The 11th Workshop on Hadron Physics in China and
Opportunities Worldwide (Hadron-China 2019)
22-28 Aug. 2019, Tianjin - China

Latest developments on Transversity and Tensor Charge

Marco Radici
INFN - Pavia

In collaboration with A. Bacchetta (Univ. Pavia)

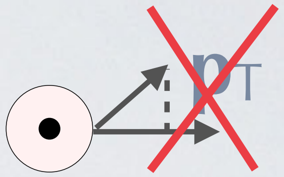
a phase transition

quark polarization

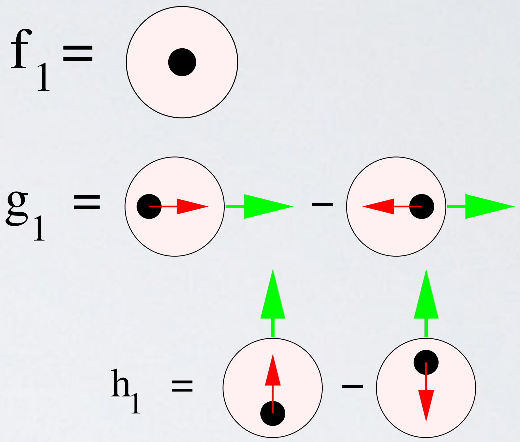
nucleon polarization

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

thousands data **hundreds** data **tens** data



collinear PDFs



chiral-odd

a phase transition

quark polarization

nucleon polarization

	U	L	T
U	f_1		$h_1^\perp$
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thousands data

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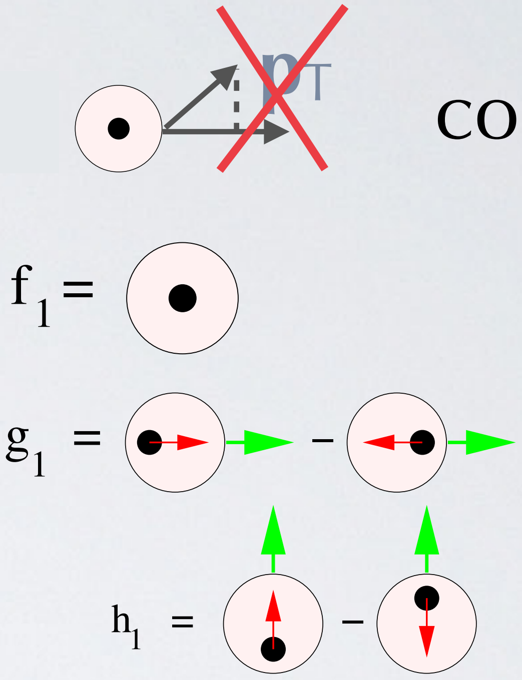
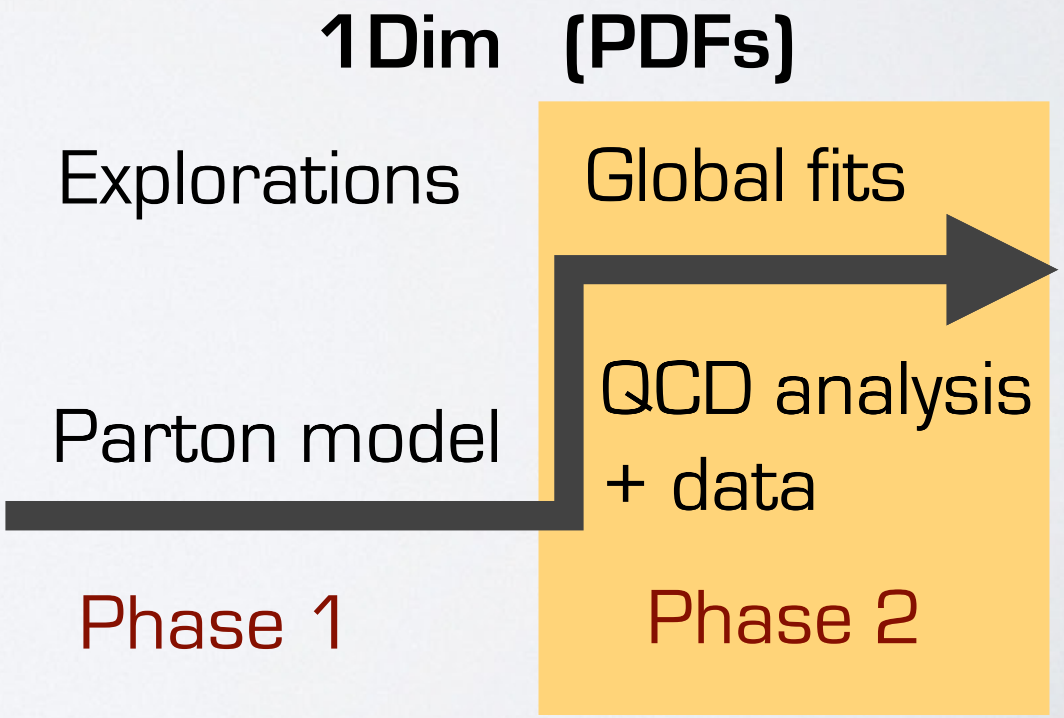


collinear PDFs

chiral-odd

first global fit
(= lepton-hadron scatt.
and hadron collisions)
of **PDF h_1**

Radici and Bacchetta,
P.R.L. ***120*** *(18) 192001*



1Dim (PDFs)

Explorations

Global fits

Parton model

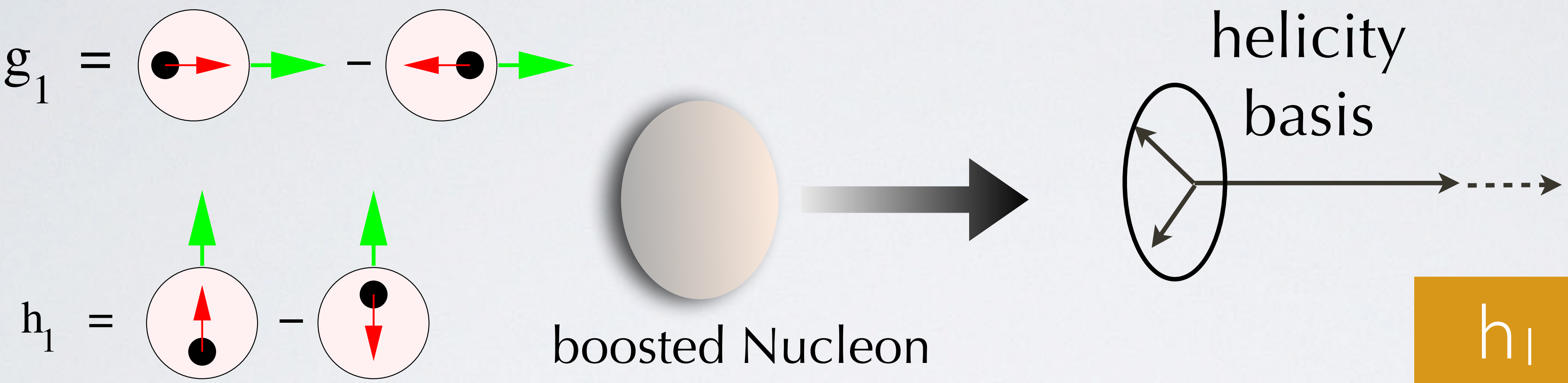
QCD analysis
+ data

Phase 1

Phase 2

Transversity : Why

- transversity is very different from helicity

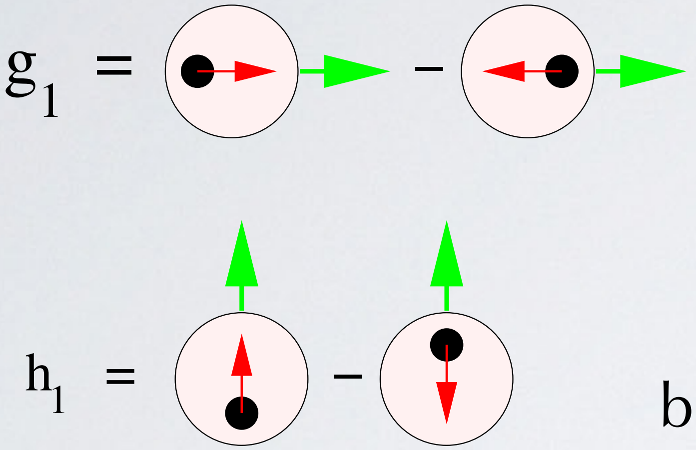


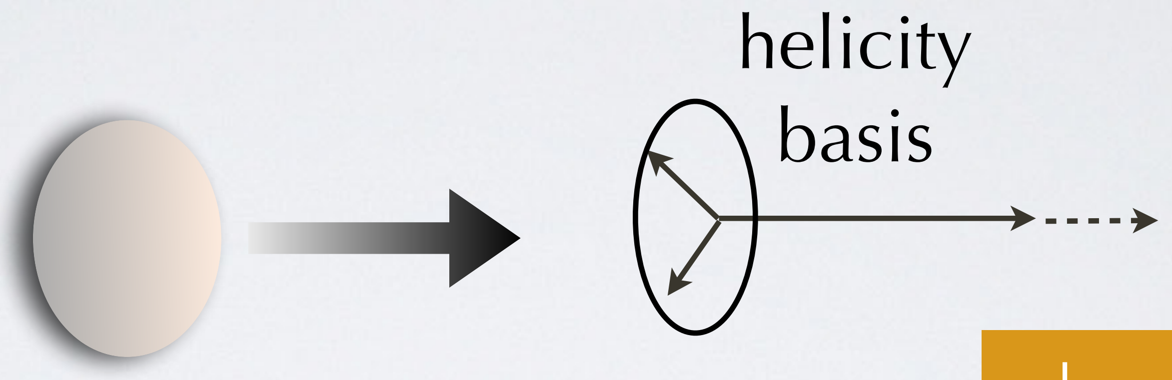
g_1 diagonal

h_1 non diagonal
(“chiral-odd”)

Transversity : Why

- transversity is very different from helicity





boosted Nucleon

g_1 diagonal

h_1 non diagonal
(“chiral-odd”)

- no h_1 for gluons
(in Nucleon)

→ pure non-singlet evolution

playground for tests of perturbative and nonperturbative QCD

Tensor Charge

- 1st Mellin moment of transversity ⇒ tensor “charge”

$$\delta q \equiv g_T^q = \int_0^1 dx \; \left[h_1^q(x,Q^2) - h_1^{\bar{q}}(x,Q^2)\right]$$

no associated conserved current in $\mathcal{L}_{\mathrm{QCD}}$

↓

tensor “charge” g_T	scales with Q^2	C-odd
axial charge g_A	conserved	C-even

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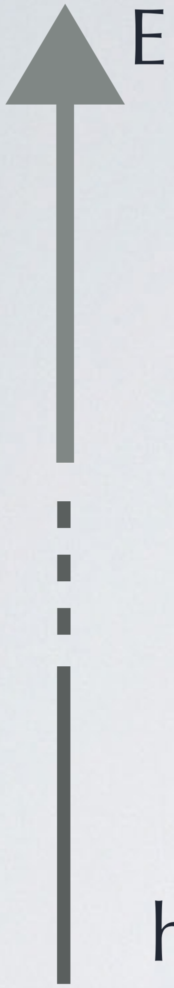
↓

tensor "charge" g_T	scales with Q^2	C-odd
axial charge g_A	conserved	C-even

tensor charge not directly accessible in $\mathcal{L}_{\mathrm{SM}}$
low-energy footprint of new physics at higher scales ?

potential for BSM discovery ?

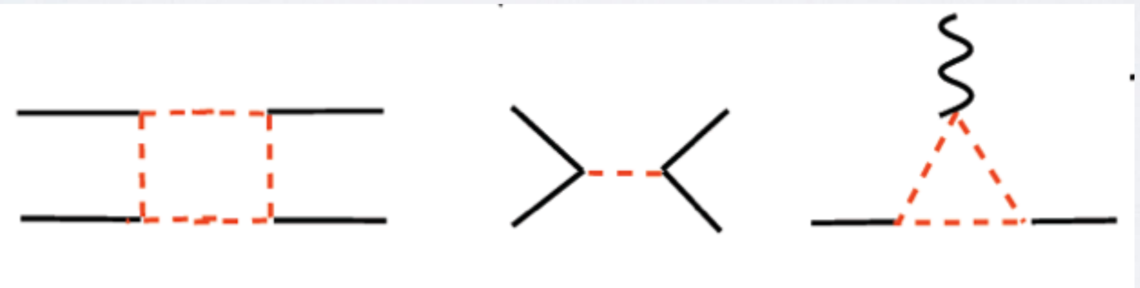
search for new physics **B**eyond **S**tandard **M**odel



M_{BSM}
high energy

direct access:
new particles

$E_{exp} \ll M_{BSM}$
low energy
high precision



indirect access:
virtual effects

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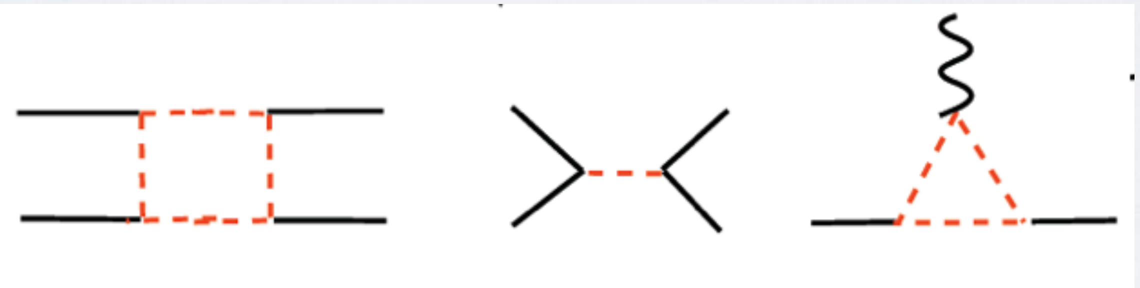
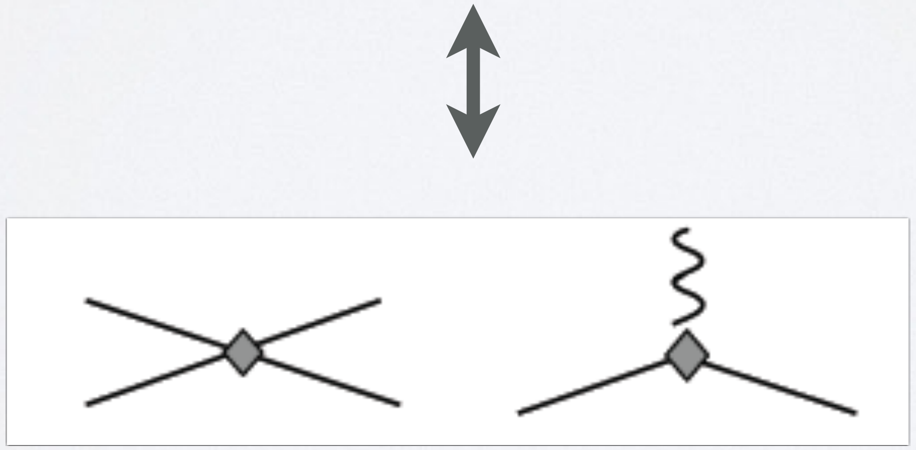
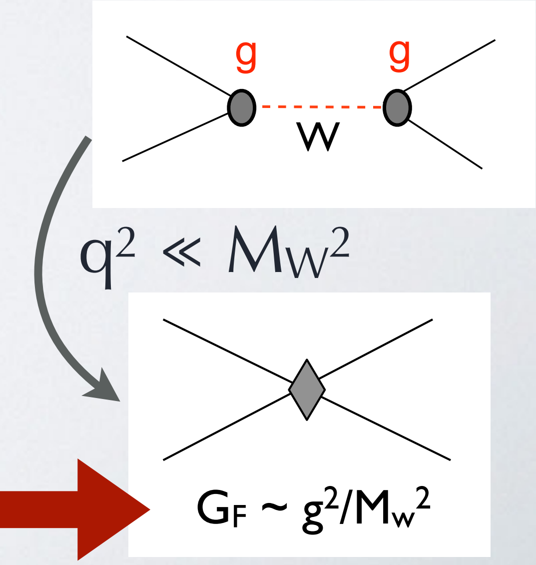


indirect access:
virtual effects

footprint:
new local
operators



Example:
weak CC
interaction

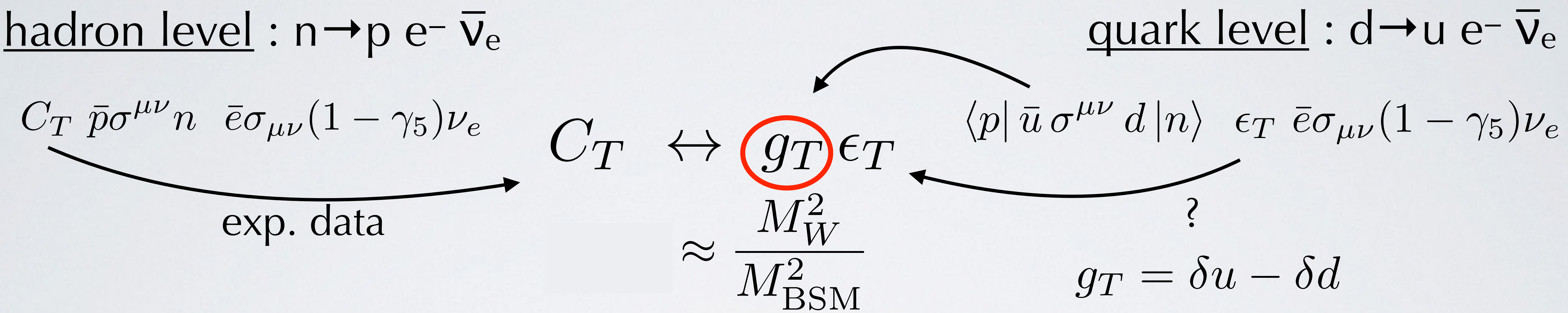


$q^2 \ll M_W^2$

$G_F \sim g^2/M_W^2$

Examples of indirect access

- **nuclear β-decay**: effective field theory including operators not in SM Lagrangian; for example, tensor operator



hadron level : $n \to p\, e^- \bar{\nu}_e$

quark level : $d \to u\, e^- \bar{\nu}_e$

$C_T\ \bar{p}\sigma^{\mu\nu}n\ \ \bar{e}\sigma_{\mu\nu}(1-\gamma_5)\nu_e$

exp. data

$$C_T \leftrightarrow \boxed{g_T}\,\epsilon_T \approx \frac{M_W^2}{M_{\mathrm{BSM}}^2}$$

$\langle p|\,\bar{u}\,\sigma^{\mu\nu}\,d\,|n\rangle\ \ \epsilon_T\ \bar{e}\sigma_{\mu\nu}(1-\gamma_5)\nu_e$

?

