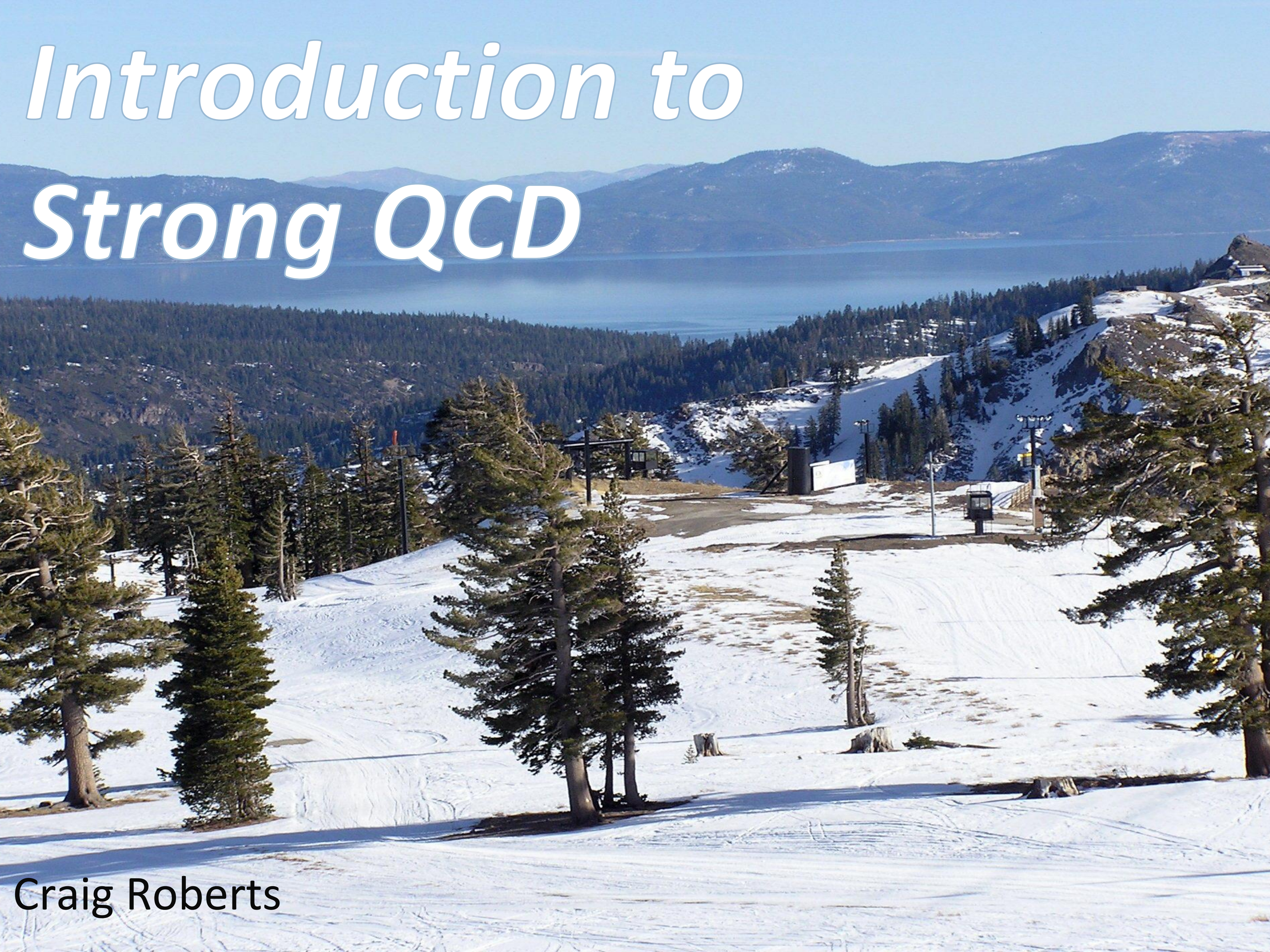


Introduction to Strong QCD

Craig Roberts

2013: Higgs and Englert

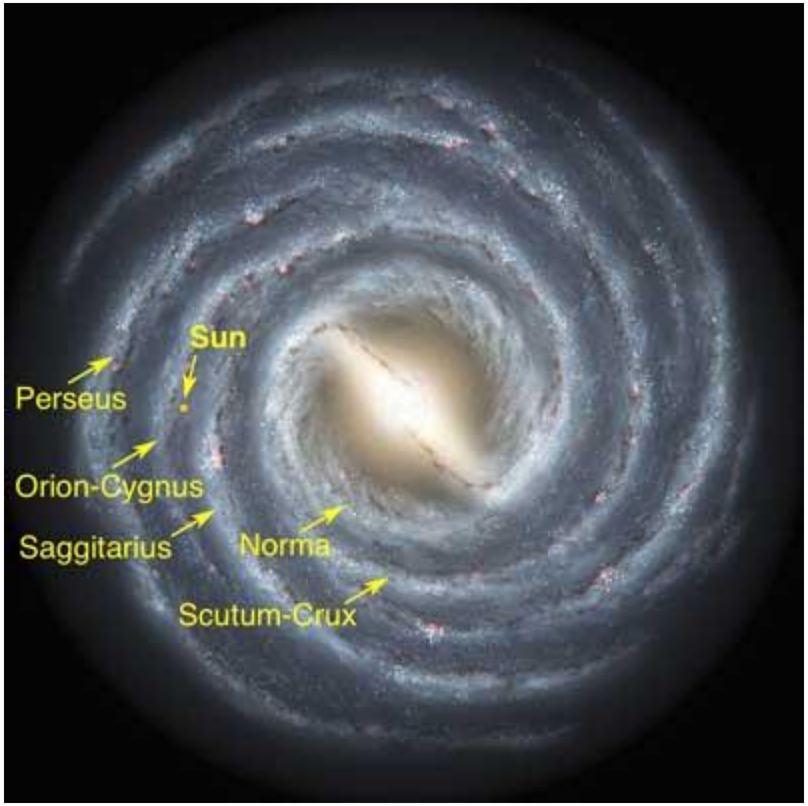
- The 2013 Nobel Prize in Physics was awarded to Peter Higgs and Francois Englert following discovery of the Higgs boson at the Large Hadron Collider.
- With this discovery the Standard Model of Particle Physics became complete.
- Its formulation and verification are a remarkable story.

Where to now?

Emergent Phenomena in the Standard Model

Existence of the Universe as we know it depends critically on the following empirical facts:

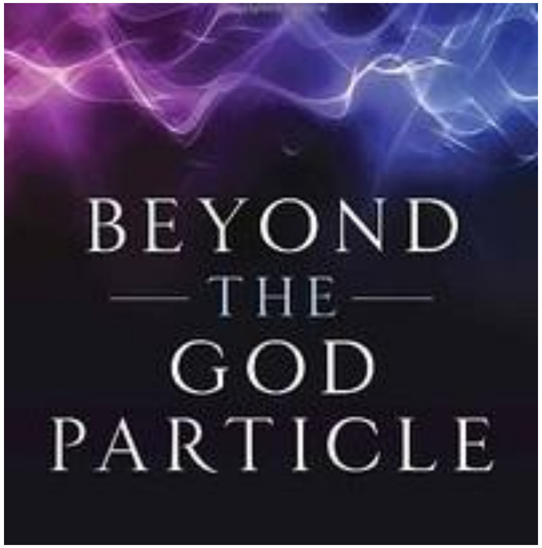
- Proton is massive, *i.e*. the mass-scale for strong interactions is vastly different to that of electromagnetism
- Proton is absolutely stable, despite being a composite object constituted from three valence quarks
- Pion is unnaturally light (not massless, but lepton-like mass), despite being a strongly interacting composite object built from a valence-quark and valence antiquark

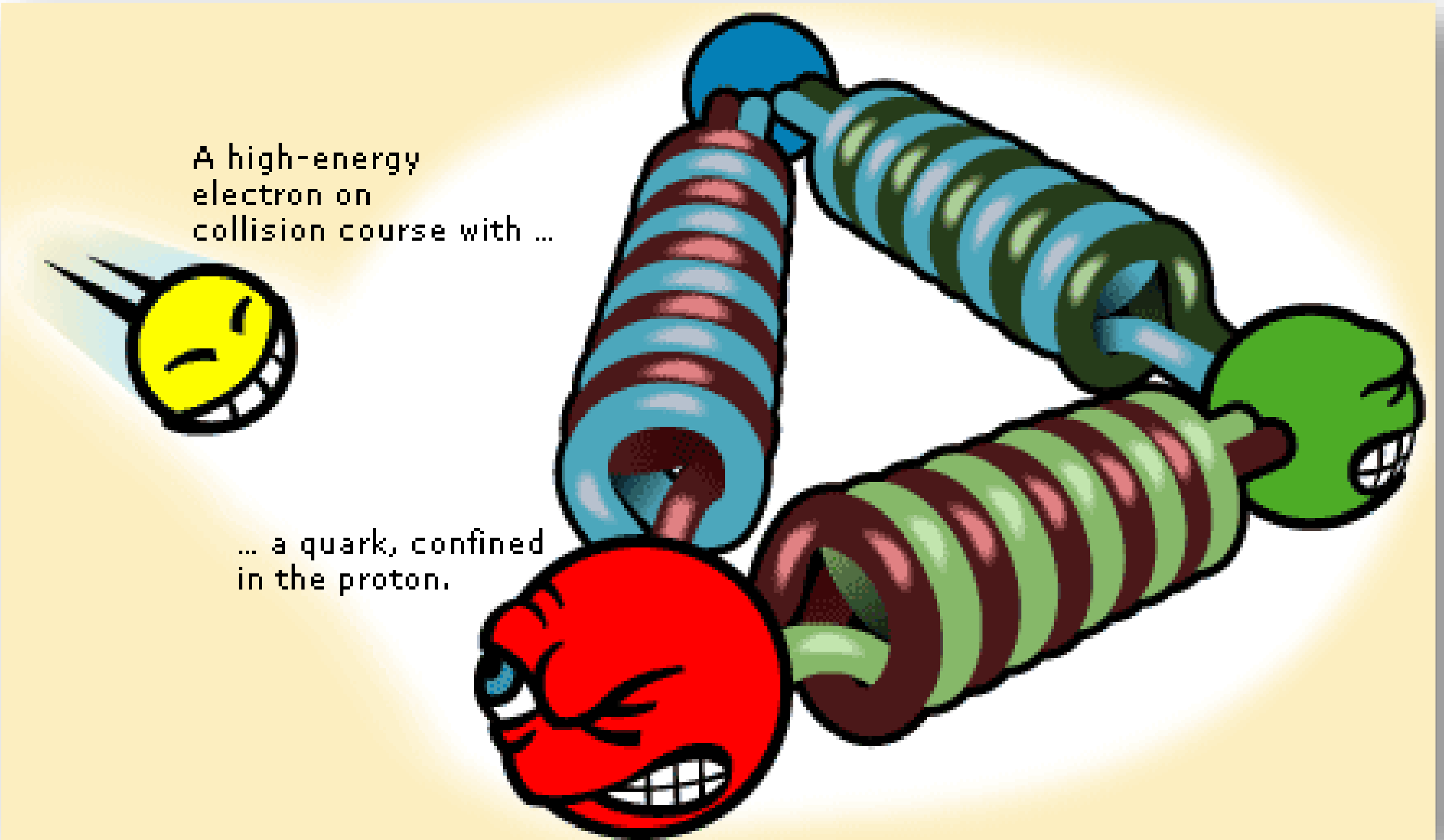


Emergence: low-level rules producing high-level phenomena, with enormous apparent complexity

Origin of Mass

- LHC has NOT found the "God Particle" because the Higgs boson is NOT the origin of mass
  - Higgs-boson only produces a little bit of mass
  - Higgs-generated mass-scales explain neither the proton's mass nor the pion's (*near*-)masslessness
  - Hence LHC has, as yet, taught us very little about the origin, structure and nature of the nuclei whose existence support the Cosmos
- Strong interaction sector of the Standard Model, *i.e*. Quantum ChromoDynamics (QCD), is the key to understanding the origin, existence and properties of (almost) all known matter





Quantum Chromodynamics

Quantum Chromodynamics (QCD)

- Quite possibly, the most remarkable theory we have ever invented
- One line and two definitions are responsible for the <u>origin</u>, <u>mass</u> and <u>size</u> of all visible matter!

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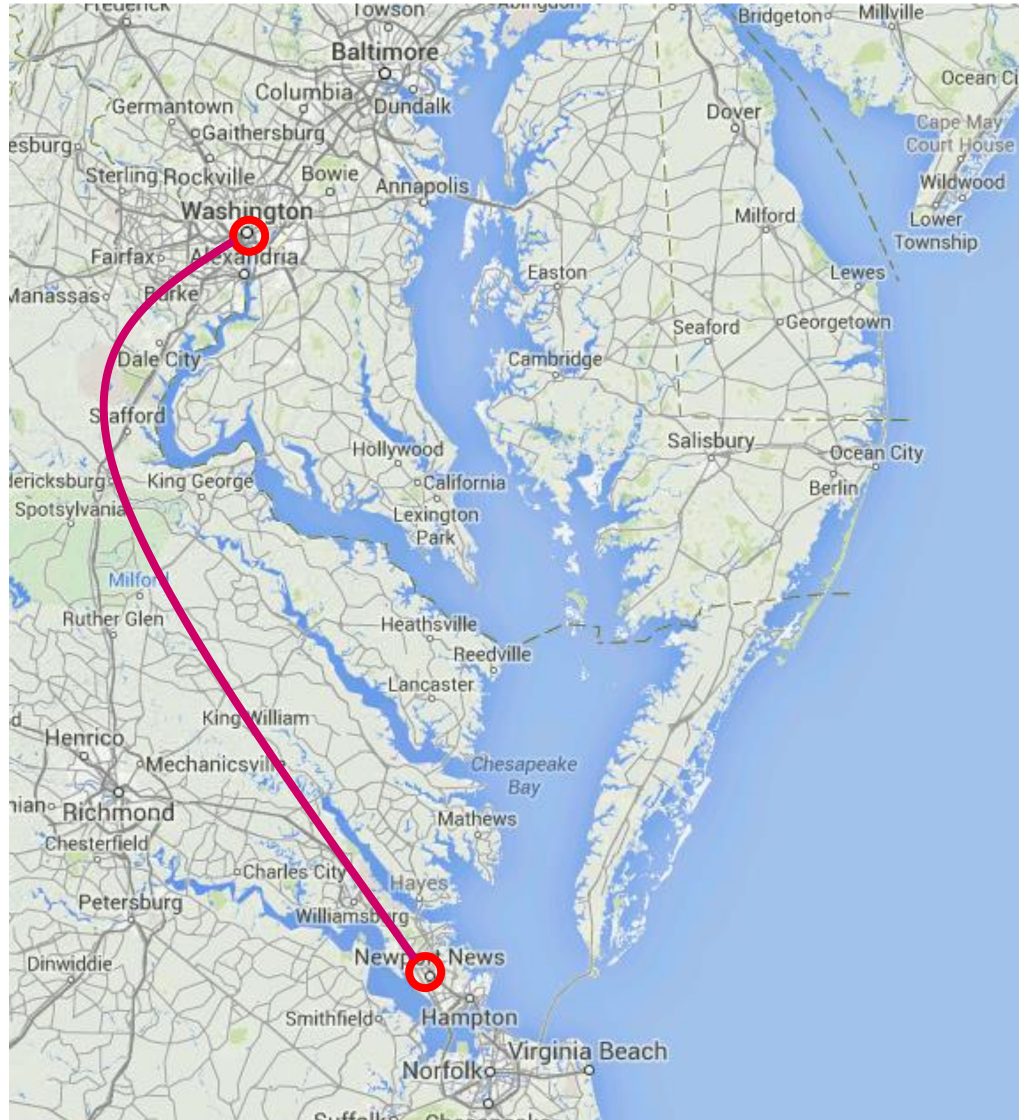
Jefferson Lab

Thomas Jefferson National Accelerator Facility (JLab)

- Driving distance:
  Washington DC to JLab
  ≈ 270km



Thomas Jefferson National Accelerator Facility (JLab)

- 1984 … DoE provided initial funding for research, development and design
- 1987 … Construction begins on Continuous Electron Beam Accelerator Facility (CEBAF) - February 13
- 1994 … Accelerator reaches design energy of 4 GeV
- Construction cost in $FY14 ≈ $1-Billion
- *Goal … Write the book about the strongest force in nature - the force that holds nuclei together - and determine how that force can be explained in terms of quarks & gluons (quantum chromodynamics - QCD).*

Thomas Jefferson National Accelerator Facility (JLab)

e.g. S. J. Brodsky and G. R. Farrar, Phys. Rev. Lett. 31, 1153 (1973)

- One of the primary reasons for building CEBAF/JLab

  Prediction: at energy-scales greater than some *a priori* unknown minimum value, $\boldsymbol{Q_0}$, cross-sections and form factors behave as

Parton model scaling

$$\mathcal{A}(\sigma^2) \overset{\sigma \gg Q_0}{\propto} \left[\frac{Q_0^2}{\sigma^2}\right]^N \ln\left[\frac{Q_0^2}{\sigma^2}\right]^{\gamma_{\mathcal{A}}}$$

QCD scaling violations

power = (number $_{\text{valence-quarks}}$ – 1 + $\Delta\lambda$)

$\Delta\lambda$=0,1, depending on whether helicity is conserved or flipped

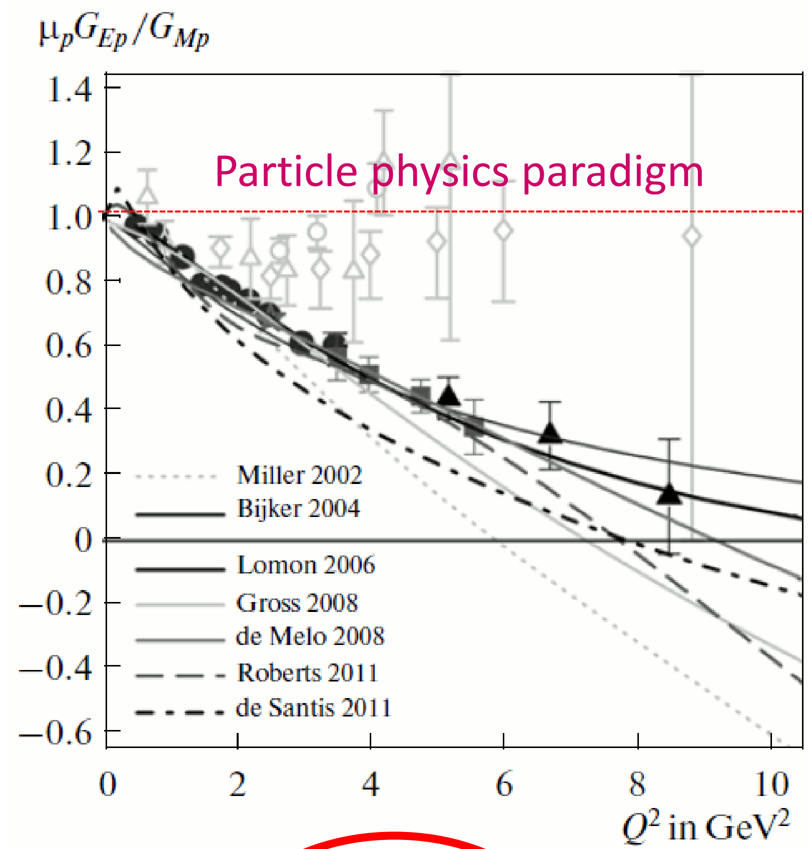
… prediction of $1/k^2$ vector-boson exchange

logarithm = distinctive feature & concrete prediction of QCD

- Initially imagined that $\boldsymbol{Q_0}$ = 1GeV!
- So, JLab was initially built to reach 4GeV.

