Nuclear dependence of the TMDs and azimuthal asymmetries in semi-inclusive deep-inelastic scattering

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Two important quantities:

- parton distribution functions (PDFs) ⟷ hadron structure
- fragmentation function (FFs) ⟷ hadronization

hadron structure (the PDFs ...)

hadronization (the FFs ...)

⟷ theoretical framework ⟷

the measurable quantities (differential cross section, azimuthal asymmetries, polarizations, ...)

↑

collinear expansion

Why do we need it? How do we do it?

Outline

- Introduction: theoretical framework for semi-inclusive DIS
  - Collinear expansion in inclusive DIS $e^- + N \to e^- + X$ and the gauge invariant parton distribution functions (PDFs);
  - Collinear expansion in semi-inclusive DIS $e^- + N \to e^- + q + X$, the gauge invariant transverse momentum dependent (TMD) parton distributions and the azimuthal asymmetries.
- Nuclear dependence of the TMDs and the azimuthal asymmetries.
  - un-polarized nuclear target;
  - polarized nuclear target;
  - a simple numerical estimation
- Summary and outlook

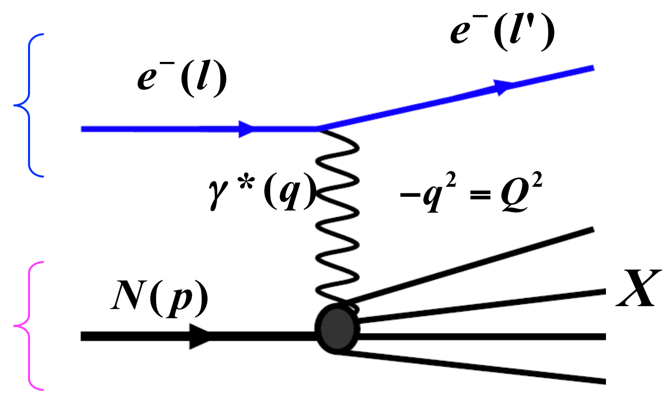
Inclusive（单举）deep inelastic scattering (DIS) $e^- + N \rightarrow e^- + X$

The differential cross section：

$$d\sigma = \frac{\alpha_{em}^2}{sQ^4} L^{\mu\nu}(l,l') W_{\mu\nu}(q,p) \frac{d^3 l'}{2E'}$$

leptonic tensor

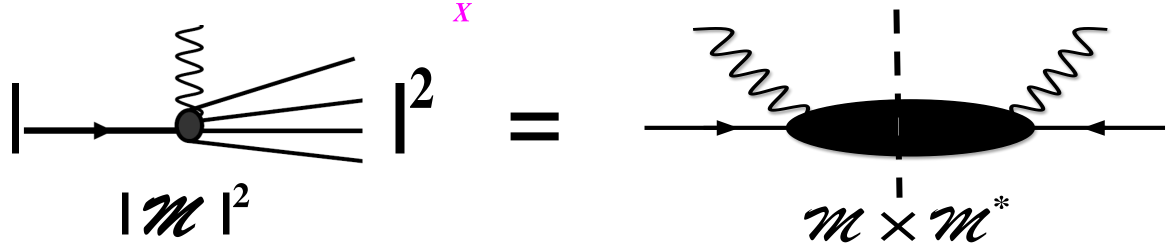
hadronic tensor



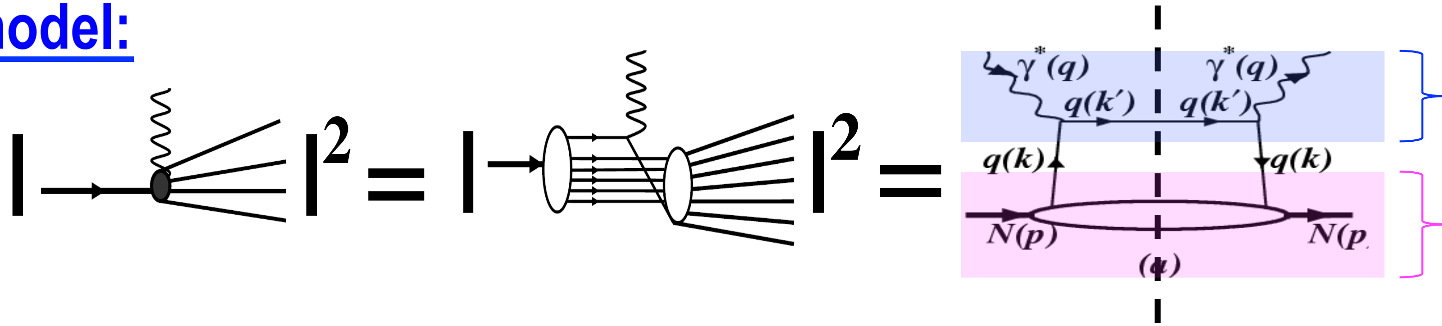
$$x_B = \frac{Q^2}{2q\cdot p}, \qquad y = \frac{q\cdot p}{l\cdot p}$$

$$\mathcal{M}(eN \rightarrow eX) = \langle eX | \hat{S} | eN \rangle \qquad \hat{S} = T e^{i\int d^4x \hat{\mathcal{H}}_I(x)} \qquad \hat{\mathcal{H}}_I(x) = J_\mu(x) A^\mu(x)$$

The hadronic tensor: $W_{\mu\nu}(q,p) = \sum_X \langle p | J_\mu(0) | X \rangle \langle X | J_\nu(0) | p \rangle (2\pi)^4 \delta^4(p+q-p_X)$



Parton model:



The hadronic tensor: $W_{\mu\nu}(q,p)=\sum_X \langle p|J_\mu(0)|X\rangle\langle X|J_\nu(0)|p\rangle(2\pi)^4\delta^4(p+q-p_X)$

We use:

$$J_\mu(x)=e\bar\psi(x)\gamma_\mu\psi(x),$$

$$|X\rangle=|X'\rangle|k'\rangle,$$

$$\psi(x)|X'\rangle|k'\rangle=u(k')e^{-ik'\cdot x}|X'\rangle$$

- $|X\rangle$ is divided into two independent parts $|k'\rangle$ and $|X'\rangle$
- $\psi(x)$ acts only on $|k'\rangle$

We obtain: $W_{\mu\nu}(q,p,S)=\int\frac{d^4k}{(2\pi)^4}Tr\left[\hat H_{\mu\nu}(k,q)\hat\phi(k,p,S)\right]$

the hard part $\hat H_{\mu\nu}(k,q)=\gamma_\mu(\not k+\not q)\gamma_\nu(2\pi)\delta_+\left((k+q)^2\right)$

the matrix element $\hat\phi(k,p,S)=\int d^4z e^{ikz}\langle p,S|\bar\psi(0)\psi(z)|p,S\rangle$

information on the partonic structure of nucleon.

Inclusive DIS and Parton Distribution Functions

$$W_{\mu\nu}(q,p) = \int \frac{d^4k}{(2\pi)^4} Tr\left[\hat{H}_{\mu\nu}(k,q)\hat{\phi}(k,p)\right]$$

$x = k^+ / p^+$

$k^\pm = \frac{1}{\sqrt{2}}(k_0 \pm k_3)$

$n = (0,1,\vec{0}_\perp)$

$\bar{n} = (1,0,\vec{0}_\perp)$

Leading twist: neglecting power suppressed terms $\sim 1/Q$

this is equivalent to take $p \approx p^+\bar{n}, \quad k \approx xp$

$\Rightarrow \hat{\phi}(k,p) = \not{p}\, f(x) + \ldots$

collinear approximation

$\Rightarrow$ The naïve parton model （部分子模型）

$$W_{\mu\nu}(q,p) \approx \left[(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}) + \frac{1}{2xq\cdot p}(q+2xp)_\mu (q+2xp)_\nu\right] f_q(x)$$

Quark distribution function: $f_q(x) = \int \frac{dz^-}{2\pi} e^{ixp^+z^-} \langle p|\bar{\psi}(0)\frac{\gamma^+}{2}\psi(z)|p\rangle$

$$\psi(z) = \sum_k [a(k)u(k)e^{-ikz} + b^\dagger(k)v(k)e^{ikz}]$$

$$f_q(x) = \langle p|a^\dagger(x)a(x) + b^\dagger(x)b(x)|p\rangle$$

no local (color) gauge invariance!

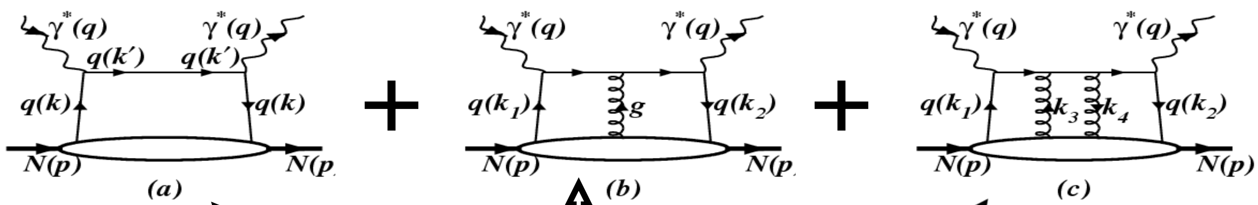
number density of quark and anti-quark.

