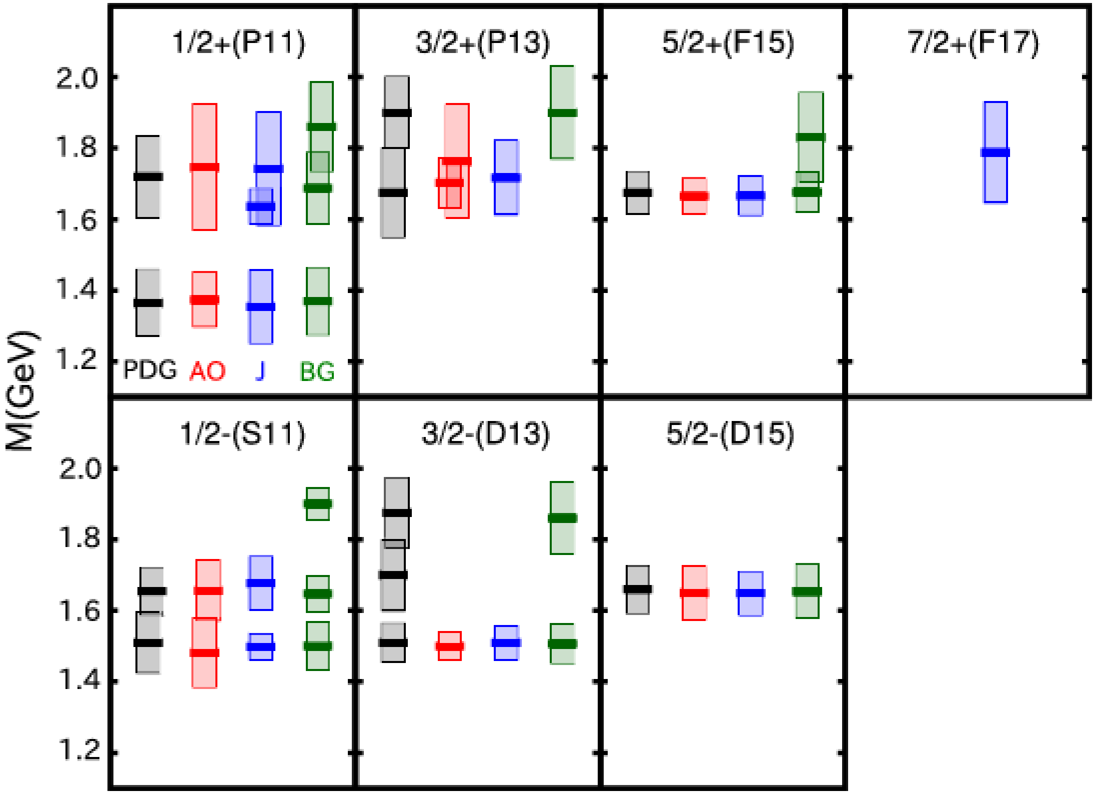
Applications of Strong QCD

Craig Roberts

Strong Interactions in the Standard Model of Particle Physics

- Extract from spectrum of nucleon states (resonances) with mass less-than 2GeV
- Experiment (PDG) compared with theory (AO, J, BG)
- Theory results are outcome of massive computational effort, analysing *22,348 independent data points*, representing *complete array of partial waves*



Nature's scale for visible, strongly-interacting matter = 1 GeV = 1.783×10^{-27} kg ≈ 2000 × m$_e$

Strong Interactions in the Standard Model of Particle Physics

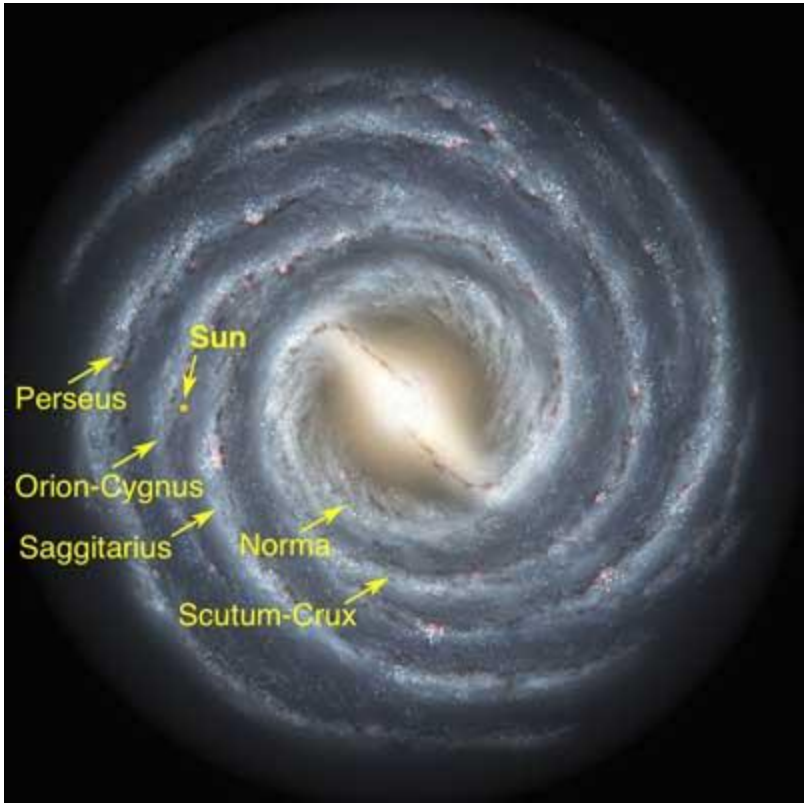
- Extract from spectrum of nucleon sta
  (reso
  mass
- Exper
  comp
  (AO, J
- Theory
  outcon
  compu
  analysi
  indepen
  represe
  array of partial waves

Nature's scale for visible, strongly-interacting matter = 1 GeV = 1.783×10^{-27} kg ≈ 2000 × m$_e$

Emergent Phenomena in the Standard Model

Existence of the Universe as we know it depends critically on the following empirical facts:

- Proton is massive, *i.e*. the mass-scale for strong interactions is vastly different to that of electromagnetism
- Proton is absolutely stable, despite being a composite object constituted from three valence quarks
- Pion is unnaturally light (not massless, but lepton-like mass), despite being a strongly interacting composite object built from a valence-quark and valence antiquark



Emergence: low-level rules producing high-level phenomena, with enormous apparent complexity

Strong Interactions in the Standard Model

$$\mathcal{L}_{\rm QCD} = \bar{\psi}_i\,(i(\gamma^\mu D_\mu)_{ij} - m\,\delta_{ij})\,\psi_j - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a$$

- Only apparent scale in chromodynamics is mass of the quark field
- Quark mass is said to be generated by Higgs boson.
- In connection with everyday matter, that mass is 1/250th of the natural (empirical) scale for strong interactions,
  viz. more-than two orders-of-magnitude smaller
- Plainly, the Higgs-generated mass is very far removed from the natural scale for strongly-interacting matter
- *Nuclear physics mass-scale* – 1 GeV – is an *emergent feature of the Standard Model*
  - No amount of staring at $L_{\rm QCD}$ can reveal that scale
- Contrast with quantum electrodynamics, *e.g*. spectrum of hydrogen levels measured in units of m_e, which appears in $L_{\rm QED}$

$$\mathcal{L}_{\rm QCD} = \bar{\psi}_i \left(i(\gamma^\mu D_\mu)_{ij} \quad \right) \psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

Whence Mass?

- Classical chromodynamics … non-Abelian local gauge theory
- Remove the current mass … there's no energy scale left
- *No dynamics in a scale-invariant theory*; only kinematics … the theory looks the same at all length-scales … there can be no clumps of anything … *hence bound-states are impossible*.
- *Our Universe can't exist*
- *Higgs boson doesn't solve this problem* …
  - normal matter is constituted from light-quarks
  - the mass of protons and neutrons, the kernels of all visible matter, are 100-times larger than anything the Higgs can produce
- *Where did it all begin?*

  … becomes … Where did it all come from?

Trace Anomaly

- Classically, in a scale invariant theory

  the *energy-momentum tensor must be traceless:* $T_{\mu\mu} \equiv 0$

- Classical chromodynamics is meaningless … must be quantised
- Regularisation and renormalisation of (ultraviolet) divergences introduces a mass-scale

  … *dimensional transmutation*: mass-dimensionless quantities become dependent on a mass-scale, ζ

- $\alpha \to \alpha(\zeta)$ in QCD's (massless) Lagrangian density, $\mathbb{L}(m=0)$

  QCD β function

  Under a scale transformation $\zeta \to e^{\sigma} \zeta$, then $\alpha \to \sigma\, \alpha\beta(\alpha)$

  $$\mathbb{L} \to \sigma\, \alpha\beta(\alpha)\, d\mathbb{L}/d\alpha$$

  $$\Rightarrow \partial_{\mu} D_{\mu} = \delta\mathbb{L}/\delta\sigma = \alpha\beta(\alpha)\, d\mathbb{L}/d\alpha = \beta(\alpha)\, \tfrac{1}{4} G_{\mu\nu} G_{\mu\nu} = T_{\rho\rho} =: \Theta_0$$

  Trace anomaly

- Straightforward, nonperturbative derivation, without need for diagrammatic analysis …

  Quantisation of renormalisable four-dimensional theory forces nonzero value for trace of energy-momentum tensor

$$\mathcal{L}_{\rm QCD} = \bar{\psi}_i \, (i(\gamma^\mu D_\mu)_{ij} \qquad\qquad) \, \psi_j - \frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu}$$

Whence?

- Classical chromodynamics … non-Abelian local gauge theory
- Local gauge invariance; but there is no confinement without a mass-scale
  - Three quarks can still be colour-singlet
  - Colour rotations will keep them colour singlets
  - But they need have no proximity to one another
    … proximity is meaningless in a scale-invariant theory
- Whence mass … equivalent to whence a mass-scale … equivalent to whence a confinement scale
- *Understanding the origin of mass in QCD is quite likely inseparable from the task of understanding confinement.*

Where is the mass?

$$T_{\mu\mu} = \frac{1}{4}\beta(\alpha(\zeta))G^a_{\mu\nu}G^a_{\mu\nu}$$

Trace Anomaly

- Knowing that a trace anomaly exists does not deliver a great deal

  … Indicates only that a mass-scale must exist
- Can one compute and/or understand the magnitude of that scale?
- One can certainly *measure* the magnitude … consider proton:

$$\langle p(P)|T_{\mu\nu}|p(P)\rangle = -P_\mu P_\nu$$

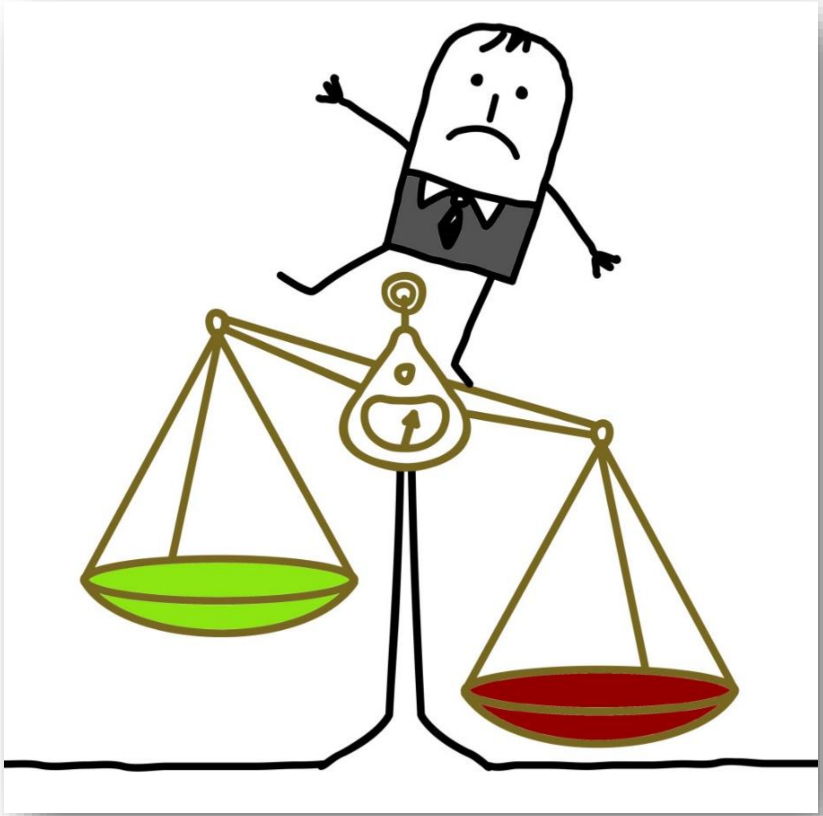
$$\langle p(P)|T_{\mu\mu}|p(P)\rangle = -P^2 = m_p^2$$

$$= \langle p(P)|\Theta_0|p(P)\rangle$$

- In the chiral limit the entirety of the proton's mass is produced by the trace anomaly, Θ_0

  … In QCD, Θ_0 measures the strength of gluon self-interactions

  … so, from one perspective, m_p is completely generated by glue.



On the other hand …

$$T_{\mu\mu} = \frac{1}{4}\beta(\alpha(\zeta))G^a_{\mu\nu}G^a_{\mu\nu}$$

Trace Anomaly

- In the chiral limit

$$\langle\pi(q)|T_{\mu\nu}|\pi(q)\rangle = -q_\mu q_\nu \;\Rightarrow\; \langle\pi(q)|\Theta_0|\pi(q)\rangle = 0$$

- Does this mean that the scale anomaly vanishes trivially in the pion state, *i.e*. gluons contribute nothing to the pion mass?
- Difficult way to obtain "zero"!
- Easier to imagine that "zero" owes to cancellations between different operator contributions to the expectation value of Θ_0.
- Of course, such precise cancellation should not be an accident.

  It could only arise naturally because of some symmetry and/or symmetry-breaking pattern.

Whence "1" and yet "0" ?

$$\langle p(P)|\Theta_0|p(P)\rangle = m_p^2 , \quad \langle \pi(q)|\Theta_0|\pi(q)\rangle = 0$$

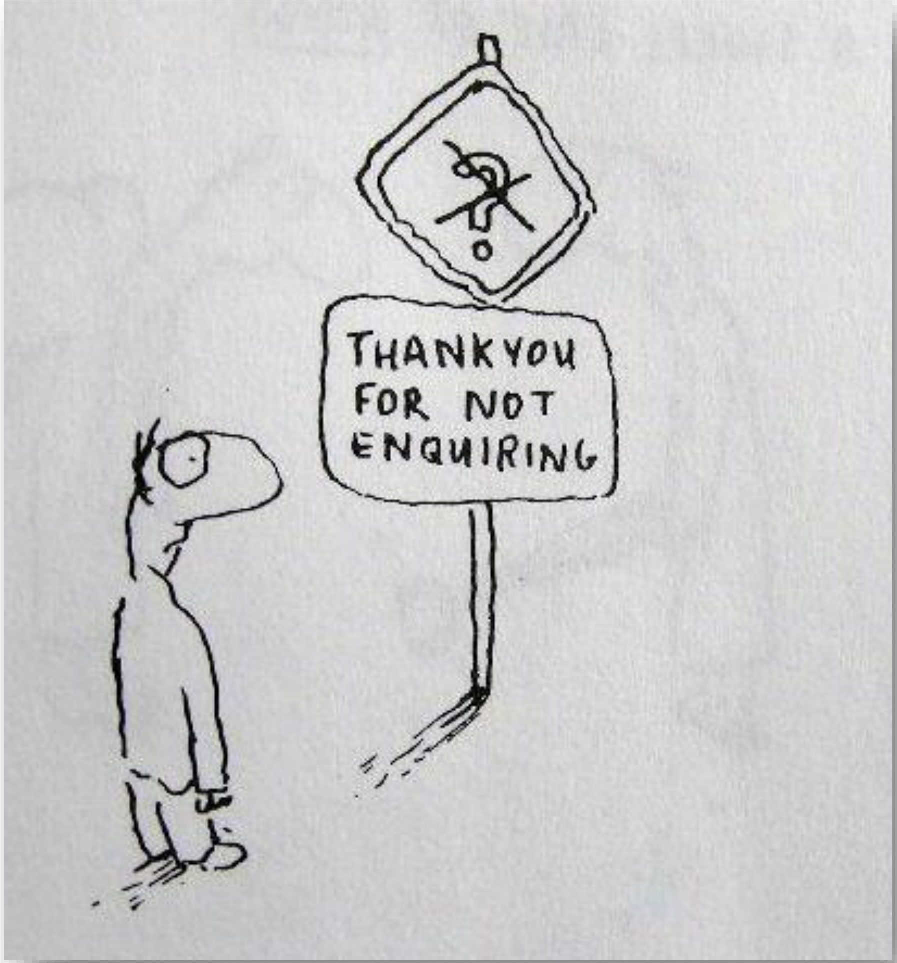
- *No statement of the question*

  "Whence the proton's mass?"

  is complete without the additional clause

  "Whence the **absence** *of a pion mass?"*

- Natural visible-matter mass-scale must emerge simultaneously with apparent preservation of scale invariance in related systems
  - Expectation value of Θ_0 in pion is always zero, irrespective of the size of the natural mass-scale for strong interactions = m_p



Enigma of Mass

Rudimentary version of this relation is apparent in Nambu's Nobel Prize work

Model independent
Gauge independent
Scheme independent

$$f_\pi E_\pi(p^2) = B(p^2)$$

The most fundamental expression of Goldstone's Theorem and DCSB

$f_\pi E_\pi(p^2) = B(p^2)$

Enigma of mass

This algebraic identity is why QCD's pion is massless in the chiral limit

- The quark level Goldberger-Treiman relation shows that DCSB has a very deep and far reaching impact on physics within the strong interaction sector of the Standard Model; viz.,

  Goldstone's theorem is fundamentally an expression of equivalence between the one-body problem and the two-body problem in the pseudoscalar channel.