$g_T = \delta u - \delta d$

isovector tensor charge

precision of 0.1% ⇒ BSM scale > [3-5] TeV

Examples of indirect access

- **nuclear β-decay**: effective field theory including operators not in SM Lagrangian; for example, tensor operator

hadron level : n→p e⁻ $\bar{\nu}_e$

quark level : d→u e⁻ $\bar{\nu}_e$

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?

$$\approx \frac{M_W^2}{M_{\rm BSM}^2}$$

$$g_T = \delta u - \delta d$$

isovector tensor charge

precision of 0.1% ⇒ BSM scale > [3-5] TeV

- **neutron EDM**: estimate CPV induced by quark chromo-EDM d_q

$$\mathcal{L}_{\rm CPV} \supset\ ie \sum_{f=u,d,s,e} d_f\ \bar{f}\,\sigma_{\mu\nu}\gamma_5\,f\ F^{\mu\nu}$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

$$d_n = \boxed{\delta u}\,d_u + \boxed{\delta d}\,d_d + \boxed{\delta s}\,d_s$$

exp. bounds

+ tensor charge

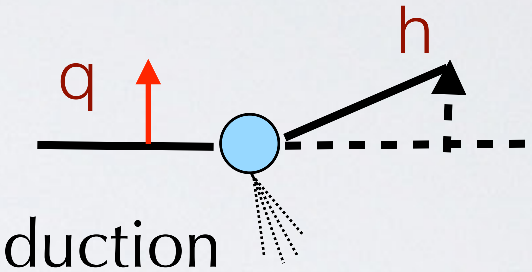
constraints on
CP violation
encoded in q EDM

extraction of transversity

transversity is chiral-odd → need a chiral-odd partner

- itself : fully polarized Drell-Yan ✘

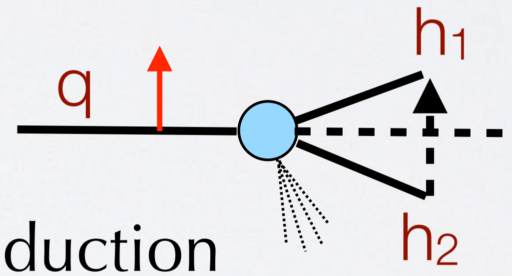
- Collins function : the Collins effect



1-hadron semi-inclusive production

TMD framework
h_1 as TMD

- IFF : the di-hadron mechanism



2-hadrons semi-inclusive production

collinear framework
h_1 as PDF

- hadron-in-jet mechanism : mixed framework **h_1 as PDF**

- lattice "quasi-h1" : using Ji's LaMET *Chen et al., N.P.* ***B911*** *(16) 246*

extraction of transversity

transversity is chiral-odd → need a chiral-odd partner

- itself : fully polarized Drell-Yan ✘

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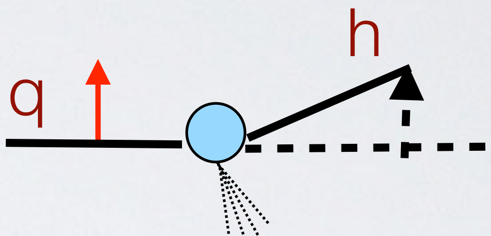

1-hadron semi-inclusive production

TMD framework
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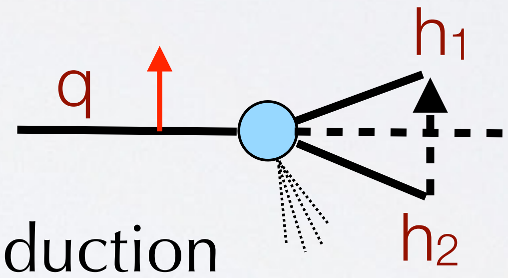


2-hadrons semi-inclusive production

collinear framework
h_1 as PDF

- hadron-in-jet mechanism : mixed framework **h_1 as PDF**

- lattice "quasi-h1" : using Ji's LaMET *Chen et al., N.P.* ***B911*** *(16) 246*

why di-hadron mechanism ?

collinear framework

- simple product of PDF and IFF

Ex.: SIDIS

$$A_{\text{SIDIS}}^{\sin(\phi_R+\phi_S)}(x,z,M_h^2) \sim -\frac{\sum_q e_q^2\ h_1^q(x)\ \frac{|\boldsymbol{R}_T|}{M_h}\ H_{1,q}^{\sphericalangle}(z,M_h^2)}{\sum_q e_q^2\ f_1^q(x)\ D_{1,q}(z,M_h^2)}$$

x-dependence of A_{SIDIS} all in PDF

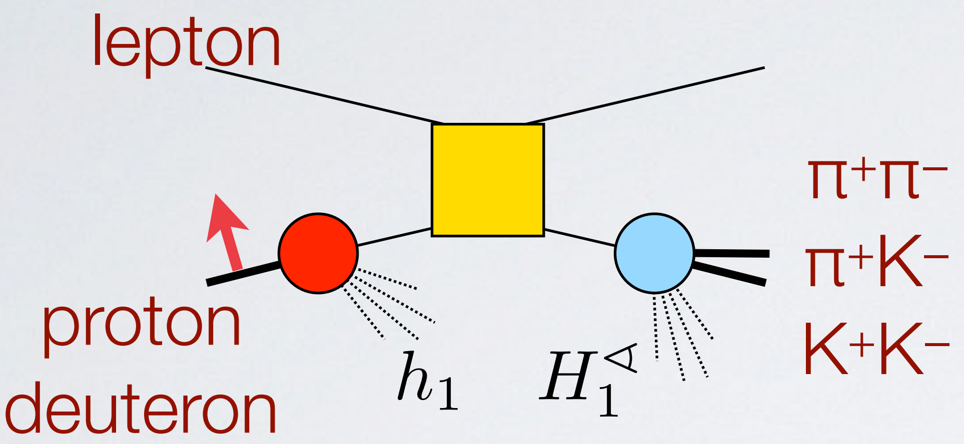
- factorization theorems for all hard processes
  → universality of h_1 $H_1^{\sphericalangle}$ mechanism

advantages of di-hadron mechanism

factorization theorems for all hard processes

data used in the global fit

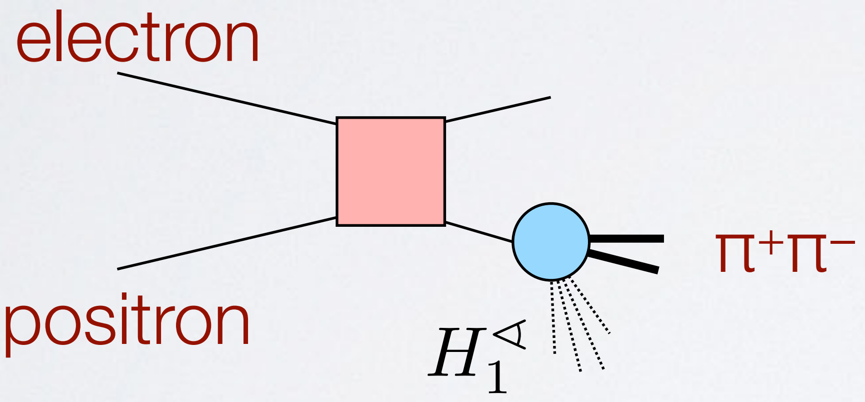
SIDIS



Airapetian et al., JHEP ***0806*** *(08) 017*

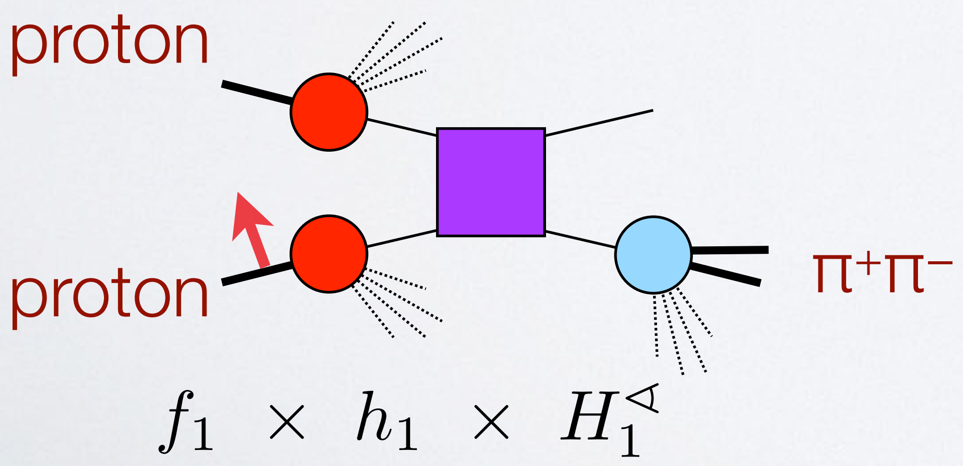
Adolph et al., P.L. ***B713*** *(12)*
Braun et al., E.P.J. Web Conf. ***85*** *(15)*

e+e−



Vossen et al., P.R.L. ***107*** *(11) 072004*

p p↑



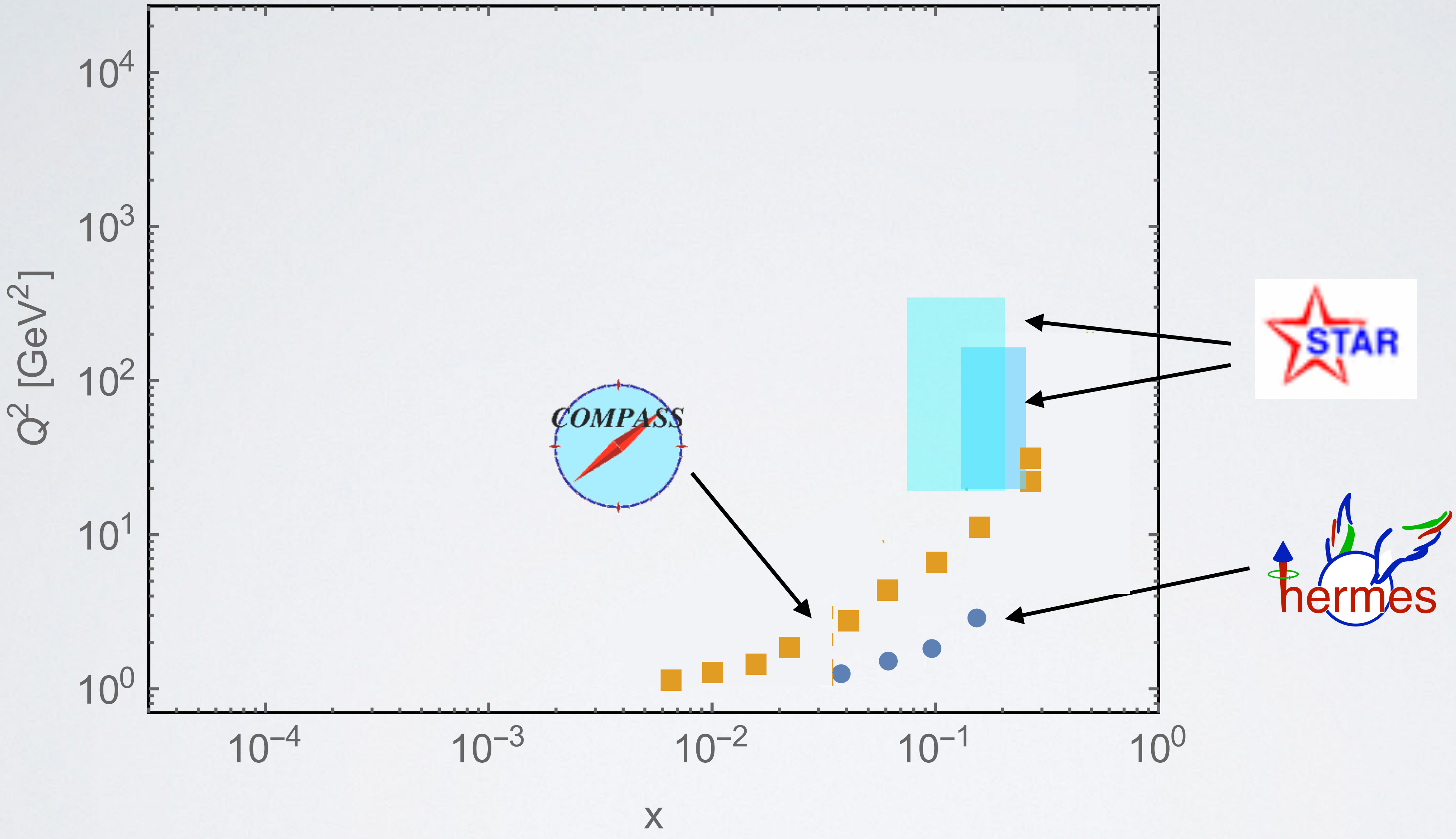
run 2006
(s=200 GeV^2)

Adamczyk et al. (STAR),
P.R.L. ***115*** *(2015) 242501*

run 2011
(s=500 GeV^2)

Adamczyk et al. (STAR),
P.L. ***B780*** *(18) 332*

the phase space

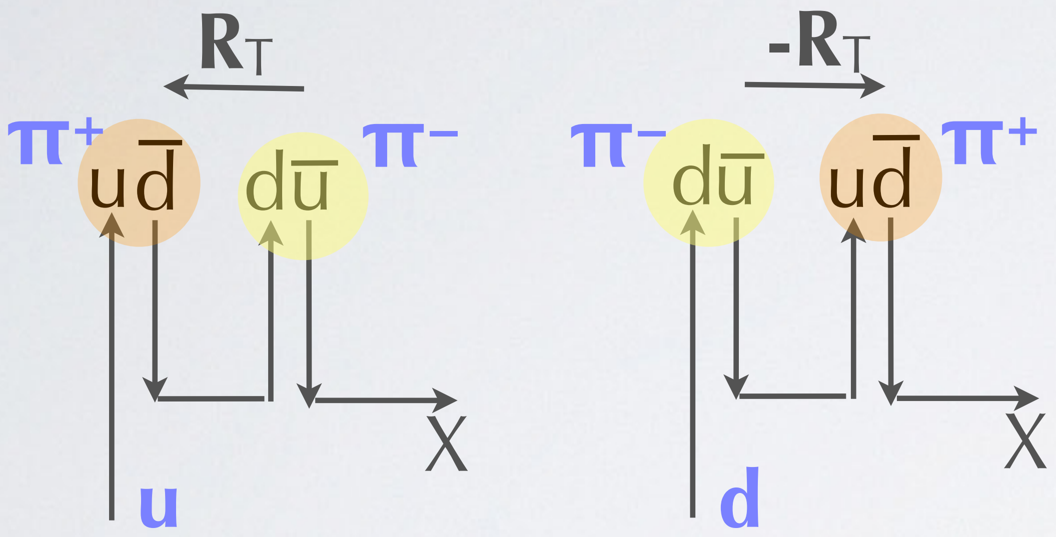


- mostly medium/high x
- guess low-x behavior (relevant for calculation of tensor charge - see later)

currently, only LO analysis

$$A_{UT}^{\sin(\phi_R+\phi_S)}(x,z,M_h^2) \propto -\frac{|\mathbf{R}|}{M_h}\,\frac{\sum_q e_q^2\, h_1^q(x)\; H_1^{\triangleleft q}(z,M_h^2)}{\sum_q e_q^2\, f_1^q(x)\; D_1^q(z,M_h^2)}$$

$\pi^+\pi^-$
tree level



$$H_1^{\triangleleft u} = -H_1^{\triangleleft d}$$

isospin symmetry

$$\left.\begin{aligned} H_1^{\triangleleft q} &= -H_1^{\triangleleft \bar{q}} \\ D_1^q &= D_1^{\bar{q}} \end{aligned}\right\}$$

charge conjugation

access only $q-\bar{q} = q_V$, q=u,d
valence flavors in SIDIS A_{UT}

theoretical uncertainties

unpolarized Di-hadron Fragmentation Function D_1

- **quark** D_1^q is **well** constrained by $e^+e^- \to (\pi^+\pi^-) X$ (Montecarlo)
- **gluon** D_1^g is **not** constrained by $e^+e^- \to (\pi^+\pi^-) X$ (currently, LO analysis)
- **no data** available yet for $p\, p \to (\pi^+\pi^-) X$

we don't know anything about the gluon D_1^g

our choice: set $D_1^g(Q_0) = \begin{cases} 0 \\ D_1^u(Q_0) / 4 \\ D_1^u(Q_0) \end{cases}$ ~ 1-hadron $D_1^g(Q_0)$

deteriorates our e^+e^- fit as $\chi^2/\mathrm{dof} = \begin{cases} 1.69 & 1.28 \\ 1.81 & 1.37 \\ 2.96 & 2.01 \end{cases}$

background ρ channels

choice of functional form

functional form whose Mellin transform can be computed analytically and complying with Soffer Bound at any x and scale Q^2

$$h_1^{q_v}(x;Q_0^2) = F^{q_v}(x)\left[\mathrm{SB}^q(x) + \overline{\mathrm{SB}}^{\bar{q}}(x)\right]$$

Soffer Bound

$$2|h_1^q(x,Q^2)| \leq 2\ \mathrm{SB}^q(x,Q^2) = |f_1^q(x,Q^2) + g_1^q(x,Q^2)|$$

MSTW08 DSSV

$$F^{q_v}(x) = \frac{N_{q_v}}{\max_x[|F^{q_v}(x)|]}\, x^{A_{q_v}}\, [1 + B_{q_v}\,\mathrm{Ceb}_1(x) + C_{q_v}\,\mathrm{Ceb}_2(x) + D_{q_v}\,\mathrm{Ceb}_3(x)]$$

Ceb_n(x) Cebyshev polynomial

10 fitting parameters

constrain parameters

$|N_{q_v}| \leq 1 \Rightarrow |F^{q_v}(x)| \leq 1$ Soffer Bound ok at any Q^2

constrain parameters : low-x trend

$$\left.\begin{array}{l}\lim\limits_{x\to 0} x\mathrm{SB}^q(x) \propto x^{a_q} \\[2ex] \lim\limits_{x\to 0} F^{q_v}(x) \propto x^{A_q}\end{array}\right\} \qquad h_1^q(x) \overset{x\,\to\,0}{\approx} x^{A_q + a_q - 1}$$

tensor charge $\delta q(Q^2) = \int_{x_{\min}}^{1} dx\, h_1^{q-\bar{q}}(x, Q^2)$

low-x behavior important

constrain parameters

δq finite => $A_q + a_q > 0$

our choice

$A_q + a_q > \frac{1}{3}$

$\left| \int_0^{x_{\min}} dx \right| \sim 1\%$ of $\left| \int_{x_{\min}}^{1} dx \right|$

for $x_{\min}=10^{-6}$ from MSTW08

Other choices

"massive" jet in DIS → h_1 at twist 3
violation of Burkardt-Cottingham s.r.
Accardi and Bacchetta, P.L. ***B773*** *(17) 632*

$\int_0^1 dx\, g_2(x) \propto \int_0^1 dx\, \frac{h_1(x)}{x} \longrightarrow A_q + a_q > 1$

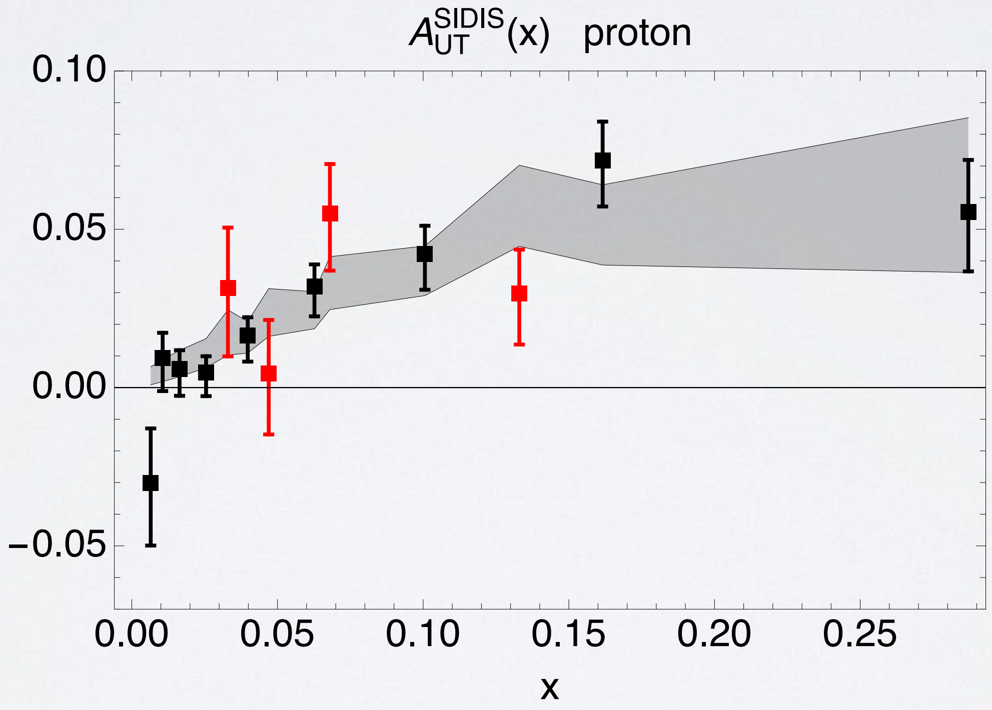
small-x dipole picture $h_1^{q_v}(x) \overset{x\,\to\,0}{\approx} x^{1-2\sqrt{\frac{\alpha_s(Q^2)N_c}{2\pi}}} \longrightarrow$ at Q_0 $A_q + a_q \sim 1$