Thomas Jefferson National Accelerator Facility (JLab)

- 1994 – 2004
  - An enormous number of fascinating experimental results
  - Including an empirical demonstration that the distribution of charge and magnetisation within the proton are completely different,
  - Suggesting that quark-quark correlations play a crucial role in nucleon structure
- *But* no sign of parton model scaling and certainly not of scaling violations

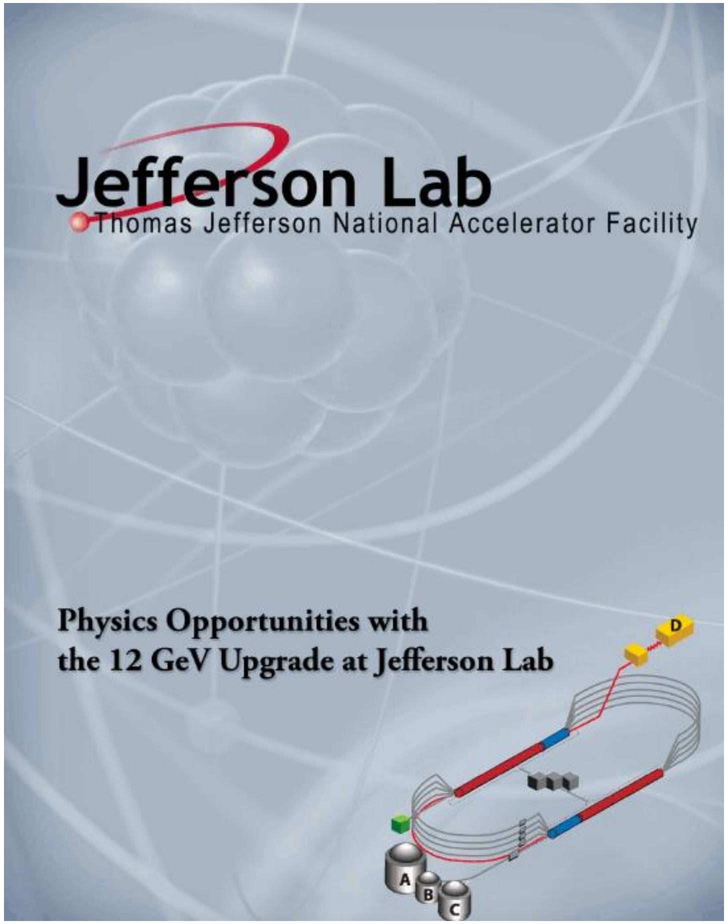


$$\frac{\mu_p G_E^p(Q^2)}{G_M^p(Q^2)} = 1$$

Particle physics paradigm

Thomas Jefferson National Accelerator Facility (JLab)

- 2004 … Mission Need Agreed on upgrade of CEBAF (JLab's accelerator) to 12GeV
- 2014 … 12GeV commissioning beams now being delivered to the experimental halls
- Final cost of upgrade is approximately $370-Million
- Physics of JLab at 12GeV arXiv:1208.1244 [hep-ex]
- *12 GeV Era has begun*



Selected Science Challenges for the coming decade

- Search for exotic hadrons – impossible in quark model
  - *Discovery would force dramatic reassessment of the distinction between the notions of matter fields and force fields*
- Exploit opportunities provided by new data on hadron elastic and transition form factors
  - *Chart infrared evolution of QCD's coupling and dressed-masses*
  - *Reveal correlations that are key to nucleon structure*
  - *Expose the facts or fallacies in contemporray descriptions of hadron structure*

Selected Science Challenges for the coming decade

- Precision experimental study of valence-quark region, and theoretical computation of distribution functions and distribution amplitudes
  - *Computation is critical*
  - *Without it, no amount of data will reveal anything about the theory underlying the phenomena of strong interaction physics*
- Explore and exploit opportunities to use precision-QCD as a probe for physics beyond the Standard Model

Overarching Science Challenge for the coming decade

Discover, theoretically

& experimentally:

- *Realisation of confinement,*
- *its relationship to DCSB*

  – the Origin of Visible Mass
- *and the link between them*

Excerpt from the top-10

The New York Times

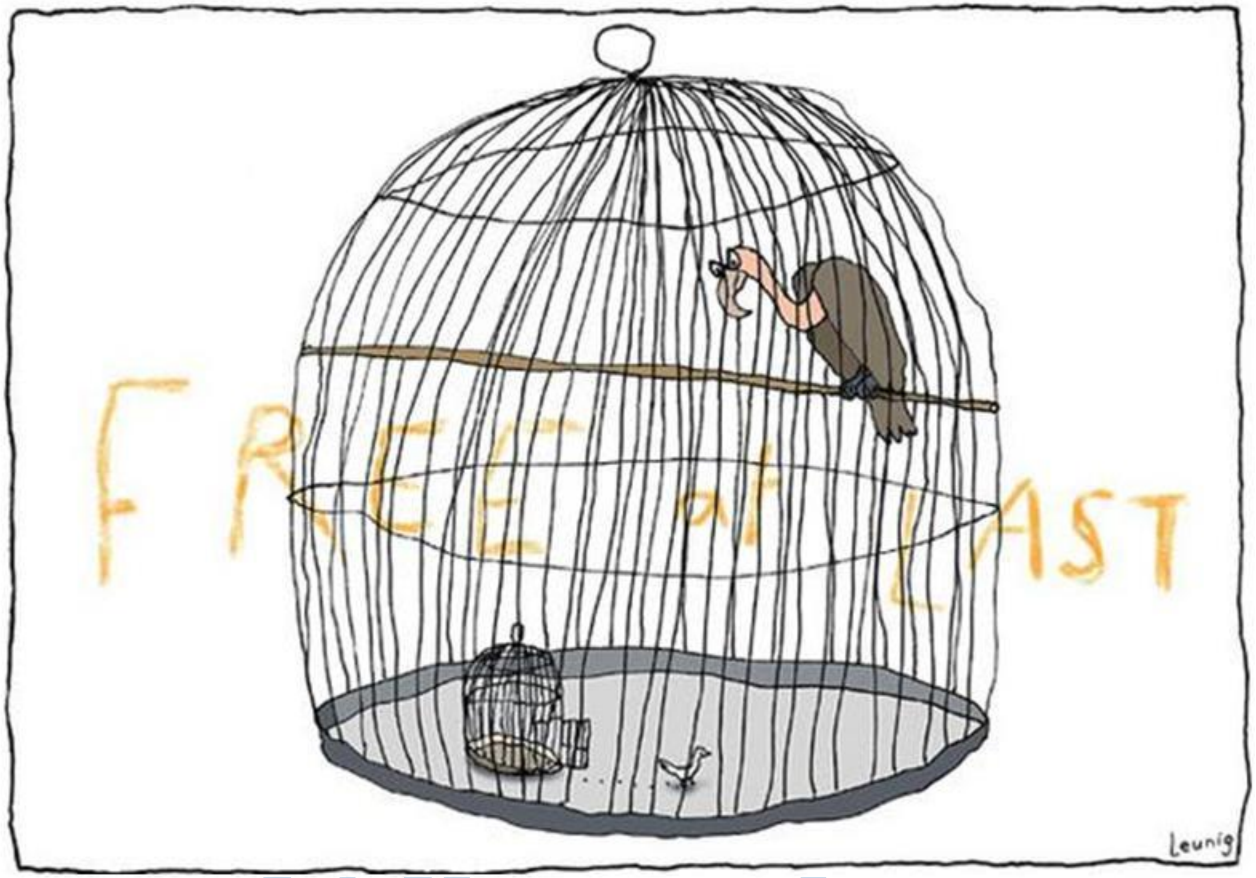
WORLD | U.S. | N.Y. / REGION | BUSINESS | TECHNOLOGY | SCIENCE | HEALTH | SPORTS | OPINION

10 Physics Questions to Ponder for a Millennium or Two

By George Johnson
Published: August 15, 2000

- ***Can we quantitatively understand quark and gluon confinement in quantum chromodynamics and the existence of a mass gap?***

  Quantum chromodynamics is the theory describing the strong nuclear force. Carried by gluons, it binds quarks into particles like protons and neutrons. Apparently, the tiny subparticles are permanently confined: one can't pull a quark or a gluon from a proton because the strong force gets stronger with distance and snaps them right back inside.



What is Confinement?

CLAY MATHEMATICS INSTITUTE

Millennium prize of $1,000,000 for proving that $SU_c(3)$ gauge theory is mathematically well-defined, which will necessarily prove or disprove a confinement conjecture

142 MILLENNIUM PRIZE PROBLEMS

YANG–MILLS EXISTENCE AND MASS GAP. *Prove that for any compact simple gauge group G, a non-trivial quantum Yang–Mills theory exists on $\mathbb{R}^4$ and has a mass gap $\Delta > 0$. Existence includes establishing axiomatic properties at least as strong as those cited in* [**45**, **35**].

5. Comments

An important consequence of the existence of a mass gap is clustering: Let $\vec{x} \in \mathbb{R}^3$ denote a point in space. We let H and $\vec{P}$ denote the energy and momentum, generators of time and space translation. For any positive constant $C < \Delta$ and for any local quantum field operator $\mathcal{O}(\vec{x}) = e^{-i\vec{P}\cdot\vec{x}}\mathcal{O}e^{i\vec{P}\cdot\vec{x}}$ such that $\langle\Omega, \mathcal{O}\Omega\rangle = 0$, one has

$$|\langle\Omega, \mathcal{O}(\vec{x})\mathcal{O}(\vec{y})\Omega\rangle| \leq \exp(-C|\vec{x} - \vec{y}|), \tag{2}$$

as long as $|\vec{x} - \vec{y}|$ is sufficiently large. Clustering is a locality property that, roughly speaking, may make it possible to apply mathematical results established on $\mathbb{R}^4$ to any 4-manifold, as argued at a heuristic level (for a supersymmetric extension of four-dimensional gauge theory) in [**49**]. Thus the mass gap not only has a physical significance (as explained in the introduction), but it may also be important in mathematical applications of four-dimensional quantum gauge theories to geometry. In addition the existence of a uniform gap for finite-volume approximations may play a fundamental role in the proof of existence of the infinite-volume limit.

There are many natural extensions of the Millennium problem. Among other things, one would like to prove the existence of an isolated one-particle state (an upper gap, in addition to the mass gap), to prove confinement, to

Confinement?

CLAY MATHEMATICS INSTITUTE

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Confinement?

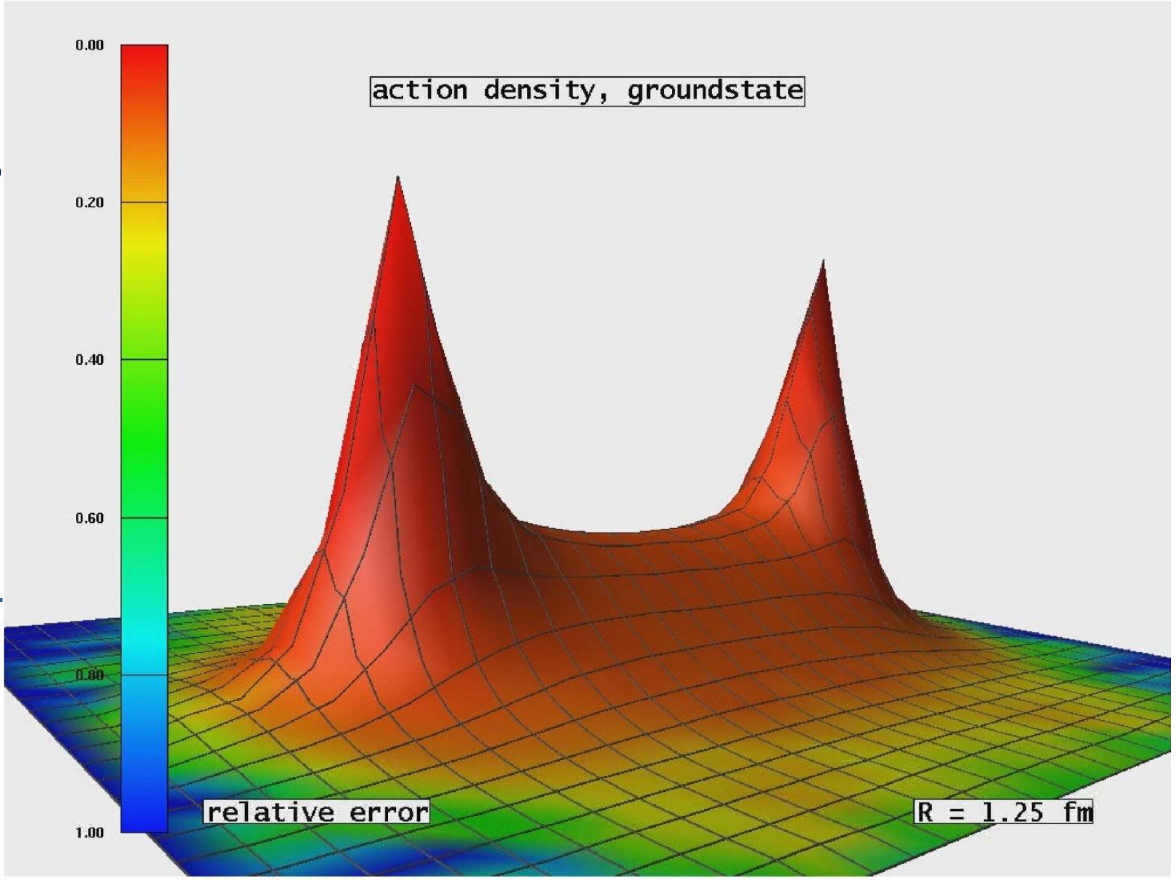
Light quarks & Confinement

- Folklore … *Hall-D Conceptual Design Report(5)*

  "The color field lines between a quark and an anti-quark form flux tubes.

  A unit area placed midway between the quarks and perpendicular to the line connecting them intercepts a constant number of field lines, independent of the distance between the quarks.

  This leads to a constant force between the quarks – and a large force at that, equal to about 16 metric tons."



Light quarks & Confinement

- *Static* picture of confinement

4 × 10^{-27} g

16 × 10^{+6} g

8 × 10^{-27} g

Light quarks & Confinement

- Problem:
  16 tonnes of force
  makes a lot of pions.

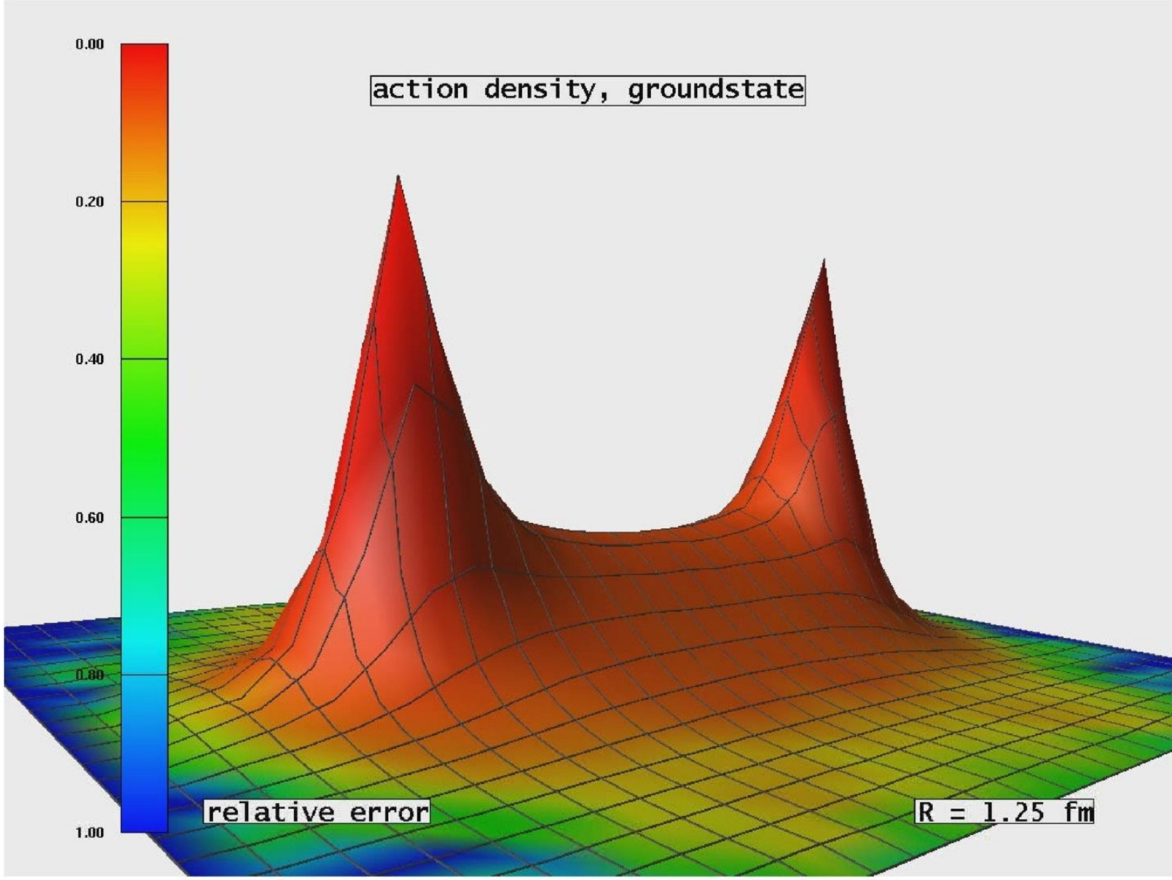
Light quarks & Confinement

- Problem:
  16 tonnes of force
  makes a lot of pions.

