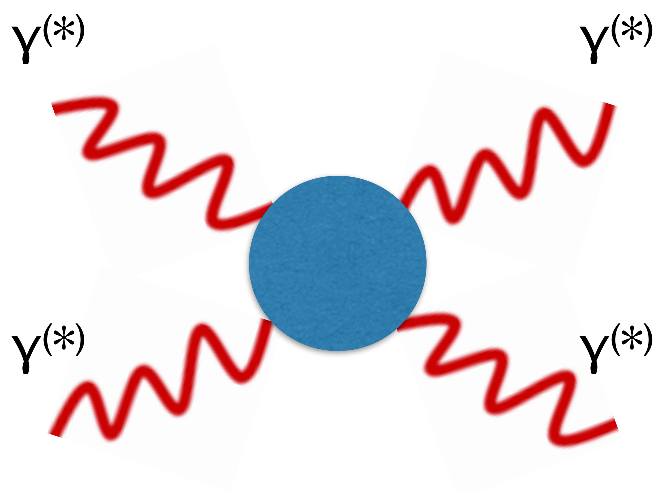


Hadronic light-by-light scattering and the muon's (g-2)

Igor Danilkin

in coll. with Marc Vanderhaeghen

July 29, 2017

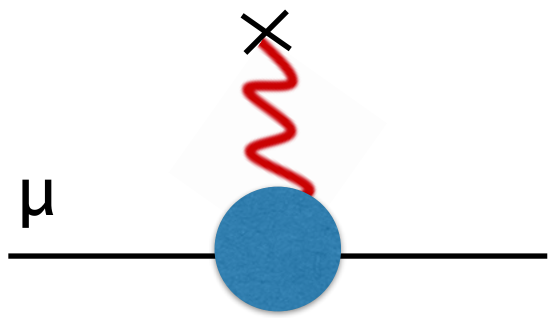
SFB 1044
THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

JOHANNES GUTENBERG UNIVERSITÄT MAINZ
JG|U

Motivation

- Magnetic moment of the muon

$$\vec{\mu} = \frac{Q}{2m}\, g\, \vec{S}$$



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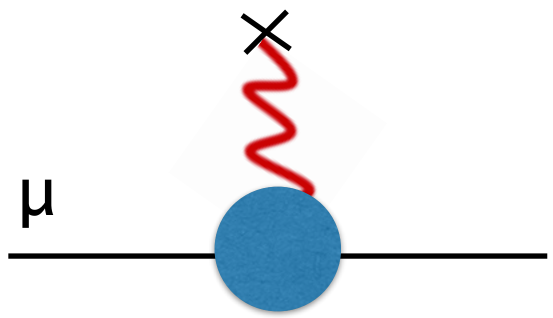
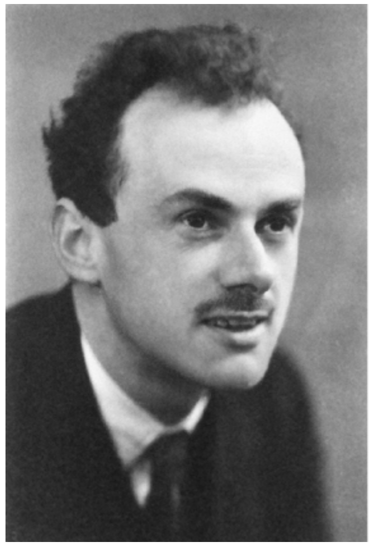
- anomalous part

$$a_\mu = \frac{(g-2)_\mu}{2}$$



Classical mechanics g=1
Dirac equation g=2



Dirac (1928)

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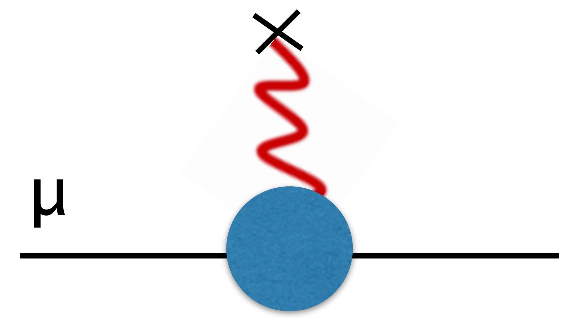

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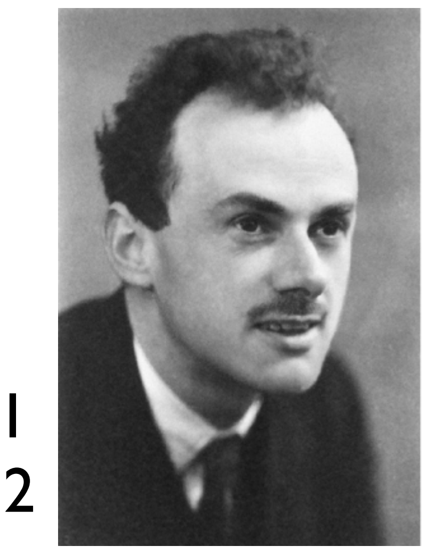
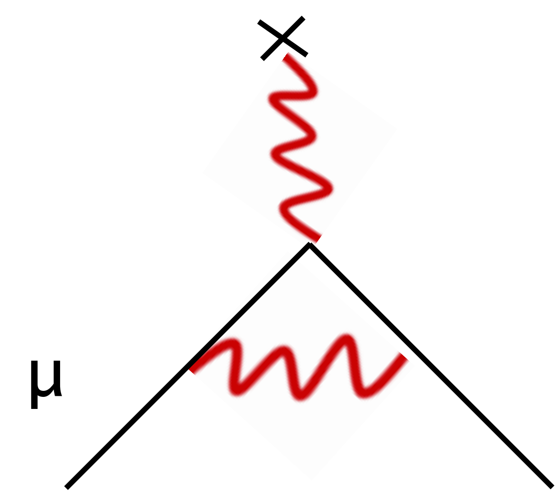


Dirac (1928)

- first correction to LO result

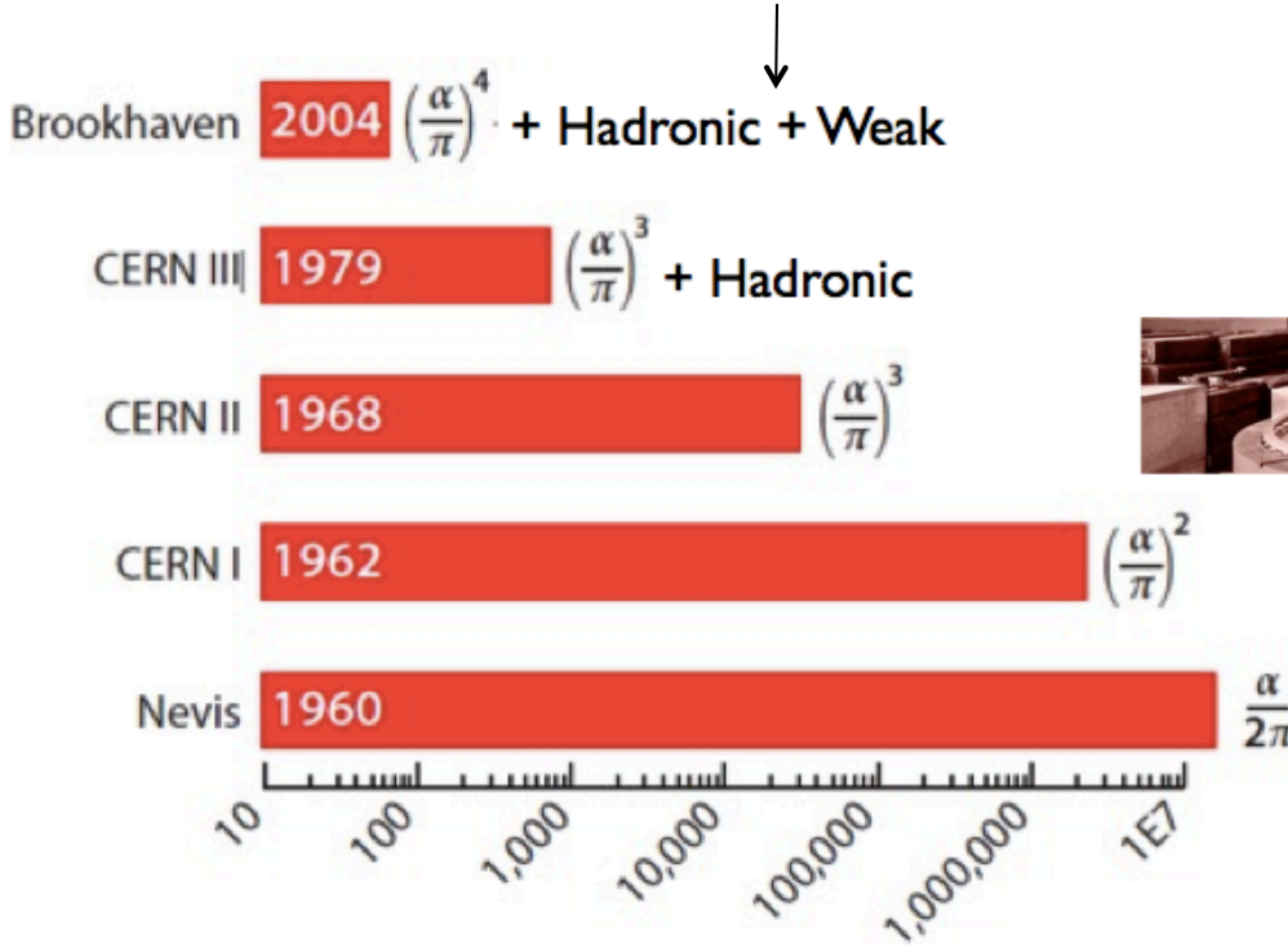
$$a_\mu = \frac{\alpha}{2\pi} + \ldots$$

Schwinger

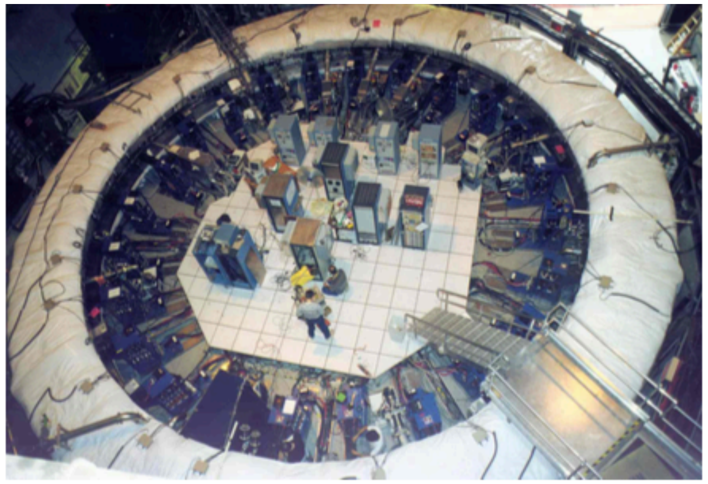


(g-2) history of relevant corrections

Contribution (theory) resolved



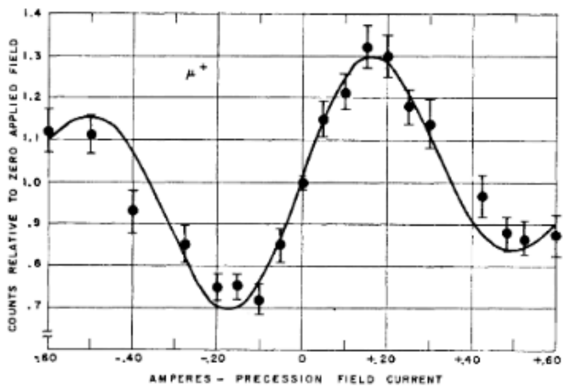
Uncertainty of measurement in 10^{-11}



Brookhaven

CERN I

Nevis



(g-2) theory vs exp

Experiment:

$$a_\mu^{exp} = (11\,659\,208.9 \pm 6.3) \times 10^{-10}$$

BNL, (2006)
PRD 73 072003

Theory:

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3 - 4 σ
deviation !

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FNAL, J-PARC experiments

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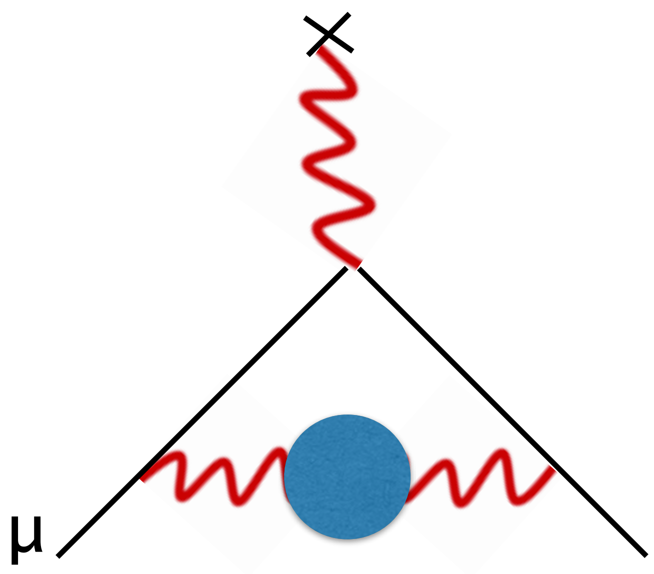

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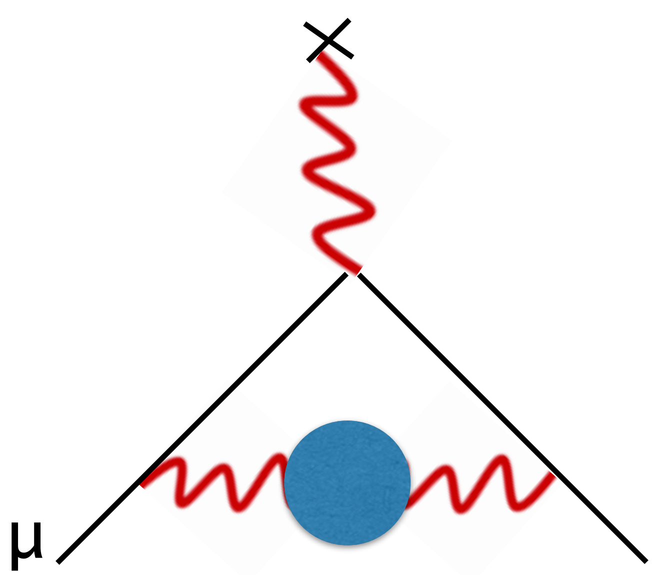

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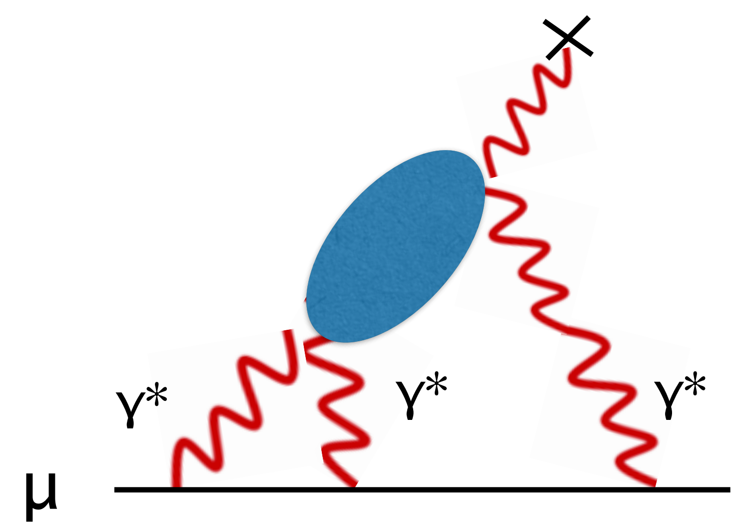

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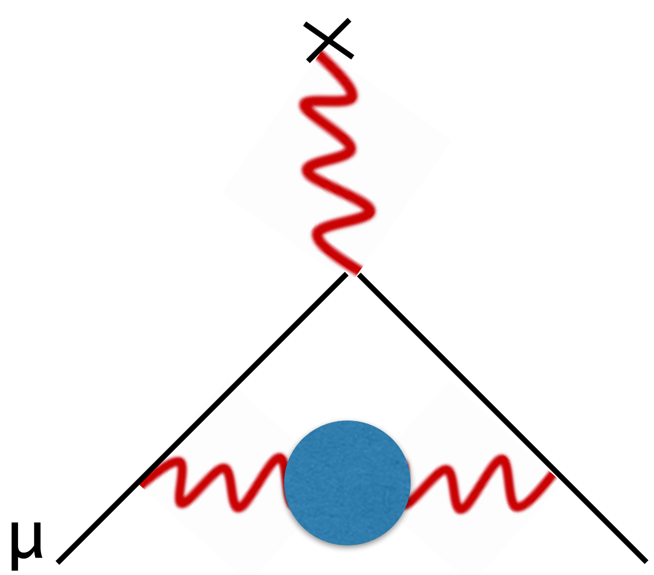

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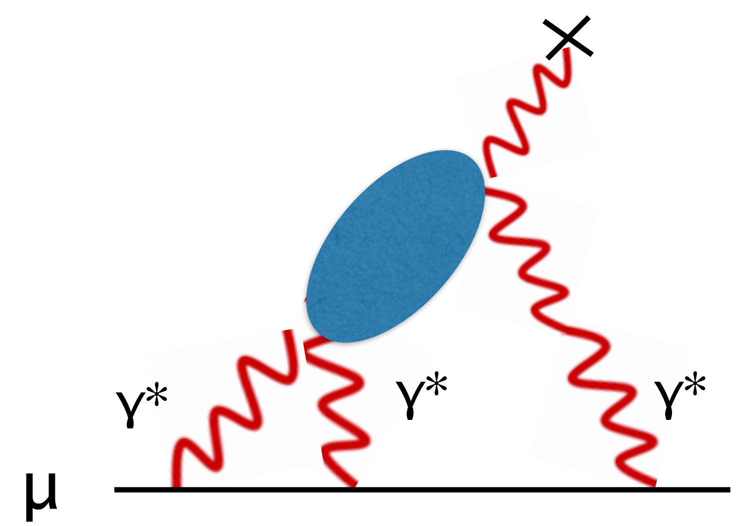

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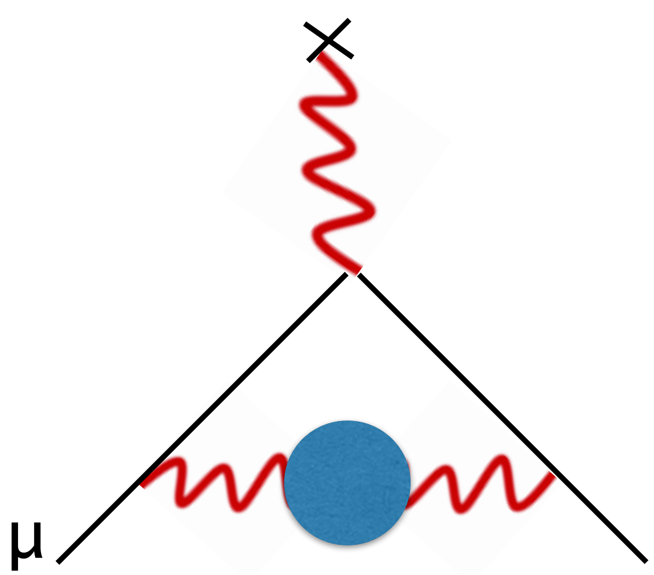

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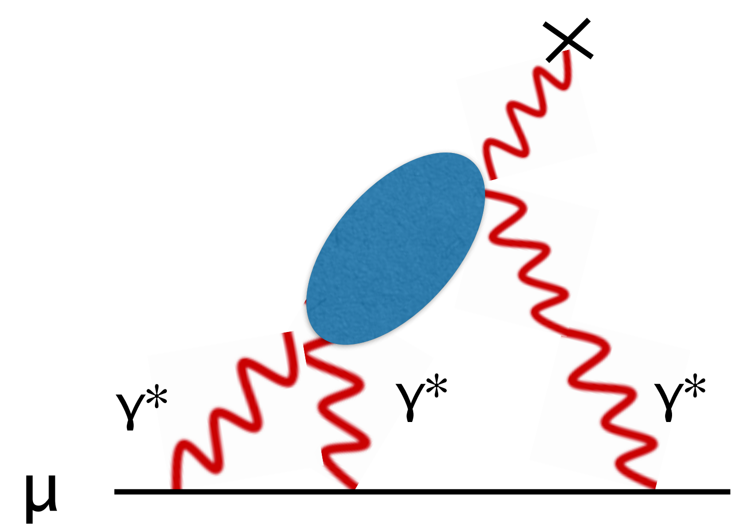

relies on measurements of **TFF** to reduce model dependence

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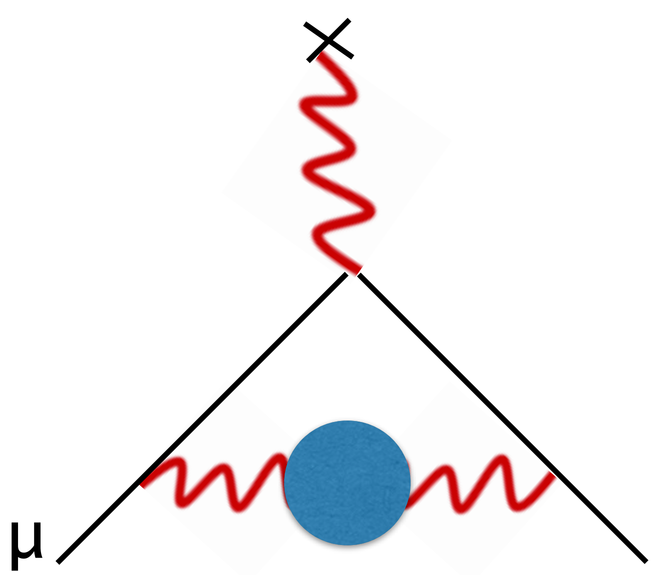

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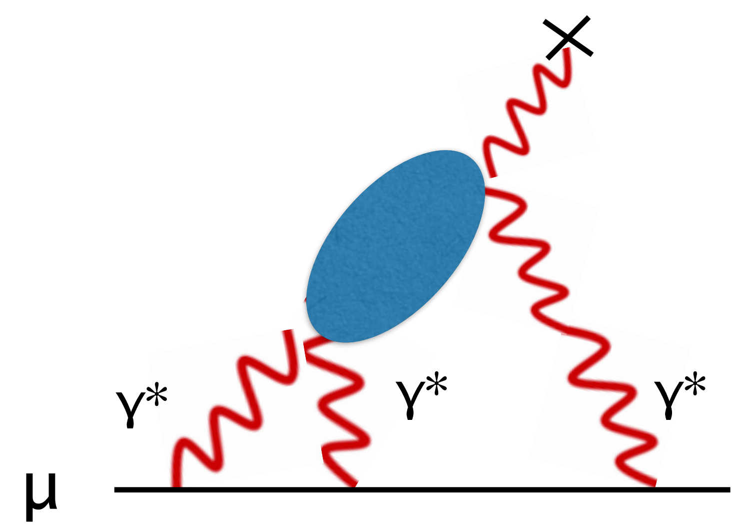

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Light by light scattering contribution

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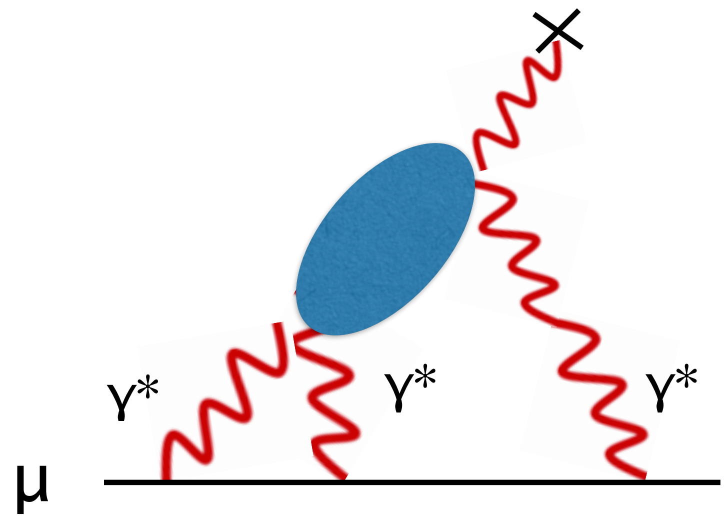

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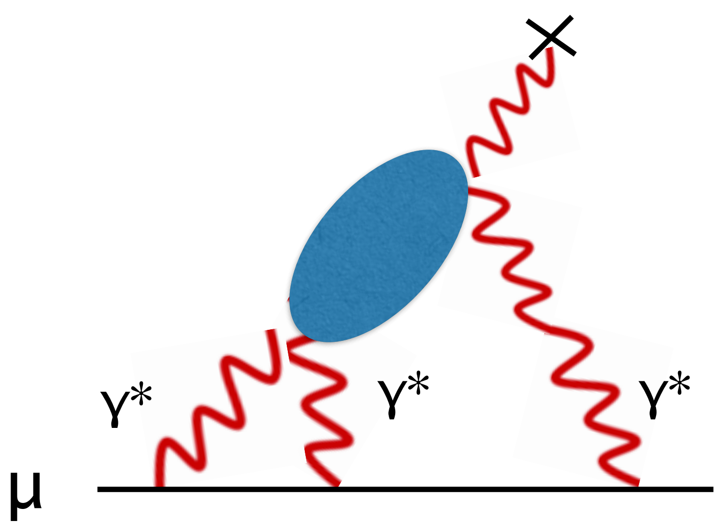

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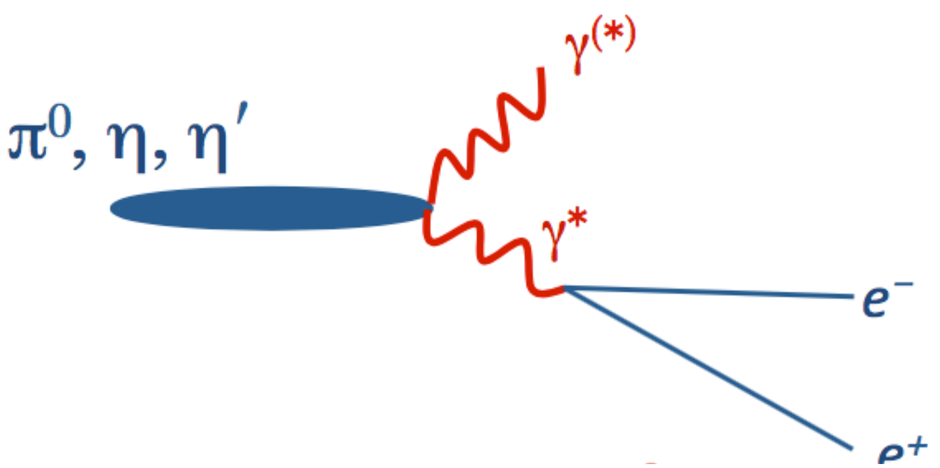
Timelike: KLOE, MAMI/A2, NA62