- This emphasises that Goldstone's theorem has a pointwise expression in QCD
- Hence, pion properties are an almost direct measure of the dressed-quark mass function.
- Thus, enigmatically, the properties of the *massless* pion are the cleanest expression of the mechanism that is responsible for almost all the visible mass in the universe.

$$\langle p(P)|\Theta_0|p(P)\rangle = m_p^2\,, \quad \langle \pi(q)|\Theta_0|\pi(q)\rangle = 0$$

Whence "0" ?

$$\langle p(P)|\Theta_0|p(P)\rangle = m_p^2\,, \quad \langle\pi(q)|\Theta_0|\pi(q)\rangle = 0$$

Whence "0" ?

The answer is algebraic

*Munczek, H. J., Phys. Rev. D **52** (1995) pp. 4736-4740*
*Bender, A., Roberts, C.D. and von Smekal, L., Phys. Lett. B **380** (1996) pp. 7-12*
*Maris, P. , Roberts, C.D. and Tandy, P.C., Phys. Lett. B **420** (1998) pp. 267-273*
*Binosi, Chang, Papavassiliou, Qin, Roberts, Phys. Rev. D **93** (2016) 096010/1-7*

Pion masslessness

- Obtain a coupled set of gap- and Bethe-Salpeter equations

Quantum field theory statement:
In the pseudsocalar channel, the dynamically generated mass of the two fermions is precisely cancelled by the attractive interactions between them – iff –

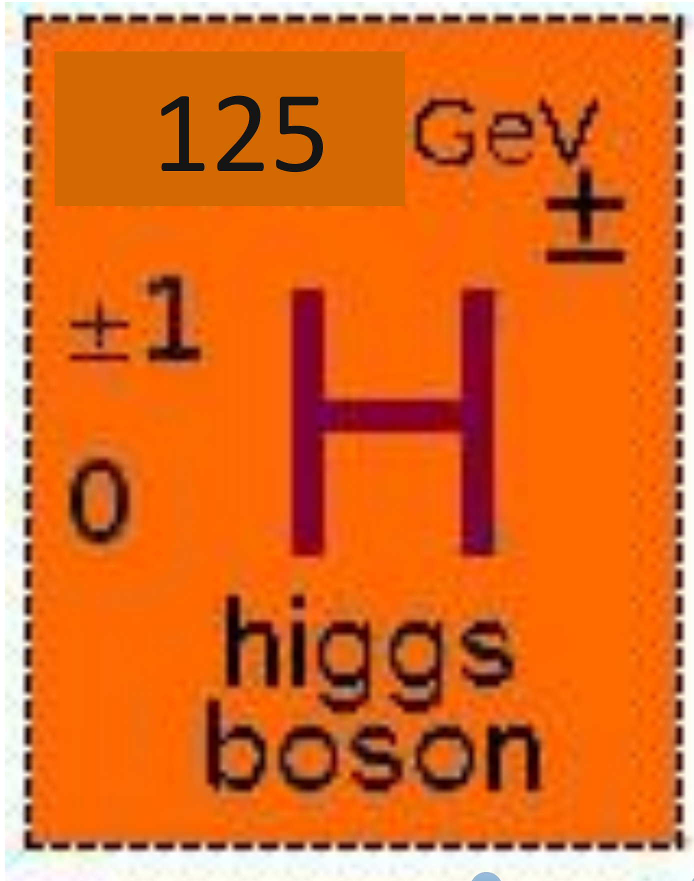
$$f_\pi E_\pi(p^2) = B(p^2)$$

- Cancellation guarantees that

$$\Rightarrow 2 M_q + U_g \equiv 0$$

t $P^2=0$ …

- Interacting, bound system remains massless



Where is it?

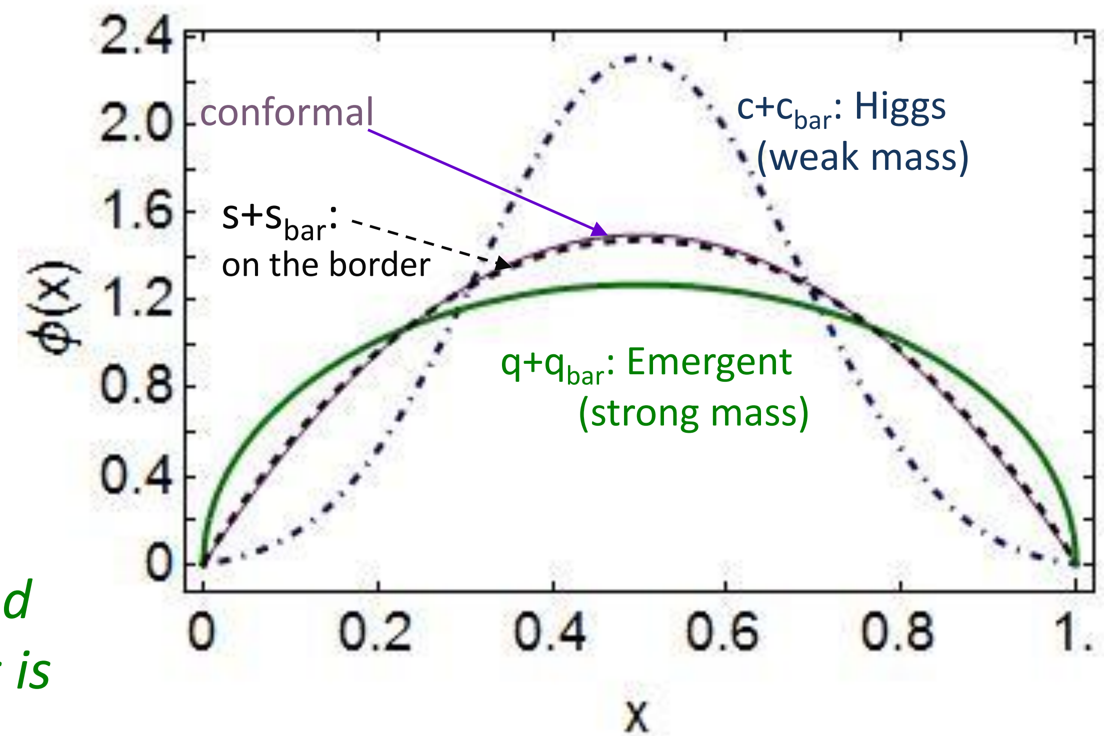
Parton distribution amplitudes of S-wave heavy-quarkonia
Minghui Ding, Fei Gao, Lei Chang, Yu-Xin Liu and Craig D. Roberts
arXiv:1511.04943 [nucl-th], Phys. Lett. B **753** (2016) pp. 330-335

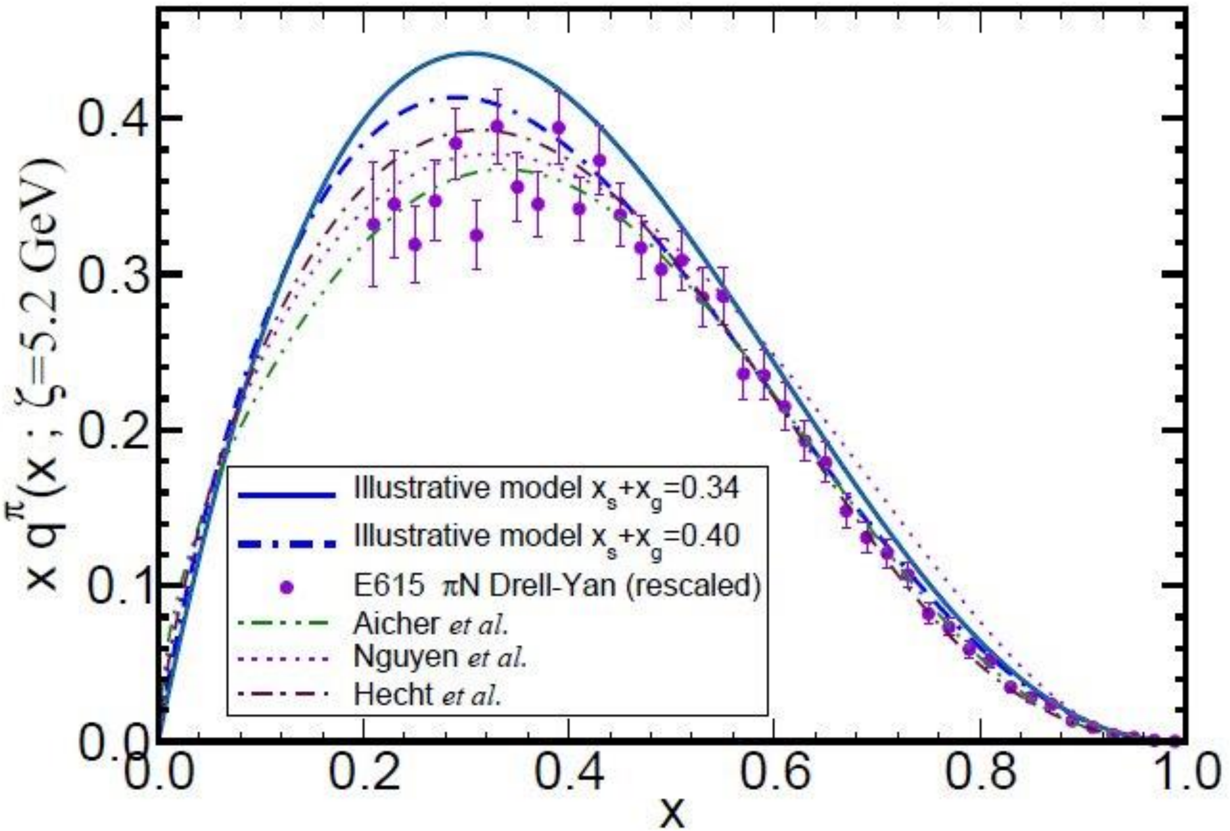
Emergent Mass *vs.* Higgs Mechanism

- When does Higgs mechanism begin to influence mass generation?
- limit $m_{quark} \to \infty$

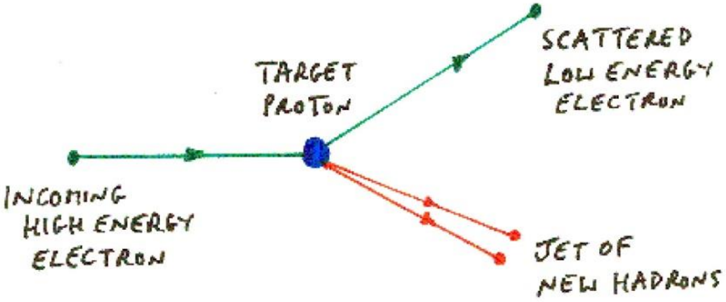
  $\varphi(x) \to \delta(x-½)$
- limit $m_{quark} \to 0$

  $\varphi(x) \sim (8/\pi)\,[x(1-x)]^{½}$
- Transition boundary lies just above $m_{strange}$
- *Comparison between distributions of light-quarks and those involving strange-quarks is good place to seek signals for strong-mass generation*





π & K Valence-quark Distribution Functions

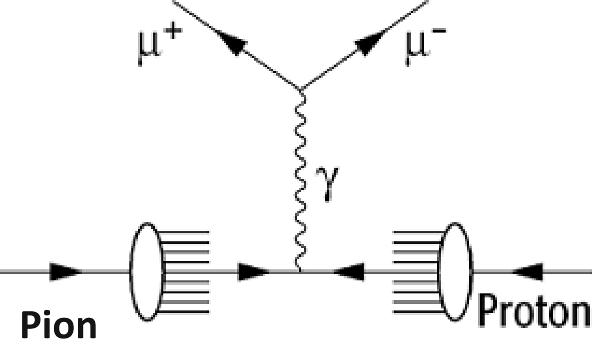


Deep inelastic scattering

- Quark discovery experiment at SLAC
  (1966-1978, Nobel Prize in 1990)
- Completely different to elastic scattering
  - *Blow the target to pieces instead of keeping only those events where it remains intact.*
- Cross-section is interpreted as a measurement of the momentum-fraction probability distribution for quarks and gluons within the target hadron: $q(x)$, $g(x)$

Probability that a quark/gluon within the target will carry a fraction x of the bound-state's light-front momentum

Distribution Functions of the Nucleon and Pion in the Valence Region, Roy J. Holt and Craig D. Roberts, arXiv:1002.4666 [nucl-th], Rev. Mod. Phys. **82** (2010) pp. 2991-3044



Empirical status of the Pion's valence-quark distributions

- Owing to absence of pion targets, the pion's valence-quark distribution functions are measured via the Drell-Yan process:

  $\pi\, p \rightarrow \mu^+\, \mu^-\, X$

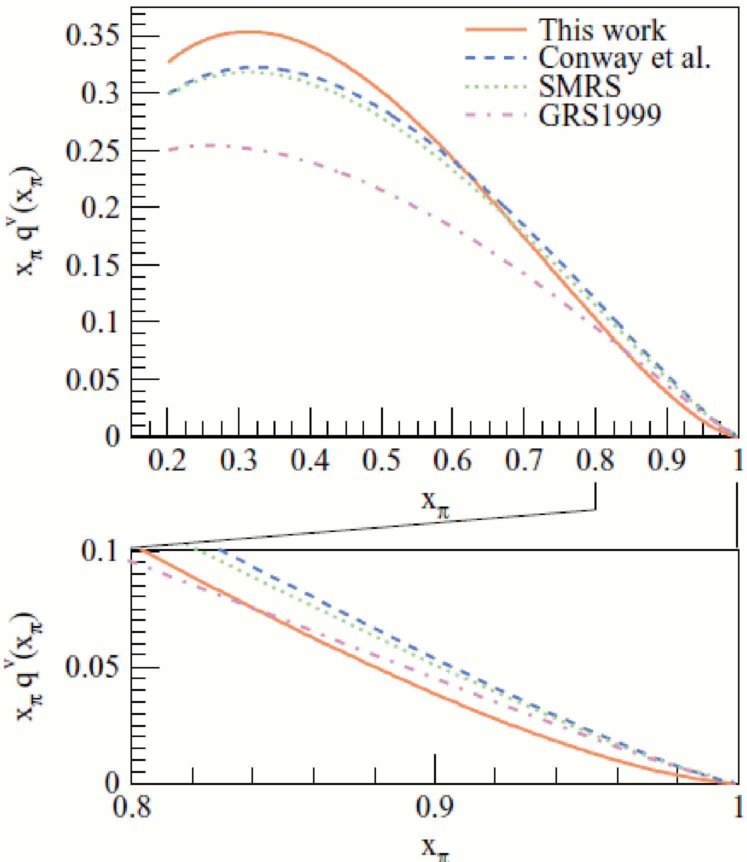
- Three experiments: CERN (1983 & 1985) and FNAL (1989). No more recent experiments because theory couldn't even explain these!
- Problem