G. *Bali* et al., [*PoS LAT2005 (2006) 308*]

Light quarks & Confinement

- In the presence of light quarks, *pair creation seems to occur non-localized and instantaneously*
- No flux tube in a theory with light-quarks.
- ***Flux-tube is not the correct paradigm for confinement in hadron physics***



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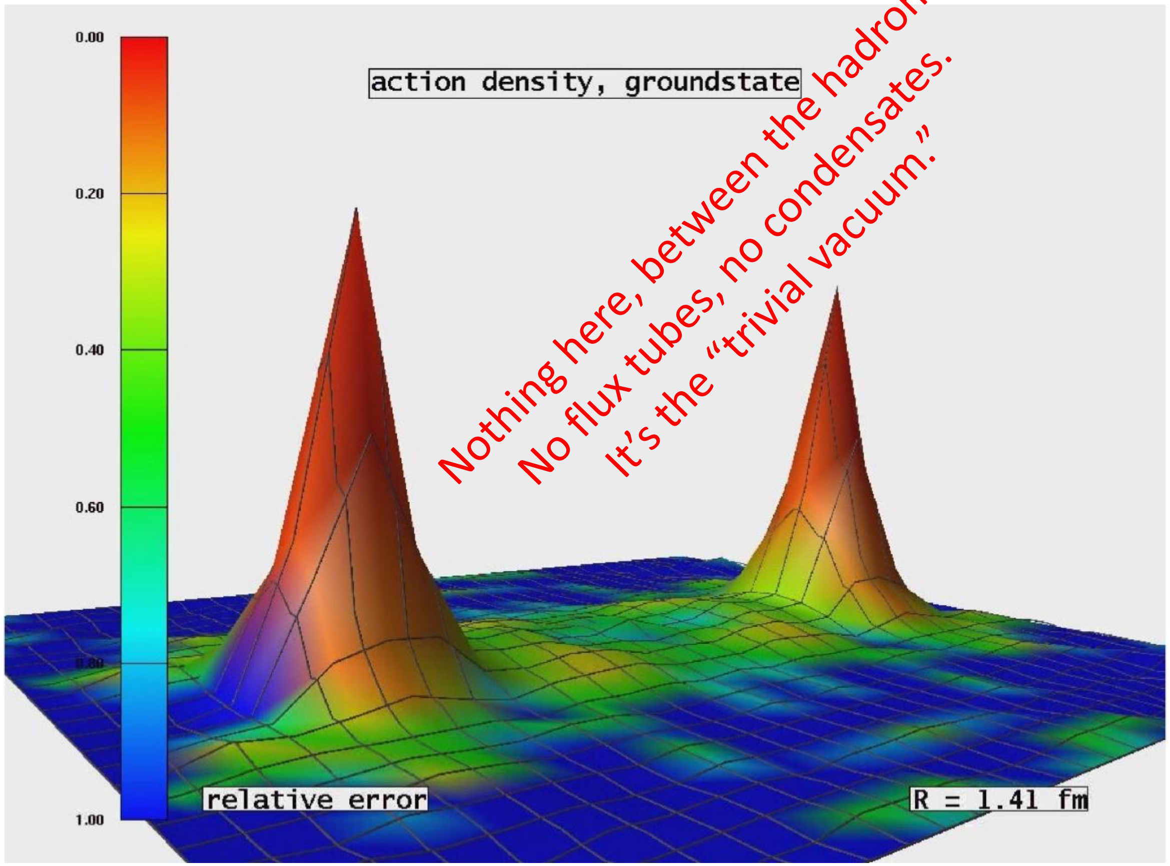
Confinement contains condensates
Brodsky, Roberts, Shrock,Tandy
arXiv:1202.2376 [nucl-th], Phys. Rev. C**85** (2012) 065202



- *Existence of mass-gap in pure-gauge theory*
- Strong evidence supporting this conjecture: lQCD predicts $\Delta \sim 1.5$ GeV
- But $\Delta^2/m_\pi^2 > 100$,

  So, can mass-gap in pure Yang-Mills play any role in understanding confinement when dynamical chiral symmetry breaking (DCSB) ensures existence of an almost-massless strongly-interacting excitation in our Universe?
- Conjecture: If *answer is* not *simply* ***no***, then it is probable that one cannot claim to provide an understanding of confinement without simultaneously explaining its connection with DCSB.
- Conjecture: *Pion must play critical role in any explanation of real-world confinement. Any discussion that omits reference to the pion's role is possibly irrelevant.*

142 MILLENNIUM PRIZE PROBLEMS

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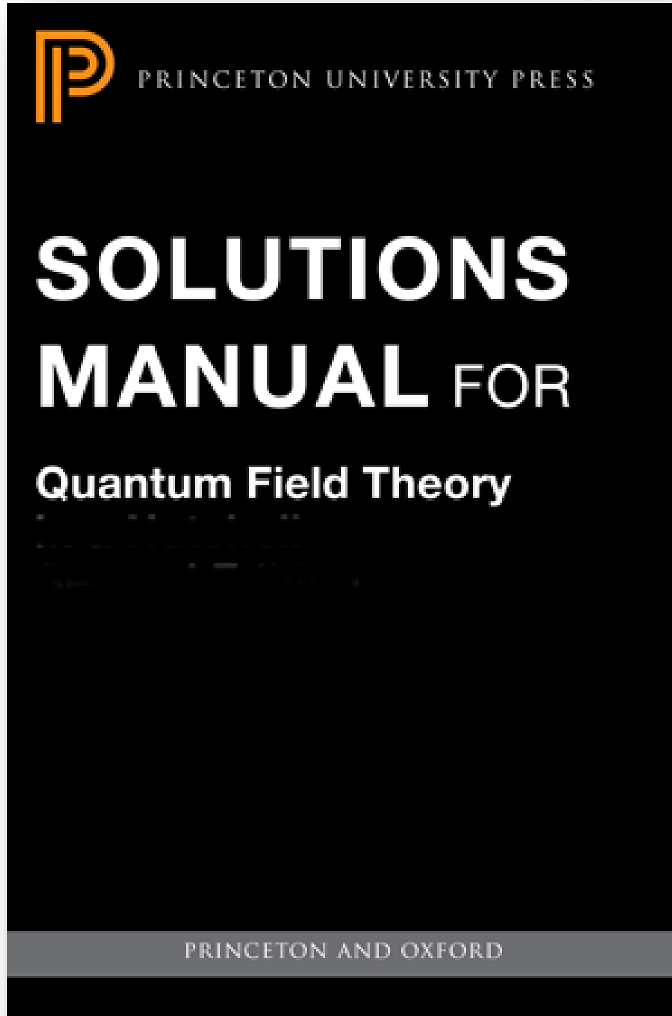


Theoretical Answers

Textbook definition: Gauge Boson

- A gauge boson is a force carrier, mediating one of Nature's fundamental interactions
- All known gauge bosons have spin "1", *i.e*. all are vector bosons.
- Owing to gauge invariance, no term of the form

$$m^2 B_\mu B_\mu$$

can appear in the gauge theory Lagrangian.

- Thus, all gauge bosons are massless in the absence of a Higgs mechanism:
  - Photon … known to be massless
  - W and Z bosons … begin life massless, but known to become massive, owing to Higgs mechanism, which is abundantly clear in the Lagrangian
  - Gluon … there is no Higgs coupling and textbooks describe them as massless

Particle Data Group

Citation: C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C, 40, 100001 (2016) and 2017 update

g or gluon

$$I(J^P) = 0(1^-)$$

SU(3) color octet

Mass $m = 0$. Theoretical value. A mass as large as a few MeV may not be precluded, see YNDURAIN 95.

VALUE	*DOCUMENT ID*		*TECN*	*COMMENT*
• • • We do not use the following data for averages, fits, limits, etc. • • •				
	ABREU	92E	DLPH	Spin 1, not 0
	ALEXANDER	91H	OPAL	Spin 1, not 0
	BEHREND	82D	CELL	Spin 1, not 0
	BERGER	80D	PLUT	Spin 1, not 0
	BRANDELIK	80C	TASS	Spin 1, not 0

gluon REFERENCES

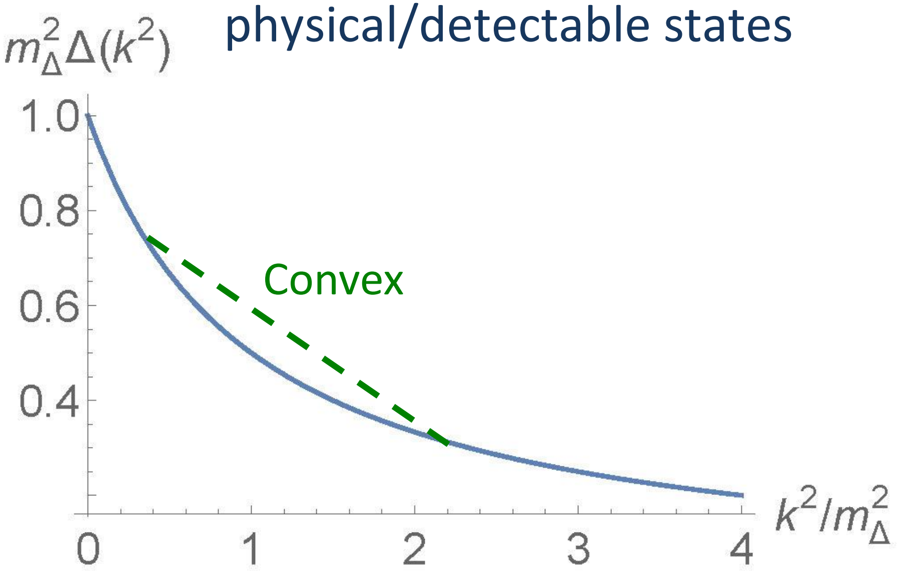
YNDURAIN	95	PL B345 524	F.J. Yndurain	(MADU)
ABREU	92E	PL B274 498	P. Abreu *et al.*	(DELPHI Collab.)
ALEXANDER	91H	ZPHY C52 543	G. Alexander *et al.*	(OPAL Collab.)
BEHREND	82D	PL B110 329	H.J. Behrend *et al.*	(CELLO Collab.)
BERGER	80D	PL B97 459	C. Berger *et al.*	(PLUTO Collab.)
BRANDELIK	80C	PL B97 453	R. Brandelik *et al.*	(TASSO Collab.)

Normal Particle

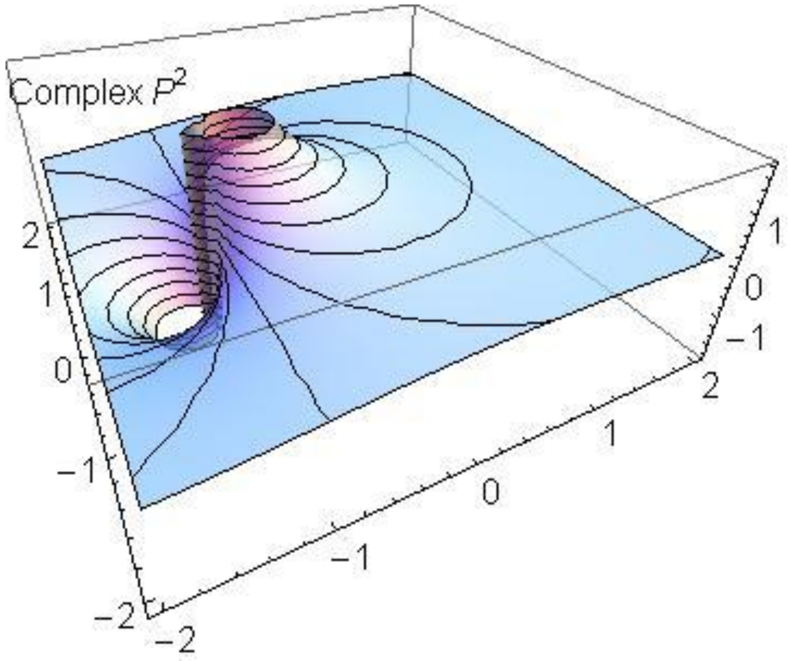
- Free-particle propagator
  - Convex function
- Spectral function is positive

$$\Delta(k^2) = \int_0^\infty ds\, \frac{\rho(s)}{s+k^2}$$

  - *ρ(s)*>0
- Corresponds to a state with positive norm
  - Will appear in the space of physical/detectable states

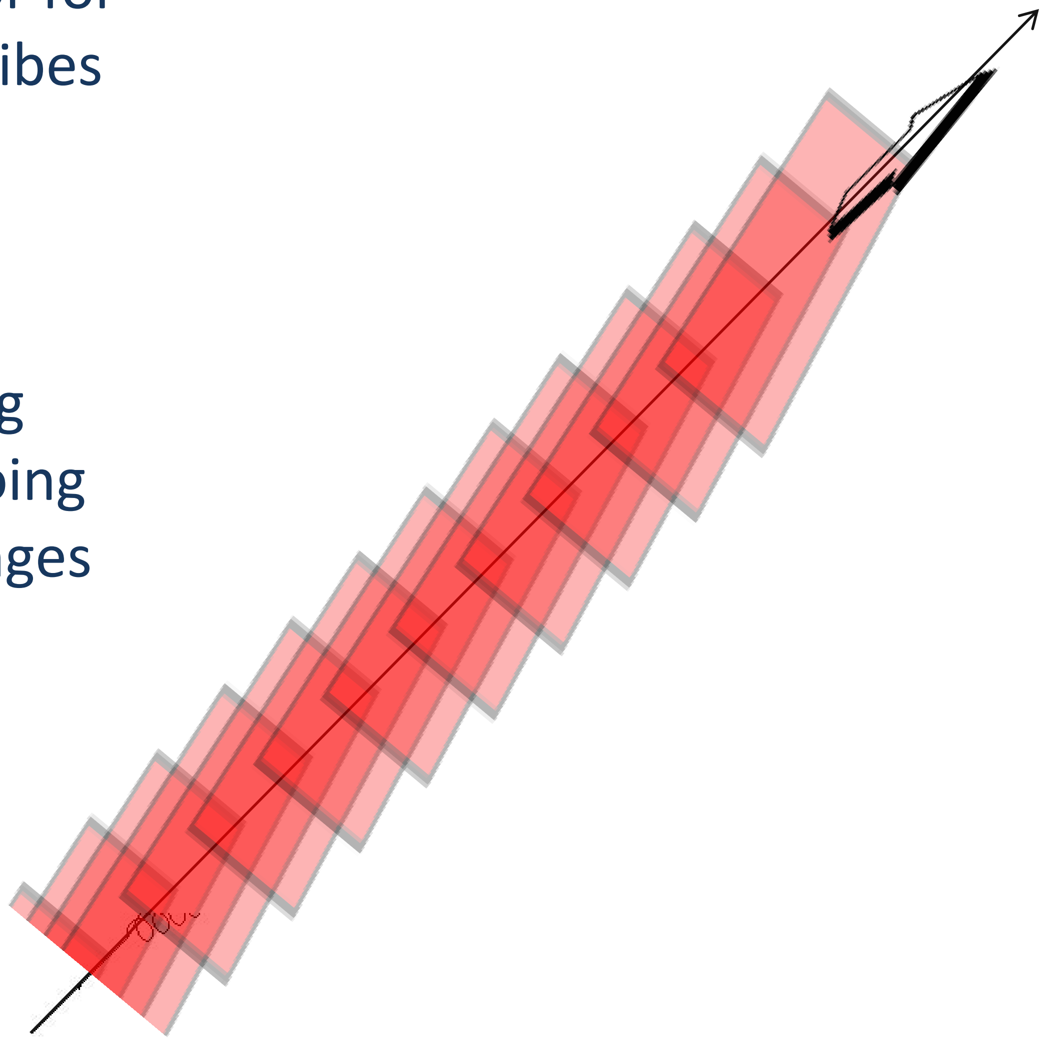


- Exhibits a simple pole on the timelike axis



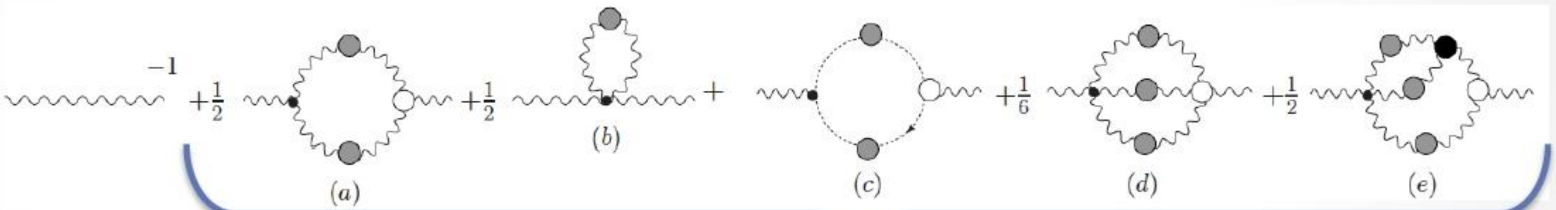
Plane wave propagation

- Feynman propagator for a free particle describes a Plane Wave
- A particle begins to propagate
- It can proceed a long way before undergoing any qualitative changes



Pinch Technique: Theory and Applications
Daniele Binosi & Joannis Papavassiliou
Phys. Rept. 479 (2009) 1-152

$$\Delta^{-1}_{\mu\nu}(q) =$$



$$\Pi_{\mu\nu}(q)$$

$$\Pi_{\mu\nu}(q) = P_{\mu\nu}(q)\Pi(q)$$

$$P_{\mu\nu}(q) = g_{\mu\nu} - q_\mu q_\nu / q^2$$

Gluon Gap Equation

Bridging a gap between continuum-QCD and ab initio predictions of hadron observables, D. Binosi *et al.*, arXiv:1412.4782 [nucl-th], Phys. Lett. B742 (2015) 183-188

In QCD: Gluons become massive!

- Running gluon mass

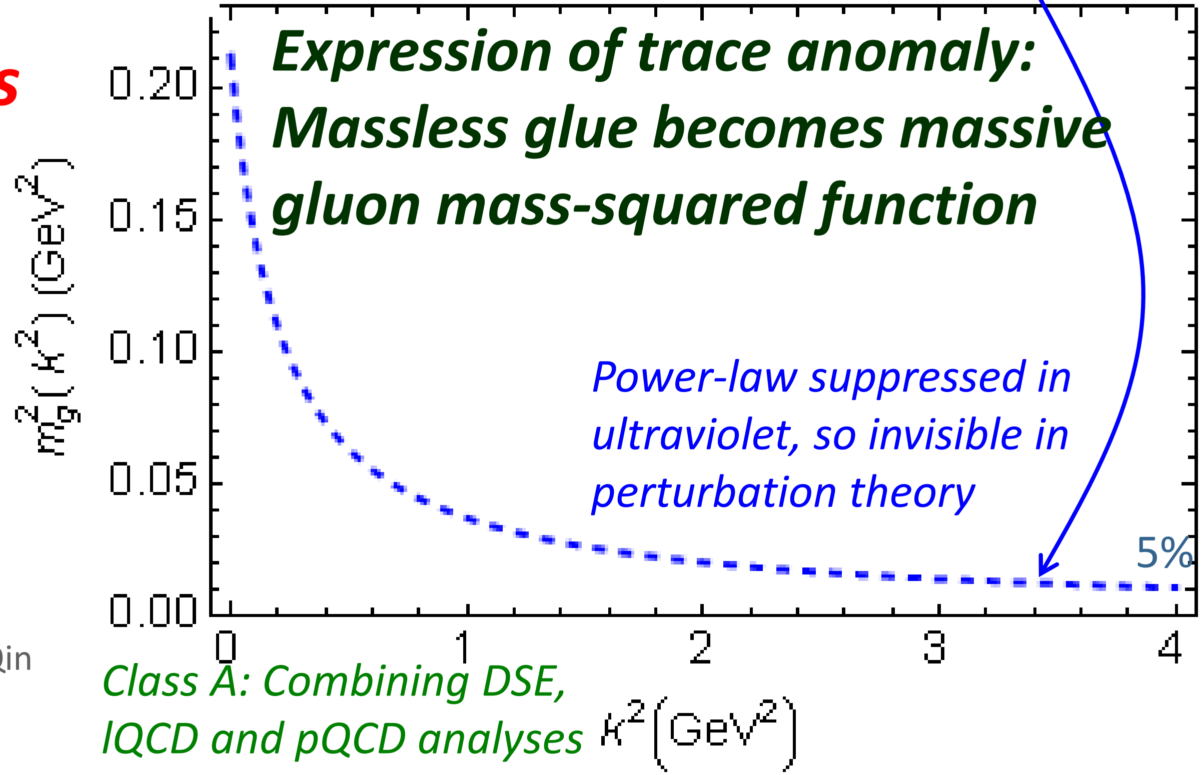
$$d(k^2) = \frac{\alpha(\zeta)}{k^2 + m_g^2(k^2;\zeta)}$$

$$\alpha_s(0) = 2.77 \approx 0.9\pi\,,\ m_g^2(0) = (0.46\,\text{GeV})^2$$

- Gluons are ***cannibals*** – a particle species whose members become massive by eating each other!