Kovchegov & Sievert, arXiv:1808.10354

statistical uncertainty

the bootstrap method

- shift each exp. point by Gaussian noise within exp. variance
- create sets of virtual points to be fitted: 50, 100, 200 sets… until average and standard deviation reproduce original exp. points (**here, 200x3=600**)
- exclude largest and smallest 5% => **90% band**



automatically accounts for correlations

Results

Our first global fit

first ever extraction of transversity from data of SIDIS and proton-proton collisions

Radici and Bacchetta, P.R.L. ***120*** *(18) 192001*

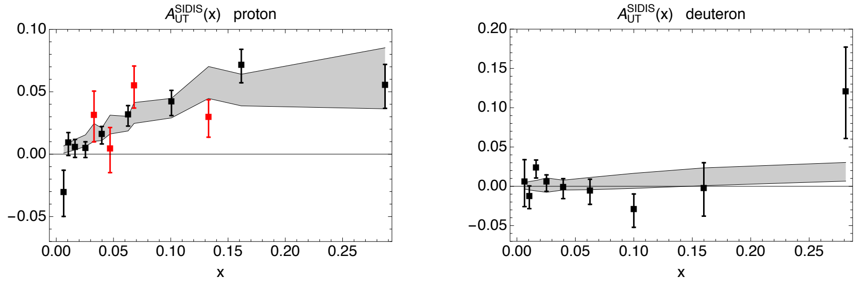
SIDIS

● COMPASS

18 data points

■ hermes

4 data points



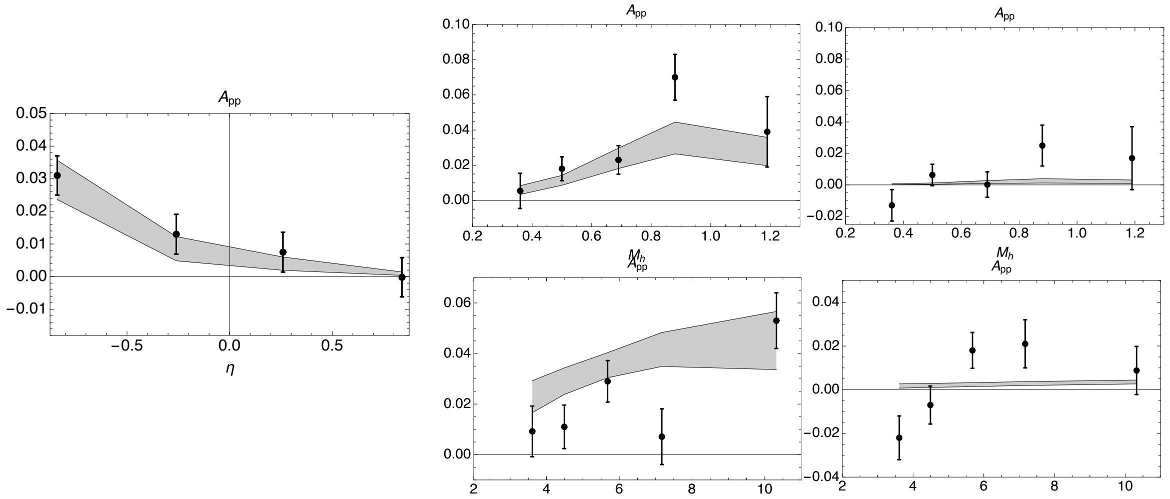
pp collisions

● STAR

run 2006

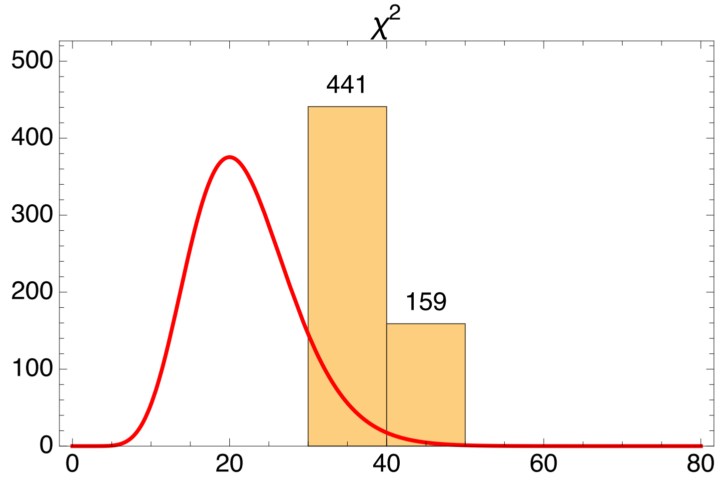
(s=200 GeV^2)

10 independent data points



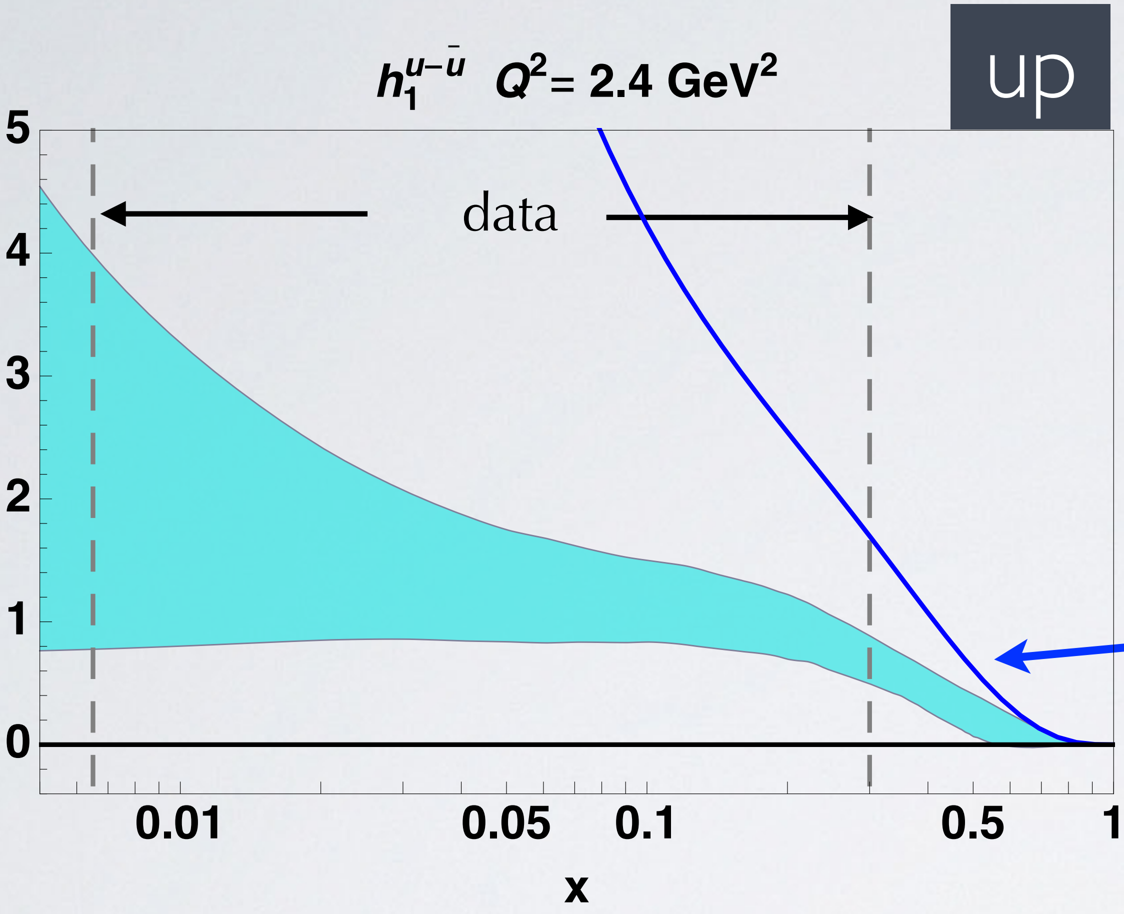
probability density function of χ^2 distribution for 22 d.o.f.

(for χ^2/dof = 1 perfect overlap)

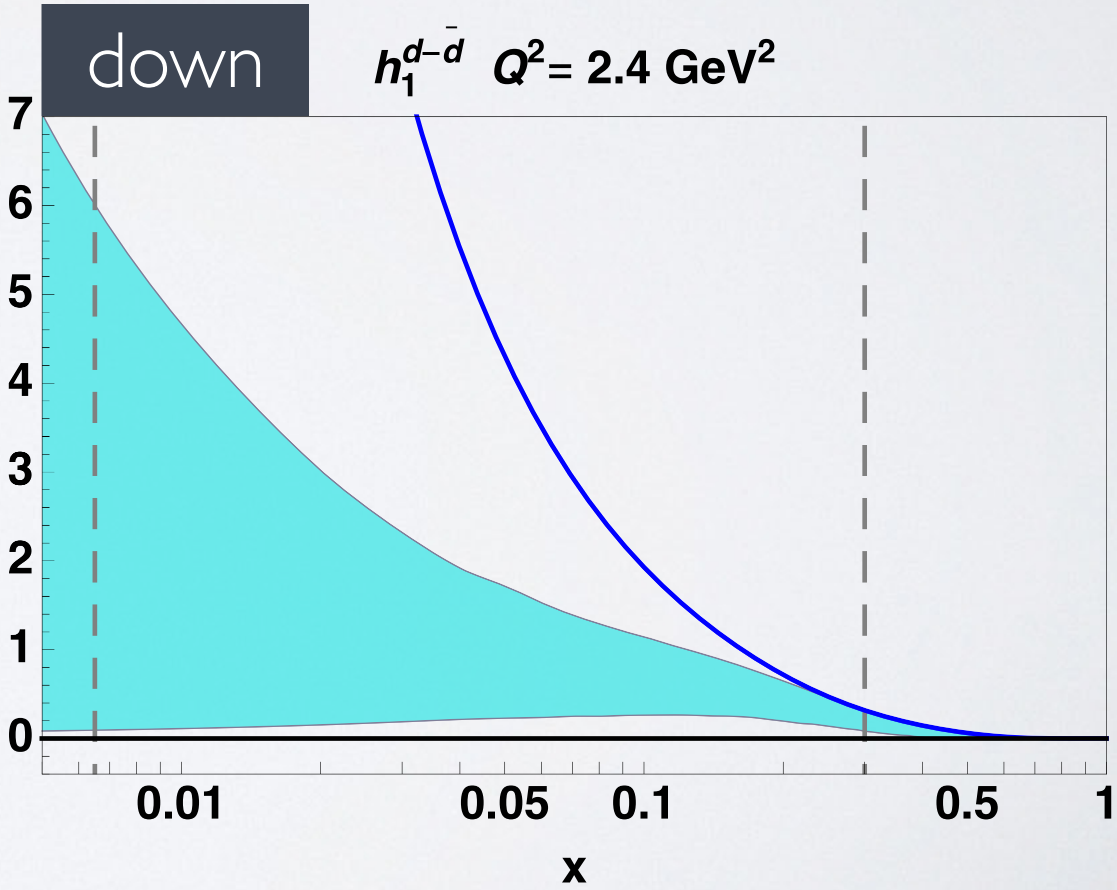


χ^2/dof = 1.76 ± 0.11

the extracted transversity

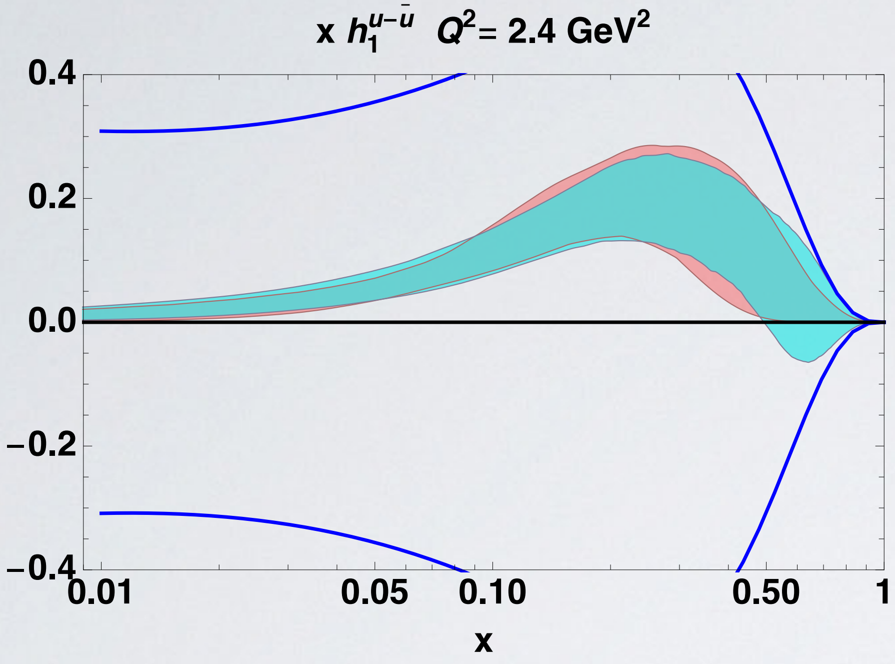


Soffer
bound



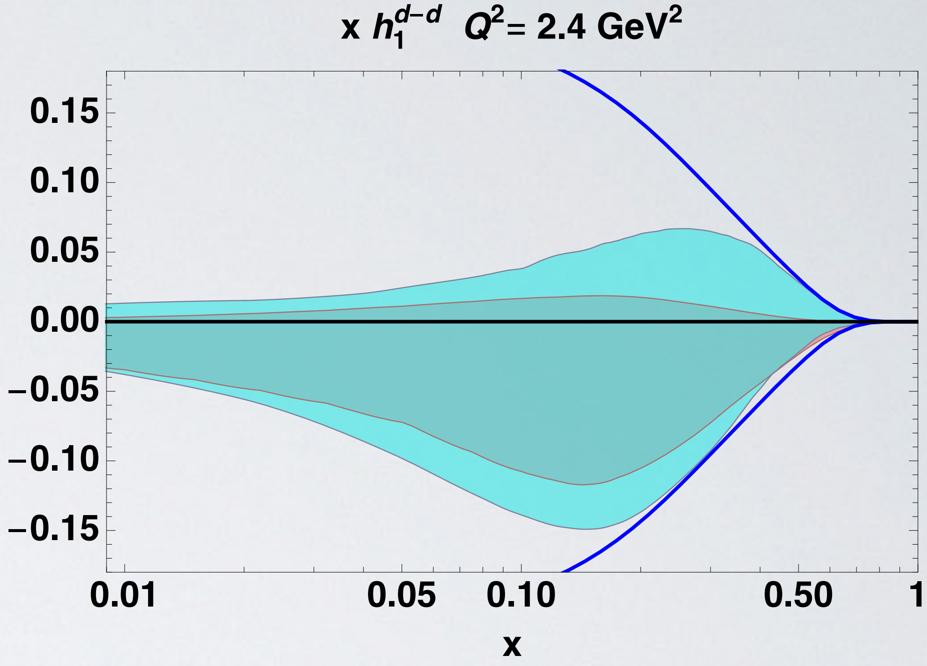
uncertainty band from
90% of 600 replicas
= max uncertainty on $D_1{}^g(Q_0)$

Comparison with other extractions



Anselmino et al.,
*P.R. D**87** (13) 094019*

Torino

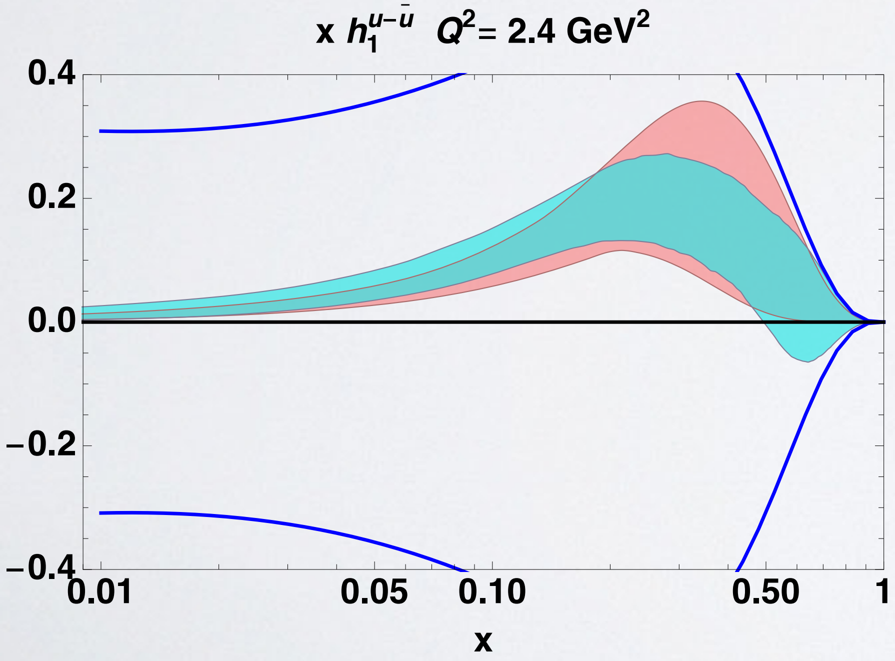


up

global fit

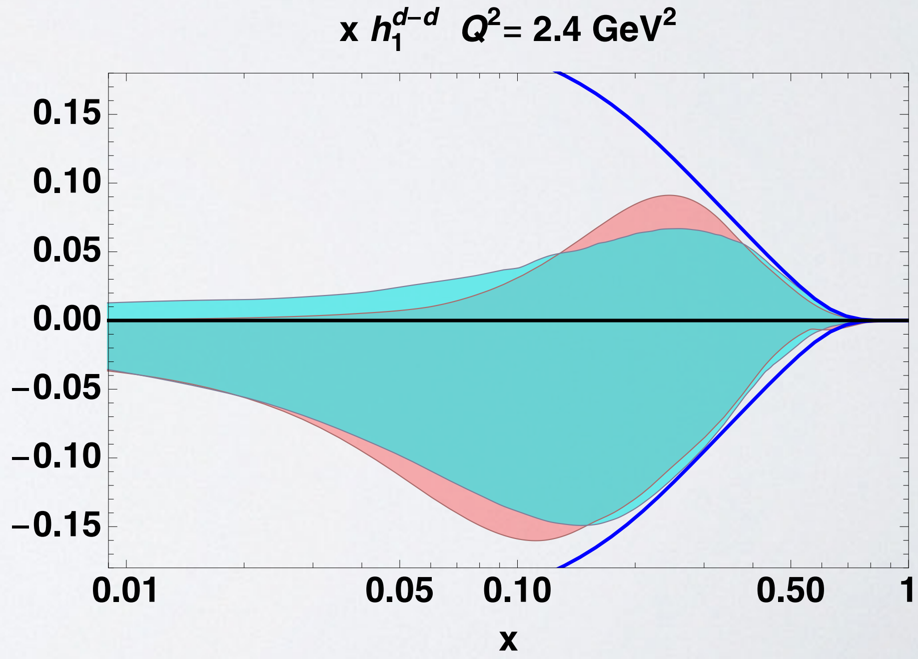
Radici and Bacchetta,
*P.R.L. **120** (18) 192001*