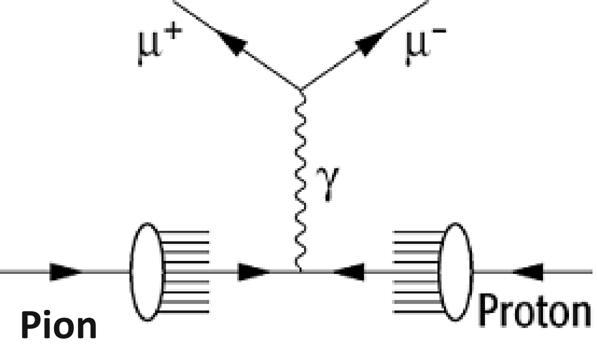
  Conway *et al*. Phys. Rev. D **39**, 92 (1989)
  Wijesooriya *et al*. Phys.Rev. C **72** (2005) 065203

  Behaviour at large-*x* inconsistent with pQCD; viz,

  expt. $(1\text{-}x)^{1+\varepsilon}$

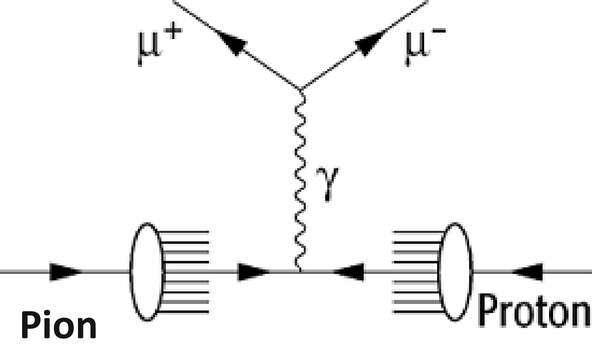
  cf. QCD $(1\text{-}x)^{2+\gamma}$





Models of the Pion's valence-quark distributions

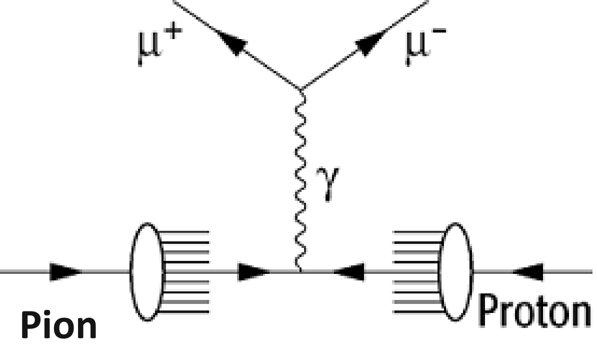
- *$(1-x)^\beta$ with $\beta=0$ (i.e., a* constant – any fraction is equally probable!)
  - Nambu–Jona-Lasinio models, when a translationally invariant regularization is used
- *$(1-x)^\beta$ with $\beta=1$*
  - *Nambu*–Jona-Lasinio NJL models with a hard cutoff
  - AdS/QCD models using light-front holography
  - Duality arguments produced by some theorists
- *$(1-x)^\beta$ with $0<\beta<2$*
  - Relativistic constituent-quark models, with power-law depending on the form of model wave function
- *$(1-x)^\beta$ with $1<\beta<2$*
  - Instanton-based models, all of which have incorrect large-k^2 behaviour



Models of the Pion's valence-quark distributions

- $(1-x)^\beta$ *with* $\beta=0$ *(i.e., a* constant – any fraction is equally probable!)
  - Nambu–Jona-Lasinio models, when a translationally invariant regularization is used
- $(1-x)^\beta$ *with* $\beta=1$
  - Nambu–Jona-Lasinio NJL models with a hard cutoff
  - AdS/QCD models using light-front holography
  - Duality arguments produced by some theorists
- $(1-x)^\beta$ *with* $0<\beta<2$
  - Relativistic constituent-quark models, with power-law depending on the form of model wave function
- $(1-x)^\beta$ *with* $1<\beta<2$
  - Instanton-based models, all of which have incorrect large-k^2 behaviour

Completely unsatisfactory. Impossible to suggest that there's even qualitative agreement!



DSE prediction of the Pion's valence-quark distributions

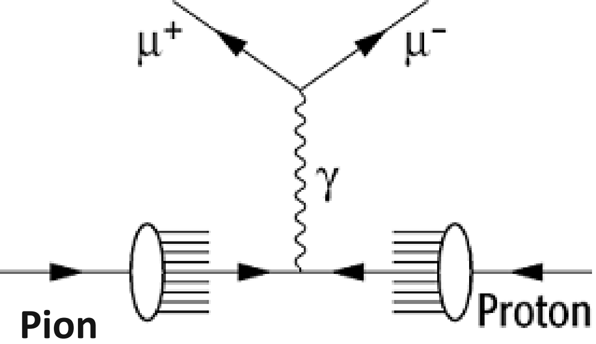
- Consider a theory in which quarks scatter via a vector-boson exchange interaction whose $k^2 >> m_G^2$ behaviour is $(1/k^2)^\beta$,
- Then at a resolving scale Q_0

$$u_\pi(x;Q_0) \sim (1-x)^{2\beta}$$

namely, the large-x behaviour of the quark distribution function is a direct measure of the momentum-dependence of the underlying interaction.

- In QCD, β=1 and hence

$$^{QCD}u_\pi(x;Q_0) \sim (1-x)^2$$



DSE prediction of the Pion's valence-quark distributions

- Consider a theory in which quarks scatter via a vector-boson exchange interaction whose $k^2 >> m_G^2$ behaviour is $(1/k^2)^\beta$,
- Then at a resolving scale Q_0

$$u_\pi(x;Q_0) \sim (1-x)^{2\beta}$$

namely, the large-x behaviour of the quark distribution function is a direct measure of the momentum-dependence of the underlying interaction.

- In QCD, β=1 and hence

$$u_\pi^{QCD}(x;Q_0) \sim (1-x)^2$$

Completely unambigous! Direct connection between experiment and theory, empowering both as tools of discovery.

π & K PDFs

- Extant data on π & K PDFs (mesonic Drell-Yan) is old: 1980-1989
- New data would be welcome:
  - persistent doubts about the Bjorken-$x \simeq 1$ behaviour of the pion's valence-quark PDF
  - single modest-quality measurement of $u^K(x)/u^\pi(x)$ cannot be considered definitive.
- Approved experiment, using tagged DIS at JLab 12, should contribute to a resolution of pion question
  - Similar technique may also serve for kaon and Jlab 12 experiment approved.
- Future:
  - new mesonic Drell-Yan measurements at modern facilities could yield valuable information on π and K PDFs (COMPASS),
  - as could two-jet experiments at the large hadron collider;
  - EIC would be capable of providing access to π and K PDFs through measurements of forward nucleon structure functions.

Basic features of the pion valence-quark distribution function,
Lei Chang, Cédric Mezrag, et al.,
arXiv:1406:5450 [nucl-th], Phys. Lett. B **737** (2014)pp. 23–29

Valence-quark distribution functions in the kaon and pion,
Chen Chen, Lei Chang et al.
arXiv:1602.01502 [nucl-th], Phys. Rev. D**93** (2016) 074021/1-11

Valence-quark PDFs within mesons

- Compute PDFs from imaginary part of virtual-photon – pion forward Compton scattering amplitude:

  $\gamma\, \pi \to \gamma\, \pi$

- Handbag diagram is insufficient. Doesn't even preserve global symmetries. Exists a class of leading-twist corrections that remedies this defect ⇒

Projection onto light-front

$$u_V^\pi(x) = N_c \mathrm{tr} \int_{dk} \delta_n^x(k_\eta^\pi) \times \partial_{k_\eta^\pi} \left[\Gamma_\pi(k_\eta^\pi, -P_\pi) S(k_\eta^\pi)\right] \Gamma_\pi(k_{\bar\eta}^\pi, P_\pi)\, S(k_{\bar\eta}^\pi)\,,$$

Partial derivative wrt relative momentum

Similar expressions for $u_V^K(x)$, $s_V^K(x)$

Measurable quantities
Directly related to
dynamically generated quark masses
& bound-state wave functions

Pion PDF

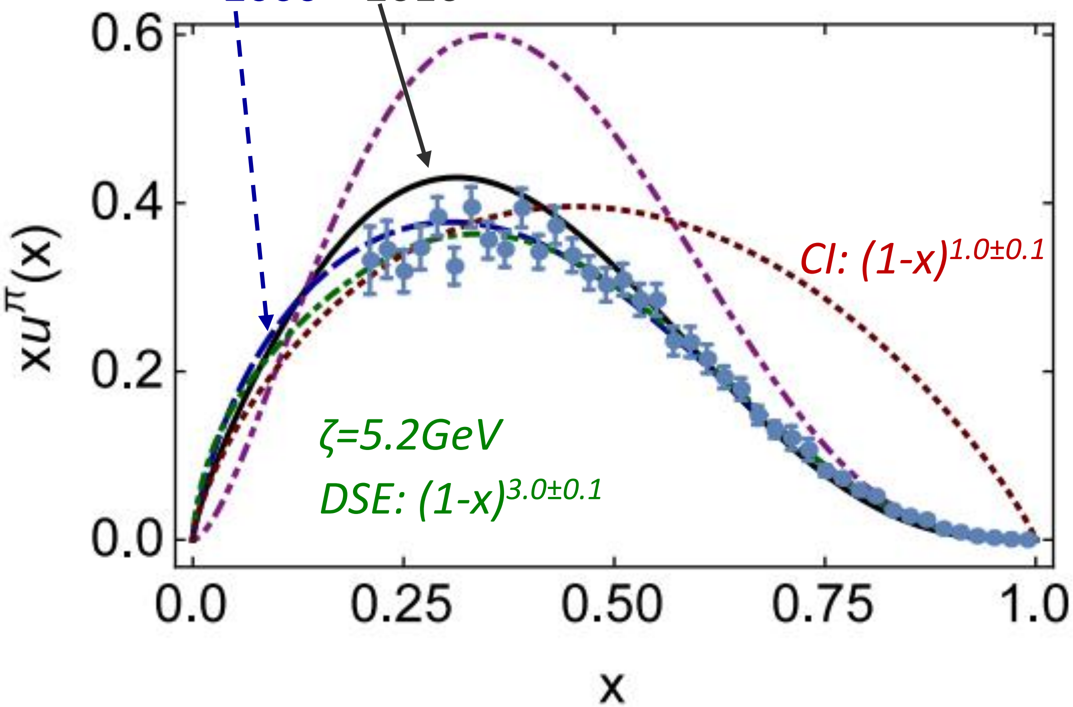


FIG. 3. $xu^{\pi}(x;\zeta_{5.2})$. Solid (black) curve, our prediction, expressed in Eqs. (32), (33); dot-dot-dashed (purple) curve, result obtained when sea-quark and gluon contributions are neglected at ζ_H, *i.e.* using $u_V^{\pi}(x)$ from Eqs. (14), (17); dashed (blue) curve first DSE prediction [38]; and data, Ref. [4], rescaled according to the reanalysis described in Ref. [40], from which the dot-dashed (green) curve is drawn. The dotted (red) curve is the result obtained using a Poincaré-covariant regularisation of a contact interaction, Eq. (36).

- Purple dot-dot-dash = prediction at ζ_H
- Data = modern reappraisal of E615: NLO analysis plus ***soft-gluon resummation*** (ASV)

QCD demands it
& phenomenology should respect that

- Solid black curve, prediction evolved to ζ=5.2GeV, the scale associated with the experiments
- Blue dashed curve = first DSE prediction, in 2000 (ζ=5.2GeV)
- Dotted red curve = result obtained with momentum-independent gluon exchange (contact interaction, ζ=5.2GeV)

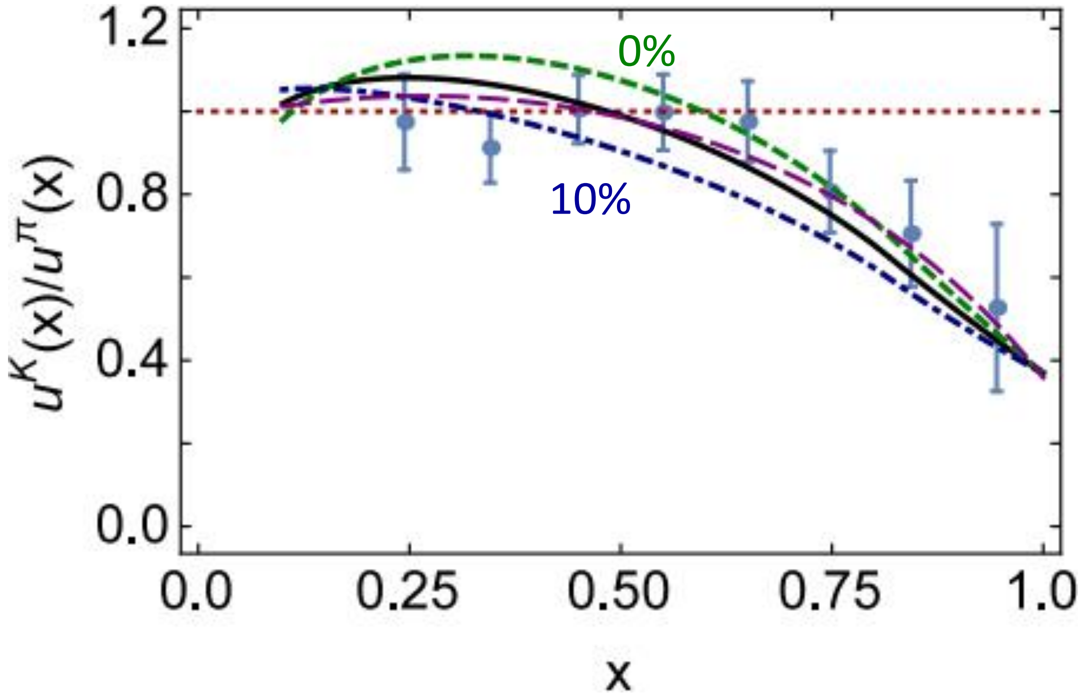
Kaon's gluon content

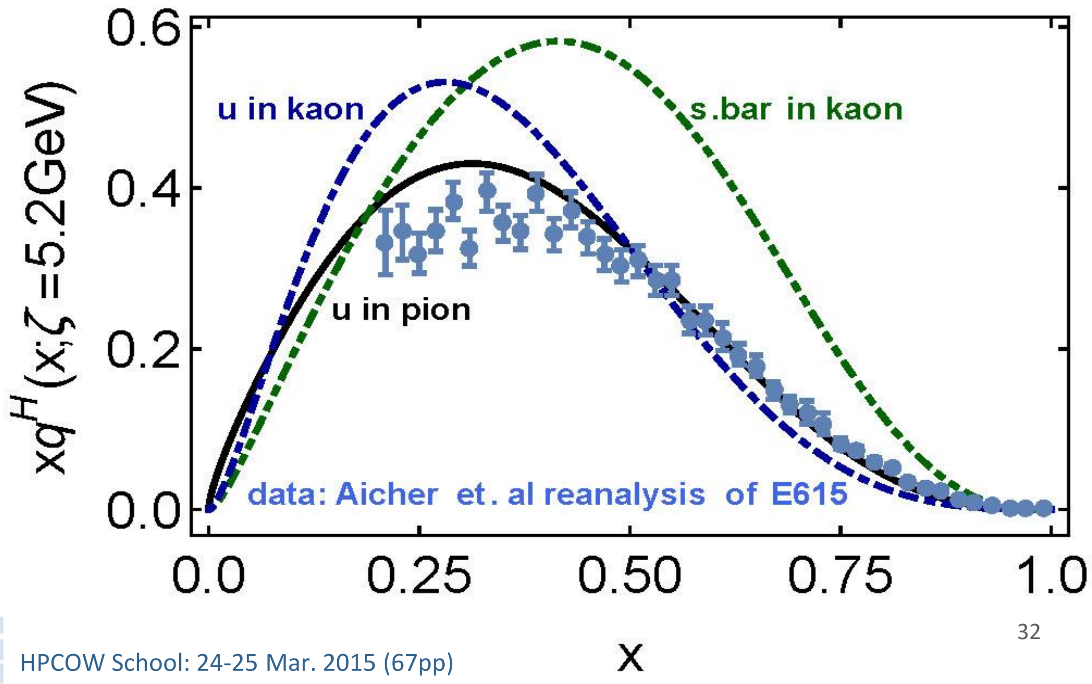
- $\langle x \rangle_g^K(\zeta_H) = 0.05 \pm 0.05$
  $\Rightarrow$ Valence quarks carry 95% of kaon's momentum at ζ_H
- DGLAP-evolved to ζ_2

q	$\langle x \rangle_q^K$	$\langle x^2 \rangle_q^K$	$\langle x^3 \rangle_q^K$
u	0.28	0.11	0.048
$\bar{s}$	0.36	0.17	0.092

Valence-quarks carry ⅔ of kaon's light-front momentum

Cf. Only ½ for the pion





Valence-quark distribution functions in the kaon and pion,
Chen Chen, Lei Chang *et al.*
arXiv:1602.01502 [nucl-th], Phys. Rev. D**93** (2016) 074021/1-11

π & K PDFs

- Marked differences between π & K gluon content
  - ζ_H:
    - Whilst $\frac{1}{3}$ of pion's light-front momentum carried by glue
    - *Only* $\frac{1}{20}$ of the kaon's light-front momentum lies with glue
  - $\zeta_2{}^2 = 4\ \text{GeV}^2$
    - Glue carries $\frac{1}{2}$ of pion's momentum and $\frac{1}{3}$ of kaon's momentum
  - Evident in differences between large-x behaviour of valence-quark distributions in these two mesons
- Signal of Nambu-Goldstone boson character of π
  - Nearly complete cancellation between one-particle dressing and binding attraction in this almost massless pseudoscalar system

$$2\ \text{Mass}_Q + U_g \approx 0$$

Valence-quark distribution functions in the kaon and pion,
Chen Chen, Lei Chang *et al*.
arXiv:1602.01502 [nucl-th], Phys. Rev. D**93** (2016) 074021/1-11

π & K PDFs

- Existing textbook description of Goldstone's theorem via pointlike modes is *simplistic*

Valence-quark distribution functions in the kaon and pion,
Chen Chen, Lei Chang *et al.*
arXiv:1602.01502 [nucl-th], Phys. Rev. D**93** (2016) 074021/1-11

π & *K* PDFs

- The appearance of Nambu-Goldstone modes in the Standard Model is far more interesting
  - Nambu-Goldstone modes are nonpointlike!
  - Intimately connected with origin of mass!
  - Possibly/Probably(?) inseparable from expression of confinement!