$\mu_g \approx ½\, m_p$

$$m_g^2(k^2) \approx \frac{\mu_g^4}{\mu_g^2 + k^2}$$



Class A: Combining DSE, lQCD and pQCD analyses of QCD's gauge sector

Interaction model for the gap equation, S.-x.Qin *et al.*, arXiv:1108.0603 [nucl-th], Phys. Rev. C **84** (2011) 042202(R) [5 pages]

Confined particle

- All QCD solutions for gluon & quark propagators exhibit an inflection point in k^2 … consequence of the running-mass function

⇒ Spectral function is NOT positive

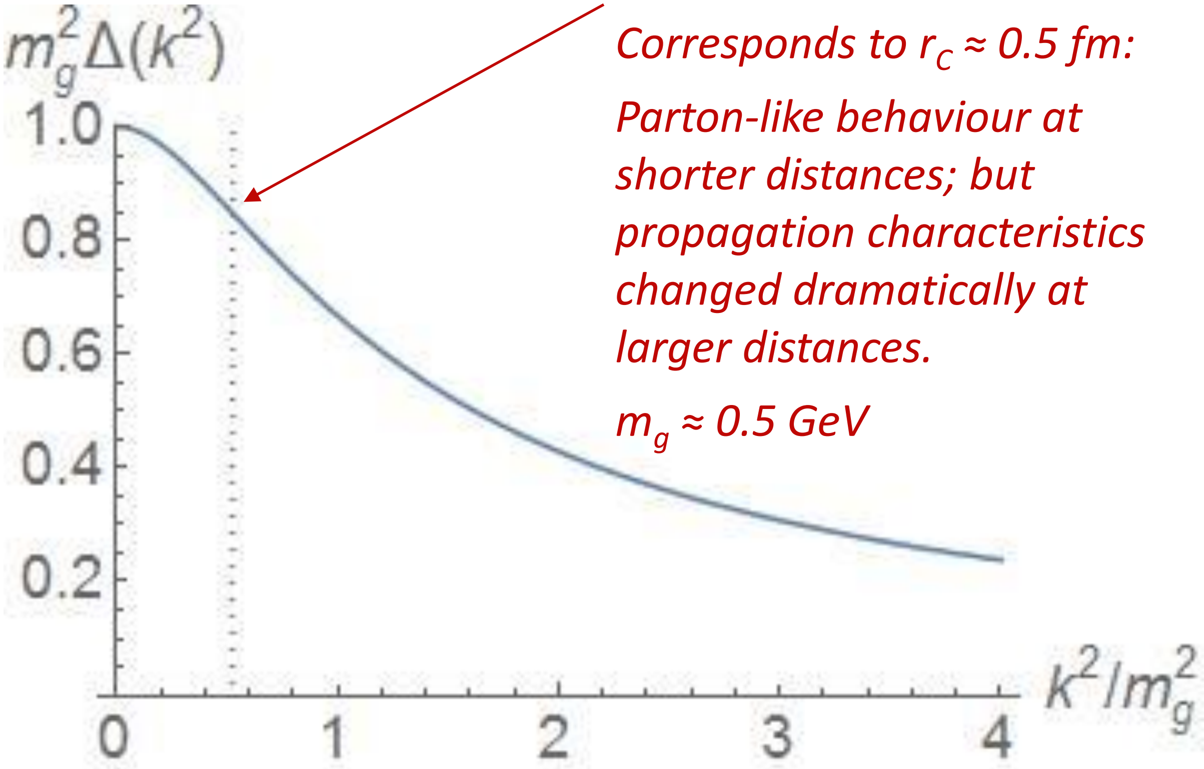
⇒ Such states have negative norm (negative probability)

⇒ Negative norm states are not observable

⇒ This object is confined!

$$\Delta(k^2) = \int_0^\infty ds\, \frac{\rho(s)}{s+k^2}$$

Sum of "probabilities"



Inflexion point

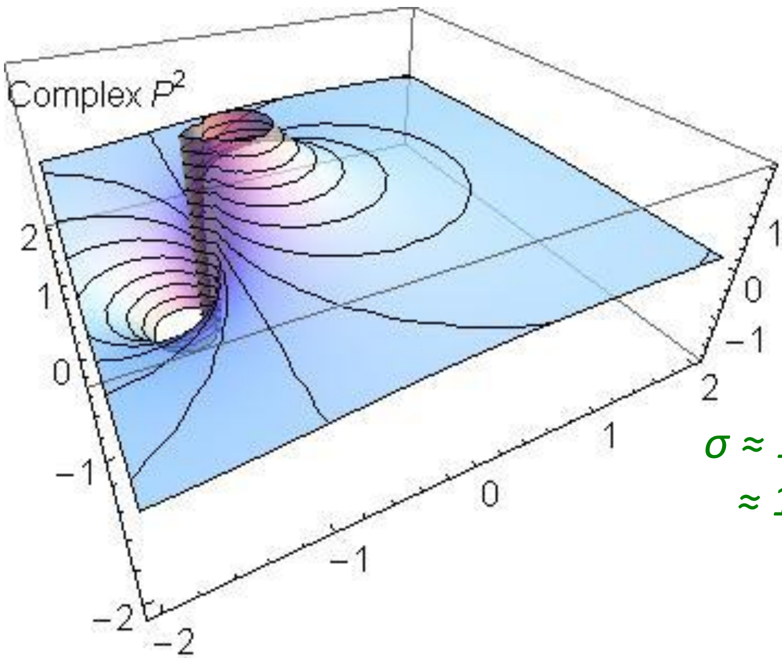
Corresponds to $r_C \approx 0.5$ fm:

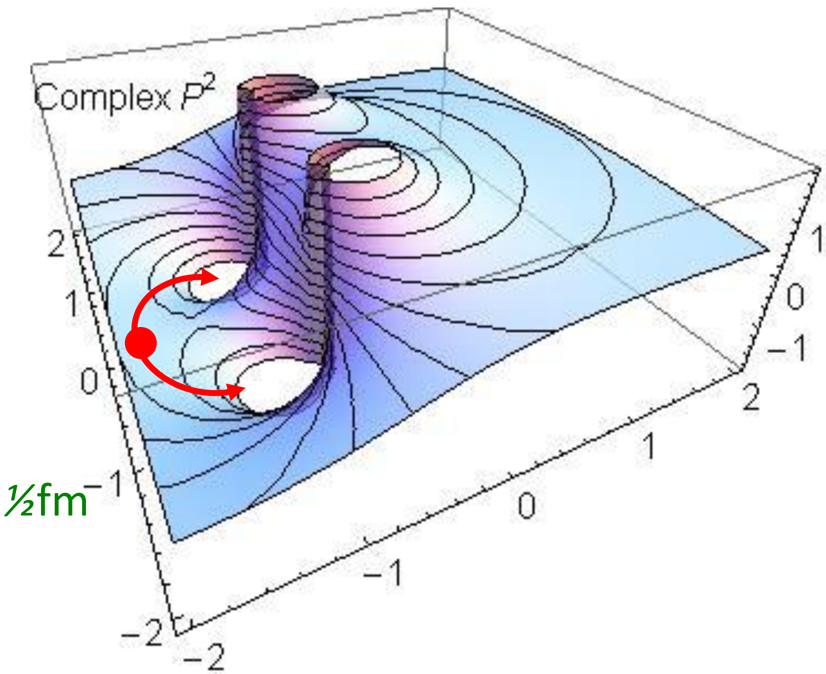
Parton-like behaviour at shorter distances; but propagation characteristics changed dramatically at larger distances.

$m_g \approx 0.5$ GeV

Confinement

- Meaning …





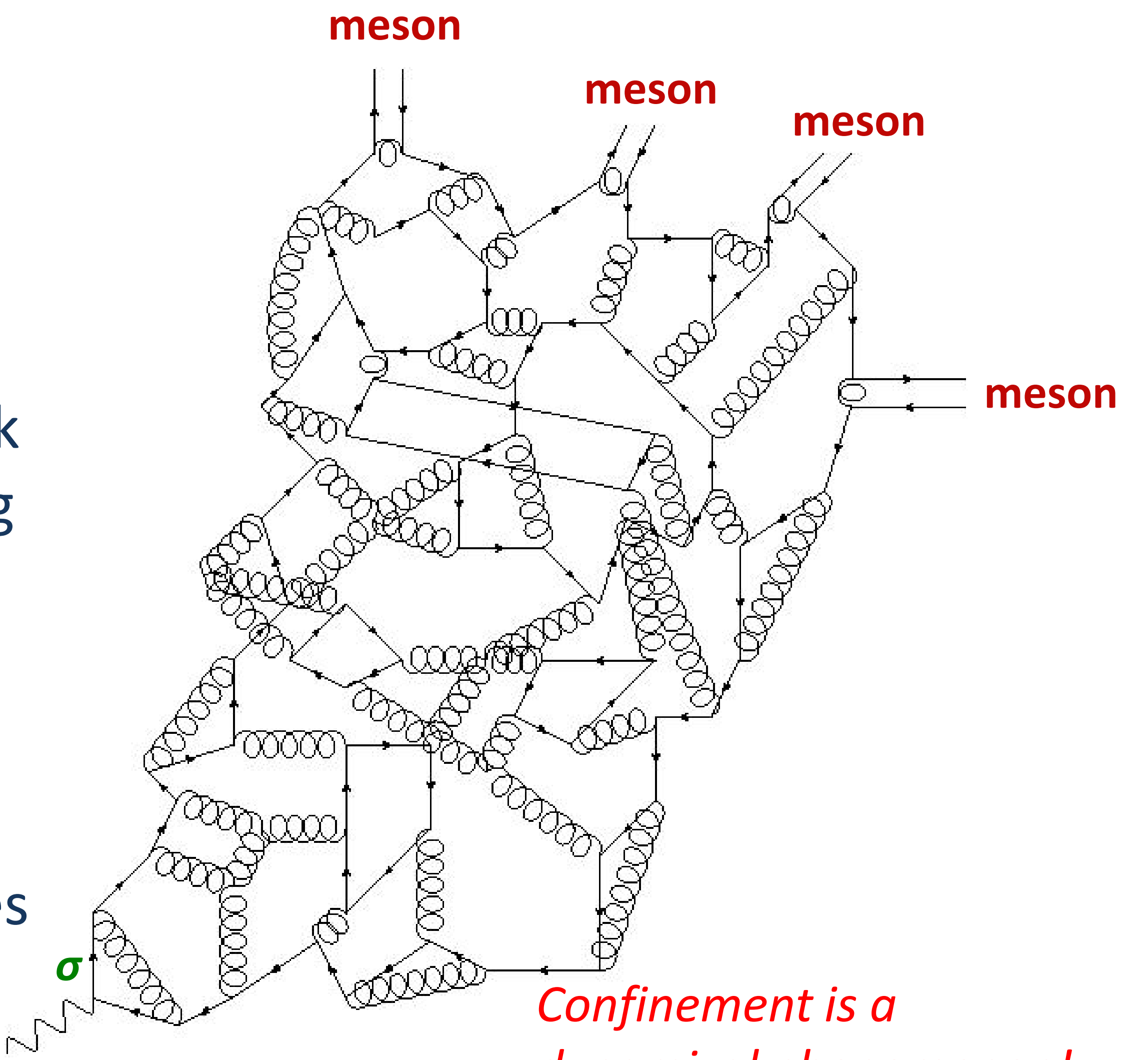
$\sigma \approx 1/Im(m)$
$\approx 1/2\Lambda_{QCD} \approx$ ½fm^{-1}

Real-particle mass-pole splits, moving into pair(s) of complex conjugate singularities, (or qualitatively analogous structures chracterised by a dynamically generated mass-scale)

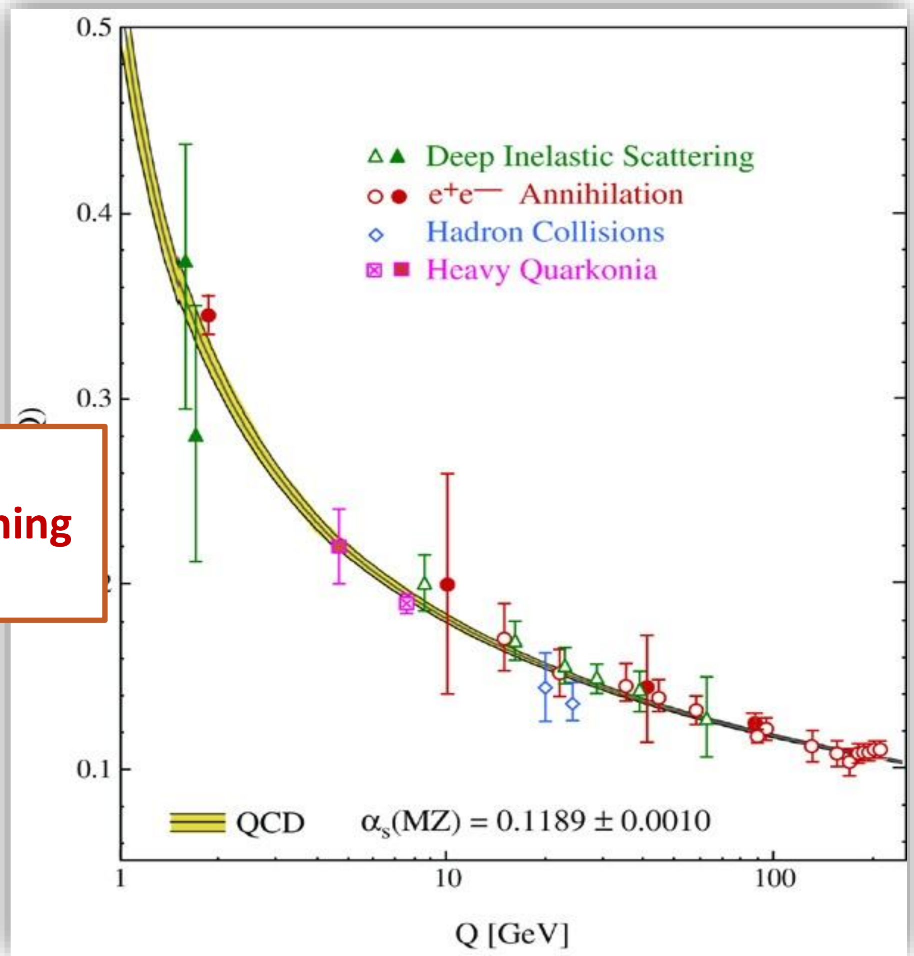
Propagation described by rapidly damped wave & hence state cannot exist in observable spectrum

Quark Fragmentation

- A quark begins to propagate
- But after each "step" of length $\boldsymbol{\sigma \approx 1/m_g}$, on average, an interaction occurs, so that the quark *loses* its identity, sharing it with other partons
- Finally, a cloud of partons is produced, which coalesces into colour-singlet final states



Confinement is a dynamical phenomenon!

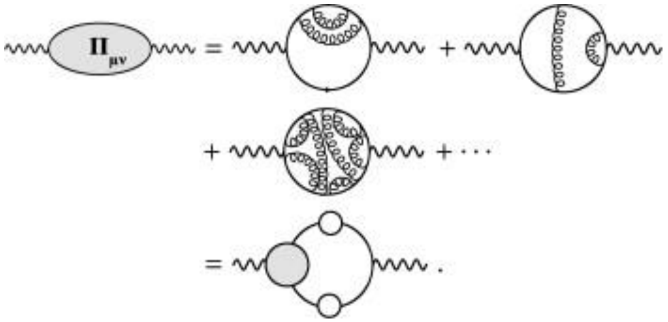


⇐
What's happening out here?!

QCD's Running Coupling

QED Running Coupling

- Quantum gauge field theories defined in four spacetime dimensions,
  - Lagrangian couplings and masses come to depend on a mass scale
  - Can often be related to the energy or momentum at which a given process occurs.
- Archetype is QED, for which there is a sensible perturbation theory.
- QED, owing to the Ward identity:
  - a single running coupling
  - measures strength of the photon-charged-fermion vertex
  - can be obtained by summing the virtual processes that dress the bare photon, *viz*. by computing the photon vacuum polarisation.



- QED's running coupling is known to great accuracy and the running has been observed directly.

Process-dependent (emergent) Effective Charge

- [Grunberg:1982fw]: process-dependent procedure

$$\int_0^{1^-} dx_{Bj} \left(g_1^p \left(x_{Bj}, Q^2\right) - g_1^n \left(x_{Bj}, Q^2\right) \right) \equiv \frac{g_A}{6} \left[1 - \frac{\alpha_{g_1}\left(Q^2\right)}{\pi} \right]$$

- an effective running coupling defined to be completely fixed by leading-order term in the perturbative expansion of a given observable in terms of the canonical running coupling.
  - Obvious difficulty/drawback = process-dependence itself.
  - Effective charges from different observables can in principle be algebraically connected to each other via an expansion of one coupling in terms of the other.
  - But, any such expansion contains infinitely many terms; and connection doesn't provide a given process-dependent charge with ability to predict another observable, since the expansion is only defined after both effective charges are independently constructed.

QCD Running Coupling

- Four individual, apparently UV-divergent interaction vertices in perturbative QCD ⇒ possibly four distinct IR couplings.
  - Naturally, if nonperturbatively there are two or more couplings, they must all become equivalent on the perturbative domain.
- Questions:
  - How many distinct running couplings exist in nonperturbative QCD?
  - How can they be computed?
  - If defined using a 3- or 4-point vertex, which arrangement of momenta defines the running? (Infinitely many choices.)
- ***Claim*: Nonperturbatively, too,**
  QCD possesses a unique running coupling.
- Alternative
  - Possibly an essentially different RGI intrinsic mass-scale for each coupling
  - Then BRST symmetry irreparably broken by nonperturbative dynamics
  - Conclusion: QCD non-renormalisable owing to IR dynamics.
- No empirical evidence to support such a conclusion: QCD does seem to be a well-defined theory at all momentum scales, possibly owing to dynamical generation of gluon and quark masses, which are large at IR momenta.

