

# **Review on Multi-Quark States from Theoretical Results**

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**Beihang University**

**Nanjing 2017/08/26**

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- **Experimental status of  $P_c(4380)$  and  $P_c(4450)$**
- History of multiquark states
- Various theoretical methods
- Our QCD sum rule studies

# Experimental status of Pc(4380) and Pc(4450)

PRL 115, 072001 (2015)

Selected for a Viewpoint in *Physics*  
PHYSICAL REVIEW LETTERS

week ending  
14 AUGUST 2015



## Observation of $J/\psi p$ Resonances Consistent with Pentaquark States in $\Lambda_b^0 \rightarrow J/\psi K^- p$ Decays

R. Aaij *et al.*\*  
(LHCb Collaboration)

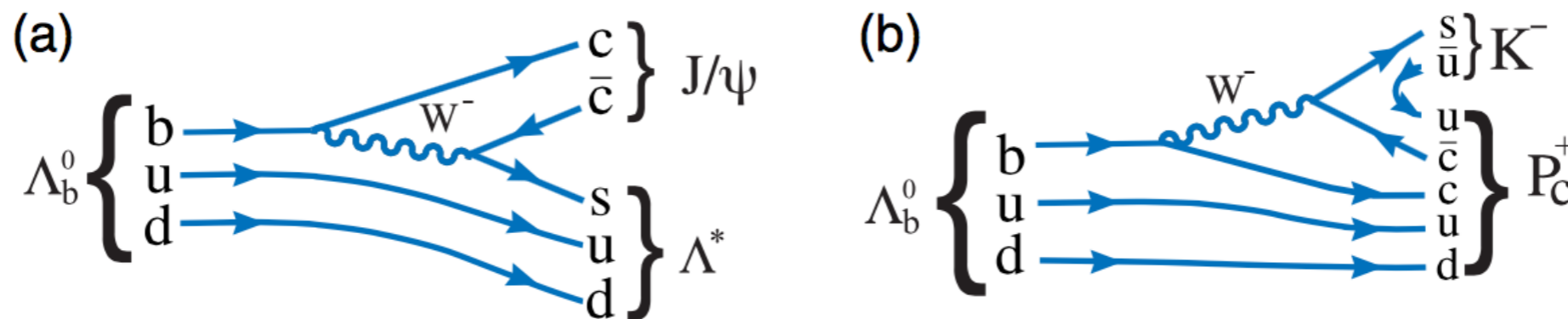


FIG. 1 (color online). Feynman diagrams for (a)  $\Lambda_b^0 \rightarrow J/\psi \Lambda^*$  and (b)  $\Lambda_b^0 \rightarrow P_c^+ K^-$  decay.

# The measured invariant mass spectra

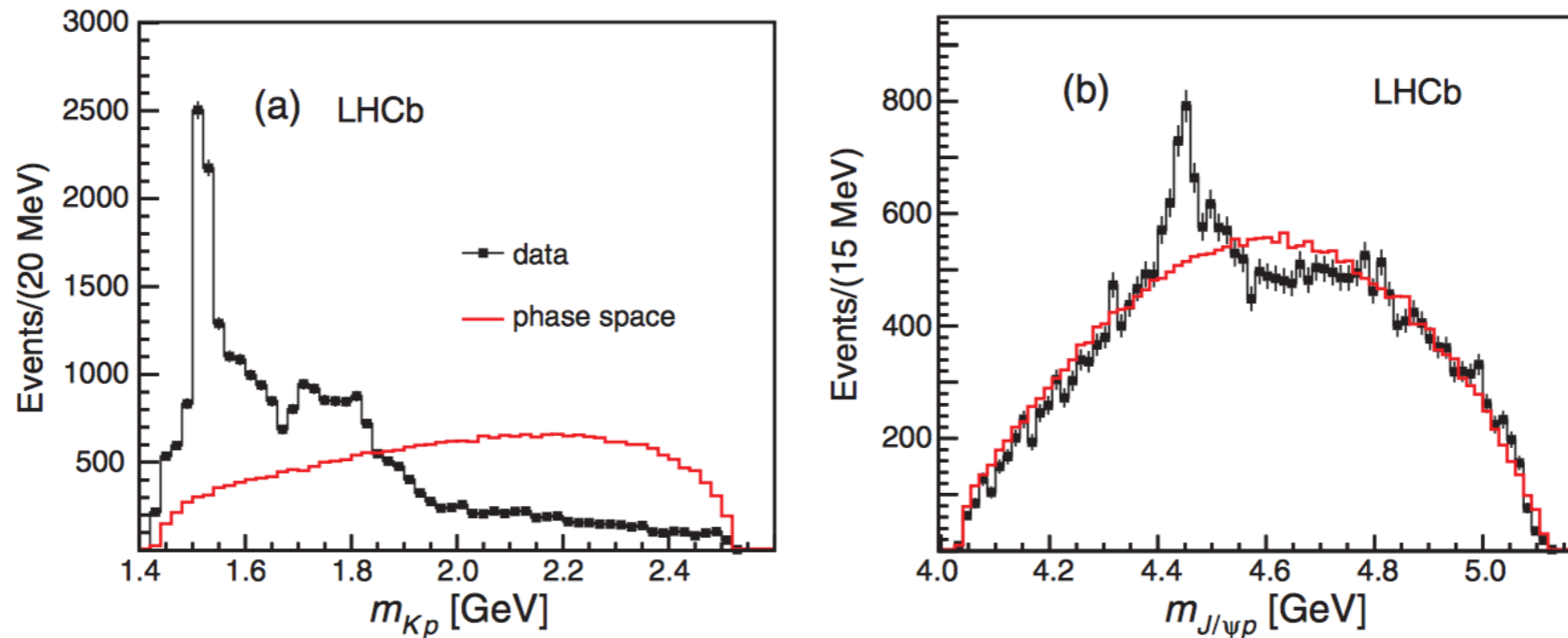
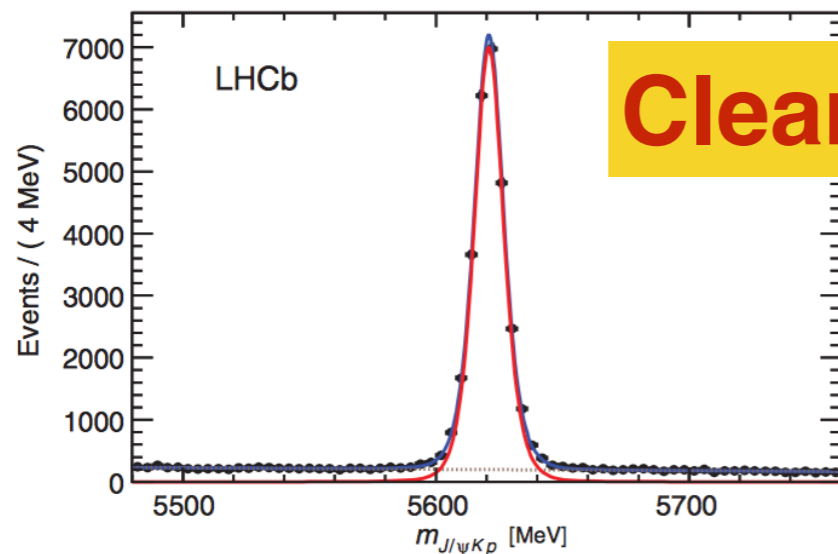


FIG. 2 (color online). Invariant mass of (a)  $K^- p$  and (b)  $J/\psi p$  combinations from  $\Lambda_b^0 \rightarrow J/\psi K^- p$  decays. The solid (red) curve is the expectation from phase space. The background has been subtracted.



**Clear signal of  $\Lambda_b$**

**LHCb performed the analysis of the above experimental data**

FIG. 4 (color online). Invariant mass spectrum of  $J/\psi K^- p$  combinations, with the total fit, signal, and background components shown as solid (blue), solid (red), and dashed lines, respectively.

# Resonance parameters of two $P_c$ states

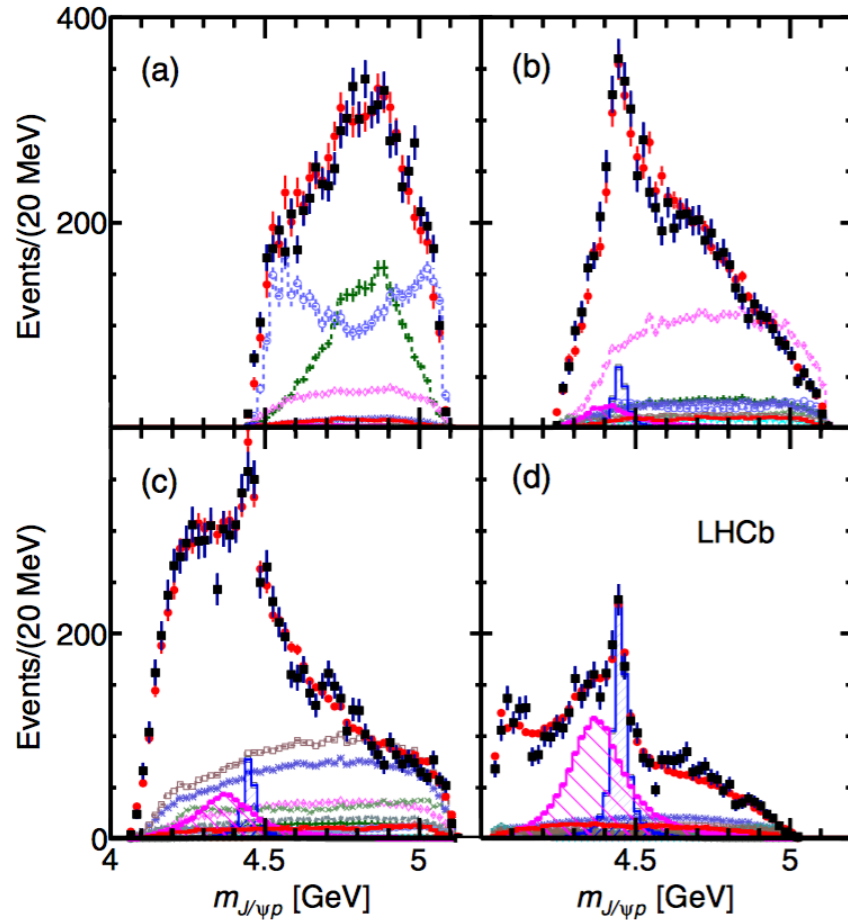
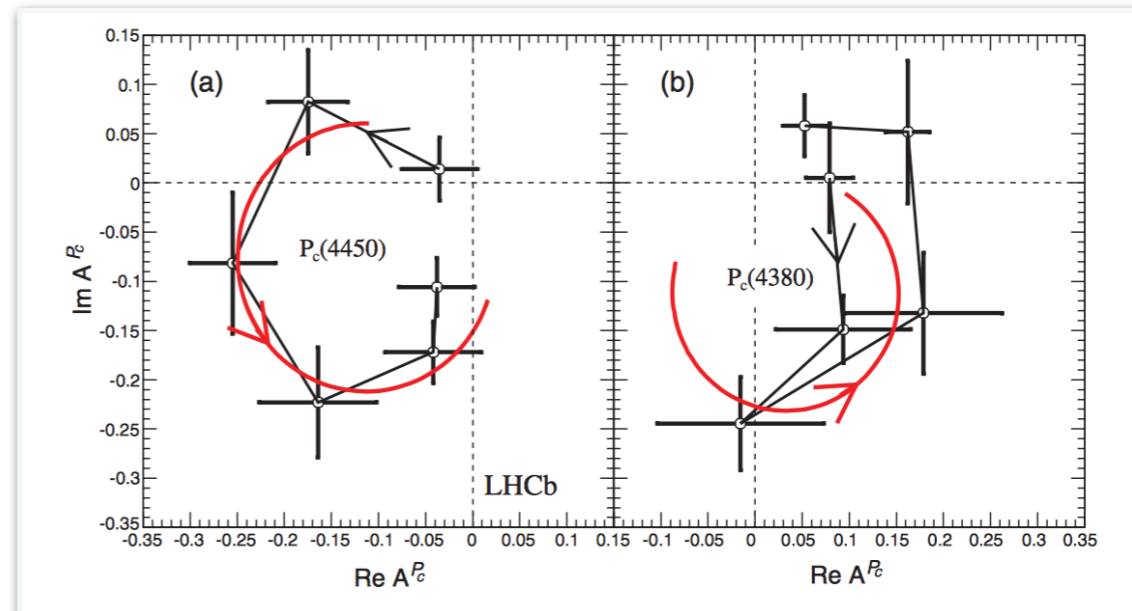