down

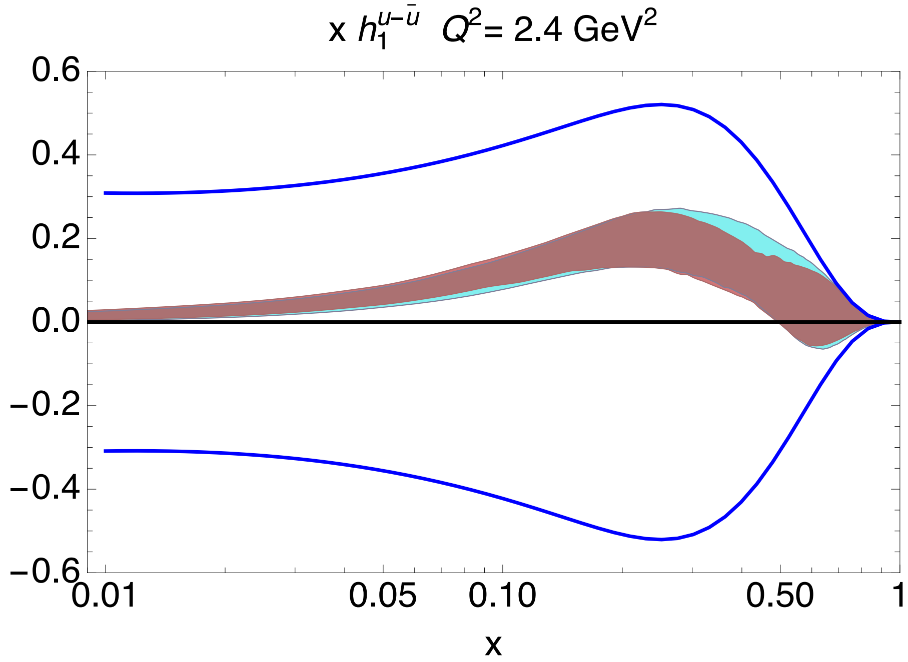


TMD

Kang et al.,
*P.R. D**93** (16) 014009*

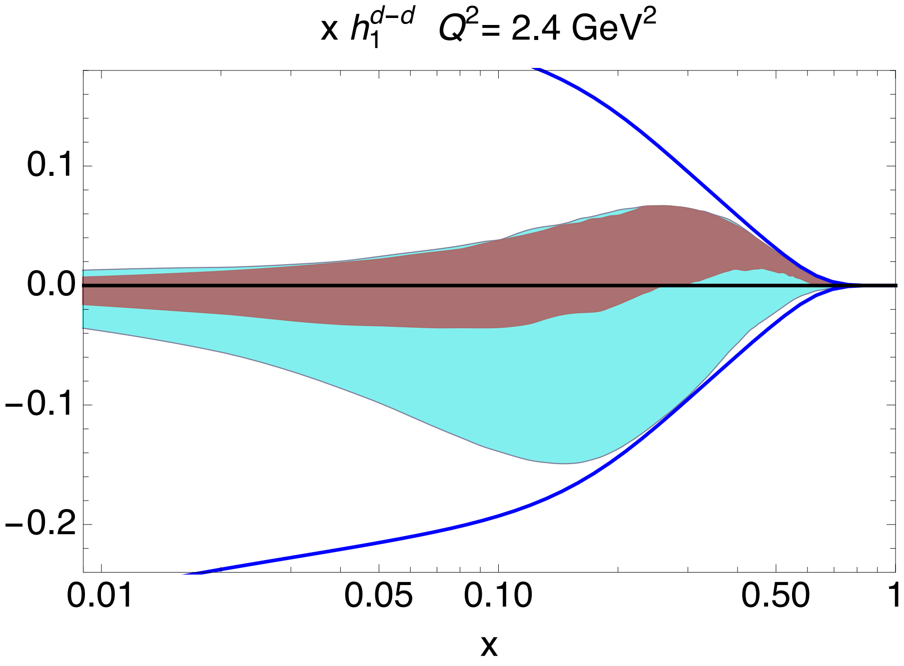


sensitivity to th. uncertainty



$D_1^g(Q_0) = 0$

$D_1^g(Q_0) = \begin{cases} 0 \\ D_1^u/4 \\ D_1^u \end{cases}$



global fit

Radici & Bacchetta,
P.R.L. ***120*** *(18) 192001*

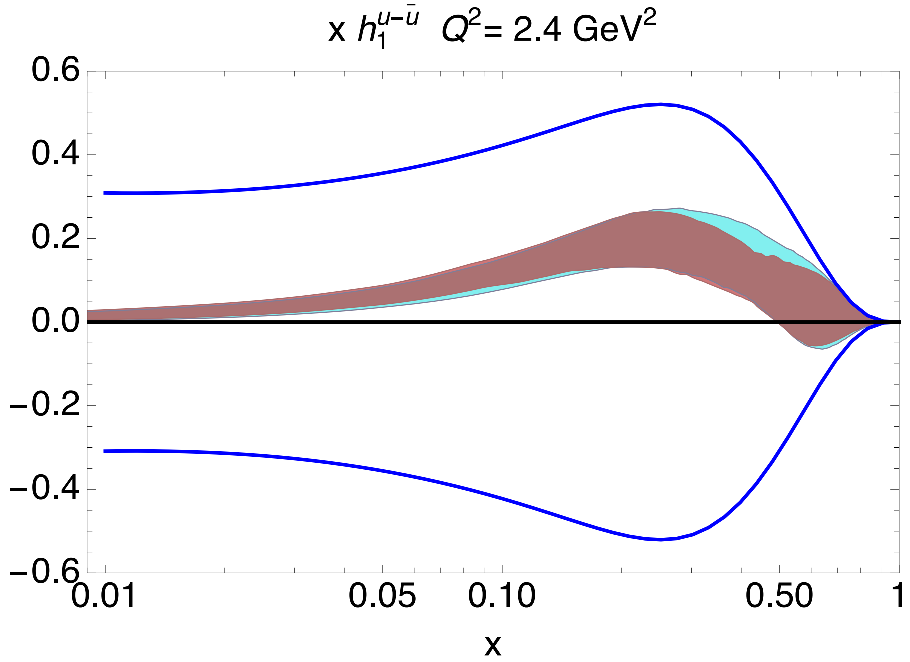
up

insensitive to uncertainty on gluon D_1

down

sensitive to uncertainty on gluon D_1

sensitivity to th. uncertainty



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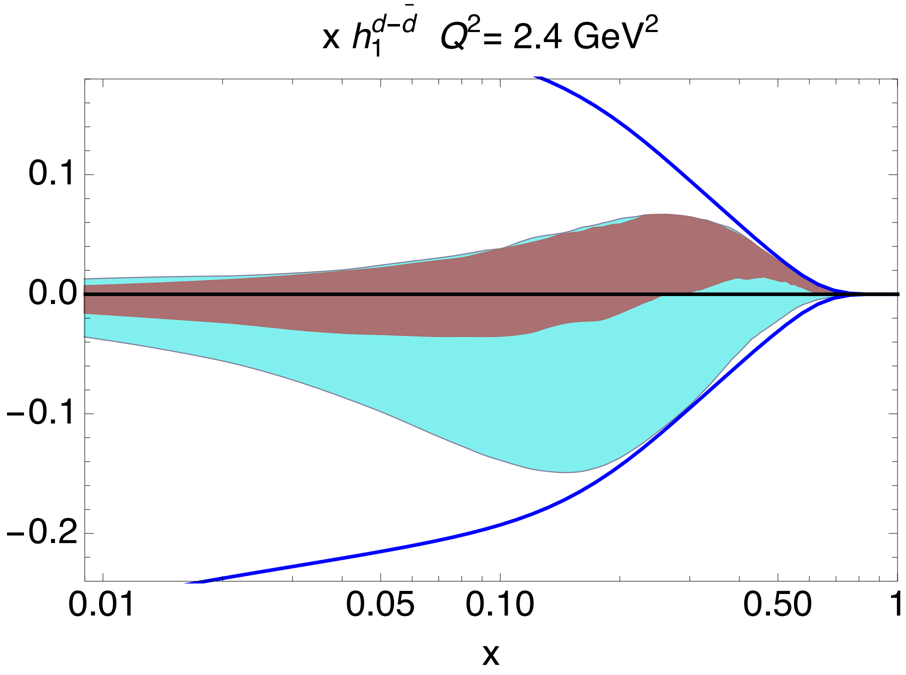


global fit

Radici & Bacchetta,
P.R.L. ***120*** *(18) 192001*

up

insensitive to uncertainty on gluon D_1

down

sensitive to uncertainty on gluon D_1

p–p : u~d , gluon @LO but **SIDIS :** u~(8x)d , gluon @NLO

need data from target more sensitive to down (deuteron, ^{3}He) and need data from multiplicities in p+p → (ππ)+X

The tensor “charge” of the proton

1st Mellin moment of transversity PDF ⇒ tensor “charge”

$$\delta q \equiv g_T^q = \int_0^1 dx\ \left[h_1^q(x,Q^2) - h_1^{\bar{q}}(x,Q^2)\right]$$

tensor charge connected to tensor operator

$$\langle P, S_p |\, \bar{q}\sigma^{\mu\nu} q\, | P, S_p \rangle = (P^\mu S_p^\nu - P^\nu S_p^\mu)\, \delta q$$
$$= (P^\mu S_p^\nu - P^\nu S_p^\mu) \int dx\, h_1^{q-\bar{q}}(x,Q^2)$$

compute on lattice

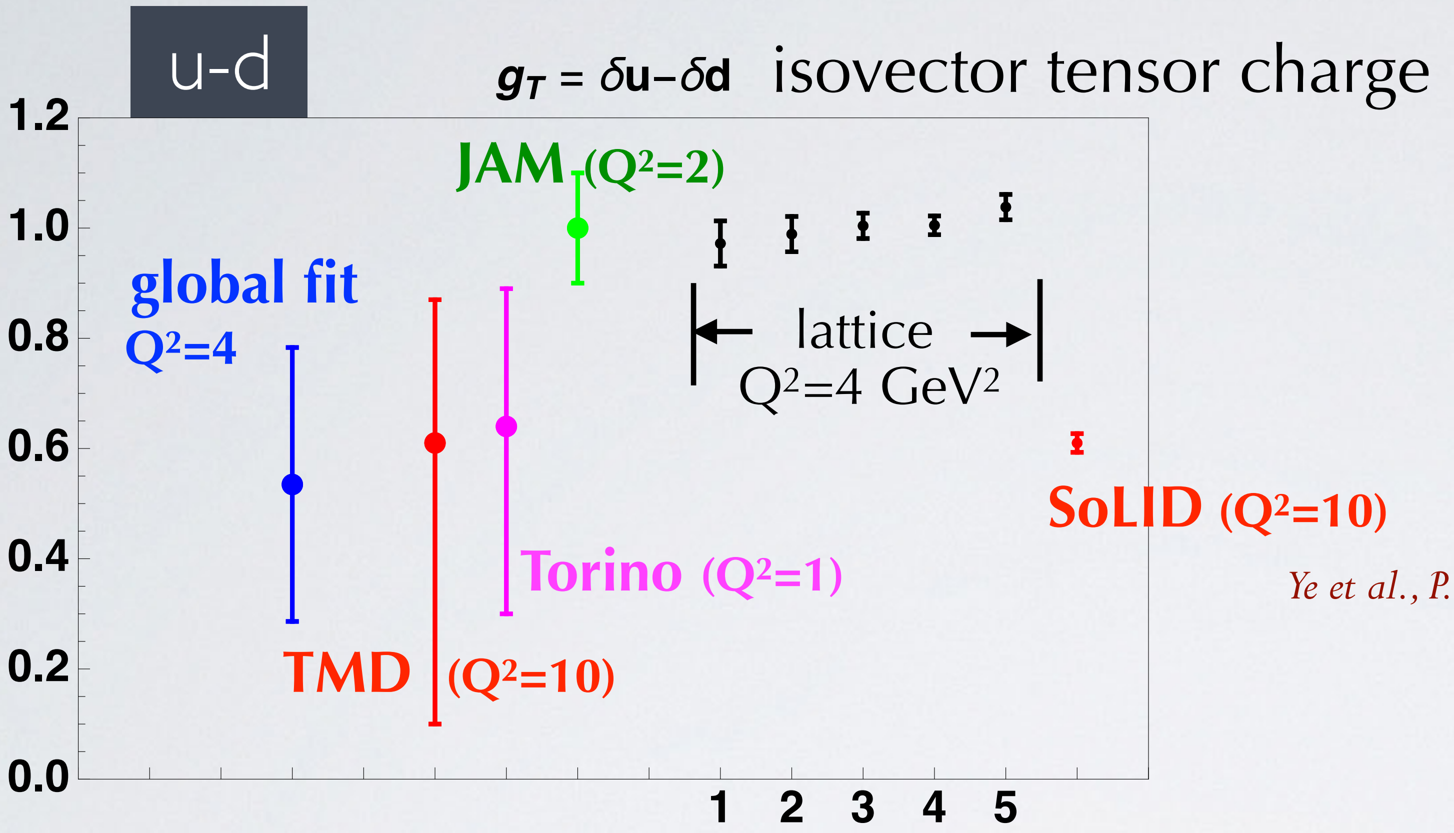
lattice δq

preferably the isovector $g_T = \delta u - \delta d$
(cancellation of “disconnected” diagrams)

extract transversity from data with transversely polarized protons

pheno δq

Results for our global fit



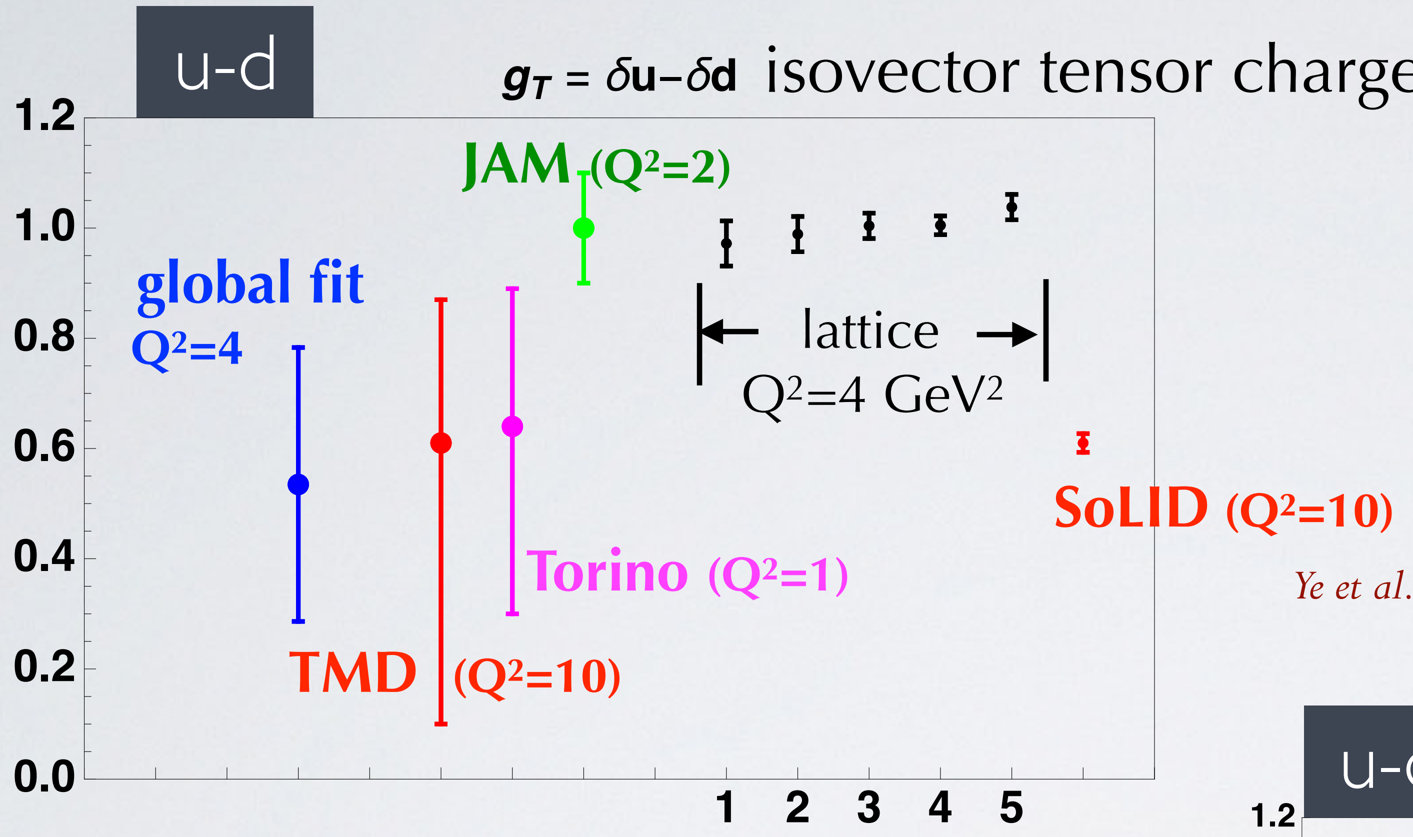
Ye et al., P.L. **B767** (17) 91

Torino, TMD, JAM from SIDIS data only

1) "MILC" '19	*Hasan et al., arXiv:1903.06487*
2) PNDME '18	*Gupta et al., P.R. D**98** (18) 034503*
3) ETMC '17	*Alexandrou et al., P.R. D**95** (17) 114514;* *E P.R. D**96** (17) 099906*
4) RQCD '14	*Bali et al., P.R. D**91** (15)*
5) LHPC '12	*Green et al., P.R. D**86** (12)*