Inclusive DIS with QCD interaction

To get the gauge invariance, we need the "multiple gluon scattering"

$$\hat{\mathscr{H}}_I(x) = \hat{\mathscr{H}}_I^{QED}(x) + \hat{\mathscr{H}}_I^{QCD}(x)$$

$W_{\mu\nu}(q,p,S) =$



$+ \cdots$

$$W_{\mu\nu}(q,p,S) = W_{\mu\nu}^{(0)}(q,p,S) + W_{\mu\nu}^{(1)}(q,p,S) + W_{\mu\nu}^{(2)}(q,p,S) + \ldots$$

$$W_{\mu\nu}^{(0)}(q,p,S) = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\hat{H}_{\mu\nu}^{(0)}(k,q)\hat{\phi}^{(0)}(k,p,S)\right]$$

$$W_{\mu\nu}^{(1)}(q,p,S) = \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4} \mathrm{Tr}\left[\hat{H}_{\mu\nu}^{(1)\rho}(k_1,k_2,q)\hat{\phi}_\rho^{(1)}(k_1,k_2,p,S)\right]$$

The hard part:

$$\hat{H}_{\mu\nu}^{(0)}(k,q) = \gamma_\mu(\not{k}+\not{q})\gamma_\nu(2\pi)\delta_+\left((k+q)^2\right)$$

$$\hat{H}_{\mu\nu}^{(1,L)\rho}(k_1,k_2,q) = \gamma_\mu \frac{(\not{k}_1+\not{q})\gamma^\rho(\not{k}_2+\not{q})}{(k_2+q)^2 - i\varepsilon}\gamma_\nu(2\pi)\delta_+\left((k_1+q)^2\right)$$

no gauge invariance!

The parton density matrix:

$$\hat{\phi}^{(0)}(k,p) = \int d^4z e^{ikz}\langle p|\bar{\psi}(0)\psi(z)|p\rangle$$

$$\hat{\phi}_\rho^{(1)}(k_1,k_2,p,S) = \int d^4z d^4y e^{ik_1y+ik_2(z-y)}\langle p,S|\bar{\psi}(0)gA_\rho(y)\psi(z)|p,S\rangle$$

Inclusive DIS with QCD interaction

Collinear approximation:

✪ Approximating the hard part at $k = xp$:

$$\hat{H}^{(0)}_{\mu\nu}(k,q) \approx \hat{H}^{(0)}_{\mu\nu}(x) \qquad \hat{H}^{(0)}_{\mu\nu}(x) \equiv \hat{H}^{(0)}_{\mu\nu}(k = xp, q)$$

$$\hat{H}^{(1)\rho}_{\mu\nu}(k_1,k_2,q) \approx \hat{H}^{(1)\rho}_{\mu\nu}(x_1,x_2) \qquad \hat{H}^{(1)\rho}_{\mu\nu}(x_1,x_2) \equiv \hat{H}^{(1)\rho}_{\mu\nu}(k_1 = x_1 p, k_2 = x_2 p, q)$$

✪ Taking only the longitudinal component of the gluon field:

$$A_\rho(y) \approx n \cdot A(y) \frac{p_\rho}{n \cdot p}$$

✪ Using the Ward identities such as,

$$p_\rho \hat{H}^{(1,L)\rho}_{\mu\nu}(x_1,x_2) = \frac{\hat{H}^{(0)}_{\mu\nu}(x_1)}{x_2 - x_1 - i\varepsilon}$$

so that all the hard parts reduce the same $\hat{H}^{(0)}_{\mu\nu}(x)$.

✪ Adding all the terms together ⟹

$$x = k^+ / p^+$$

$$k^\pm = \frac{1}{\sqrt{2}}(k_0 \pm k_3)$$

$$n = (0,1,\vec{0}_\perp)$$

$$\bar{n} = (1,0,\vec{0}_\perp)$$

Inclusive DIS with QCD interaction

$$W_{\mu\nu}(q,p,S) \approx \tilde{W}^{(0)}_{\mu\nu}(q,p,S)$$

$$\tilde{W}^{(0)}_{\mu\nu}(q,p,S) = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\hat{\Phi}^{(0)}(k,p,S)\hat{H}^{(0)}_{\mu\nu}(x)\right]$$

only leading twist contribution can be derived from it.

$$\hat{\Phi}^{(0)}(k,p,S) = \int d^4z e^{ikz} \langle p,S|\bar{\psi}(0)\mathcal{L}(0,z)\psi(z)|p,S\rangle$$

$$\text{C.f.:}\quad \hat{\phi}(k,p,S) = \int d^4z e^{ikz} \langle p,S|\bar{\psi}(0)\psi(z)|p,S\rangle$$

Un-integrated parton distribution/correlation functions:
Contain QCD interactions, gauge invariant !

gauge link

$$\mathcal{L}(0,z) = \mathcal{L}^{\dagger}(-\infty,0)\mathcal{L}(-\infty,z)$$

$$\mathcal{L}(-\infty,z) = Pe^{ig\int_{-\infty}^{z^-} dy^- A^+(0,y^-,\vec{z}_\perp)}$$

$$= 1 + ig\int_0^{z^-} dy^- A^+(0,y^-,\vec{0}_\perp) + (ig)^2\int_0^{z^-} dy^- A^+(0,y^-,\vec{0}_\perp)\int_0^{y^-} dy'^- A^+(0,y'^-,\vec{0}_\perp) + \ldots$$

Inclusive DIS with QCD interaction

Collinear expansion:

Ellis, Furmanski, Petronzio, (1982)
Qiu, Sterman (1990,1991)

✪ Expanding the hard part at $k = xp$:

$$\hat{H}^{(0)}_{\mu\nu}(k,q) = \hat{H}^{(0)}_{\mu\nu}(x) + \frac{\partial \hat{H}^{(0)}_{\mu\nu}(x)}{\partial k^\rho} \omega_\rho{}^{\rho'} k_{\rho'} + \ldots\ldots$$

$$\hat{H}^{(1)\rho}_{\mu\nu}(k_1,k_2,q) = \hat{H}^{(1)\rho}_{\mu\nu}(x_1,x_2) + \frac{\partial \hat{H}^{(1)\rho}_{\mu\nu}(x_1,x_2)}{\partial k_1^\sigma} \omega_\sigma{}^{\sigma'} k_{1\sigma'} + \ldots\ldots$$

$$\hat{H}^{(0)}_{\mu\nu}(x) \equiv \hat{H}^{(0)}_{\mu\nu}(k = xp, q)$$

$$\frac{\partial \hat{H}^{(0)}_{\mu\nu}(x)}{\partial k^\rho} \equiv \left.\frac{\partial \hat{H}^{(0)}_{\mu\nu}(k,q)}{\partial k^\rho}\right|_{k=xp}$$

$$x = k^+ / p^+$$

$$\omega_\rho{}^{\rho'} \equiv g_\rho{}^{\rho'} - \bar{n}_\rho n^{\rho'}$$

$$\omega_\rho{}^{\rho'} k_{\rho'} = (k - xp)_\rho$$

$$k^\pm = \frac{1}{\sqrt{2}}(k_0 \pm k_3)$$

$$n = (0,1,\vec{0}_\perp)$$

$$\bar{n} = (1,0,\vec{0}_\perp)$$

✪ Decomposition of the gluon field:

$$A_\rho(y) = n \cdot A(y) \frac{p_\rho}{n \cdot p} + \omega_\rho{}^{\rho'} A_{\rho'}(y)$$

✪ Using the Ward identities such as,

$$\frac{\partial \hat{H}^{(0)}_{\mu\nu}(x)}{\partial k^\rho} = -\hat{H}^{(1)\rho}_{\mu\nu}(x,x), \qquad p_\rho \hat{H}^{(1,L)\rho}_{\mu\nu}(x_1,x_2) = \frac{\hat{H}^{(0)}_{\mu\nu}(x_1)}{x_2 - x_1 - i\varepsilon}$$

to replace the derivatives etc.

✪ Adding all the terms with the same hard part together ⟹

Inclusive DIS with QCD interaction

$$W_{\mu\nu}(q,p,S) = \tilde{W}^{(0)}_{\mu\nu}(q,p,S) + \tilde{W}^{(1)}_{\mu\nu}(q,p,S) + \tilde{W}^{(2)}_{\mu\nu}(q,p,S) + \ldots$$

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twist-2, twist-3 and twist-4 contributions

$$\tilde{W}^{(0)}_{\mu\nu}(q,p,S) = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\hat{\Phi}^{(0)}(k,p,S)\hat{H}^{(0)}_{\mu\nu}(x)\right]$$

$$\hat{\Phi}^{(0)}(k,p,S) = \int d^4z e^{ikz} \langle p,S|\bar{\psi}(0)\mathcal{L}(0,z)\psi(z)|p,S\rangle$$

---

twist-3, twist-4 and even higher twist contributions

$$\tilde{W}^{(1)}_{\mu\nu}(q,p,S) = \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4} \mathrm{Tr}\left[\hat{\Phi}^{(1)}_{\rho'}(k_1,k_2,p,S)\hat{H}^{(1)\rho}_{\mu\nu}(x_1,x_2)\omega_\rho^{\ \rho'}\right]$$

$$\hat{\Phi}^{(1)}_{\rho}(k_1,k_2,p,S) = \int d^4z d^4y e^{ik_1y+ik_2(z-y)} \langle p,S|\bar{\psi}(0)\mathcal{L}(0,y)D_\rho(y)\mathcal{L}(y,z)\psi(z)|p,S\rangle$$

$$D_\rho(y) = -i\partial_\rho + gA_\rho(y)$$

Un-integrated parton distribution/correlation functions:
Contain QCD interactions, gauge invariant !

gauge link

A consistent framework for inclusive DIS $e + N \rightarrow e + X$
including higher twist contributions.

