Glueball Spectrum in QCD

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- An important issue in QCD is the glueball. QCD must have the glueballs, but there are few candidates. The reason is two-fold.

  1. Theoretically there is NO consensus on how to construct the glueballs. We have to understand npQCD first.

  2. Experimentally it is difficult to identify them, because most of them probably mix with quarkoniums.

History of npQCD/Confinement

- Pauli first raised this issue, pointing out the problems of the massless Yang-Mills theory.
- Nambu and Mandelstam conjectured the dual Meissner effect as the confinement mechanism.
- In 1977 Savvidy calculated the effective action of SU(2) QCD, and has "almost" proved the monopole condensation (a milestone in npQCD). Unfortunately the Savvidy vacuum was unstable.

Millennium Problem

- There have been huge efforts to cure the instability of Savvidy vacuum, but the instability is only a minor problem.
- Mean time 'tHooft proposed the Abelian dominance as the confinement mechanism.
- The lattice QCD has been able to demonstrate the confinement, but unable to show what is the confinement mechanism. Moreover, the lattice results were gauge dependent.
- The problem is directly related to the origin of mass of the universe, because the proton mass comes from confinement.

New Development

- Recently there have been two important progresses.

  1. For the first time the lattice QCD could pinpoint what is responsible for the confinement gauge independently.

  2. A new calculation of the QCD effective action succeeded to demonstrate the stable monopole condensation which generates the dimensional transmutation.

Central Issues

1. How can we establish the Abelian dominance? What is the Abelian part of QCD, and how different is this from the non-Abelian part?
2. What is wrong about the Savvidy action?
3. What makes the color confinement, Abelian dominance or monopole condensation? How can we verify this? What are the observable consequences?

Contents

Binding Gluons and Valence Gluons

A. Abelian Projection: SU(2)

- Choose the Abelian direction $\hat{n}$, and find the restricted potential which makes it covariant constant,

$$D_\mu \hat{n} = \partial_\mu \hat{n} + g\vec{A}_\mu \times \hat{n} = 0,$$

$$\vec{A}_\mu \to \hat{A}_\mu = A_\mu \hat{n} - \frac{1}{g}\hat{n} \times \partial_\mu \hat{n} = \mathcal{A}_\mu + \mathcal{C}_\mu,$$

$$\mathcal{A}_\mu = A_\mu \hat{n}, \quad \mathcal{C}_\mu = -\frac{1}{g}\hat{n} \times \partial_\mu \hat{n}, \quad A_\mu = \hat{n} \cdot \vec{A}_\mu.$$

Anatomy of QCD

- Obtain the gauge independent Abelian decomposition known as Cho decompositon or Cho-Duan-Ge (CDG) decomposition

$$\vec{A}_\mu = \hat{A}_\mu + \vec{X}_\mu,$$
$$\vec{X}_\mu = {X_\mu}^1 \hat{n}_1 + {X_\mu}^2 \hat{n}_2, \quad (\hat{n} \cdot \vec{X}_\mu = 0).$$

1. $\hat{A}_\mu$ has the full SU(2) gauge degrees of freedom, and describes the neutral binding gluon.
2. $\vec{X}_\mu$ transforms covariantly, and describes the colored valence gluons.

Two Types of Gluons!

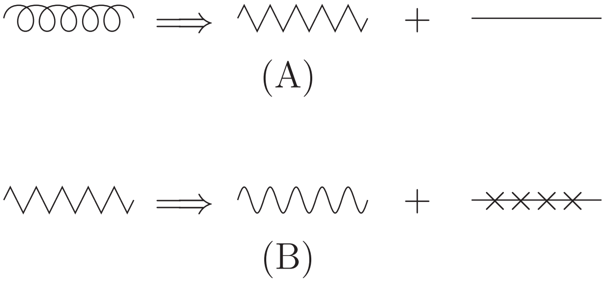

Figure : The Abelian decomposition. The gauge potential is decomposed to the binding and valence potentials in (A), and the binding potential is further decomposed to the Maxwellian and Diracian part in (B).