Process-dependent Effective Charge

S.J. Brodsky, H.J. Lu, Phys. Rev. D 51 (1995) 3652
S.J. Brodsky, G.T. Gabadadze, A.L. Kataev, H.J. Lu, Phys. Lett. B 372 (1996) 133
A. Deur, V. Burkert, Jian-Ping Chen, Phys.Lett. B 650 (2007) 244-248

- α_{g1} – Bjorken sum rule

$$\int_0^{1^-} dx_{Bj} \left(g_1^p\left(x_{Bj}, Q^2\right) - g_1^n\left(x_{Bj}, Q^2\right) \right) \equiv \frac{g_A}{6}\left[1 - \frac{\alpha_{g_1}\left(Q^2\right)}{\pi}\right]$$

$g_1^{p,n}$ are spin-dependent proton and neutron structure functions

g_A is the nucleon flavour-singlet axial-charge

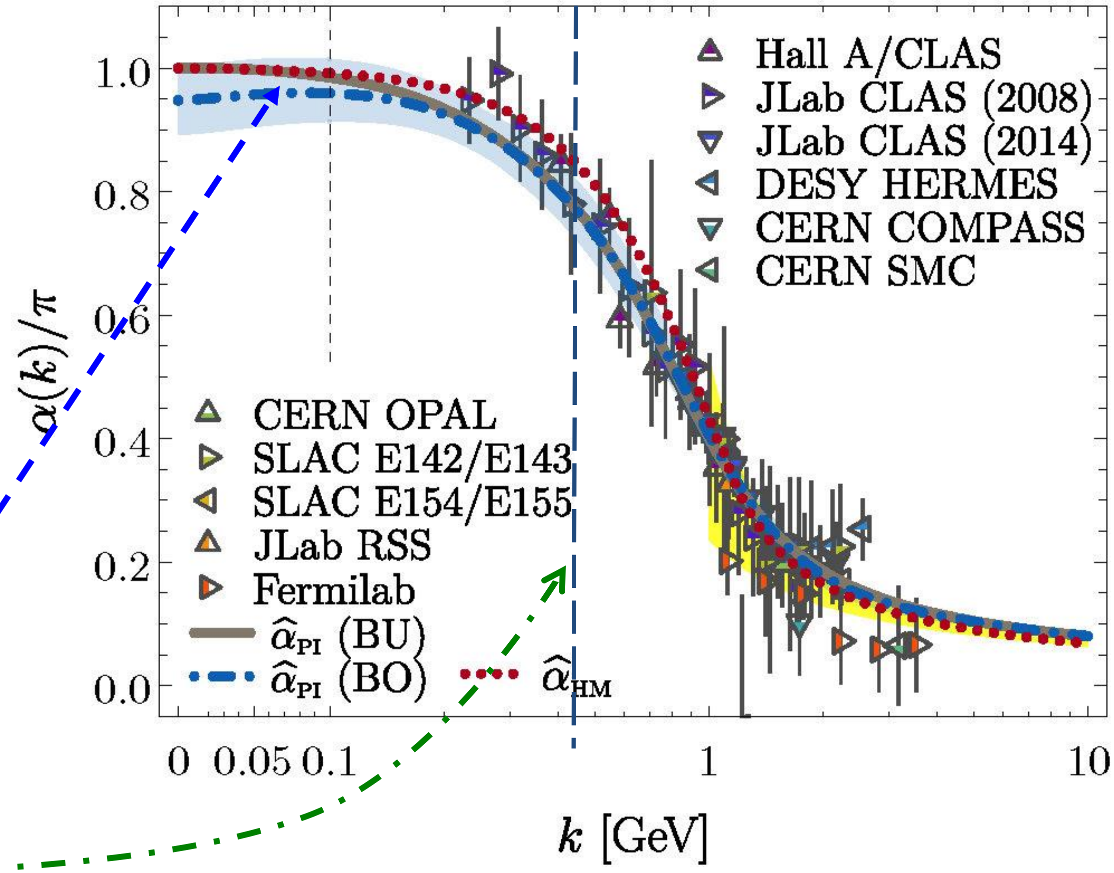
- Merits, *e.g*.
  - Existence of data for a wide range of k^2
  - Tight sum-rules constraints on the behaviour of the integral at the IR and UV extremes of k^2
  - isospin non-singlet ⇒ suppression of contributions from numerous processes that are hard to compute and hence might muddy interpretation of the integral in terms of an effective charge
    - Δ resonance
    - Disconnected (gluon mediated) diagrams

Process-independent effective-charge in QCD

- Modern continuum & lattice methods for analysing gauge sector enable

  "Gell-Mann – Low"

  running charge to be defined in QCD
- Combined continuum and lattice analysis of QCD's gauge sector yields a parameter-free prediction
- N.B. Qualitative change in $\hat{\alpha}_{PI}(k)$ at $k \approx ½\, m_p$



Process independent strong running coupling
Binosi, Mezrag, Papavassiliou, Roberts, Rodriguez-Quintero
arXiv:1612.04835 [nucl-th], Phys. Rev. D 96 (2017) 054026/1-7
The QCD Running Coupling,
A. Deur, S. J. Brodsky and G. F. de Teramond,
Prog. Part. Nucl. Phys. **90** (2016) 1-74

QCD Effective Charge

- Near precise agreement between process-independent

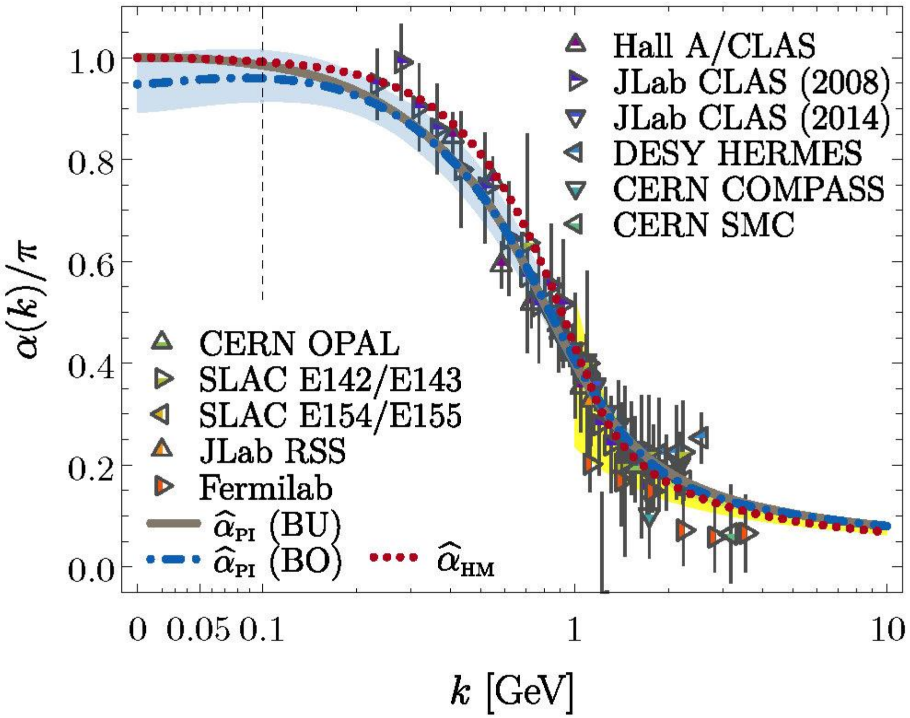
  $\hat{\alpha}_{PI}$ and α_{g1}
  & $\hat{\alpha}_{PI} \approx \alpha_{HM}$

- Perturbative domain:

$$\alpha_{g_1}(k^2) = \alpha_{\overline{\rm MS}}(k^2)(1 + 1.14\,\alpha_{\overline{\rm MS}}(k^2) + \ldots)\,,$$
$$\widehat{\alpha}_{\rm PI}(k^2) = \alpha_{\overline{\rm MS}}(k^2)(1 + 1.09\,\alpha_{\overline{\rm MS}}(k^2) + \ldots)\,,$$

  difference = (1/20) $\alpha_{\overline{MS}}^2$

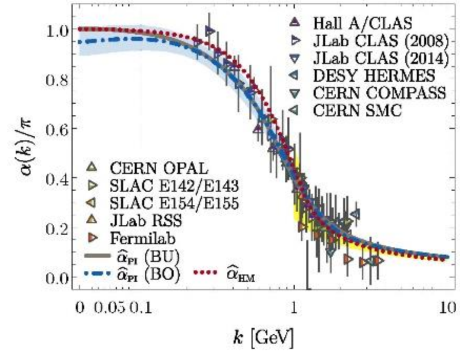
- Parameter-free prediction:
  - Curve completely determined by results obtained for gluon & ghost two-point functions using continuum and lattice-regularised QCD.



Data = process dependent effective charge [Grunberg:1982fw]:

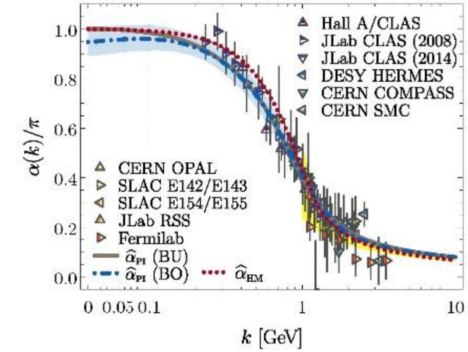
α_{g1}, defined via Bjorken Sum Rule

QCD Effective Charge



- $\hat{\alpha}_{PI}$ is a new type of effective charge
  - direct analogue of the Gell-Mann–Low effective coupling in QED, *i.e.* completely determined by the gauge-boson two-point function.
- $\hat{\alpha}_{PI}$ is
  - process-independent
  - appears in every one of QCD's dynamical equations of motion
  - known to unify a vast array of observables
- $\hat{\alpha}_{PI}$ possesses an infrared-stable fixed-point
  - Nonperturbative analysis demonstrating absence of a Landau pole in QCD
- QCD is IR finite, owing to dynamical generation of gluon mass-scale, which also serves to eliminate the Gribov ambiguity
- Asymptotic freedom ⇒ QCD is well-defined at UV momenta
- ***QCD is therefore unique amongst known 4D quantum field theories***
  - ***Potentially, defined & internally consistent at all momenta***

QCD Effective Charge



- $\hat{\alpha}_{PI}$ is a new type of effective charge
  - direct analogue of the Gell-Mann–Low effective coupling in QED, *i.e.* completely determined by the gauge … tion.
- $\hat{\alpha}_{PI}$ is
  - pro…
  - app…
  - know…
- $\hat{\alpha}_{PI}$ posse…
  - Nonp… n QCD
- QCD is IR … of gluon mass-scale, which als… … the Gribov ambiguity
- Asymptotic freedom ⇒ QCD is well-defined at UV momenta
- ***QCD is therefore unique amongst known 4D quantum field theories***
  - ***Potentially, defined & internally consistent at all momenta***

Conceivably, therefore, QCD can serve as a basis for theories that take physics beyond the Standard Model

Spontaneous(Dynamical) Chiral Symmetry Breaking = Mass Generation

The 2008 Nobel Prize in Physics was divided, one half awarded to Yoichiro Nambu

> *"for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics"*

Nambu - Jona-Lasinio Model

Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. I
Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122 (1961) 345–358

Dynamical Model Of Elementary Particles Based On An Analogy With Superconductivity. II
Y. Nambu, G. Jona-Lasinio, Phys.Rev. 124 (1961) 246-254

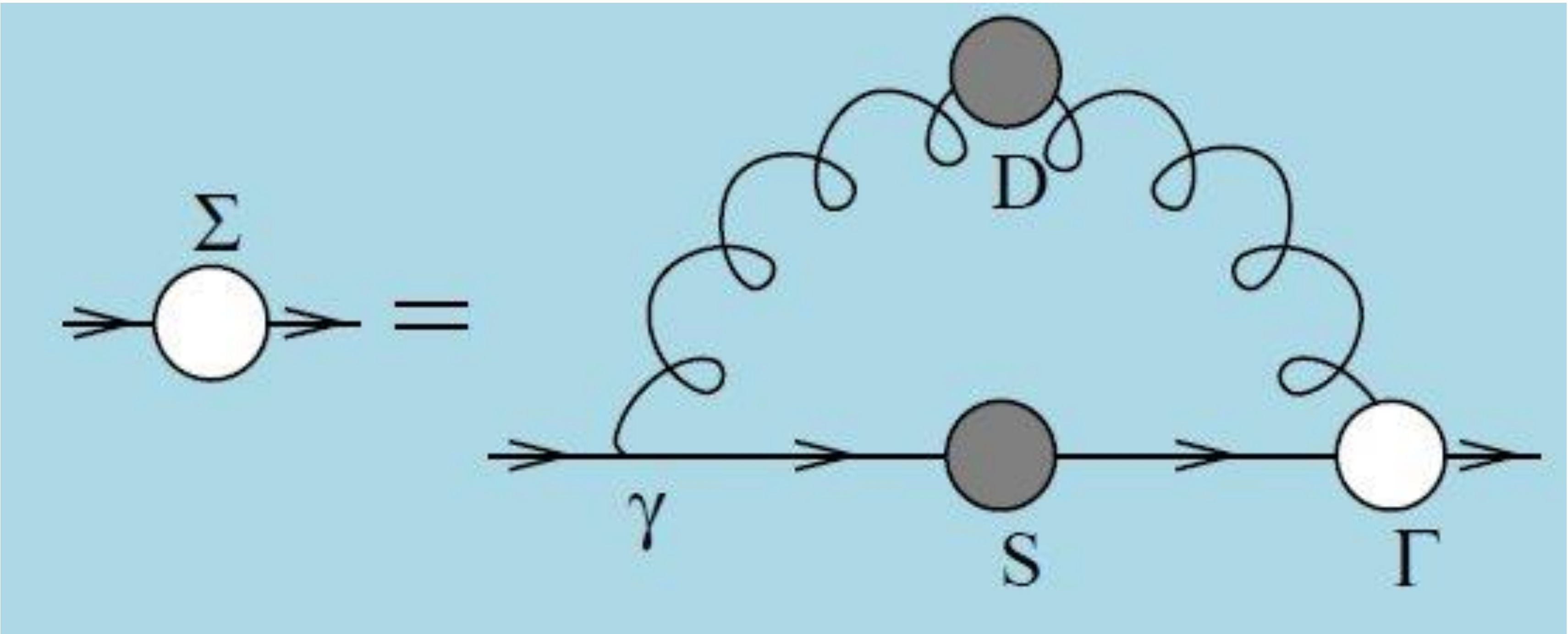
- Treats a massless (chirally-invariant) four-fermion Lagrangian & solves the gap equation in Hartree-Fock approximation (analogous to rainbow truncation)

The following Lagrangian density will be assumed ($\hbar=c=1$):

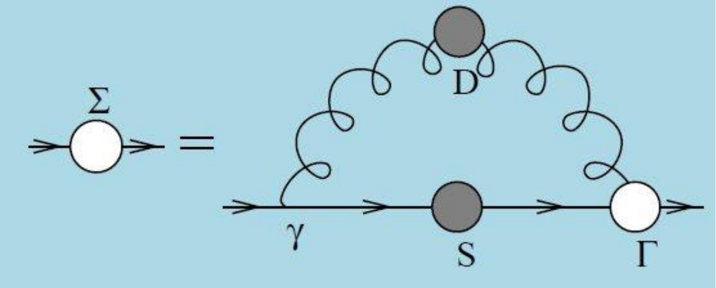
$$L=-\bar{\psi}\gamma_\mu\partial_\mu\psi+g_0[(\bar{\psi}\psi)^2-(\bar{\psi}\gamma_5\psi)^2]. \qquad (2.6)$$

The coupling parameter g_0 is positive, and has dimensions $[\text{mass}]^{-2}$. The γ_5 invariance property of the interaction is evident from Eq. (2.5). According to the

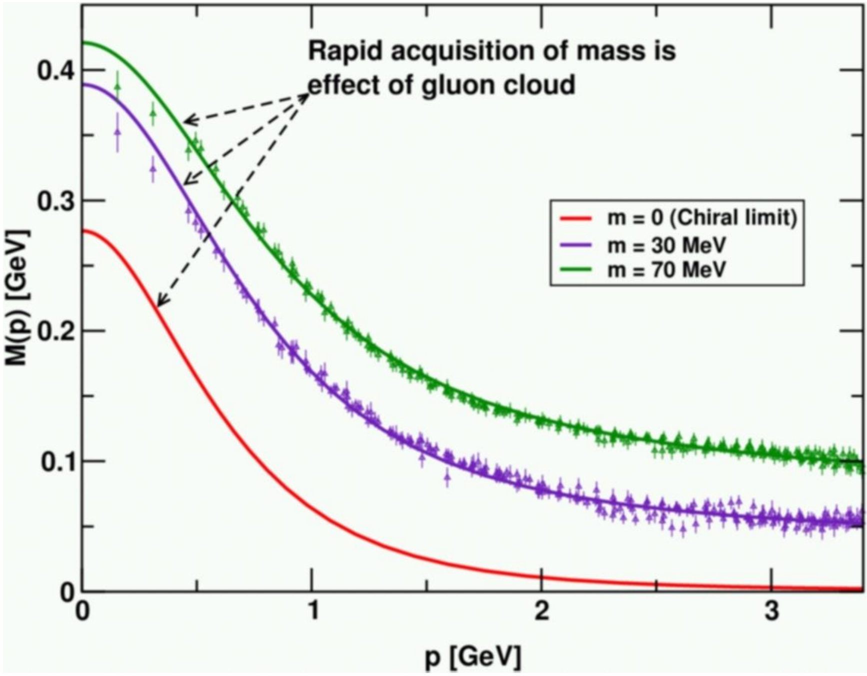
$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Quark Gap Equation



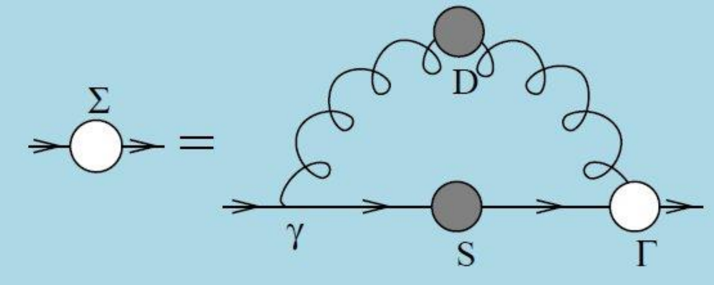
Dynamical Chiral Symmetry Breaking



- *DCSB is a fact in QCD*
  - ***Dynamical***, not spontaneous
    - Add nothing to *QCD* ,

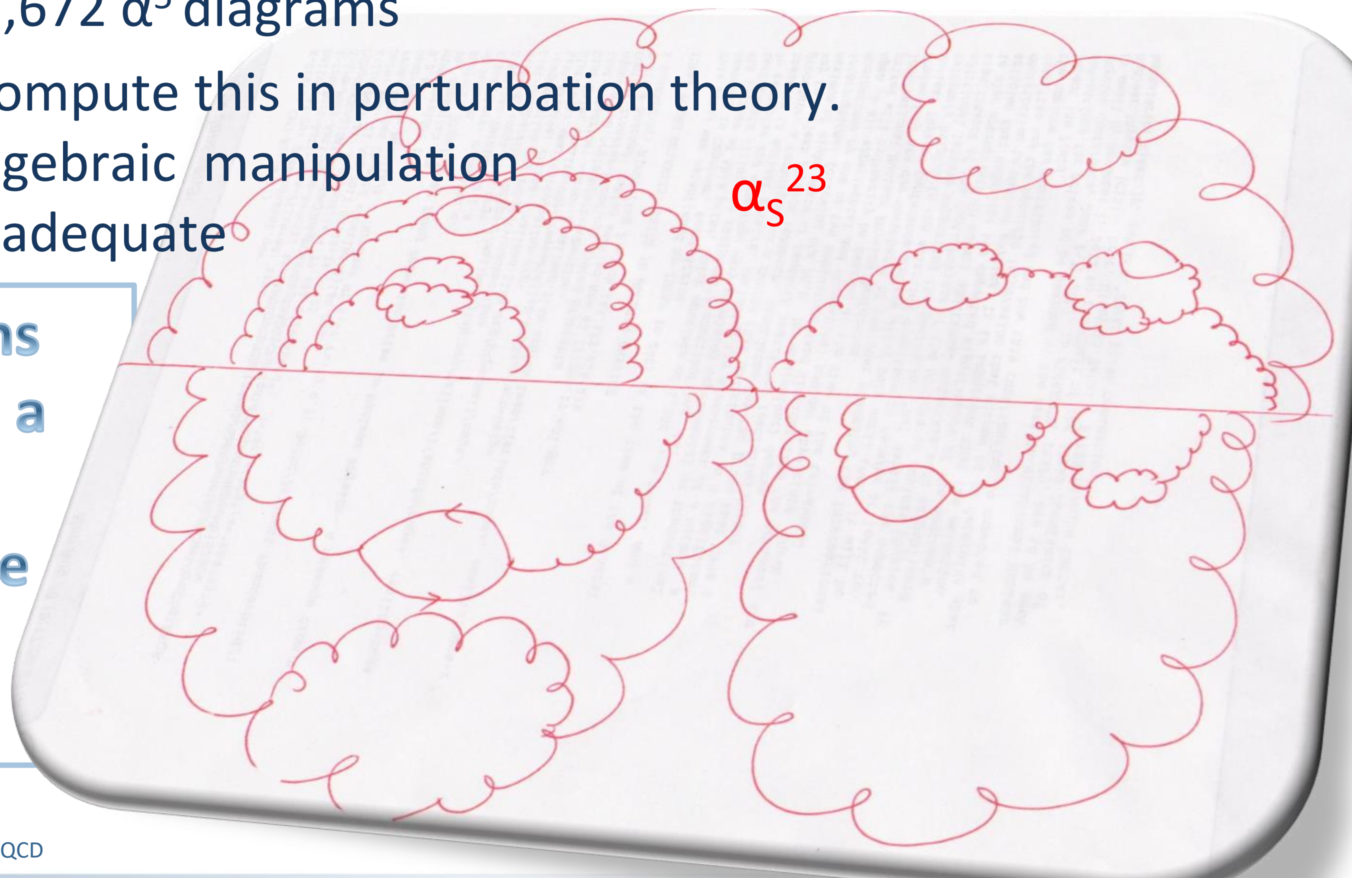
      No Higgs field, nothing!