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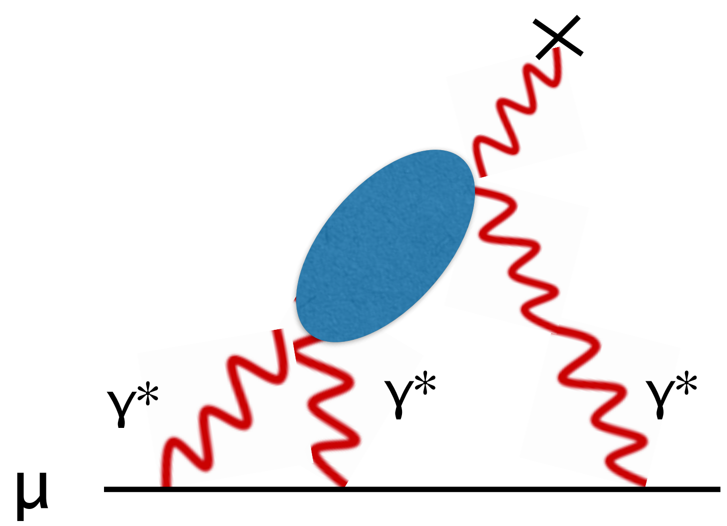

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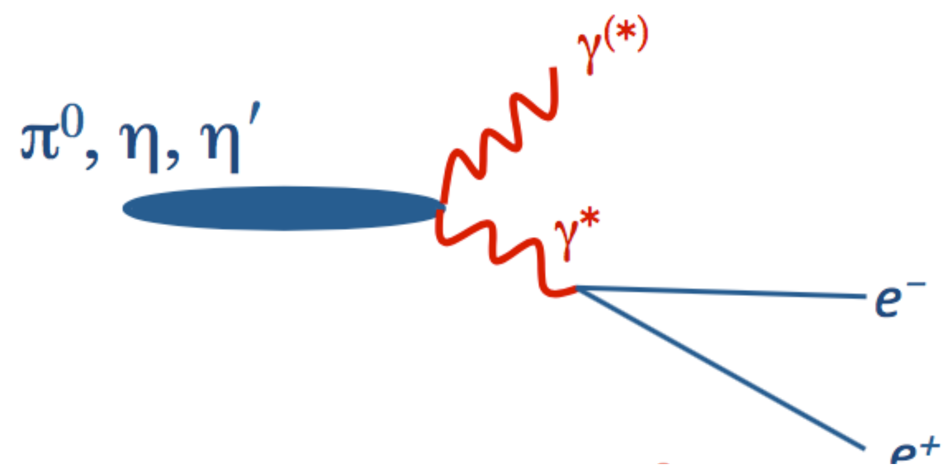
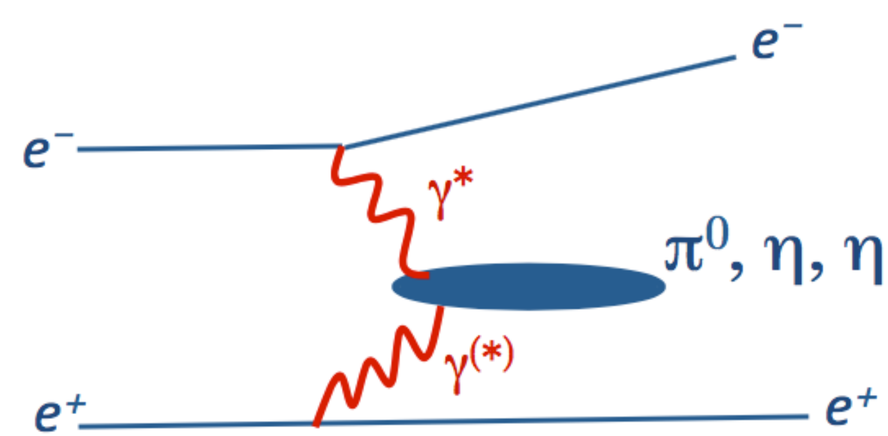


Spacelike: CLEO, BaBar, Belle, BESIII



Light by light scattering contribution

HLbL contributions to a_μ in units 10^{-10}

Authors	π^0, η, η'	$\pi\pi$, KK	scalars	axial vectors	quark loops	Total
BPaP(96)	8.5(1.3)	-1.9(1.3)	-0.68(0.20)	0.25(0.10)	2.1(03)	8.3(3.2)
HKS(96)	8.3(0.6)	-0.5(0.8)	—	0.17(0.17)	1.0(1.1)	9.0(1.5)
KnN(02)	8.3(1.2)	—	—	—	—	8.0(4.0)
MV(04)	11.4(1.0)	—	—	2.2(0.5)	—	13.6(2.5)
PdRV(09)	11.4(1.3)	-1.9(1.9)	-0.7(0.7)	1.5(1.0)	0.23	**10.5(2.6)**
N/JN(09)	9.9(1.6)	-1.9(1.3)	-0.7(0.2)	2.2(0.5)	2.1(0.3)	11.6(3.9)
J(15)	9.9(1.6)	-1.9(1.3)	-0.7(0.2)	**0.75(0.27)**	2.1(0.3)	**10.2(3.9)**

B=Bjnens, Pa=Pallante, P=Prades, H=Hayakawa, K=Kinoshita, S=Sanda, Kn=Knecht, N=Nyffeler, M=Melnikov, V=Vainshtein, dR=de Rafael, J=Jegerlehner

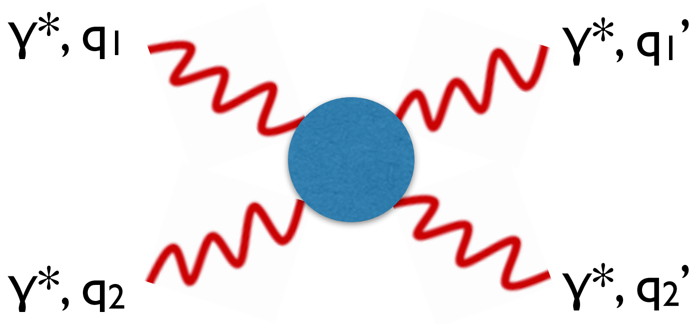
How to improve on the present calculations?

1. Space like doubly-virtual measurement of π^0 TFF at BESIII (Q_1^2, Q_2^2~0.5-1 GeV^2)
2. Dispersive analysis for ππ, KK, … loops contribution to (g-2)

Colangelo, Hoferichter, Procura, Stoffer, (2017)

Pauk, Vanderhaeghen, (2014)

Light by light scattering

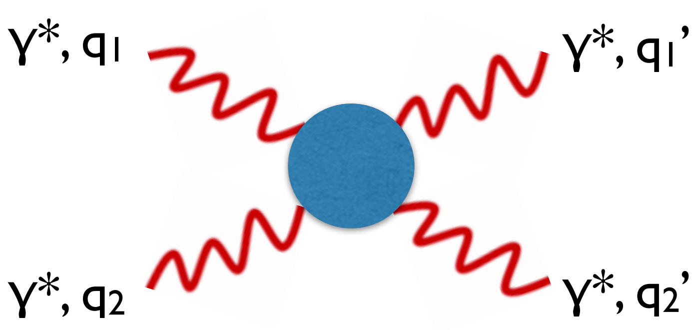


$$\lambda_i = \pm 1, 0$$

$$q_i^2 = -Q_i^2$$

Light by light scattering

Helicity amplitudes



$$M_{\lambda_1'\lambda_2'\lambda_1\lambda_2} = M^{\mu\nu\alpha\beta}\epsilon^*_\mu(\lambda_1')\,\epsilon^*_\nu(\lambda_2')\,\epsilon_\alpha(\lambda_1)\,\epsilon_\beta(\lambda_2)$$

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Light by light scattering

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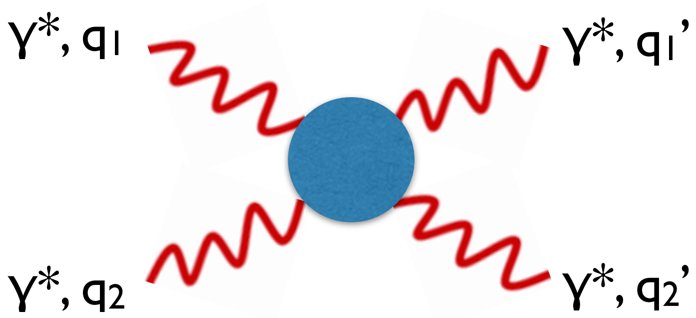

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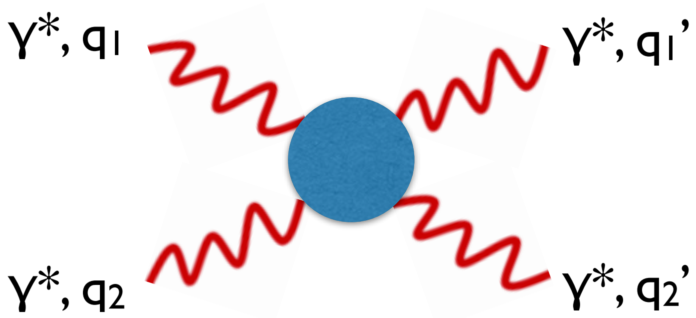
Forward scattering $q_1 = q_1',\, q_2 = q_2'$

$$s = (q_1 + q_2)^2$$

$$t = (q_1 - q_1')^2 = 0$$

Light by light scattering



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P and T symmetry: **81** → **8** independent amplitudes

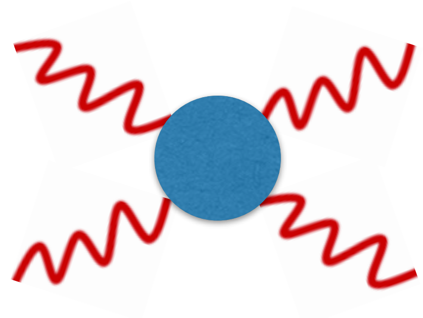
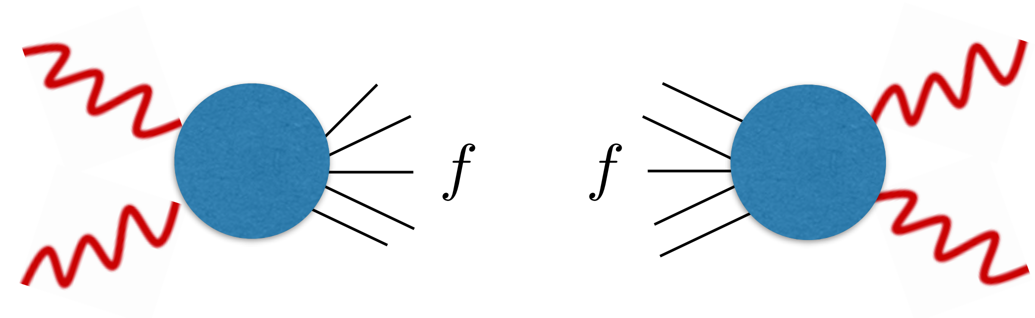
$$M_{++,++},\ M_{+-,+-},\ M_{++,--}$$

$$M_{00,00},\ M_{+0,+0},\ M_{0+,0+}$$

$$M_{++,00},\ M_{0+,-0}$$

Light by light scattering

Unitarity

2 Im  $= \sum_f \int d\Pi_f$ 

For the forward scattering (optical theorem):

$$\mathrm{Im}\, M_{++,++} = 2\sqrt{X}\, \sigma_0$$

$$\mathrm{Im}\, M_{+-,+-} = 2\sqrt{X}\, \sigma_2$$

$$\mathrm{Im}\, M_{++,--} = 2\sqrt{X}\, (\sigma_{\parallel} - \sigma_{\perp})$$

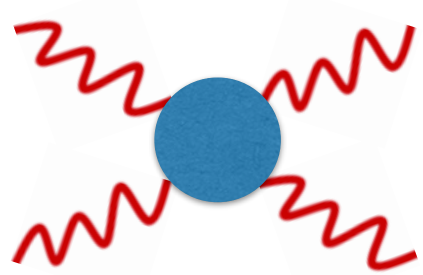
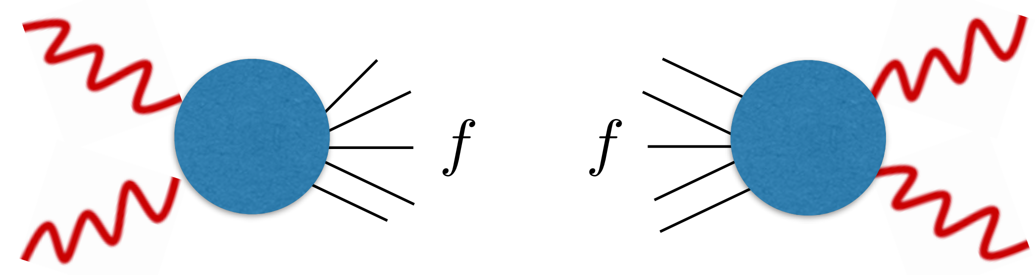
$\dots$

X - flux factor

Observables in: $e^+e^- \to e^-e^+ f$

Light by light scattering

Unitarity

2 Im  $= \sum_f \int d\Pi_f$ 

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Observables in: $e^+e^- \to e^-e^+ f$

Analyticity (fixed *t* dispersion relation)

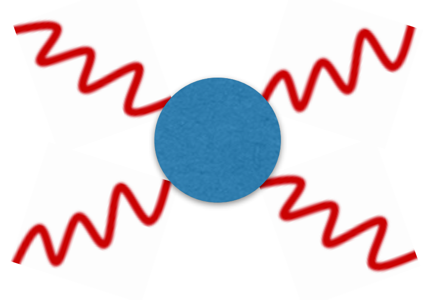
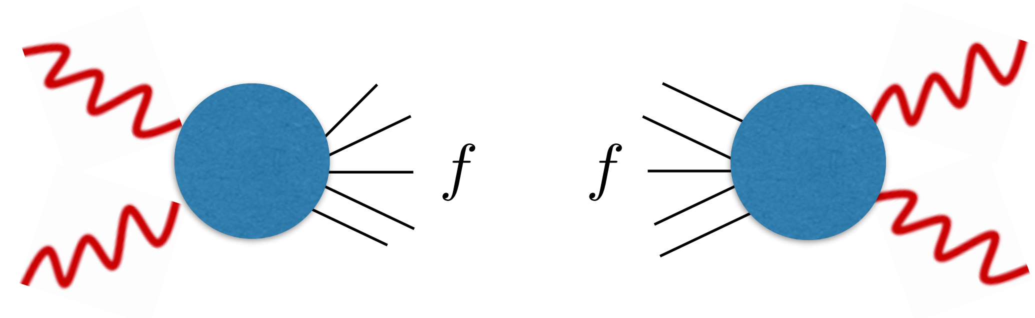
$$M_{++,--}(\nu) = \int_{\nu_0}^{\infty} \frac{d\nu'}{\pi} \frac{2\,\nu'\,\mathrm{Im} M_{++,--}(\nu')}{\nu'^2 - \nu^2 + i\,0}\,, \qquad \nu = \frac{s-u}{4}$$

...

(modulo subtractions)

Light by light scattering

Unitarity

2 Im  $= \sum_f \int d\Pi_f$ 

For the forward scattering (optical theorem):

$$\mathrm{Im}\, M_{++,++} = 2\sqrt{X}\,\sigma_0$$
$$\mathrm{Im}\, M_{+-,+-} = 2\sqrt{X}\,\sigma_2$$
$$\mathrm{Im}\, M_{++,--} = 2\sqrt{X}\,(\sigma_{\parallel} - \sigma_{\perp})$$
$$\ldots$$

X - flux factor

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(modulo subtractions)

Matching around $\nu = 0$ to the LbL Lagrangian

$$\mathcal{L} = c_1(F_{\mu\nu}F^{\mu\nu})^2 + c_2(F_{\mu\nu}\tilde{F}^{\mu\nu})^2 + \ldots$$

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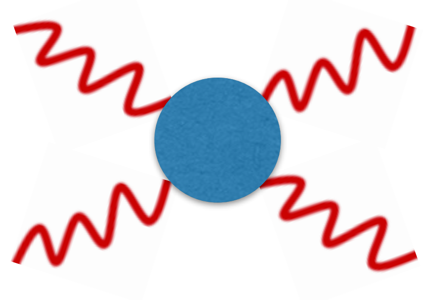
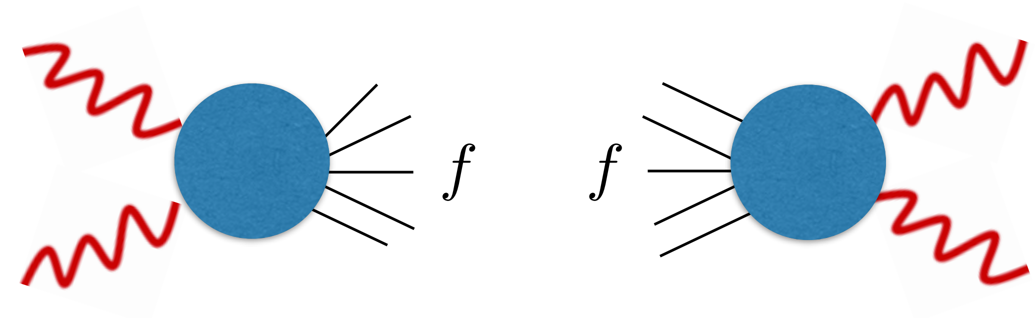
2 Im  $= \sum_f \int d\Pi_f$ 

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yield a number of **constraints on cross sections**

Light by light sum rules

Three super convergence relations

Gerasimov, Moulin (1975), Brodsky, Schmidt (1995)

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)} \left[\sigma_2 - \sigma_0\right]_{Q_2^2=0}$$

Pascalutsa, Vanderhaeghen (2010)

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)^2} \left[\sigma_\parallel + \sigma_{LT} + \frac{(s+Q_1^2)}{Q_1 Q_2} \tau_{TL}^a \right]_{Q_2^2=0}$$

$$0 = \int_{s_0}^{\infty} ds \left[\frac{\tau_{TL}(s, Q_1^2, Q_2^2)}{Q_1 Q_2}\right]_{Q_2^2=0}$$

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These sum rules have been tested in perturbative QFT both at tree-level and one loop level:

scalar QED	ϕ^4 theory
spinor QED	ϕ^4 theory + resum.

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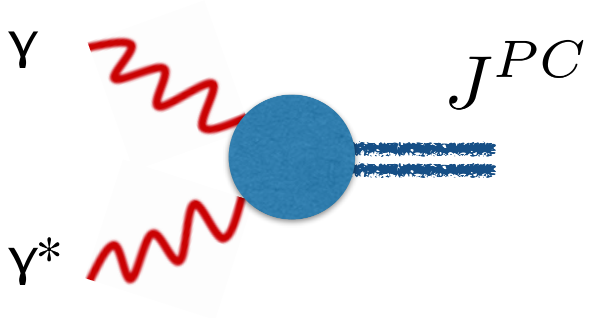
Light by light sum rules: Meson production



C=+1: $J^{PC}=0^{-+}, 0^{++}, 1^{++}, 2^{++}, \ldots$

$Q^2 \neq 0$

Landau-Yang theorem

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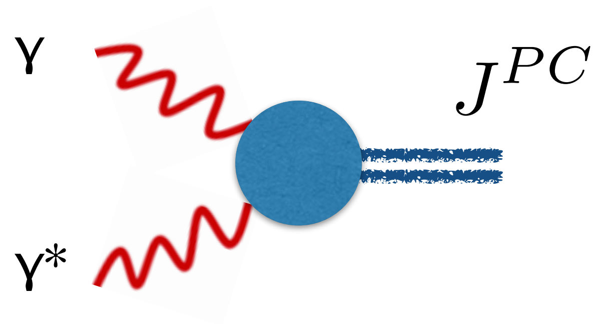

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Narrow width approximation

$$\sigma\left(\gamma^*\gamma \to J^P(\Lambda)\right) = \delta(s-m^2)\, 8\pi^2 \frac{(2J+1)\,\Gamma_{\gamma\gamma}\left(J^P\right)}{m}\left(1+\frac{Q^2}{m^2}\right)\left[\mathrm{T}^{(\Lambda)}\left(Q^2\right)\right]^2$$

Sum rules will relate 2γ width or TFFs:

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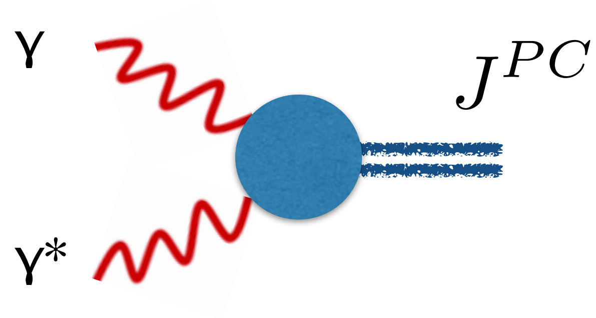

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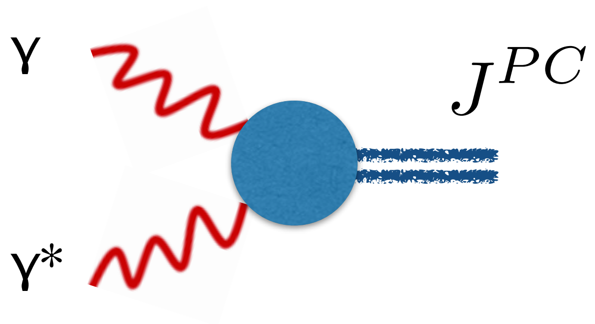

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Dominant contributions

State	m (MeV)	$\Gamma_{\gamma\gamma}$(keV)		SR_1 $(Q^2 = 0)$ (nb)
η	547.862±0.017	0.516±0.020		-193±7
η'	957±0.06	4.35±0.25		-304±17
$f_2(1270)$	1275.5±0.8	2.93±0.40	(Λ=2)	434±60
			(Λ=0)	≈0
$f_2(1565)$	1562±13	0.70±0.14		56±11
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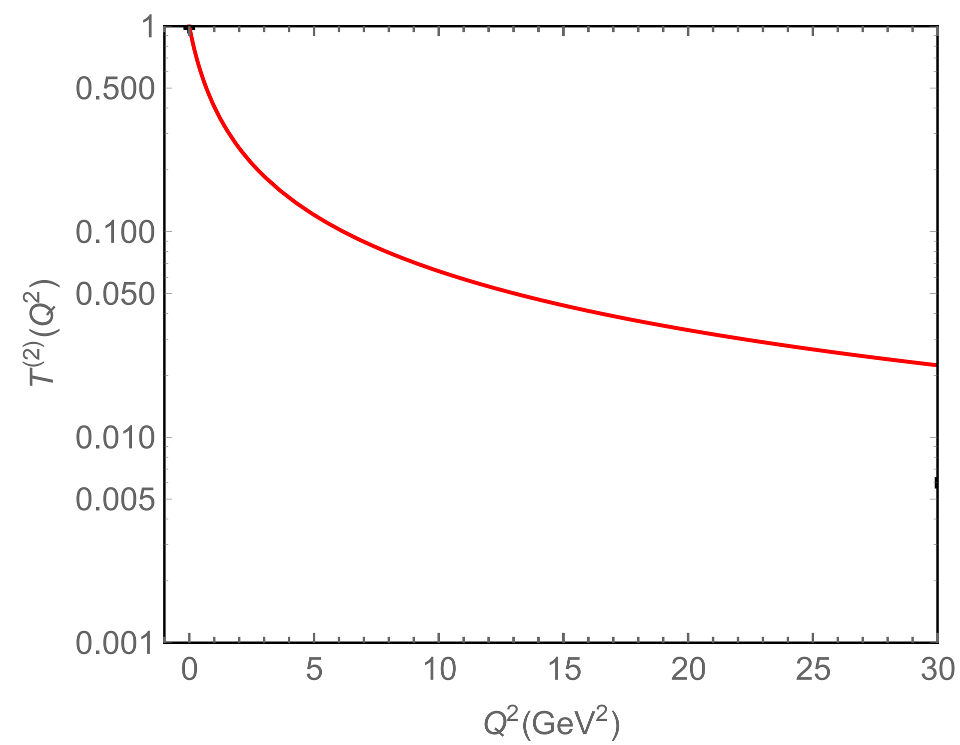

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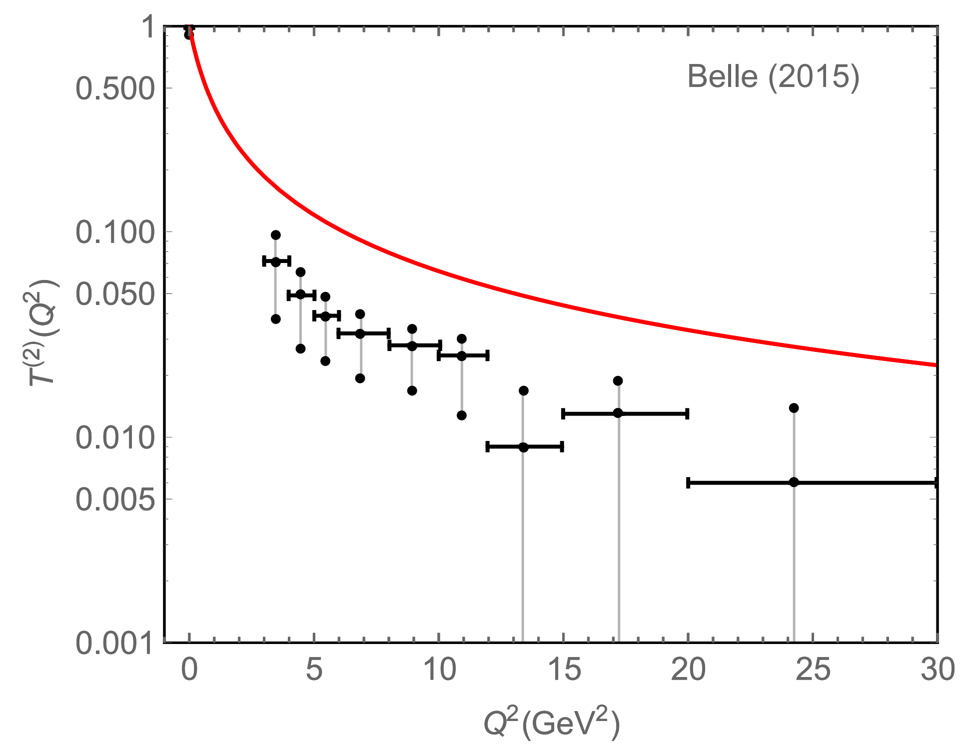

