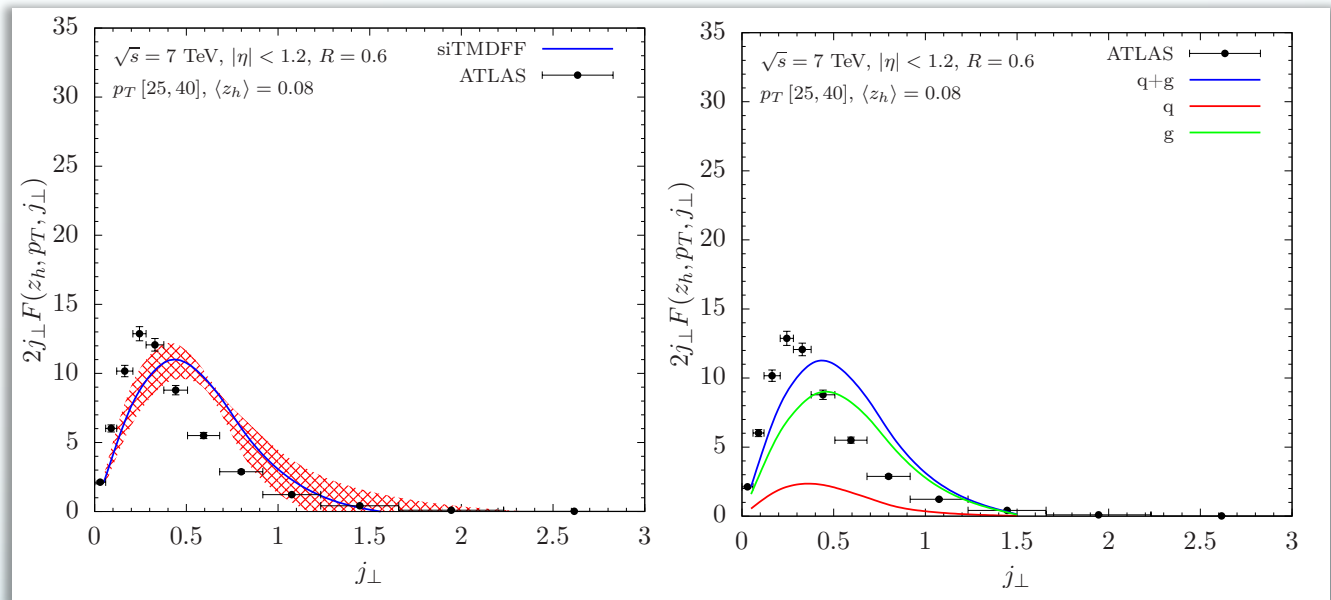
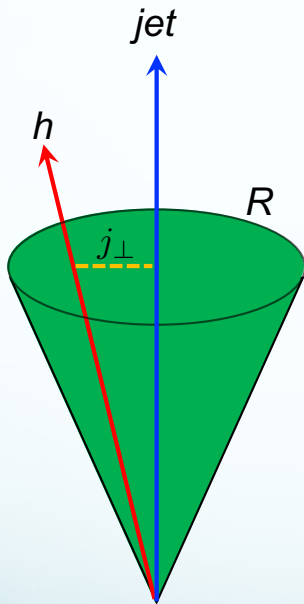
- Single spin asymmetry of hadron in p+p collisions



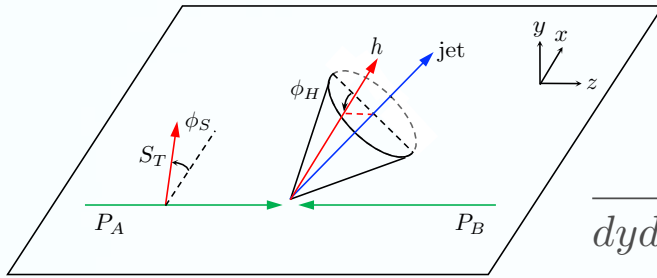
**We can indeed understand this spin asymmetry after all**

# Jet TMDs: novel opportunities beyond standard processes

- Study transverse momentum distribution of hadrons inside a fully reconstructed jet
  - Probe TMD fragmentation functions
  - Sensitive to gluon TMDs: for inclusive jet production at the LHC