FIG. 8 (color online).  $m_{J/\psi p}$  in various intervals of  $m_{Kp}$  for the fit with two  $P_c^+$  states: (a)  $m_{Kp} < 1.55$  GeV, (b)  $1.55 < m_{Kp} < 1.70$  GeV, (c)  $1.70 < m_{Kp} < 2.00$  GeV, and (d)  $m_{Kp} > 2.00$  GeV. The data are shown as (black) squares with error bars, while the (red) circles show the results of the fit. The blue and purple histograms show the two  $P_c^+$  states. See Fig. 7 for the legend.

	$P_c(4380)^+$	$P_c(4450)^+$
Significance	$9\sigma$	$12\sigma$
Mass (MeV)	$4380 \pm 8 \pm 29$	$4449.8 \pm 1.7 \pm 2.5$
Width (MeV)	$205 \pm 18 \pm 86$	$39 \pm 5 \pm 19$
Fit fraction(%)	$8.4 \pm 0.7 \pm 4.2$	$4.1 \pm 0.5 \pm 1.1$
$\mathcal{B}(\Lambda_b^0 \rightarrow P_c^+ K^-;$ $P_c^+ \rightarrow J/\psi p)$	$(2.56 \pm 0.22 \pm 1.28^{+0.46}_{-0.36}) \times 10^{-5}$	$(1.25 \pm 0.15 \pm 0.33^{+0.22}_{-0.18}) \times 10^{-5}$

Branching ratio results are submitted to Chin. Phys. C (arXiv:1509.00292)  
 Ref:  $\mathcal{B}(B^0 \rightarrow Z^-(4430)K^+; Z^- \rightarrow J/\psi\pi^-) = (3.4 \pm 0.5^{+0.9}_{-1.9} \pm 0.2) \times 10^{-5}$

Argand diagrams show the resonance behavior of two  $P_c$  states





- The LHCb experiment at CERN's Large Hadron Collider has reported **the discovery of a class of particles** known as **pentaquarks**.

Posted by Corinne Pralavorio on 14 Jul 2015. Last updated 14 Jul 2015, 10:19.

*Voir en français*



Possible layout of the quarks in a pentaquark particle. The five quarks might be tightly bound (left). They might also be assembled into a meson (one quark and one antiquark) and a baryon (three quarks), weakly bound together (Image: Daniel Dominguez)

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- Experimental status of  $P_c(4380)$  and  $P_c(4450)$
- **History of multiquark states**
- Various theoretical methods
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# The history of multiquark states



Phys.Lett. 8 (1964) 214-215

Volume 8, number 3

PHYSICS LETTERS

1 February 1964

## A SCHEMATIC MODEL OF BARYONS AND MESONS \*

M. GELL-MANN

*California Institute of Technology, Pasadena, California*

Received 4 January 1964

...

A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon  $b$  if we assign to the triplet  $t$  the following properties: spin  $\frac{1}{2}$ ,  $z = -\frac{1}{3}$ , and baryon number  $\frac{1}{3}$ . We then refer to the members  $u^{\frac{2}{3}}$ ,  $d^{-\frac{1}{3}}$ , and  $s^{-\frac{1}{3}}$  of the triplet as "quarks"  $q$  and the members of the anti-triplet as anti-quarks  $\bar{q}$ . Baryons can now be constructed from quarks by using the combinations  $(qqq)$ ,  $(qqq\bar{q})$ , etc., while mesons are made out of  $(q\bar{q})$ ,  $(qq\bar{q}\bar{q})$ , etc. It is assumed that the lowest baryon configuration  $(qqq)$  gives just the representations **1**, **8**, and **10** that have been observed, while



8419/TH.412  
21 February 1964

AN  $SU_3$  MODEL FOR STRONG INTERACTION SYMMETRY AND ITS BREAKING

II \*)

G. Zweig

CERN---Geneva

\*) Version I is CERN preprint 8182/TH.401, Jan. 17, 1964.

...

- 6) In general, we would expect that baryons are built not only from the product of three aces,  $AAA$ , but also from  $\bar{A}AAAA$ ,  $\bar{A}AAAAA$ , etc., where  $\bar{A}$  denotes an anti-ace. Similarly, mesons could be formed from  $\bar{A}A$ ,  $\bar{A}AAA$  etc. For the low mass mesons and baryons we will assume the simplest possibilities,  $\bar{A}A$  and  $AAA$ , that is, "deuces and treys".