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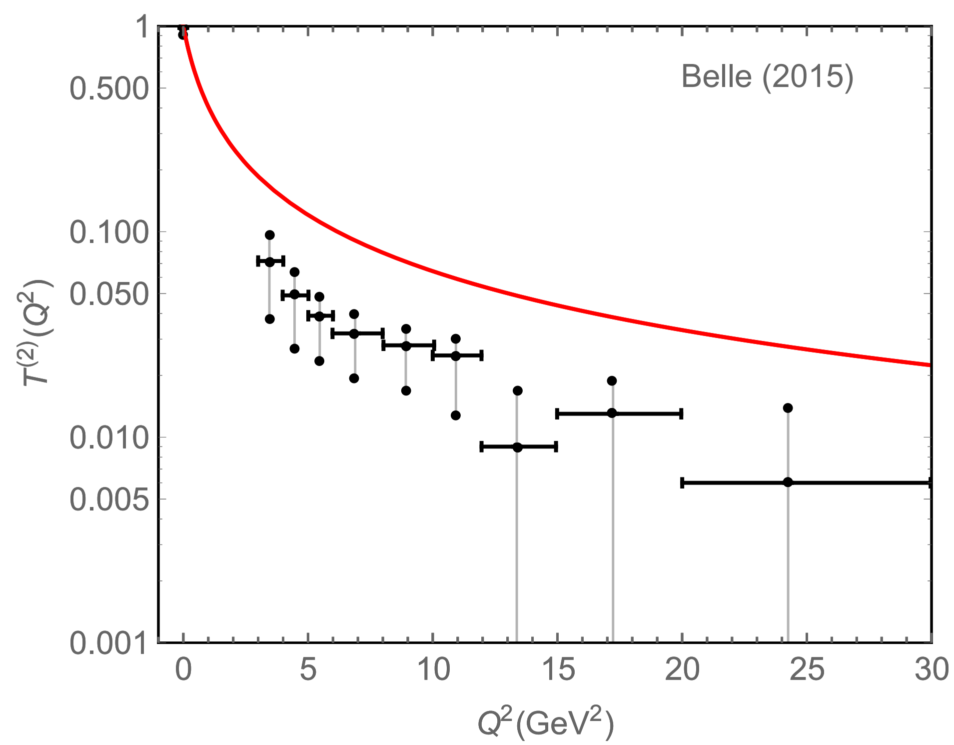
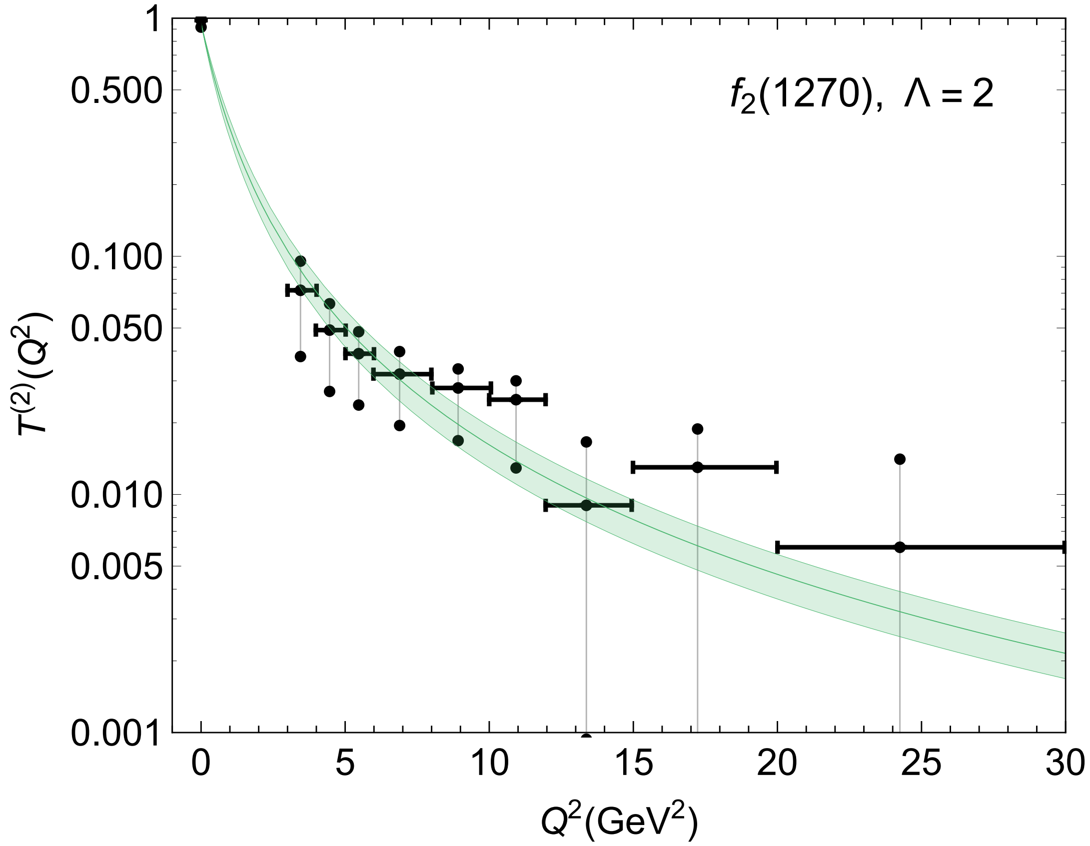
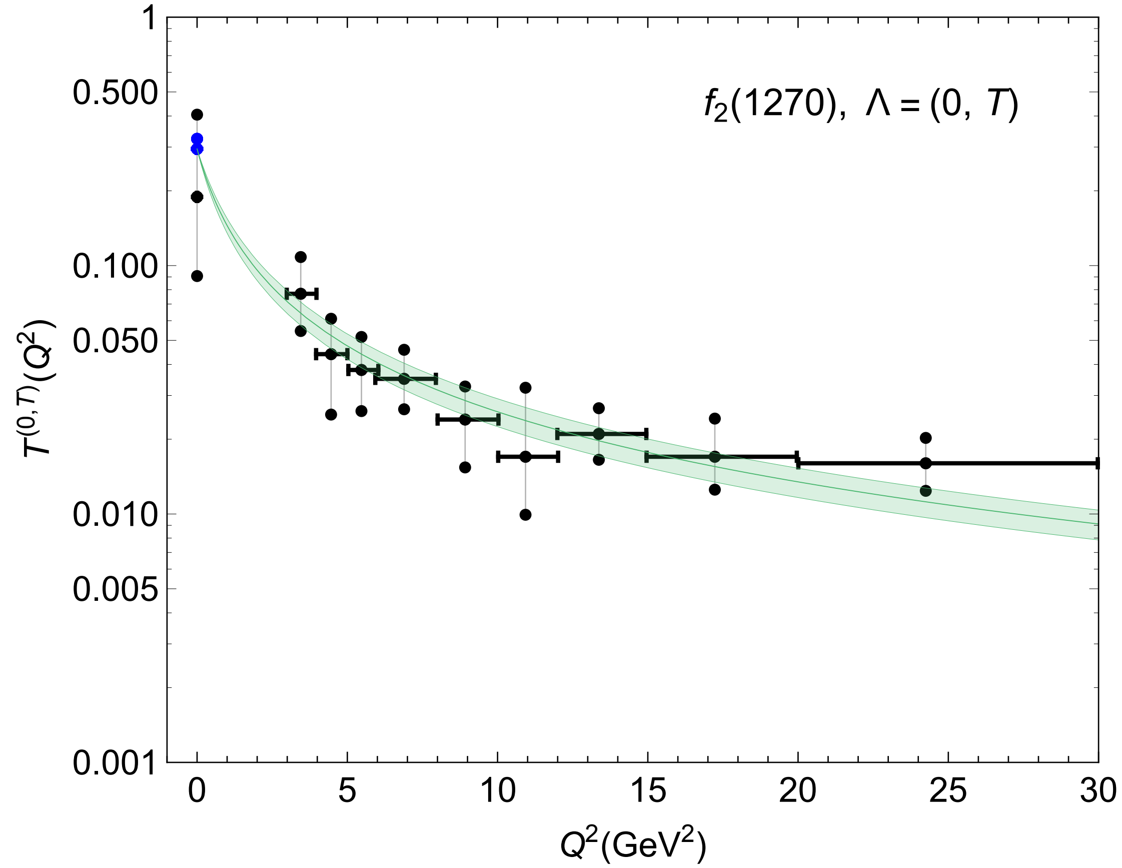


Pascalutsa, Pauk
Vanderhaeghen
(2012)

Belle (2015)

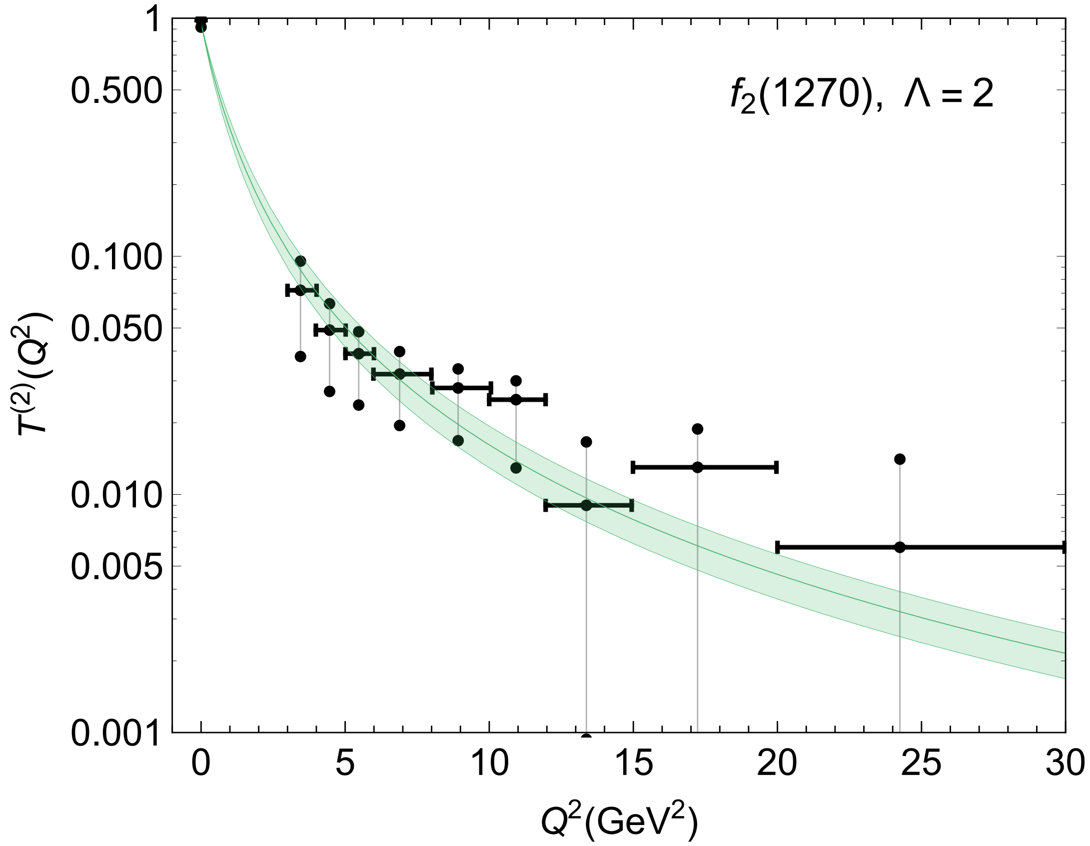


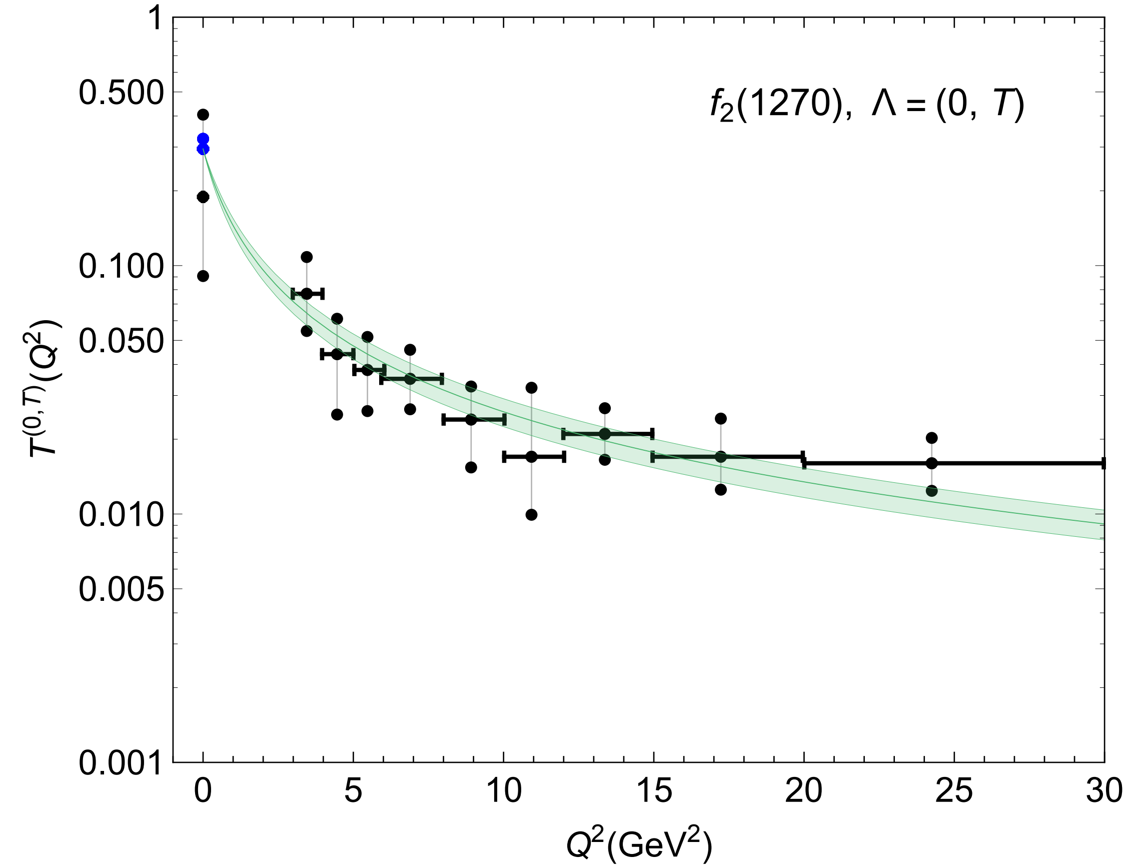
$f_2(1270)$, $\Lambda = 2$
$T^{(2)}(Q^2)$
$Q^2(\mathrm{GeV}^2)$
1
0.500
0.100
0.050
0.010
0.005
0.001
0 5 10 15 20 25 30

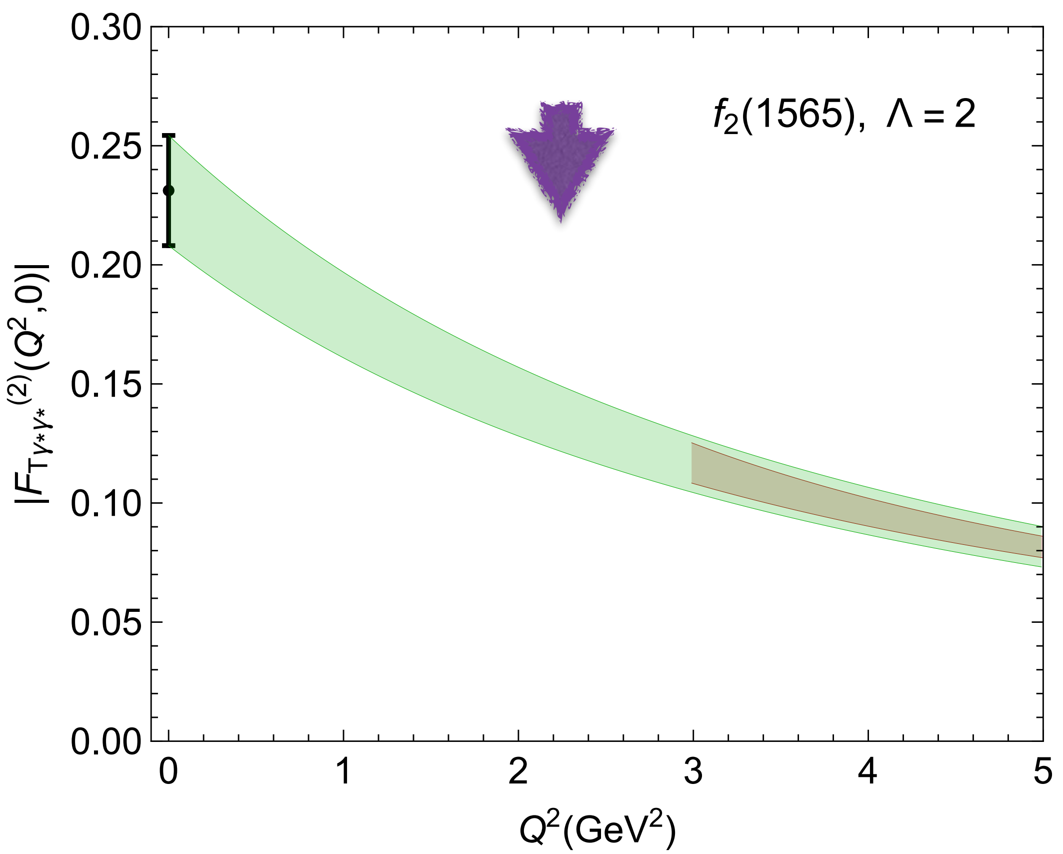


$f_2(1270)$, $\Lambda = (0, T)$
$T^{(0,T)}(Q^2)$
$Q^2(\mathrm{GeV}^2)$
1
0.500
0.100
0.050
0.010
0.005
0.001
0 5 10 15 20 25 30

Belle (2015)







Prediction:

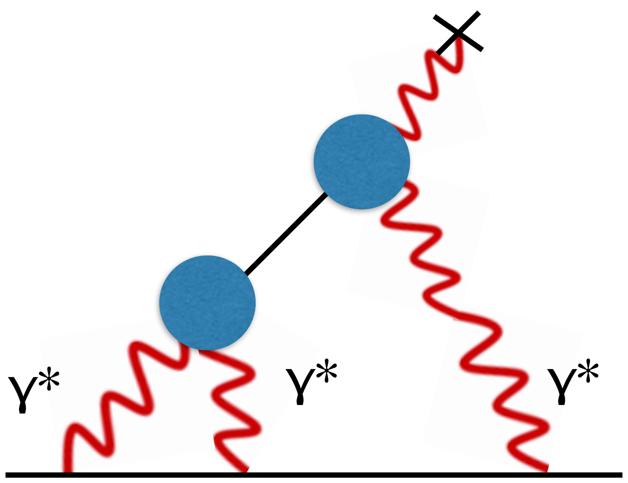
$f_2(1565)$

$$\lambda_{\Lambda=2} = 2719 \pm 53\,\mathrm{MeV}$$

I.D., Vanderhaeghen (2016)

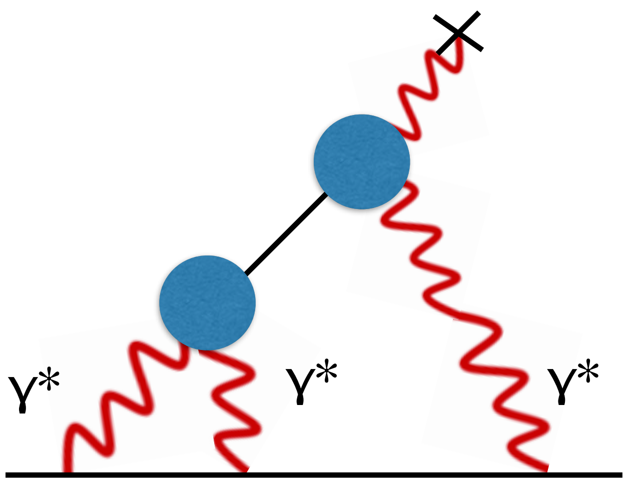
future Belle data

Meson contributions to (g-2)



$$a_\mu^{LbL} = \lim_{k\to 0} ie^6 \int \frac{d^4q_1}{(2\pi)^4} \int \frac{d^4q_2}{(2\pi)^4}\, T^{\mu\nu\lambda\sigma}(q_1, k-q_1-q_2, q_2)\, \Pi_{\mu\nu\lambda\sigma}(q_1, k-q_1-q_2, q_2)$$
$$\frac{1}{q_1^2}\,\frac{1}{q_2^2}\,\frac{1}{(k-q_1-q_2)^2}\,\frac{1}{(p+q_1)^2-m^2}\,\frac{1}{(p'-q_2)^2-m^2}$$

Meson contributions to (g-2)



Lepton tensor: well known

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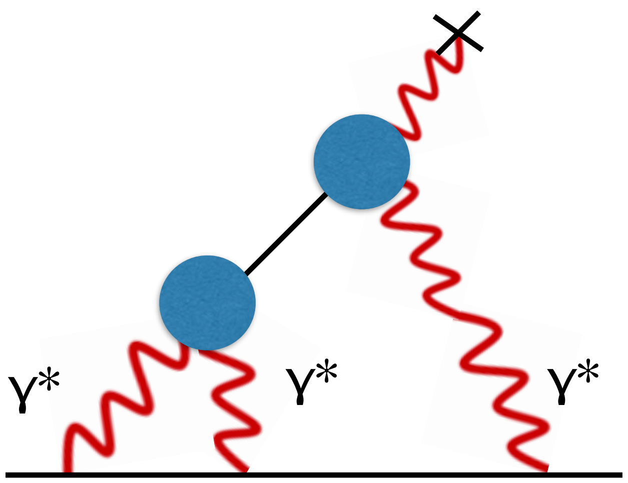

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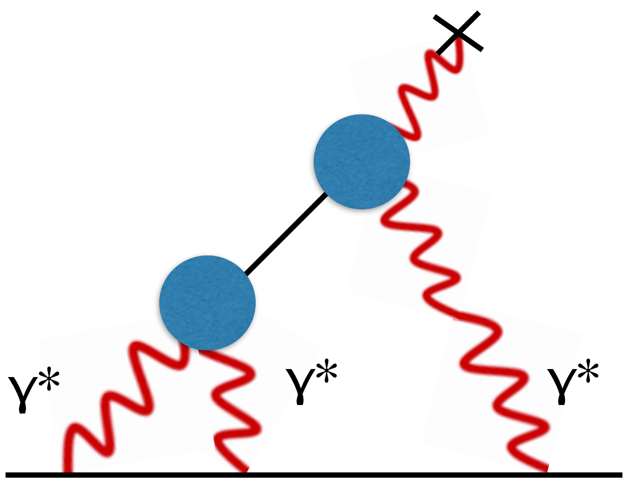

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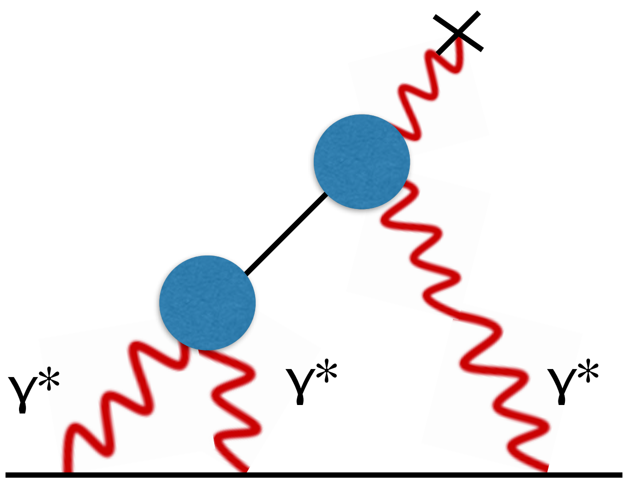
Results (excluding low energy region):

$$a_\mu[f_2(1270), f_2(1565)] = (0.1 \pm 0.01) \times 10^{-10}$$

Meson contributions to (g-2)



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$$a_\mu^{LbL} = \lim_{k\to 0} ie^6 \int \frac{d^4 q_1}{(2\pi)^4} \int \frac{d^4 q_2}{(2\pi)^4} T^{\mu\nu\lambda\sigma}(q_1, k-q_1-q_2, q_2)\, \Pi_{\mu\nu\lambda\sigma}(q_1, k-q_1-q_2, q_2)$$
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Results (excluding low energy region):

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New evaluation of axial vector contributions (satisfying Landau-Yang theorem)

$$a_\mu[f_1(1285), f_1(1420)] = (0.64 \pm 0.20) \times 10^{-10}$$
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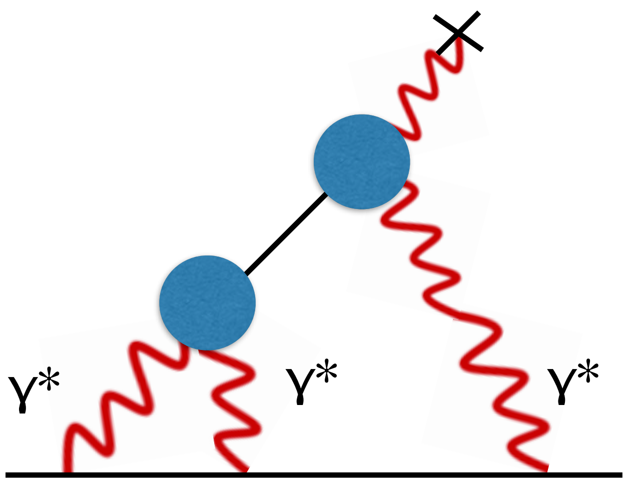
Pauk, Vdh (2013)
Jegerlehner (2015)

Meson contributions to (g-2)



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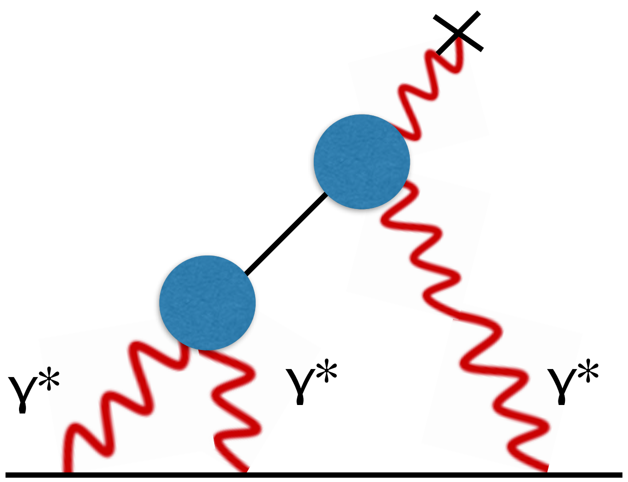