Results for our global fit

$g_T = \delta u - \delta d$ isovector tensor charge



1) **"MILC" '19**	*Hasan et al., arXiv:1903.06487*
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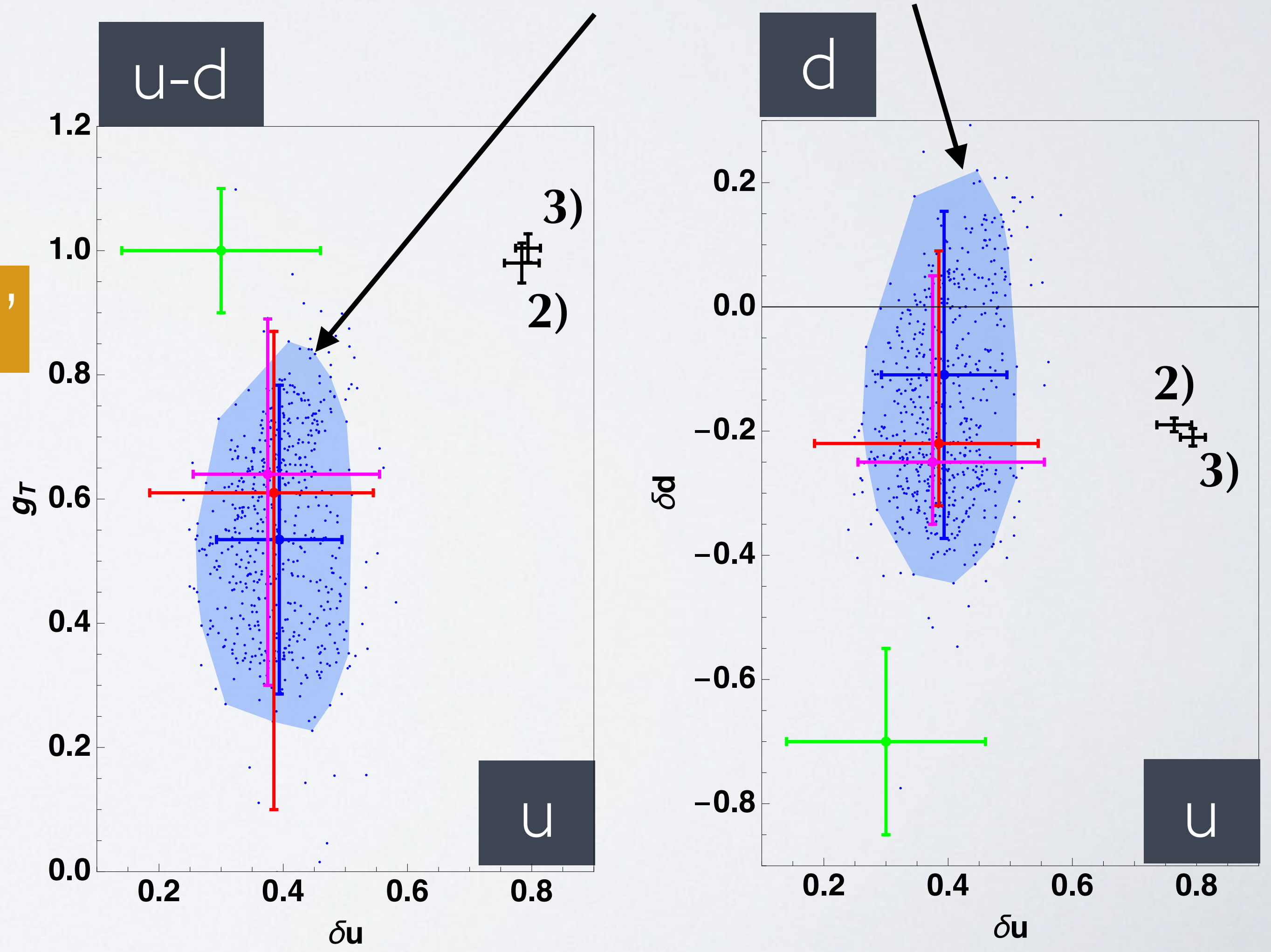
*Ye et al., P.L. **B767** (17) 91*

Torino, **TMD**, **JAM** from SIDIS data only

JAM includes constraint from "lattice g_T"

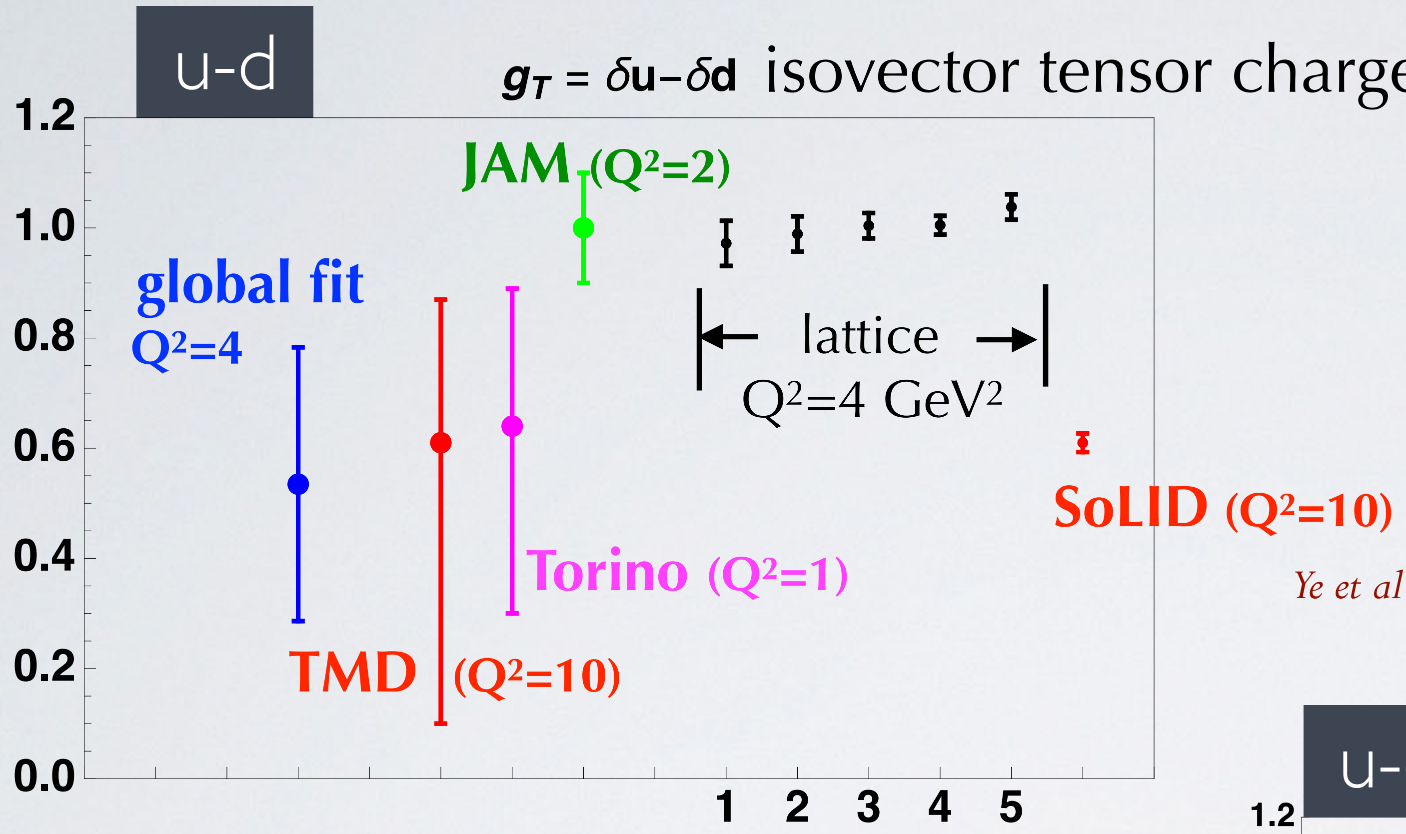
*Lin et al., P.R.L. **120** (18) 152502*

shaded area = 90% C.L.



But if we look also at **δ**u and **δ**d …

Results for our global fit



Torino, **TMD**, **JAM** from SIDIS data only

JAM includes constraint from "lattice g_T"

Lin et al., P.R.L. ***120*** *(18) 152502*

no simultaneous compatibility between "pheno δq" and "lattice δq"

1)	**"MILC" '19**	*Hasan et al., arXiv:1903.06487*
2)	**PNDME '18**	*Gupta et al., P.R. D***98** *(18) 034503*
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Ye et al., P.L. ***B767*** *(17) 91*

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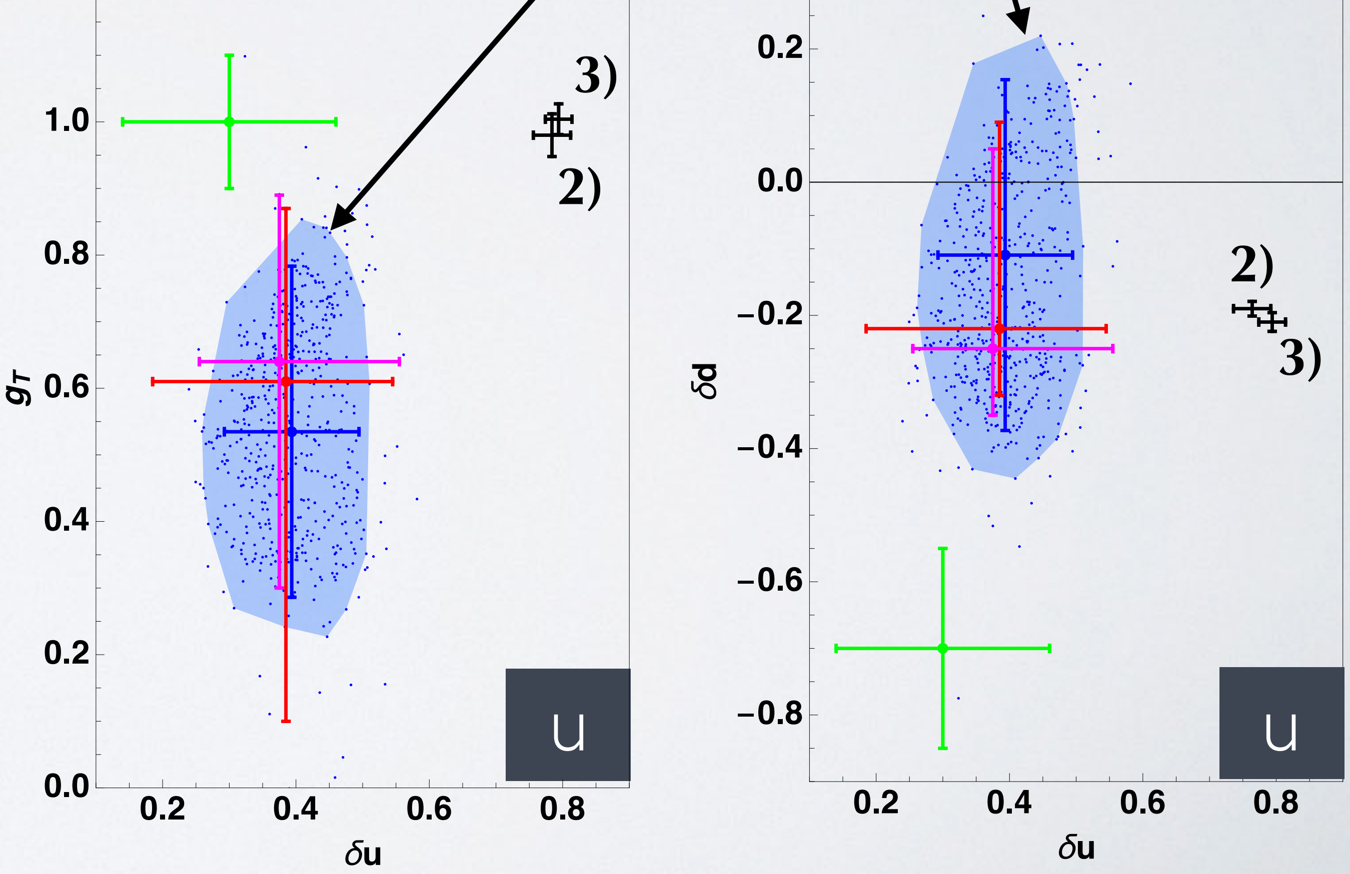


pheno vs. lattice tensor charge

main problem of "pheno δq" is extrapolating outside data..

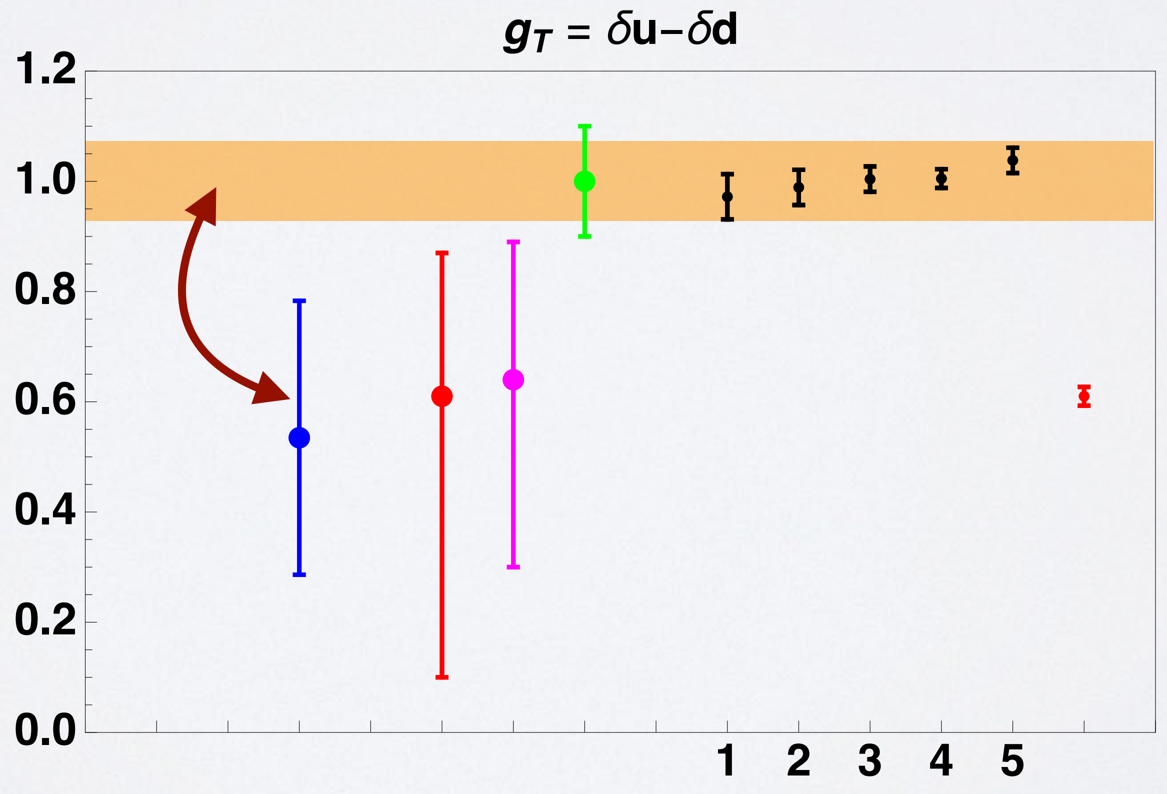
$$\delta q = \int_0^{x_{\min}} dx\, h_1^{q-\bar{q}} + \int_{x_{\min}}^{x_{\max}} dx\, h_1^{q-\bar{q}} + \int_{x_{\max}}^{1} dx\, h_1^{q-\bar{q}}$$

constraining "pheno g_T" with "lattice g_T"
as **JAM** Collaboration did ?

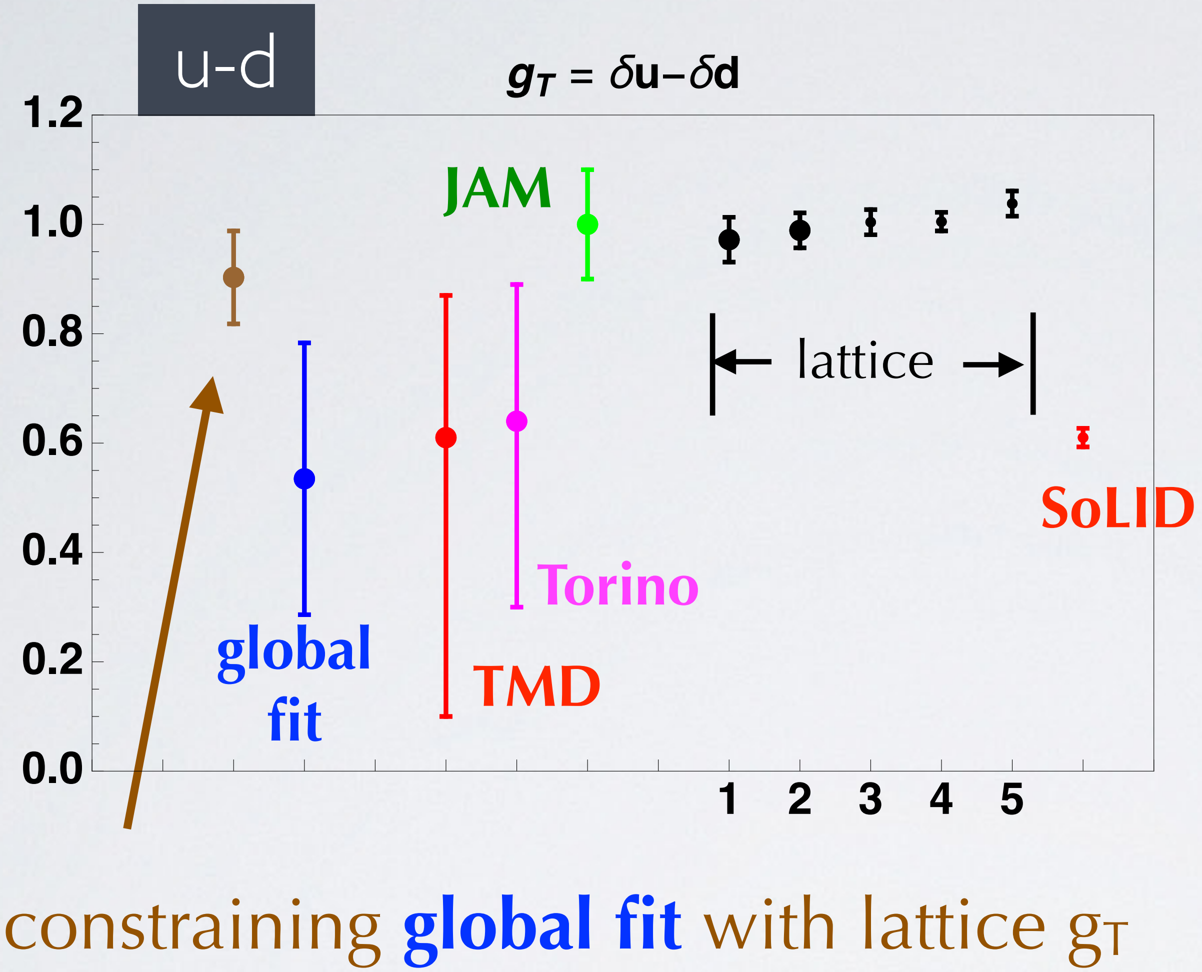
*P.R.L. **120** (18) 152502, arXiv:1710.09858*

$\overline{g_T}^{\text{latt}} = 1.004 \pm 0.057$

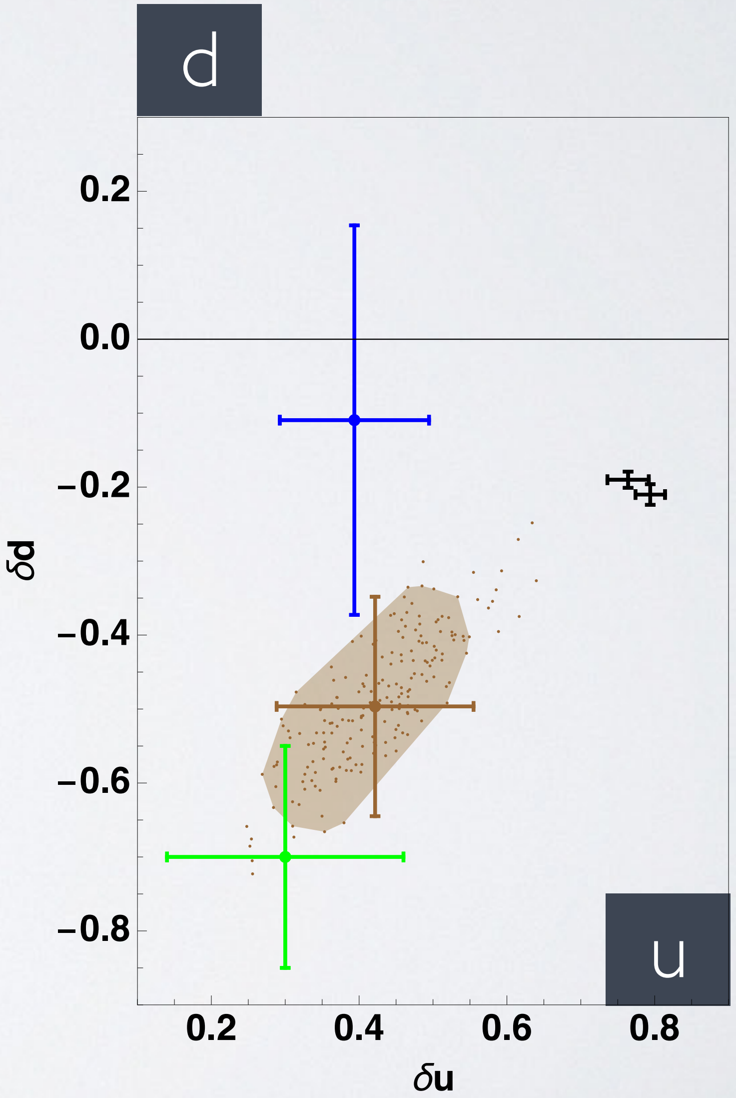
are they compatible?