- Difference between gluon content of *π* & *K* is measurable … using well-designed EIC
- Write a definitive new chapter in future textbooks on the Standard Model

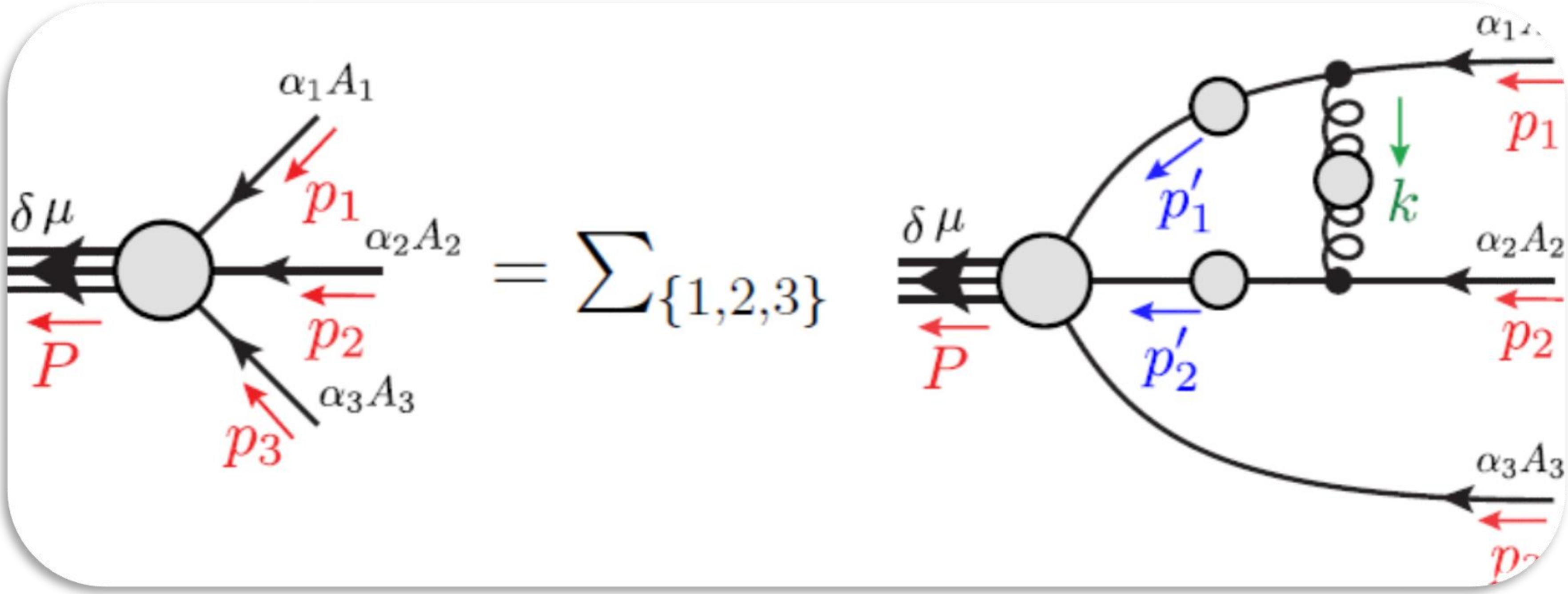
Electron Ion Collider: The Next QCD Frontier

New Challenge

- Three valence-body problem
- Baryons in QCD
  - Three valence quarks
- Spectrum and properties of hybrid and exotic mesons

  exotic mesons: quantum numbers not possible for quantum mechanical quark-antiquark systems
  hybrid mesons: normal quantum numbers but non-quark-model decay pattern
  BOTH suspected of having "constituent gluon" content

  - Valence-quark + valence-antiquark+valence-gluon(?)



Baryons as a 3-valence-body problem

Unification of Meson & Baryon Properties

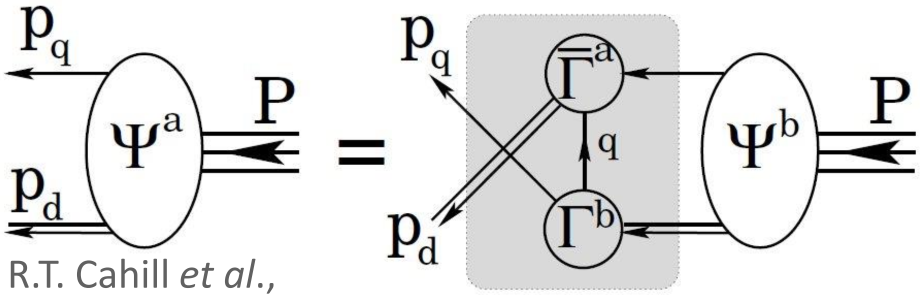
- Correlate the properties of meson and baryon ground- and excited-states within a *single, symmetry-preserving framework*
  - Symmetry-preserving means:

    Poincaré-covariant & satisfy relevant Ward-Takahashi identities

R.T. Cahill *et al.*,
Austral. J. Phys. 42 (1989) 129-145

DSEs & Baryons

- *Dynamical chiral symmetry breaking* (DCSB)
  – has enormous impact on meson properties.
  - *Must be included in description and prediction of baryon properties*.
- *DCSB* is essentially a quantum field theoretical effect. In quantum field theory
  - Meson appears as pole in four-point quark-antiquark Green function → Bethe-Salpeter Equation
  - *Nucleon appears as a pole in a six-point quark Green function → Faddeev Equation*.
- Poincaré covariant Faddeev equation sums all possible exchanges and interactions that can take place between three dressed-quarks

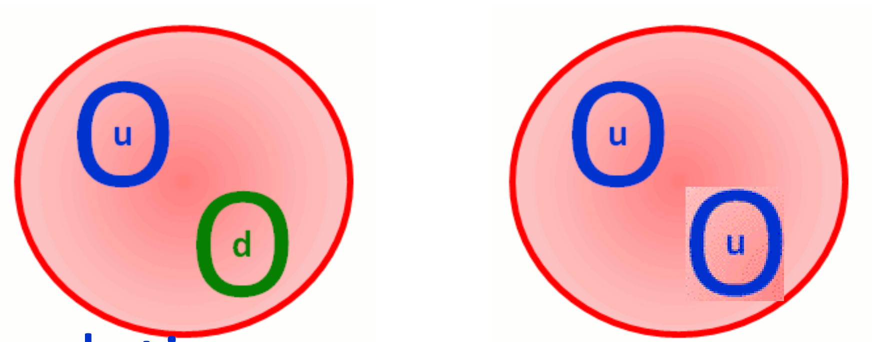


R.T. Cahill *et al.*,
Austral. J. Phys. 42 (1989) 129-145

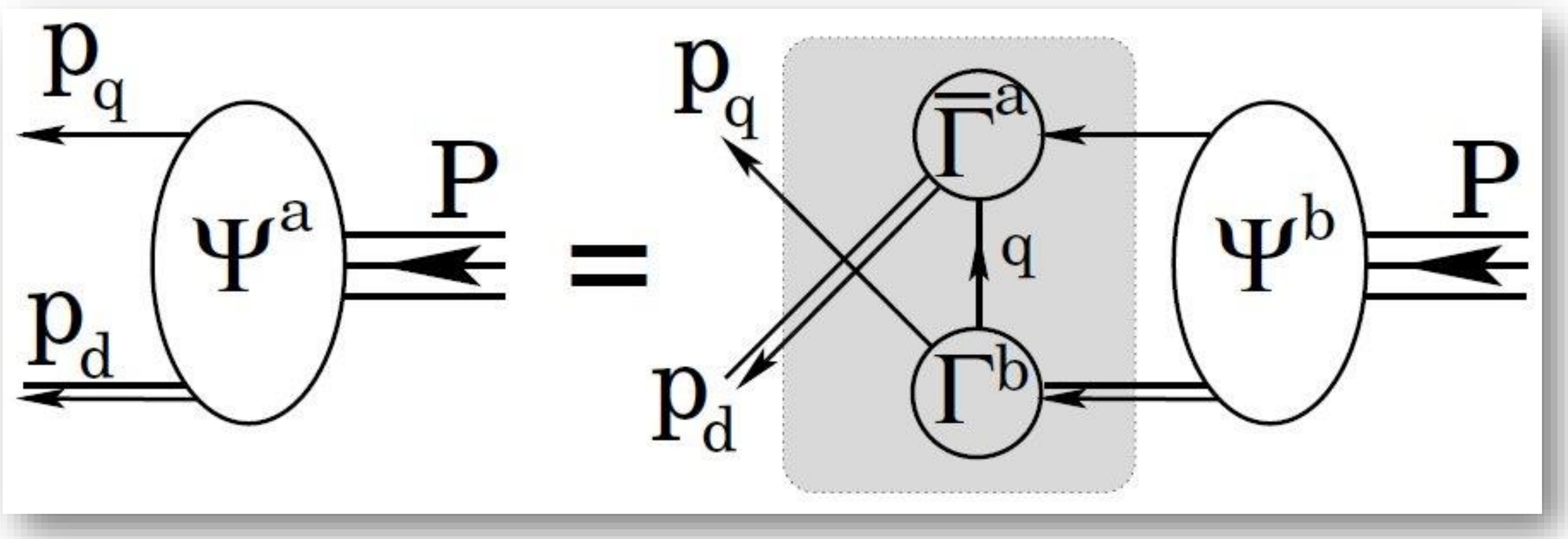
Baryon Structure

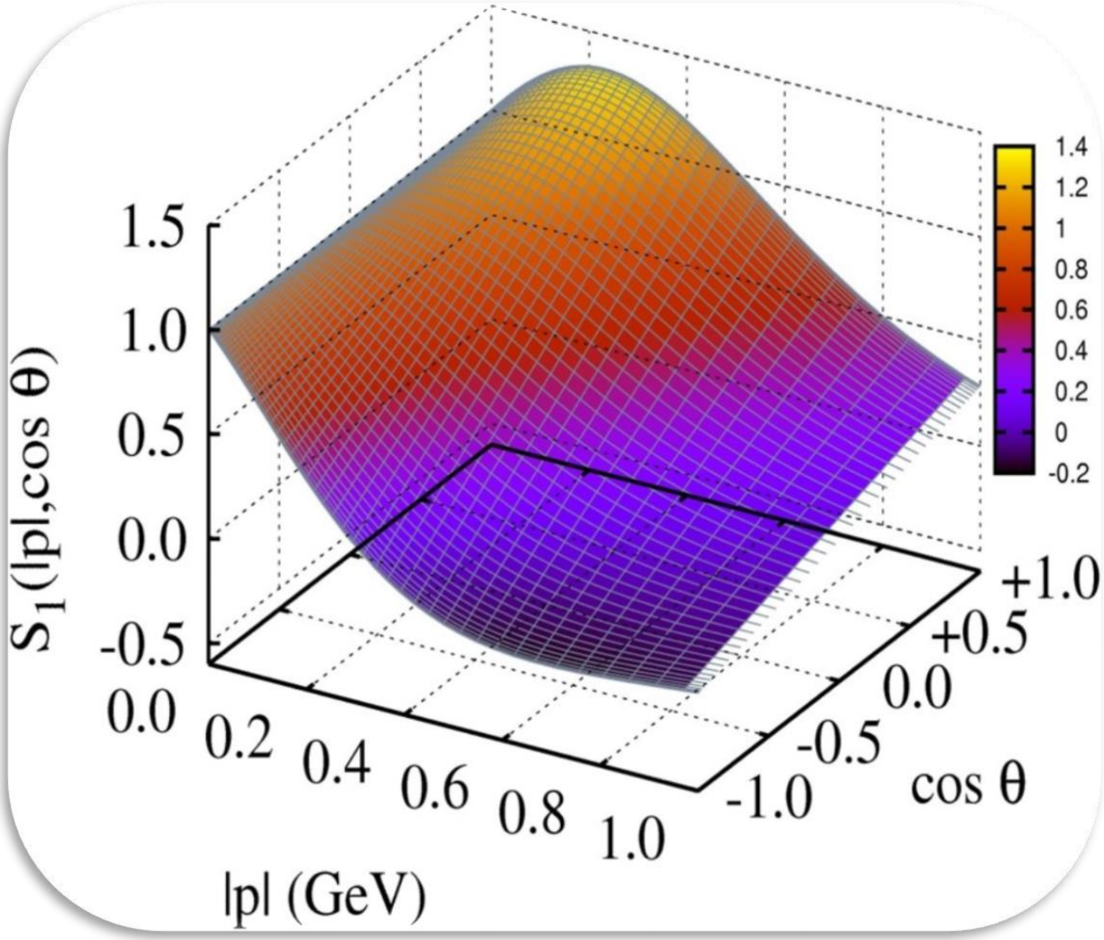
- Poincaré covariant Faddeev equation sums all possible exchanges and interactions that can take place between three dressed-quarks
- Confinement and DCSB are readily expressed
- ***Prediction***: owing to *DCSB in QCD, strong diquark correlations exist within baryons*
- Diquark correlations are not pointlike
  - Typically, $r_{0+} \sim r_\pi$ & $r_{1+} \sim r_\rho$ (actually 10% larger)
  - They have soft form factors

Faddeev Equation



- Proton (prediction)
  - Isoscalar+scalar [ud] correlations
  - Isovector+pseudovector {uu}, {ud} correlations





Proton's Wave Function

Light-cone distribution amplitudes of the nucleon and negative parity nucleon resonances from lattice QCD
V. M. Braun *et al*., Phys. Rev. D 89 (2014) 094511

Light-cone distribution amplitudes of the baryon octet
G. S. Bali *et al*. JHEP 1602 (2016) 070