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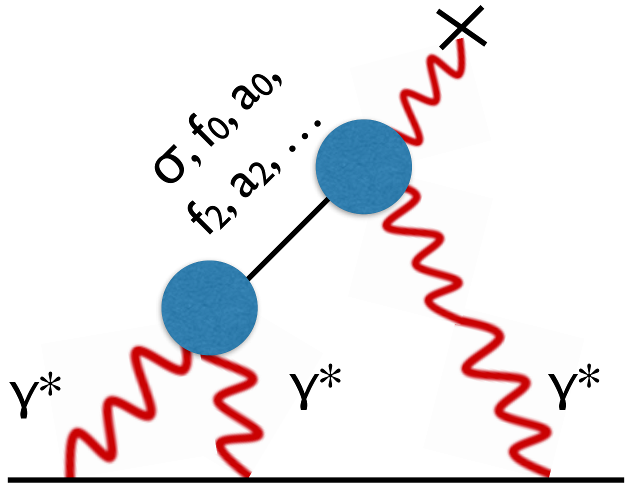
FNAL, J-PARC experiments

Improvements: Multi-meson production

Important contributions beyond **pseudo-scalar** poles



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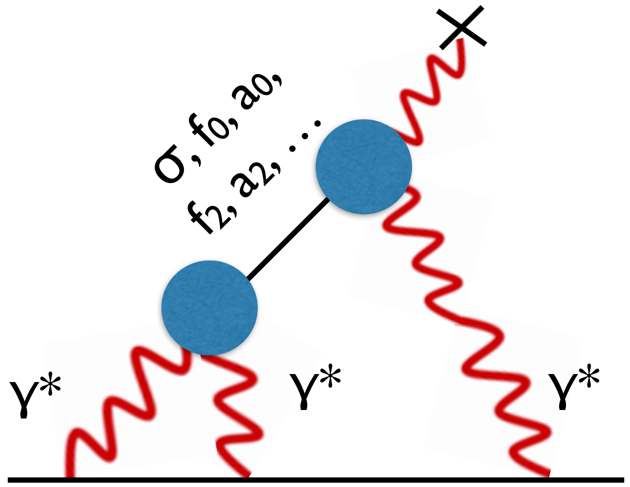
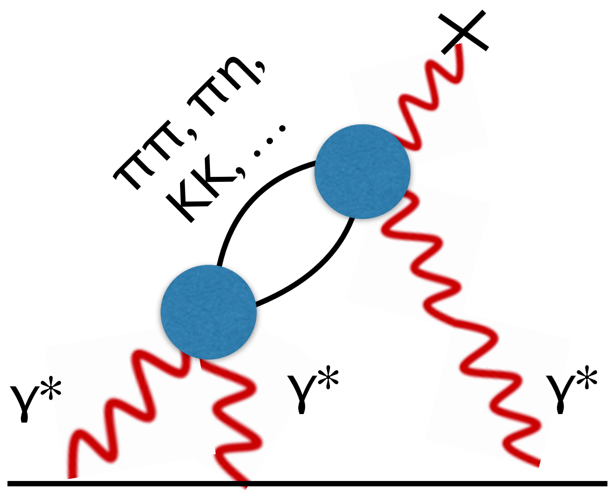


dispersive analysis for
ππ, πη, … loops



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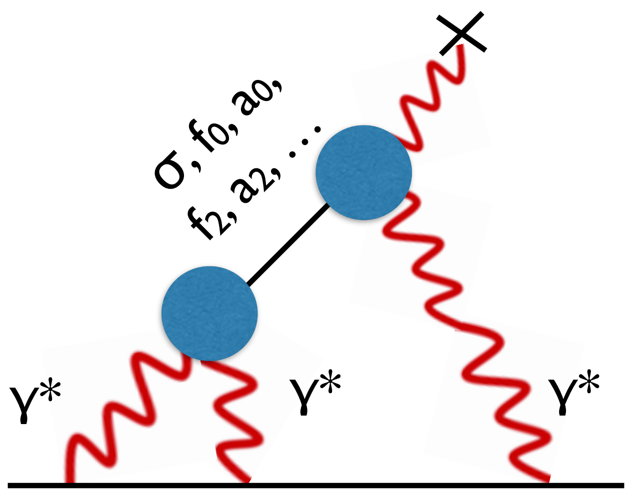

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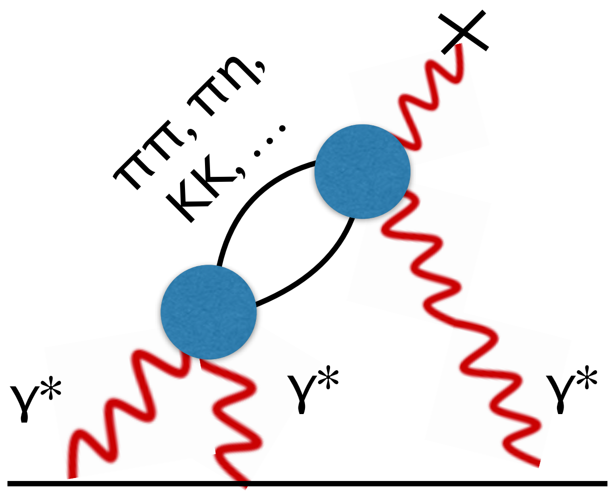

Pauk, Vanderhaeghen, (2014)

Colangelo, Hoferichter, Procura, Stoffer, (2017)

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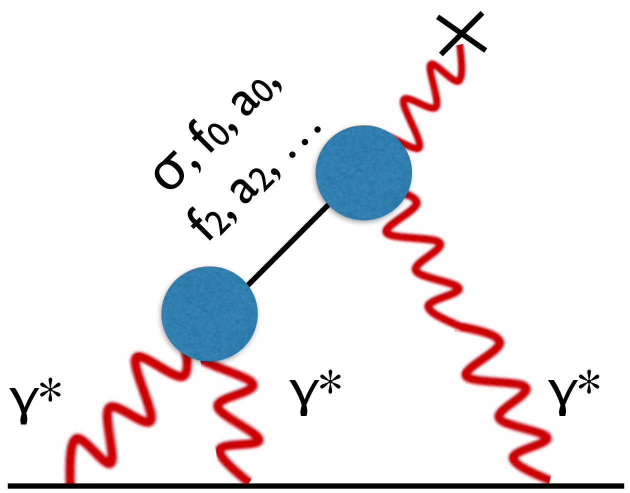

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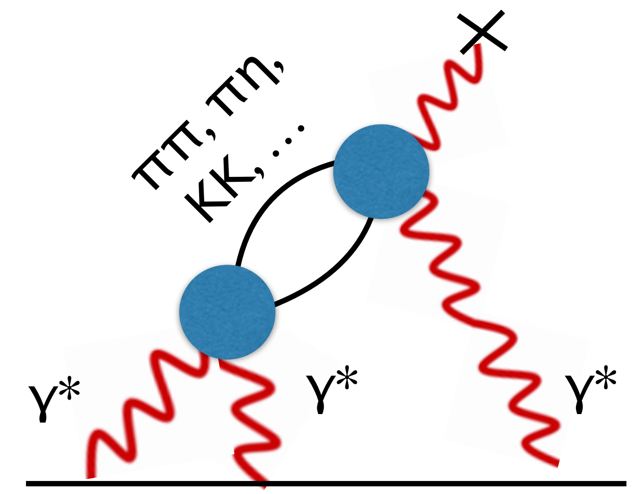
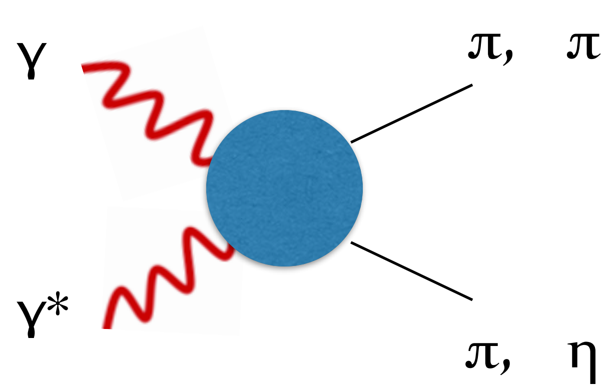


Important ingredient: γγ*→ππ, πη, …

Pauk, Vanderhaeghen, (2014)

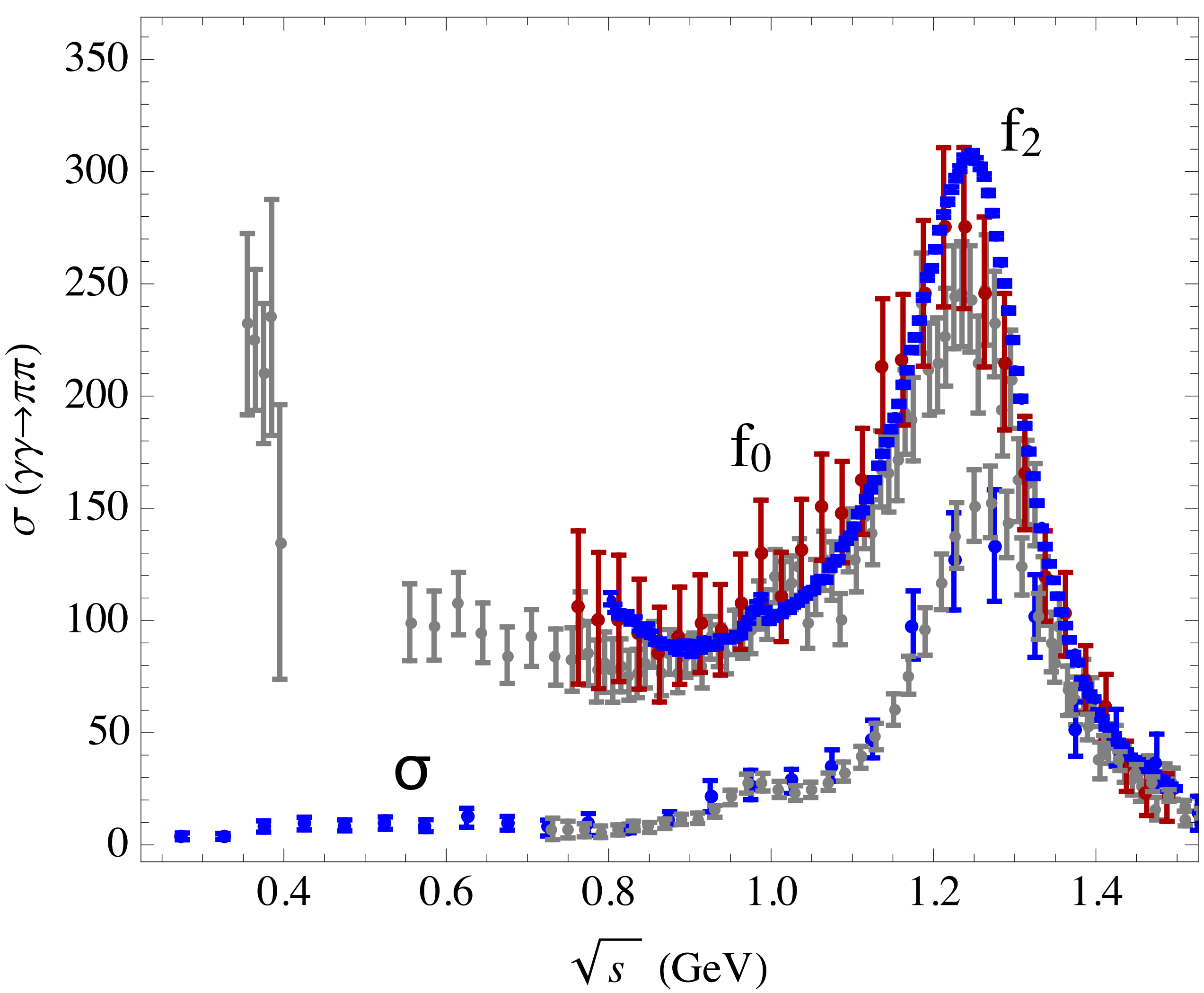
Colangelo, Hoferichter, Procura, Stoffer, (2017)



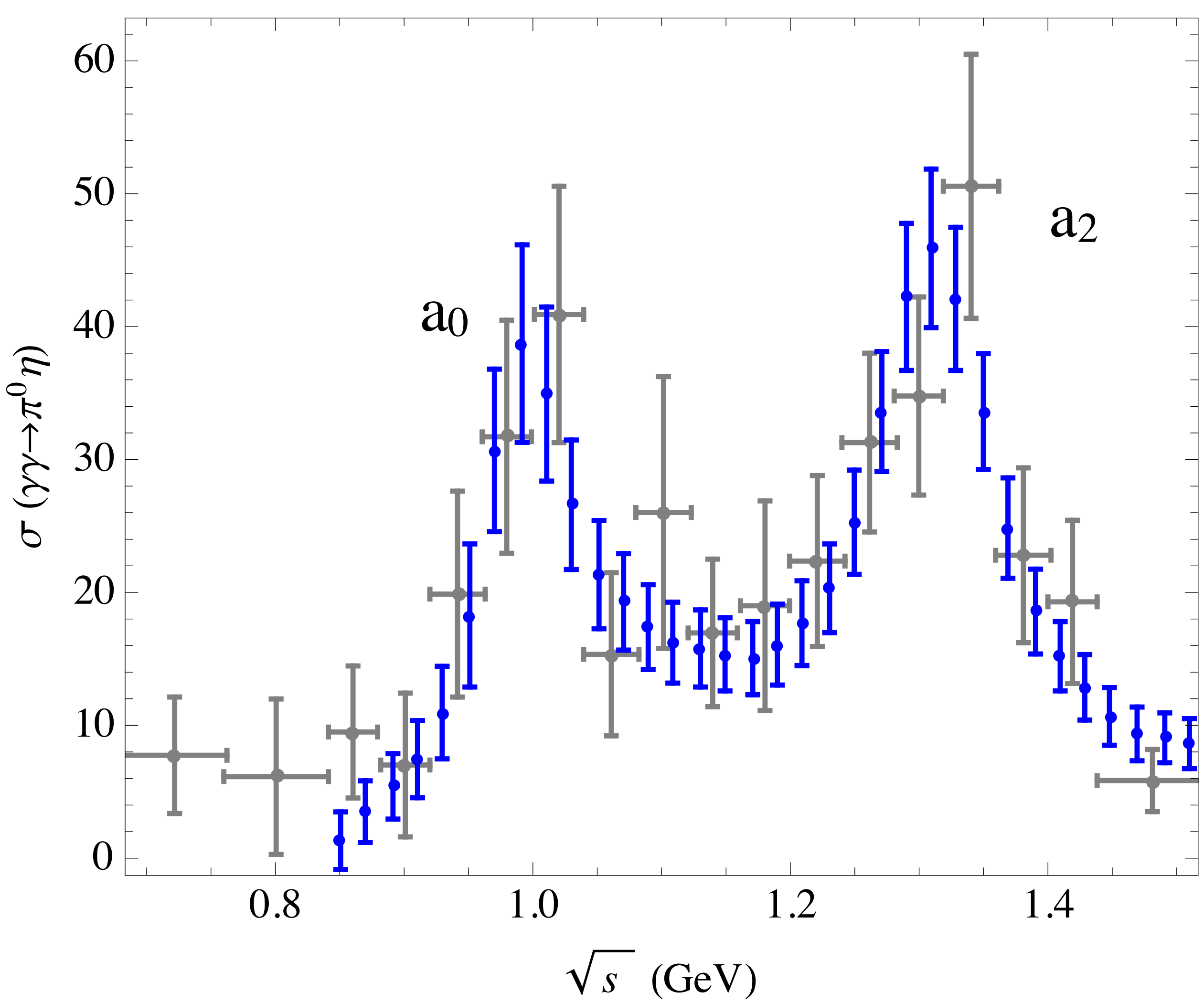
γγ→ππ, KK, ηη, πη (Belle: 07,08, 09,10, ..)
γγ*→ππ, πη (BESIII in progress)

Experimental data

$\gamma\gamma \to \pi^+\pi^-, \pi^0\pi^0$



$\gamma\gamma \to \pi^0\eta$



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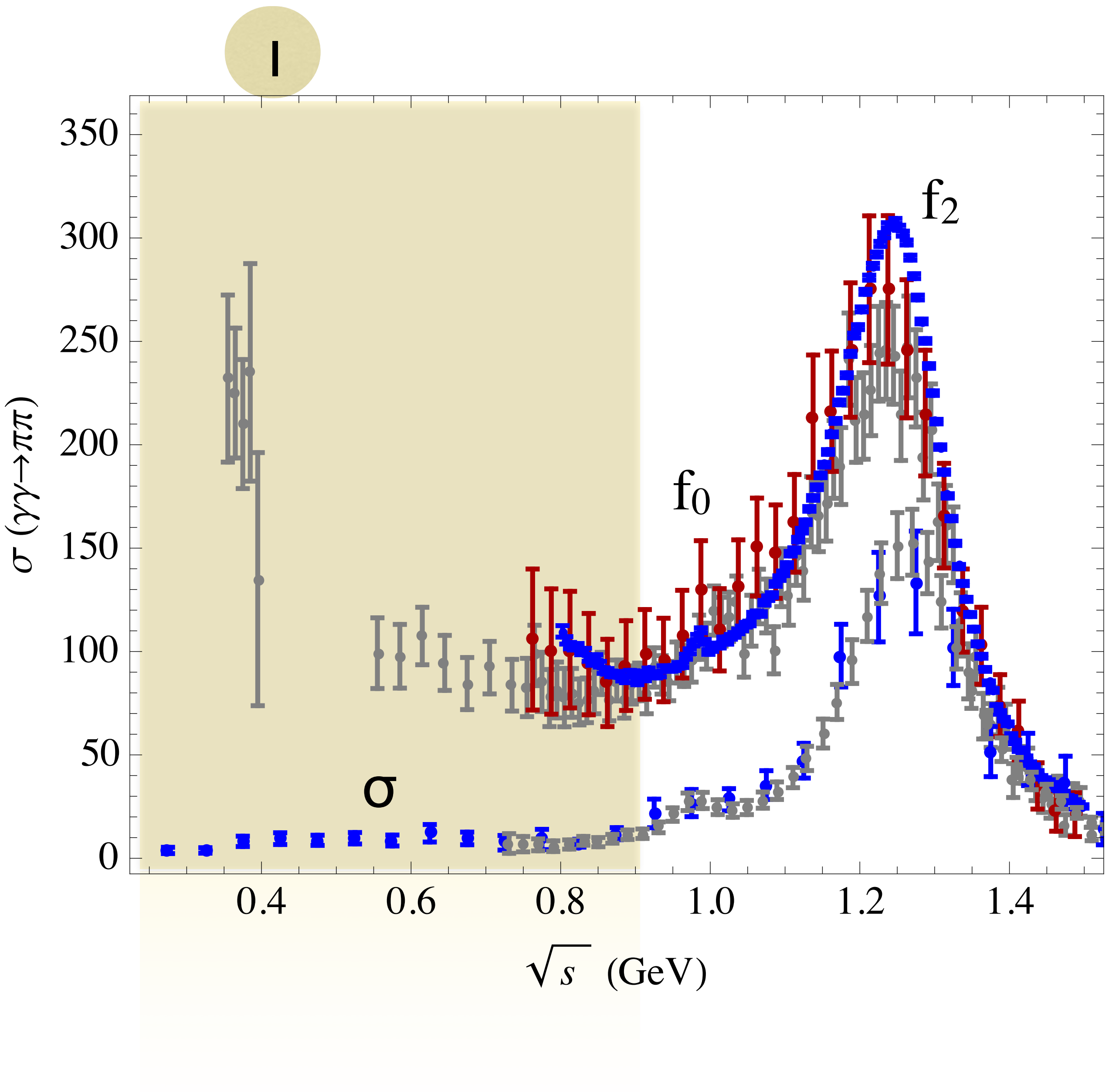

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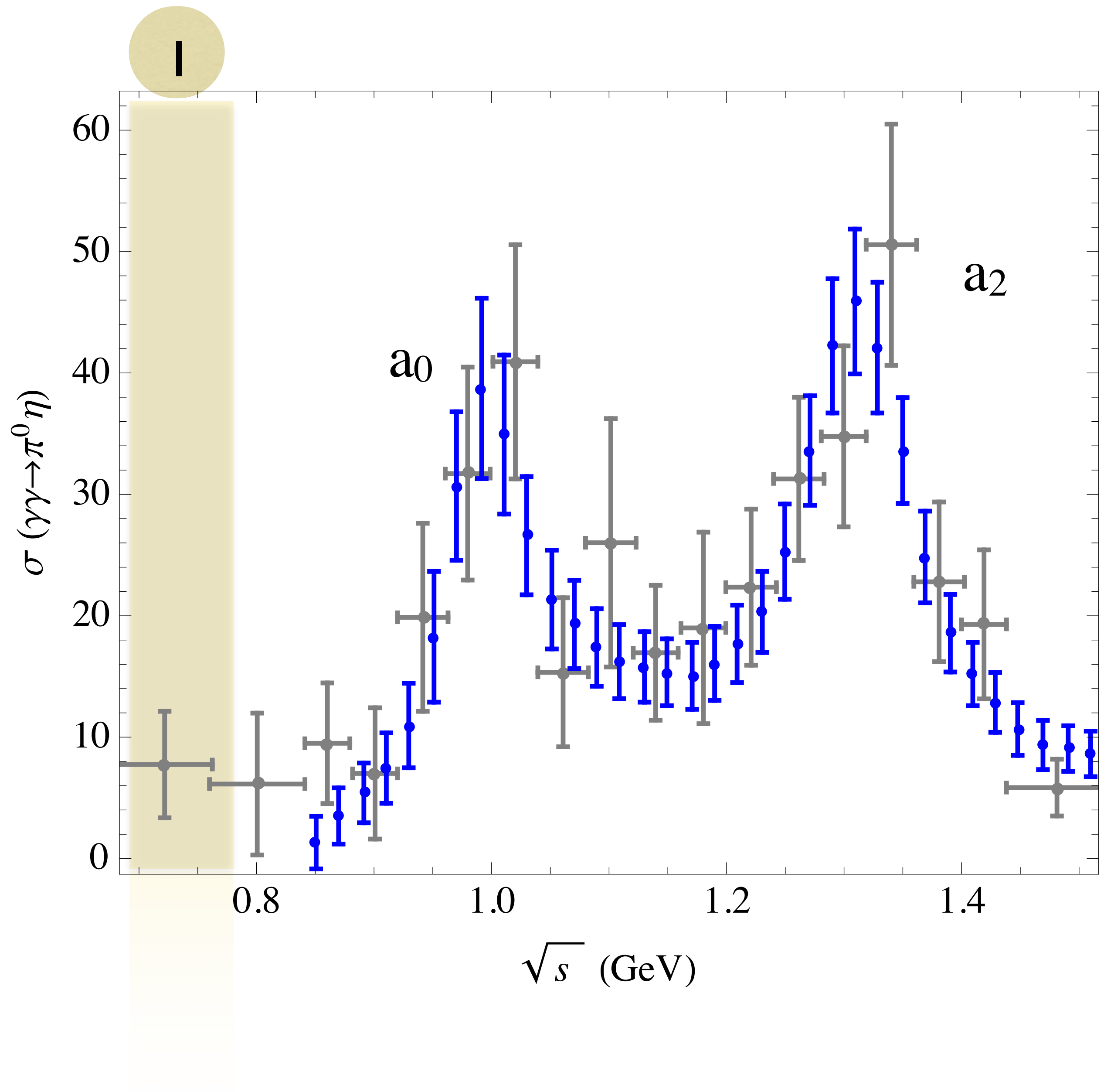

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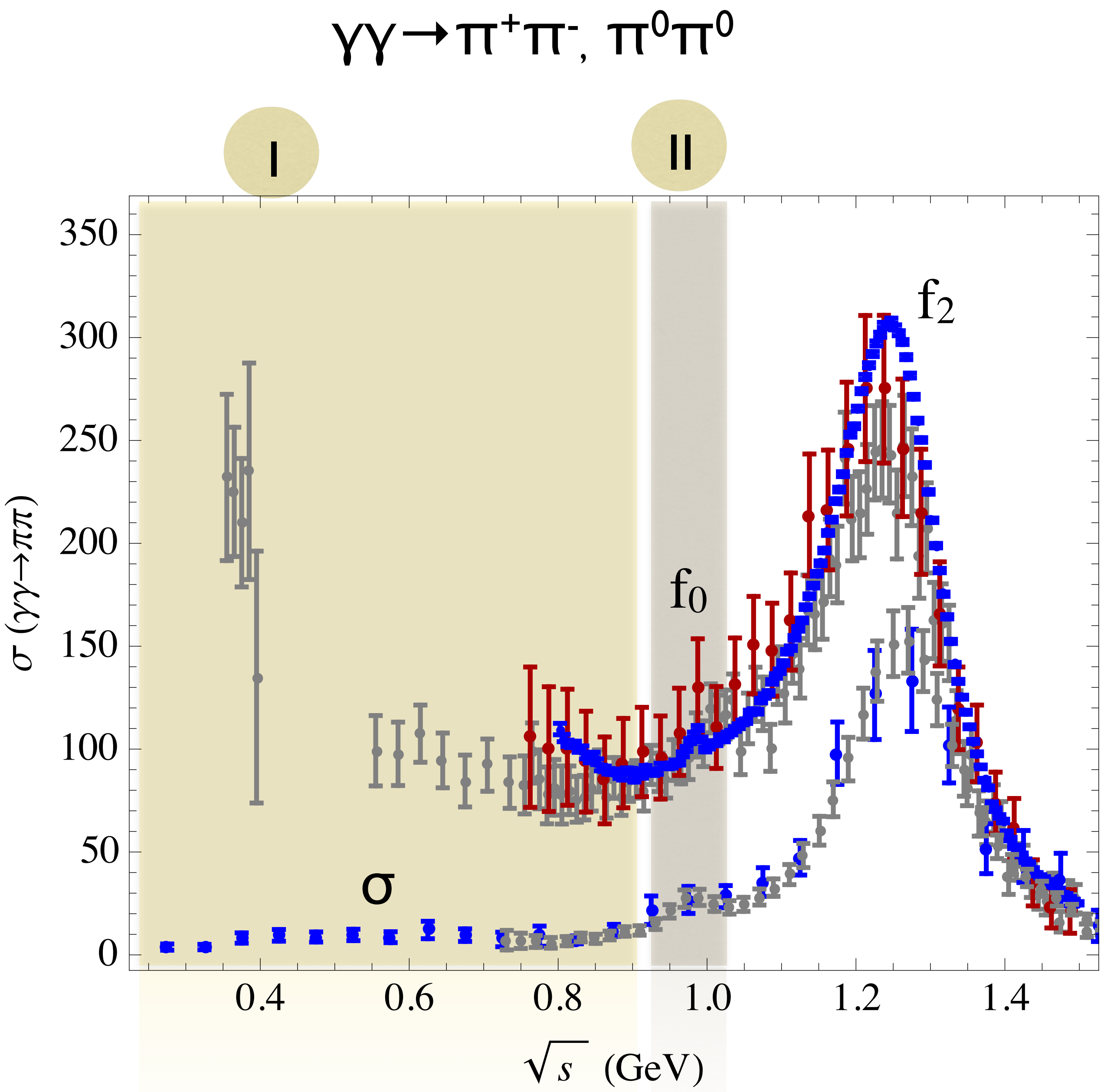