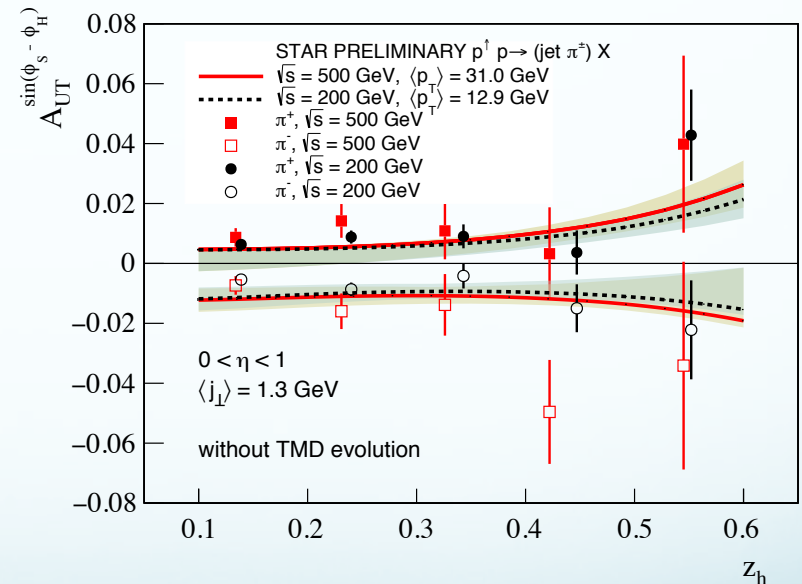
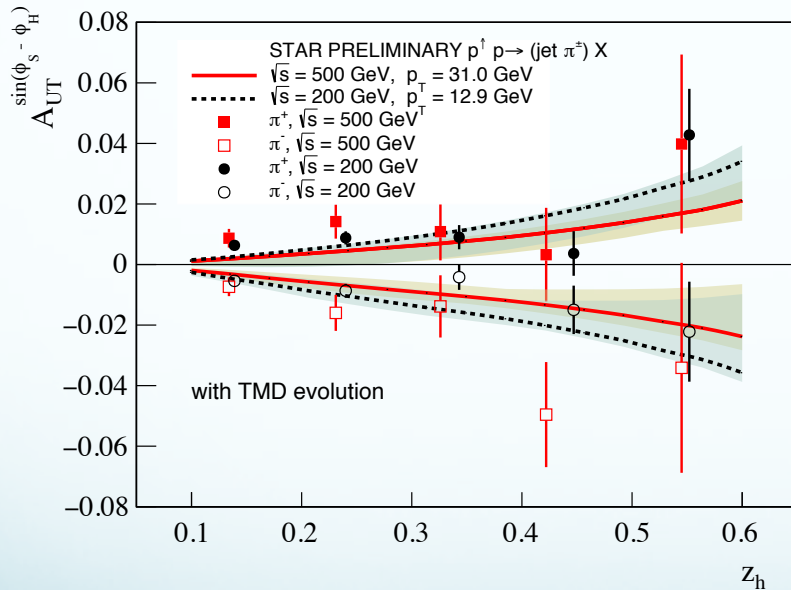


# Collins azimuthal asymmetry



$$p^\uparrow [\vec{S}_\perp(\phi_S)] + p \rightarrow [\text{jet } h(\phi_H)] + X$$

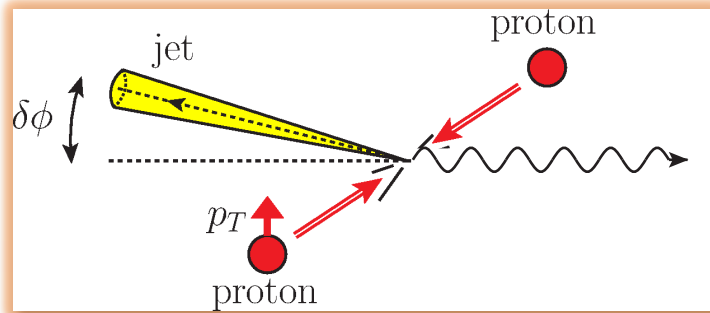
$$\frac{d\sigma}{dy d^2 p_\perp^{\text{jet}} dz d^2 j_T} = F_{UU} + \sin(\phi_S - \phi_H) F_{UT}^{\sin(\phi_S - \phi_H)}$$



- Test universality of Collins function between  $e+p$ ,  $e+e$ , and  $p+p$
- Test TMD evolution