- $\hat{A}_\mu$ is essentially Abelian, but has a dual structure. It has the topological (Diracian) $\mathcal{C}_\mu$ and the non-topological (Maxwellian) $\mathcal{A}_\mu$,

$$\vec{H}_{\mu\nu} = \partial_\mu \mathcal{C}_\nu - \partial_\nu \mathcal{C}_\mu + g\ \mathcal{C}_\mu \times \mathcal{C}_\nu = H_{\mu\nu} \hat{n},$$
$$H_{\mu\nu} = -\frac{1}{g} \hat{n} \cdot (\partial_\mu \hat{n} \times \partial_\nu \hat{n}).$$

- $\hat{F}_{\mu\nu}$ becomes Abelian but has two potentials, "electric" A_μ and "magnetic" $\tilde{C}_\mu$,

$$\hat{F}_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu + g \hat{A}_\mu \times \hat{A}_\nu = (F_{\mu\nu} + H_{\mu\nu}) \hat{n},$$
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad H_{\mu\nu} = \partial_\mu \tilde{C}_\nu - \partial_\nu \tilde{C}_\mu,$$
$$\tilde{C}_\mu = -\frac{1}{g} \hat{n}_1 \cdot \partial_\mu \hat{n}_2.$$

B. Vacuum Decomposition

- Vacuum isometry: Impose the maximal isometry

$$\forall_i \ D_\mu \hat{n}_i = 0 \quad \Rightarrow \quad \vec{F}_{\mu\nu} = 0.$$

Solve it and construct the most general vacuum potential

$$\hat{\Omega}_\mu = \frac{1}{2g} \epsilon_{ij}{}^{k} \ (\hat{n}_i \cdot \partial_\mu \hat{n}_j) \ \hat{n}_k = -\frac{1}{g} \hat{n} \times \partial_\mu \hat{n} - \tilde{C}_\mu \hat{n}.$$

- With the S^3 compactification of R^3, $\hat{n}$ characterizes the Hopf invariant $\Pi_3(S^3) \simeq \Pi_3(S^2)$ which describes the topologically distinct vacua $|n\rangle$ with integer vacuum number n.

- With $\hat{\Omega}_\mu$ we have the vacuum decomposition

$$\vec{A}_\mu = \hat{A}_\mu + \vec{X}_\mu = \hat{\Omega}_\mu + \vec{B}_\mu + \vec{X}_\mu = \hat{\Omega}_\mu + \vec{Z}_\mu,$$
$$\hat{A}_\mu = \hat{\Omega}_\mu + \vec{B}_\mu, \quad \vec{Z}_\mu = \vec{B}_\mu + \vec{X}_\mu,$$
$$\vec{B}_\mu = (A_\mu + \tilde{C}_\mu)\hat{n}.$$

Notice that $\vec{B}_\mu$ inherits the dual structure of $\hat{A}_\mu$. Moreover, $\hat{\Omega}_\mu$ (just like $\hat{A}_\mu$) has the full gauge degrees of freedom.

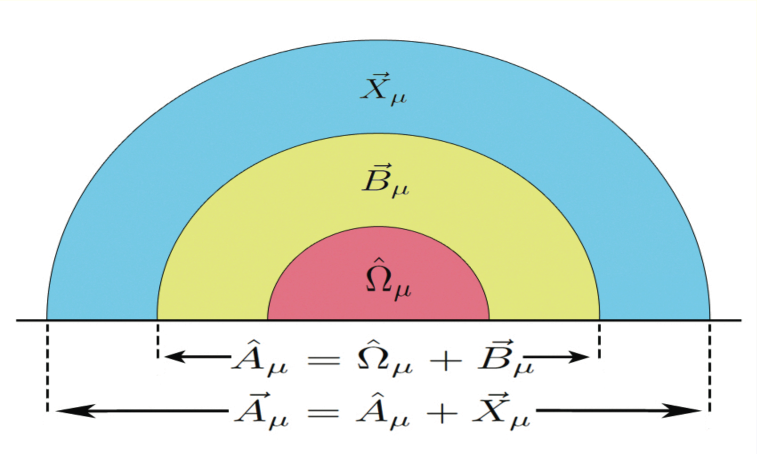

Figure : The affine structure of non-Abelian connection space: It has two proper subspaces $\hat{A}_\mu$ and $\hat{\Omega}_\mu$ which form their own non-Abelian connection spaces. Moreover, $\hat{A}_\mu$ (and $\vec{B}_\mu$) has a dual structure.

A. Restricted QCD (RCD)

- Define RCD which describes the Abelian sub-dynamics by $\hat{A}_\mu$,

$$\mathcal{L}_{RCD} = -\frac{1}{4}\hat{F}_{\mu\nu}{}^2 = -\frac{1}{4}(F_{\mu\nu} + H_{\mu\nu})^2$$
$$= -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2g}F_{\mu\nu}\hat{n}\cdot(\partial_\mu\hat{n}\times\partial_\nu\hat{n}) - \frac{1}{4g^2}(\partial_\mu\hat{n}\times\partial_\nu\hat{n})^2.$$

  It has the full $SU(2)$ gauge degrees of freedom, yet is simpler than the Yang-Mills theory.