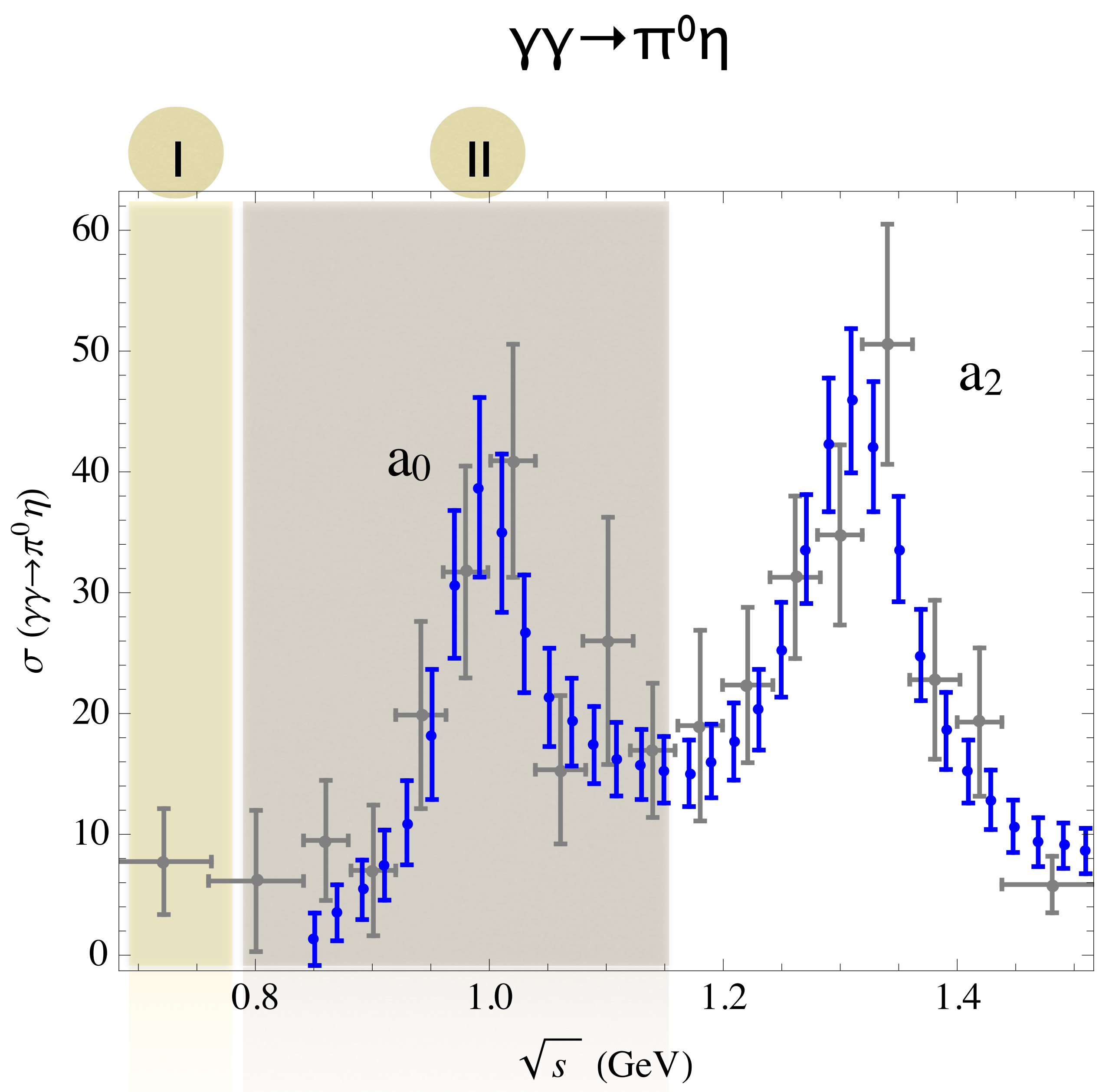

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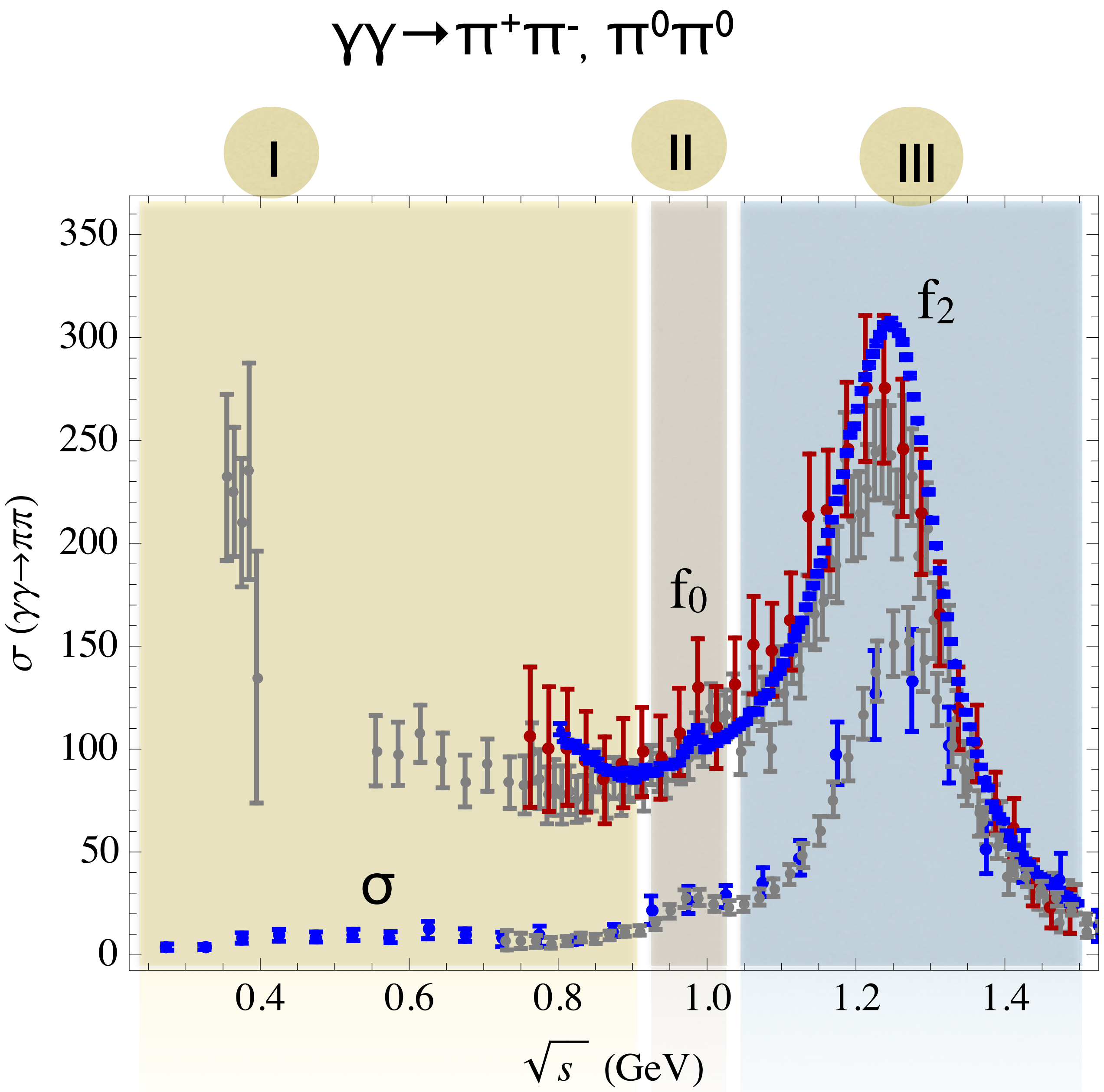

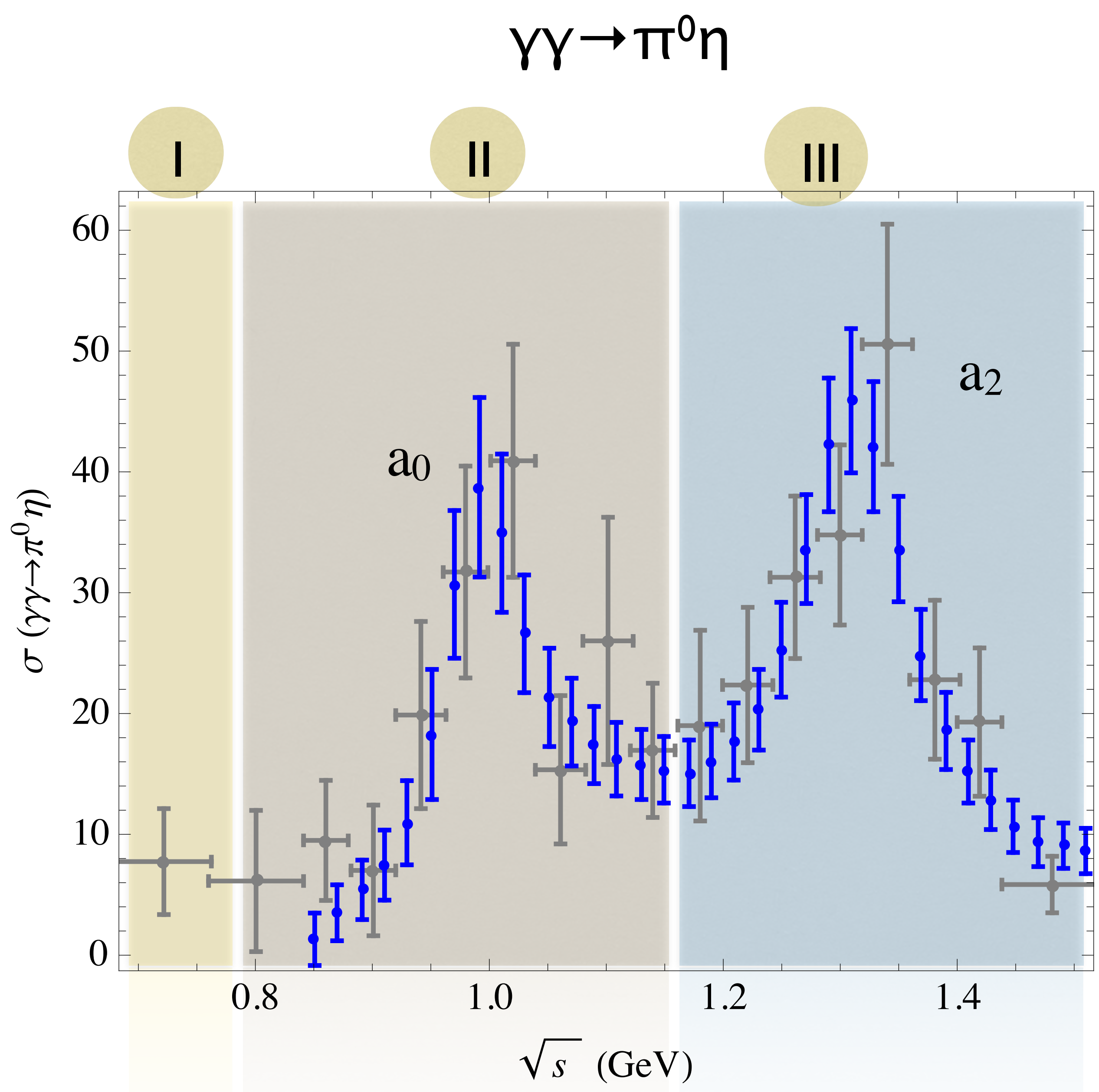

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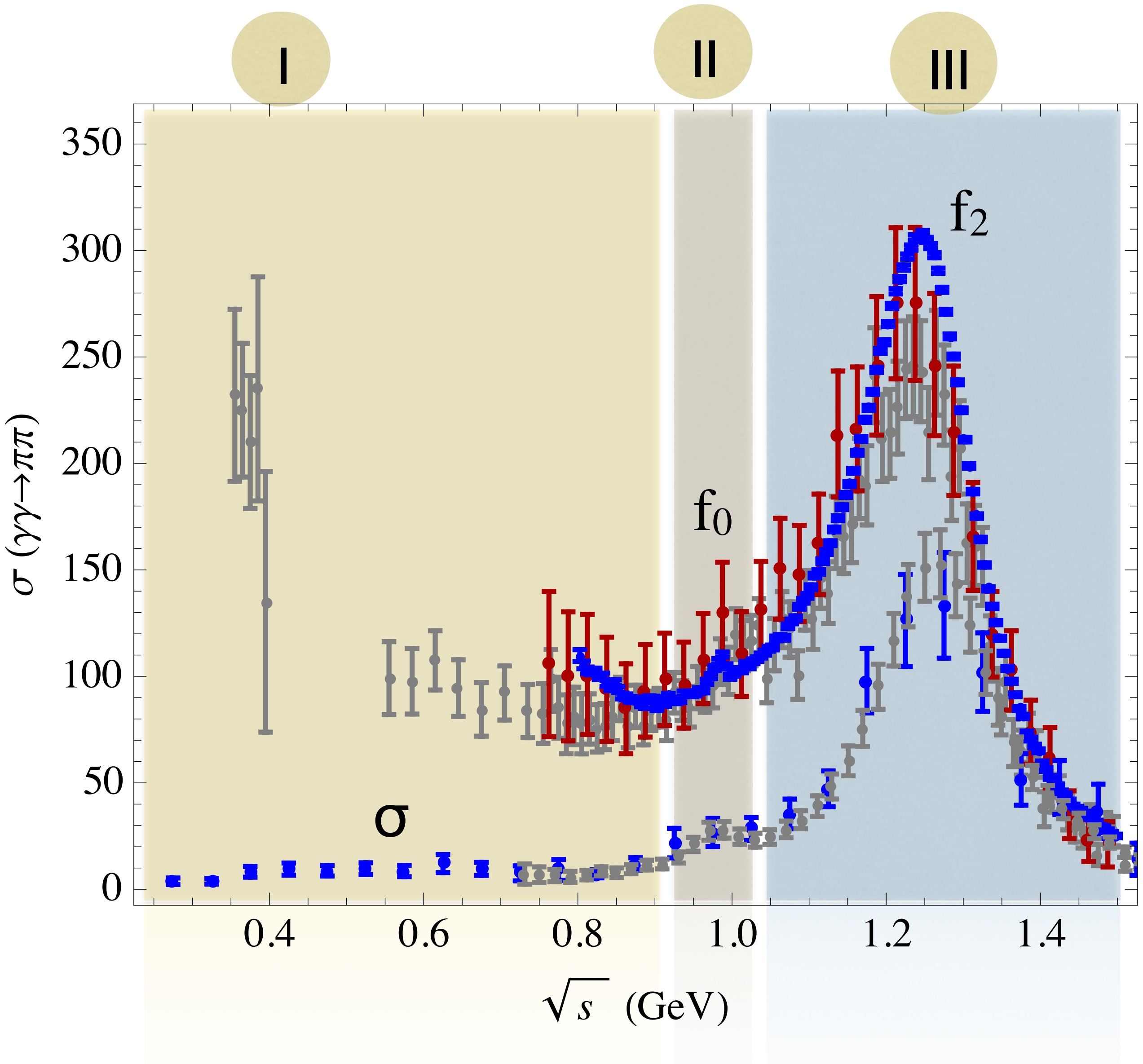
$\gamma\gamma \to \pi^+\pi^-$, $\pi^0\pi^0$



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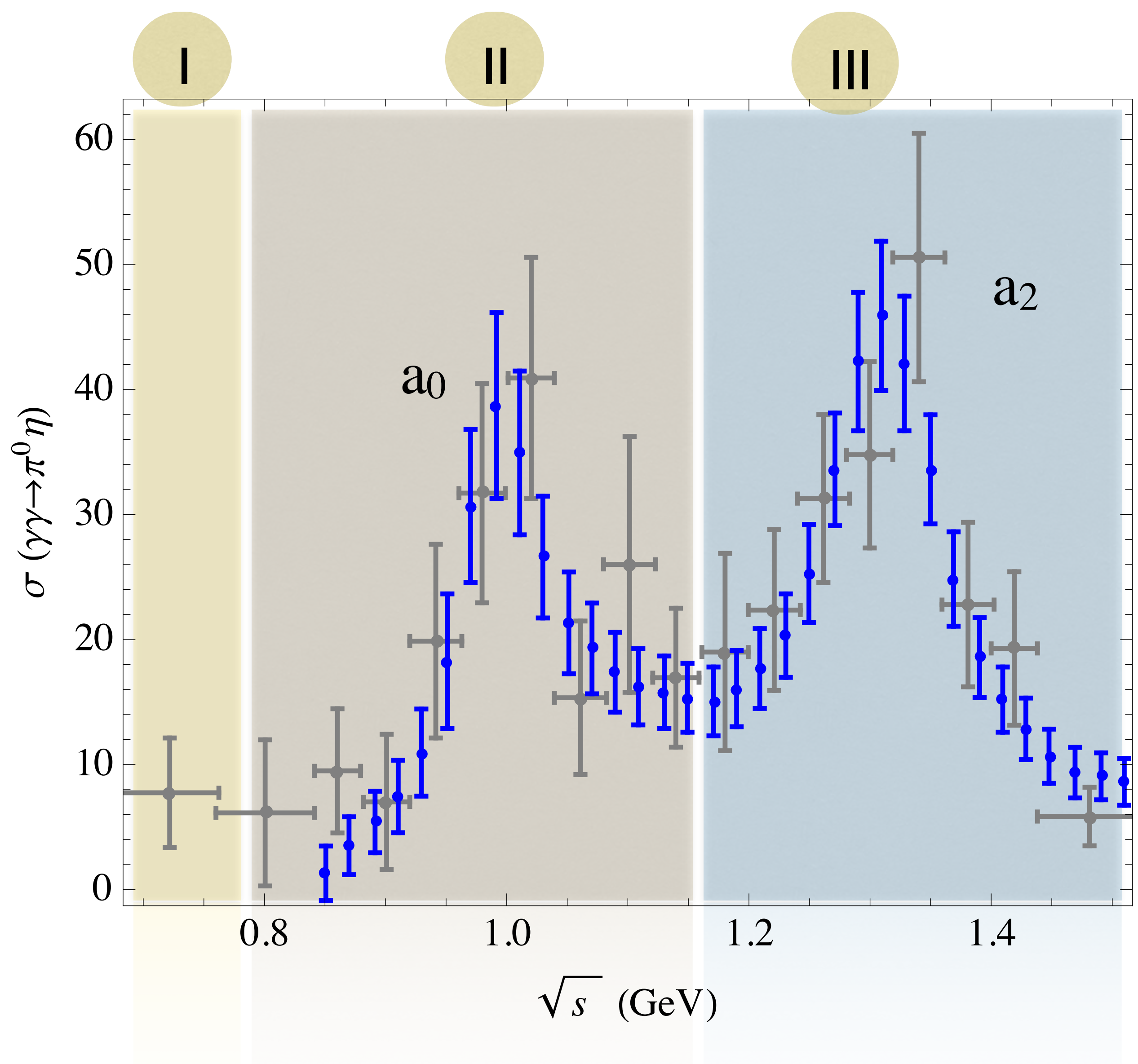

$\gamma\gamma \to \pi^+\pi^-$: Mark II ('90), CELLO ('92), Belle ('07)
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$\gamma\gamma \to \eta\eta$: Belle ('10)
$\gamma\gamma \to KK$: ARGUS ('90), TASSO ('85), CELLO ('89), Belle ('13)

Ongoing experiment
$\gamma\gamma^* \to \pi\pi$, $\pi\eta$: BES III

What has been done so far?

$Q^2 = 0$	Approach	Inelasticity	Number of fitted parameters to $\sigma_{\gamma\gamma\to MM}$	Range of applicability
[Hoferichter et. al. 2011]	Roy-Steiner	$\pi\pi$	0	$\sqrt{s} < 0.98$ GeV
[Morgan et. al. 1998]	Disp, Omnes	$\pi\pi$	0	$\sqrt{s} \lesssim 0.6$ GeV
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[Current work]	Disp, Omnes	$\pi\pi$, KK $\pi\eta$, KK	0	$\sqrt{s} < 1.4$ GeV
$Q^2 \neq 0$				
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Only dispersive analyses are shown

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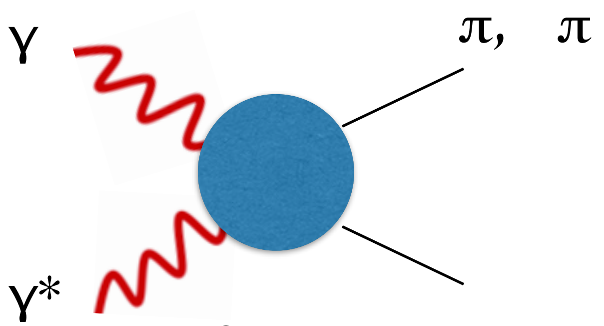
Cross section



$q_1^2 = 0$

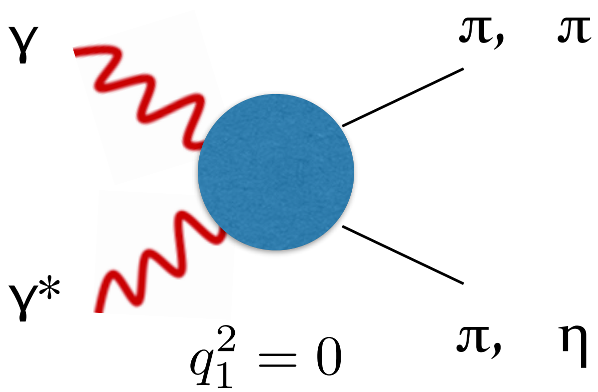
$q_2^2 = -Q^2$

C=+1: $J^{PC}=0^{++}, 2^{++}, 1^{-+}, \ldots.$

$Q^2 \neq 0$

Landau-Yang theorem

Cross section



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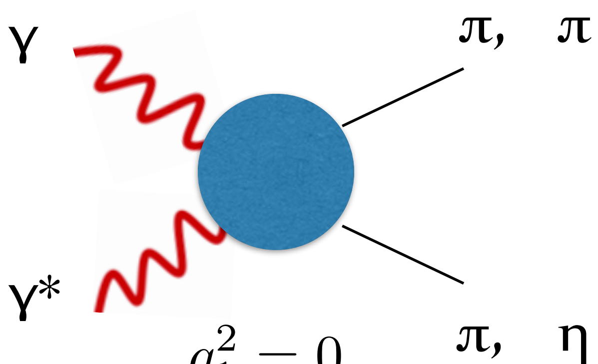
Helicity amplitudes

$$H_{\lambda_1\lambda_2} = H^{\mu\nu}\epsilon_\mu(\lambda_1)\,\epsilon_\nu(\lambda_2), \quad \lambda_1 = \pm 1,\ \lambda_2 = \pm 1, 0$$

Cross section



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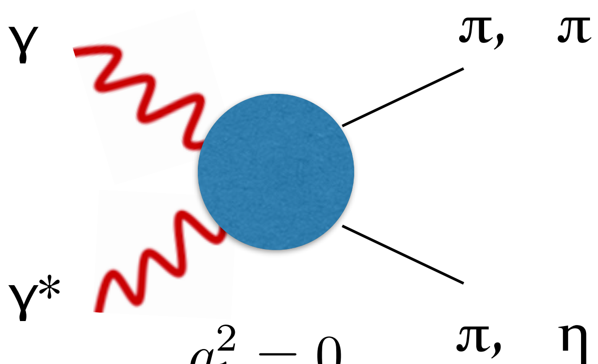
P symmetry: **6** → **3** independent amplitudes

$$H_{++}, H_{+-}, H_{+0}$$

Cross section



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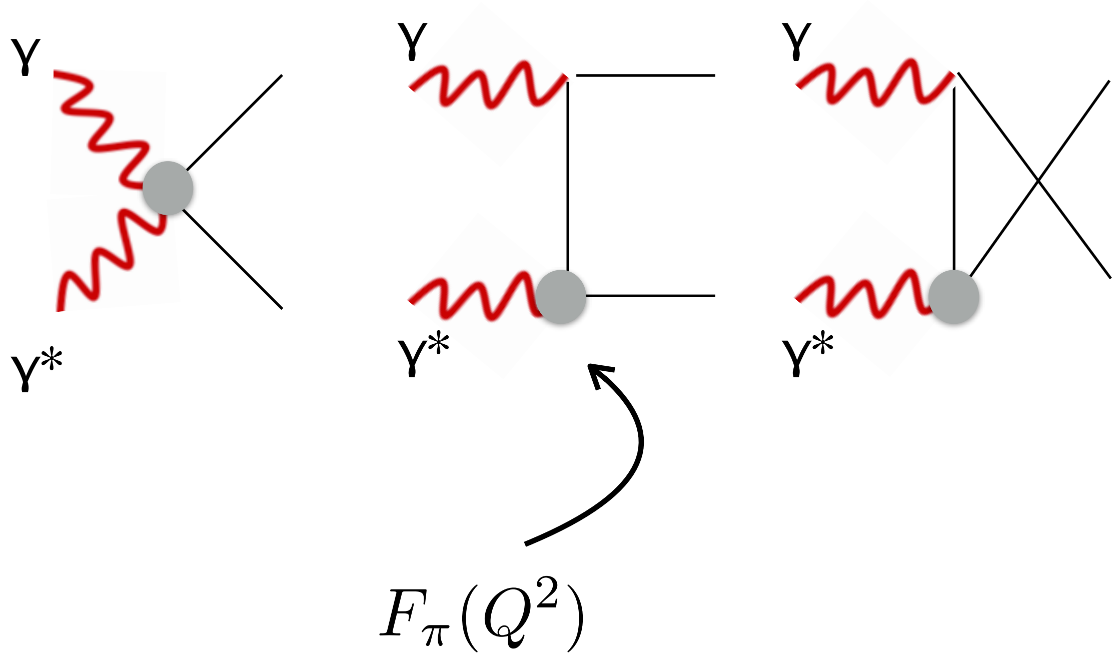
Differential cross section

$$\frac{d\sigma}{d\,\cos\theta} = \pi\alpha^2 \frac{\rho(s)}{4\,(s+Q^2)}\,\left(|H_{++}|^2 + |H_{+-}|^2 + |H_{+0}|^2\right)$$

Born amplitudes ($Q^2 \neq 0$)