# V+jet: factorized form

- No TMD factorization Collins-Qiu 07, Rogers-Mulders 10



Strategy to move forward: ignore the d.o.f that breaks factorization

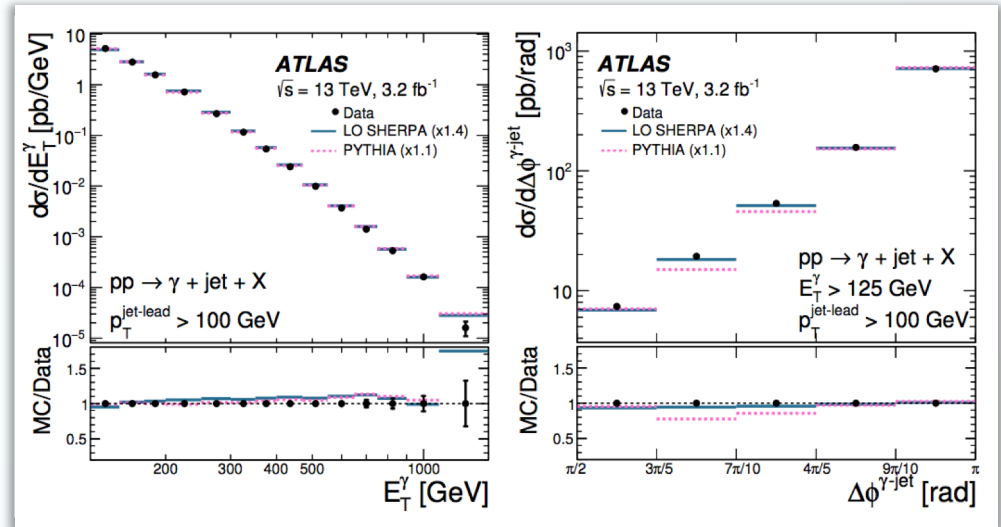
- Within SCET, one can derive a factorized form in terms of
  - TMD PDFs: collinear d.o.f. Buffing, Kang, Lee, Liu, arXiv:1812.07549
  - Soft functions: soft d.o.f.
  - Jet function: jet production

$$\begin{aligned}
 \frac{d\sigma}{dy_J dy_\gamma dp_\perp d^2\vec{q}_\perp} &= \sum_{a,b,c} \int d\phi_J \int \prod_i^4 d^2\vec{k}_{i\perp} \delta^{(2)}(\vec{q}_\perp - \sum_i^4 \vec{k}_{i\perp}) \\
 &\quad \times f_a^{\text{unsub}}(x_a, k_{1\perp}^2) f_b^{\text{unsub}}(x_b, k_{2\perp}^2) S_{n\bar{n}n_J}^{\text{global}}(\vec{k}_{3\perp}) \\
 &\quad \times S_{n_J}^{cs}(\vec{k}_{4\perp}, R) H_{ab\rightarrow c\gamma}(p_\perp) J_c(p_\perp R)
 \end{aligned}$$

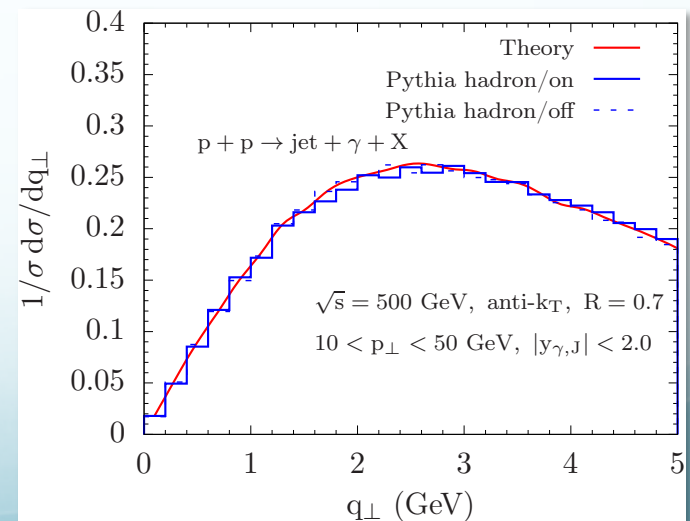
- Two soft functions: global soft + collinear soft (along jet direction)

# Phenomenology

- No data as a function of imbalance, but lots of LHC data on other variables for photon+jet
  - Pythia describes them well