Constraining our global fit with “lattice g_T”



constraining **global fit** with lattice g_T



confirm JAM results:
constraining “pheno g_T” with “lattice gT”
at the price of
incompatibility for δu and δd

Tension "pheno" - "lattice"

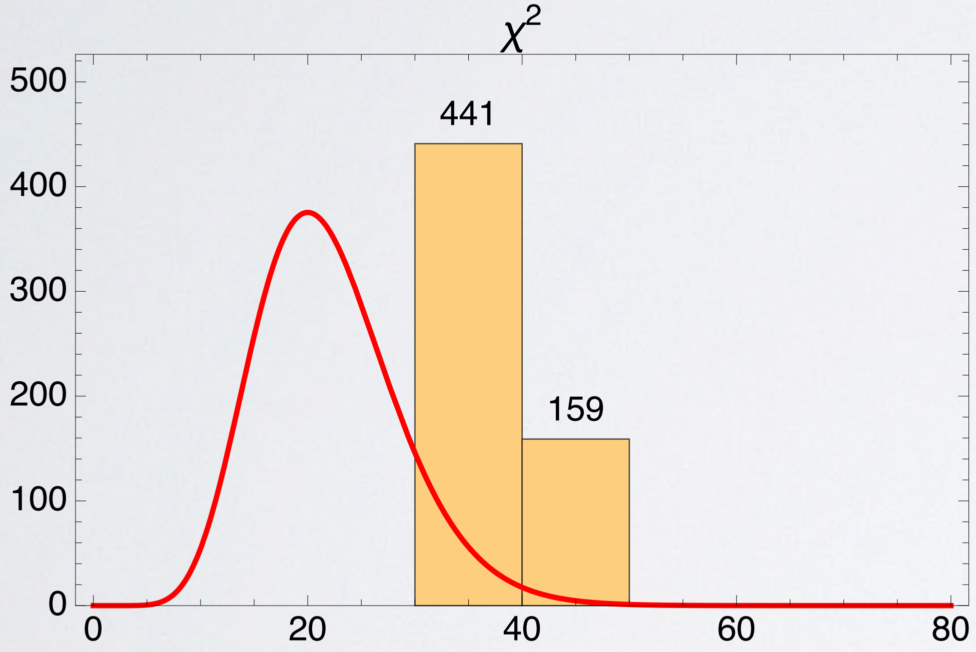
if we constrain our **global fit** with lattice results for all components of tensor charge (up, down, isovector) the χ^2 clearly deteriorate

$\overline{g_T}^{\mathrm{latt}} = 1.004 \pm 0.057$

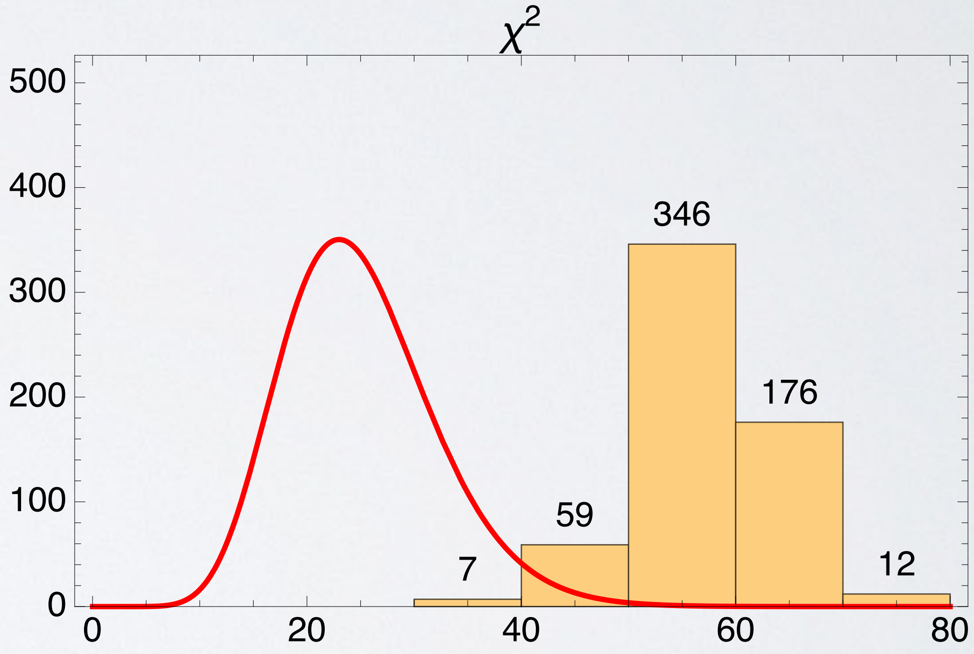
$\overline{\delta u}^{\mathrm{latt}} = 0.782 \pm 0.031$

$\overline{\delta d}^{\mathrm{latt}} = -0.218 \pm 0.026$

$\chi^2/\mathrm{dof} = 1.76 \pm 0.11$



$\chi^2/\mathrm{dof} = 2.29 \pm 0.25$



22 d.o.f.

probability density function of
χ^2 distribution for

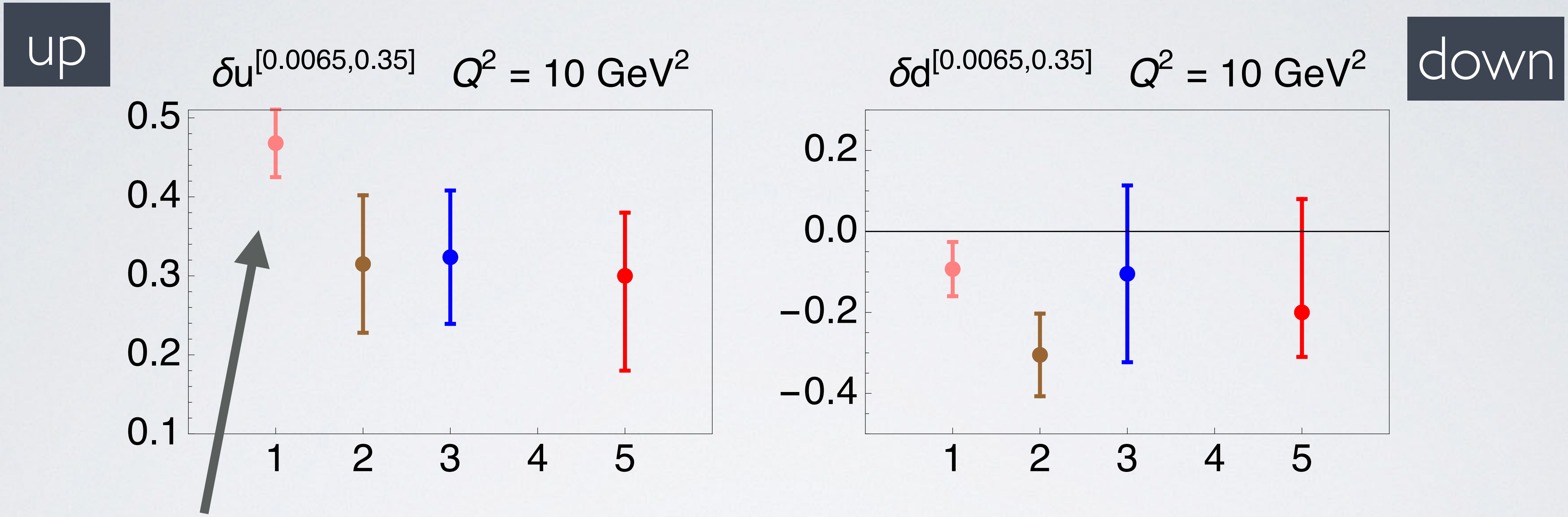
25 d.o.f.

statistically very unlikely ….

truncated tensor charge

truncated

$\delta q^{[0.0065,0.35]} \quad Q^2 = 10$



expect stability
when integrating
on x-range of
exp. data…

1) global fit + constrain g_T , δu , δd

2) global fit + constrain g_T

3) global fit '17 *Radici & Bacchetta, P.R.L.* ***120*** *(18) 192001*

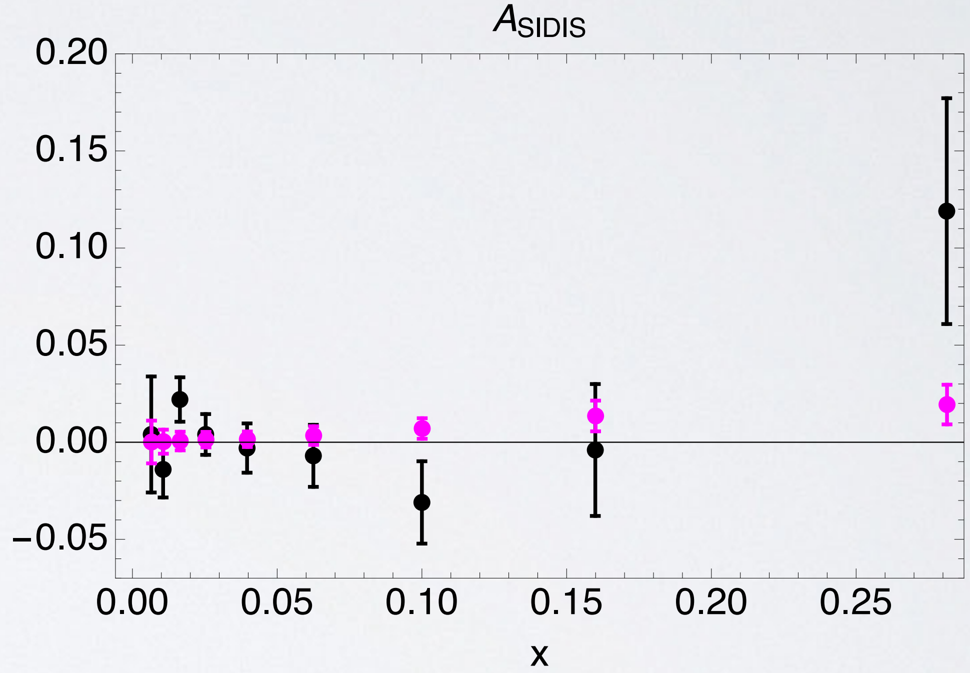
5) "TMD fit" *Kang et al., P.R. D**93** (16) 014009*

Compass pseudo-data

add to data of our global fit
a new set of SIDIS pseudo-data for **deuteron** target

● *Adolph et al., P.L.* ***B713*** *(12)*

● pseudodata
arXiv:1812.07281

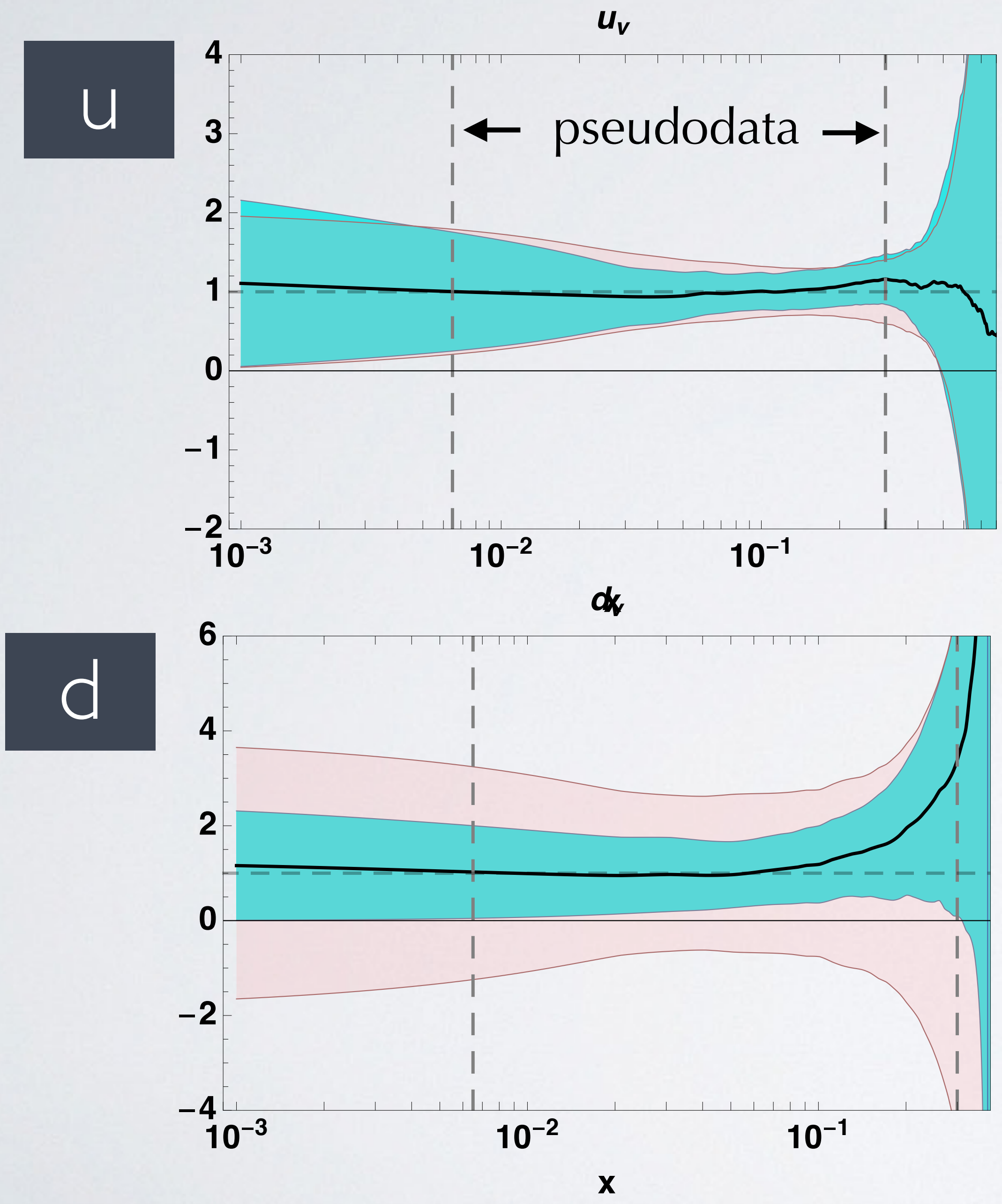


statistical error ~ 0.6 x [error in 2010 proton data]
<A> = average value of replicas in previous global fit

study impact on precision of previous global fit

Adding Compass pseudodata

range [0.0065, x , 0.28]



$$\left[\frac{h_{1\mathrm{min}}}{\langle h_1\rangle_{\substack{\text{global}\\ \text{fit}}}} \quad \frac{\langle h_1\rangle}{\langle h_1\rangle_{\substack{\text{global}\\ \text{fit}}}} \quad \frac{h_{1\mathrm{max}}}{\langle h_1\rangle_{\substack{\text{global}\\ \text{fit}}}} \right]$$

deuteron target
→ better precision on down

global fit

global fit + pseudodata

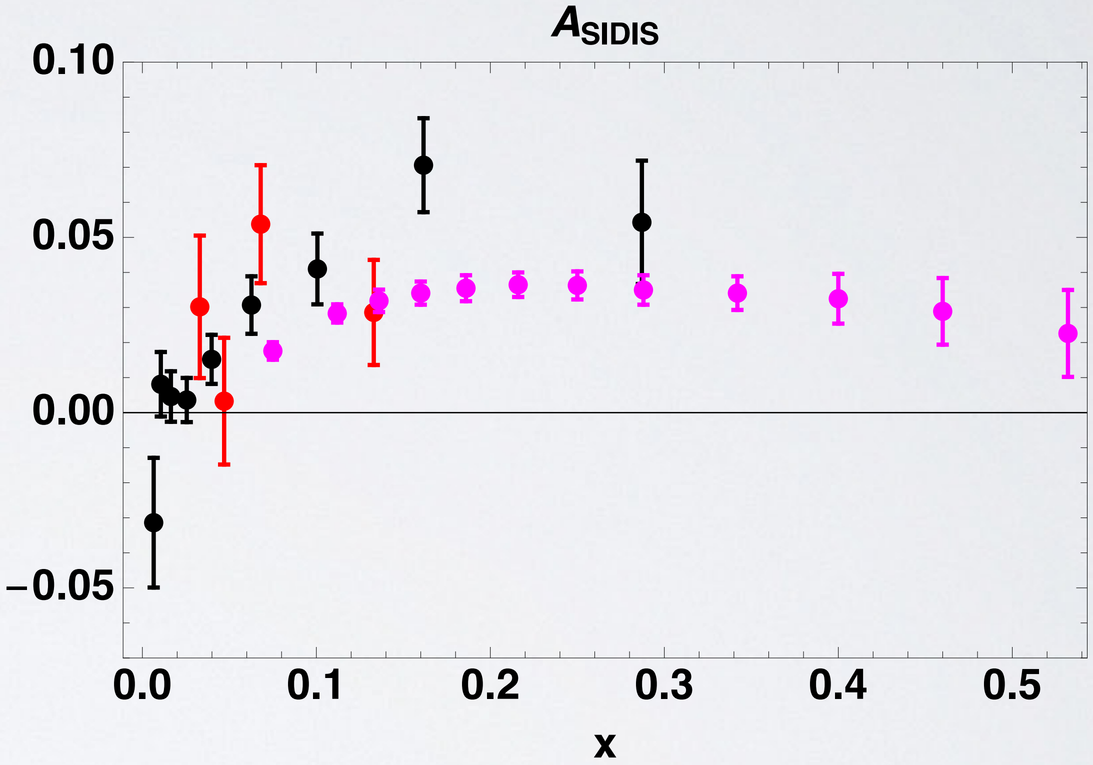
CLAS12 pseudo-data

add to data of our global fit
a new set of SIDIS pseudo-data for **proton** target

● COMPASS *Adolph et al., P.L.* ***B713*** *(12)*

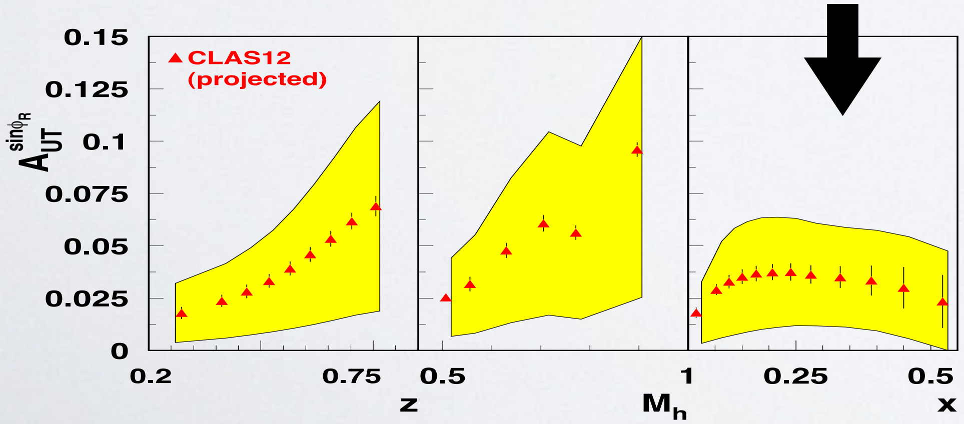
● hermes *Airapetian et al.,*
JHEP ***0806*** *(08) 017*