Vertex ππγ*

$$\langle \pi^+ | j_\mu(0) | \pi^+(p') \rangle = (p + p')_\mu \, F_\pi(Q^2)$$

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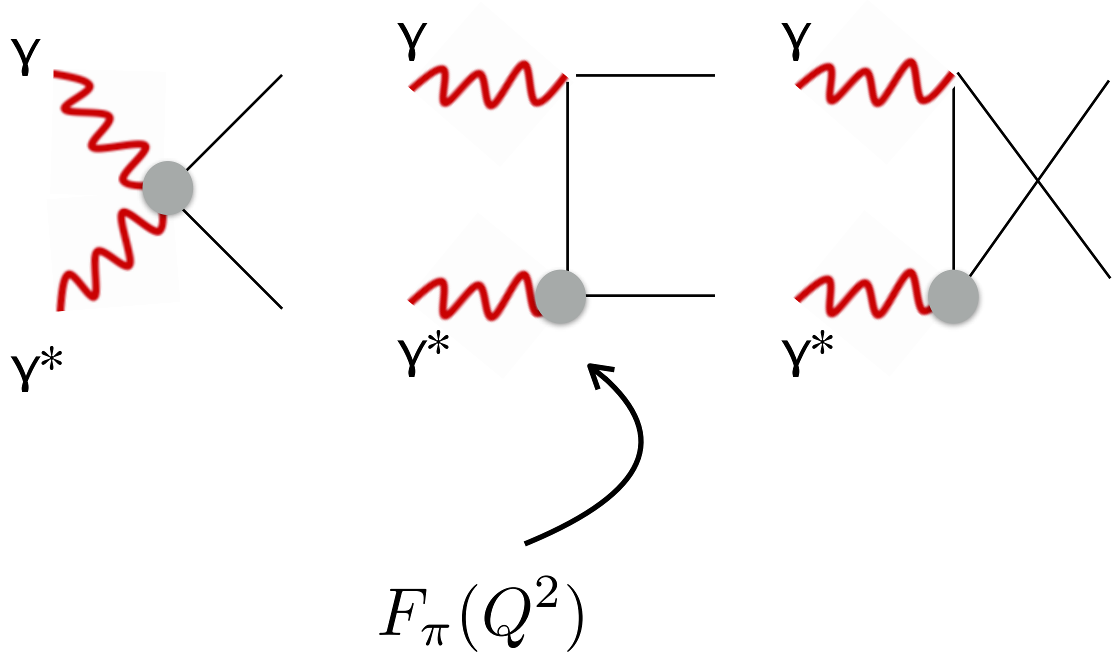

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Space-like region

$$F_\pi(Q^2) = \frac{1}{1 + Q^2/M_\rho^2}$$

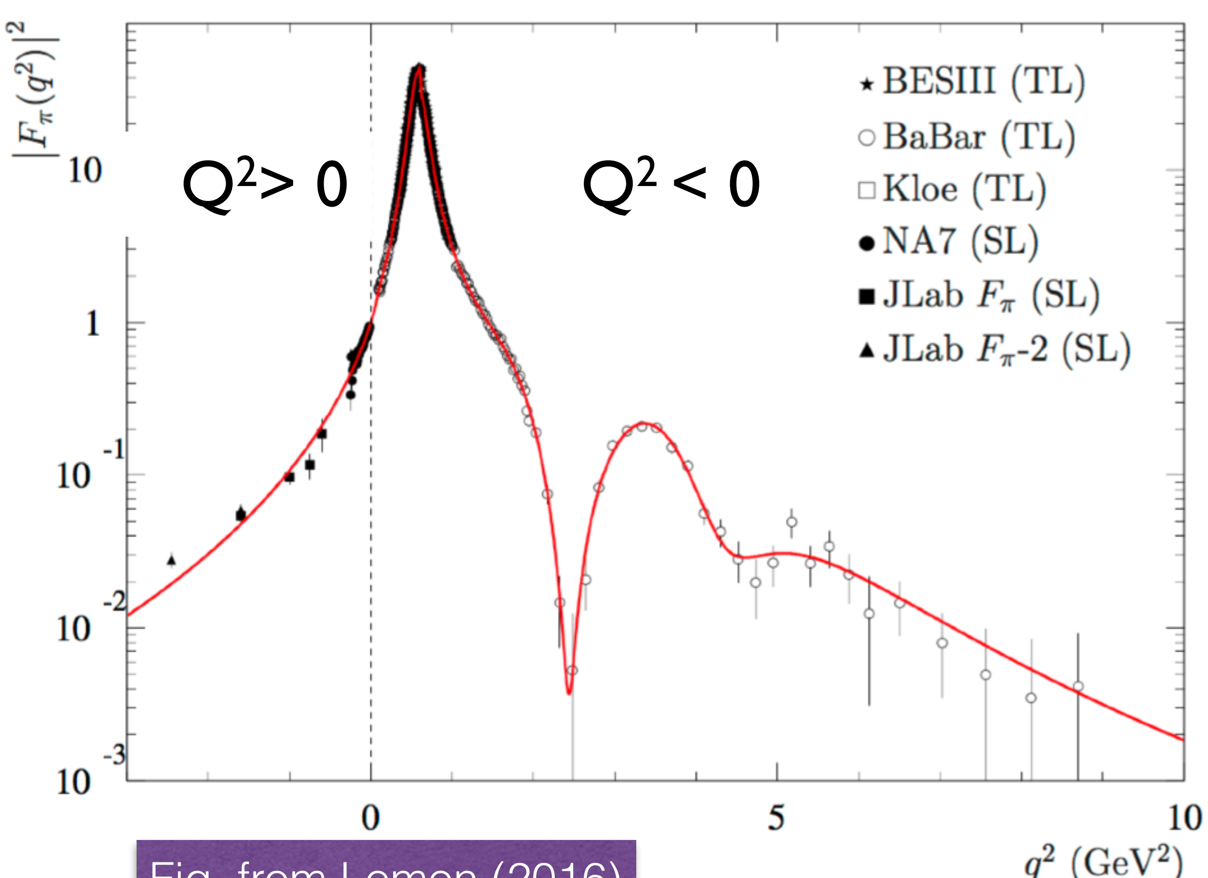
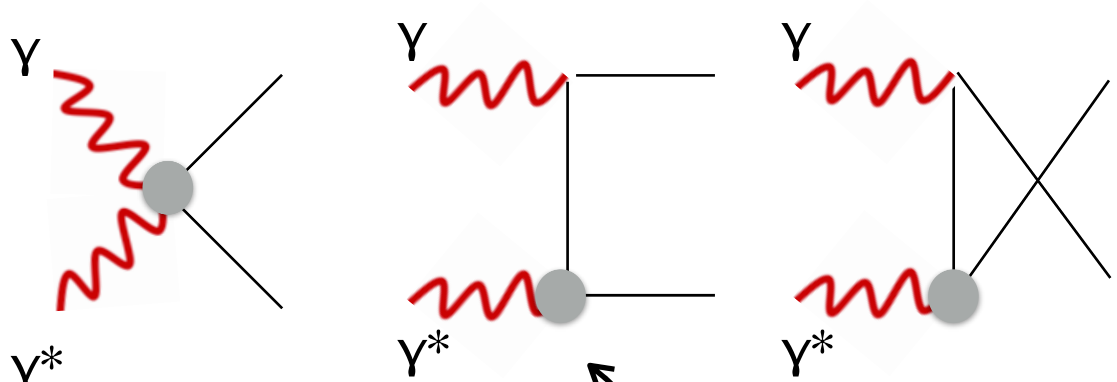


Fig. from Lomon (2016)

Born amplitudes ($Q^2 \neq 0$)



$F_\pi(Q^2)$

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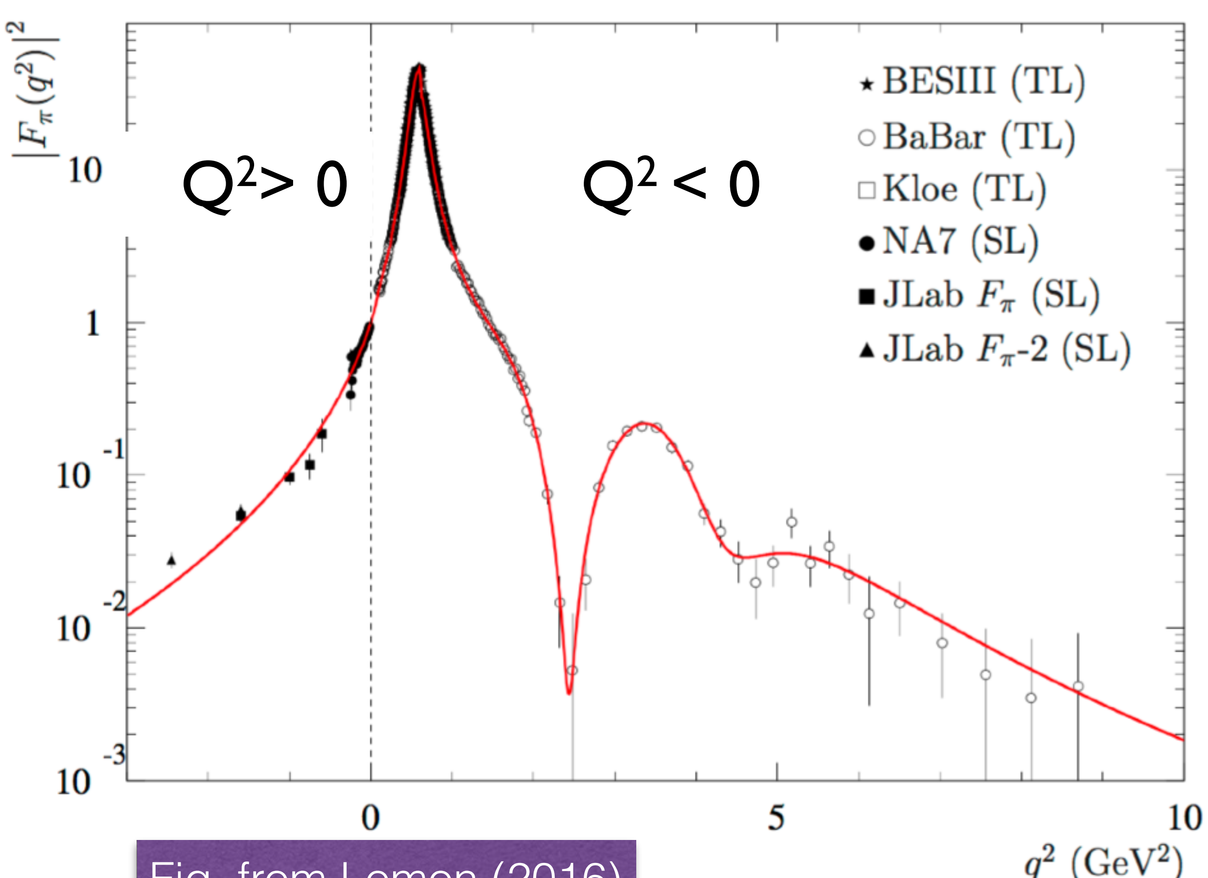

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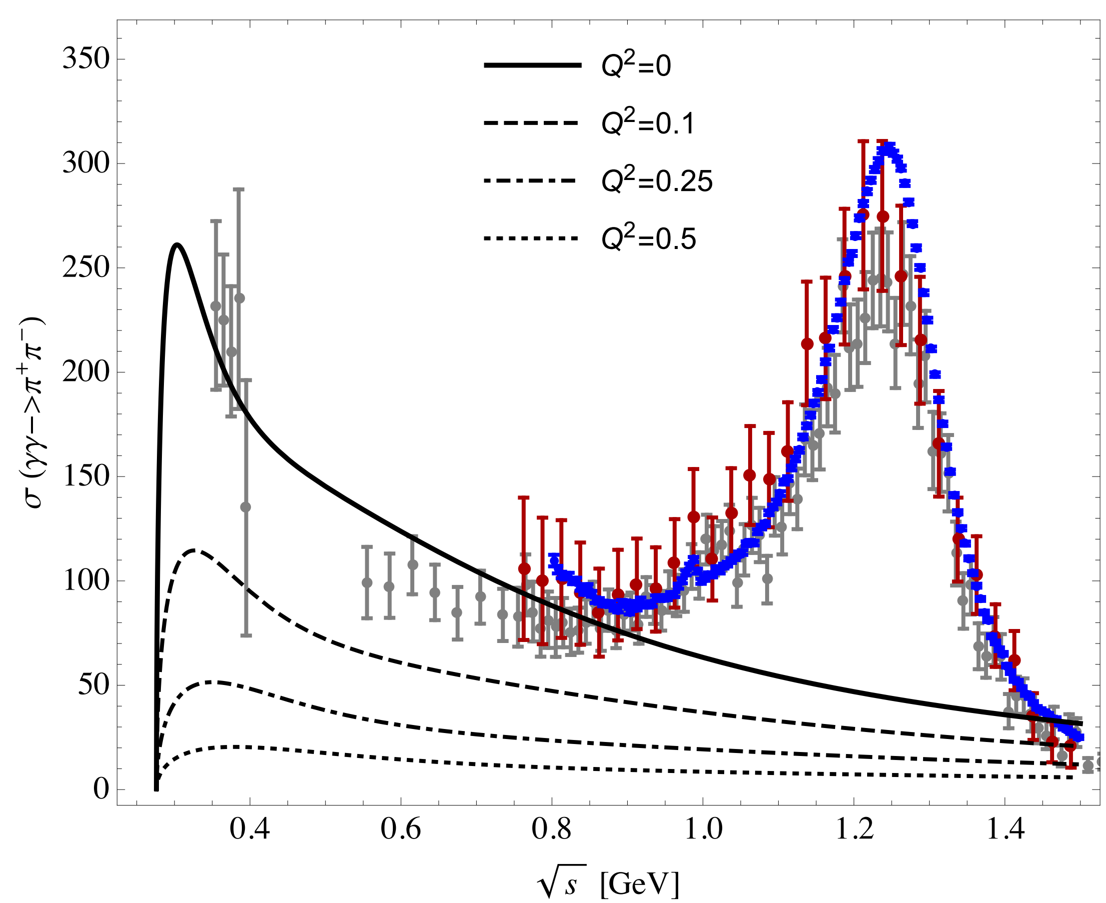
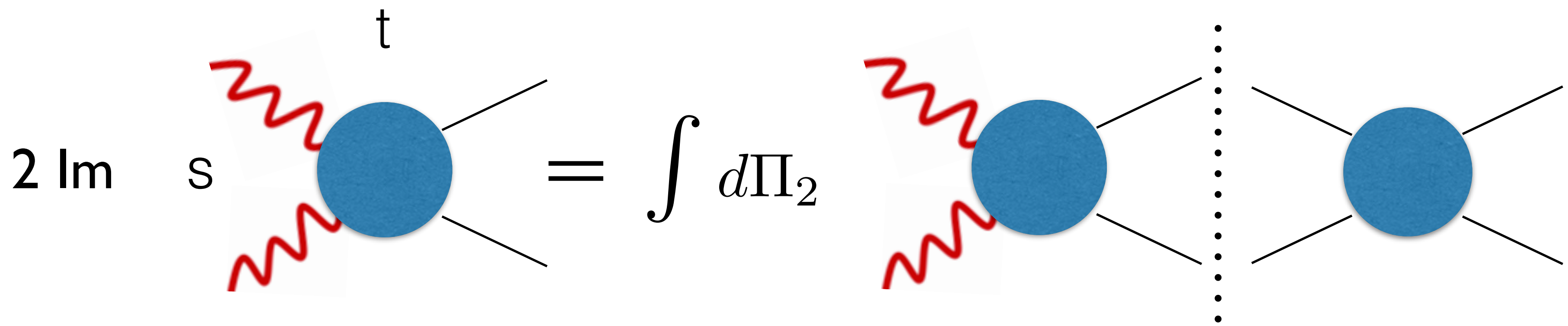


Unitarity



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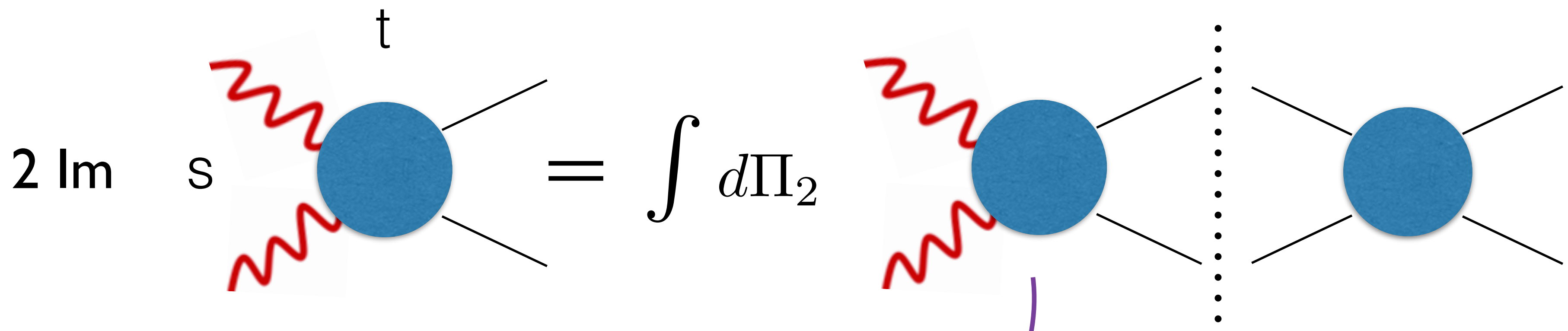
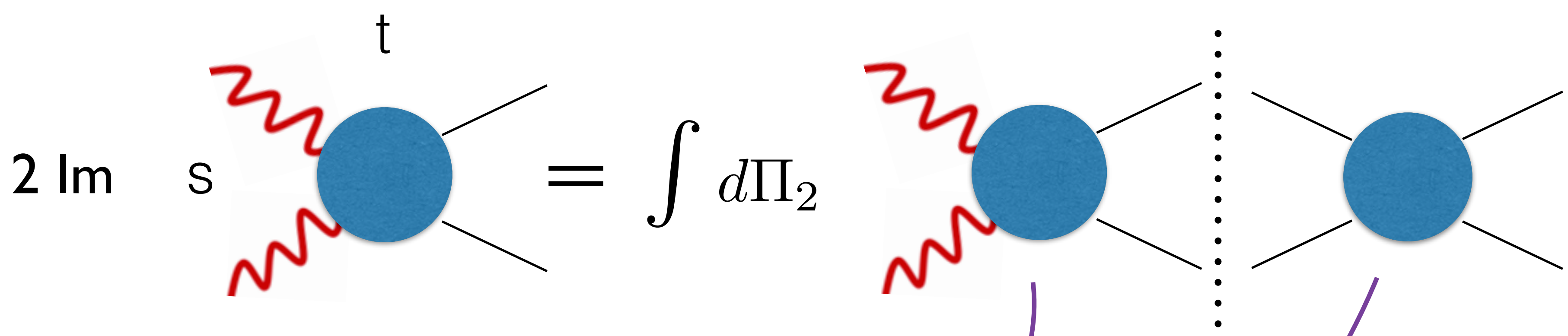


Partial wave expansion

$$H_{\lambda_1\lambda_2}(s,t) = \sum_{J=0}^{\infty}(2J+1)\, h_{J,\lambda_1\lambda_2}(s)\, d^J_{\lambda_1-\lambda_2,0}(\theta)$$

Unitarity



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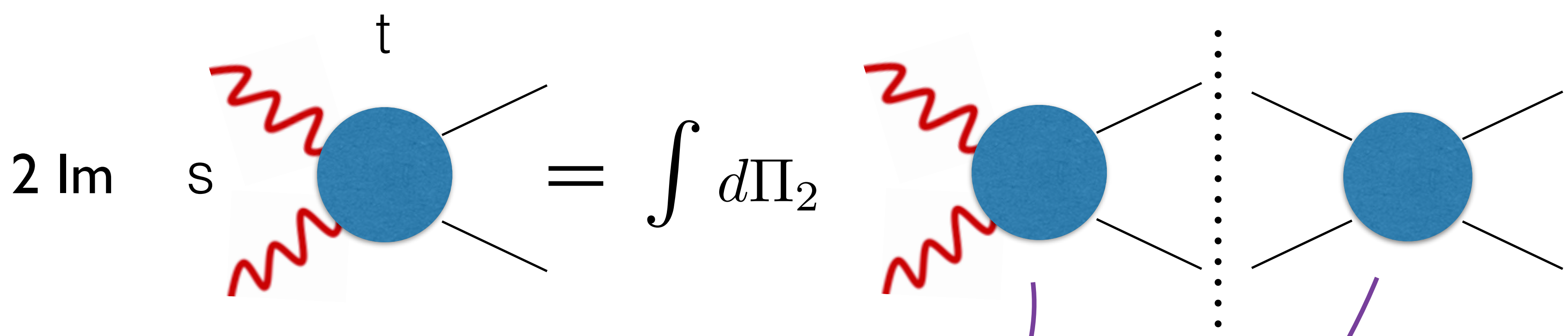
Unitarity



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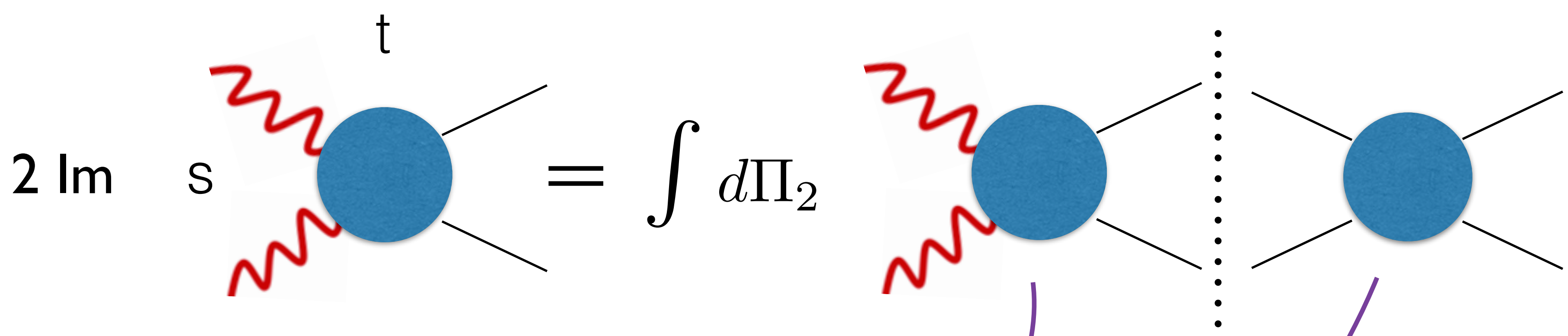
These "diagonalise unitarity" and contain resonance information

Definite: J, λ_1, λ_2

$$\mathrm{Im}\, h_{\gamma\gamma^*\to\pi\pi}(s) = h_{\gamma\gamma^*\to\pi\pi}(s)\, \rho_{\pi\pi}(s)\, t^*_{\pi\pi\to\pi\pi}(s)$$

Unitarity



Partial wave expansion

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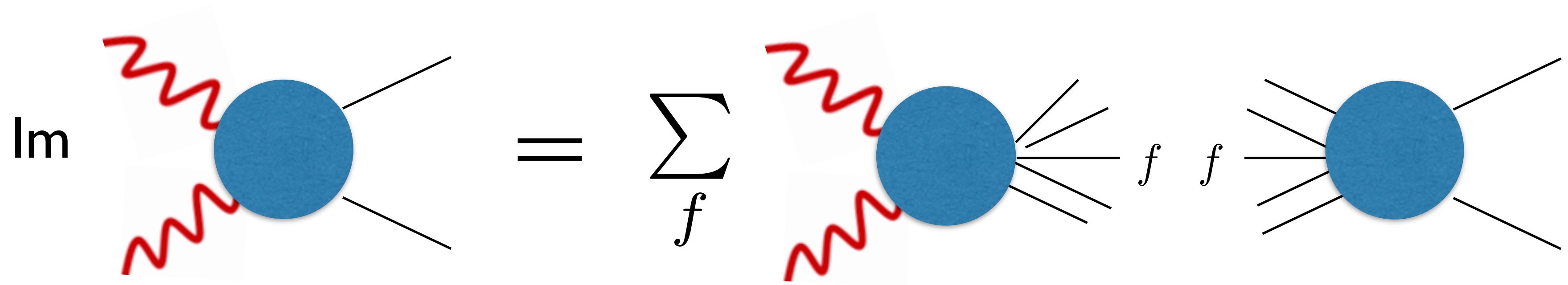
Coupled channel Unitarity

Coupled-channel unitarity

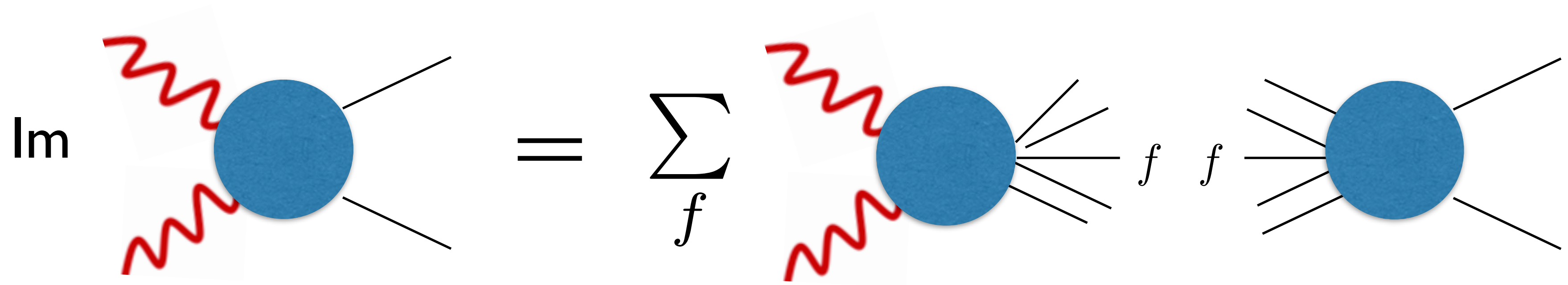
Definite: J, λ_1, λ_2



$$\mathrm{Im}\, h_{\gamma\gamma^*,b}(s) = \sum_f h_{\gamma\gamma^*,f}(s)\, \rho_f(s)\, t^*_{fb}(s)$$

Coupled-channel unitarity

Definite: J, λ_1, λ_2



$$\operatorname{Im} h_{\gamma\gamma^*,b}(s) = \sum_f h_{\gamma\gamma^*,f}(s)\,\rho_f(s)\,t^*_{fb}(s)$$

$$\begin{aligned}\operatorname{Im} h_{\gamma\gamma^*,1}(s) &= \rho_1\, h_{\gamma\gamma^*,1}\, t^*_{11} + \rho_2\, h_{\gamma\gamma^*,2}\, t^*_{21}\\ \operatorname{Im} h_{\gamma\gamma^*,2}(s) &= \rho_1\, h_{\gamma\gamma^*,1}\, t^*_{12} + \rho_2\, h_{\gamma\gamma^*,2}\, t^*_{22}\end{aligned}$$