- It has the monopole degrees explicitly, and provides an ideal platform to study the monopole dynamics gauge independently.

B. Extended QCD (ECD)

- From $\vec{F}_{\mu\nu} = \hat{F}_{\mu\nu} + \hat{D}_\mu \vec{X}_\nu - \hat{D}_\nu \vec{X}_\mu + g\vec{X}_\mu \times \vec{X}_\nu$ we have

$$\mathcal{L}_{QCD} = -\frac{1}{4}\vec{F}_{\mu\nu}^2 = -\frac{1}{4}\hat{F}_{\mu\nu}^2$$

$$-\frac{1}{4}(\hat{D}_\mu \vec{X}_\nu - \hat{D}_\nu \vec{X}_\mu)^2 - \frac{g}{2}\hat{F}_{\mu\nu} \cdot (\vec{X}_\mu \times \vec{X}_\nu) - \frac{g^2}{4}(\vec{X}_\mu \times \vec{X}_\nu)^2.$$

1. QCD can be interpreted as RCD made of binding gluons $\hat{A}_\mu$ which has valence gluons $\vec{X}_\mu$ as colored source.
2. $\hat{A}_\mu$ and $\vec{X}_\mu$ can be viewed as slow-varying classical background and fast-moving quantum fluctuation.
3. This is a gauge independent decomposition.

Attention!

- ECD tells that QCD is (a sugar coating of) an Abelian gauge theory coupled to massless charged vector fields, which is a “sick” (and terrible) theory with all kinds of diseases (infra-red instability, etc.).
- What saves QCD from this sickness is the dual structure, in particular the monopole potential, of RCD.
- This dual structure makes QCD a wonderful theory again, generating the monopole condensation and the mass gap.

- ECD has the classical (background) gauge symmetry

$$\delta \hat{A}_\mu = \frac{1}{g}\hat{D}_\mu \vec{\alpha}, \quad \delta \vec{X}_\mu = -\vec{\alpha} \times \vec{X}_\mu.$$

Moreover, it has the quantum (fast) gauge symmetry

$$\delta \hat{A}_\mu = \frac{1}{g}(\hat{n} \cdot D_\mu \vec{\alpha})\hat{n}, \quad \delta \vec{X}_\mu = \frac{1}{g}\hat{n} \times (D_\mu \vec{\alpha} \times \hat{n}),$$

which assures the valence gluons to be massless.

- ECD assures the Abelian dominance, because the valence gluons become the confined prisners.

C. Color Reflection Invariance

- Consider the color reflection which changes the color of the valence gluons

$$(\hat{n}_1, \hat{n}_2, \hat{n}) \rightarrow (\hat{n}_1, -\hat{n}_2, -\hat{n}).$$

Under the color reflection we have

$$\hat{A}_\mu \rightarrow -A_\mu \hat{n} - \frac{1}{g}\hat{n} \times \partial_\mu \hat{n} = -\mathcal{A}_\mu + \mathcal{C}_\mu.$$

$$A_\mu = \hat{n} \cdot \vec{A}_\mu \rightarrow A_\mu, \quad \tilde{C}_\mu = -\frac{1}{g}\hat{n}_1 \cdot \partial_\mu \hat{n}_2 \rightarrow -\tilde{C}_\mu,$$

$$\vec{X}_\mu \rightarrow X_\mu^1\ \hat{n}_1 - X_\mu^2\ \hat{n}_2, \quad X_\mu \rightarrow X_\mu^* = \frac{1}{\sqrt{2}}(X_\mu^1 - iX_\mu^2).$$

- So we have two different Abelian decompositions related by the color reflection, which describes the same physics.
- After the Abelian decomposition it becomes the only remaining symmetry of the non-Abelian gauge symmetry.
- The gauge invariance is simplified to the descrete color reflection invariance in ECD which is much easier to handle.