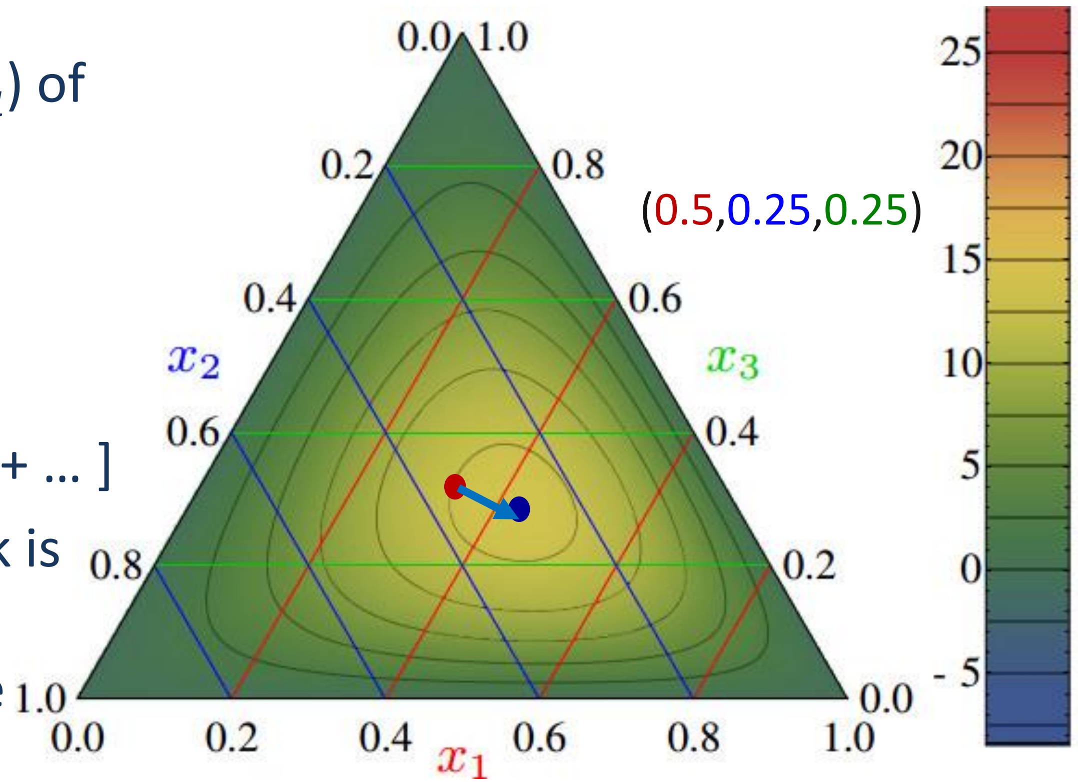
Nucleon PDAs & lQCD

- First lQCD results for n=0, 1 moments of the leading twist PDA of the nucleon are available
- Used to constrain strength (a_{11}) of the leading-order term in a conformal expansion of the nucleon's PDA:

$\Phi(x_1,x_2,x_3)$

$= 120\, x_1\, x_2\, x_3\, [\, 1 + a_{11}\, P_{11}(x_1,x_2,x_3) + \ldots\,]$

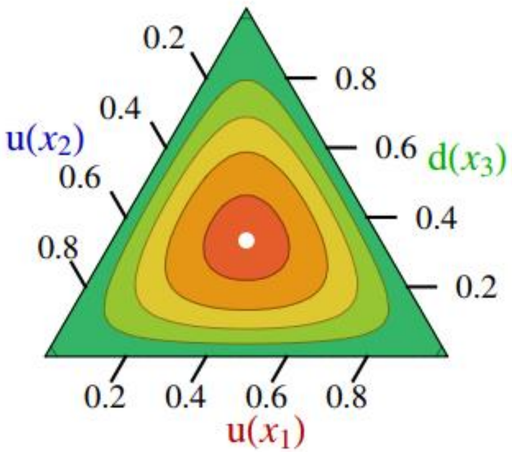
- Shift in location of central peak is *consistent* with existence of diquark correlations within the nucleon



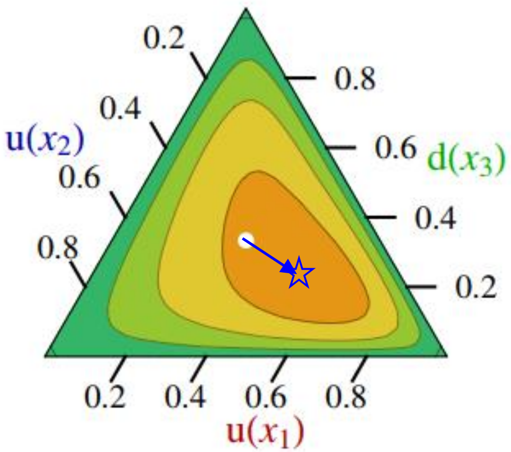
Parton distribution amplitudes: revealing diquarks in the proton and Roper resonance, Cédric Mezrag, Jorge Segovia, Lei Chang and Craig D. Roberts
arXiv:1711.09101 [nucl-th]

PDAs of Nucleon & its 1st Radial Excitation

- ➢ Methods used for mesons can be extended to compute pointwise behaviour of baryon PDAs



conformal



nucleon

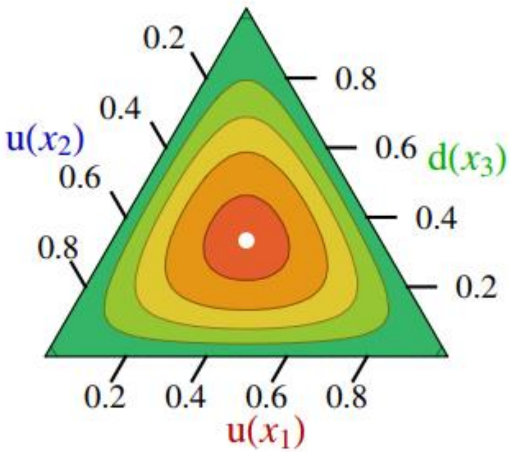
Diquark clustering skews the distribution toward the dressed-quark bystander, which therefore carries more of the proton's light-front momentum

Parton distribution amplitudes: revealing diquarks in the proton and Roper resonance, Cédric Mezrag, Jorge Segovia, Lei Chang and Craig D. Roberts
arXiv:1711.09101 [nucl-th]

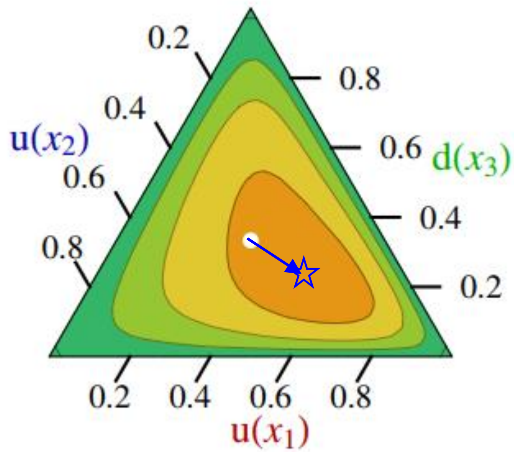
PDAs of Nucleon & its 1st Radial Excitation

- Methods used for mesons can be extended to compute pointwise behaviour of baryon PDAs

Just like QM & PDAs of meson radial excitations

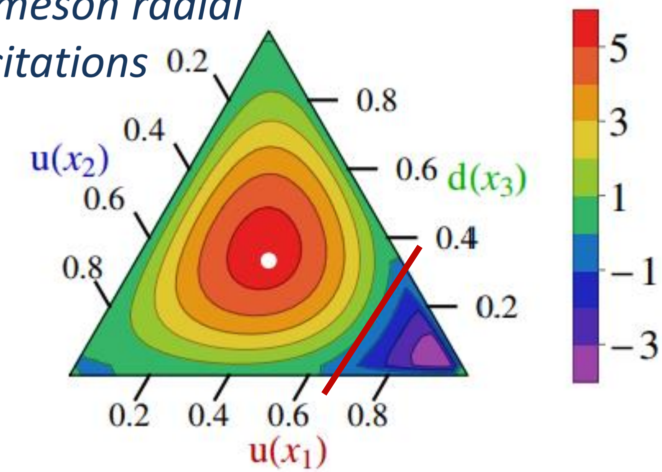


conformal



nucleon

Diquark clustering skews the distribution toward the dressed-quark bystander, which therefore carries more of the proton's light-front momentum



Roper's quark core

Excitation's PDA is not positive definite … there is a prominent locus of zeros in the lower-right corner of the barycentric plot

Diquark correlations in the nucleon

- Agreement between continuum and lattice results

  ONLY when nucleon contains scalar & axial-vector diquark correlations

- Nucleon with only a scalar-diquark, omitting the axial-vector diquark, ruled-out by this confluence between continuum and lattice results

TABLE I. A – Eq. (13) interpolation parameters for the proton and Roper PDAs in Fig. 2. B – Computed values of the first four moments of the PDAs. Our error on f_N reflects a scalar diquark content of $65 \pm 5\%$; and values in rows marked with "$\not\supset$ av" were obtained assuming the baryon is constituted solely from a scalar diquark. (All results listed at $\zeta = 2\,\text{GeV}$.)

A	$n_{\hat{\varphi}}$	α	β	w_{01}	w_{11}	w_{02}	w_{12}	w_{22}
p	65.8	1.47	1.28	0.096	0.094	0.15	−0.053	0.11
R	14.4	1.42	0.78	−0.93	0.22	−0.21	−0.057	−1.24

B	$10^3 f_N/\text{GeV}^2$	$\langle x_1 \rangle_u$	$\langle x_2 \rangle_u$	$\langle x_3 \rangle_d$
conformal PDA		0.333	0.333	0.333
lQCD [17]	2.84(33)	0.372(7)	0.314(3)	0.314(7)
lQCD [18]	3.60(6)	0.358(6)	0.319(4)	0.323(6)
herein proton	3.78(14)	0.379(4)	0.302(1)	0.319(3)
herein proton $\not\supset$ av	2.97	0.412	0.295	0.293
herein Roper	5.17(32)	0.245(13)	0.363(6)	0.392(6)
herein Roper $\not\supset$ av	2.63	0.010	0.490	0.500

Parton distribution amplitudes: revealing diquarks in the proton and Roper resonance, Cédric Mezrag, Jorge Segovia, Lei Chang and Craig D. Roberts
arXiv:1711.09101 [nucl-th]

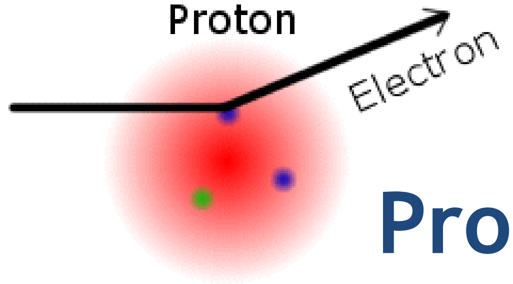
Nucleon and Roper PDAs

No humps or bumps in leading-twist PDAs of ground-state S-wave baryons

- The proton's PDA is a broad, concave function
  - maximum shifted relative to peak in QCD's conformal limit expression
  - Magnitude of shift signals presence of

    both scalar & axial-vector diquark correlations in the nucleon
    - scalar generates around 60% of the proton's normalisation.
- The radial-excitation (Roper) is constituted similarly
  - Pointwise form of its PDA
    - Negative on a material domain
  - Is result of marked interferences between the contributions from both scalar and axial-vector diquarks
    - particularly, the locus of zeros, which

      highlights its character as a radial excitation.
- These features originate with the emergent phenomenon of dynamical chiral symmetry breaking in the Standard Model.



Nucleon Structure
Probed in scattering experiments

- Electron is a good probe because it is structureless

Structureless fermion, or simply structured fermion, F_1=1 & F_2=0, so that G_E=G_M and hence distribution of charge and magnetisation within this fermion are identical

- Proton's electromagnetic current

$$J_\mu(P',P) = ie\,\bar{u}_p(P')\,\Lambda_\mu(Q,P)\,u_p(P)\,,$$
$$= \; ie\,\bar{u}_p(P')\left(\gamma_\mu F_1(Q^2) + \frac{1}{2M}\,\sigma_{\mu\nu}\,Q_\nu\,F_2(Q^2)\right)u_p(P)$$

F_1 = Dirac form factor

F_2 = Pauli form factor

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2}F_2(Q^2)\,,\quad G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

G_E = Sachs Electric form factor
If a nonrelativistic limit exists, this relates to the charge density

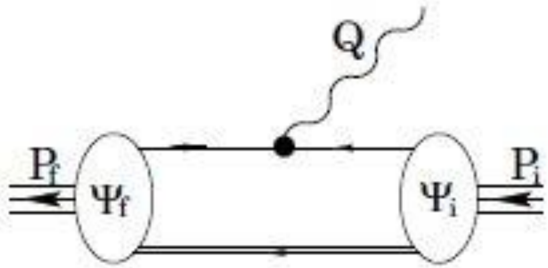
G_M = Sachs Magntic form factor
If a nonrelativistic limit exists, this relates to the magnetisation density

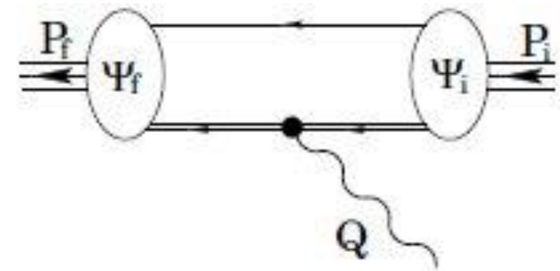
Nucleon form factors

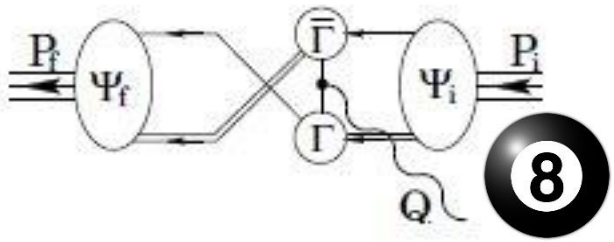
- For the nucleon & Δ-baryon and Roper-resonance, studies of the Faddeev equation exist that are based on the 1-loop renormalisation-group-improved interaction that was used efficaciously in the study of mesons
  - *Toward unifying the description of meson and baryon properties*
    G. Eichmann, I.C. Cloët, R. Alkofer, A. Krassnigg and C.D. Roberts
    arXiv:0810.1222 [nucl-th], Phys. Rev. C **79** (2009) 012202(R) (5 pages)
  - *Survey of nucleon electromagnetic form factors*
    I.C. Cloët, G. Eichmann, B. El-Bennich, T. Klähn and C.D. Roberts
    arXiv:0812.0416 [nucl-th], Few Body Syst. **46** (2009) pp. 1-36
  - *Nucleon electromagnetic form factors from the Faddeev equation*
    G. Eichmann, arXiv:1104.4505 [hep-ph]
  - *Nucleon and Δ elastic and transition form factors,* Jorge Segovia, Ian C. Cloët, Craig D. Roberts and Sebastian M. Schmidt
    arXiv:1408.2919 [nucl-th], Few Body Syst. **55** (2014) pp. 1185-1222
- Analyses retain the scalar and axial-vector diquark correlations, known to be necessary and sufficient for reliable description

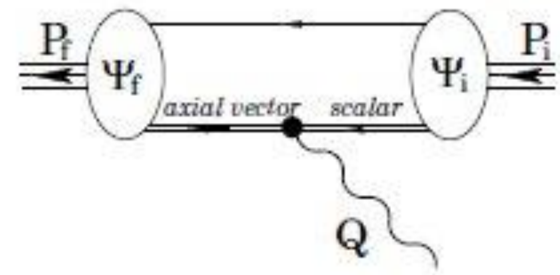
Photon-nucleon current

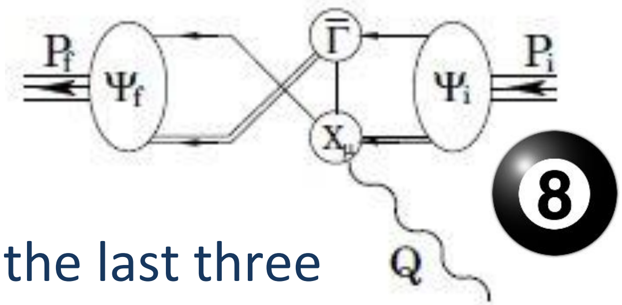
- To compute form factors, one needs a photon-nucleon current
- Composite nucleon must interact with photon via nontrivial current constrained by Ward-Green-Takahashi identities
- DSE → BSE → Faddeev equation plus current → nucleon form factors
- In a realistic calculation, the last three diagrams represent 8-dimensional integrals, which can be evaluated using Monte-Carlo techniques