      Effect achieved purely through quark+gluon dynamics.
  - It's the most important mass generating mechanism for visible matter in the Universe.
    - Responsible for ≈98% of the proton's mass.
    - Higgs mechanism is (*almost*) irrelevant to light-quarks.

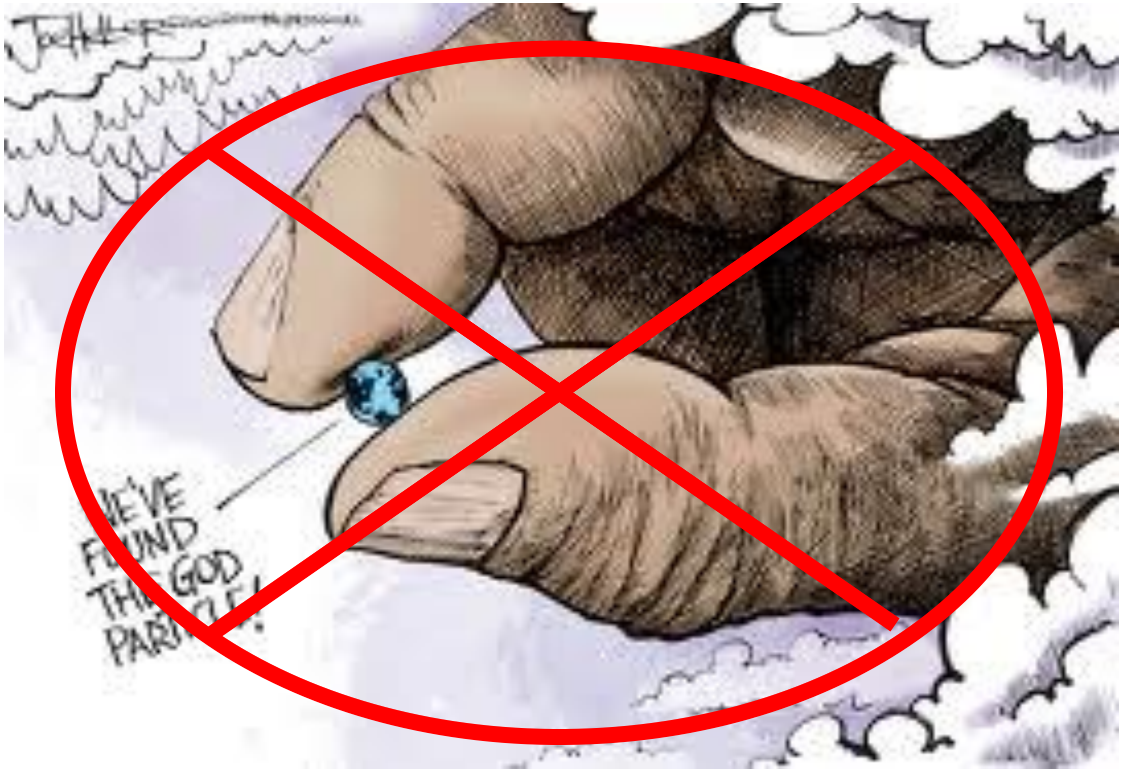


Where does the mass come from?

- Deceptively simply picture
- Corresponds to the sum of a countable infinity of diagrams.
  NB. QED has 12,672 α^5 diagrams
- Impossible to compute this in perturbation theory.
  The standard algebraic manipulation tools are just inadequate



Just one of the terms that are summed in a solution of the simplest, reasonable truncation of QCD's gap equation



DCSB:

Mass from Nothing

Ward-Green-Takahashi identities

- Quantum mechanics is typically formulated using a Hamiltonian
  - There are conservation laws, associated with operators that commute with the Hamiltonian
- But quantum mechanics can only treat a finite number of bodies
  - The particles retain their identity despite interacting with a "potential"
- Quantum field theory is the only known way to reconcile quantum mechanics and special relativity
  - Particles are replaced by fields
  - Conservation laws are expressed via
    - Noether currents
    - Ward-Green-Takahasi identities

Express relationships between correlations that involve a different number of "particles"

- Simplest Ward identity
  - QED Z_1=Z_2

General structure: WGTIs relate (n+1)-body to n-body problems

⇒ interaction-induced divergences in 3-point photon-fermion-antifermion vertex are identical to those for the 2-point single fermion propagator

Ward Identity

…udies and the latter in studying atmospheric ionization at ground level. These increases in ionization are considered to be due to radioactive matter brought down with the rain. Between 0935 and 1900 hr. GMT on November 29 at Ottawa precipitation was falling. The precipitation started as snow and changed to rain about 1400 hr. Compared with the results of Doan and Wait and McNish the 35 percent increase in the soft component registered at Ottawa by counters seems too high to be explained in the same way, unless there was an exceptionally high density of radioactive matter in the atmosphere at the time. An alternative, but not very likely explanation, might be that there was a burst of hard gamma-rays or some other radiation which would increase the number of soft shower particles without any appreciable effect on the hard component.

An interesting feature of the November 19 increase is the difference between the measurements at the various stations, particularly between Resolute and Godhaven (geomagnetic latitude 80°). These two stations are about 900 miles apart and the differences confirm previous indications that sudden increments in cosmic-ray intensity occur over a limited area. The lack of a sudden decrease after the increment is unusual, since a decrease has been reported on previous occasions.

The cooperation of the Department of Transport of the Government of Canada is appreciated for supplying facilities at Resolute and for weather information.

[1] A. Dauvillier, Comptes Rendus **229**, 1096 (1949).
[2] Forbush, Stinchcomb, and Schein, Bull. Am. Phys. Soc. **25**, No. 1, 15 (1950).
[3] I. L. Chakraborty and S. D. Chatterjee, Ind. J. Phys. **23**, 525 (1949).
[4] Forbush, Gill, and Vallarta, Rev. Mod. Phys. **21**, 44 (1949).
[5] R. L. Doan, Phys. Rev. **49**, 107 (1936).
[6] G. R. Wait and A. G. McNish, Monthly Weather Rev. **62**, 1 (1934).

An Identity in Quantum Electrodynamics

J. C. Ward
The Clarendon Laboratory, Oxford, England
February 27, 1950

It has been recently proved by Dyson[1] that all divergencies in the S-matrix of electrodynamics may be removed by a renormalization of mass and charge. Dyson defines certain fundamental divergent operators Γ_μ, S_F', D_F' and gives a procedure for the calculation of their finite parts $\Gamma_{\mu 1}$, S_{F1}', D_{F1}' by a process of successive approximation. It is then shown that

$$\Gamma_\mu = Z_1^{-1}\Gamma_{\mu 1}(e_1), \quad S_F' = Z_2 S_{F1}(e_1), \quad D_F' = Z_3 D_{F1}(e_1),$$
$$e_1 = Z_1^{-1} Z_2 Z_3^{\frac{1}{2}} e,$$

where Z_1, Z_2, and Z_3 are certain infinite constants and e_1 is the renormalized electronic charge. Dyson conjectured that $Z_1 = Z_2$, and it is proposed here to give a formal proof of this relation.

In the first place, with any proper electron self-energy part W, may be associated a set of proper vertex parts V^i obtained by inserting a photon line in one of the electron lines of W. Now consider the operators $\Lambda_\mu(V^i, p, p)$ in which the two external electron momentum variables p have been set equal, and the external photon variable made to vanish. Then $\Lambda_\mu(V^i, p, p)$ may be obtained from $\Sigma(W, p)$ by replacing S_F by $S_F\gamma_\mu S_F$ at one electron line of W. Because of the identity

$$-(1/2\pi)\partial S_F/\partial p_\mu = S_F\gamma_\mu S_F,$$

on summing $\Lambda_\mu(V^i, p, p)$ over all vertex parts V^i associated with W, one finds

$$\Sigma_{V^i}\Lambda_\mu(V^i, p, p) = -(1/2\pi)(\partial\Sigma(W, p)/\partial p_\mu).$$

(One can verify that any closed loop in W gives zero total effect.) Finally summing over all proper electron self-energy parts W, one finds

$$\Lambda_\mu(p, p) = -(1/2\pi)(\partial\Sigma^*(p)/\partial p_\mu).$$

Now substitute this identity into Eqs. (91) and (95) of reference 1. One finds

$$\Lambda_\mu = Z_1^{-1}[(1-Z_1)\gamma_\mu + \Lambda_{\mu C}], \quad \Sigma^* = Z_2^{-1}[(Z_2-1)S_F^{-1} + S_F^{-1}S_C/2\pi].$$

We have

$$-(1/2\pi)Z_2^{-1}\{(Z_2-1)2\pi\gamma_\mu + \gamma_\mu S_C + (\gamma_\lambda p_\lambda - iK_0)(\partial S_C/\partial p_\mu)\}$$
$$= Z_1^{-1}\{(1-Z_1)\gamma_\mu + \Lambda_{\mu C}(p, p)\}.$$

Now put

$$\gamma_\lambda p_\lambda = iK_0, \quad (p_\lambda)^2 = -K_0^2.$$

The convergent parts of these equations then vanish and there is left the relation

$$-(1/2\pi)Z_2^{-1}(Z_2-1)2\pi\gamma_\mu = Z_1^{-1}(1-Z_1)\gamma_\mu$$

which reduces immediately to $Z_1 = Z_2$.

[1] F. J. Dyson, Phys. Rev. **75**, 1736 (1949).

The Partial Molal Entropy of Superfluid in Pure He⁴ below the λ-Point

O. K. Rice
Department of Chemistry, University of North Carolina, Chapel Hill, North Carolina
March 3, 1950

In a recent article[1] (the notation of which is retained here, except that subscripts $4n$ and $4s$ refer to normal fluid and superfluid, respectively, in place of 1 and 2), I have considered the thermodynamics of liquid helium on the two-fluid theory, taking account of the fact that if two "phases" or "components," the normal fluid and the superfluid, exist together they must be in equilibrium with each other. On this basis, using the assumed relation[2] which states that the total molal entropy S at any temperature is the mole fraction x_{4n} of normal fluid times the molal entropy S_λ at the λ-point

$$S = x_{4n}S_\lambda = (1-x_{4s})S_\lambda, \tag{1}$$

using the empirical relation for S as a function of temperature

$$S = S_\lambda(T/T_\lambda)^r \tag{2}$$

(with $r \sim 5.6$), and assuming that the partial molal enthalpy of superfluid, $\bar{H}_{4s}$, is independent of temperature (at essentially constant pressure), and independent of x_{4s} (i.e., there is no heat of mixing), I derived the equation for the partial molal entropy of superfluid

$$\bar{S}_{4s} = S_\lambda x_{4n}/(r+1). \tag{3}$$

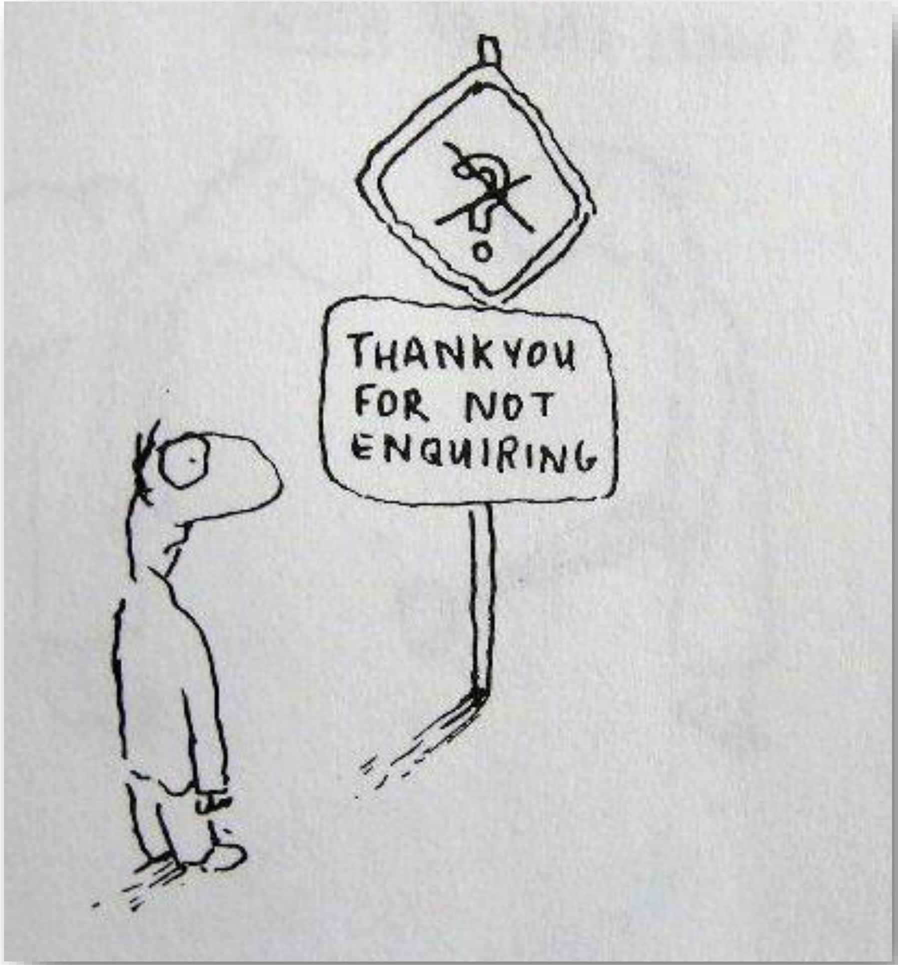
However, as I remarked in reference 1, there are some approximations involved in this procedure. Equation (1) is based on the assumption that below T_λ the entropy is contributed solely by the normal fluid, whose molal entropy is always set equal to the constant S_λ, thus neglecting any temperature dependence. Furthermore, there is an implied inconsistency, since Eq. (1) assumes no entropy of mixing while Eq. (3) implies that there is a mixing entropy. In fact, in the following letter we shall show that we may derive a somewhat different expression for S from Eq. (3). We shall, therefore, discard Eq. (1) and turn to a consideration of the enthalpies.

If $\bar{H}_{4s}$ is independent of x_{4s}, then $\bar{H}_{4n}$ must be also, and we have $\bar{H}_{4n} = H_{4n}$, where H_{4n} is the enthalpy of pure normal helium. We can write for the total molal enthalpy[3]

$$H = x_{4n}H_{4n}. \tag{4}$$

We will now proceed to derive an expression for $\bar{S}_{4n}$ in a somewhat more direct way than in reference 1, using Eq. (4) in place of Eq. (1). Since $F = H - TS$ and $\mu_{4s} = \bar{H}_{4s} - T\bar{S}_{4s} = -T\bar{S}$… the condition for internal equilibrium, $F = \mu_{4s}$, gives

$$\bar{S}_{4s} = S - H/T.$$



Enigma of Mass

Maris, Roberts and Tandy
*nucl-th/9707003, Phys.Lett. B**420** (1998) 267-273*

Pion's Goldberger -Treiman relation

- Pion's Bethe-Salpeter amplitude

  Solution of the Bethe-Salpeter equation

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\Big]$$

- Dressed-quark propagator $S(p) = \dfrac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$

- Axial-vector Ward-Takahashi identity entails

$$f_\pi E_\pi(k;P=0) = B(k^2)$$

Owing to DCSB & Exact in Chiral QCD

***Miracle*: two body problem solved, almost completely, once solution of one body problem is known**

Rudimentary version of this relation is apparent in Nambu's Nobel Prize work

Model independent
Gauge independent
Scheme independent

$$f_\pi E_\pi(p^2) = B(p^2)$$

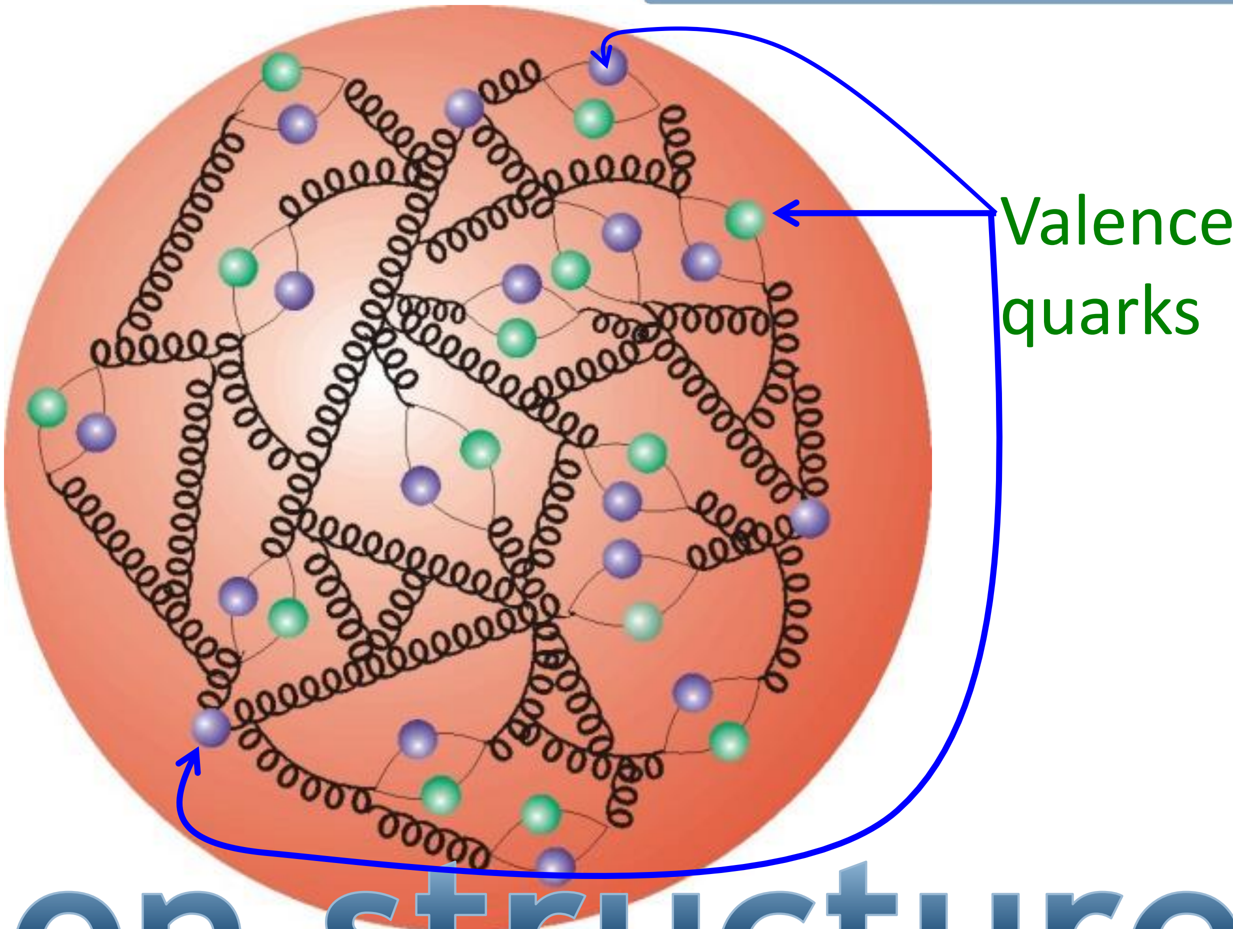
The most fundamental expression of Goldstone's Theorem and DCSB