Inclusive DIS with QCD interaction

$$W_{\mu\nu}(q,p,S)=\tilde{W}^{(0)}_{\mu\nu}(q,p,S)+\tilde{W}^{(1)}_{\mu\nu}(q,p,S)+\tilde{W}^{(2)}_{\mu\nu}(q,p,S)+\dots$$

twist-2, twist-3 and twist-4 contributions

$$\tilde{W}^{(0)}_{\mu\nu}(q,p,S)=\int\frac{p^+dx}{2\pi}\mathrm{Tr}\left[\hat{\Phi}^{(0)}(x;p,S)\hat{H}^{(0)}_{\mu\nu}(x)\right]$$

$$\hat{\Phi}^{(0)}(x;p,S)=\int\frac{d^4k}{(2\pi)^3}\delta(k^+-xp^+)\hat{\Phi}^{(0)}(k;p,S)=\int dz^-e^{ixp^+z^-}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,z^-)\psi(z^-)|p,S\rangle$$

twist-3, twist-4 and even higher twist contributions

$$\tilde{W}^{(1)}_{\mu\nu}(q,p,S)=\int\frac{p^+dx_1}{2\pi}\frac{p^+dx_2}{2\pi}\mathrm{Tr}\left[\hat{\Phi}^{(1)}_{\rho'}(x_1,x_2;p,S)\hat{H}^{(1)\rho}_{\mu\nu}(x_1,x_2)\omega_\rho^{\ \rho'}\right]$$

$$\hat{\Phi}^{(1)}_{\rho}(x_1,x_2;p,S)=\int dz^-dy^-e^{ix_1p^+y^-+ix_2p^+(z^--y^-)}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,y^-)D_\rho(y^-)\mathcal{L}(y^-,z^-)\psi(z^-)|p,S\rangle$$

$$D_\rho(y)=-i\partial_\rho+gA_\rho(y^-)$$

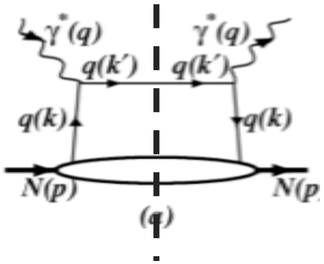
Integrated parton distribution/correlation functions:
Contain QCD interactions, gauge invariant !

gauge link

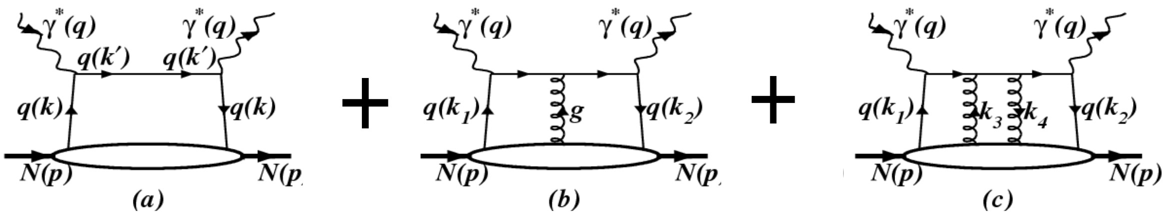
A consistent framework for inclusive DIS $e+N\rightarrow e+X$ including higher twist contributions.

Inclusive DIS with QCD interaction

Lesson I: Quark distribution is not merely



but



…

i.e., it contains "intrinsic motion" and "multiple gluon scattering".

Intrinsic transverse momentum and multiple gluon scattering always mix up with each other to give us the finally observed effects.

Lesson II: "Multiple gluon scattering" is contained in the gauge link.

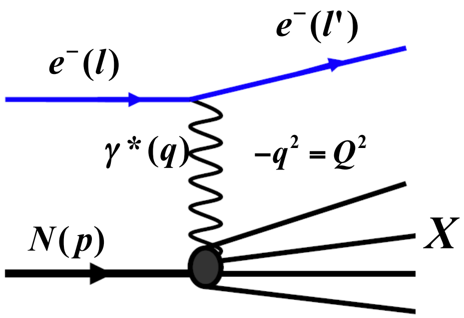
Collinear expansion is the necessary procedure to establish the relationship between the differential cross section and the gauge invariant parton distributions.

Collinear expansion leads to a formalism that can be used to study leading as well as higher twist contributions order by order in a systematic way.

From inclusive to semi-inclusive DIS: the TMDs

From inclusive to semi-inclusive DIS: Much more can be studied !

In inclusive DIS $e^- N \to e^- X$



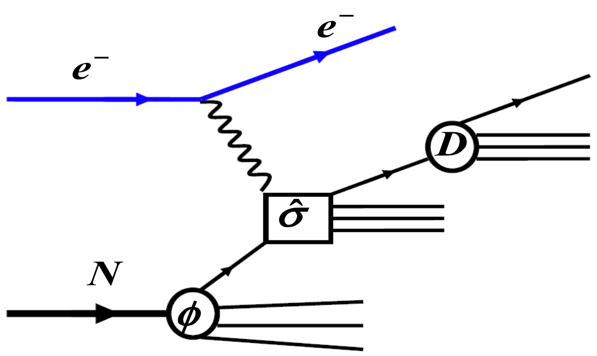
only longitudinal distributions such as,

$$f_q(x) = \int \frac{dz^-}{2\pi} e^{ixp^+z^-} \langle p | \bar{\psi}(0) \frac{\gamma^+}{2} \mathcal{L}(0,z) \psi(z) | p \rangle$$

$$\Delta f_q(x) = \int \frac{dz^-}{2\pi} e^{ixp^+z^-} \langle p | \bar{\psi}(0) \frac{\gamma_5\gamma^+}{2} \mathcal{L}(0,z) \psi(z) | p \rangle$$

are studied.

In semi-inclusive DIS $e^- N \to e^- h X$



both longitudinal & transverse, i.e. the TMDs such as,

$$f_q(x,k_\perp) = \int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{ixp^+z^- - i\vec{k}_\perp \cdot \vec{z}_\perp} \langle N | \bar{\psi}(0) \frac{\gamma^+}{2} \mathcal{L}(0,z) \psi(z) | N \rangle$$

$$\Delta f_q(x,k_\perp) = \int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{ixp^+z^- - i\vec{k}_\perp \cdot \vec{z}_\perp} \langle N | \bar{\psi}(0) \frac{\gamma_5\gamma^+}{2} \mathcal{L}(0,z) \psi(z) | N \rangle$$

and many others can be studied.

Semi-Inclusive DIS with QCD interaction

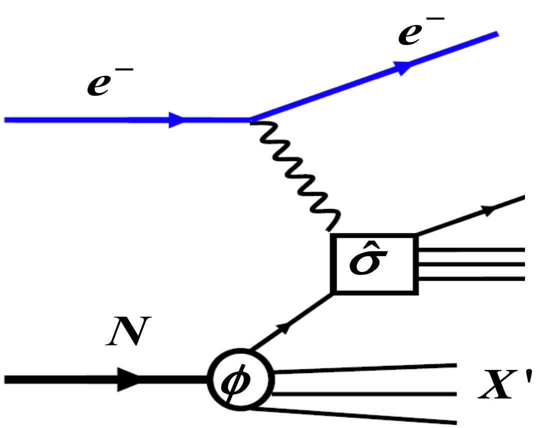
The first thing to do:

hadron structure
(the TMDs ...)

the measurable quantities
(the differential cross section,
the azimuthal asymmetries, ...)

collinear expansion in SIDIS?

YES for e+N ➔ e + q (jet) + X !



no fragmentation, simple and clear,
collinear expansion can be applied!

ZTL & X.N. Wang, PRD (2007);
J.H. Gao, ZTL, X.N. Wang, PRC (2010);
Y.K. Song, J.H. Gao, ZTL, X.N. Wang, PRD (2011);
Y.K. Song, J.H. Gao, ZTL, X.N. Wang, PRD (2014).

Semi-Inclusive DIS with QCD interaction

For semi-inclusive DIS $e+N \to e+q+X$

$$W_{\mu\nu}^{(si)}(q,p,S,k')=\tilde{W}_{\mu\nu}^{(0,si)}(q,p,S,k')+\tilde{W}_{\mu\nu}^{(1,si)}(q,p,S,k')+\tilde{W}_{\mu\nu}^{(2,si)}(q,p,S,k')+...$$

twist-2, twist-3 and twist-4 contributions

$$\tilde{W}_{\mu\nu}^{(0,si)}(q,p,S,k')=\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}[\hat{\Phi}^{(0)}(k,p,S)\hat{H}_{\mu\nu}^{(0)}(x)]\ (2E_{k'})(2\pi)^3\delta^3(\vec{k}'-\vec{k}-\vec{q})$$

$$\hat{\Phi}^{(0)}(k,p,S)=\int d^4z e^{ikz}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,z)\psi(z)|p,S\rangle$$

twist-3, twist-4 and even higher twist contributions

$$\tilde{W}_{\mu\nu}^{(1,si)}(q,p,S,k')=\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\sum_{c=L,R}\mathrm{Tr}[\hat{\Phi}_{\rho'}^{(1)}(k_1,k_2,p,S)\ \hat{H}_{\mu\nu}^{(1,c)\rho}(x_1,x_2)\ \omega_\rho^{\ \rho'}]\ (2E_{k'})(2\pi)^3\delta^3(\vec{k}'-\vec{k}_c-\vec{q})$$

$$\hat{\Phi}_\rho^{(1)}(k_1,k_2,p,S)=\int d^4z d^4y e^{ik_1y+ik_2(z-y)}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,y)D_\rho(y)\mathcal{L}(y,z)\psi(z)|p,S\rangle$$

→ A consistent framework for semi-inclusive DIS $e+N \to e+q+X$ including higher twist contributions.