The multiquark states were predicted at the birth of Quark Model



# Quark Model

LIGHT UNFLAVORED (S = C = B = 0)		STRANGE (S = ±1, C = B = 0)		CHARMED, STRANGE (C = S = ±1)		c $\bar{c}$ J <sup>G</sup> (J <sup>PC</sup> )					
J <sup>G</sup> (J <sup>PC</sup> )		J <sup>G</sup> (J <sup>PC</sup> )		I(J <sup>P</sup> )		J <sup>G</sup> (J <sup>PC</sup> )					
• π <sup>±</sup>	1 <sup>-</sup> (0 <sup>-</sup> )	• π <sub>2</sub> (1670)	1 <sup>-</sup> (2 <sup>-+</sup> )	• K <sup>±</sup>	1/2(0 <sup>-</sup> )	• D <sub>s</sub> <sup>±</sup>	0(0 <sup>-</sup> )	• η <sub>c</sub> (1S)	0 <sup>+</sup> (0 <sup>-+</sup> )		
• π <sup>0</sup>	1 <sup>-</sup> (0 <sup>-+</sup> )	• ϕ(1680)	0 <sup>-</sup> (1 <sup>-</sup> )	• K <sup>0</sup>	1/2(0 <sup>-</sup> )	• D <sub>s</sub> <sup>*±</sup>	0(? <sup>?</sup> )	• J/ψ(1S)	0 <sup>-</sup> (1 <sup>-</sup> )		
• η	0 <sup>+</sup> (0 <sup>-+</sup> )	• ρ <sub>3</sub> (1690)	1 <sup>+</sup> (3 <sup>-</sup> )	• K <sub>S</sub> <sup>0</sup>	1/2(0 <sup>-</sup> )	• D <sub>s0</sub> <sup>±</sup> (2317) <sup>±</sup>	0(0 <sup>+</sup> )	• χ <sub>c0</sub> (1P)	0 <sup>+</sup> (0 <sup>++</sup> )		
• f <sub>0</sub> (600)	0 <sup>+</sup> (0 <sup>++</sup> )	• ρ(1700)	1 <sup>+</sup> (1 <sup>-</sup> )	• K <sub>L</sub> <sup>0</sup>	1/2(0 <sup>-</sup> )	• D <sub>s1</sub> (2460) <sup>±</sup>	0(1 <sup>+</sup> )	• χ <sub>c1</sub> (1P)	0 <sup>+</sup> (1 <sup>++</sup> )		
• ρ(770)	1 <sup>+</sup> (1 <sup>-</sup> )	• a <sub>2</sub> (1700)	1 <sup>-</sup> (2 <sup>++</sup> )	• K <sub>D</sub> <sup>0</sup> (800)	1/2(0 <sup>+</sup> )	• D <sub>s1</sub> (2536) <sup>±</sup>	0(1 <sup>+</sup> )	• h <sub>c</sub> (1P)	? <sup>?</sup> (1 <sup>-+</sup> )		
• ω(782)	0 <sup>-</sup> (1 <sup>-</sup> )	• f <sub>0</sub> (1710)	0 <sup>+</sup> (0 <sup>++</sup> )	• K*(892)	1/2(1 <sup>-</sup> )	• D <sub>s2</sub> (2573) <sup>±</sup>	0(? <sup>?</sup> )	• χ <sub>c2</sub> (1P)	0 <sup>+</sup> (2 <sup>++</sup> )		
• η'(958)	0 <sup>+</sup> (0 <sup>-+</sup> )	• η(1760)	0 <sup>+</sup> (0 <sup>-+</sup> )	• K <sub>1</sub> (1270)	1/2(1 <sup>+</sup> )	• D <sub>s1</sub> (2700) <sup>±</sup>	0(1 <sup>-</sup> )	• η <sub>c</sub> (2S)	0 <sup>+</sup> (0 <sup>-+</sup> )		
• f <sub>0</sub> (980)	0 <sup>+</sup> (0 <sup>++</sup> )	• π(1800)	1 <sup>-</sup> (0 <sup>-+</sup> )	• K <sub>1</sub> (1400)	1/2(1 <sup>+</sup> )	BOTTOM (B = ±1)		• ψ(2S)	0 <sup>-</sup> (1 <sup>-</sup> )		
• a <sub>0</sub> (980)	1 <sup>-</sup> (0 <sup>++</sup> )	• f <sub>2</sub> (1810)	0 <sup>+</sup> (2 <sup>++</sup> )	• K*(1410)	1/2(1 <sup>-</sup> )			• B <sup>±</sup>	1/2(0 <sup>-</sup> )	• ψ(3770)	0 <sup>-</sup> (1 <sup>-</sup> )
• ϕ(1020)	0 <sup>-</sup> (1 <sup>-</sup> )	• X(1835)	? <sup>?</sup> (? <sup>-+</sup> )	• K <sub>D</sub> <sup>0</sup> (1430)	1/2(0 <sup>+</sup> )	• B <sup>0</sup>	1/2(0 <sup>-</sup> )	• X(3872)	0 <sup>?</sup> (? <sup>++</sup> )		
• h <sub>1</sub> (1170)	0 <sup>-</sup> (1 <sup>-+</sup> )	• ϕ <sub>3</sub> (1850)	0 <sup>-</sup> (3 <sup>-</sup> )	• K <sub>2</sub> <sup>±</sup> (1430)	1/2(2 <sup>+</sup> )	B <sup>±</sup> /B <sup>0</sup> ADMIXTURE B <sup>±</sup> /B <sup>0</sup> /B <sub>s</sub> <sup>0</sup> /b-baryon ADMIXTURE V <sub>cb</sub> and V <sub>ub</sub> CKM Ma- trix Elements		• χ <sub>c2</sub> (2P)	0 <sup>+</sup> (2 <sup>++</sup> )		
• b <sub>1</sub> (1235)	1 <sup>+</sup> (1 <sup>-+</sup> )	• η <sub>2</sub> (1870)	0 <sup>+</sup> (2 <sup>-+</sup> )	• K(1460)	1/2(0 <sup>-</sup> )			• B*	1/2(1 <sup>-</sup> )	• X(3940)	? <sup>?</sup> ( <sup>???</sup> )
• a <sub>1</sub> (1260)	1 <sup>-</sup> (1 <sup>++</sup> )	• π <sub>2</sub> (1880)	1 <sup>-</sup> (2 <sup>-+</sup> )	• K <sub>2</sub> (1580)	1/2(2 <sup>-</sup> )			• B <sub>J</sub> (5732)	? <sup>?</sup> (? <sup>?</sup> )	• X(3945)	? <sup>?</sup> ( <sup>???</sup> )
• f <sub>2</sub> (1270)	0 <sup>+</sup> (2 <sup>++</sup> )	• ρ(1900)	1 <sup>+</sup> (1 <sup>-</sup> )	• K <sub>2</sub> (1630)	1/2(? <sup>?</sup> )			• B <sub>1</sub> (5721) <sup>0</sup>	1/2(1 <sup>+</sup> )	• ψ(4040)	0 <sup>-</sup> (1 <sup>-</sup> )
• f <sub>1</sub> (1285)	0 <sup>+</sup> (1 <sup>++</sup> )	• f <sub>2</sub> (1910)	0 <sup>+</sup> (2 <sup>++</sup> )	• K <sub>1</sub> (1650)	1/2(1 <sup>+</sup> )	• B <sub>2</sub> (5747) <sup>0</sup>	1/2(2 <sup>+</sup> )	• ψ(4160)	0 <sup>-</sup> (1 <sup>-</sup> )		
• η(1295)	0 <sup>+</sup> (0 <sup>-+</sup> )	• f <sub>2</sub> (1950)	0 <sup>+</sup> (2 <sup>++</sup> )	• K*(1680)	1/2(1 <sup>-</sup> )	BOTTOM, STRANGE (B = ±1, S = ∓1)		• X(4260)	? <sup>?</sup> (1 <sup>-</sup> )		
• π(1300)	1 <sup>-</sup> (0 <sup>-+</sup> )	• ρ <sub>3</sub> (1990)	1 <sup>+</sup> (3 <sup>-</sup> )	• K <sub>2</sub> (1770)	1/2(2 <sup>-</sup> )			• B <sub>s</sub> <sup>0</sup>	0(0 <sup>-</sup> )	• X(4360)	? <sup>?</sup> (1 <sup>-</sup> )
• a <sub>2</sub> (1320)	1 <sup>-</sup> (2 <sup>++</sup> )	• f <sub>2</sub> (2010)	0 <sup>+</sup> (2 <sup>++</sup> )	• K <sub>3</sub> <sup>±</sup> (1780)	1/2(3 <sup>-</sup> )	• B <sub>s</sub> <sup>±</sup>	0(1 <sup>-</sup> )	• ψ(4415)	0 <sup>-</sup> (1 <sup>-</sup> )		
• f <sub>0</sub> (1370)	0 <sup>+</sup> (0 <sup>++</sup> )	• f <sub>0</sub> (2020)	0 <sup>+</sup> (0 <sup>++</sup> )	• K <sub>2</sub> (1820)	1/2(2 <sup>-</sup> )	BOTTOM, CHARMED		b $\bar{b}$			
• h <sub>1</sub> (1380)	? <sup>-</sup> (1 <sup>-+</sup> )	• a <sub>4</sub> (2040)	1 <sup>-</sup> (4 <sup>++</sup> )	• K(1830)	1/2(0 <sup>-</sup> )			• B <sub>s1</sub> (5830) <sup>0</sup>	1/2(1 <sup>+</sup> )	• η <sub>b</sub> (1S)	0 <sup>+</sup> (0 <sup>-+</sup> )
• π <sub>1</sub> (1400)	1 <sup>-</sup> (1 <sup>-+</sup> )	• f <sub>4</sub> (2050)	0 <sup>+</sup> (4 <sup>++</sup> )	• K <sub>D</sub> <sup>0</sup> (1950)	1/2(0 <sup>+</sup> )	• B <sub>s2</sub> (5840) <sup>0</sup>	1/2(2 <sup>+</sup> )	• T(1S)	0 <sup>-</sup> (1 <sup>-</sup> )		
• η(1405)	0 <sup>+</sup> (0 <sup>-+</sup> )	• π <sub>2</sub> (2100)	1 <sup>-</sup> (2 <sup>-+</sup> )	• K <sub>2</sub> <sup>±</sup> (1980)	1/2(2 <sup>+</sup> )	• B <sub>sJ</sub> (5850)	? <sup>?</sup> (? <sup>?</sup> )	• χ <sub>b0</sub> (1P)	0 <sup>+</sup> (0 <sup>++</sup> )		
• f <sub>1</sub> (1420)	0 <sup>+</sup> (1 <sup>++</sup> )	• f <sub>0</sub> (2100)	0 <sup>+</sup> (0 <sup>++</sup> )	• K <sub>4</sub> <sup>±</sup> (2045)	1/2(4 <sup>+</sup> )	BOTTOM, STRANGE (B = ±1, S = ∓1)		• χ <sub>b1</sub> (1P)	0 <sup>+</sup> (1 <sup>++</sup> )		
• ω(1420)	0 <sup>-</sup> (1 <sup>-</sup> )	• f <sub>2</sub> (2150)	0 <sup>+</sup> (2 <sup>++</sup> )	• K <sub>2</sub> (2250)	1/2(2 <sup>-</sup> )			• B <sub>s</sub> <sup>0</sup>	0(0 <sup>-</sup> )	• χ <sub>b2</sub> (1P)	0 <sup>+</sup> (2 <sup>++</sup> )
• f <sub>2</sub> (1430)	0 <sup>+</sup> (2 <sup>++</sup> )	• ρ(2150)	1 <sup>+</sup> (1 <sup>-</sup> )	• K <sub>3</sub> (2320)	1/2(3 <sup>+</sup> )	• B <sub>s1</sub> (5830) <sup>0</sup>	1/2(1 <sup>+</sup> )	• T(2S)	0 <sup>-</sup> (1 <sup>-</sup> )		
• a <sub>0</sub> (1450)	1 <sup>-</sup> (0 <sup>++</sup> )	• ϕ(2170)	0 <sup>-</sup> (1 <sup>-</sup> )	• K <sub>3</sub> (2320)	1/2(3 <sup>+</sup> )	• B <sub>s2</sub> (5840) <sup>0</sup>	1/2(2 <sup>+</sup> )	• T(1D)	0 <sup>-</sup> (2 <sup>-</sup> )		
• ρ(1450)	1 <sup>+</sup> (1 <sup>-</sup> )	• f <sub>0</sub> (2200)	0 <sup>+</sup> (0 <sup>++</sup> )	• K <sub>5</sub> <sup>±</sup> (2380)	1/2(5 <sup>-</sup> )	• B <sub>sJ</sub> (5850)	? <sup>?</sup> (? <sup>?</sup> )	• χ <sub>b0</sub> (2P)	0 <sup>+</sup> (0 <sup>++</sup> )		
• η(1475)	0 <sup>+</sup> (0 <sup>-+</sup> )	• f <sub>J</sub> (2220)	0 <sup>+</sup> (2 <sup>++</sup> )	• K <sub>4</sub> (2500)	1/2(4 <sup>-</sup> )	BOTTOM, CHARMED		• χ <sub>b1</sub> (2P)	0 <sup>+</sup> (1 <sup>++</sup> )		
• f <sub>0</sub> (1500)	0 <sup>+</sup> (0 <sup>++</sup> )	• η(2225)	0 <sup>+</sup> (0 <sup>-+</sup> )	• K(3100)	? <sup>?</sup> ( <sup>???</sup> )			• T(3S)	0 <sup>-</sup> (1 <sup>-</sup> )	• χ <sub>b2</sub> (2P)	0 <sup>+</sup> (2 <sup>++</sup> )
• f <sub>1</sub> (1510)	0 <sup>+</sup> (1 <sup>++</sup> )	• ρ <sub>3</sub> (2250)	1 <sup>+</sup> (3 <sup>-</sup> )	CHARMED				• T(3S)	0 <sup>-</sup> (1 <sup>-</sup> )		

**Multiquark hadrons. I. Phenomenology of  $Q^2\bar{Q}^2$  mesons\***R. J. Jaffe<sup>†</sup>*Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305**and Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139*

(Received 15 July 1976)

The spectra and dominant decay couplings of  $Q^2\bar{Q}^2$  mesons are presented as calculated in the quark-bag model. Certain known  $0^+$  mesons [ $\epsilon(700), S^*, \delta, \kappa$ ] are assigned to the lightest cryptoexotic  $Q^2\bar{Q}^2$  nonet. The usual quark-model  $0^+$  nonet ( $Q\bar{Q} L=1$ ) must lie higher in mass. All other  $Q^2\bar{Q}^2$  mesons are predicted to be broad, heavy, and usually inelastic in formation processes. Other  $Q^2\bar{Q}^2$  states which may be experimentally prominent are discussed.