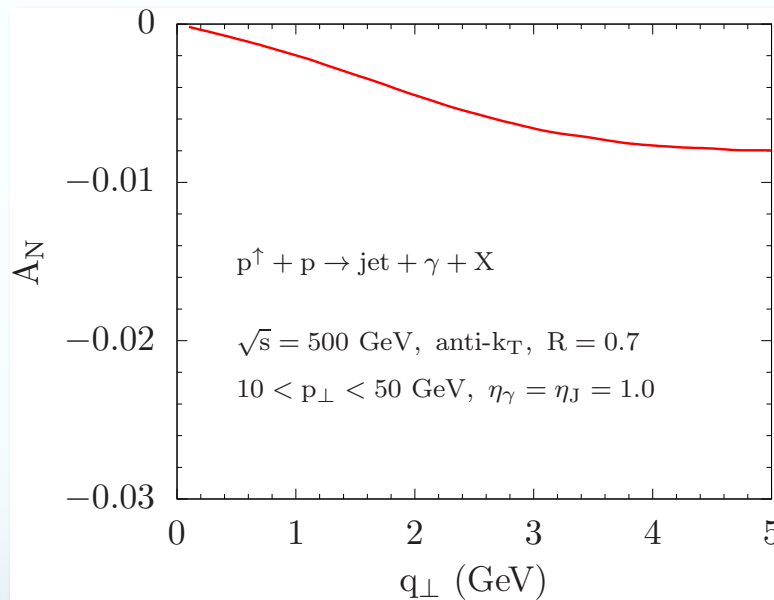


- Compare with Pythia
  - Extremely well
  - Indicate TMD factorization breaking small??



# Phenomenology at RHIC

- Prediction for Sivers asymmetry is around 1% level
  - Sivers functions in SIDIS from our earlier extraction 1401.5078
  - TMD evolution has a strong effect (suppress asymmetry), but not so much for unpolarized cross section

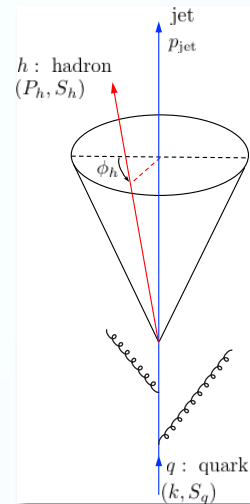
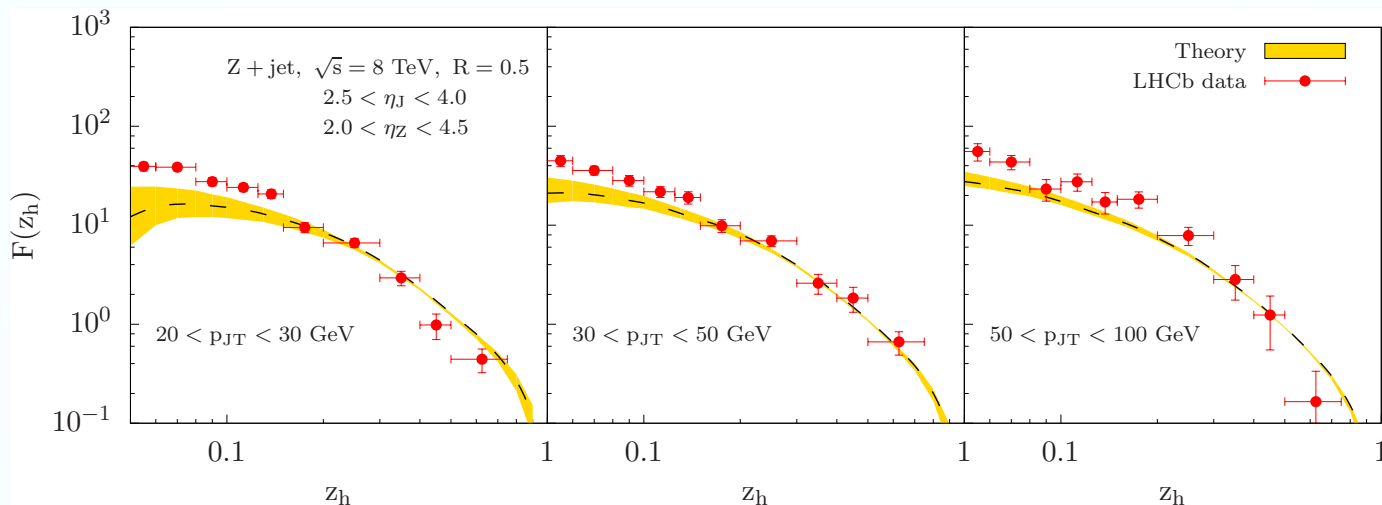