- In summary, the Abelian decomposition tells the followings:

  1. It tells that QCD has two types of gluons, and transforms QCD to ECD.

  2. It assures the Abelian dominance.

  3. It simplifies the gauge invariance to the color reflection invariance.

  4. It tells that the monopole potential is gauge invariant and parity conserving, and separates it gauge independently.

A. Lattice QCD: A New Result

- The lattice calculations based on the maximal Abelian gauge has demonstrated the monopole condensation and/or Abelian dominance. Unfortunately they suffer the critical defects:

  1. They were gauge dependent.
  2. They could not tell what is the confinement mechanism.

- With the gauge independent Abelian decomposition, LQCD can demonstrate that the monopole is responsible for the confinement gauge independently.

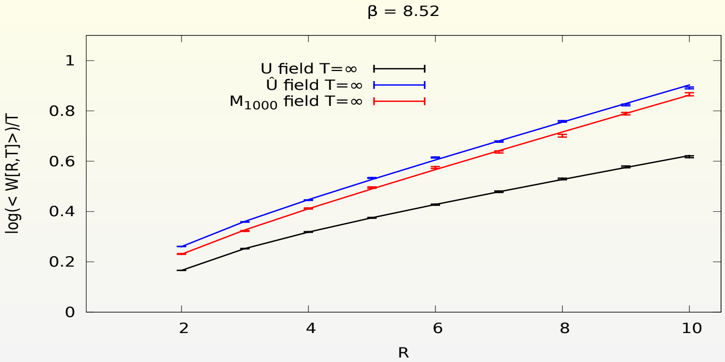

Figure : The Abelian dominance versus the monopole dominance in the SNU-KU lattice calculation. The result is obtained with the gauge independent CDG decomposition. Here the red, blue, and black lines represent the monopole, Abelian, and full potentials.

B. Savvidy Action: A Review

- Savvidy (and others) tried to calculate the one-loop effective action of QCD, but made two critical mistakes.

  1. They have chosen a wrong background which was not gauge invariant nor parity conserving, and completely neglected the topological structure of QCD

  2. They failed to implement the gauge invariance in thecalculation of the functional determinant of the gluon loop integral.

- With this they have

$$\Delta S = i \ \ln \ \text{Det} \ [(-\hat{D}^2 + 2gH)(-\hat{D}^2 - 2gH)] \\ -2i \ \ln \ \text{Det} \ (-\hat{D}^2),$$

$$\Delta \mathcal{L} = \frac{1}{16\pi^2} \int_0^\infty \frac{dt}{t^{3-\epsilon}} \frac{gHt/\mu^2}{\sinh(gHt/\mu^2)} \\ \times \Big[\exp(-2gHt/\mu^2) + \exp(+2gHt/\mu^2) - 1 \Big].$$

Infra-red Divergence

- Instability of Savvidy Vacuum: With the ζ-function regularization they obtain the Savvidy-Nielsen-Olesen (SNO) effective potential which contains the well-known imaginary part,

$$V_{SNO} = \frac{H^2}{2}\Big[1 + \frac{11g^2}{24\pi^2}\big(\ln\frac{H}{\mu^2} - c\big)\Big] + i\frac{g^2H^2}{8\pi}.$$

Obviously this destabilizes the magnetic condensation.

- The SNO instability originates from the tachyonic modes of the functional determinant in the gluon loop integral.

- The imaginary part of the SNO effective potential can easily be removed by adopting the regularization by causality. However, this does not resolve the problem.
- We have to explain what is wrong with the ζ-function regularization. We can not say the causality confines the color.
- The real problem with the SNO vacuum is not the instability, but the violation of the gauge invariance.

C. Gauge Invariant Calculation of Effective Action

- To remove these defects, choose the color reflection invariant and parity conserving monopole background

$$\hat{F}^{(b)}_{\mu\nu} = \bar{H}_{\mu\nu}\hat{n}, \quad \bar{H}_{\mu\nu} = H\delta^1_{[\mu}\delta^2_{\nu]}.$$

- Perform (not red and blue gluon loop but) the gauge invariant loop integral made of the color reflection invariant gluon pairs

$$|C, C_3\rangle = |0, 0\rangle = \frac{|XX\rangle + |XX\rangle}{\sqrt{2}}.$$

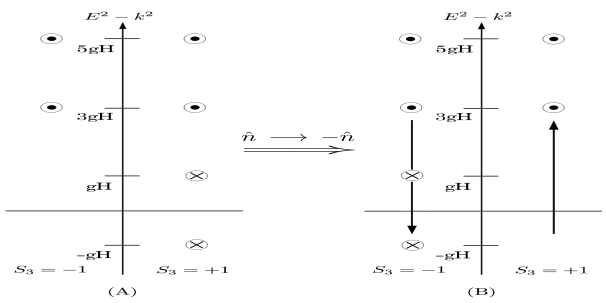

Figure : The gauge invariant eigenvalues of the gluon functional determinant. Notice that the C projection excludes the lowest two eigenmodes, in particular the tachyonic modes.