Semi-Inclusive DIS with QCD interaction

For semi-inclusive DIS $e+N\to e+q+X$

$$W_{\mu\nu}^{(si)}(q,p,S,k')=\tilde{W}_{\mu\nu}^{(0,si)}(q,p,S,k')+\tilde{W}_{\mu\nu}^{(1,si)}(q,p,S,k')+\tilde{W}_{\mu\nu}^{(2,si)}(q,p,S,k')+\ldots$$

twist-2, twist-3 and twist-4 contributions

$$\tilde{W}_{\mu\nu}^{(0,si)}(q,p,S,k'_\perp)=\int\frac{p^+dx}{2\pi}\frac{d^2k_\perp}{(2\pi)^2}\mathrm{Tr}[\hat{\Phi}^{(0)}(x,k_\perp;p,S)\hat{H}_{\mu\nu}^{(0)}(x)]\,(2\pi)^2\delta^2(\vec{k}'_\perp-\vec{k}_\perp)$$

$$\hat{\Phi}^{(0)}(x,k_\perp;p,S)=\int dz^-d^2z_\perp e^{ixp^+z^--i\vec{k}_\perp\cdot\vec{z}_\perp}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,z)\psi(z)|p,S\rangle$$

twist-3, twist-4 and even higher twist contributions

$$\tilde{W}_{\mu\nu}^{(1,si)}(q,p,S,k'_\perp)=\int\frac{p^+dx_1d^2k_{\perp1}}{(2\pi)^3}\frac{p^+dx_2d^2k_{\perp2}}{(2\pi)^3}\sum_{c=L,R}\mathrm{Tr}[\hat{\Phi}_{\rho'}^{(1)}(x_1,k_{\perp1},x_2,k_{\perp2};p,S)\,\hat{H}_{\mu\nu}^{(1,c)\rho}(x_1,x_2)\,\omega_\rho^{\ \rho'}]\,(2\pi)^2\delta^2(\vec{k}'_\perp-\vec{k}_{c\perp})$$

$$\hat{\Phi}_\rho^{(1)}(x_1,k_{\perp1},x_2,k_{\perp2};p,S)=\int dz^-d^2z_\perp dy^-d^2y_\perp e^{ik_1y+ik_2(z-y)}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,y)D_\rho(y)\mathcal{L}(y,z)\psi(z)|p,S\rangle$$

➡ A consistent framework for semi-inclusive DIS $e+N\to e+q+X$ including higher twist contributions.

SIDIS $e+p \rightarrow e+q+X$: differential cross-section to 1/Q

A complete twist-3 result for polarized $e(\lambda_l)+N(\lambda,S_\perp)\rightarrow e+q+X$

$$\frac{d\sigma}{dxdyd^2k_\perp}=\frac{2\pi\alpha_{em}^2}{Q^2y}(W_{UU}+\lambda_l W_{LU}+S_\perp W_{UT}+\lambda_l W_{LU}+\lambda_l\lambda W_{LL}+\lambda_l S_\perp W_{LT})$$

$$W_{UU}(x,k_\perp,\phi)=A(y)f_q(x,k_\perp)-\frac{2x|\vec{k}_\perp|}{Q}B(y)f_q^\perp(x,k_\perp)\cos\phi$$

$$W_{UT}(x,k_\perp,\phi,\phi_s)=\frac{|\vec{k}_\perp|}{M}A(y)f_{1T}^\perp(x,k_\perp)\sin(\phi-\phi_s)+\frac{2xM}{Q}B(y)\left\{\frac{k_\perp^2}{2M^2}f_T^\perp(x,k_\perp)\sin(2\phi-\phi_s)+f_T(x,k_\perp)\sin\phi_s\right\}$$

$$W_{LU}(x,k_\perp,\phi)=-\frac{2x|\vec{k}_\perp|}{Q}D(y)g^\perp(x,k_\perp)\sin\phi$$

$$W_{UL}(x,k_\perp,\phi)=-\frac{2x|\vec{k}_\perp|}{Q}B(y)f_L^\perp(x,k_\perp)\sin\phi$$

$$W_{LL}(x,k_\perp,\phi)=C(y)g_{1L}(x,k_\perp)-\frac{2x|\vec{k}_\perp|}{Q}D(y)g_L^\perp(x,k_\perp)\cos\phi$$

$$W_{LT}(x,k_\perp,\phi,\phi_s)=\frac{|\vec{k}_\perp|}{M}C(y)g_{1T}^\perp(x,k_\perp)\cos(\phi-\phi_s)-\frac{2xM}{Q}D(y)\left[g_T(x,k_\perp)\cos\phi_s-\frac{k_\perp^2}{2M^2}g_T^\perp(x,k_\perp)\cos(2\phi-\phi_s)\right]$$

$$A(y)=1+(1-y)^2$$
$$B(y)=2(2-y)\sqrt{1-y}$$
$$C(y)=y(2-y)$$
$$D(y)=2y\sqrt{1-y}$$

Y.K. Song, J.H. Gao, ZTL, X.N. Wang, PRD (2014).

SIDIS $e + p \rightarrow e + q + X$: differential cross-section to $1/Q^2$

A complete twist-4 result for un-polarized $e + N \rightarrow e + q + X$

$$\frac{d\sigma}{dxdyd^2k_\perp} = \frac{2\pi\alpha_{em}^2}{Q^2 y}\Bigg\{A(y) f_q(x,k_\perp) - B(y)\frac{|\vec{k}_\perp|}{Q} x f_q^\perp(x,k_\perp)\cos\phi +$$

$$-4(1-y)\frac{|\vec{k}_\perp|^2}{Q^2} x\left[\varphi_{\perp 2}^{(1)}(x,k_\perp) - \tilde{\varphi}_{\perp 2}^{(1)}(x,k_\perp)\right]\cos 2\phi$$

$$+8(1-y)\left(\frac{|\vec{k}_\perp|^2}{Q^2} x\left[\varphi_{\perp 2}^{(1)}(x,k_\perp) - \tilde{\varphi}_{\perp 2}^{(1)}(x,k_\perp)\right] + \frac{2x^2 M^2}{Q^2} f_{q(-)}(x,k_\perp)\right)$$

$$-2\left[1+(1-y)^2\right]\frac{|\vec{k}_\perp|^2}{Q^2} x\varphi_{\perp 2}^{(2,L)}(x,k_\perp)\Bigg\}$$

Y.K. Song, J.H. Gao, ZTL, X.N. Wang, PRD (2011).

SIDIS $e + p \rightarrow e + q + X$: the involved TMDs up to twist-3

There are 12 TMDs involved here up to twist-3, and they are defined via

$$\hat{\Phi}^{(0)}(x,k_\perp;p,S) = \int dz^- d^2z_\perp e^{ixp^+z^- - i\vec{k}_\perp\cdot\vec{z}_\perp} \langle N | \bar{\psi}(0)\mathcal{L}(0,z)\psi(z) | N \rangle$$

$$\hat{\Phi}^{(0)} = (\gamma^\alpha \Phi_\alpha^{(0)} + \gamma_5\gamma^\alpha \tilde{\Phi}_\alpha^{(0)})/2 + ...$$

$$\Phi_\alpha^{(0)} = p_\alpha(f_1 - \frac{\varepsilon_\perp^{kS}}{M} f_{1T}^\perp) + k_{\perp\alpha} f^\perp - M\varepsilon_{\perp\alpha S_\perp} f_T - \frac{1}{M}(k_{\perp\alpha}k_\perp^\beta - \frac{1}{2}k_\perp^2 d_\alpha{}^\beta)\varepsilon_{\perp\beta S_\perp} f_T^\perp - \lambda\varepsilon_{\perp\alpha k} f_L^\perp + ...$$

$$\tilde{\Phi}_\alpha^{(0)} = -p_\alpha(\lambda g_{1L} - \frac{k_\perp\cdot S_\perp}{M} g_{1T}^\perp) + \varepsilon_{\perp\alpha k} g^\perp - MS_{\perp\alpha} g_T + \frac{1}{M}(k_{\perp\alpha}k_\perp^\beta - \frac{1}{2}k_\perp^2 d_\alpha{}^\beta)S_{\perp\beta} g_T^\perp - \lambda k_{\perp\alpha} g_L^\perp + ...$$

Twist-2: f_1, $f_{1T}^\perp$; g_{1L}, $g_{1T}^\perp$ (4)

Twist-3: $f^\perp$, f_T, $f_T^\perp$, $f_L^\perp$; $g^\perp$, g_T, $g_T^\perp$, $g_L^\perp$ (8)