Buffing, Kang, Lee, Liu, arXiv:1812.07549

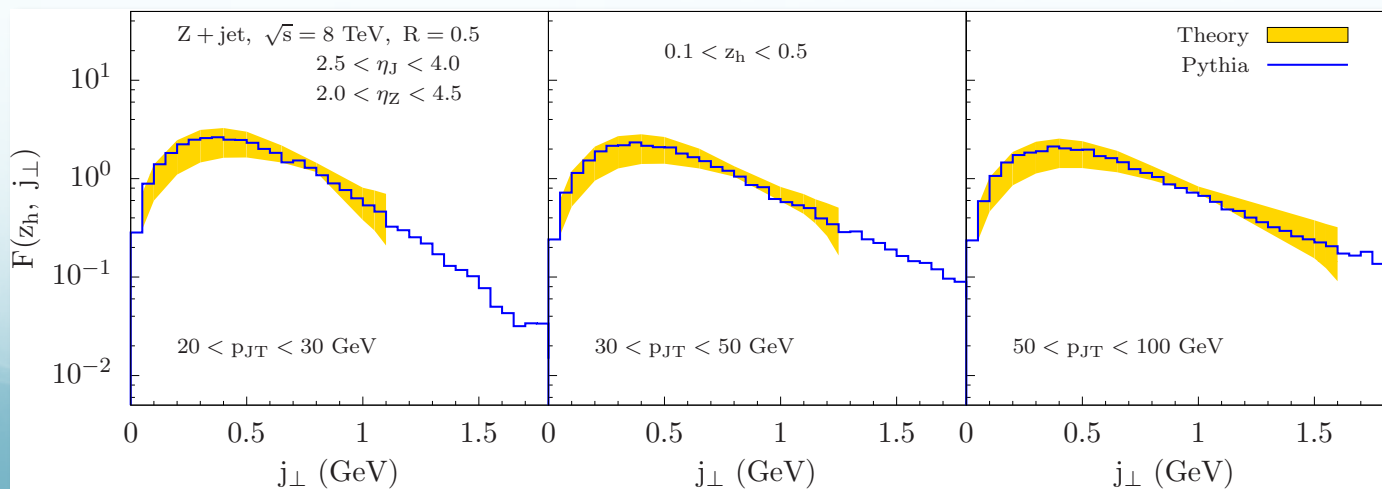
# LHCb: Z-tagged jet – quark jet

Kang, Lee, Terry, Xing, arXiv:1906.07187

## ■ $z_h$ distribution



## ■ $j_T$ distribution does now work well with Pythia





# Very active theoretical research

- DOE TMD Collaboration

https://sites.google.com/a/lbl.gov/tmdwiki/



Search this site

## Navigation

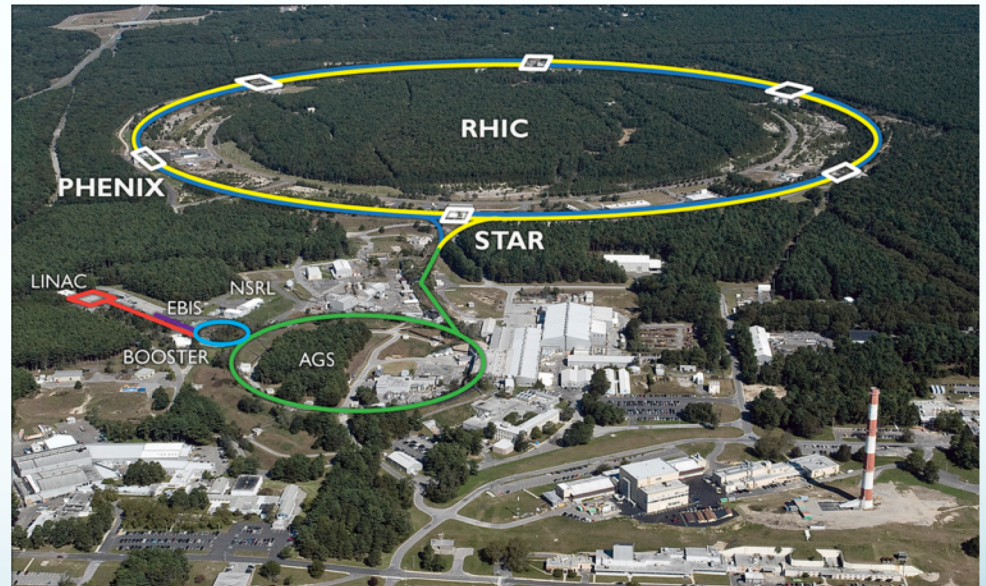
**Main**

Overview  
Collaboration calendar  
Meetings  
Publications  
Conference Talks  
Code Packages  
Jobs