The hadron with four quarks plus one antiquark was developed by Strottman in 1979

**Multiquark baryons and the MIT bag model**

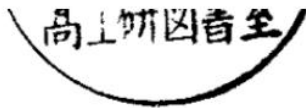
D. Strottman

*Theoretical Division, Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico 87545*

(Received 4 December 1978)

The calculation of masses of  $q^4\bar{q}$  and  $q^5\bar{q}^2$  baryons is carried out within the framework of Jaffe's approximation to the MIT bag model. A general method for calculating the necessary  $SU(6) \supset SU(3) \otimes SU(2)$  coupling coefficients is outlined and tables of the coefficients necessary for  $q^4\bar{q}$  and  $q^5\bar{q}^2$  calculations are given. An expression giving the decay amplitude of an arbitrary multiquark state to arbitrary two-body final states is given in terms of  $SU(3)$  Racah and  $9-\lambda\mu$  recoupling coefficients. The decay probabilities for low-lying  $1/2^-$   $q^4\bar{q}$  baryons are given and compared with experiment. All low-lying  $1/2^-$  baryons are found to belong to the same  $SU(6)$  representation and all known  $1/2^-$  resonances below 1900 MeV may be accounted for without the necessity of introducing  $P$ -wave states. The masses of many exotic states are predicted including a  $1/2^-$   $Z_0^*$  at 1650 MeV and  $1/2^-$  hypercharge  $-2$  and  $+3$  states at 2.25 and 2.80 GeV, respectively. The agreement with experiment for the  $3/2^-$  and  $5/2^-$  baryons is less good. The lowest  $q^5\bar{q}^2$  state is predicted to be a  $1/2^+$   $\Lambda^*$  at 1900 MeV.

# The name **pentaquark** was first proposed by Lipkin in 1987



WIS-87/32/May-PH

New Possibilities for Exotic Hadrons - Anticharmed Strange Baryons\*

Harry J. Lipkin  
Department of Nuclear Physics  
Weizmann Institute of Science  
76100 Rehovot, Israel  
Submitted to Physics Letters

**PLB 195 (1987) 484**

May 20, 1987

**ABSTRACT**

## **$Y = 2$ STATES IN SU(6) THEORY\***

Freeman J. Dyson† and Nguyen-Huu Xuong

Department of Physics, University of California, San Diego, La Jolla, California

(Received 30 November 1964)

Two-baryon states. – The SU(6) theory of strongly interacting particles<sup>1,2</sup> predicts a classification of two-baryon states into multiplets according to the scheme

$$\underline{56} \otimes \underline{56} = \underline{462} \oplus \underline{1050} \oplus \underline{1134} \oplus \underline{490}. \quad (1)$$

We now propose the hypothesis that all low-lying resonant states of the two-baryon system belong to the 490 multiplet.<sup>3</sup> This means that six zero-strangeness states shown in Table I should be observed. In all these states odd  $T$  goes with even  $J$  and vice versa.

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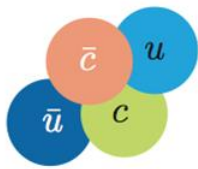
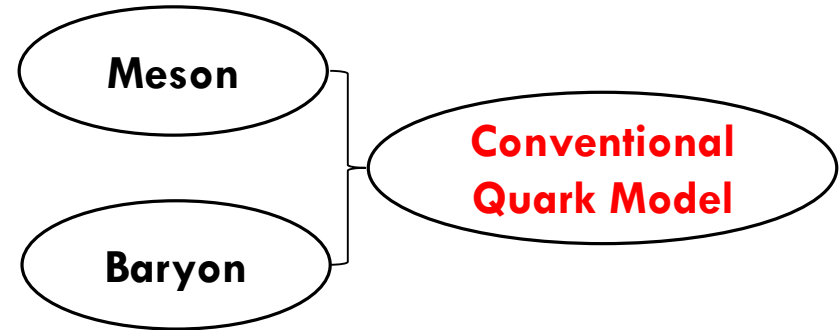
# Categorizations



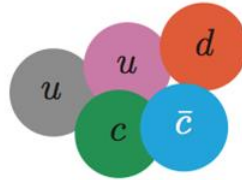
meson( $q\bar{q}$ )



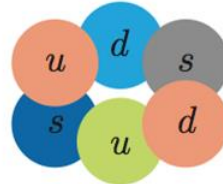
baryon( $qqq$ )



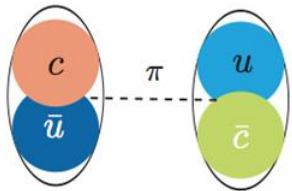
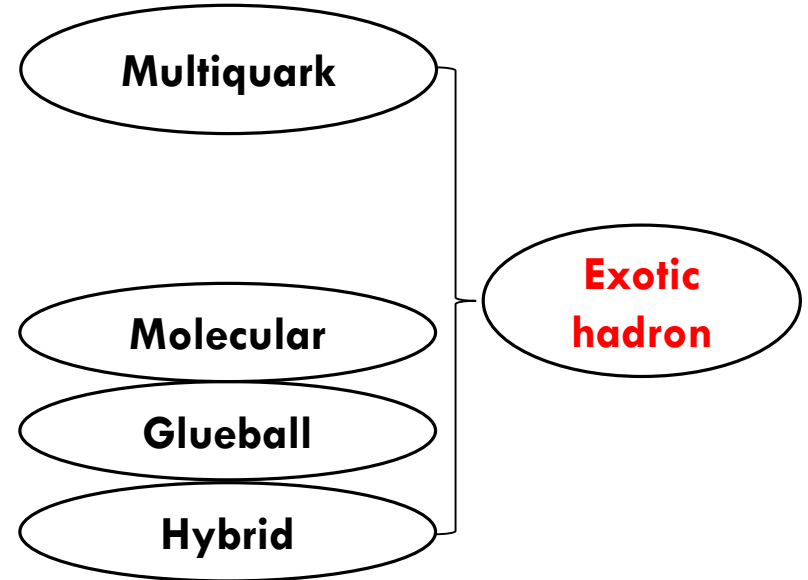
tetraquark



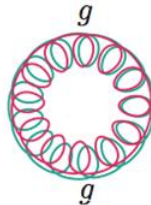
pentaquark



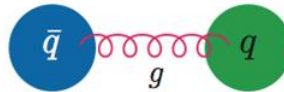
dibaryon



molecule



glueball



hybrid

- Identifying exotic states is one of the most important issues of particle physics
- Various experimental signals provide us good platform to identify exotic state

# Theoretical explanations of experimental signals

## Resonant

### ● Conventional hadrons

### ● Exotic states

#### ➤ **Molecular states:**

loosely bound states composed of a pair of mesons/baryons; probably bounded by the pion exchange.

#### ➤ **Multiquark states:**

bound states of four/five/six quarks; bounded by colored-force between quarks; there are many states within the same multiplet.

#### ➤ **Hybrids:**

bound states composed of a pair of quarks and one valance gluon.

**Y(4260)**

## Non-Resonant

Many exotic states lie very close to open-charm threshold; It's quite possible that some threshold enhancements are not real resonances.

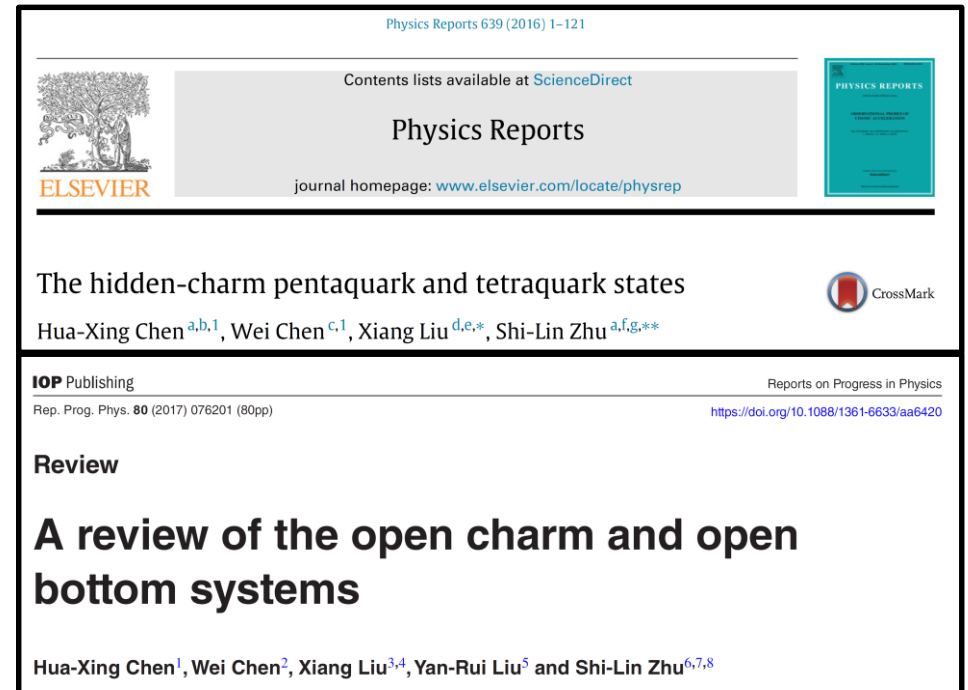
- **Kinematical effect**
- **Opening of new threshold**
- **Cusp effect**
- **Final state interaction**
- **Interference between continuum and charmonium states**
- **Triangle singularity due to the special kinematics**

# Theoretical methods/models

- Various quark models
- Various effective methods
- Chiral unitary model
- One boson exchange model
- Diquark/triquark model
- **QCD sum rules**
- Lattice QCD
- .....

➤ Various non-resonant explanations

➤ Many methods/models to study productions and decay patterns of exotic hadrons



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PHYSICS REPORTS

The hidden-charm pentaquark and tetraquark states

Hua-Xing Chen<sup>a,b,1</sup>, Wei Chen<sup>c,1</sup>, Xiang Liu<sup>d,e,\*</sup>, Shi-Lin Zhu<sup>a,f,g,\*\*</sup>

IOF Publishing

Rep. Prog. Phys. **80** (2017) 076201 (80pp)

Reports on Progress in Physics

<https://doi.org/10.1088/1361-6633/aa6420>

Review

**A review of the open charm and open bottom systems**

Hua-Xing Chen<sup>1</sup>, Wei Chen<sup>2</sup>, Xiang Liu<sup>3,4</sup>, Yan-Rui Liu<sup>5</sup> and Shi-Lin Zhu<sup>6,7,8</sup>

# Theoretical methods/models

- Various quark models
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  - Chiral unitary model
  - One boson exchange model
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  - **QCD sum rules**
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  - Various non-resonant explanations
  - Many methods/models to study productions and decay patterns of exotic hadrons
- From **Hadron** Level
- From **Quark** Level

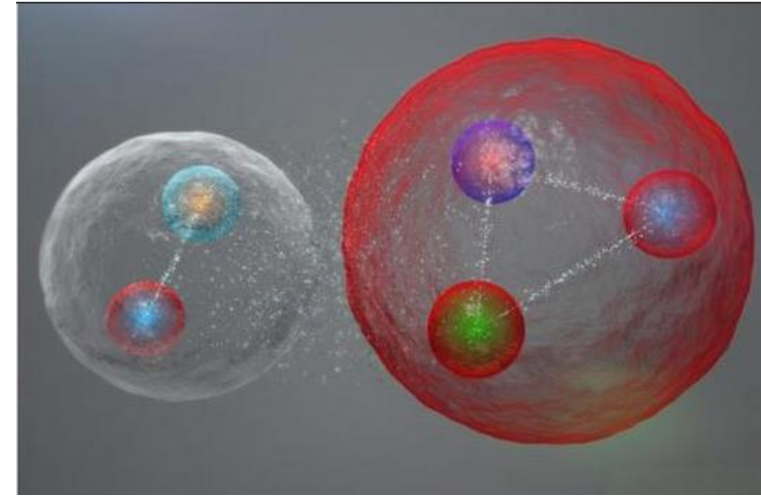


## From **Hadron Level**

The methods/models at the **hadron** level usually take **the molecular picture**

- Chiral unitary model
- One boson exchange model

.....