Spin independent: f_1, $f^\perp$; $g^\perp$ $(3 = 1 + 2)$

Longitudinal spin independent: $f_L^\perp$; g_{1L}, $g_L^\perp$ $(3 = 1 + 2)$

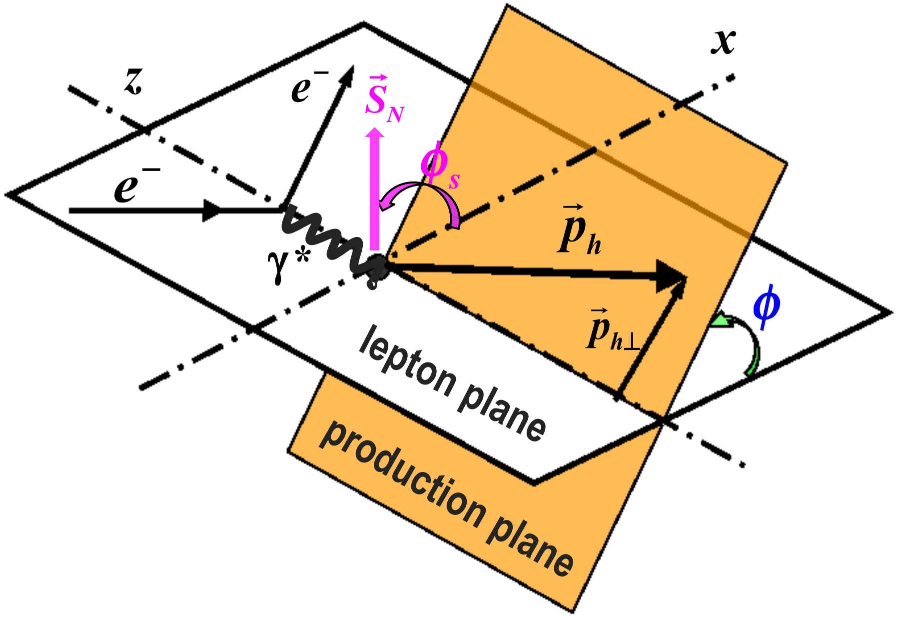
Transverse spin independent: $f_{1T}^\perp$, f_T, $f_T^\perp$; $g_{1T}^\perp$, g_T, $g_T^\perp$ $(6 = 2 + 4)$

SIDIS $e + p \rightarrow e + q + X$: Azimuthal asymmetries

<u>At the leading twist (twist-2):</u>

$$\langle \sin(\phi - \phi_s) \rangle_{UT} = S_\perp \frac{|\vec{k}_\perp|}{2M} \frac{f_{1T}^\perp(x,k_\perp)}{f_q(x,k_\perp)}$$

$$\langle \cos(\phi - \phi_s) \rangle_{LT} = \lambda_l S_\perp \frac{|\vec{k}_\perp|}{2M} \frac{C(y)}{A(y)} \frac{g_{1T}^\perp(x,k_\perp)}{f_q(x,k_\perp)}$$



Take g=0, i.e., take away the effects of "multiple gluon scattering", we have,

$$f_{1T}^\perp(x,k_\perp)\Big|_{g=0} = 0 \qquad \langle \sin(\phi - \phi_s) \rangle_{UT}\Big|_{g=0} = 0$$

$$g_{1T}^\perp(x,k_\perp)\Big|_{g=0} = 0 \qquad \langle \cos(\phi - \phi_s) \rangle_{LT}\Big|_{g=0} = 0$$

SIDIS $e + p \rightarrow e + q + X$: Azimuthal asymmetries

Twist-3:

$$\langle \sin\phi \rangle_{LU} = \lambda_l \frac{|\vec{k}_\perp|}{Q} \frac{D(y)}{A(y)} \frac{x g_\perp(x,k_\perp)}{f_q(x,k_\perp)} \xrightarrow{g=0} 0$$

$$\langle \sin(2\phi - \phi_s) \rangle_{UT} = S_\perp \frac{k_\perp^2}{MQ} \frac{B(y)}{A(y)} \frac{x f_T^\perp(x,k_\perp)}{f_q(x,k_\perp)} \xrightarrow{g=0} 0$$

$$\langle \sin\phi_s \rangle_{UT} = S_\perp \frac{2xM}{Q} \frac{B(y)}{A(y)} \frac{f_T(x,k_\perp) - \frac{k_\perp^2}{2M^2} f_T^\perp(x,k_\perp)}{f_q(x,k_\perp)} \xrightarrow{g=0} 0$$

$$\langle \cos\phi \rangle_{UU} = -\frac{|\vec{k}_\perp|}{Q} \frac{B(y)}{A(y)} \frac{x f^\perp(x,k_\perp)}{f_1(x,k_\perp)} \xrightarrow{g=0} -\frac{B(y)}{A(y)} \frac{|\vec{k}_\perp|}{Q}$$

$$\langle \cos\phi \rangle_{LL} = -\frac{|\vec{k}_\perp|}{Q} \frac{B(y) x f^\perp(x,k_\perp) + \lambda_l \lambda D(y) g_L^\perp(x,k_\perp)}{A(y) f_1(x,k_\perp) + \lambda_l \lambda C(y) g_{1L}(x,k_\perp)} \xrightarrow{g=0} -\frac{|\vec{k}_\perp|}{Q} \frac{B(y) f_1(x,k_\perp) + \lambda_l \lambda D(y) g_{1L}(x,k_\perp)}{A(y) f_1(x,k_\perp) + \lambda_l \lambda C(y) g_{1L}(x,k_\perp)}$$

Twist-4:

$$\langle \cos 2\phi \rangle_{UU} = -\frac{|\vec{k}_\perp|^2}{Q^2} \frac{C(y)}{A(y)} \frac{x\left[\varphi_{\perp 2}(x,k_\perp) - \tilde{\varphi}_{2\perp}(x,k_\perp)\right]}{f_q(x,k_\perp)} \xrightarrow{g=0} -\frac{|\vec{k}_\perp|^2}{Q^2} \frac{C(y)}{A(y)}$$

SIDIS $e + A \rightarrow e + q + X$: Nuclear dependence

Replace N by A, i.e., consider $e + A \rightarrow e + q + X$, we obtain
the same results as long as we replace the TMDs in N by the corresponding TMDs in A.

E.g.:

$$\langle \sin(\phi - \phi_s) \rangle_{UT}^{eA} = S_{\perp A} \frac{|\vec{k}_\perp|}{2M} \frac{f_{1T}^{\perp A}(x,k_\perp)}{f_q^A(x,k_\perp)} \qquad \langle \cos(\phi - \phi_s) \rangle_{LT}^{eA} = \lambda_l S_{\perp A} \frac{|\vec{k}_\perp|}{2M} \frac{C(y)}{A(y)} \frac{g_{1T}^{\perp A}(x,k_\perp)}{f_q^A(x,k_\perp)}$$

$$\langle \cos\phi \rangle_{UU}^{eA} = -\frac{|\vec{k}_\perp|}{Q} \frac{B(y)}{A(y)} \frac{x f_q^{\perp A}(x,k_\perp)}{f_q^A(x,k_\perp)} \qquad \langle \sin\phi \rangle_{LU}^{eA} = \lambda_l \frac{|\vec{k}_\perp|}{Q} \frac{D(y)}{A(y)} \frac{x g_\perp^A(x,k_\perp)}{f_q^A(x,k_\perp)}$$

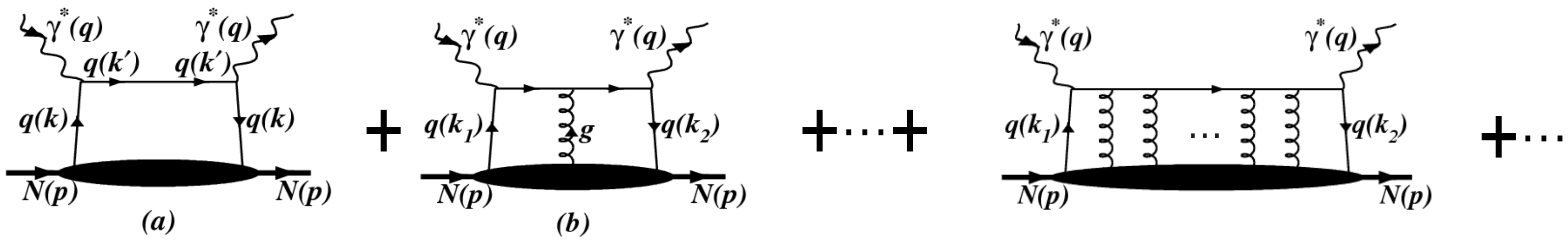
$$\langle \sin(2\phi - \phi_s) \rangle_{UT}^{eA} = S_{\perp A} \frac{k_\perp^2}{MQ} \frac{B(y)}{A(y)} \frac{x f_T^{\perp A}(x,k_\perp)}{f_q^A(x,k_\perp)}$$

$$f_q^A(x,k_\perp) = \int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{ixp^+z^- - i\vec{k}_\perp \cdot \vec{z}_\perp} \langle A | \bar{\psi}(0) \frac{\gamma^+}{2} \mathcal{L}(0,z) \psi(z) | A \rangle$$

Only vector polarization part of A is considered: $S_A = (0, \vec{S}_A)$ in the rest frame of A.