**Topical Collaboration for the Coordinated Theoretical Approach to  
Transverse Momentum Dependent Hadron Structure in QCD**

# Active experimental programs

- US: Jefferson Lab 12 GeV + RHIC spin program
- COMPASS + HERMES + BELLE + BES



# Electron Ion Collider

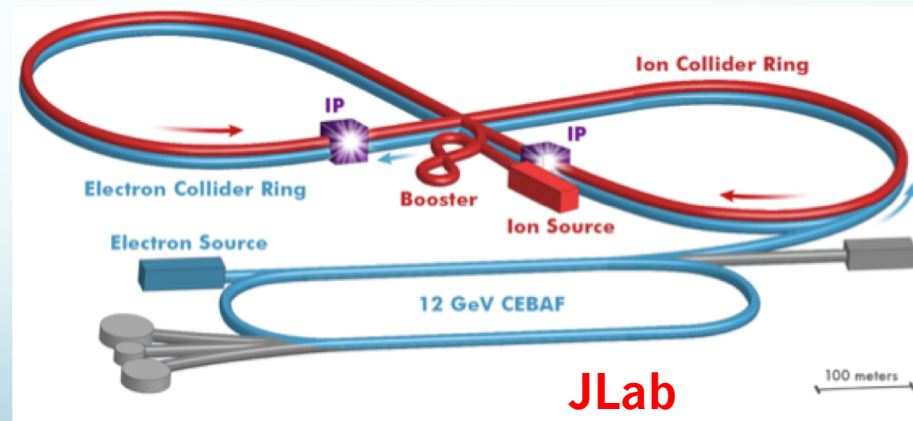
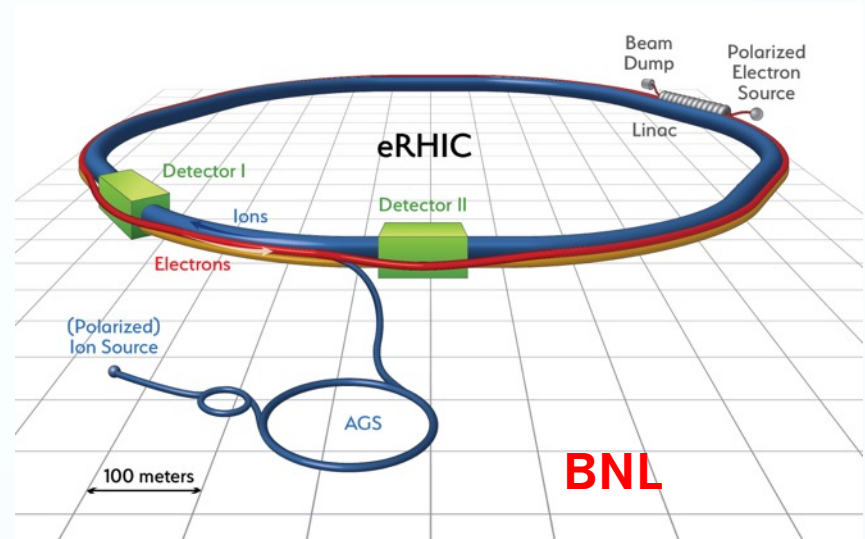


Brookhaven National Laboratory in New York is a potential host for the Electron-Ion Collider.

## NUCLEAR PHYSICS

# Billions-dollar collider gets thumbs up

Proposed US electron-ion smasher wins endorsement from influential nuclear-science panel.



# Summary

- Study on TMDs are extremely active in the past few years, lots of progress have been made, especially for quark TMDs
- Nucleon as a QCD “laboratory”: in particular topics/ideas that are similar to those in AMO/Condensed Matter Physics
  - Quantum correlation: spin-spin correlation, spin-orbit correlation, orbital motion, quantum phase interference effects ...
  - 3D imaging of the nucleon at the most fundamental level
- Exciting opportunities: lots of experiments activities/measurements being/to be performed/planned in current and future experimental facilities (most importantly, **the EIC**)
  - Especially gluon TMDs, jet TMDs

Thank you!