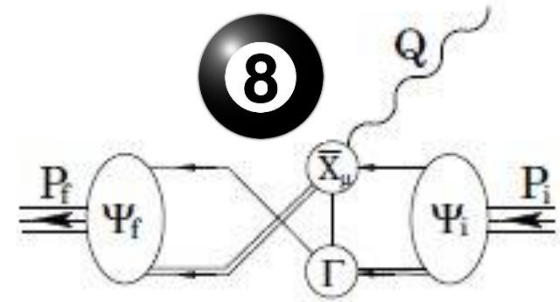












Oettel, Pichowsky, Smekal
Eur.Phys.J. A8 (2000) 251-281

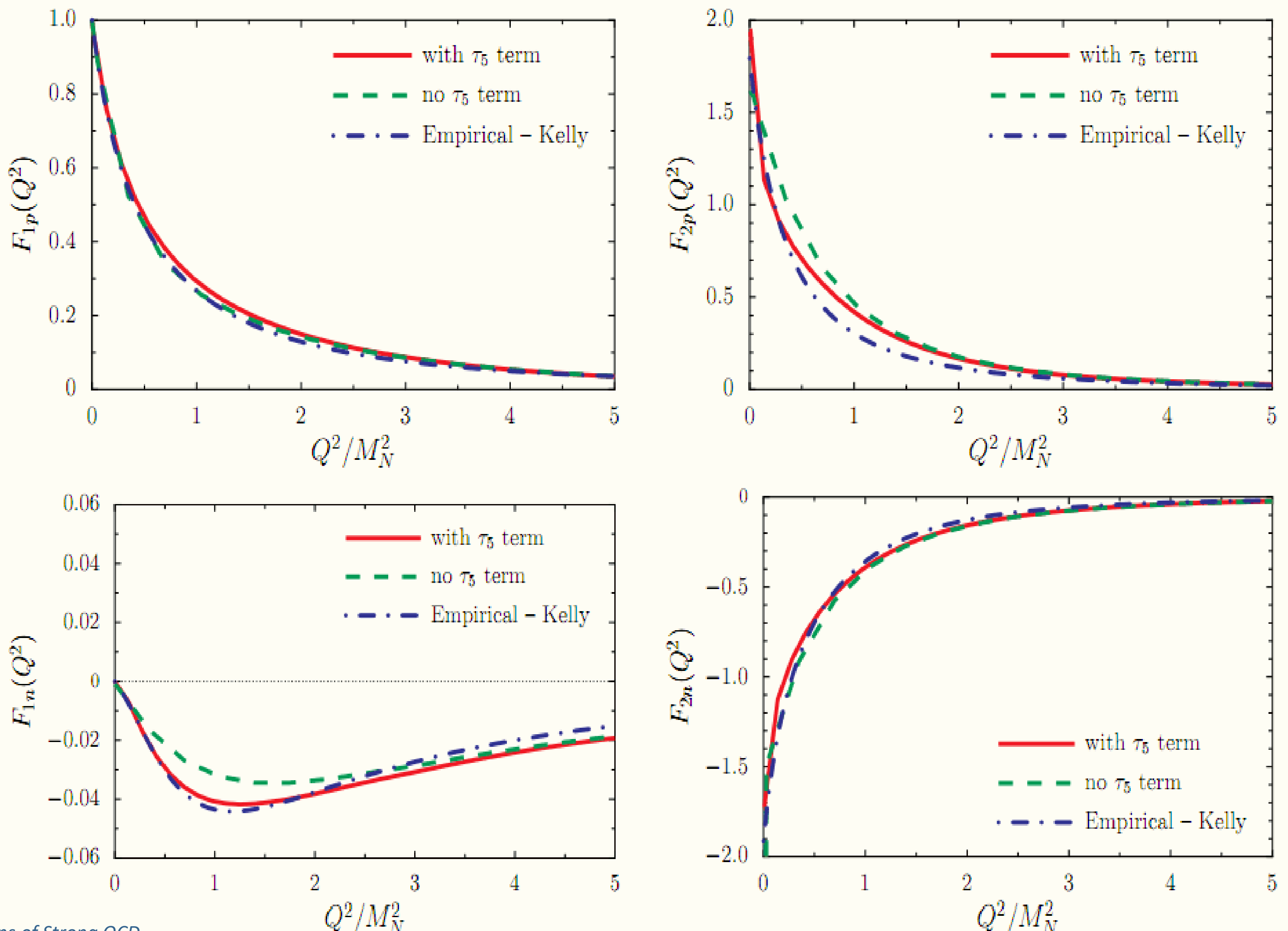
Survey of nucleon electromagnetic form factors
I.C. Cloët et al, arXiv:0812.0416 [nucl-th],
Few Body Syst. **46** (2009) pp. 1-36

Nucleon Form Factors

Unification of meson and nucleon form factors.

Very good description.

Quark's momentum-dependent anomalous magnetic moment has observable impact & materially improves agreement in all cases.



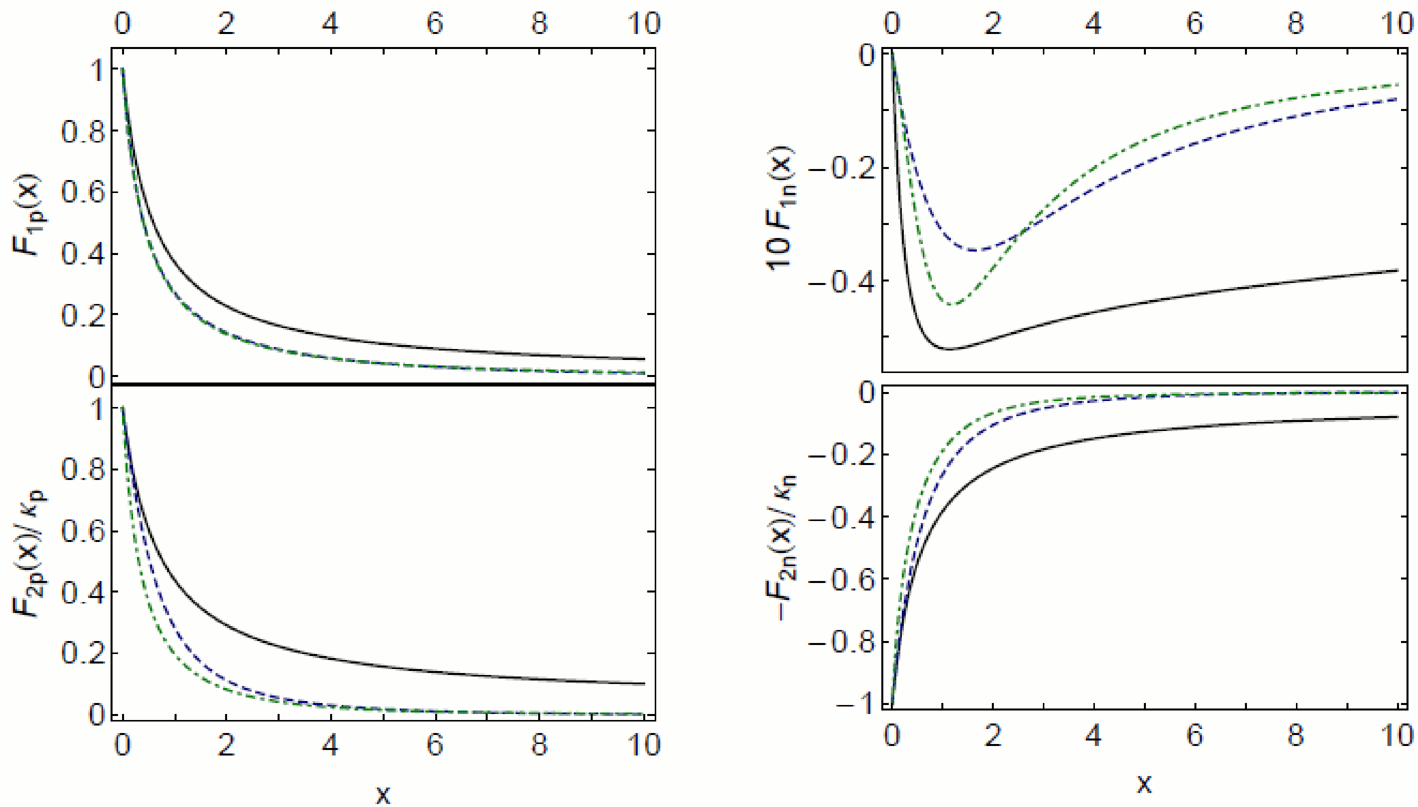
Nucleon and Roper electromagnetic elastic and transition form factors, D. J. Wilson, I. C. Cloët, L. Chang and C. D. Roberts, arXiv:1112.2212 [nucl-th], Phys. Rev. C**85** (2012) 025205 [21 pages]

Nucleon Form Factors

Black solid curve = contact interaction

Blue dashed curve = momentum-dependent interaction

Green dot-dashed curve = parametrisation of experimental data



Momentum independent Faddeev amplitudes, paired with momentum-independent dressed-quark mass and diquark Bethe-Salpeter amplitudes, produce harder form factors, which are readily distinguished from experiment

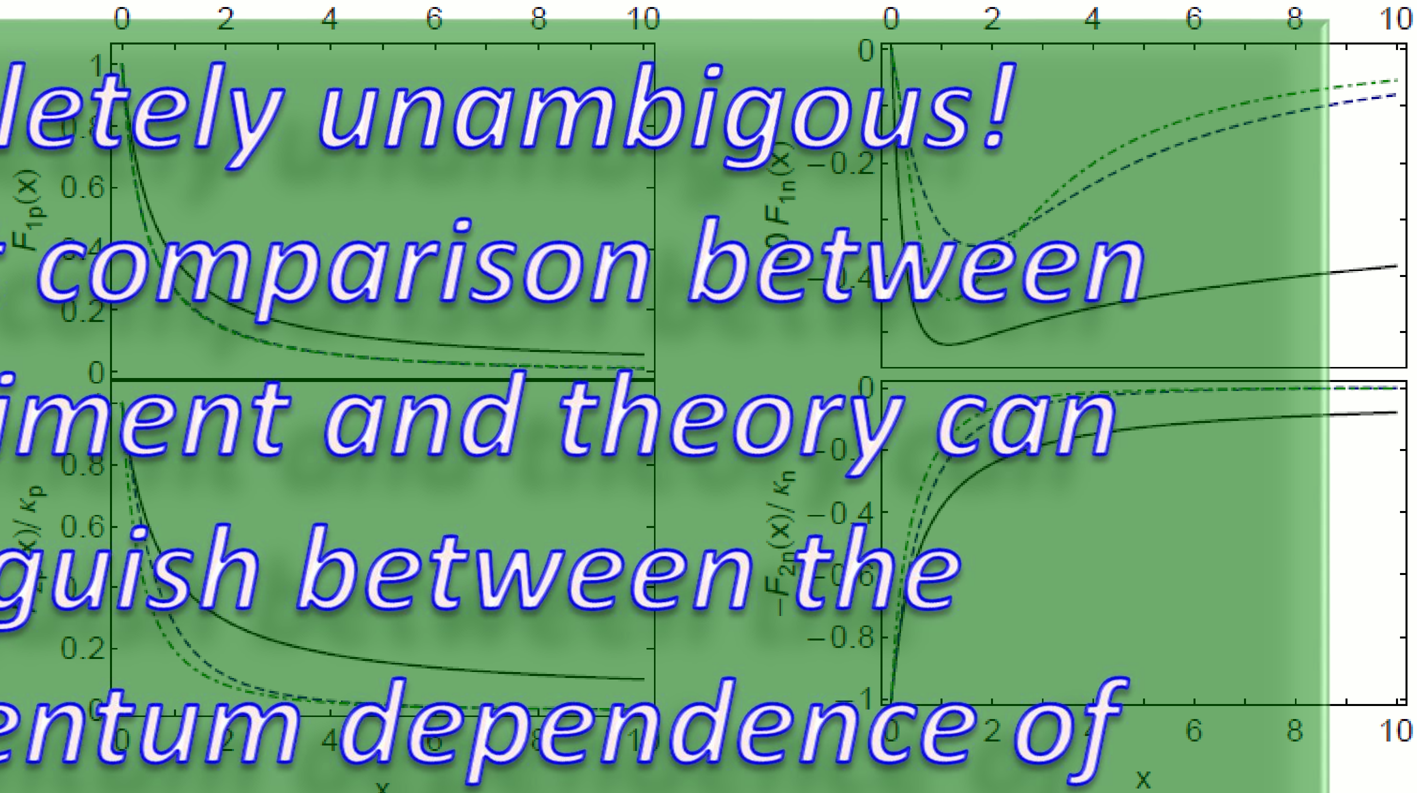
Nucleon and Roper electromagnetic elastic and transition form factors, D. J. Wilson, I. C. Cloët, L. Chang and C. D. Roberts, arXiv:1112.2212 [nucl-th], Phys. Rev. C**85** (2012) 025205 [21 pages]

Nucleon Form Factors

Black solid curve = contact interaction

Blue dashed curve = momentum-dependent interaction

Green dot-dashed curve = parametrisation of experimental data



Completely unambigous!

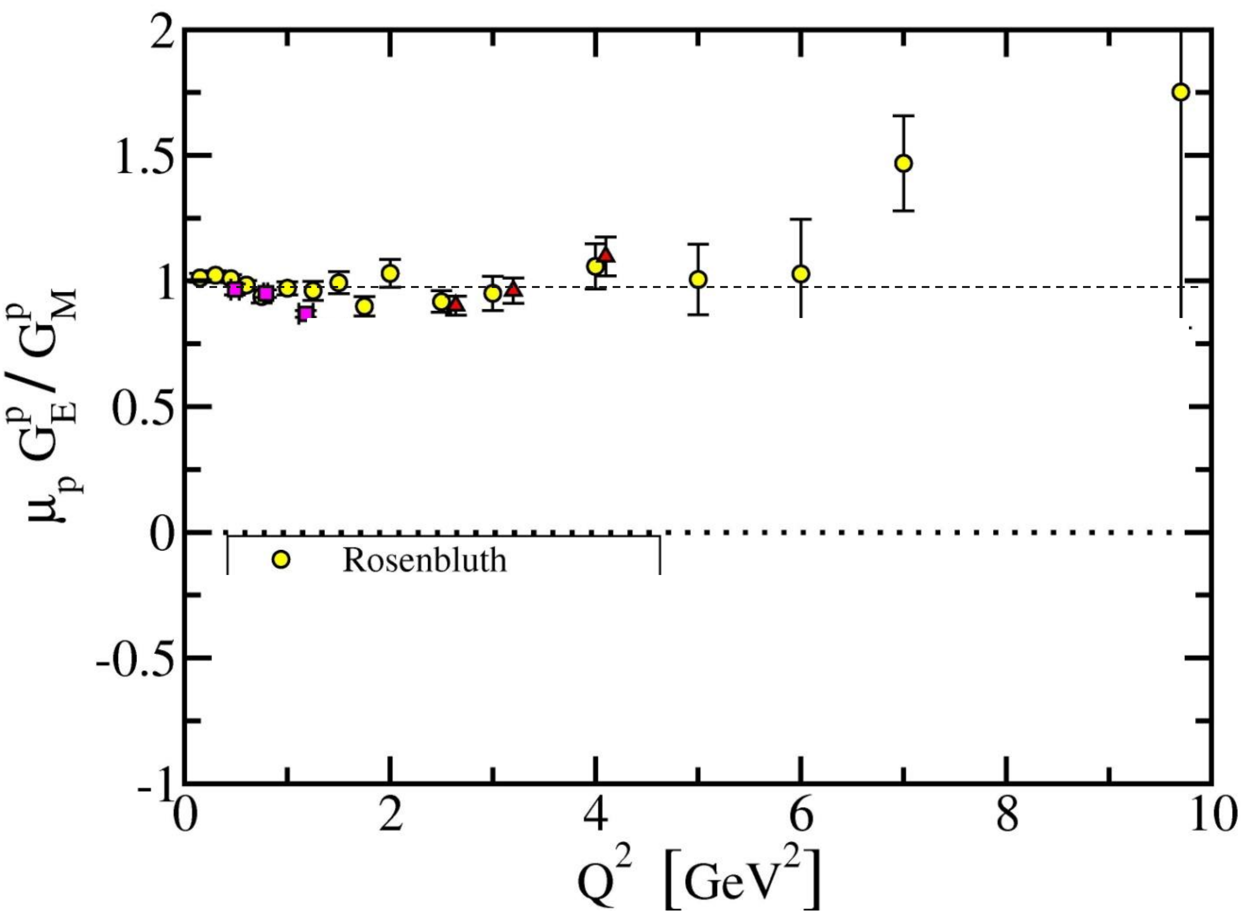
Direct comparison between experiment and theory can distinguish between the momentum dependence of strong-interaction theory