## From **Hadron Level**

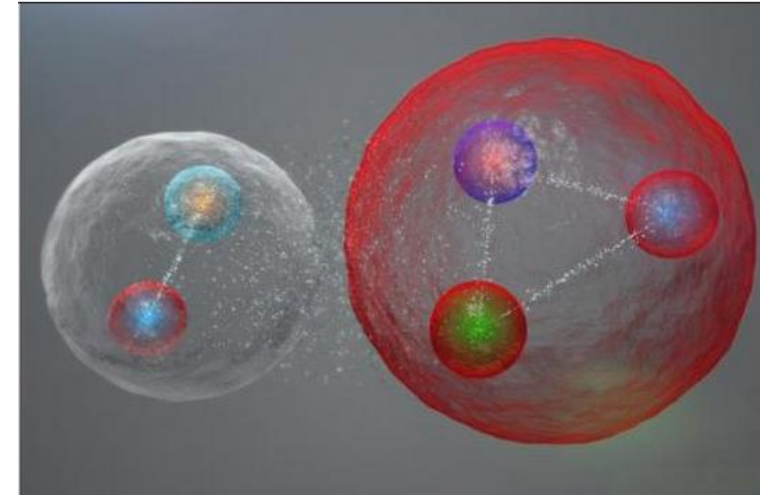
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## The **Molecular Picture**

The masses of  $P_c(4380)$  and  $P_c(4450)$  are close to the  $\Sigma_c(2455)D^*$  and  $\Sigma_c^*(2520)D^*$  thresholds, respectively.



**D mesons**

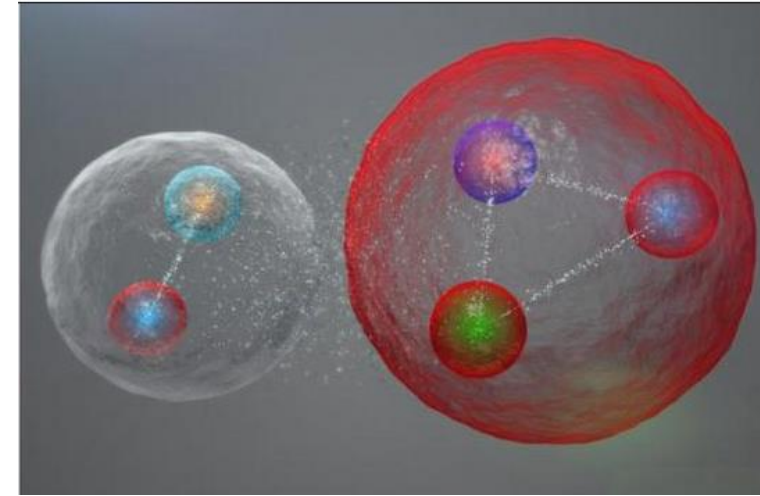
**$\Sigma_c$  baryons**

## From **Hadron Level**

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**D mesons**

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**Deuteron: loosely bound state of proton and neutron**

**Nucleon force: short-range, mid-range, long-range**

$\rho$  and  $\omega$  exchanges

Scalar  $\sigma$  with mass  
around 600 MeV

Pion exchange

## Prediction of narrow $N^*$ and $\Lambda^*$ resonances with hidden charm above 4 GeV

Jia-Jun Wu<sup>1,2</sup>, R. Molina<sup>2,3</sup>, E. Oset<sup>2,3</sup> and B. S. Zou<sup>1,3</sup>

1. *Institute of High Energy Physics, CAS, Beijing 100049, China*

2. *Departamento de Física Teórica and IFIC, Centro Mixto Universidad de Valencia-CSIC, Institutos de Investigación de Paterna, Aptdo. 22085, 46071 Valencia, Spain*

3. *Theoretical Physics Center for Science Facilities, CAS, Beijing 100049, China*

(Dated: June 25, 2010)

The interaction between various charmed mesons and charmed baryons are studied within the framework of the coupled channel unitary approach with the local hidden gauge formalism. Several meson-baryon dynamically generated narrow  $N^*$  and  $\Lambda^*$  resonances with hidden charm are predicted with mass above 4 GeV and width smaller than 100 MeV. The predicted new resonances definitely

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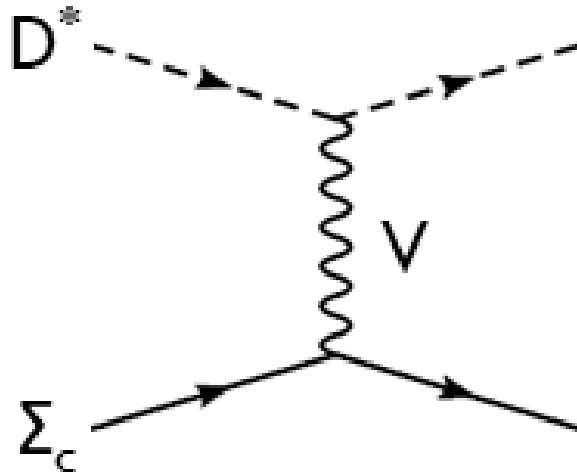
1. *Institute of High Energy Physics, CAS, Beijing 100049, China*

2. *Departamento de Física Teórica and IFIC, Centro Mixto Universidad de Valencia-CSIC, Institutos de Investigación de Paterna, Aptdo. 22085, 46071 Valencia, Spain*

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$$\mathcal{L}_{VVV} = ig \langle V^\mu [V^\nu, \partial_\mu V_\nu] \rangle$$

$$\mathcal{L}_{PPV} = -ig \langle V^\mu [P, \partial_\mu P] \rangle$$

$$\mathcal{L}_{BBV} = g (\langle \bar{B} \gamma_\mu [V^\mu, B] \rangle + \langle \bar{B} \gamma_\mu B \rangle \langle V^\mu \rangle)$$

$$V_{ab}(P_1 B_1 \rightarrow P_2 B_2) = \frac{C_{ab}}{4f^2} (E_{P_1} + E_{P_2}),$$

$$V_{ab}(V_1 B_1 \rightarrow V_2 B_2) = \frac{C_{ab}}{4f^2} (E_{V_1} + E_{V_2}) \vec{\epsilon}_1 \cdot \vec{\epsilon}_2,$$

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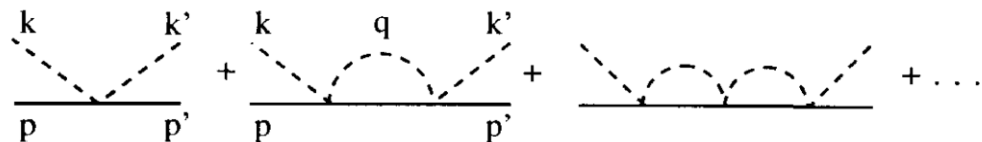
1. Institute of High Energy Physics, CAS, Beijing 100049, China

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$$T = V + VGT = (1 - VG)^{-1}V,$$

TABLE I: Coefficients  $C_{ab}$  in Eq. (2) for  $(I, S) = (1/2, 0)$

	$\bar{D}\Sigma_c$	$\bar{D}\Lambda_c^+$	$\eta_c N$	$\pi N$	$\eta N$	$\eta' N$	$K\Sigma$	$K\Lambda$
$\bar{D}\Sigma_c$	-1	0	$-\sqrt{3/2}$	-1/2	$-1/\sqrt{2}$	1/2	1	0
$\bar{D}\Lambda_c^+$		1	$\sqrt{3/2}$	-3/2	$1/\sqrt{2}$	-1/2	0	1

$(I, S)$	$z_R$ (MeV)	$g_a$		
(1/2, 0)	4269	$\bar{D}\Sigma_c$	$\bar{D}\Lambda_c^+$	
		2.85	0	
(0, -1)	4213	$\bar{D}_s\Lambda_c^+$	$\bar{D}\Xi_c$	$\bar{D}\Xi'_c$
	4403	1.37	3.25	0
		0	0	2.64

TABLE III: Pole positions  $z_R$  and coupling constants  $g_a$  for the states from  $PB \rightarrow PB$ .

$(I, S)$	$z_R$ (MeV)	$g_a$		
(1/2, 0)	4418	$\bar{D}^*\Sigma_c$	$\bar{D}^*\Lambda_c^+$	
		2.75	0	
(0, -1)	4370	$\bar{D}_s^*\Lambda_c^+$	$\bar{D}^*\Xi_c$	$\bar{D}^*\Xi'_c$
	4550	1.23	3.14	0
		0	0	2.53

TABLE IV: Pole position and coupling constants for the bound states from  $VB \rightarrow VB$ .

# Possible hidden-charm molecular baryons composed of an anti-charmed meson and a charmed baryon<sup>\*</sup>

YANG Zhong-Cheng(杨忠诚)<sup>1</sup> SUN Zhi-Feng(孙志峰)<sup>2,4</sup> HE Jun(何军)<sup>1,3;1)</sup>  
LIU Xiang(刘翔)<sup>2,4;2)</sup> ZHU Shi-Lin(朱世琳)<sup>1;3)</sup>

In this work, we have employed the OBE model to study whether there exist the loosely bound hidden-charm molecular states composed of an S-wave anti-charmed meson and an S-wave charmed baryon. Our numerical results indicate that there do not exist  $\Lambda_c \bar{D}$  and  $\Lambda_c \bar{D}^*$  molecular states due to the absence of bound state solution, which is an interesting observation in this work. Additionally, we notice the bound state solutions only for five hidden-charm states, i.e.,  $\Sigma_c \bar{D}^*$  states with  $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ ,  $\frac{1}{2}(\frac{3}{2}^-)$ ,  $\frac{3}{2}(\frac{1}{2}^-)$ ,  $\frac{3}{2}(\frac{3}{2}^-)$  and  $\Sigma_c \bar{D}$  state with  $\frac{3}{2}(\frac{1}{2}^-)$ . We also extend the same

## A new page for hadron physics: Identifying exotic hidden-charm pentaquarks

Rui Chen<sup>1,2</sup> and Xiang Liu<sup>1,2\*</sup>

<sup>1</sup>*Research Center for Hadron and CSR Physics, Lanzhou University & Institute of Modern Physics of CAS, Lanzhou 730000, China*

<sup>2</sup>*School of Physical Science and Technology, Lanzhou University, Lanzhou 730000, China*