Rudimentary version of this relation is apparent in Nambu's Nobel Prize work

Model independent
Gauge independent
Scheme independent

$$f_\pi E_\pi(p^2) \Leftrightarrow B(p^2)$$

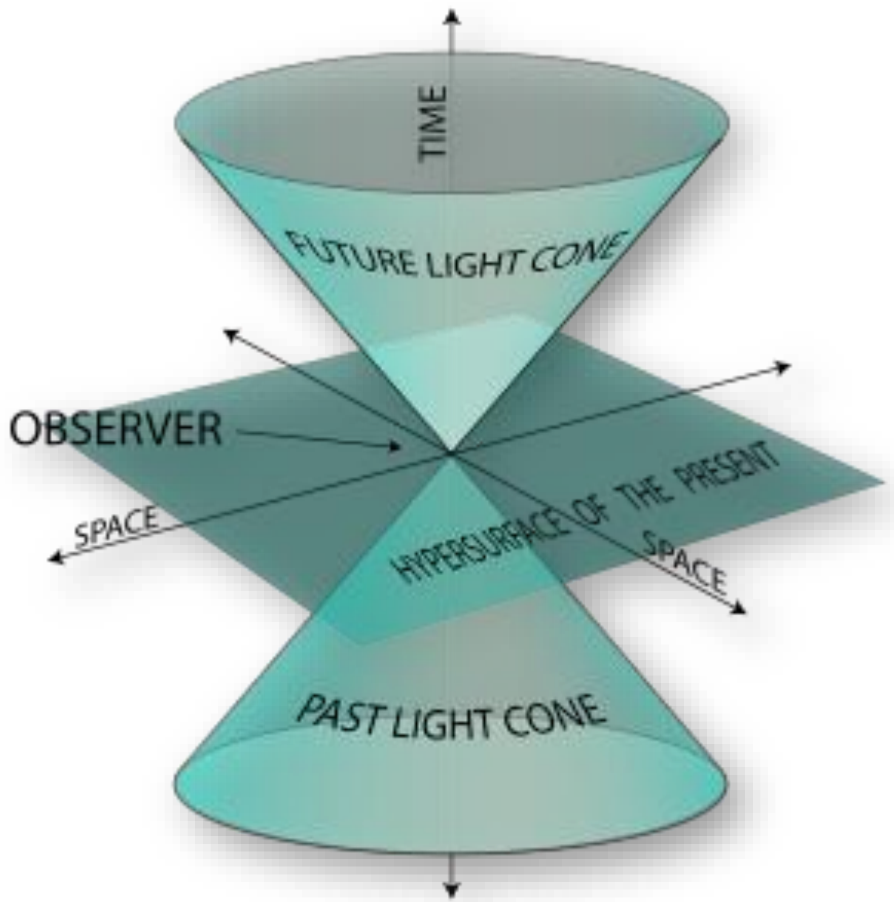
Pion exists if, and only if, mass is dynamically generated



Parton structure of hadrons

Parton Structure of Hadrons

- Valence-quark structure of hadrons
  - Definitive of a hadron.
    After all, it's how we distinguish a proton from a neutron
  - Expresses charge; flavour; baryon number; and other Poincaré-invariant macroscopic quantum numbers
  - Via evolution, determines background at LHC
- Foreseeable future will bring precision experimental study of (far) valence region, and theoretical computation of distribution functions and distribution amplitudes
  - *Computation is critical*
  - *Without it, no amount of data will reveal anything about the theory underlying the phenomena of strong interaction physics*

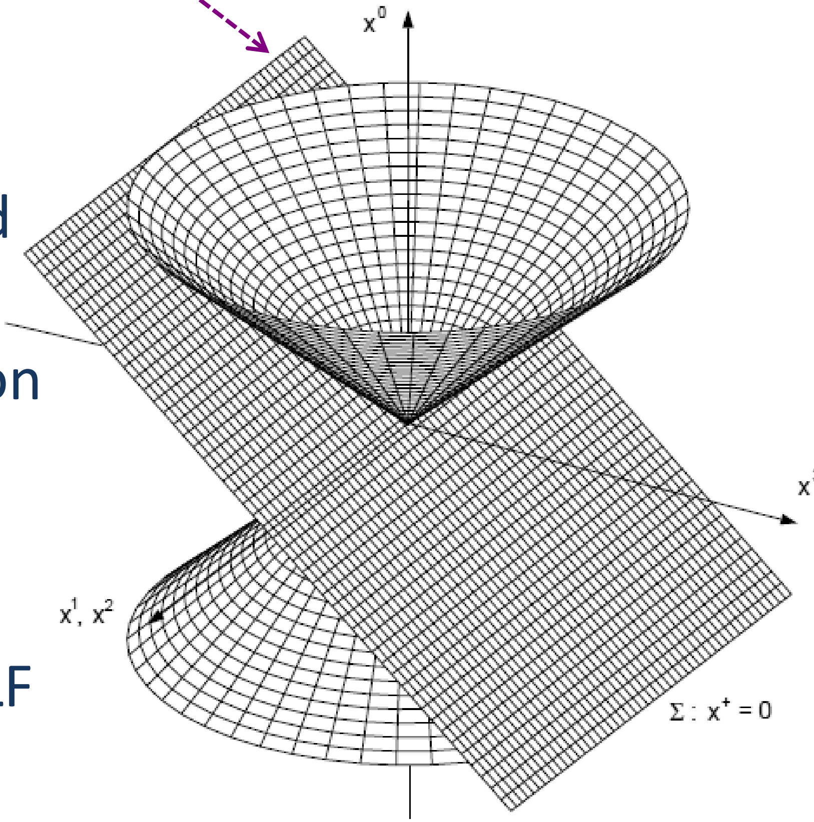


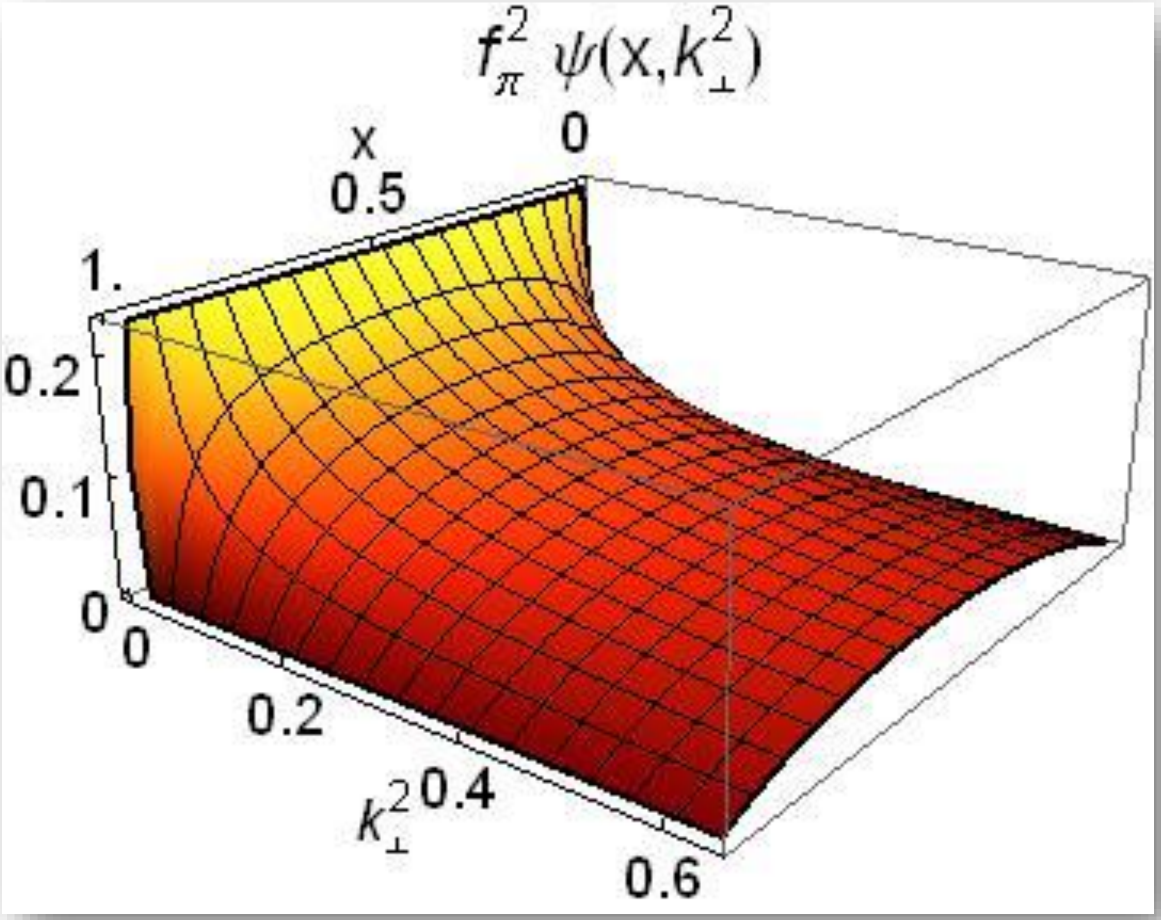
Why light-front?

Light-front Quantisation

- Hamiltonian formulation of quantum field theory.
  - Fields are specified on a particular initial surface:

    Light front $x^+ = x^0 + x^3 = 0$
- Using LF quantisation:
  - ✓ quantum mechanics-like wave functions can be defined;
  - ✓ quantum-mechanics-like expectation values can be defined and evaluated
  - ✓ Parton distributions are correlation functions at equal LF-time x^+ ; namely, within the initial surface $x^+ = 0$, and can thus be expressed directly in terms of ground state LF wavefunctions





Pion's Wave Function

Imaging dynamical chiral symmetry breaking: pion wave function on the light front, Lei Chang, et al., arXiv:1301.0324 [nucl-th], Phys. Rev. Lett. **110** (2013) 132001 (2013) [5 pages].

Pion's valence-quark Distribution Amplitude

- Methods have been developed that enable direct computation of the pion's light-front wave function
- $\varphi_\pi(x)$ = twist-two parton distribution amplitude = projection of the pion's Poincaré-covariant wave-function onto the light-front

$$\varphi_\pi(x) = Z_2 \mathrm{tr}_{CD} \int \frac{d^4k}{(2\pi)^4} \delta(n \cdot k - x n \cdot P)\, \gamma_5 \gamma \cdot n\, S(k) \Gamma_\pi(k;P) S(k-P)$$

- Results have been obtained with the DCSB-improved DSE kernel, which unifies matter & gauge sectors

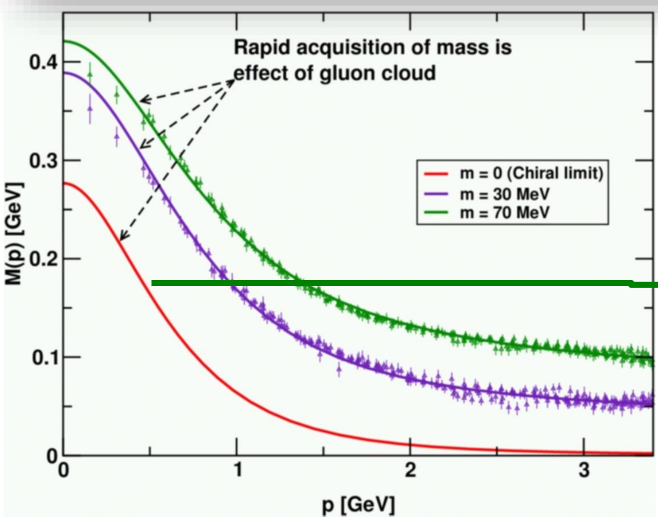
$$\varphi_\pi(x) \propto x^\alpha (1-x)^\alpha, \text{ with } \alpha \approx 0.5$$

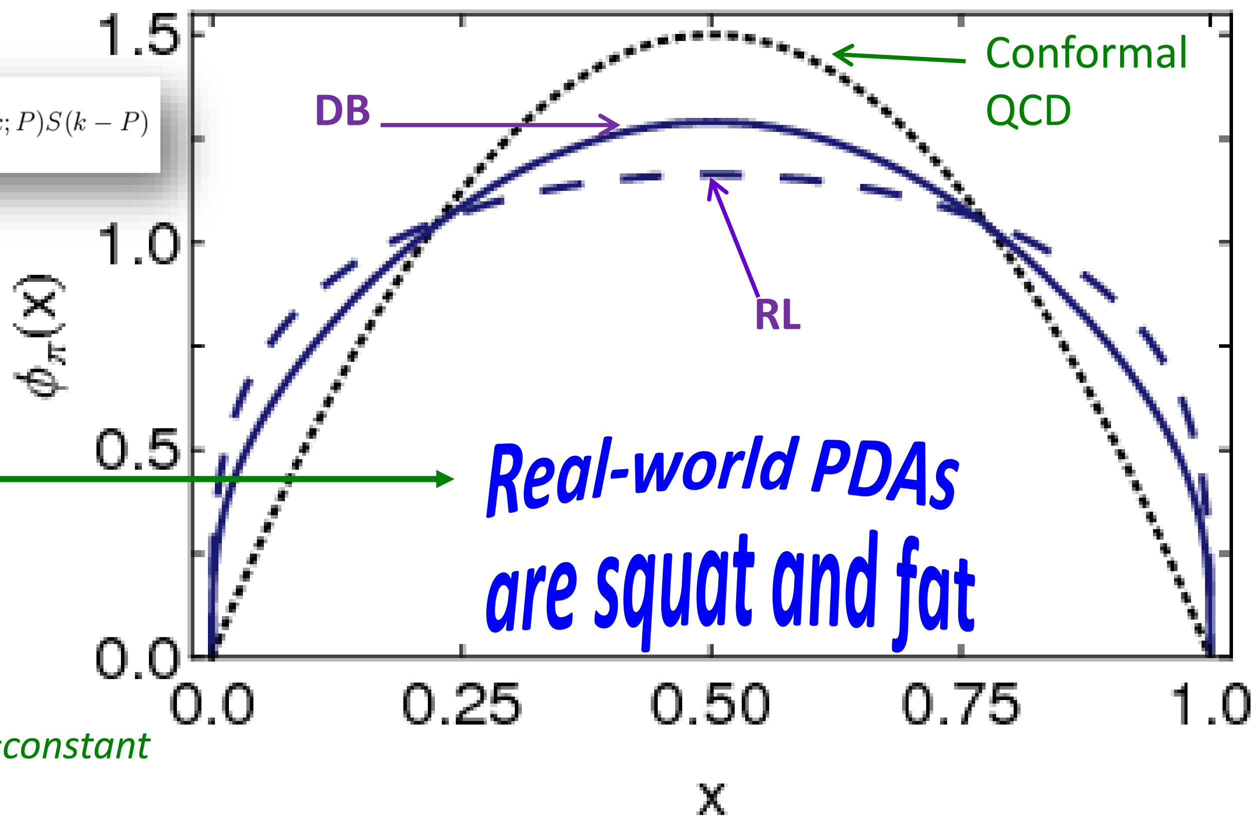
Imaging dynamical chiral symmetry breaking: pion wave function on the light front, Lei Chang, *et al.*, arXiv:1301.0324 [nucl-th], Phys. Rev. Lett. **110** (2013) 132001 (2013) [5 pages].

Pion's valence-quark Distribution Amplitude

- Continuum prediction:
  marked broadening of $\varphi_\pi(x)$, which owes to DCSB

$$\varphi_\pi(x) = Z_2 \mathrm{tr}_{CD} \int \frac{d^4k}{(2\pi)^4} \delta(n\cdot k - x n\cdot P)\, \gamma_5 \gamma\cdot n\, S(k) \Gamma_\pi(k;P) S(k-P)$$





A theory that produces M(p)=constant also produces φ(x)=constant

Imaging dynamical chiral symmetry breaking: pion wave function on the light front, Lei Chang, *et al.*, arXiv:1301.0324 [nucl-th], Phys. Rev. Lett. **110** (2013) 132001 (2013) [5 pages].