Momentum independent Faddeev amplitudes paired with momentum-independent dressed-quark mass and diquark Bethe-Salpeter amplitudes, produce harder form factors, which are readily distinguished from experiment

$$\frac{\mu_p G_E^p(Q^2)}{G_M^p(Q^2)}$$

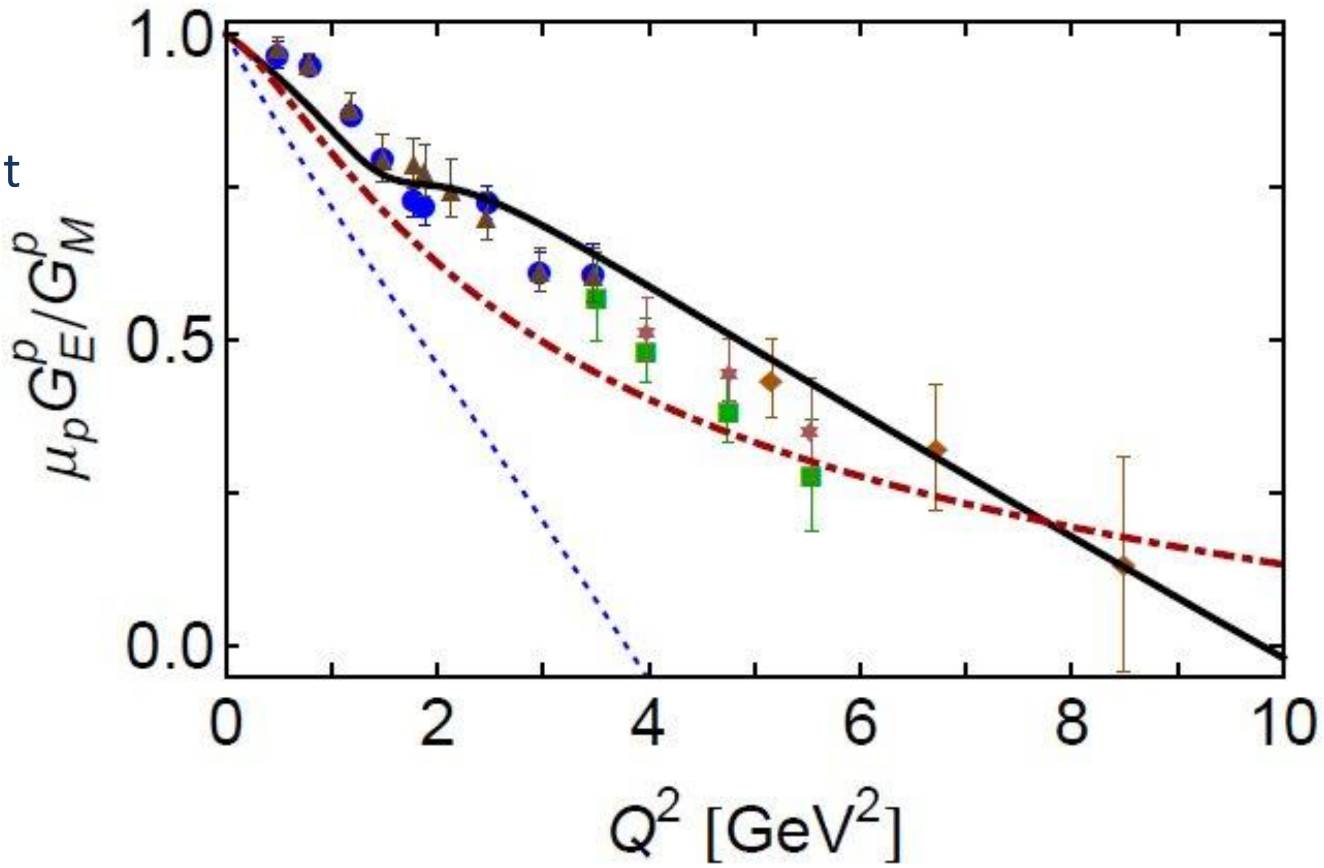
Ratio of proton's electromagnetic form factors

- Data before 1999
  - Looks like the structure of the proton is simple
- The properties of JLab (high luminosity) enabled a new technique to be employed.
- First data released in 1999 and paint a VERY DIFFERENT PICTURE



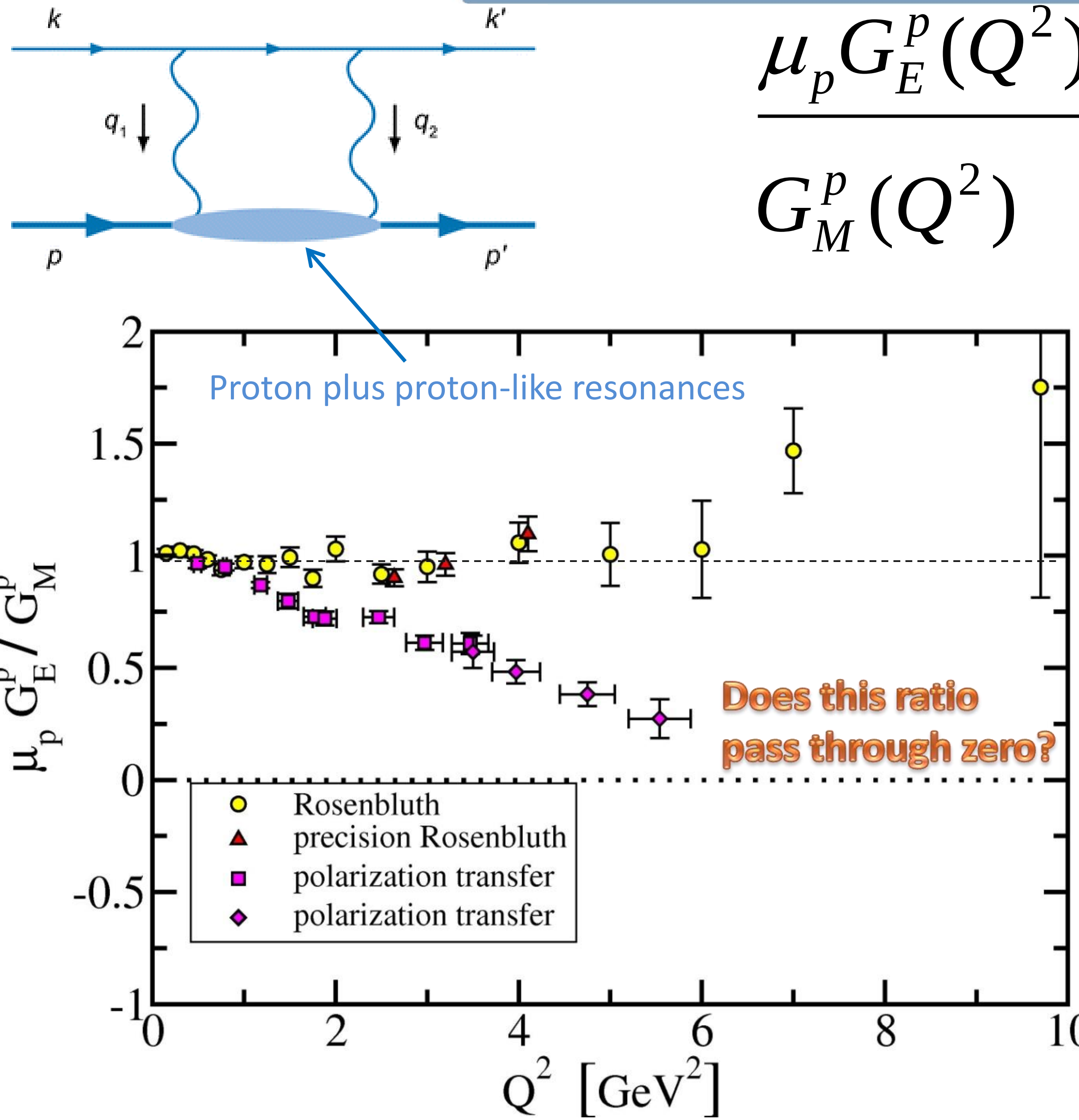
$$\frac{\mu_p G_E^p(Q^2)}{G_M^p(Q^2)}$$

- DSE
  - Solid: $M(p^2)$ result
  - Dashed: M constant
- Dot-dashed = 2004 parametrisation of data



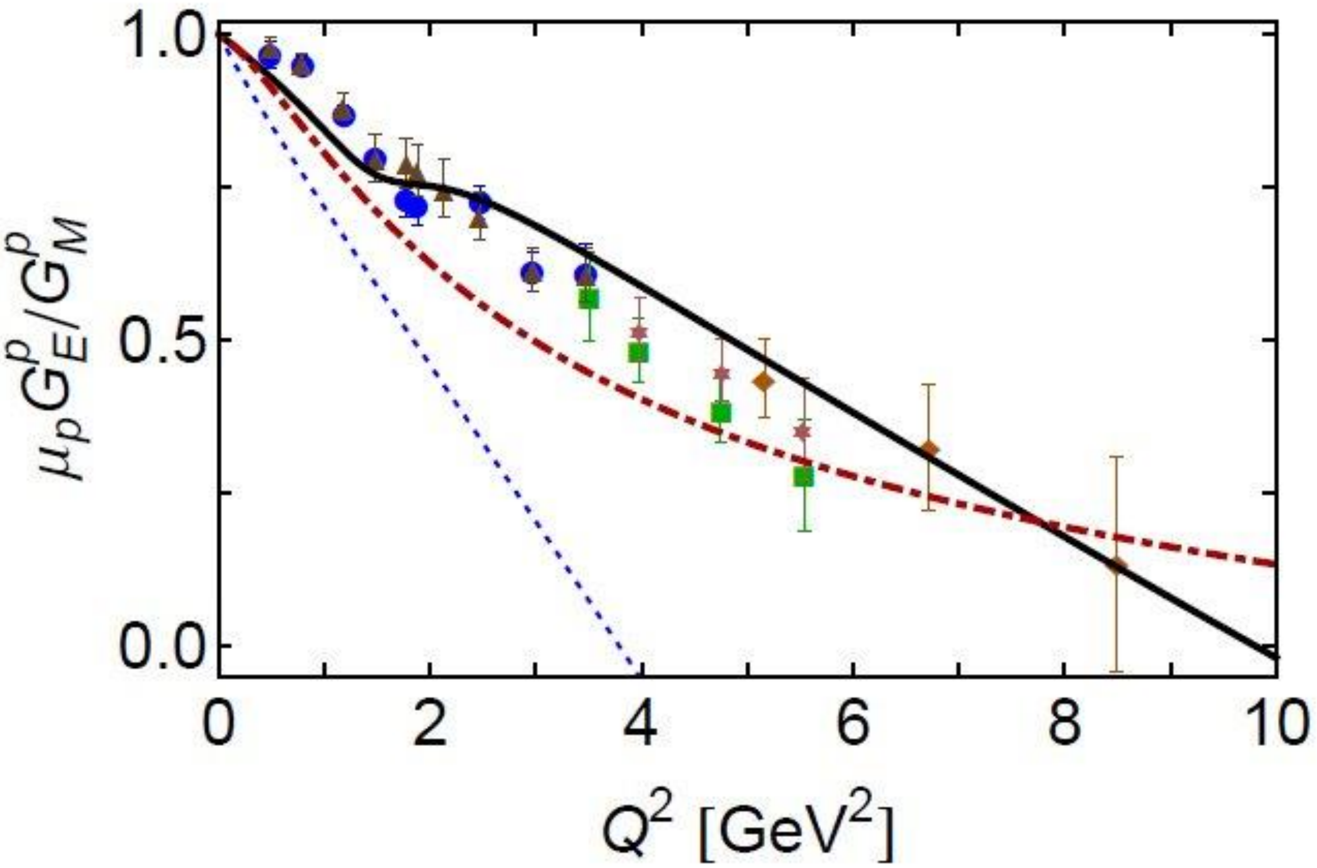
$$\frac{\mu_p G_E^p(Q^2)}{G_M^p(Q^2)}$$

- DSE studies indicate that the *proton has a very rich internal structure*
- The JLab data, obtained using the polarisaton transfer method, are an accurate indication of the behaviour of this ratio
- The pre-1999 data (Rosenbluth) receive large corrections from so-called 2-photon exchange contributions



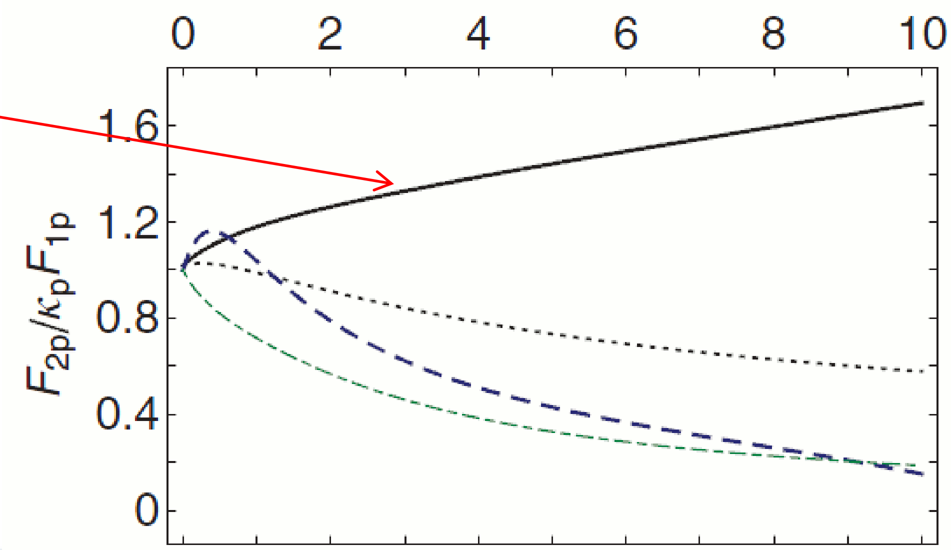
$$\frac{\mu_p G_E^p(Q^2)}{G_M^p(Q^2)}$$

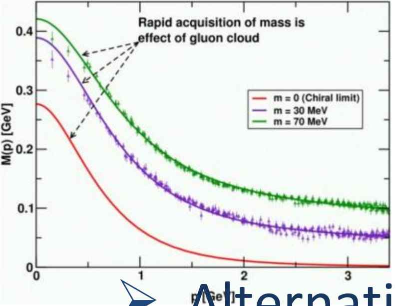
- DSE: there is plainly a chance that G_E can theoretically pass through zero
- But, is a zero unavoidable?



Origin of the zero & its location

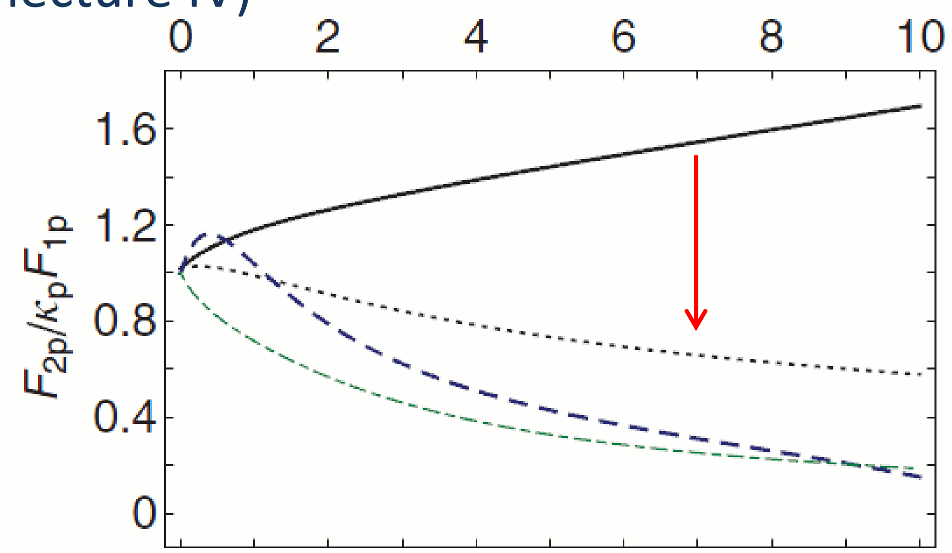
- The Pauli form factor is a gauge of the distribution of magnetization within the proton. Ultimately, this magnetisation is carried by the dressed quarks and influenced by correlations amongst them, which are expressed in the Faddeev wave function.
- If the dressed quarks are described by a momentum-independent mass function, *M*=constant, then they behave as Dirac particles with constant Dirac values for their magnetic moments and produce a hard Pauli form factor





Origin of the zero & its location

- Alternatively, suppose that the dressed quarks possess a momentum-dependent mass function, $M=M(p^2)$, which is large at infrared momenta but vanishes as their momentum increases.
- At small momenta they will then behave as constituent-like particles with a large magnetic moment, but their mass and magnetic moment will drop toward zero as the probe momentum grows. (Remember: Massless fermions do not possess a measurable magnetic moment – lecture IV)
- Such dressed quarks produce a proton Pauli form factor that is large for $Q^2 \sim 0$ but drops rapidly on the domain of transition between nonperturbative and perturbative QCD, to give a very small result at large Q^2.