Xue-Qian Li<sup>†</sup>

*School of Physics, Nankai University, Tianjin 300071, China*

Shi-Lin Zhu<sup>1,2,3‡</sup>

<sup>1</sup>*School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China*

<sup>2</sup>*Collaborative Innovation Center of Quantum Matter, Beijing 100871, China*

<sup>3</sup>*Center of High Energy Physics, Peking University, Beijing 100871, China*



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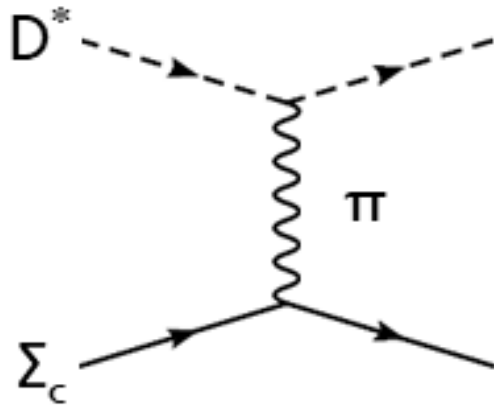
Shi-Lin Zhu<sup>1,2,3‡</sup>

<sup>1</sup>School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China

<sup>2</sup>Collaborative Innovation Center of Quantum Matter, Beijing 100871, China

<sup>3</sup>Center of High Energy Physics, Peking University, Beijing 100871, China

The authors investigated the **one-pion-exchange-based potentials** for the  $\Sigma_c \bar{D}^*$  and  $\Sigma_c^* \bar{D}^*$  systems:



$$\mathcal{L}_{\bar{D}^* \bar{D}^* \mathbb{P}} = i \frac{2g}{f_\pi} v^\alpha \varepsilon_{\alpha\mu\nu\lambda} \bar{D}_a^{*\mu\dagger} D_b^{*\lambda} \partial_\nu \mathbb{P}_{ab},$$

$$\mathcal{L}_{\mathcal{B}_6 \mathcal{B}_6 \mathbb{P}} = i \frac{g_1}{2f_\pi} \varepsilon^{\mu\nu\lambda\kappa} v_\kappa \text{Tr} \left[ \bar{\mathcal{B}}_6 \gamma_\mu \gamma_\lambda \partial_\nu \mathbb{P} \mathcal{B}_6 \right],$$

$$\mathcal{L}_{\mathcal{B}_6^* \mathcal{B}_6^* \mathbb{P}} = -i \frac{3g_1}{2f_\pi} \varepsilon^{\mu\nu\lambda\kappa} v_\kappa \text{Tr} \left[ \bar{\mathcal{B}}_{6\mu}^* \partial_\nu \mathbb{P} \mathcal{B}_{6\nu}^* \right],$$

# One boson exchange model

arXiv:1507.03704

The effective potential in  
momentum space

Scattering amplitude

$$V(\mathbf{Q}) \approx -\frac{\mathcal{M}}{\sqrt{2E_A} \sqrt{2E_B} \sqrt{2E_C} \sqrt{2E_D}}$$

Fourier transformation

$V(\mathbf{r})$  → The effective potential in coordinate space

The effective potential in momentum space

Scattering amplitude

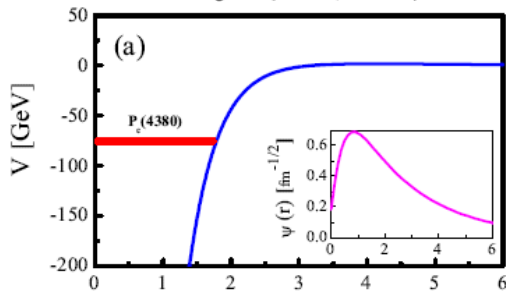
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Fourier transformation

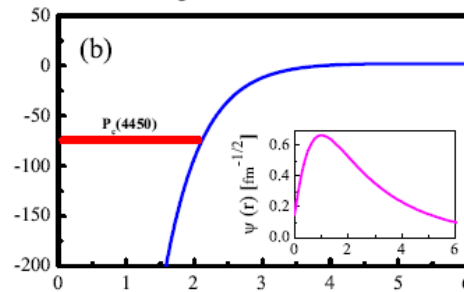
$V(\mathbf{r})$  → The effective potential in coordinate space

## Solving Schrödinger equation

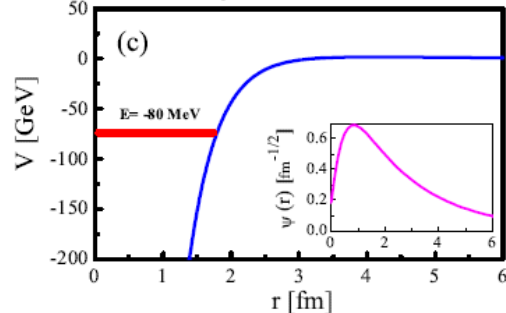
$\Sigma_c \bar{D}^* (I=1/2, J=3/2)$



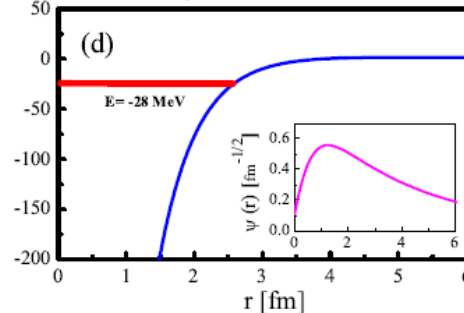
$\Sigma_c^* \bar{D}^* (I=1/2, J=5/2)$



$\Sigma_c \bar{D}^* (I=3/2, J=1/2)$



$\Sigma_c^* \bar{D}^* (I=3/2, J=1/2)$



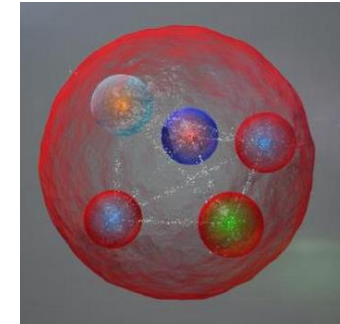
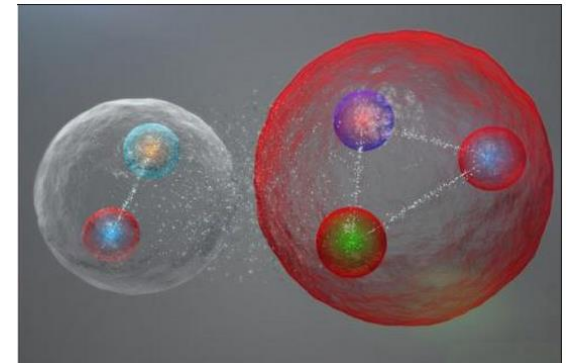
➤  $P_c(4380)$  and  $P_c(4450)$  are first identified as the hidden-charm molecular pentaquarks  $\Sigma_c \bar{D}^*$  with  $(I=1/2, J=3/2)$ , and  $\Sigma_c^* \bar{D}^*$  with  $(I=1/2, J=3/2)$ , respectively.

➤ Their study indicates that there should exist a  $\Sigma_c \bar{D}^*$  state with  $(I=3/2, J=1/2)$  and a  $\Sigma_c^* \bar{D}^*$  state with  $(I=3/2, J=1/2)$ .

## From **Quark** Level

The methods/models at the **quark** level can take both **the molecular picture** and **the (compact) multiquark picture**

- Diquark/triquark model
- QCD sum rules
- Lattice QCD
- .....

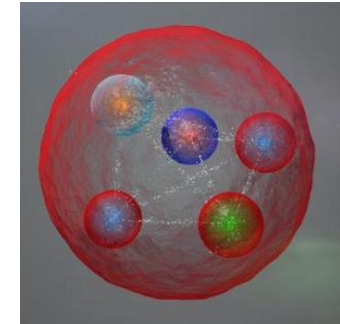
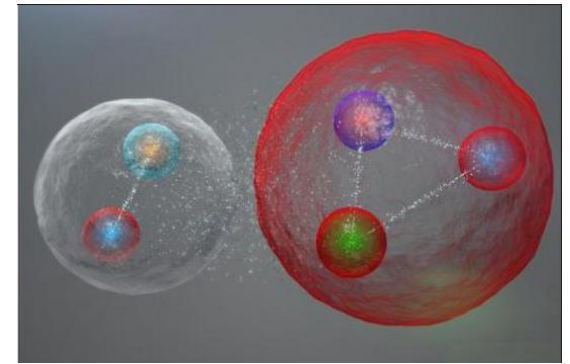


# From **Quark** Level

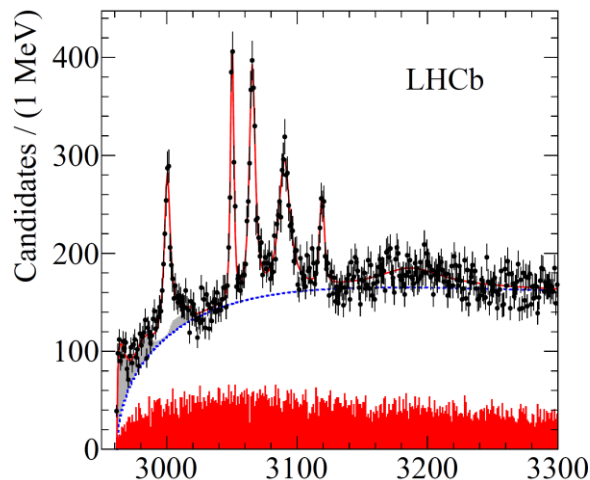
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- Lattice QCD

.....



These studies help to understand **the internal structure** of hadrons



Recent LHCb Experiment [arXiv:1703.04639]

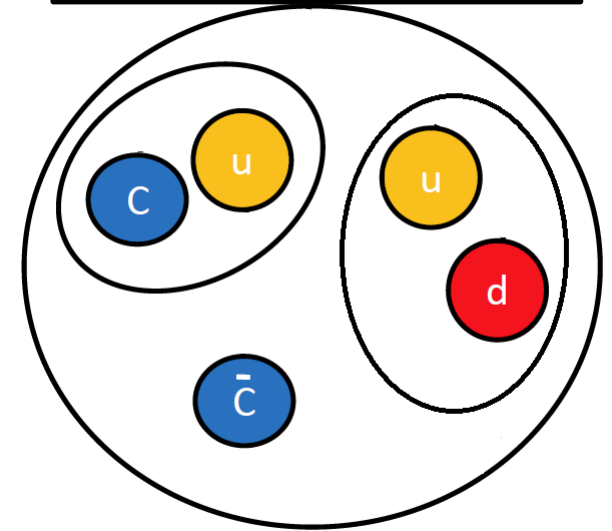
**fine structure of QCD?**

## The New Pentaquarks in the Diquark Model

L. Maiani,<sup>1</sup> A.D. Polosa,<sup>1</sup> and V. Riquer<sup>1</sup>

<sup>1</sup>*Dipartimento di Fisica and INFN, 'Sapienza' Università di Roma  
P.le Aldo Moro 5, I-00185 Roma, Italy*

Pentaquark baryons are a natural expectation of an extended picture of hadrons where diquarks are the fundamental units. The parity/mass pattern observed, when compared with exotic mesons, appears as the footprint of a compact five-quark structure. What has been learned from the  $X, Y, Z$  phenomenology informs about the newly found pentaquark structure and suggests further experimental tests and directions to be explored.