Pion's valence-quark Distribution Amplitude

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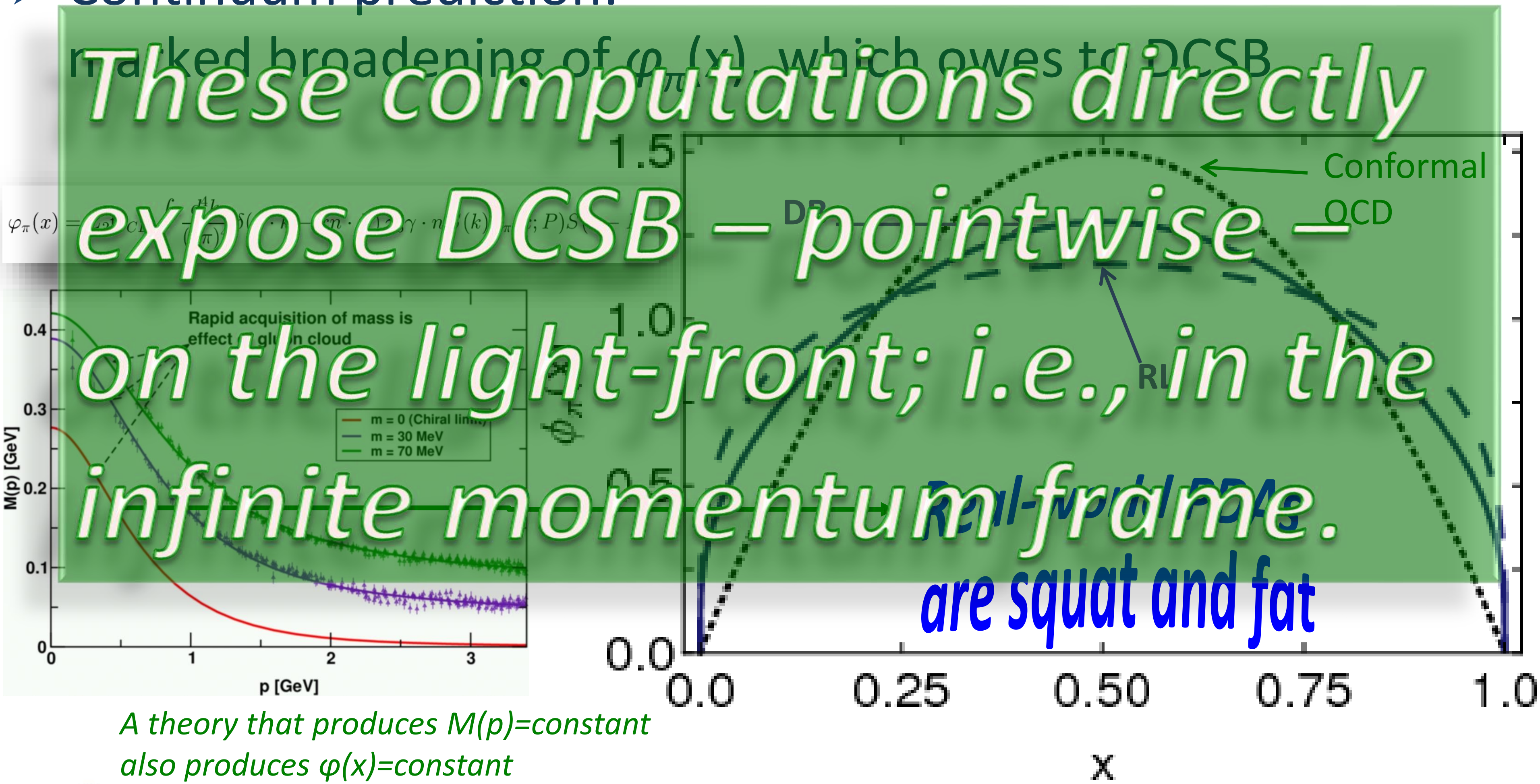
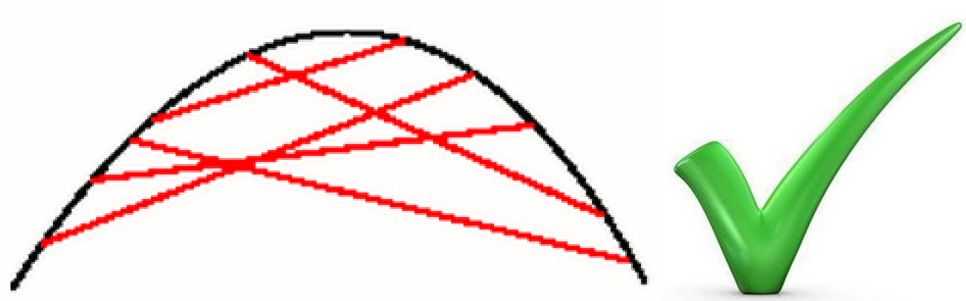


A theory that produces M(p)=constant also produces φ(x)=constant

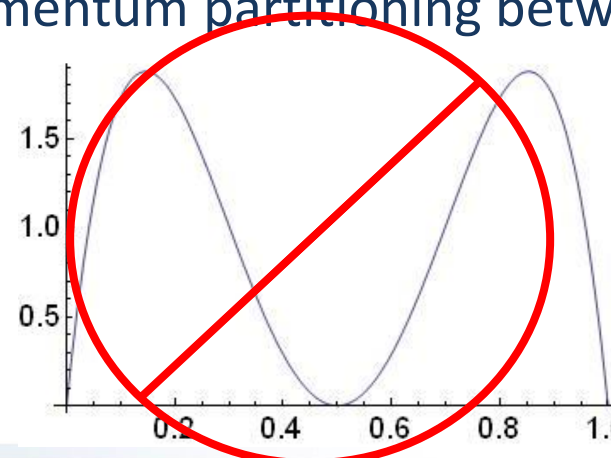
Features of Ground-state PDAs

- A diverse array of studies have shown that ground-state S-wave meson PDAs are broad, concave functions

Concave function: no line segment lies above any point on the graph



- Camel-humped distributions – popular for many years – are physically unreasonable because they correspond to bound-state amplitudes that disfavour equal momentum partitioning between valence-quark degrees of freedom



Hard Exclusive Processes & PDAs

- In the theory of strong interactions, the cross-sections for many *hard* exclusive hadronic reactions can be expressed in terms of the PDAs of the hadrons involved
- Example: pseudoscalar-meson elastic electromagnetic form factor

$$\exists Q_0 > \Lambda_{\rm QCD} \,\big|\, Q^2 F_P(Q^2) \stackrel{Q^2>Q_0^2}{\approx} 16\pi \alpha_s(Q^2) f_P^2 w_\varphi^2,$$

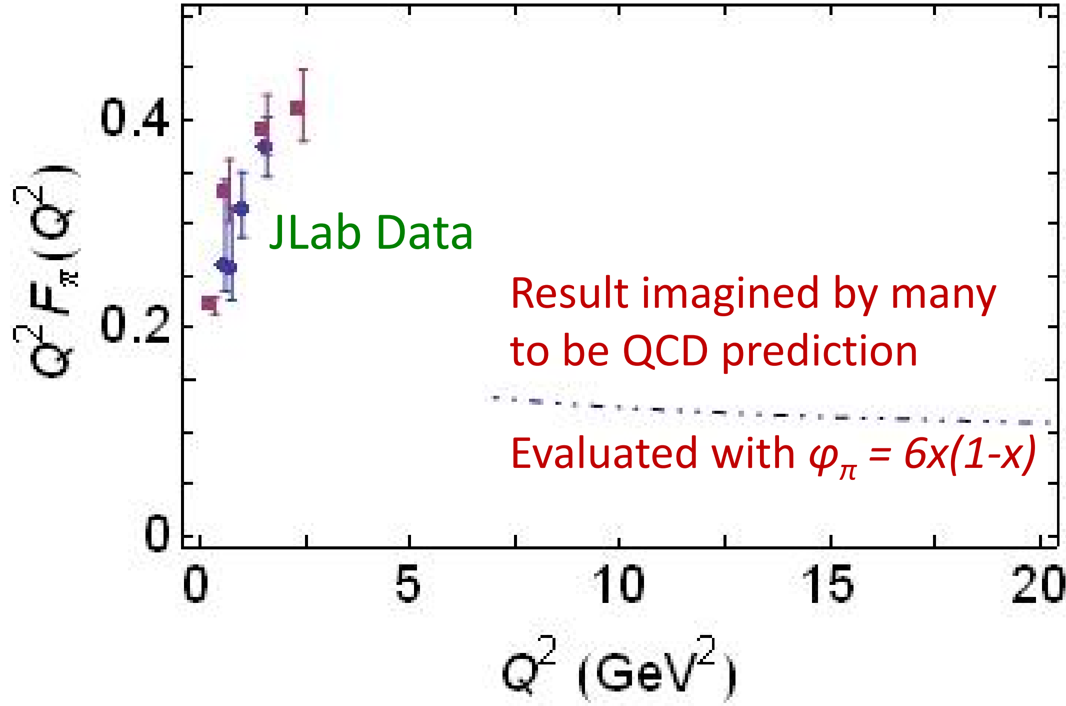
$$w_\varphi = \frac{1}{3}\int_0^1 dx\, \frac{1}{x}\varphi^P(x),$$

  - $\alpha_S(Q^2)$ is the strong running coupling,
  - $\varphi_\pi(u)$ is the meson's twist-two valence-quark PDA
  - f_P is the meson's leptonic decay constant

It was promised that JLab would verify this fundamental prediction

Pion electromagnetic form factor

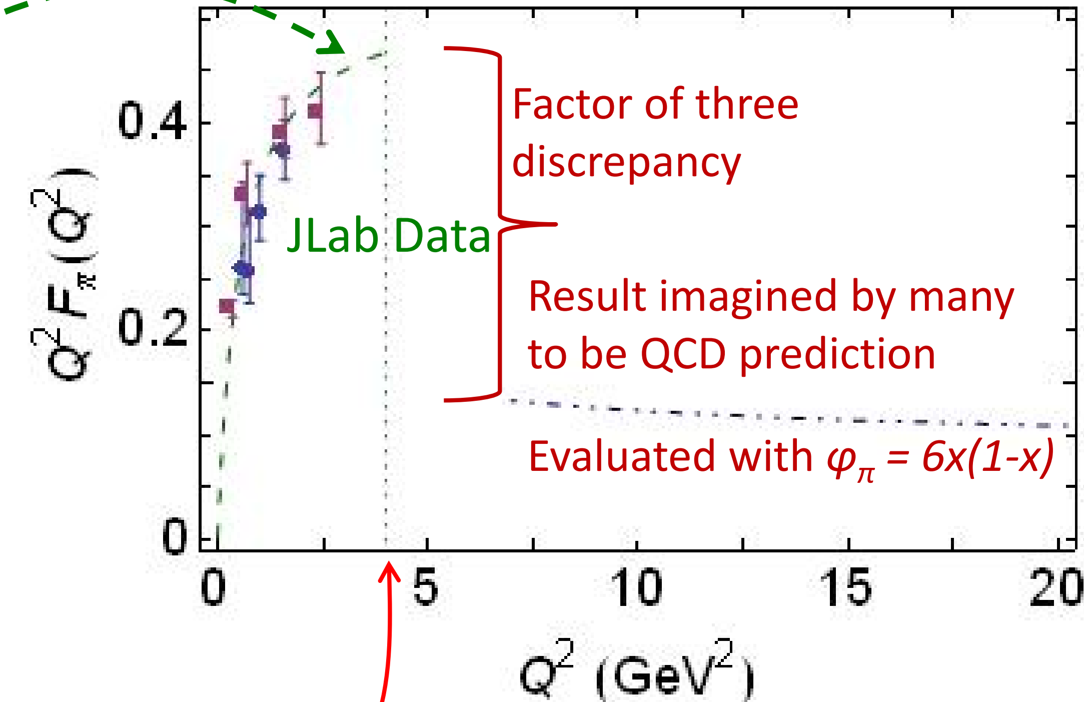
- In 2001 – seven years after beginning operations, Jefferson Lab provided the first high precision pion electroproduction data for F_π between Q^2 values of 0.6 and 1.6 $(GeV/c)^2$.



- 2006 & 2007 – new result, at $Q^2=2.45$ $(GeV/c)^2$
- Authors of the publications stated: “still far from the transition to the Q^2 region where the pion looks like a simple quark-antiquark pair”
  - disappointment and surprise

Pion electromagnetic form factor

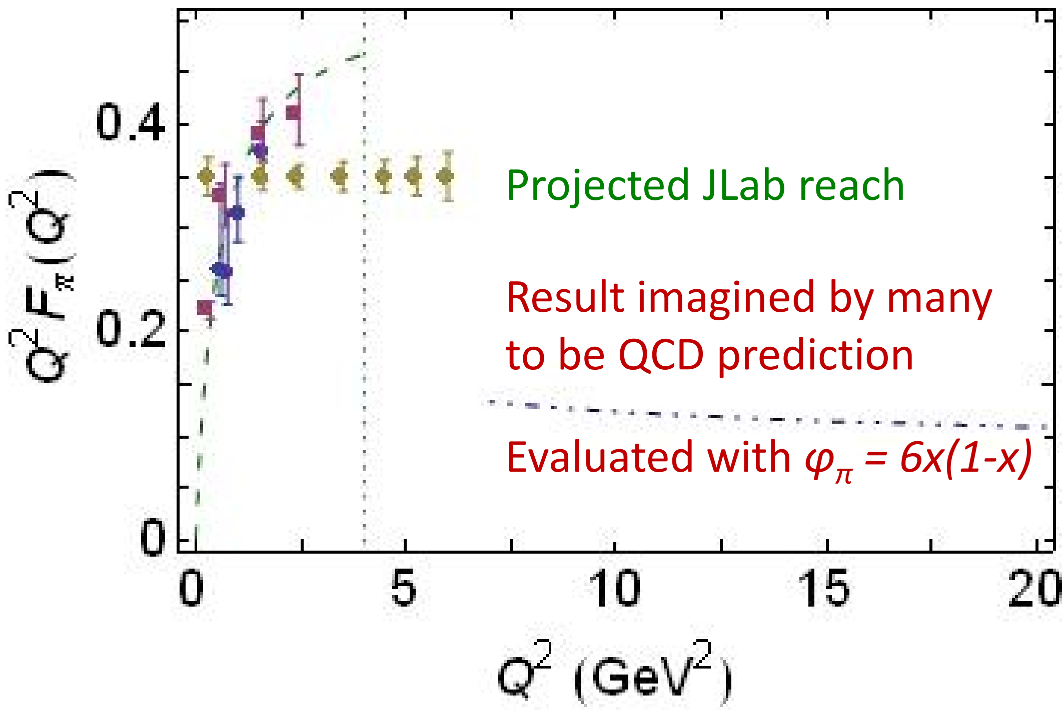
- Year 2000 *prediction* for $F_\pi(Q^2)$
  - *P.Maris & P.C. Tandy, Phys.Rev. C62 (2000) 055204*
- Problem … used brute-force computational method … unable to compute for Q^2>4GeV^2



- Shape of prediction suggested to many that one might *never* see parton model scaling and QCD scaling violations

Pion electromagnetic form factor

- Plans were made and an experiment approved that use the higher-energy electron beam at the 12 GeV Upgrade at Jefferson Lab.
- The Upgrade will allow an extension of the F_π measurement up to a value of Q^2 of about 6 $(GeV/c)^2$, which will probe the pion at double the resolution.

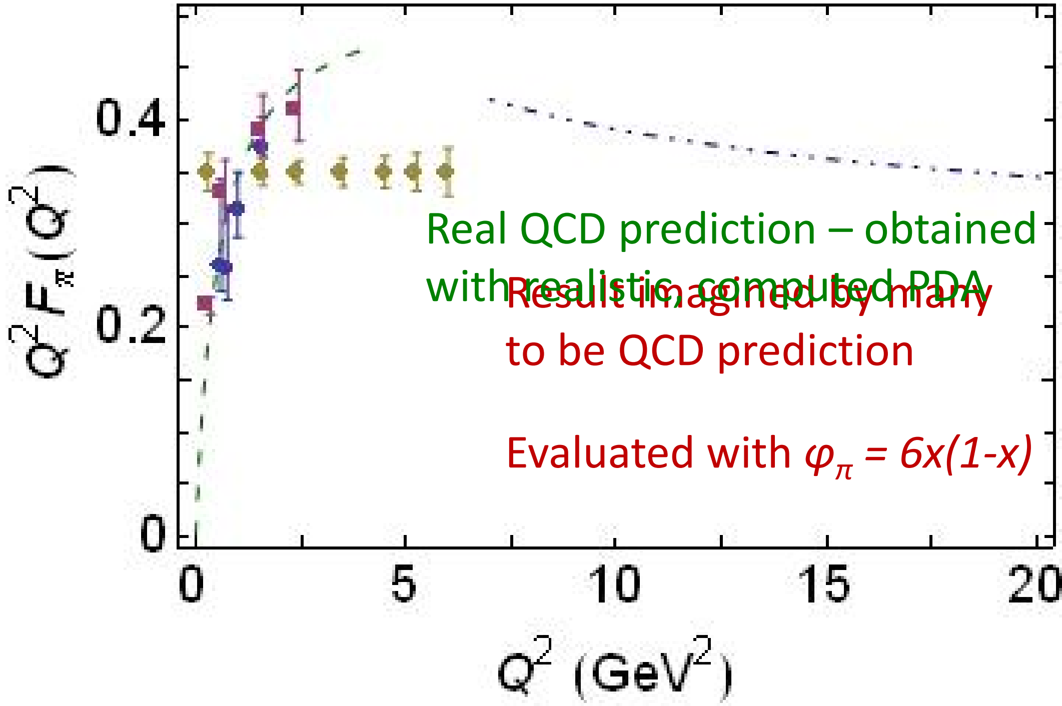


- Will there be any hint of a trend toward the asymptotic pQCD prediction?

New Algorithm
New Insights

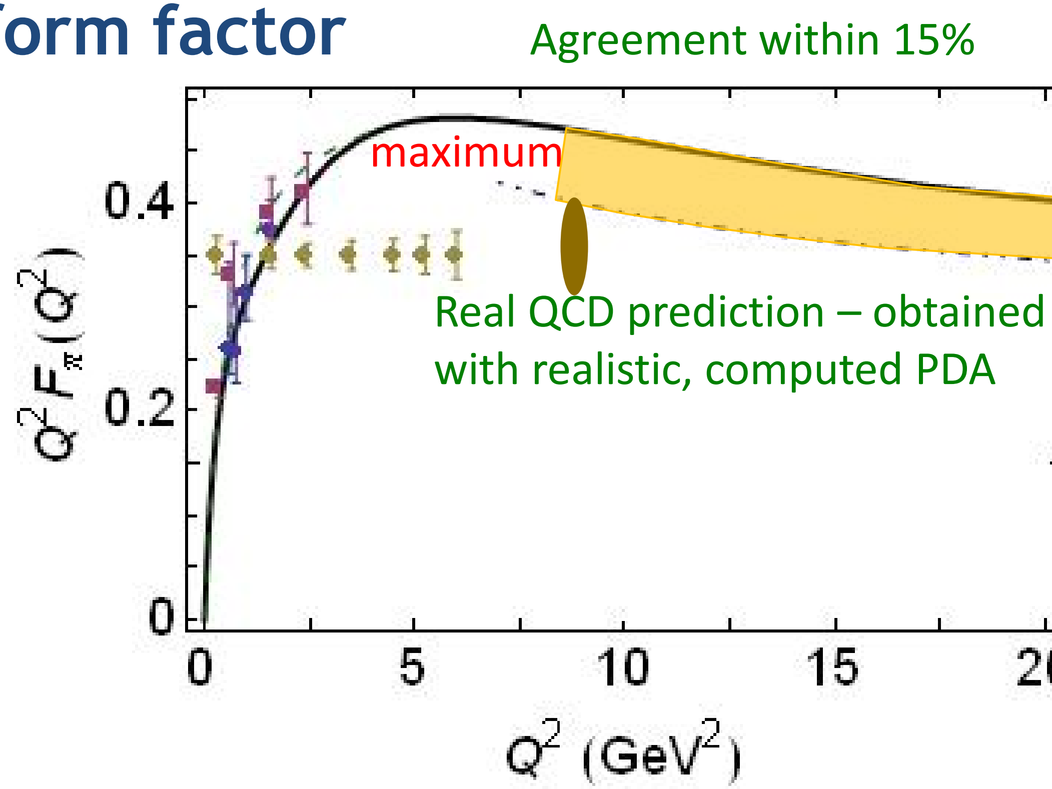
Pion electromagnetic form factor

- Solution – Part 1
  - Compare data with the real QCD prediction; i.e. the result calculated using the broad pion PDA predicted by modern analyses of continuum QCD



Pion electromagnetic form factor

- Solution – Part 1
  - Compare data with the real QCD prediction; i.e. the result calculated using the broad pion PDA predicted by modern analyses of continuum QCD
- Solution – Part 2
  - Algorithm used to compute the PDA can also be employed to compute $F_\pi(Q^2)$ directly, to arbitrarily large Q^2



- Predictions:
  - JLab will see maximum
  - Experiments to 8GeV^2 will see parton model scaling and QCD scaling violations for the *first time* in a hadron form factor

Pion electromagnetic form factor at spacelike momenta
L. Chang et al., arXiv:1307.0026 [nucl-th], Phys. Rev. Lett. 111, 141802 (2013)

Implications

- Verify the theory of factorisation in hard exclusive processes, with dominance of hard contributions to the pion form factor for $Q^2 > 8\text{GeV}^2$.
- Notwithstanding that, normalisation of $F_\pi(Q^2)$ is fixed by a pion wave-function whose dilation with respect to $\varphi_\pi^{asy}(x)=6x(1-x)$ is a definitive signature of DCSB
  - ***Empirical measurement of the strength of DCSB in the Standard Model – the origin of visible mass***
- Close the book on a story that began thirty-five years ago
- Paves the way for a dramatic reassessment of pictures of proton & neutron structure, which is already well underway

To be continued