● clas pseudodata C12-12-009



A 12 GeV Research Proposal to Jefferson Lab (PAC 39)

Measurement of transversity with dihadron production
in SIDIS with transversely polarized target



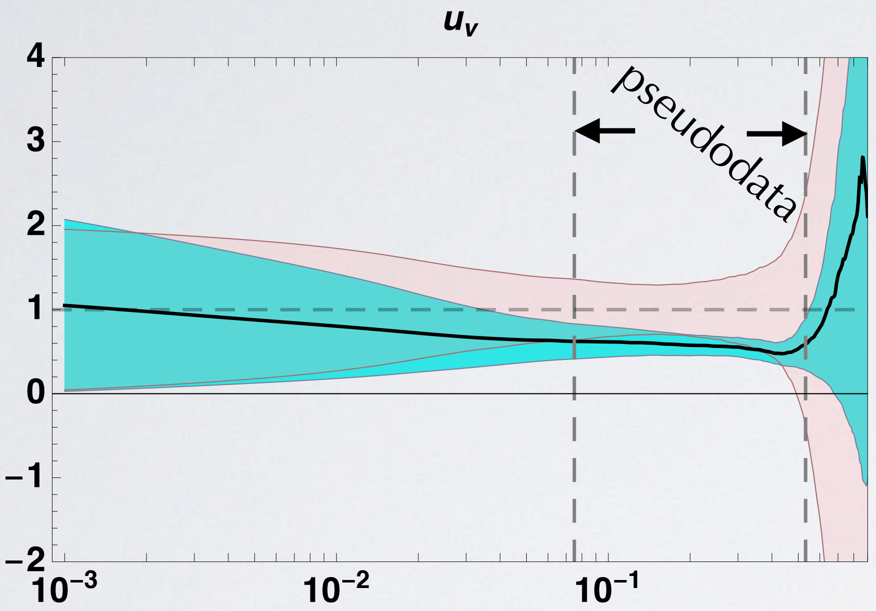
study impact on precision
of published global fit

Adding CLAS12 pseudodata

range [0.075, x , 0.53]

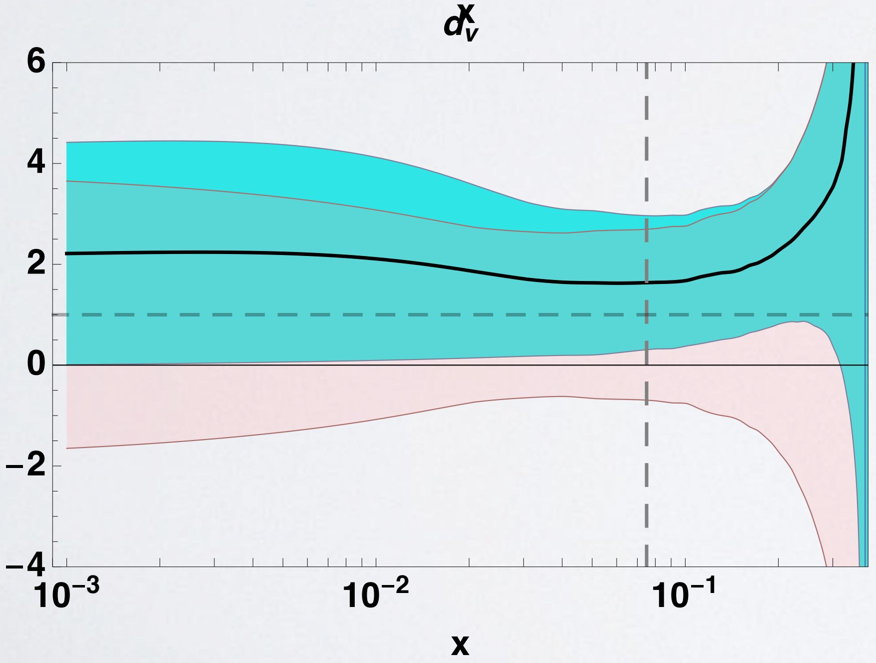
proposal C12-12-009

u



$$\left[\frac{h_{1\mathrm{min}}}{\langle h_1\rangle_{\substack{\text{global}\\ \text{fit}}}} \quad \frac{\langle h_1\rangle}{\langle h_1\rangle_{\substack{\text{global}\\ \text{fit}}}} \quad \frac{h_{1\mathrm{max}}}{\langle h_1\rangle_{\substack{\text{global}\\ \text{fit}}}} \right]$$

d



proton target
→ better precision on up

global fit

global fit + pseudodata

PRELIMINARY

add to data of our global fit
the set of STAR data at s=500 GeV^2

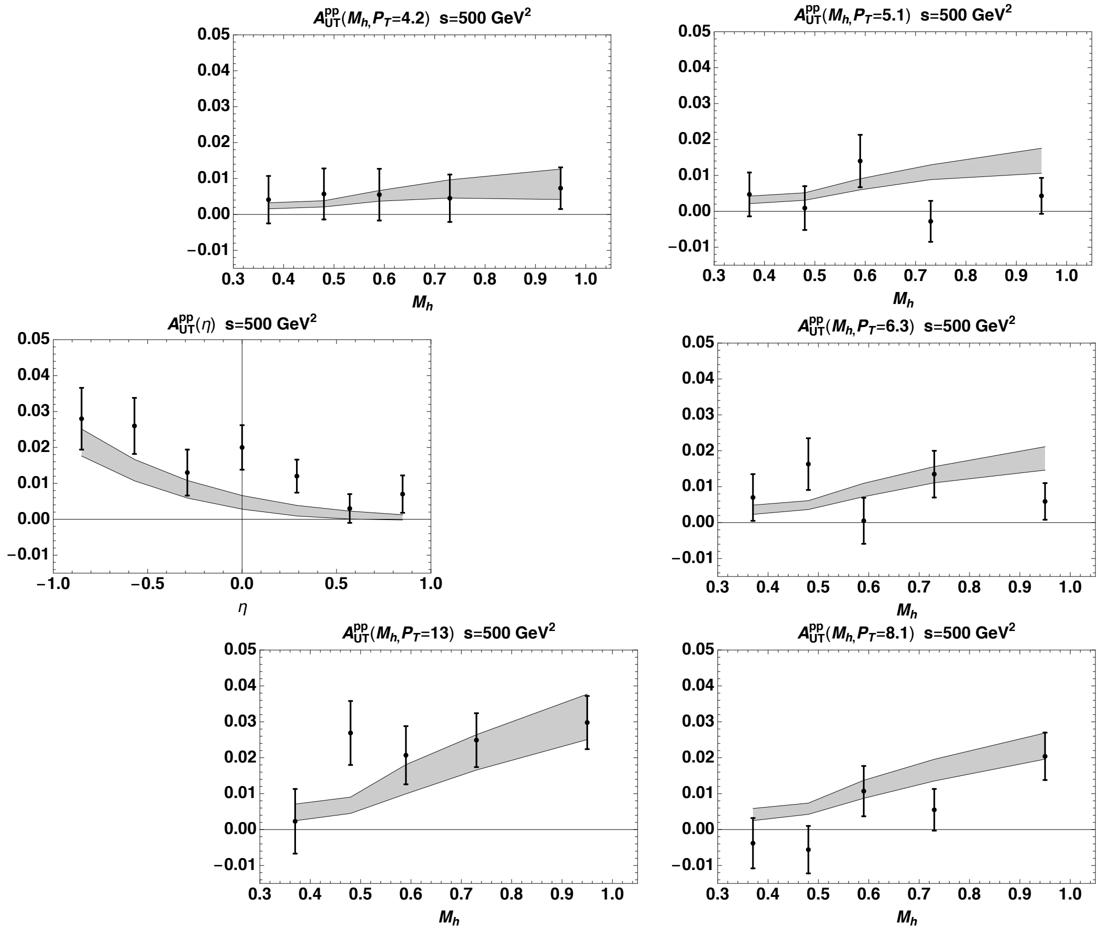
Adamczyk et al. (STAR),
P.L. ***B780*** *(18) 332*

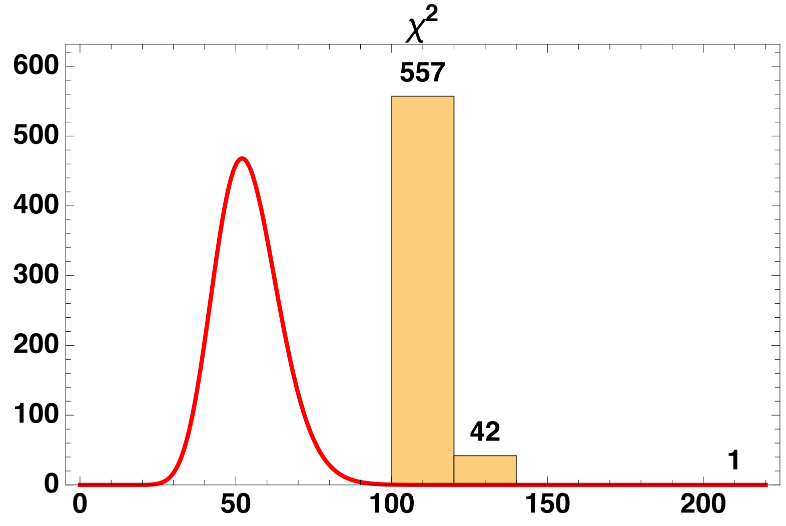
SIDIS **22** data points

pp collisions

run 2006 (s=200 GeV^2) **10** indep. data points

run 2011 (s=500 GeV^2) **32** indep. data points





(for χ^2/dof = 1 perfect overlap)

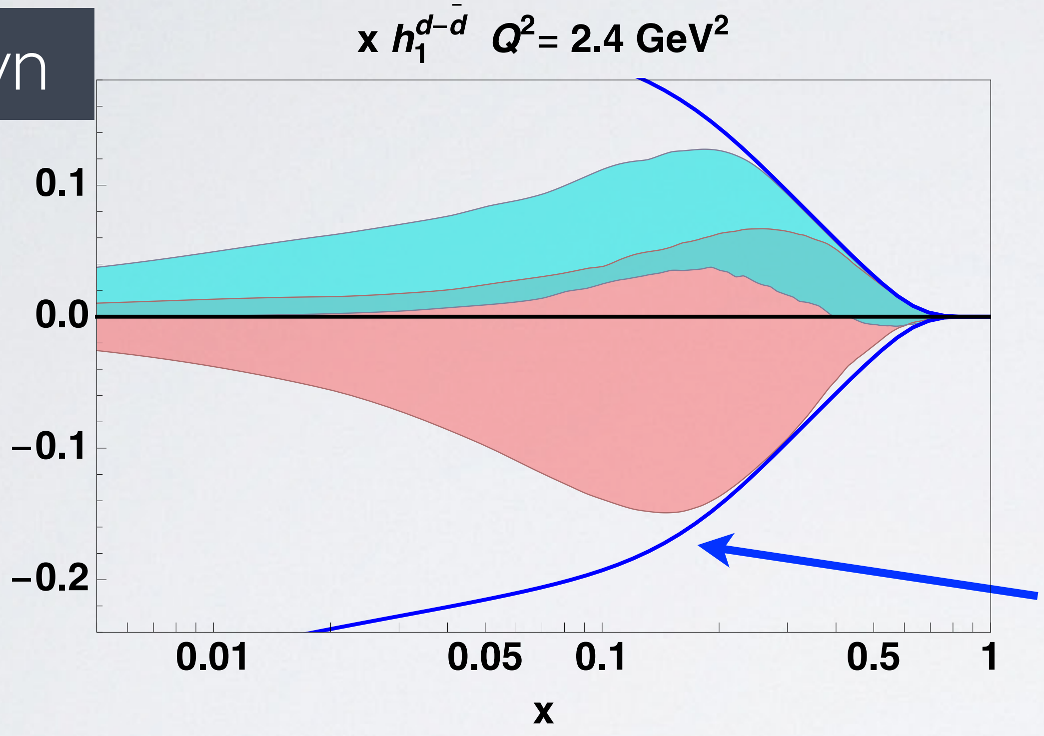
probability density function of
χ^2 distribution for 54 d.o.f.

χ^2/dof = 2.12 ± 0.09

PRELIMINARY

up basically not modified

down

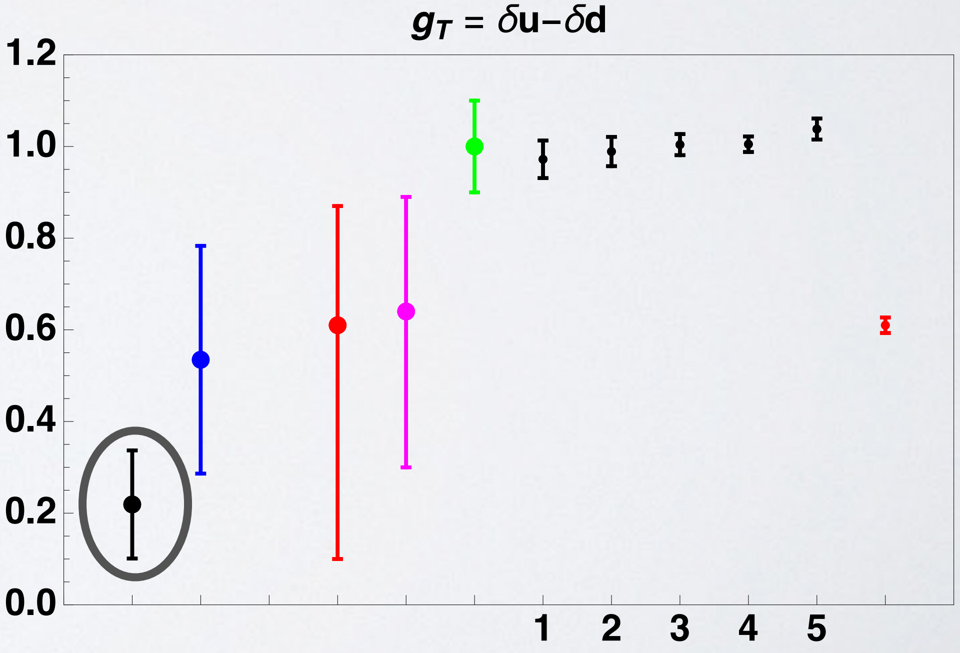


Soffer bound

global fit

Radici and Bacchetta, P.R.L. ***120*** *(18) 192001*

global fit + STAR s=500 data



more precise and fully positive

→ positive δd

→ smaller g_T = δu-δd !

unexpected opposite trend :
need h_1 for sea quarks ?

Conclusions

- **global fit** for h_1 is now possible as for f_1 & g_1

- uncertainty on **gluon channel in pp** collisions drives uncertainty on h_1^{down} , **need data** on **deuteron/^{3}He** and on ($\pi\pi$) **multiplicities**

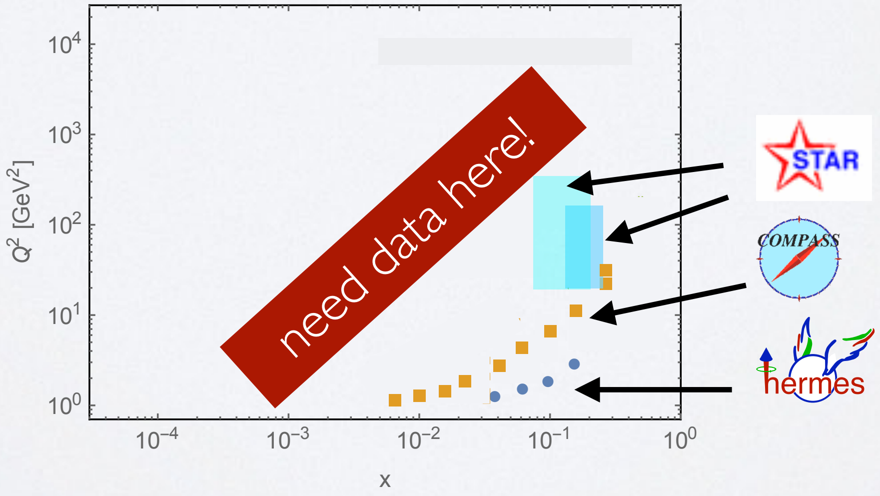
- **NO** simultaneous **compatibility with lattice** for tensor charge in up, down, and isovector channels

- it is possible to **force compatibility** but it is **statistically very unlikely**

- adding **STAR** s=500 data gives puzzling results: need sea quarks?