# Diquark/triquark model

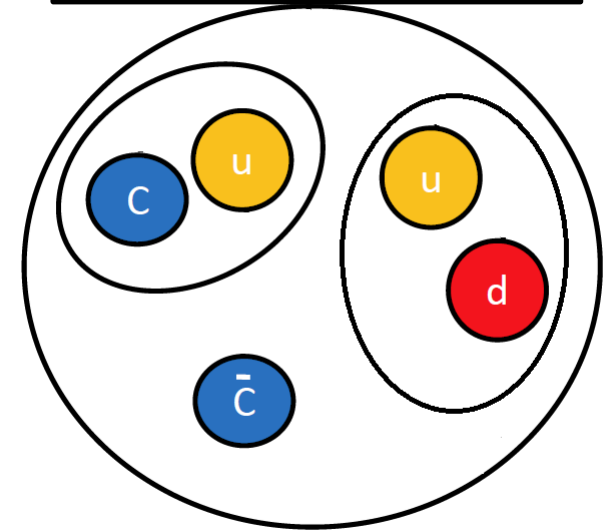
## The New Pentaquarks in the Diquark Model

L. Maiani,<sup>1</sup> A.D. Polosa,<sup>1</sup> and V. Riquer<sup>1</sup>

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arXiv:1507.03704



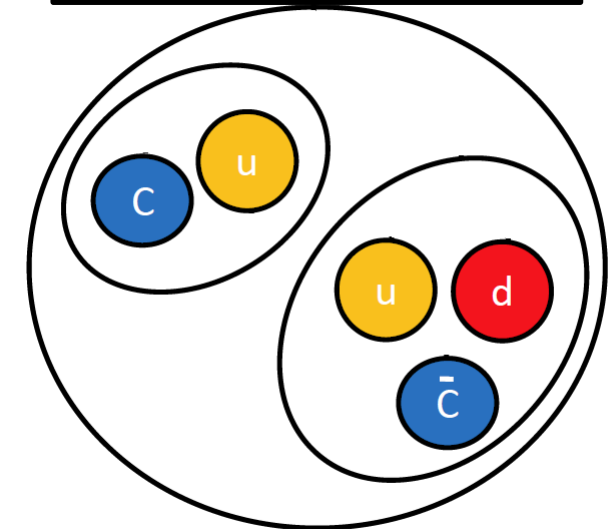
## The Pentaquark Candidates in the Dynamical Diquark Picture

Richard F. Lebed\*

*Department of Physics, Arizona State University, Tempe, Arizona 85287-1504, USA  
(Dated: July, 2015)*

Starting with the dynamical picture of the exotic  $c\bar{c}$ -containing states  $XYZ$  as the confinement-induced hadronization of a rapidly separating pair of a compact diquark and antidiquark, we describe the pentaquark candidates  $P_c^+(4380)$  and  $P_c^+(4450)$  in terms of a confined but rapidly separating color-antitriplet diquark  $cu$  and color-triplet "triquark"  $\bar{c}(ud)$ . This separation explains the relatively small  $P_c^+$  widths, despite these 5-quark systems lying far above both the  $J/\psi p$  and  $\Lambda_c \bar{D}^{(*)0}$  thresholds. The  $P_c^+$  states are predicted to form isospin doublets with neutral partners  $P_c^0$ .

arXiv:1507.05867



# CONTENTS

- Experimental status of  $P_c(4380)$  and  $P_c(4450)$
- History of multiquark states
- Various theoretical methods
- **Our QCD sum rule studies**



# A $[J/\psi p]$ current

flavor contents

$$[\bar{c}_d(x)\gamma_\mu c_d(x)][\epsilon^{abc}(u_a^T(x)C d_b(x))\gamma_5 u_c(x)]$$

color indices

## A $[J/\psi p]$ current

The diagram illustrates the structure of a  $[J/\psi p]$  current. It features a central equation with two horizontal lines above and below it, each divided into four segments by vertical lines. The top line is labeled "flavor contents" and the bottom line is labeled "color indices".

$$[\bar{c}_d(x)\gamma_\mu c_d(x)][\varepsilon^{abc}(u_a^T(x)C d_b(x))\gamma_5 u_c(x)]$$

flavor contents

color indices

The Pauli principle is automatically satisfied

## Two Configurations:

$$[\bar{c}_d c_d][\epsilon^{abc} q_a q_b q_c] \text{ and } [\bar{c}_d q_d][\epsilon^{abc} c_a q_b q_c]$$

These two configurations, **as if they are local**, can be related to each other through

- **The Fierz transformation**

$$\begin{aligned} (\bar{s}_a u_b)(\bar{s}_b d_a) = & -\frac{1}{4} \{ (\bar{s}_a u_a)(\bar{s}_b d_b) + (\bar{s}_a \gamma_\mu u_a)(\bar{s}_b \gamma^\mu d_b) + \frac{1}{2} (\bar{s}_a \sigma_{\mu\nu} u_a)(\bar{s}_b \sigma^{\mu\nu} d_b) \\ & - (\bar{s}_a \gamma_\mu \gamma_5 u_a)(\bar{s}_b \gamma^\mu \gamma_5 d_b) + (\bar{s}_a \gamma_5 u_a)(\bar{s}_b \gamma_5 d_b) \}. \end{aligned}$$

- **The color rearrangement**

$$\delta^{de} \epsilon^{abc} = \delta^{da} \epsilon^{ebc} + \delta^{db} \epsilon^{aec} + \delta^{dc} \epsilon^{abe}$$

# Configuration $[\bar{c}_d c_d][\epsilon^{abc} q_a q_b q_c]$

- There are three independent local light baryon fields of flavor-octet and having a positive parity:

H. X. Chen, V. Dmitrasinovic, A. Hosaka, K. Nagata and S. L. Zhu, Phys. Rev. D 78, 054021 (2008)

$$\begin{aligned} N_1^N &= \epsilon_{abc} \epsilon^{ABD} \lambda_{DC}^N (q_A^{aT} C q_B^b) \gamma_5 q_C^c, \\ N_2^N &= \epsilon_{abc} \epsilon^{ABD} \lambda_{DC}^N (q_A^{aT} C \gamma_5 q_B^b) q_C^c, \\ N_{3\mu}^N &= \epsilon_{abc} \epsilon^{ABD} \lambda_{DC}^N (q_A^{aT} C \gamma_\mu \gamma_5 q_B^b) \gamma_5 q_C^c, \end{aligned}$$

- Together with light baryon fields having negative parity and the charmonium fields:

$$\begin{aligned} &\bar{c}_d c_d [0^+], \bar{c}_d \gamma_5 c_d [0^-], \\ &\bar{c}_d \gamma_\mu c_d [1^-], \bar{c}_d \gamma_\mu \gamma_5 c_d [1^+], \bar{c}_d \sigma_{\mu\nu} c_d [1^\pm], \end{aligned}$$

- We can construct the currents of the configuration  $[\bar{c}_d c_d][\epsilon^{abc} q_a q_b q_c]$ .
- Those containing  $J=3/2$  components are  $[\bar{c}_d c_d][N_{3\mu}^N]$ ,  $[\bar{c}_d \gamma_5 c_d][N_{3\mu}^N]$ ,  $[\bar{c}_d \gamma_\mu c_d][N_{1,2}^N]$ ,  $[\bar{c}_d \gamma_\mu \gamma_5 c_d][N_{1,2}^N]$ ,  $[\bar{c}_d \gamma_\mu c_d][N_{3\nu}^N]$ ,  $[\bar{c}_d \gamma_\mu \gamma_5 c_d][N_{3\nu}^N]$ ,  $[\bar{c}_d \sigma_{\mu\nu} c_d][N_{1,2}^N]$ ,  $[\bar{c}_d \sigma_{\mu\nu} c_d][N_{3\rho}^N]$ ,
- Three of them of  $J=3/2$  &  $5/2$  couple well to the combination of  $J/\psi$  and **proton**

$$\begin{aligned} \eta_{1\mu}^{c\bar{c}uud} &= [\bar{c}_d \gamma_\mu c_d][\epsilon_{abc} (u_a^T C d_b) \gamma_5 u_c], \\ \eta_{2\mu}^{c\bar{c}uud} &= [\bar{c}_d \gamma_\mu c_d][\epsilon_{abc} (u_a^T C \gamma_5 d_b) u_c], \\ \eta_{3\{\mu\nu\}}^{c\bar{c}uud} &= [\bar{c}_d \gamma_\mu c_d][\epsilon_{abc} (u_a^T C \gamma_\nu \gamma_5 d_b) u_c] + \{\mu \leftrightarrow \nu\}. \end{aligned}$$



# QCD SUM RULE

- In sum rule analyses, we consider **two-point correlation functions**:

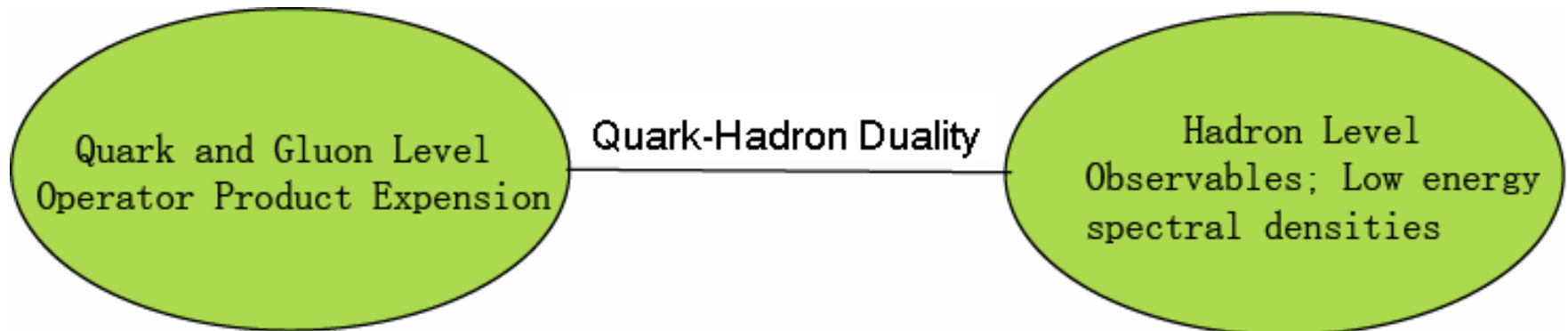
$$\begin{aligned}\Pi(q^2) &\stackrel{\text{def}}{=} i \int d^4x e^{iqx} \langle 0 | T \eta(x) \eta^\dagger(0) | 0 \rangle \\ &\approx \sum_n \langle 0 | \eta | n \rangle \langle n | \eta^\dagger | 0 \rangle\end{aligned}$$

where  $\eta$  is the current which can couple to **hadronic states**.