1 = ππ
2 = KK

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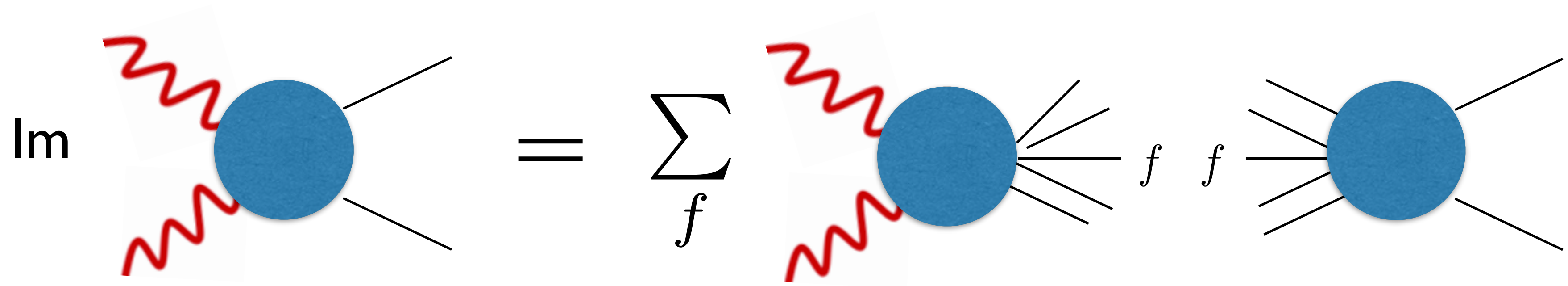

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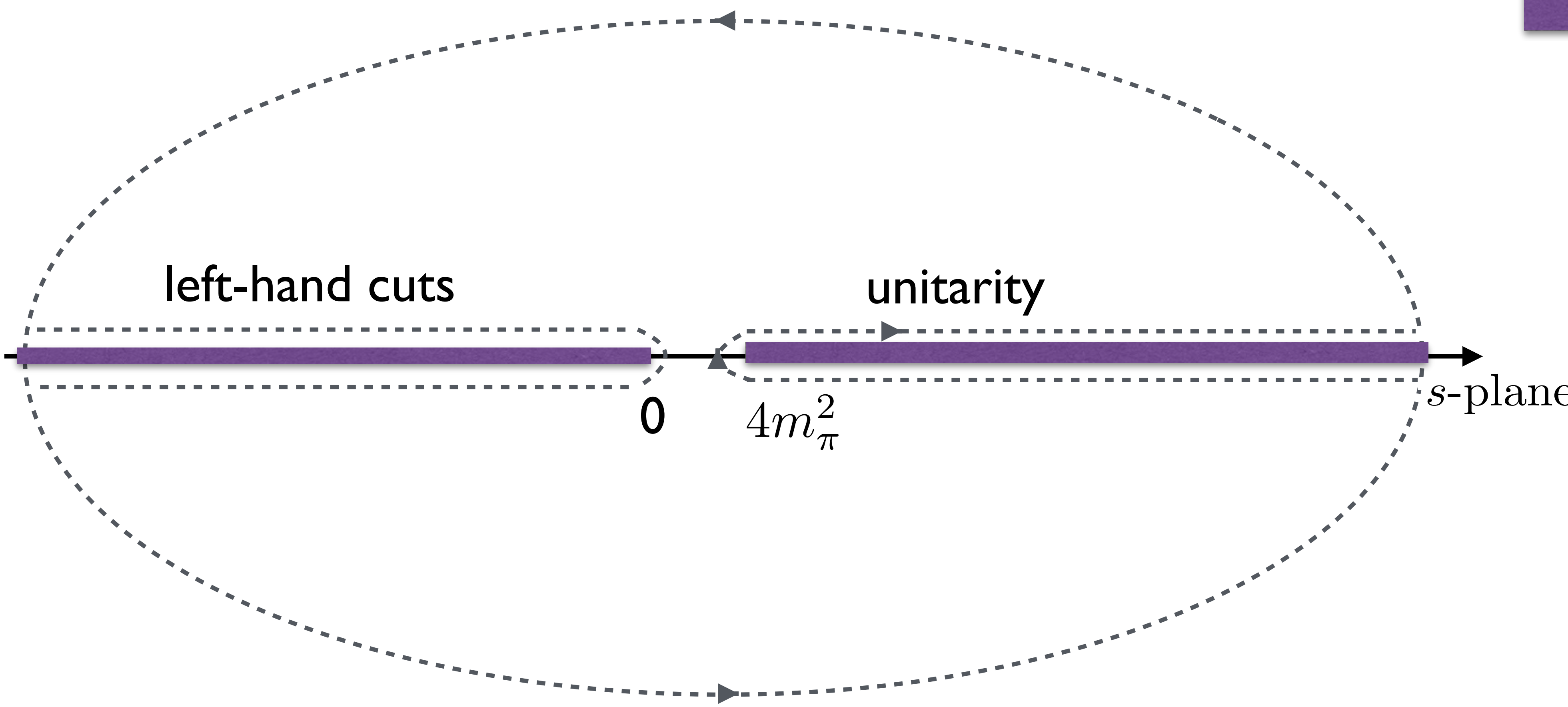
1 = $\pi\pi$
2 = KK

Entire dynamical information that does not depend on the underlying theory (e.g. QCD) comes from **unitarity**

Dispersion relation

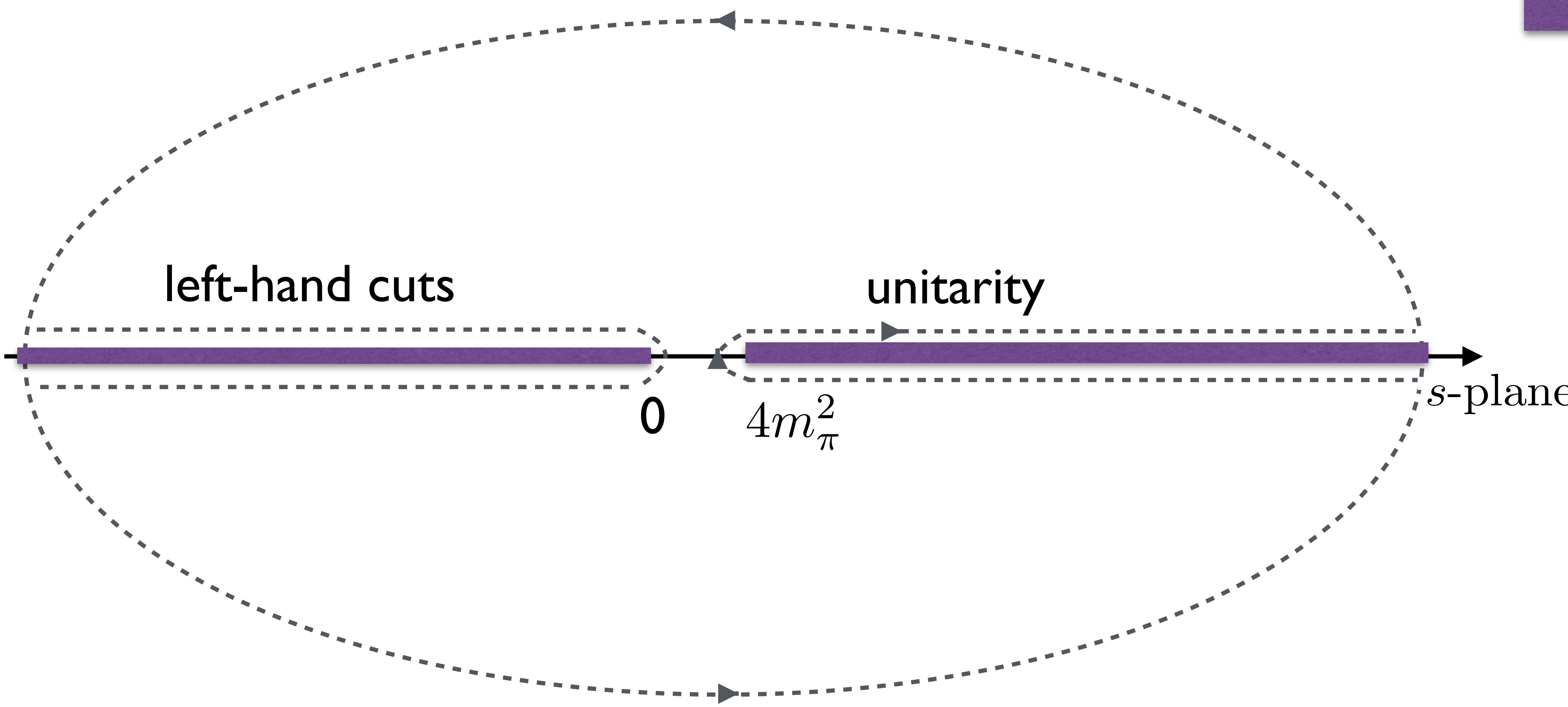
Definite: J, λ_1, λ_2



$$h(s) = \frac{1}{2\pi i}\int_C ds' \frac{h(s')}{s'-s} = \int_{-\infty}^{0} \frac{ds'}{\pi}\frac{\mathrm{Im}\, h(s')}{s'-s} + \int_{4m_\pi^2}^{\infty} \frac{ds'}{\pi}\frac{\mathrm{Im}\, h(s')}{s'-s}$$

Dispersion relation

Definite: J, λ_1, λ_2



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analyticity relates scattering amplitude at different energies

Dispersion relation

Left and right-hand cuts

Definite: J, λ_1, λ_2

$$h(s) = \int_{-\infty}^{0} \frac{ds'}{\pi} \frac{\operatorname{Im} h(s')}{s' - s} + \int_{4m_\pi^2}^{\infty} \frac{ds'}{\pi} \frac{\operatorname{Im} h(s')}{s' - s}$$

Dispersion relation

Left and right-hand cuts

Definite: J, λ_1, λ_2

$$h(s) = \int_{-\infty}^{0} \frac{ds'}{\pi} \frac{\mathrm{Im}\, h(s')}{s'-s} + \int_{4m_\pi^2}^{\infty} \frac{ds'}{\pi} \frac{\mathrm{Im}\, h(s')}{s'-s}$$

Looking for a solution in the form (N/D technique)

$$h(s) = h^{Born}(s) + \Omega(s)\, N(s)$$

$$s > 4m_\pi^2$$
$$\mathrm{Im}\,\Omega(s) = \Omega(s)\,\rho(s)\,t^*(s)$$
$$\mathrm{Im}\, h(s) = h(s)\,\rho(s)\,t^*(s)$$

Omnes (1958)
Morgan et. al. (1998)

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Left and right-hand cuts

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Omnes (1958)
Morgan et. al. (1998)

Dispersive integral (twice subtracted) for J=0

$$h(s) = h^{Born}(s) + \Omega(s)\Big(a + b\,s + \frac{s^2}{\pi}\int_{-\infty}^{-s_L} \frac{ds'}{s'^2} \frac{\mathrm{Im}(h(s'))\Omega^{-1}(s)}{s'-s} - \frac{s^2}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^2} \frac{h^{Born}(s')\,\mathrm{Im}(\Omega^{-1}(s'))}{s'-s}\Big)$$

similar eq. for coupled-channel (ππ,KK)

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Definite: J, λ_1, λ_2

$$h(s) = \int_{-\infty}^{0} \frac{ds'}{\pi} \frac{\mathrm{Im}\, h(s')}{s' - s} + \int_{4m_\pi^2}^{\infty} \frac{ds'}{\pi} \frac{\mathrm{Im}\, h(s')}{s' - s}$$

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Q^2 - dependent

see also Moussallam (2013)

similar eq. for coupled-channel (ππ,KK)

Dispersion relation

Dispersive integral for J=0

$$h(s) = h^{Born}(s) + \Omega(s)\Big(a + b\,s + \frac{s^2}{\pi}\int_{-\infty}^{-s_L} \frac{ds'}{s'^2}\frac{\mathrm{Im}(h(s'))\Omega^{-1}(s)}{s'-s} - \frac{s^2}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^2}\frac{h^{Born}(s')\,\mathrm{Im}(\Omega^{-1}(s'))}{s'-s}\Big)$$

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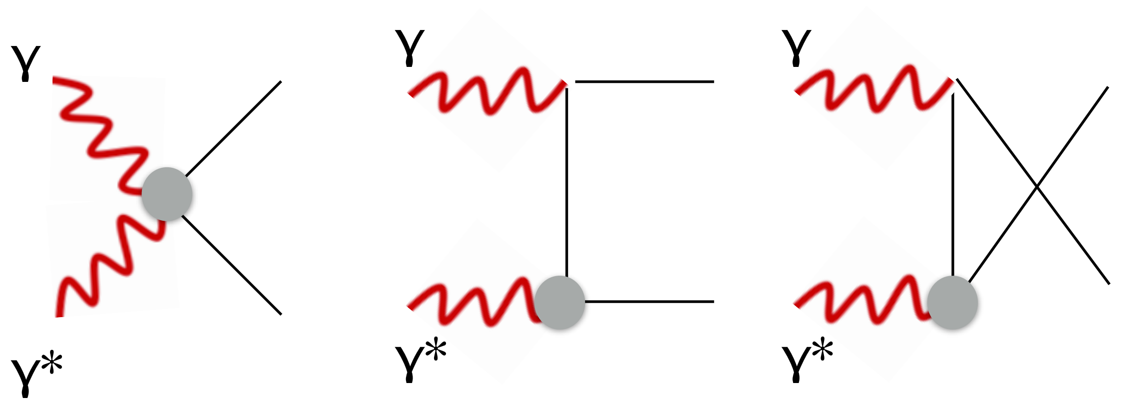

Dispersion relation

Dispersive integral for J=0

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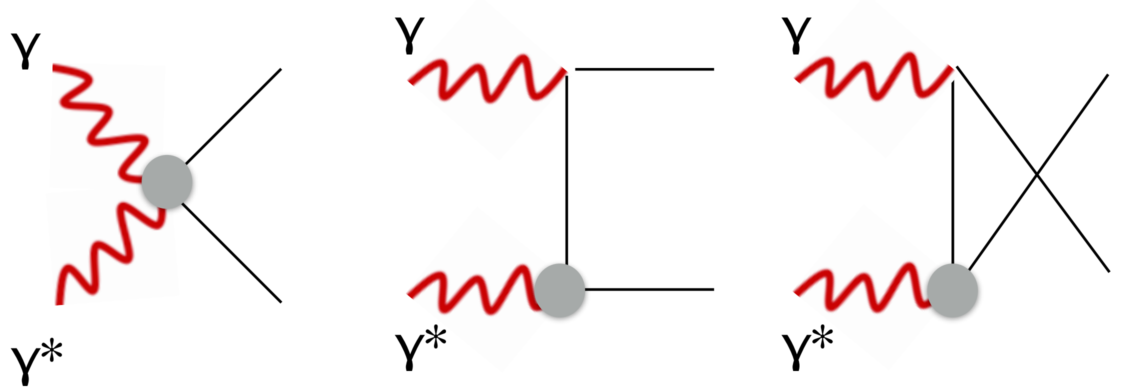

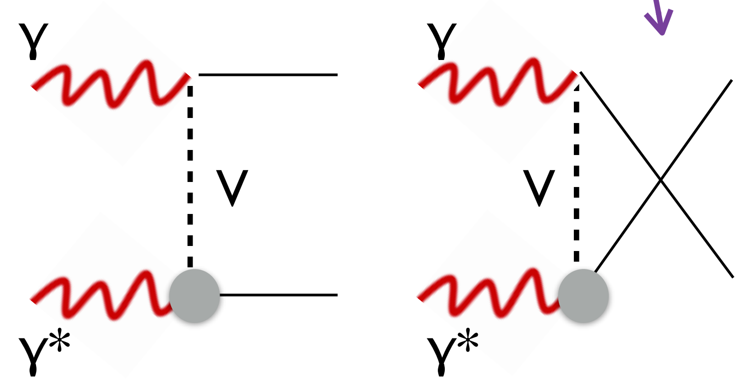

Dispersion relation

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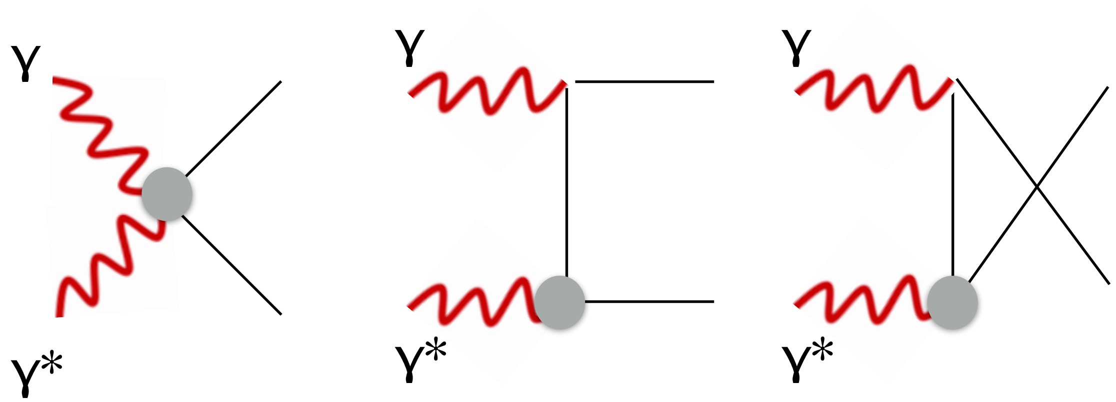
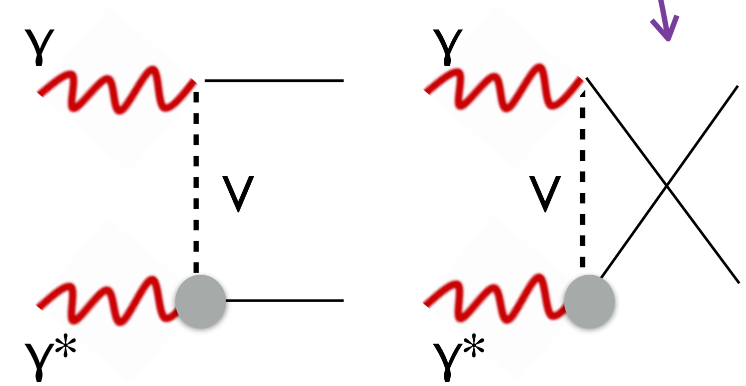
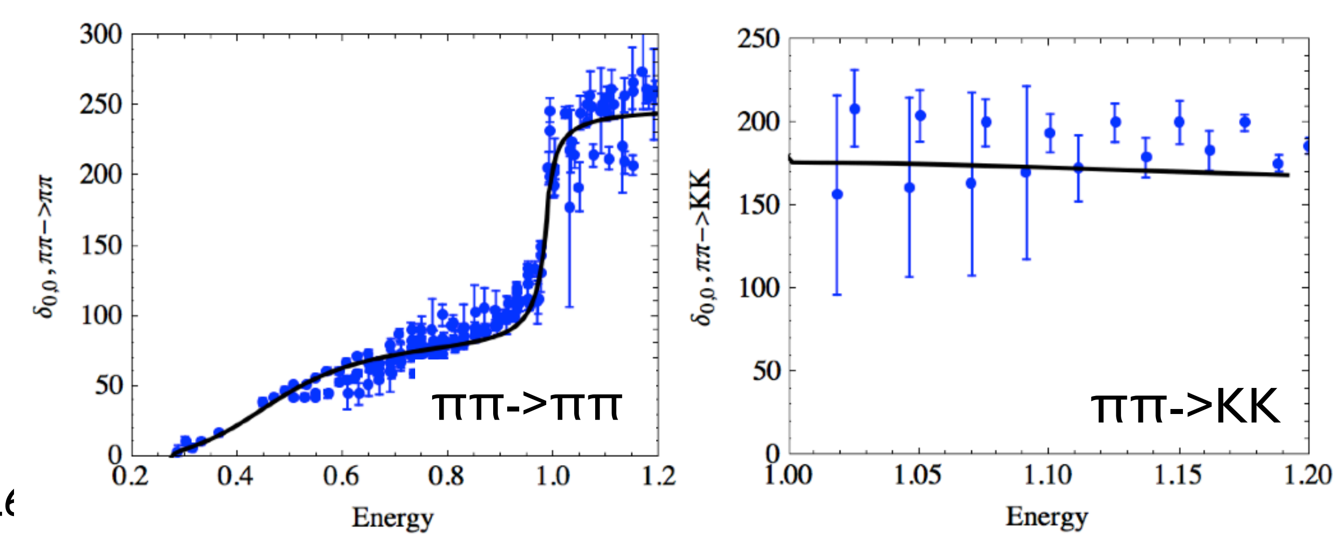




Coupled channel Omnes

$$\Omega(s) = \begin{pmatrix} \Omega_{\pi\pi\to\pi\pi} & \Omega_{\pi\pi\to K\bar{K}} \\ \Omega_{K\bar{K}\to\pi\pi} & \Omega_{K\bar{K}\to K\bar{K}} \end{pmatrix}$$

N/D technique + left-hand cuts (conformal map)



Subtraction constants

Dispersive integral for J=0

$$h(s) = h^{Born}(s) + \Omega(s)\Big(a + b\,s + \frac{s^2}{\pi}\int_{-\infty}^{-s_L} \frac{ds'}{s'^2}\frac{\text{Im}(h(s'))\Omega^{-1}(s)}{s'-s}$$

$$- \frac{s^2}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^2}\frac{h^{Born}(s')\,\text{Im}(\Omega^{-1}(s'))}{s'-s}\Big)$$

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Soft photon limit (q_1=0)

$$H_{\lambda_1\lambda_2} \to H^{Born}_{\lambda_1\lambda_2}$$
$$s = -Q^2,\; t = u = m_\pi^2$$

Subtraction constants

Dispersive integral for J=0

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Soft photon limit (q_1=0)

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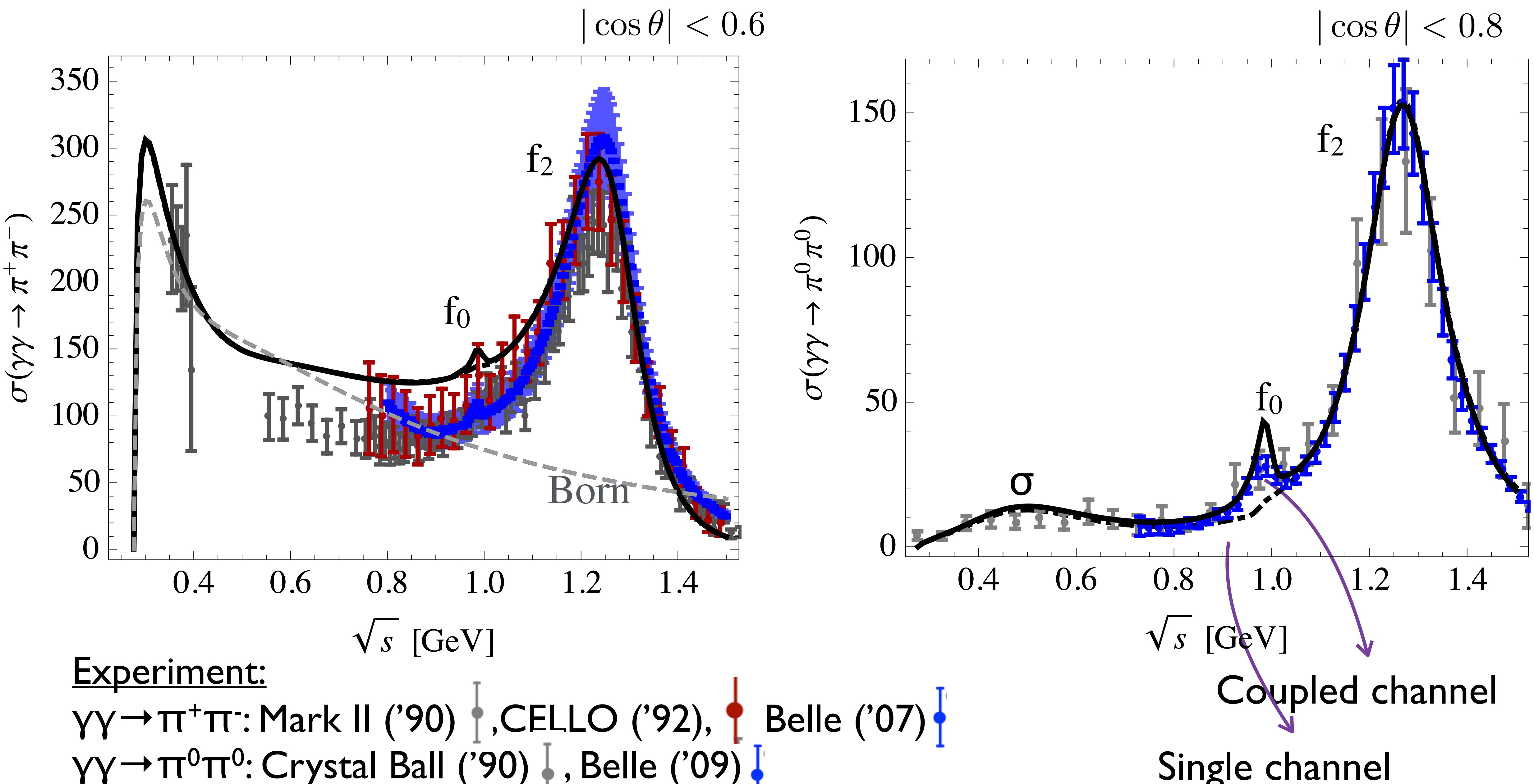
$$s = -Q^2,\ t = u = m_\pi^2$$

For space like photons: generalized polarizabilities

$$\pm\frac{2\alpha}{m_\pi}\frac{H^n_{+\pm}}{s+Q^2} = (\alpha_1 \mp \beta_1)_{\pi^0} + \ldots$$

$$\pm\frac{2\alpha}{m_\pi}\frac{(H^c_{+\pm} - H^{Born}_{+\pm})}{s+Q^2} = (\alpha_1 \mp \beta_1)_{\pi^+} + \ldots$$

$\gamma\gamma \to \pi\pi$ (Q^2=0)