- With the C projection we have

$$\ln\ \mathrm{Det}^{1/2}\ K = \ln\ \mathrm{Det}\ [(-\hat{D}^2 + 2gH)(-\hat{D}^2 + 2gH)],$$
$$\Delta\mathcal{L} = \frac{1}{16\pi^2}\int_0^\infty \frac{dt}{t^{3-\epsilon}}\frac{gHt/\mu^2}{\sinh(gHt/\mu^2)}$$
$$\times\Big[\exp(-2gHt/\mu^2)+\exp(-2gHt/\mu^2)-1\Big].$$

- The C projection removes the tachyonic modes and restores the gauge invariance in QCD. It plays the role of the GSO projection which removes tachyons and assures supersymmetry and modular invariance in NSR string theory.

No Infra-red Divergence!

D. Monopole Condensation and Asymptotic Freedom

- With the C projection the effective potential becomes real

$$V = \frac{H^2}{2}\Big[1 + \frac{11g^2}{24\pi^2}\big(\ln\frac{gH}{\mu^2} - c\big)\Big].$$

- Define the running coupling $\bar{g}$ by $\frac{\partial^2 V}{\partial H^2}\Big|_{H=\bar{\mu}^2} = \frac{g^2}{\bar{g}^2}$ and find

$$\frac{1}{\bar{g}^2} = \frac{1}{g^2} + \frac{11}{24\pi^2}\big(\ln\frac{\bar{\mu}^2}{\mu^2} - c + \frac{3}{2}\big), \quad \beta(\bar{\mu}) = \bar{\mu}\frac{\partial \bar{g}}{\partial \bar{\mu}} = -\frac{11\bar{g}^3}{24\pi^2}.$$

Asymptotic Freedom

- Find the renormalized potential

$$V_{ren} = \frac{H^2}{2}\Big[1 + \frac{11\bar{g}^2}{24\pi^2}\big(\ln\frac{H}{\bar{\mu}^2} - \frac{3}{2}\big)\Big],$$

which has a non-trivial minimum at

$$\langle H \rangle = \bar{\mu}^2 \exp\Big(-\frac{24\pi^2}{11\bar{g}^2} + 1\Big).$$

- This is identical to the Savvidy potential, except that the monopole condensation is stable.

Dynamical Symmetry Breaking!

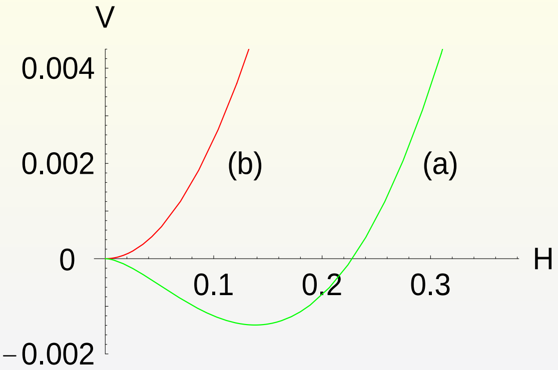

Figure : The one-loop effective potential of SU(2) QCD. Here blue and red curves represent the effective potential and the classical potential.

- In general for arbitray constant monopole background $\bar{H}_{\mu\nu}$ we find

$$\mathcal{L}_{eff} = \begin{cases} -\frac{H^2}{2} - \frac{11g^2H^2}{48\pi^2}\Big(\ln\frac{gH}{\mu^2} - c\Big), & E = 0 \\ -\frac{E^2}{2} + \frac{11g^2E^2}{48\pi^2}\Big(\ln\frac{gE}{\mu^2} - c\Big) - i\frac{11g^2E^2}{96\pi}, & H = 0 \end{cases}$$

$$c = 1 - \ln\, 2 - \frac{24}{11}\zeta'(-1, \frac{3}{2}) = 0.94556...$$

- The effective action is invariant under the dual transformation

$$H \rightarrow -iE, \qquad E \rightarrow iH.$$

This electric-magnetic duality is a fundamental symmetry of the effective action of gauge theory, Abelian and non-Abelian.

- What did they do wrong? They calculated the effective action of an Abelian gauge theory coupled to the massless charged vector fields. This is a sick theory, not QCD.