- adding **Compass** and **CLAS12** SIDIS **pseudodata** increases precision of down and up, respectively

- ultimate resource: **EIC**

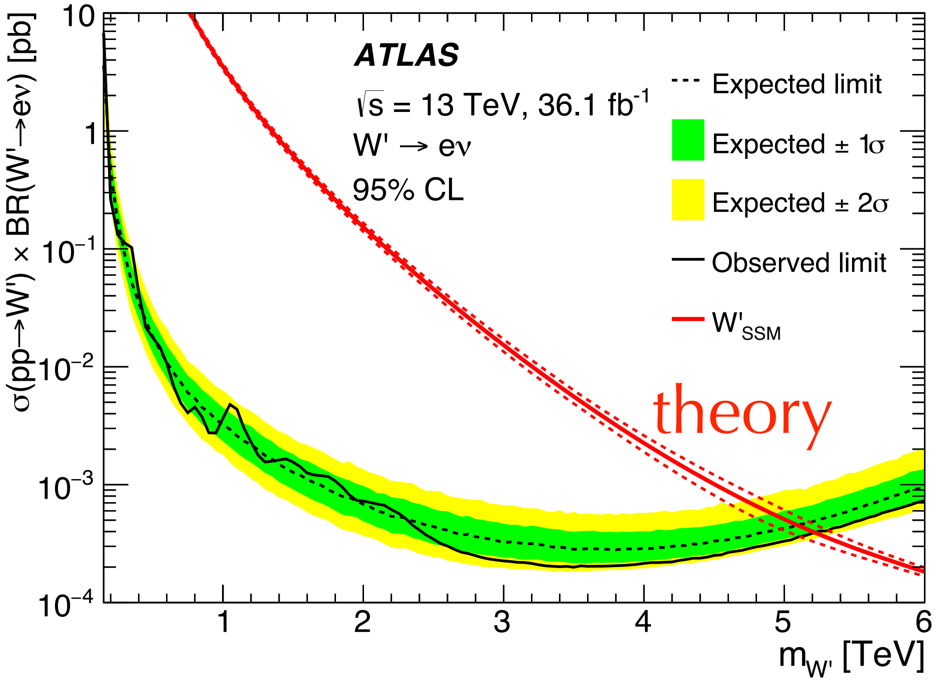


Back-up

Examples of direct access

- **$p\,p \to e^- \nu + X$** search for $W' \to e^- \nu$ with W' heavy partner of W

limits on cross section

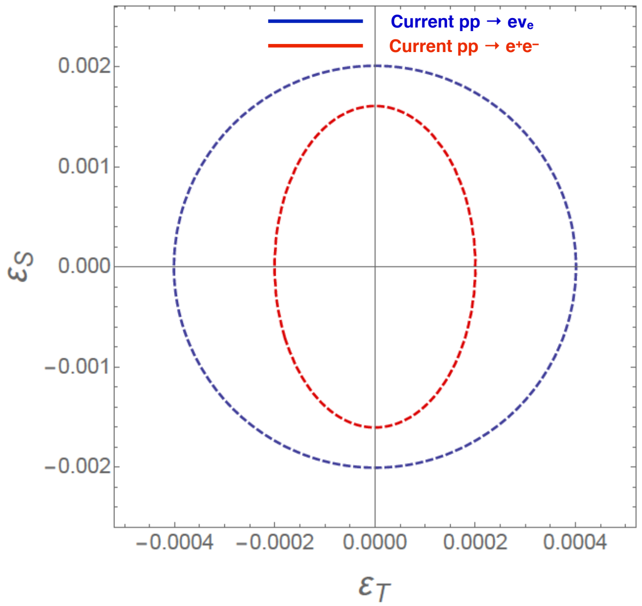


$$\frac{\text{data}}{\text{background}}$$

Aaboud et al. (ATLAS), E.P.J. ***C78*** *(18) 401*

$M_{W'} > 5.1\text{–}5.2$ TeV at 95% C.L.

puts contraints on BSM operators including scalar (ε_S) & tensor (ε_T)



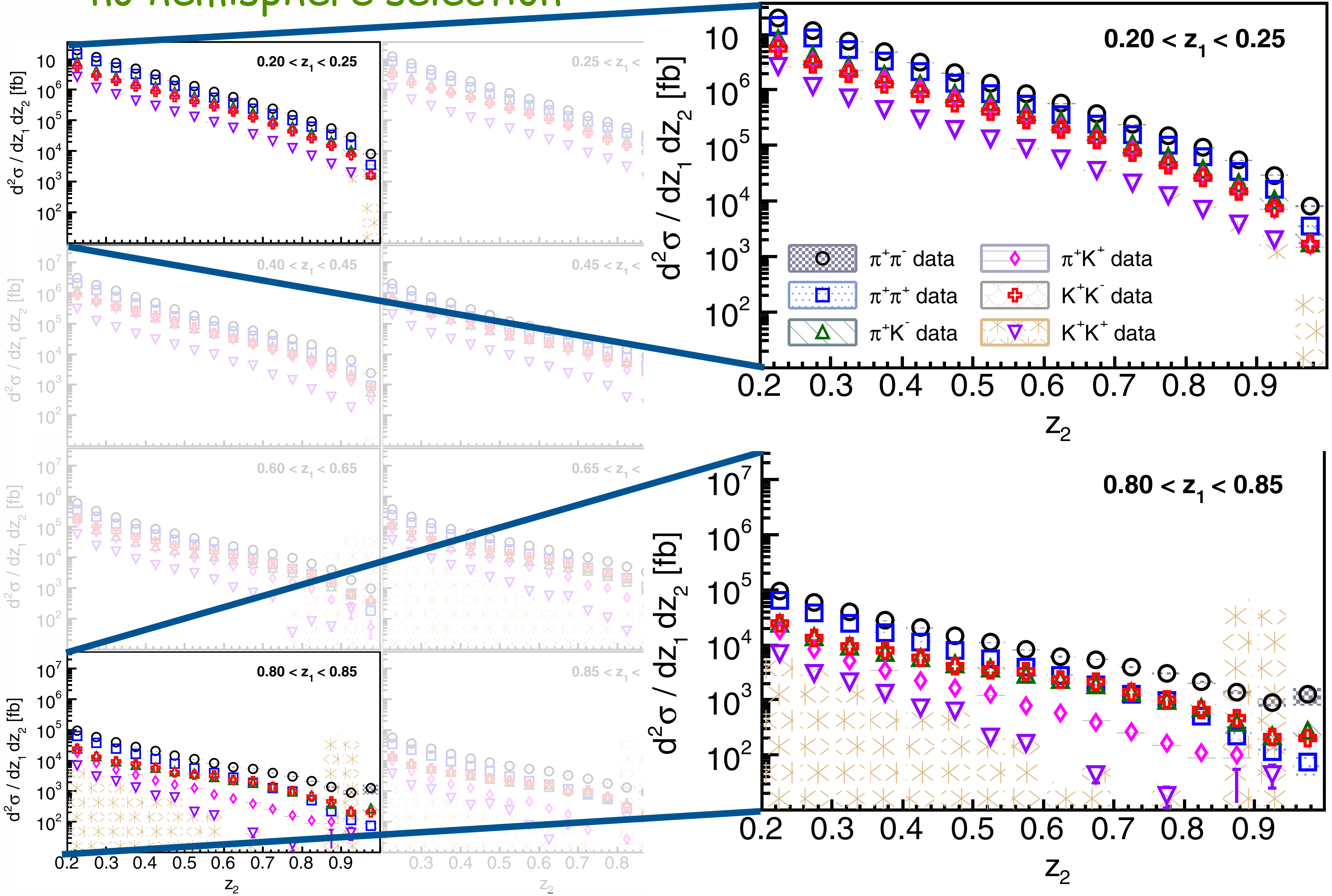
Gupta et al. (PNDME),
*P.R. D**98** (18) 034503*

constraints reinforced including
$p\,p \to Z' \to e^- e^+ + X$

e^+e^- cross section for (hh) from all hemispheres

hadron-pair production

no hemisphere selection



$D_1\ (z_2)$ @low z_1 ≠ $D_1\ (z_2)$ @high z_1

e^+e^- cross section for (hh) in different hemispheres

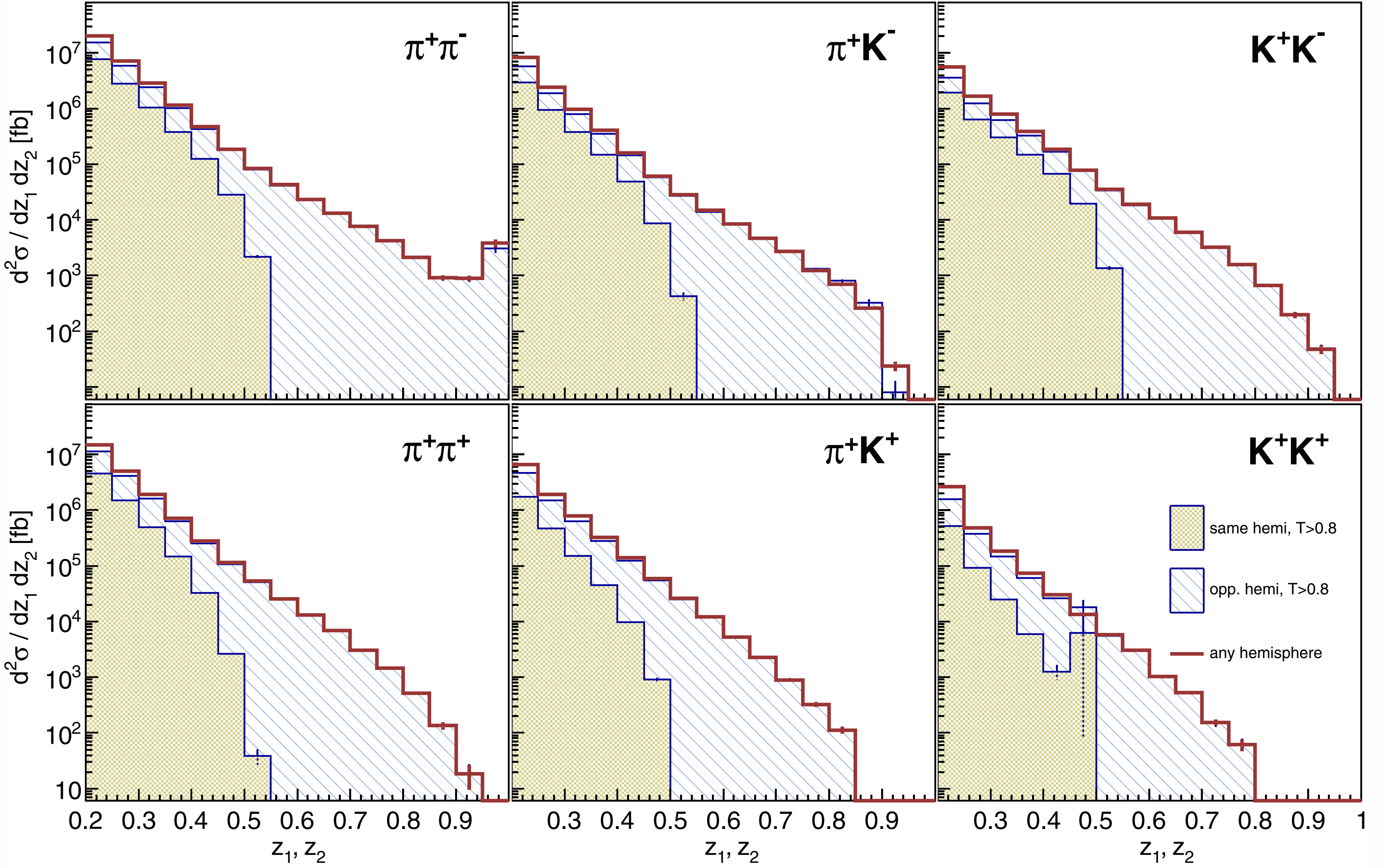
hadron-pairs: topology comparison

- any hemisphere vs. opposite- & same-hemisphere pairs
  - same-hemisphere pairs with kinematic limit at $z_1=z_2=0.5$

[Phys. Rev. D92 (2015) 092007]

opposite hemisphere
$0 < z_1=z_2 < 1$

same hemisphere
$0 < z_1+z_2 < 1$
$0 < z_1=z_2 < 0.5$



gunar.schnell @ desy.de 18 FF18 - Feb 20th, 2018

$$\mathrm{DiFF}\,(z_1, z_2) \neq D_1(z_1)\; D_1(z_2)$$

hadronic collisions in Mellin space

dσ (η,M_h,P_T) typical cross section for $a+b^\uparrow \to c^\uparrow+d$ process

$$\frac{d\sigma_{UT}}{d\eta} \propto \int d|\mathbf{P}_T| dM_h \sum_{a,b,c,d} \int \frac{dx_a dx_b}{8\pi^2 \bar{z}} f_1^a(x_a) \boxed{h_1^b(x_b)} \frac{d\hat{\sigma}_{ab^\uparrow \to c^\uparrow d}}{d\hat{t}} H_1^{\sphericalangle\, c}(\bar{z}, M_h)$$

to be computed thousands times… usual trick: use **Mellin anti-transform**

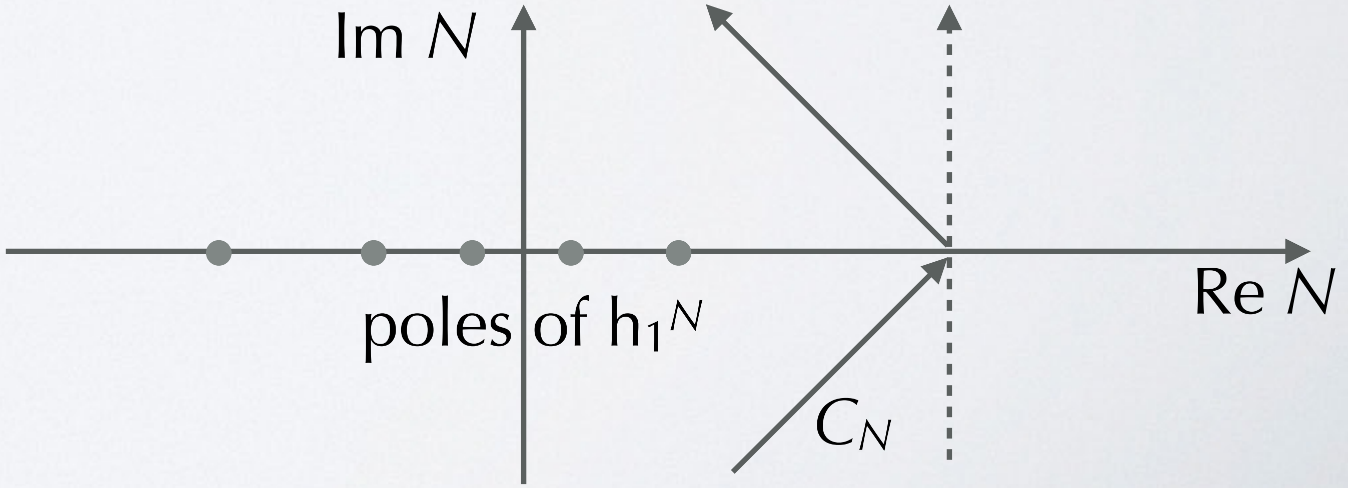
$$h_1(x, Q^2) = \int_{\mathcal{C}_N} dN \; x^{-N} \; h_1^N(Q^2) \qquad N \in \mathbb{C}$$

Stratmann & Vogelsang,
*P.R. D**64** (01) 114007*

$$\frac{d\sigma_{UT}}{d\eta} \propto \sum_b \boxed{\int_{\mathcal{C}_N} dN} \int d|\mathbf{P}_T| \boxed{h_{1b}^N(P_T^2)} \int dM_h \underbrace{\sum_{a,c,d} \int \frac{dx_a dx_b}{8\pi^2 \bar{z}} f_1^a(x_a)\, x_b^{-N} \frac{d\hat{\sigma}_{ab^\uparrow \to c^\uparrow d}}{d\hat{t}} H_1^{\sphericalangle\, c}(\bar{z}, M_h)}_{F_b(N, \eta, |\mathbf{P}_T|, M_h)}$$

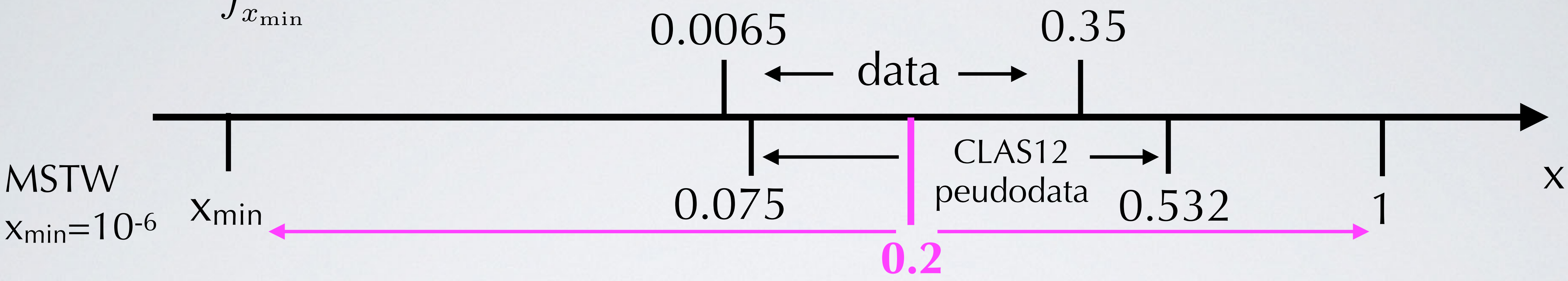
pre-compute F_b only one time
on contour C_N

this **speeds up** convergence
and facilitates ∫ d*N* , provided
that **h_1^N is known analytically**



break down of Mellin moment

$$\delta q(Q^2) = \int_{x_{\rm min}}^{1} dx\, h_1^{q-\bar{q}}(x, Q^2)$$



0.0065 ← data → 0.35

0.075 ← CLAS12 peudodata → 0.532

MSTW

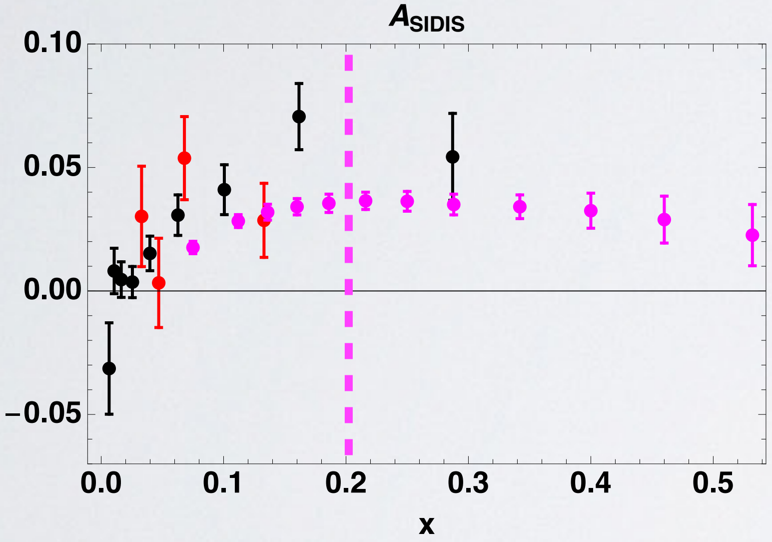
xmin=10^{-6}

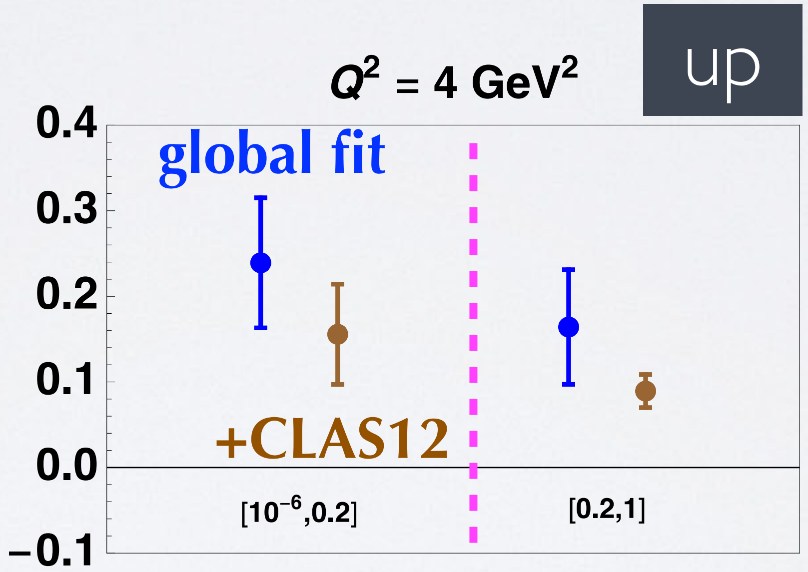
xmin

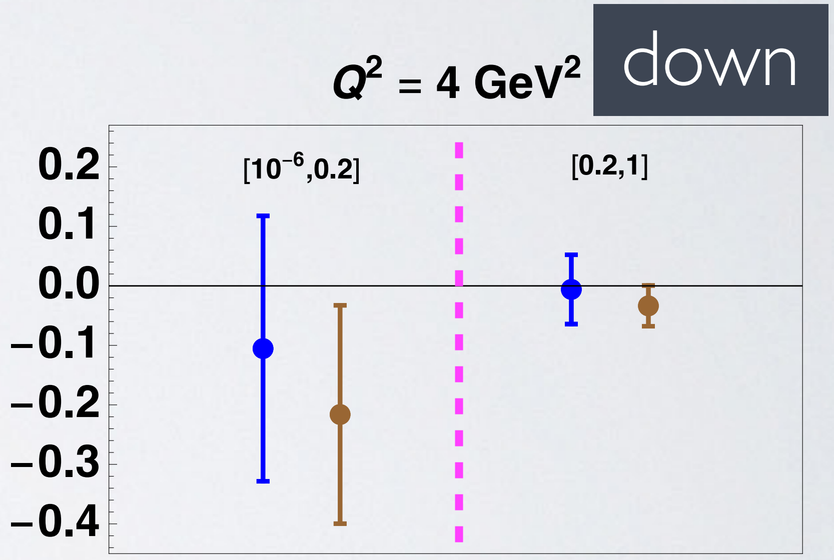
1

x

0.2







impact of CLAS12 pseudodata at large x (>0.2)
gives ~50% of up tensor charge
relative error $\Delta g_T/g_T$ from 82% → 43%