SIDIS $e + A \rightarrow e + q + X$: Nuclear dependence of the TMDs

Gauge link comes from:



Replace N by A, the gluons can connect to different nucleons in A.

Nuclear enhancement

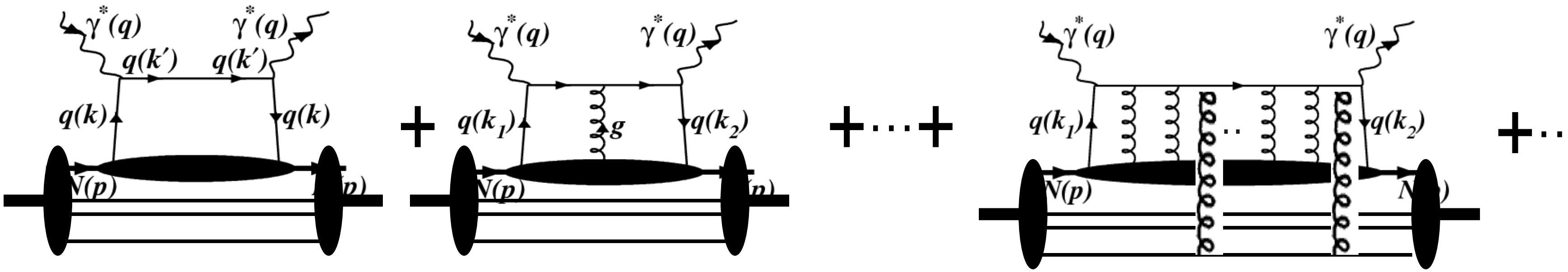
Transverse momentum broadening

Transverse momentum broadening should be contained in the gauge link

ZTL, X.N. Wang & J. Zhou, PRD (2008);
J.H. Gao, ZTL, X.N. Wang, PRC (2010);
Y.K. Song, J.H. Gao, ZTL, X.N. Wang, PRD (2011);
Y.K. Song, J.H. Gao, ZTL, X.N. Wang, PRD (2014).
Y.K. Song, ZTL, X.N. Wang, PRD (2014).

SIDIS $e + A \rightarrow e + q + X$: Nuclear dependence of the TMDs

Gauge link comes from:



Replace N by A, the gluons can connect to different nucleons in A.

⟹ Nuclear enhancement
Transverse momentum broadening

Transverse momentum broadening should be contained in the gauge link

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Y.K. Song, J.H. Gao, ZTL, X.N. Wang, PRD (2014).
Y.K. Song, ZTL, X.N. Wang, PRD (2014).

Transverse momentum broadening in nucleus

We take the "maximal two gluon approximation":

either zero or two gluons are connected to one spectator nucleon.

$$\Longrightarrow \quad f_q^A(x,k_\perp) = \frac{A}{\pi\,\Delta_{2F}} \int d^2 l_\perp e^{-(\vec{k}_\perp - \vec{l}_\perp)^2/\Delta_{2F}} f_q^N(x,l_\perp)$$

$$\Delta_{2F} = \int d\xi_N^- \hat{q}_F(\xi_N)$$ transverse momentum broadening squared

$$\hat{q}_F(\xi_N) = \frac{2\pi^2\alpha_s}{N_c} \rho_N^A(\xi_N)[x f_g^N(x)]|_{x=0}$$ quark transport parameter in *A*

gluon distribution function in nucleon *N*

number density of nucleon in nucleus *A*

A direct consequences of the "multiple gluon scattering" in the gauge link!

Nuclear dependence of the TMDs: unpolarized

Un-polarized *A*:

Under the "maximal two gluon approximation":

$$f_q^A(x,k_\perp) \approx \frac{A}{\pi\,\Delta_{2F}} \int d^2l_\perp e^{-(\vec{k}_\perp - \vec{l}_\perp)^2/\Delta_{2F}} f_q^N(x,l_\perp)$$

$$k_\perp^2 f^{\perp A}(x,k_\perp) \approx \frac{A}{\pi\,\Delta_{2F}} \int d^2l_\perp e^{-(\vec{k}_\perp - \vec{l}_\perp)^2/\Delta_{2F}} (k_\perp \cdot l_\perp) f^{\perp N}(x,l_\perp)$$

$$k_\perp^2 g^{\perp A}(x,k_\perp) \approx \frac{A}{\pi\,\Delta_{2F}} \int d^2l_\perp e^{-(\vec{k}_\perp - \vec{l}_\perp)^2/\Delta_{2F}} (k_\perp \cdot l_\perp) g^{\perp N}(x,l_\perp)$$

Nuclear dependence of the TMDs: unpolarized

An example: take a Gaussian for the transverse momentum dependence

$$f_q^N(x,k_\perp) = \frac{1}{\pi\alpha} f_q^N(x)\, e^{-\vec{k}_\perp^2/\alpha},$$

$$f^{\perp N}(x,k_\perp) = \frac{1}{\pi\beta} f^{\perp N}(x)\, e^{-\vec{k}_\perp^2/\beta}, \qquad g^{\perp N}(x,k_\perp) = \frac{1}{\pi\tilde{\beta}} g^{\perp N}(x)\, e^{-\vec{k}_\perp^2/\tilde{\beta}}$$

$\Rightarrow$

$$f_q^A(x,k_\perp) \approx \frac{A}{\pi(\alpha+\Delta_{2F})} f_q^N(x)\, e^{-\vec{k}_\perp^2/(\alpha+\Delta_{2F})}$$

twist-2 distribution

$$f^{\perp A}(x,k_\perp) \approx \frac{A}{\pi(\beta+\Delta_{2F})}\left(\frac{\beta}{\beta+\Delta_{2F}}\right) f^{\perp N}(x)\, e^{-\vec{k}_\perp^2/(\beta+\Delta_{2F})}$$

$$g^{\perp A}(x,k_\perp) \approx \frac{A}{\pi(\tilde{\beta}+\Delta_{2F})}\left(\frac{\tilde{\beta}}{\tilde{\beta}+\Delta_{2F}}\right) g^{\perp N}(x)\, e^{-\vec{k}_\perp^2/(\tilde{\beta}+\Delta_{2F})}$$

twist-3 TMDs, suppression factor!

Nuclear dependence of the azimuthal asymmetries: unpolarizded

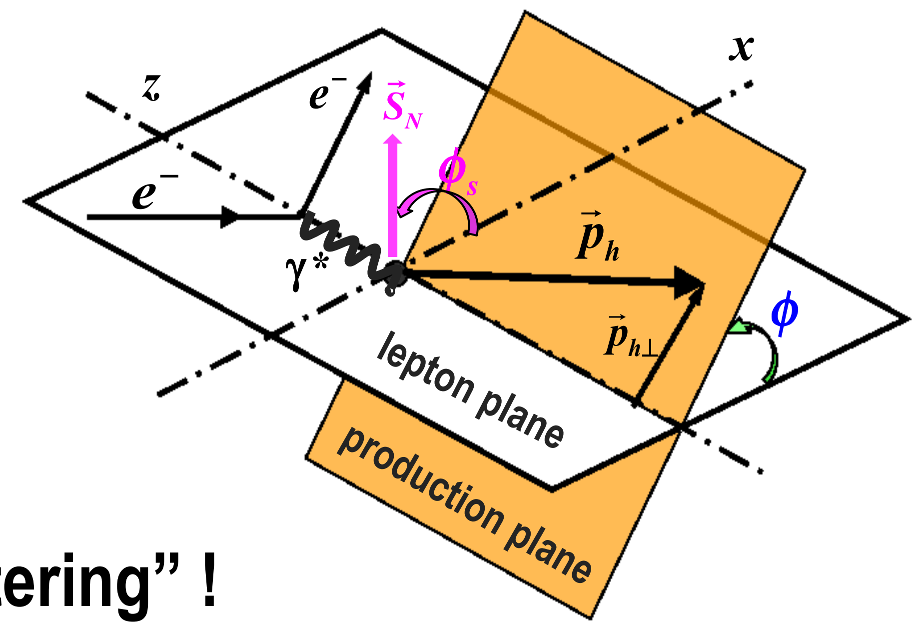
Under the Gaussian ansatz: for the case that $\alpha = \beta = \gamma$

$$\frac{\langle \cos\phi \rangle_{UU}^{eA}}{\langle \cos\phi \rangle_{UU}^{eN}} \approx \frac{\alpha}{\alpha + \Delta_{2F}}$$

suppressed!

$$\frac{\langle \sin\phi \rangle_{LU}^{eA}}{\langle \sin\phi \rangle_{LU}^{eN}} \approx \frac{\alpha}{\alpha + \Delta_{2F}}$$

$$\frac{\langle \cos 2\phi \rangle_{UU}^{eA}}{\langle \cos 2\phi \rangle_{UU}^{eN}} \approx \left(\frac{\alpha}{\alpha + \Delta_{2F}} \right)^2$$



Another place to study the effects of "multiple gluon scattering" !

Nuclear dependence of the TMDs: longitudinally polarized

In the case of a polarized nucleus *A*:

Take the approximation: each nucleon has an average polarization of $2J_A/A$.