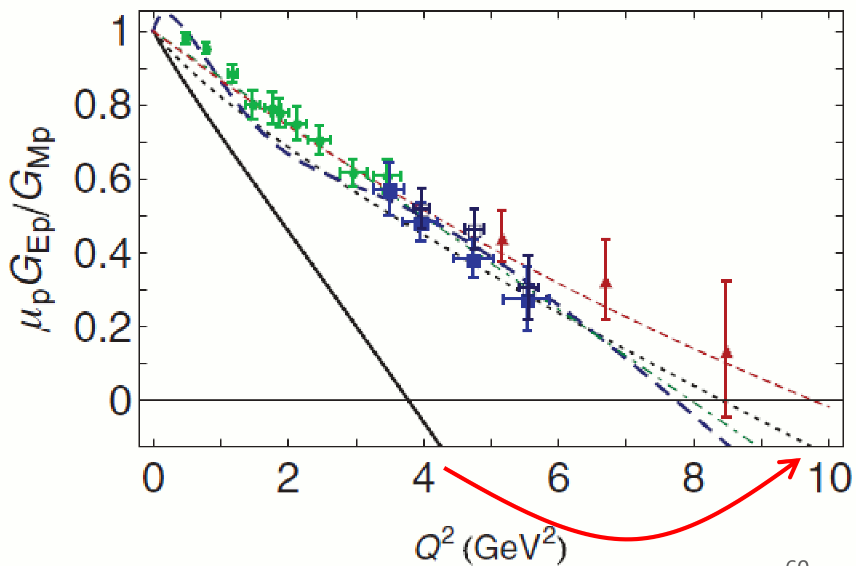
Origin of the zero & its location

- The precise form of the Q^2 dependence will depend on the evolving nature of the angular momentum correlations between the dressed quarks.
- From this perspective, existence, and location if so, of the zero in

$$\mu_p G_{Ep}(Q^2)/G_{Mp}(Q^2)$$

  are a fairly direct measure of the location and width of the transition region between the nonperturbative and perturbative domains of QCD as expressed in the momentum dependence of the dressed-quark mass function.
- Hard, M=constant

  → Soft, $M=M(p^2)$

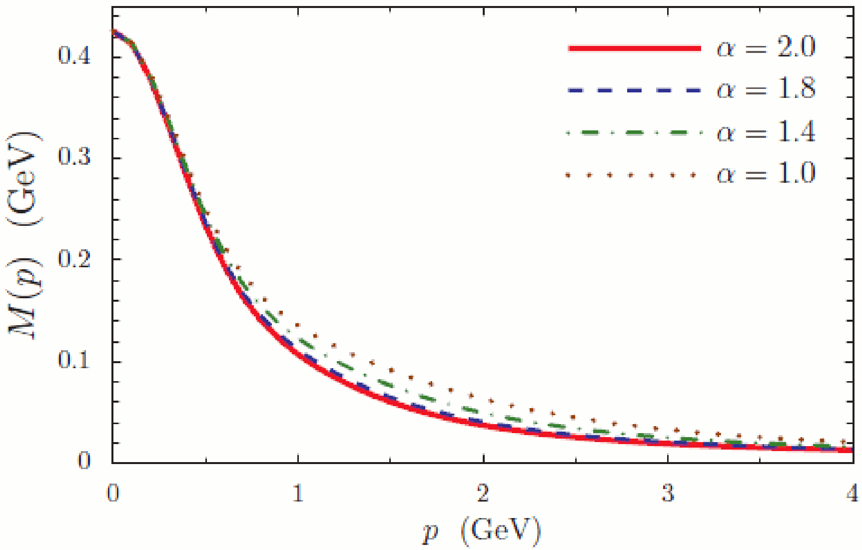


Origin of the zero & its location

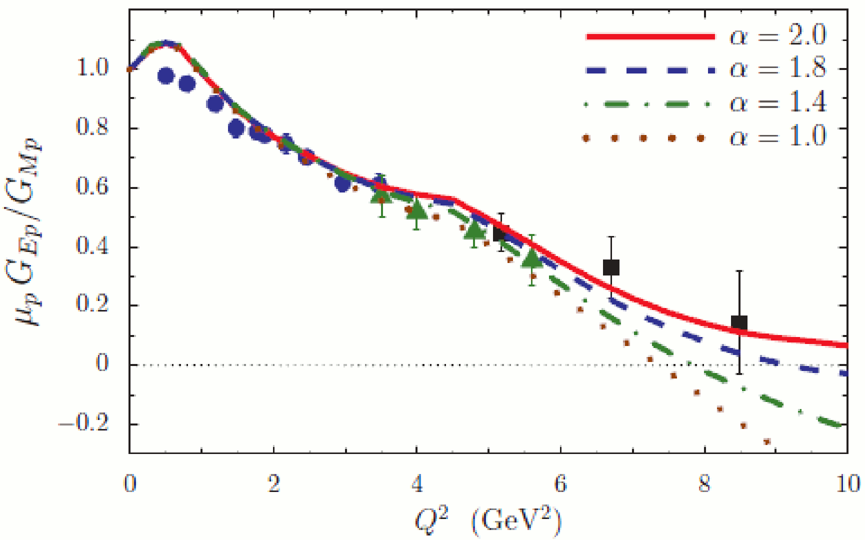
- One can anticipate that a mass function which rapidly becomes partonic—namely, is very soft—will not produce a zero
- We've seen that a constant mass function produces a zero at a small value of Q^2
- And also seen and know that a mass function which resembles that obtained in the best available DSE studies and via lattice-QCD simulations produces a zero at a location that is consistent with extant data.
- There is opportunity here for very constructive feedback between future experiments and theory.

I.C. Cloët, C.D. Roberts, A.W. Thomas: Revealing dressed-quarks via the proton's charge distribution, arXiv:1304.0855 [nucl-th], Phys. Rev. Lett. **111** (2013) 101803

Visible Impacts of DCSB



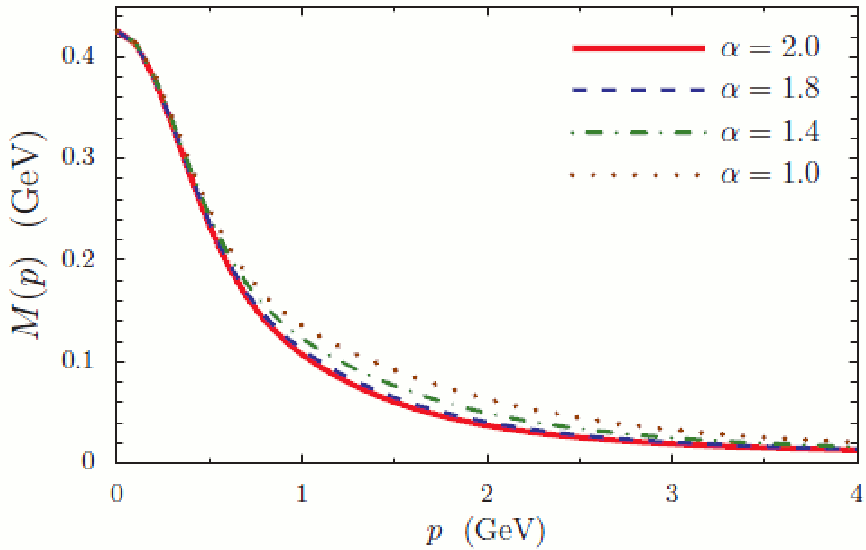
$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



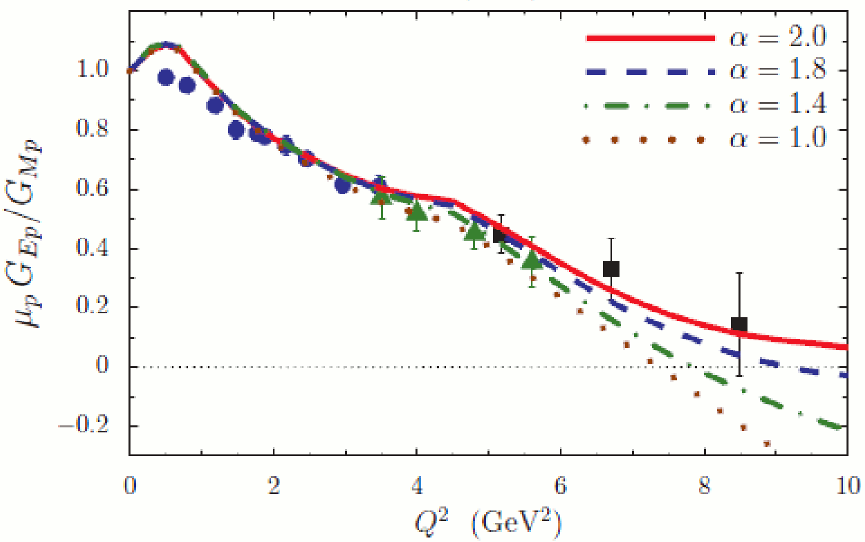
- Apparently small changes in M(p) within the domain 1<p(GeV)<3 have striking effect on the proton's electric form factor
- The possible existence and location of the zero is determined by behaviour of $Q^2F_2^p(Q^2)$, proton's Pauli form factor
- Like the pion's PDA, $Q^2F_2^p(Q^2)$ measures the rate at which dressed-quarks become parton-like:
  - ✓ $F_2^p=0$ for bare quark-partons
  - ✓ Therefore, G_E^p can't be zero on the bare-parton domain

I.C. Cloët, C.D. Roberts, A.W. Thomas: Revealing dressed-quarks via the proton's charge distribution,
arXiv:1304.0855 [nucl-th], Phys. Rev. Lett. **111** (2013) 101803

Visible Impacts of DCSB



$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



- Follows that the
  - ✓ possible existence
  - ✓ and location

  of a zero in the ratio of proton elastic form factors

  $$[\mu_p G_{Ep}(Q^2)/G_{Mp}(Q^2)]$$

  are a direct measure of the nature of the quark-quark interaction in the Standard Model.

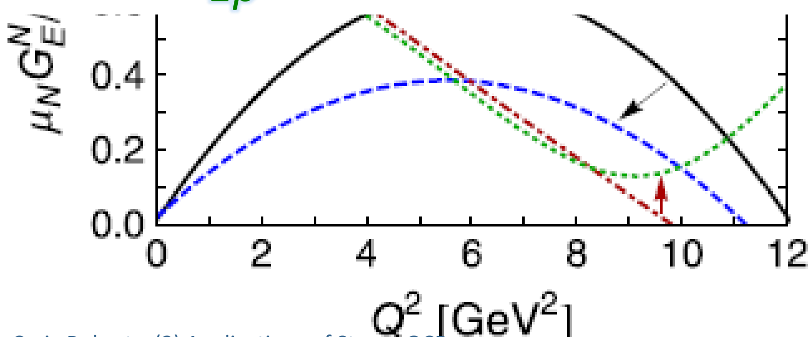
J. Segovia, I.C. Cloët, C.D. Roberts, S.M. Schmidt: Nucleon and Δ Elastic and Transition Form Factors, arXiv:1408.2919 [nucl-th], Few Body Syst. **55** (2014) 1185 [on-line]

Electric Charge

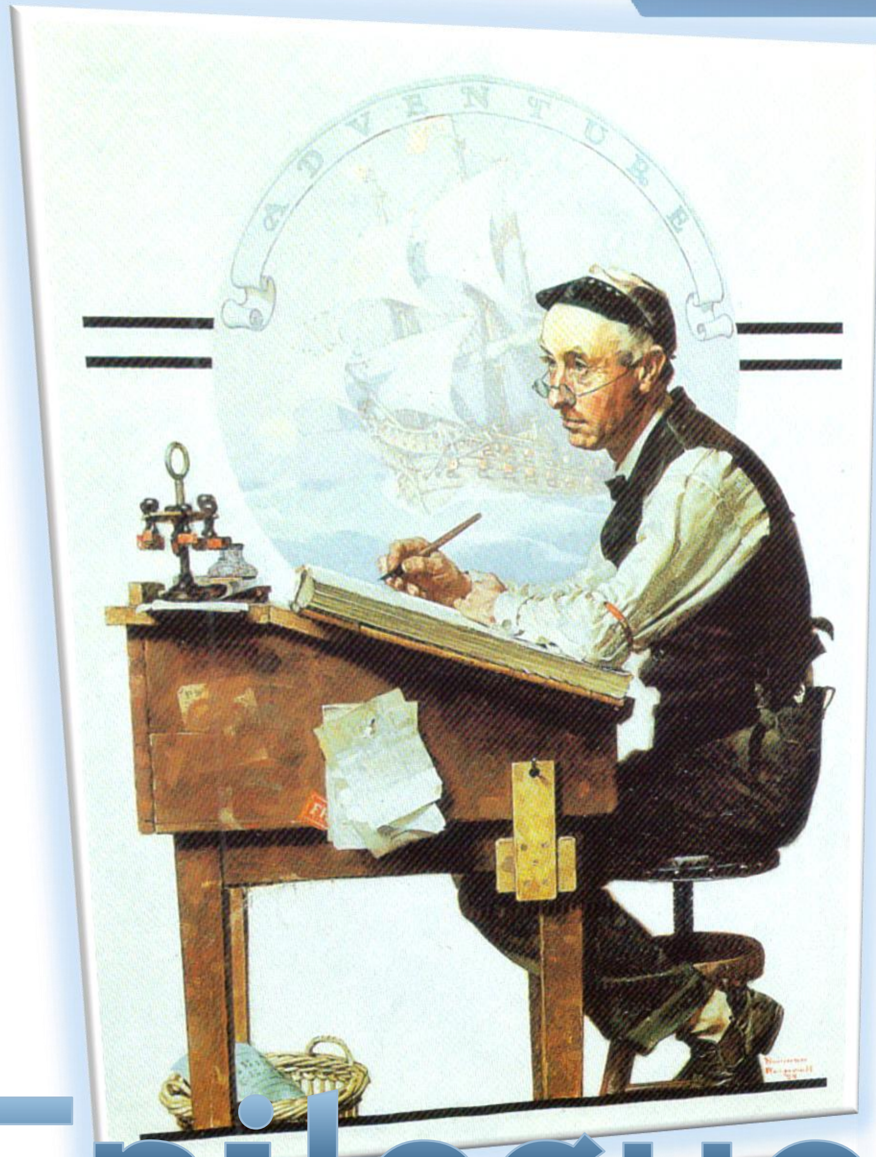
- Proton: if one accelerates the rate at which the dressed-quark sheds its cloud of gluons to become a parton, then zero in G_{ep} is pushed to larger Q^2
- Opposite for neutron!
- Explained by presence of diquark correlations

Leads to *Prediction neutron:proton*

$G_{En}(Q^2) > G_{Ep}(Q^2)$ at $Q^2 > 4 GeV^2$



- These features entail that at x≈ 5 the electric form factor of the neutral neutron will become larger than that of the unit-charge proton!
- JLab12 will probe this prediction



Epilogue

Epilogue

- Emergence:
  - Confinement and dynamical chiral symmetry breaking in the Standard Model
    - Are they related?
    - Are they the same?
      - Role of the pion seems to be key in answering these questions
  - Conformal anomaly
    - Can have neither confinement nor DCSB if scale invariance of (classical) chromodynamics is not broken by quantisation
    - Know a mass-scale must exist, but only experience/experiment reveals its value
    - Once size known, continuum and lattice-regularised *quantum* chromodynamics ⇒ *gluons and quarks acquire momentum-dependent masses*
      - Values are large in the infrared $m_g \propto 500$ MeV ≈ $m_p/2$ & $M_q \propto 350$ MeV ≈ $m_p/3$
        - » Seem to be the foundation for DCSB
        - » Can be argued to explain confinement as a dynamical phenomenon, tied to fragmentation functions

Epilogue

- Reductive explanation
  - Fundamental equivalence of the one- and two-body problems in the matter-sector
    - Quark gap equation ≡ Pseudoscalar meson Bethe-Salpeter equation
  - Entails that properties of the pion & kaon
    Nature's lightest observable strong-interaction excitations
    are (possibly) the cleanest means by which to probe the origin and manifestations of mass in the Standard Model
  - Numerous predictions (meson & baryon PDAs, PDFs, form factors, etc.) that can be tested at contemporary and planned facilities
    - JLab 12GeV
    - EIC
  - Refining those predictions *before experiments begin* will require combination of all existing nonperturbative approaches to strong interaction dynamics in the Standard Model