- By using the **dispersion relation**, we can obtain the **spectral density**

$$\Pi(q^2) = \int_{s_<}^{\infty} \frac{\rho(s)}{s - q^2 - i\epsilon} ds$$

- In QCD sum rule, we can calculate these matrix elements from QCD (**OPE**) and relate them to observables by using **dispersion relation**.



## Quark and Gluon Level

(Convergence of OPE)

$$\Pi_{OPE}(q^2) \xrightarrow[\substack{\text{dispersion relation} \\ s = -q^2}]{\hspace{10em}} \rho_{OPE}(s) = a_n s^n + a_{n-1} s^{n-1}$$

Quark-Hadron Duality

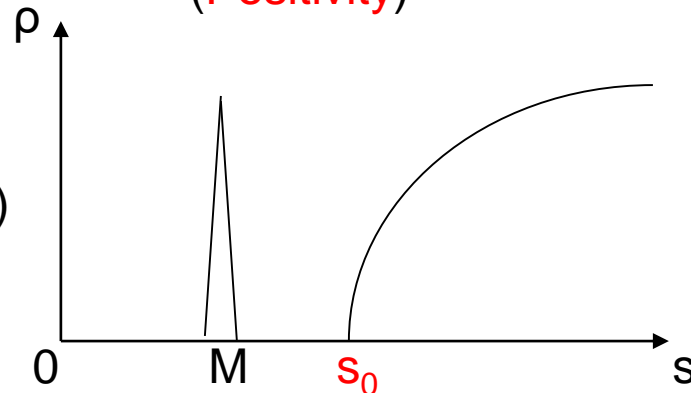
## Hadron Level

$$\Pi_{phys}(q^2) = f_P^2 \frac{\not{q} + M}{q^2 - M^2} \longleftrightarrow \rho_{phys}(s) = \lambda_x^2 \delta(s - M_x^2) + \dots$$

(Positivity)

(for baryon case)

(Sufficient amount of Pole contribution)



# QCD Sum Rule

- **Borel transformation** to suppress the higher order terms:

$$\Pi(M_B^2) \equiv f^2 e^{-M^2/M_B^2} = \int_{s_0}^{s_0} e^{-s/M_B^2} \rho(s) ds$$

- **Two** parameters

$$M_B, s_0$$

We need to choose certain region of  $(M_B, s_0)$ .

- **Criteria**

1. Stability
2. Convergence of OPE
3. Positivity of spectral density
4. Sufficient amount of pole contribution



Y. Chung, H. G. Dosch, M. Kremer and D. Schall, Nucl. Phys. B 197, 55 (1982)

D. Jido, N. Kodama and M. Oka, Phys. Rev. D 54, 4532 (1996)

Y. Kondo, O. Morimatsu and T. Nishikawa, Nucl. Phys. A 764, 303 (2006)

K. Ohtani, P. Gubler and M. Oka, Phys. Rev. D 87, no. 3, 034027 (2013)

# Parity of Pentaquark

- Assuming  $J$  is a pentaquark current,  $\gamma_5 J$  is its partner having the opposite parity.
- They can couple to the same physical state through

$$\langle 0 | J | P(q) \rangle = f_P u(q),$$

$$\langle 0 | \gamma_5 J | P(q) \rangle = f_P \gamma_5 u(q).$$

- The same pentaquark current  $J$  can couple to states of both positive and negative parities through

$$\langle 0 | J | P(q) \rangle = f_P u(q),$$

$$\langle 0 | J | P'(q) \rangle = f_{P'} \gamma_5 u'(q).$$

where  $|P(q)\rangle$  has the same parity as  $J$ , while  $|P'(q)\rangle$  has the opposite parity.

$$f_P^2 \frac{\not{q} + M}{q^2 - M^2}$$

$$f_{P'}^2 \frac{-\not{q} + M}{q^2 - M^2}$$

The sum rule results obtained using  $J_{\mu}^{\bar{D}^* \Sigma_c} (J=3/2)$  are

$$M_{[\bar{D}^* \Sigma_c], 3/2^-} = 4.37^{+0.18}_{-0.12} \text{ GeV} .$$

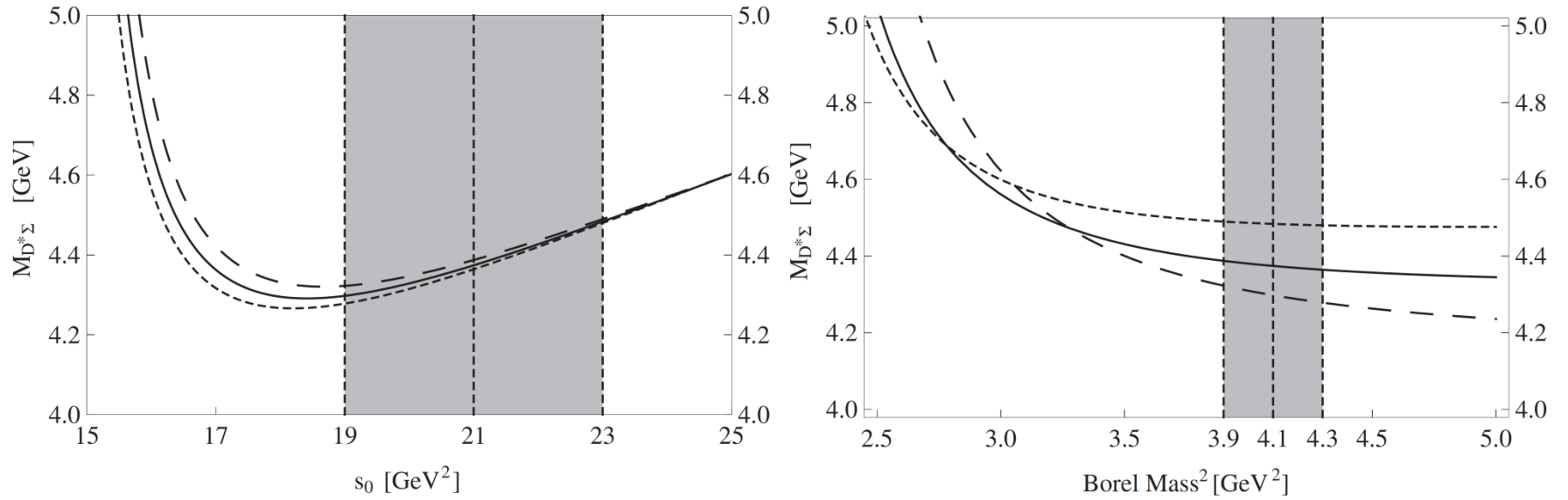


FIG. 1: The variation of  $M_{[\bar{D}^* \Sigma_c], 3/2^-}$  with respect to the threshold value  $s_0$  (left) and the Borel mass  $M_B$  (right). In the left figure, the

The sum rule results obtained using  $J_{\{\mu\nu\}}^{\bar{D}\Sigma_c^*}$  and  $J_{\{\mu\nu\}}^{\bar{D}^*\Lambda_c}$  are not useful.  
 However, their mixing gives a reliable mass sum rule ( $J=5/2$ )

$$J_{\{\mu\nu\}}^{\bar{D}\Sigma_c^* \& \bar{D}^*\Lambda_c} = \sin \theta \times J_{\{\mu\nu\}}^{\bar{D}\Sigma_c^*} + \cos \theta \times J_{\{\mu\nu\}}^{\bar{D}^*\Lambda_c}$$

$$\tan \theta = -1.25$$

$$M_{[\bar{D}\Sigma_c^* \& \bar{D}^*\Lambda_c], 5/2^+} = 4.47^{+0.19}_{-0.12} \text{ GeV}.$$

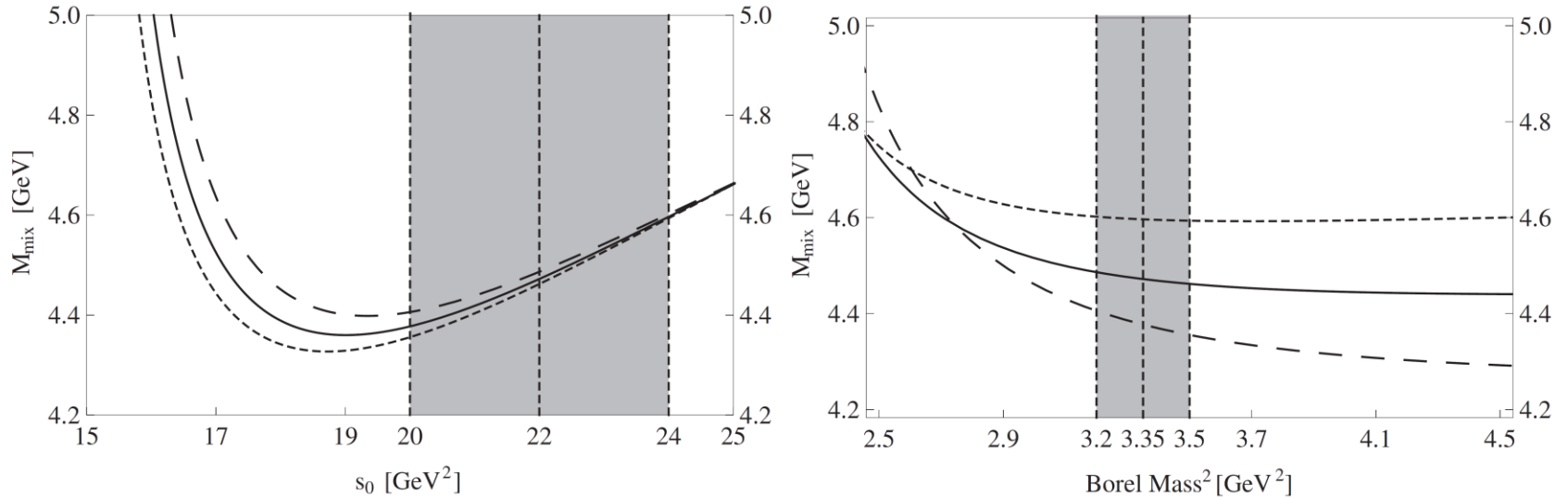


FIG. 2: The variation of  $M_{[\bar{D}\Sigma_c^* \& \bar{D}^*\Lambda_c], 5/2^+}$  with respect to the threshold value  $s_0$  (left) and the Borel mass  $M_B$  (right).

# Summary

- Besides the mass spectrum, production and decay properties are also important to understand **exotic hadrons**.
- We still need more theoretical and experimental joint efforts.
- This is a research field full of challenges and opportunities.

**Thank you very much!**