Under the "maximal two gluon approximation":

For the longitudinally polarized nuclei:

$$g_{1L}^{A}(x,k_\perp) \approx \frac{2J_A}{\pi\,\Delta_{2F}} \int d^2 l_\perp e^{-(\vec{k}_\perp-\vec{l}_\perp)^2/\Delta_{2F}} g_{1L}^{N}(x,l_\perp)$$

$$k_\perp^2 f_L^{\perp A}(x,k_\perp) \approx \frac{2J_A}{\pi\,\Delta_{2F}} \int d^2 l_\perp e^{-(\vec{k}_\perp-\vec{l}_\perp)^2/\Delta_{2F}} (k_\perp \cdot l_\perp) f_L^{\perp N}(x,l_\perp)$$

$$k_\perp^2 g_L^{\perp A}(x,k_\perp) \approx \frac{2J_A}{\pi\,\Delta_{2F}} \int d^2 l_\perp e^{-(\vec{k}_\perp-\vec{l}_\perp)^2/\Delta_{2F}} (k_\perp \cdot l_\perp) g_L^{\perp N}(x,l_\perp)$$

Nuclear dependence of the TMDs: longitudinally polarized

With the Gaussian ansatz

$$g_{1L}^{N}(x,k_\perp) = \frac{1}{\pi\alpha_L} g_{1L}^{N}(x) e^{-\vec{k}_\perp^2/\alpha_L},$$

$$f_L^{\perp N}(x,k_\perp) = \frac{1}{\pi\beta_L} f_L^{\perp N}(x) e^{-\vec{k}_\perp^2/\beta_L} \qquad g_L^{\perp N}(x,k_\perp) = \frac{1}{\pi\tilde{\beta}_L} g_L^{\perp N}(x) e^{-\vec{k}_\perp^2/\tilde{\beta}_L}$$

$\Rightarrow$

$$g_{1L}^{A}(x,k_\perp) \approx \frac{2J_A}{\pi(\alpha_L+\Delta_{2F})} g_{1L}^{N}(x) e^{-\vec{k}_\perp^2/(\alpha_L+\Delta_{2F})}$$

twist-2 distribution

$$f_L^{\perp A}(x,k_\perp) \approx \frac{2J_A}{\pi(\beta_L+\Delta_{2F})}\left(\frac{\beta_L}{\beta_L+\Delta_{2F}}\right) f_L^{\perp N}(x) e^{-\vec{k}_\perp^2/(\beta_L+\Delta_{2F})}$$

$$g_L^{\perp A}(x,k_\perp) \approx \frac{2J_A}{\pi(\tilde{\beta}_L+\Delta_{2F})}\left(\frac{\tilde{\beta}_L}{\tilde{\beta}_L+\Delta_{2F}}\right) g_L^{\perp N}(x) e^{-\vec{k}_\perp^2/(\beta_L+\Delta_{2F})}$$

twist-3

Very similar to those in the un-polarized case.

Nuclear dependence of the TMDs: transversely polarized

For the transversely polarized nuclei:

$$\varepsilon_\perp^{kS} f_{1T}^{\perp A}(x,k_\perp) \approx \frac{2J_A}{\pi\,\Delta_{2F}} \int d^2l_\perp e^{-(\vec{k}_\perp-\vec{l}_\perp)^2/\Delta_{2F}} \varepsilon_\perp^{lS} f_{1T}^{\perp N}(x,l_\perp)$$

$$(k_\perp \cdot S_\perp) g_{1T}^{\perp A}(x,k_\perp) \approx \frac{2J_A}{\pi\,\Delta_{2F}} \int d^2l_\perp e^{-(\vec{k}_\perp-\vec{l}_\perp)^2/\Delta_{2F}} (l_\perp \cdot S_\perp) g_{1T}^{\perp N}(x,l_\perp)$$

$$\varepsilon_\perp^{kS}(k_\perp \cdot S_\perp) f_T^{\perp A}(x,k_\perp) \approx \frac{2J_A}{\pi\,\Delta_{2F}} \int d^2l_\perp e^{-(\vec{k}_\perp-\vec{l}_\perp)^2/\Delta_{2F}} \varepsilon_\perp^{lS}(l_\perp \cdot S_\perp) f_T^{\perp N}(x,l_\perp)$$

$$\varepsilon_\perp^{kS}(k_\perp \cdot S_\perp) g_T^{\perp A}(x,k_\perp) \approx \frac{2J_A}{\pi\,\Delta_{2F}} \int d^2l_\perp e^{-(\vec{k}_\perp-\vec{l}_\perp)^2/\Delta_{2F}} \varepsilon_\perp^{lS}(l_\perp \cdot S_\perp) g_T^{\perp N}(x,l_\perp)$$

$$\varepsilon_\perp^{kS}(k_\perp \cdot S_\perp) f_T^{A}(x,k_\perp) \approx \frac{2J_A}{\pi\,\Delta_{2F}} \int d^2l_\perp e^{-(\vec{k}_\perp-\vec{l}_\perp)^2/\Delta_{2F}} \Big\{ \varepsilon_\perp^{kS}(k_\perp \cdot S_\perp) f_T^{N}(x,l_\perp) - \\ - \Big[(k_\perp \cdot S_\perp)(k_\perp \cdot l_\perp)\varepsilon_\perp^{lS} - (k_\perp \cdot S_\perp)\frac{l_\perp^2}{2}\varepsilon_\perp^{kS} - (l_\perp \cdot S_\perp)\frac{k_\perp^2}{2}\varepsilon_\perp^{lS} \Big] \frac{f_T^{\perp N}(x,l_\perp)}{M^2} \Big\}$$

$$\varepsilon_\perp^{kS}(k_\perp \cdot S_\perp) g_T^{A}(x,k_\perp) \approx \frac{2J_A}{\pi\,\Delta_{2F}} \int d^2l_\perp e^{-(\vec{k}_\perp-\vec{l}_\perp)^2/\Delta_{2F}} \Big\{ \varepsilon_\perp^{kS}(k_\perp \cdot S_\perp) g_T^{N}(x,l_\perp) - \\ + \Big[(k_\perp \cdot S_\perp)(k_\perp \cdot l_\perp)\varepsilon_\perp^{lS} - (k_\perp \cdot S_\perp)\frac{l_\perp^2}{2}\varepsilon_\perp^{kS} - (l_\perp \cdot S_\perp)\frac{k_\perp^2}{2}\varepsilon_\perp^{lS} \Big] \frac{g_T^{\perp N}(x,l_\perp)}{M^2} \Big\}$$

Nuclear dependence of the TMDs: transversely polarized

With the Gaussian ansatz

$$f_{1T}^{\perp N}(x,k_\perp) = \frac{1}{\pi\alpha_T} f_{1T}^{\perp N}(x) e^{-\vec{k}_\perp^2/\alpha_T}, \qquad g_{1T}^{\perp N}(x,k_\perp) = \frac{1}{\pi\tilde{\alpha}_T} g_{1T}^{\perp N}(x) e^{-\vec{k}_\perp^2/\tilde{\alpha}_T},$$

$$f_T^{N}(x,k_\perp) = \frac{1}{\pi\beta_T} f_T^{N}(x) e^{-\vec{k}_\perp^2/\beta_T}, \qquad g_T^{N}(x,k_\perp) = \frac{1}{\pi\tilde{\beta}_T} g_T^{N}(x) e^{-\vec{k}_\perp^2/\tilde{\beta}_T},$$

$$f_T^{\perp N}(x,k_\perp) = \frac{1}{\pi\gamma_T} f_T^{\perp N}(x) e^{-\vec{k}_\perp^2/\gamma_T}, \qquad g_T^{\perp N}(x,k_\perp) = \frac{1}{\pi\tilde{\gamma}_T} g_T^{\perp N}(x) e^{-\vec{k}_\perp^2/\tilde{\gamma}_T},$$

⇨

twist-2:

$$f_{1T}^{\perp A}(x,k_\perp) \approx \frac{2J_A}{\pi(\alpha_T+\Delta_{2F})}\left(\frac{\alpha_T}{\alpha_T+\Delta_{2F}}\right) f_{1T}^{N}(x) e^{-\vec{k}_\perp^2/(\alpha_T+\Delta_{2F})}$$

$$g_{1T}^{\perp A}(x,k_\perp) \approx \frac{2J_A}{\pi(\tilde{\alpha}_T+\Delta_{2F})}\left(\frac{\tilde{\alpha}_T}{\tilde{\alpha}_T+\Delta_{2F}}\right) g_{1T}^{N}(x) e^{-\vec{k}_\perp^2/(\tilde{\alpha}_T+\Delta_{2F})}$$

twist-3:

$$f_T^{A}(x,k_\perp) \approx \frac{2J_A}{\pi(\beta_T+\Delta_{2F})} f_T^{N}(x) e^{-\vec{k}_\perp^2/(\beta_T+\Delta_{2F})}$$

$$g_T^{A}(x,k_\perp) \approx \frac{2J_A}{\pi(\tilde{\beta}_T+\Delta_{2F})} g_T^{N}(x) e^{-\vec{k}_\perp^2/(\tilde{\beta}_T+\Delta_{2F})}$$

$$f_T^{\perp A}(x,k_\perp) \approx \frac{2J_A}{\pi(\gamma_T+\Delta_{2F})}\left(\frac{\gamma_T}{\gamma_T+\Delta_{2F}}\right)^2 f_T^{\perp N}(x) e^{-\vec{k}_\perp^2/(\gamma_T+\Delta_{2F})}$$