<u>Experiment:</u>

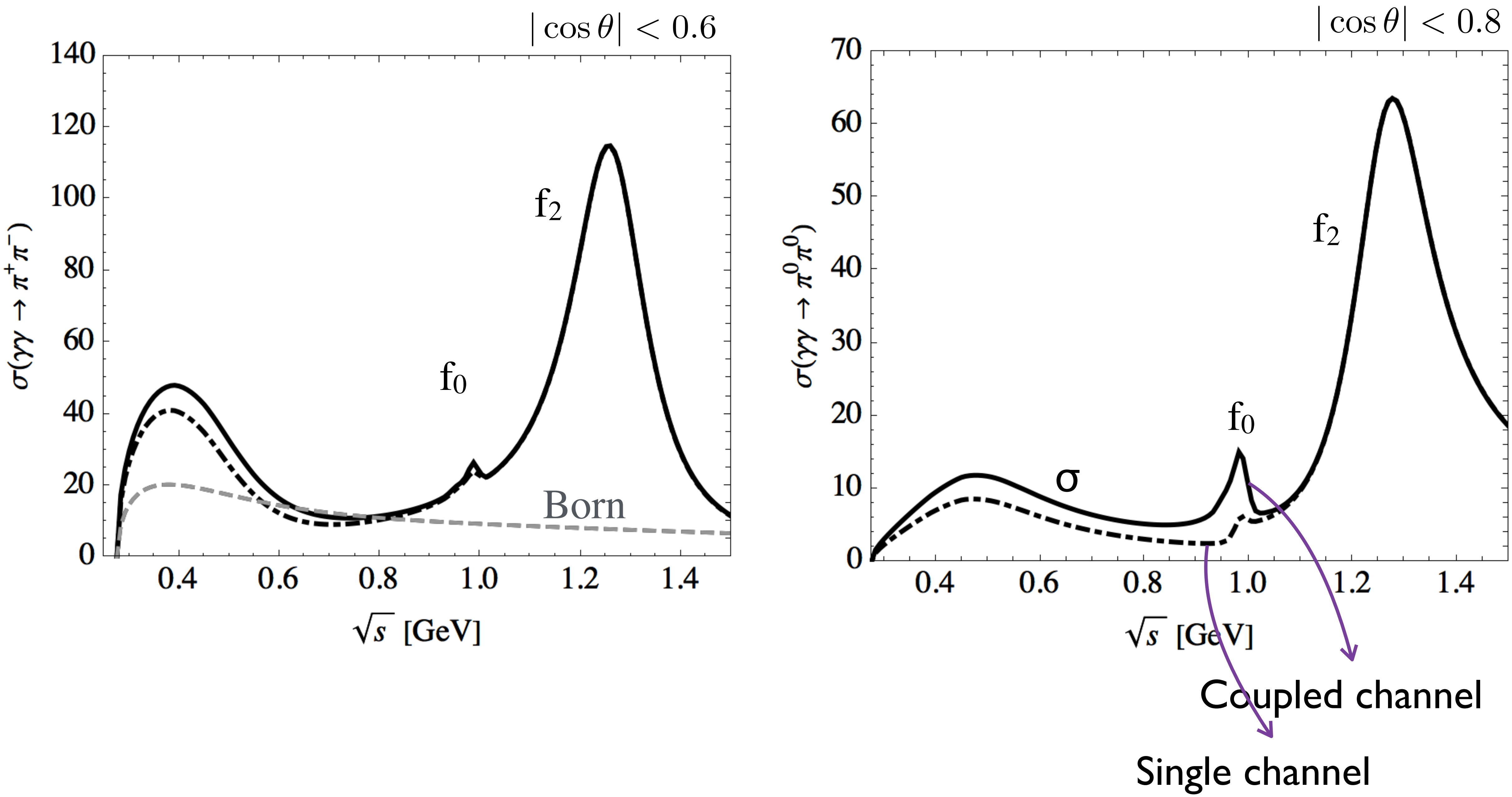
$\gamma\gamma \to \pi^+\pi^-$: Mark II ('90), CELLO ('92), Belle ('07)

$\gamma\gamma \to \pi^0\pi^0$: Crystal Ball ('90), Belle ('09)

I.D., Vanderhaeghen (work in progress)

see also Dai ('14), Hoferichter ('11), Garcia-Martin et. al ('10)

$\gamma\gamma \rightarrow \pi\pi$ (Q^2=0.5)

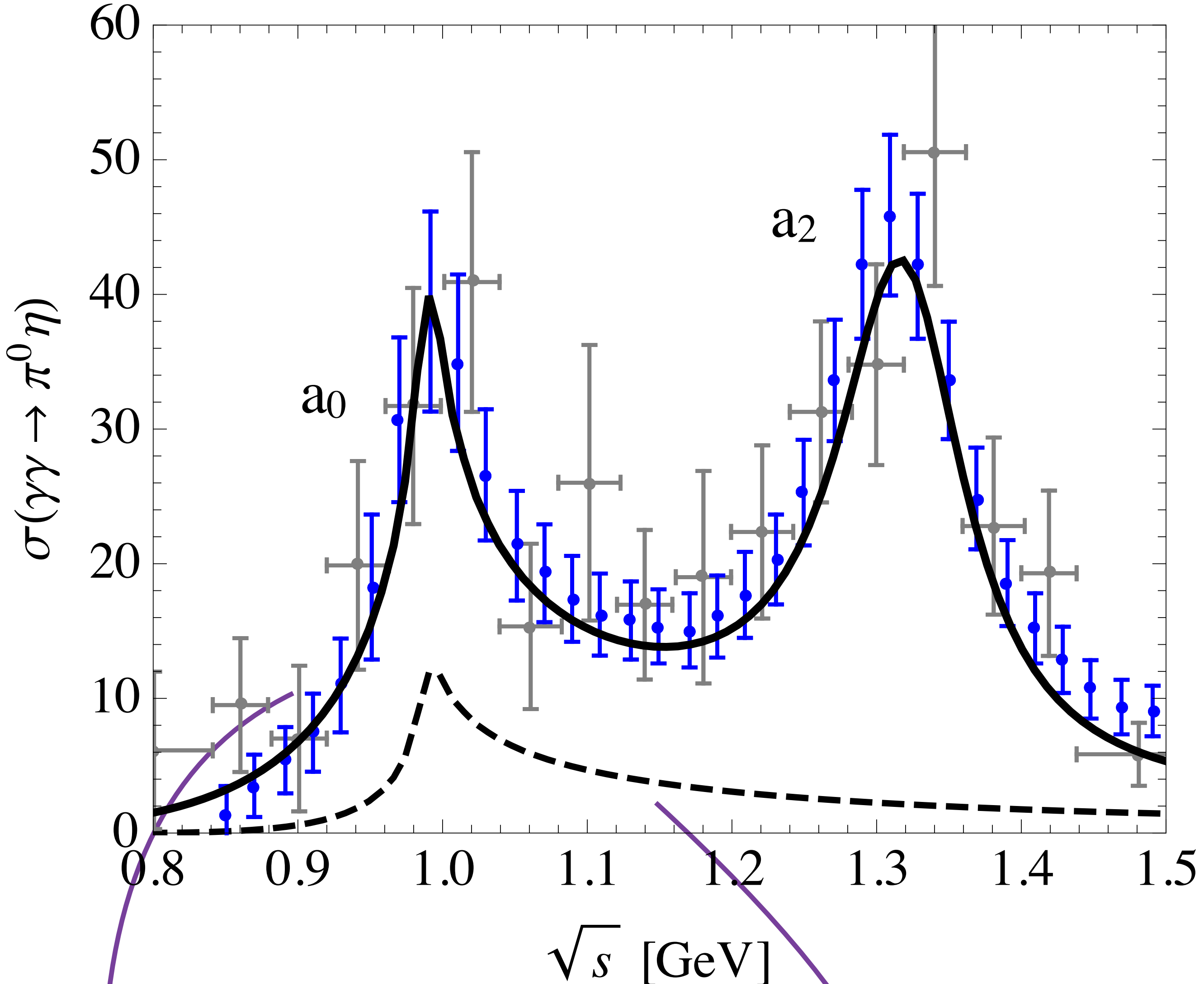


Currently results for Q^2=0.5 without VM in the left-hand cut…

Ongoing experiment: BES III

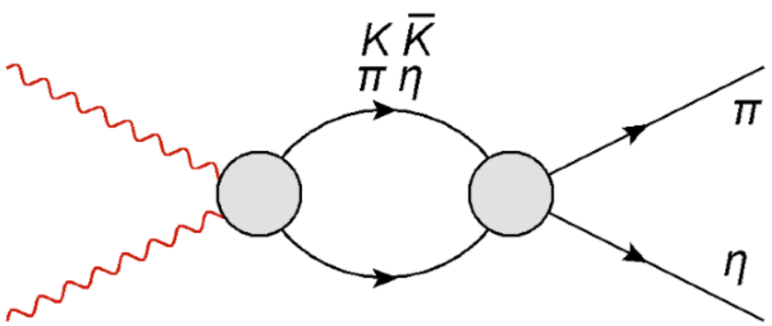
I.D., Vanderhaeghen (work in progress)

$\gamma\gamma \to \pi\eta$ (Q^2=0)



Coupled channel:
with VM

Coupled channel:
no VM



$$\Omega(s) = \begin{pmatrix} \Omega_{\pi\eta\to\pi\eta} & \Omega_{\pi\eta\to K\bar{K}} \\ \Omega_{K\bar{K}\to\pi\eta} & \Omega_{K\bar{K}\to K\bar{K}} \end{pmatrix}$$

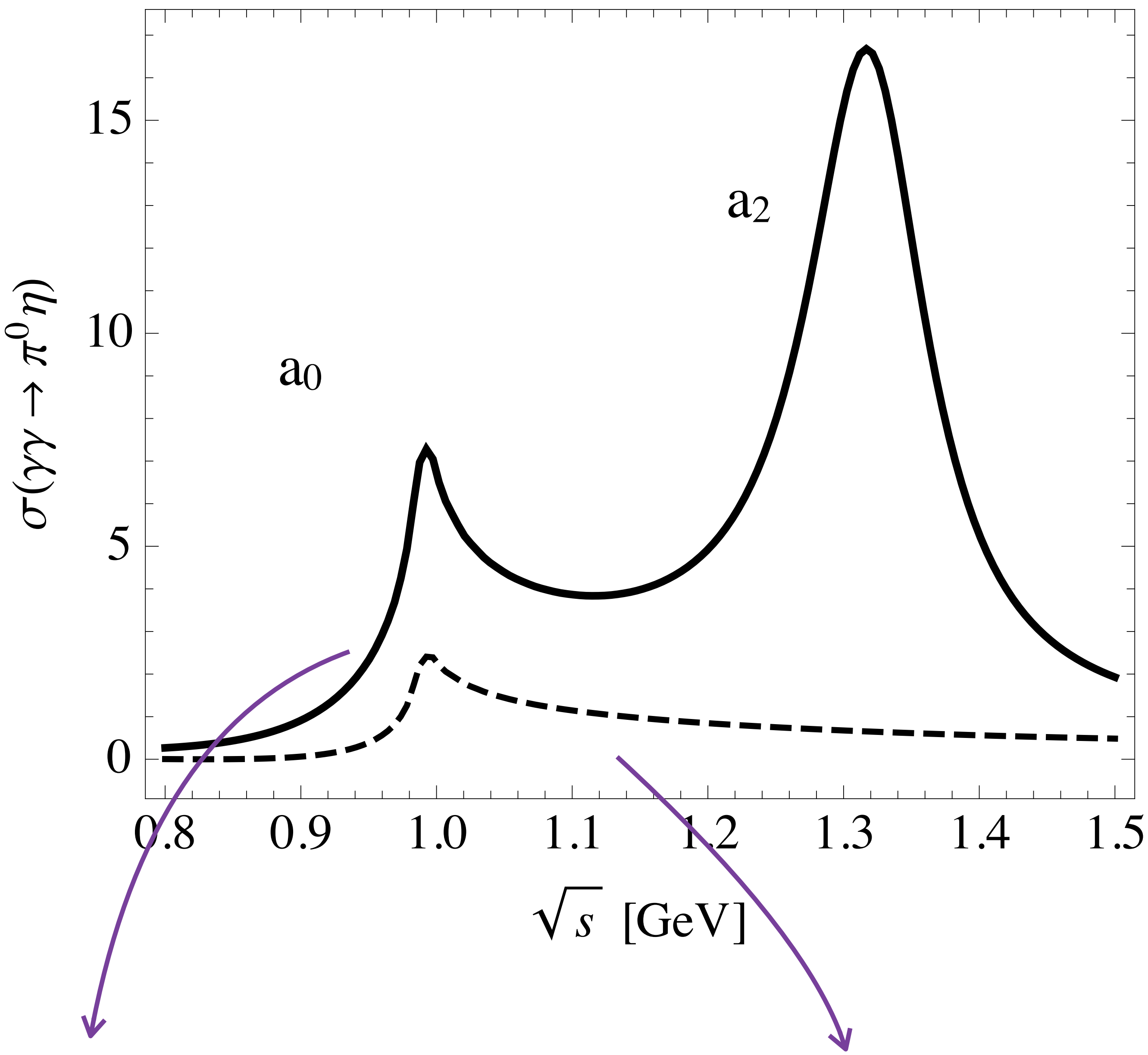
I.D., Gil, Lutz (2011), (2013)

Coupled-channel dispersive treatment for J=0 is **crucial**

a_2(1230) described as a Breit Wigner resonance

I.D., Deineka, Vanderhaeghen (work in progress)

$\gamma\gamma \to \pi\eta$ $(Q^2=0.5)$



Coupled channel:
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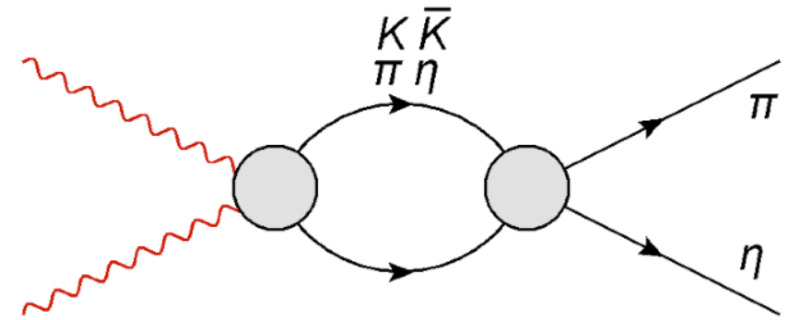

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Coupled-channel dispersive treatment for J=0 is **crucial**

a_2(1230) described as a Breit Wigner resonance

I.D., Deineka,
Vanderhaeghen
(work in progress)

Summary and Outlook

- In light of the **new Belle data (2015)** for $f_2(1270)$ TFFs and using LbL sum rules we **predicted** (Λ=2) TFF for $f_2(1565)$

- **Update for meson contributions to (g-2) LbL**
  Tensor mesons contributions found to be small compared to anticipated exp. uncertainty $1.6*10^{-10}$

  Axial vector mesons contributions (satisfying Landau-Yang theorem constraint) evaluated by 2 groups and found to be between $(0.64 - 0.75 \pm 0.27)10^{-10}$

- **Next steps?**
  Need to take into account $f_0(500)$ and non resonant contributions in a dispersive approach

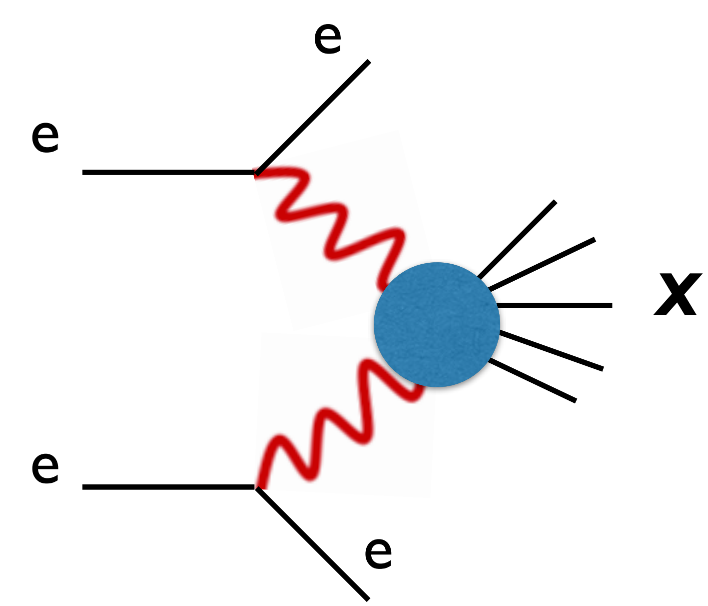
- Main ingredients: **$\gamma\gamma^* \to \pi\pi$, $\pi\eta$,**… (work in progress). Can be used in **different** (g-2) dispersive approaches.

  It is important to **validate** dispersive treatment of **$\gamma\gamma^* \to \pi\pi$, $\pi\eta$,**… with upcoming BES III data

Extra slides

Light by light scattering

Observables in experiment $e^+e^- \to e^-e^+X$



$$
\begin{aligned}
d\sigma \quad &= \quad \frac{\alpha^2}{16\pi^4\, Q_1^2\, Q_2^2}\, \frac{2\sqrt{X}}{s(1-4m^2/s)} \cdot \frac{d^3\vec{p}_1^{\,\prime}}{E_1'} \cdot \frac{d^3\vec{p}_2^{\,\prime}}{E_2'} \\
&\times \quad \{4\,\rho_1^{++}\,\rho_2^{++}\,\sigma_{TT} + \rho_1^{00}\,\rho_2^{00}\,\sigma_{LL} + 2\,\rho_1^{++}\,\rho_2^{00}\,\sigma_{TL} + 2\,\rho_1^{00}\,\rho_2^{++}\,\sigma_{LT} \\
&+2\,\left(\rho_1^{++}-1\right)\,\left(\rho_2^{++}-1\right)\,\left(\cos 2\tilde{\phi}\right)\,\tau_{TT} + 8\left[\frac{\left(\rho_1^{00}+1\right)\,\left(\rho_2^{00}+1\right)}{\left(\rho_1^{++}-1\right)\,\left(\rho_2^{++}-1\right)}\right]^{1/2}\left(\cos\tilde{\phi}\right)\tau_{TL} \\
&+h_1h_2\,4\left[\left(\rho_1^{00}+1\right)\,\left(\rho_2^{00}+1\right)\right]^{1/2}\tau_{TT}^{a} + h_1h_2\,8\left[\left(\rho_1^{++}-1\right)\,\left(\rho_2^{++}-1\right)\right]^{1/2}\left(\cos\tilde{\phi}\right)\,\tau_{TL}^{a}\Big\}\,,
\end{aligned}
$$

Born amplitudes ($Q^2 \neq 0$)

Differential cross section

$$\frac{d\sigma}{d\,\cos\theta} = \pi\alpha^2 \frac{\rho(s)}{4\left(s+Q^2\right)}\;\left(|H_{++}|^2 + |H_{+-}|^2 + |H_{+0}|^2\right)$$

Born amplitudes ($Q^2 \neq 0$)

Differential cross section

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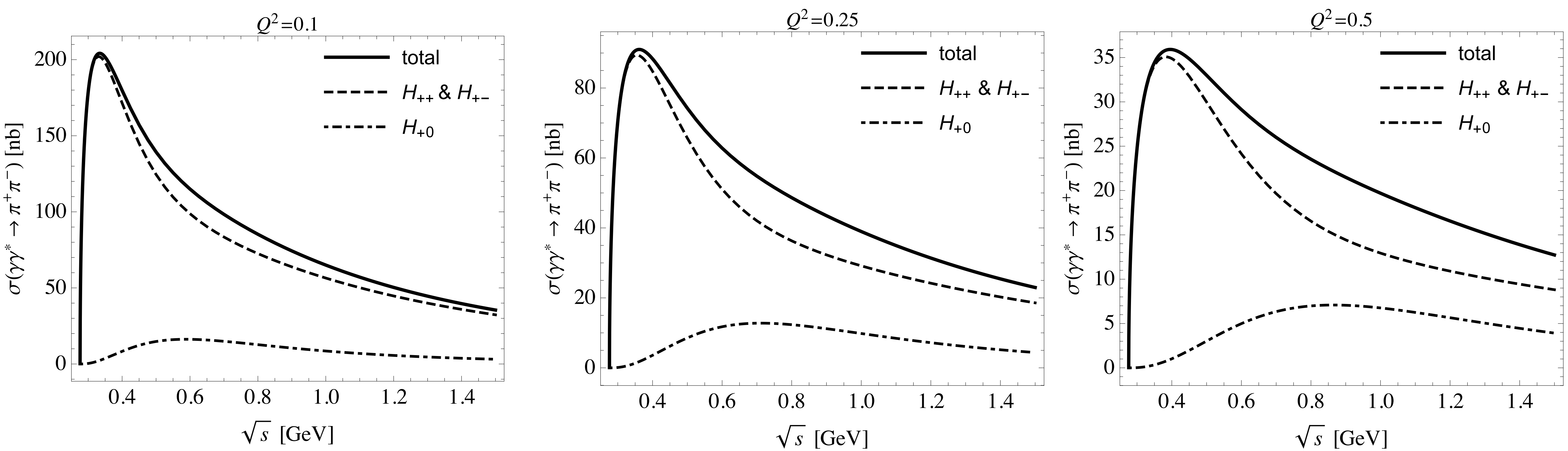

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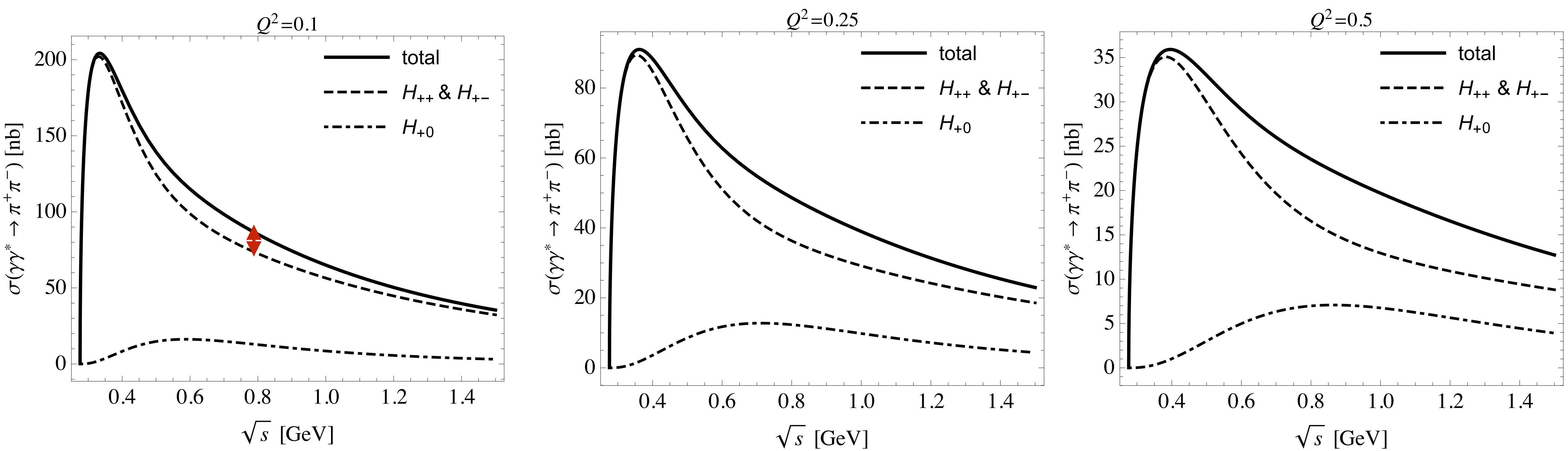

$\sqrt{s}$ =0.8 GeV: 15%

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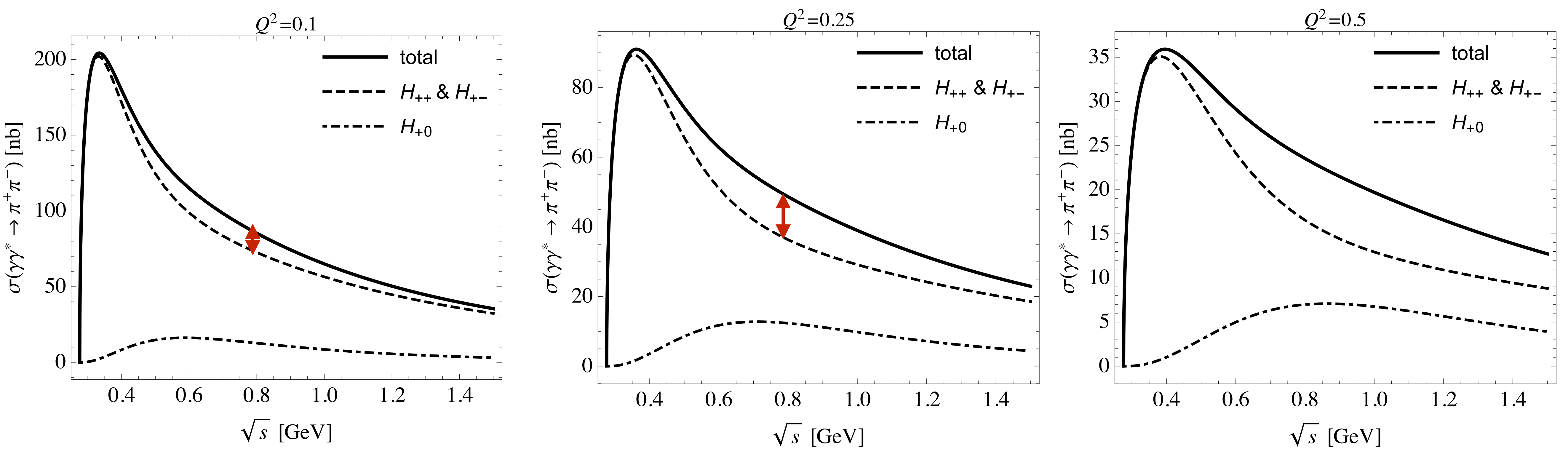

$\sqrt{s}$ =0.8 GeV: 15% 25%

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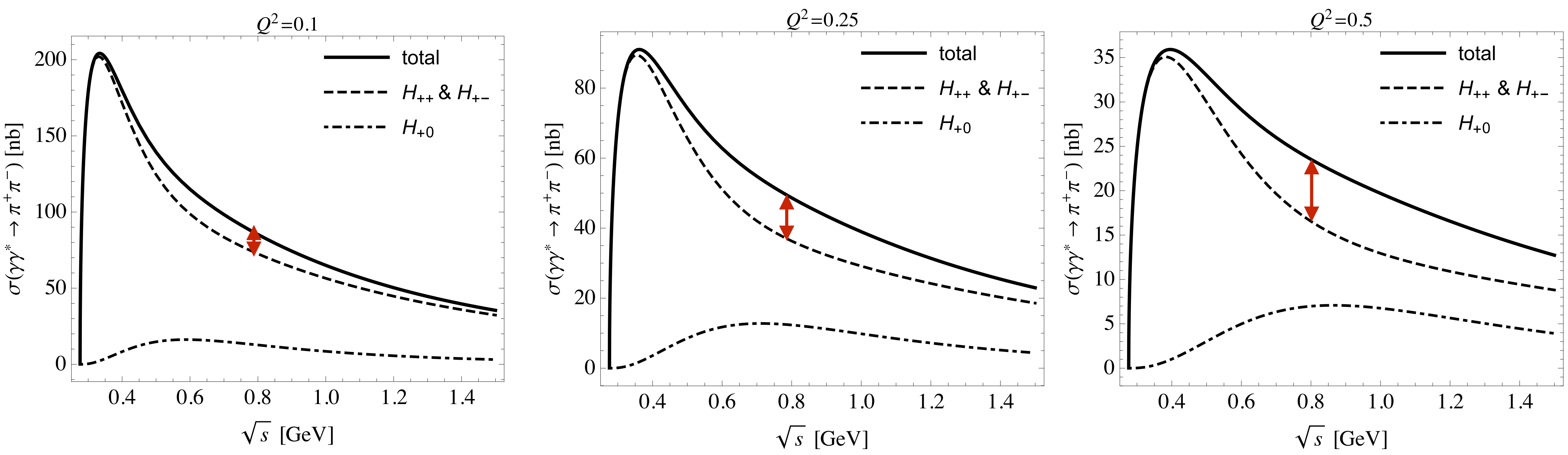

$\sqrt{s}$ =0.8 GeV: 15% 25% 30%