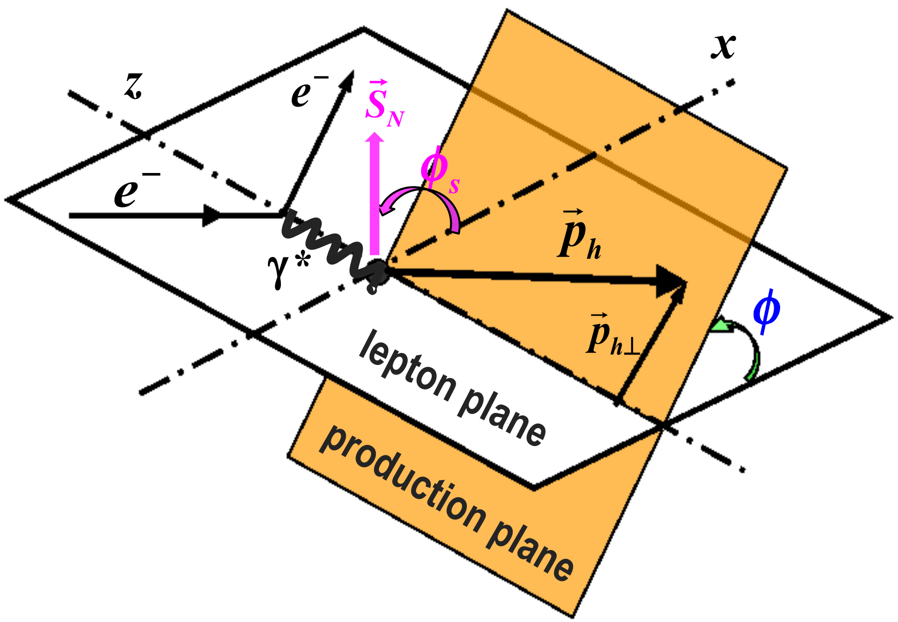
$$g_T^{\perp A}(x,k_\perp) \approx \frac{2J_A}{\pi(\tilde{\gamma}_T+\Delta_{2F})}\left(\frac{\tilde{\gamma}_T}{\tilde{\gamma}_T+\Delta_{2F}}\right)^2 g_T^{\perp N}(x) e^{-\vec{k}_\perp^2/(\tilde{\gamma}_T+\Delta_{2F})}$$

Under the Gaussian ansatz: for the case that $\alpha = \beta = \gamma$

$$\frac{\langle \sin(\phi - \phi_s) \rangle_{UT}^{eA}}{\langle \sin(\phi - \phi_s) \rangle_{UT}^{eN}} = \frac{\langle \cos(\phi - \phi_s) \rangle_{UT}^{eA}}{\langle \cos(\phi - \phi_s) \rangle_{UT}^{eN}} \approx \frac{2J_A}{A} \frac{\alpha}{\alpha + \Delta_{2F}}$$

$$\frac{\langle \sin\phi \rangle_{UL}^{eA}}{\langle \sin\phi \rangle_{UL}^{eN}} \approx \frac{2J_A}{A} \frac{\alpha}{\alpha + \Delta_{2F}}$$

$$\frac{\langle \sin\phi_s \rangle_{UT}^{eA}}{\langle \sin\phi_s \rangle_{UT}^{eN}} = \frac{\langle \cos\phi_s \rangle_{LT}^{eA}}{\langle \cos\phi_s \rangle_{LT}^{eN}} \approx \frac{2J_A}{A}$$



A numerical estimation of the transverse momentum broadening Δ_{2F}

The transverse momentum broadening: $\Delta_{2F} = \int d\xi_N^- \hat{q}_F(\xi_N)$

$$\hat{q}_F(\xi_N) = \frac{2\pi^2\alpha_s}{N_c}\rho_N^A(\xi_N)[xf_g^N(x)]|_{x=0}$$

A rough estimation:

The quark transport parameter: $\hat{q}_F(\xi_N) = \hat{q}_F(0)\rho_N^A(\xi_N)/\rho_N^A(0)$

Take a hard sphere for $\rho_N^A(\xi_N) = \begin{cases} A/(4\pi R_A^3/3) & \text{for} \quad r < R_A \\ 0 & \text{for} \quad r > R_A \end{cases}$ $\quad R_A = r_0 A^{1/3}$

$$\Longrightarrow \quad \Delta_{2F} = \int d\xi_N^- \hat{q}_F(\xi_N) \approx 3\sqrt{2}\hat{q}(0) r_0 A^{1/3}/4$$

$\hat{q}(0)$: quark transport parameter at the center of the nucleus
extracted from the analysis of suppression of leading hadrons in SIDIS with nuclei.

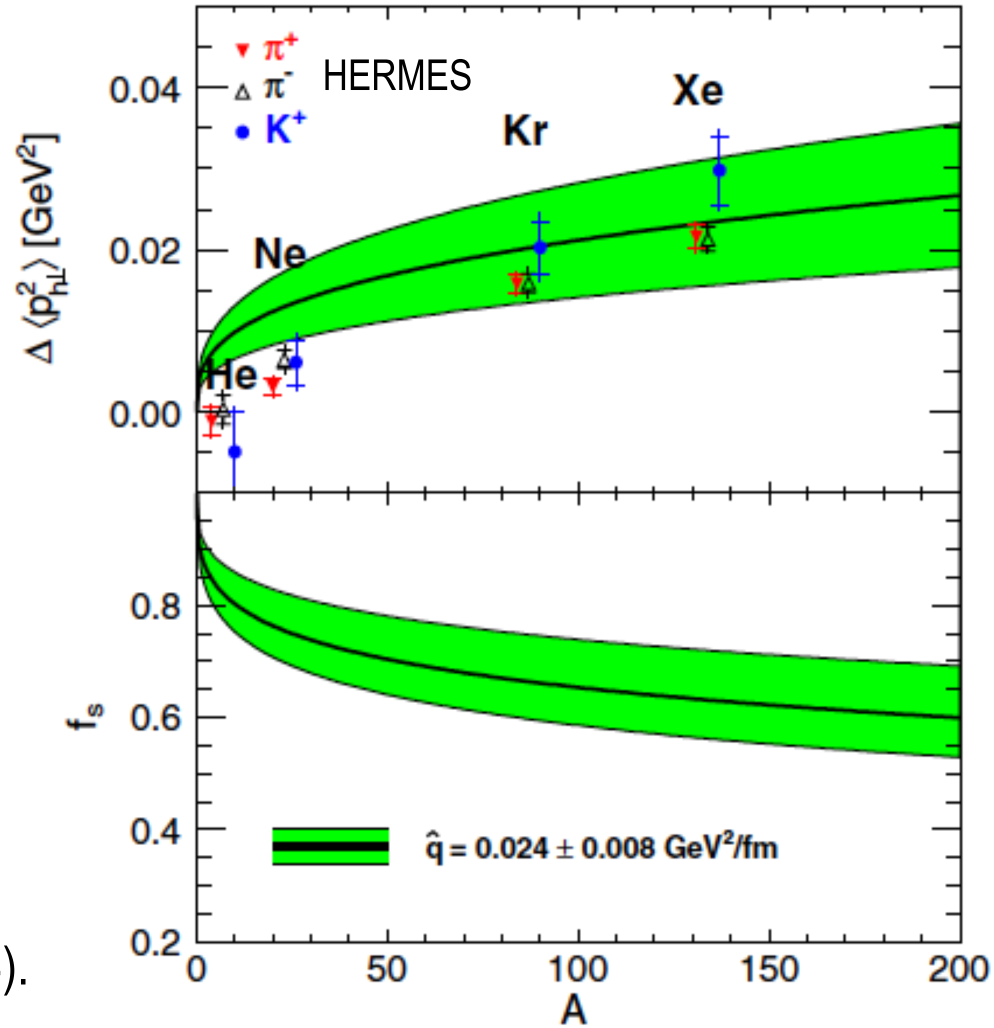
W.T. Deng, and X.N. Wang, PRC 81, 024902 (2010);
N.B. Chang, W.T. Deng, and X.N. Wang, PRC 89, 034911 (2014).

A numerical estimation of the transverse momentum broadening Δ_{2F}

Comparison with data on $\Delta\left\langle p_{h\perp}^2 \right\rangle = \left\langle p_{h\perp}^2 \right\rangle^{eA\to ehX} - \left\langle p_{h\perp}^2 \right\rangle^{ep\to ehX}$

$$p_{h\perp} \approx z k_\perp + p_{F\perp}$$

$$\Delta\left\langle p_{h\perp}^2 \right\rangle \approx z^2 \Delta_{2F}$$



The suppression factor

$$f_s = \alpha / (\alpha + \Delta_{2F})$$

Y.K. Song, ZTL, X.N. Wang, PRD (2014).

Summary and Outlook

- We show that a direct consequence of the "multiple gluon scattering" contained in the gauge link is a significant transverse momentum broadening in nucleus, for leading and higher twist TMDs;
- Such transverse momentum broadening leads in general to suppression of azimuthal asymmetries in SIDIS with nuclear targets;
- A rough numerical estimation of the broadening is made and comparison with the data available is given;
- More interesting measurements are expected, which may provide another place to study the effects of "multiple gluon scattering".

Thank you for your attention!