- In physics something is wrong when we encounter tachyons.

1. In Higgs mechanism we have tachyon when we choose the false vacuum.
2. In NSR string we have tachyonic vacuum when we do not make the GSO projection.

SU(3) QCD

- SU(3) QCD is more complicated, but the generic feature remains the same. The Abelian decomposition devides the gluons to the binding gluons (the "neurons") and the valence gluons (the " chromons"), and transforms QCD to ECD.
- Choosing the monopole background we can calculate the effective action, imposing the color reflection invariance.
- Remarkably the Abelian decomposition reduces the calculation of the effective action to that of three SU(2) subgroups.

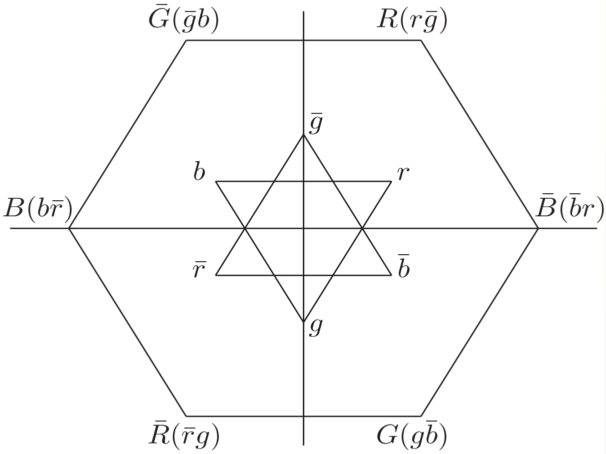

Figure : The color assignment of the valence gluons.

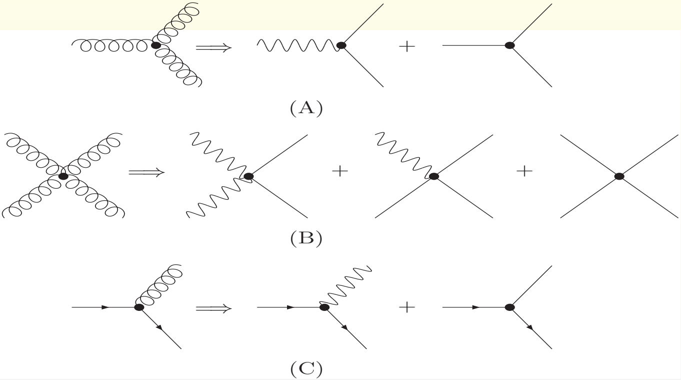

Figure : The decomposition of QCD interactions in ECD. The three point vertex is decomposed to two vertices in (A), the four point vertex is decomposed to three vertices in (B), and the quark-gluon vertex is decomposed to two vetrices in (C).

- The decomposition of the QCD vertices has deep implications.

  1. It makes the color conservation explicit. There is no three-point vertex made of two or three neuron legs, and no four-point vertex made of three or four neuron legs.

  2. It tells that there are two types of gluon jets, the neuron-chromon jets and the chromon jets which can be distinguished by experiments. So we can test the Abelian decomposition by experiment.

Two Types of Gluon Jets!

- Explicitly we have the following SU(3) effective action

$$\Delta\mathcal{L} = \lim_{\epsilon\to 0} \frac{g^2}{8\pi^2} \sum_p \int_0^\infty \frac{dt}{t^{3-\epsilon}} \frac{H_p E_p t^2/\mu^4}{\sinh(gH_p t/\mu^2)\sin(gE_p t/\mu^2)} \Big[\exp(-2gH_p t/\mu^2) + \exp(+2igE_p t/\mu^2) - 1\Big].$$

- With this we have

$$\mathcal{L}_{eff} = \begin{cases} -\sum_p \Big(\frac{H_p^2}{3} + \frac{11g^2 H_p^2}{48\pi^2}(\ln\frac{gH_p}{\mu^2} - c)\Big), & (E_p = 0) \\ \sum_p \Big(\frac{E_p^2}{3} + \frac{11g^2 E_p^2}{48\pi^2}(\ln\frac{gE_p}{\mu^2} - c) \\ \qquad - i\frac{11g^2}{96\pi}E_p^2\Big). & (H_p = 0) \end{cases}$$

- For the monopole background the effective potential is given by

$$V = \frac{3}{4}\sum_p H_p^2 + \frac{11g^2}{48\pi^2}\sum_p H_p^2 \ln\big(\frac{gH_p}{\mu^2} - c\big).$$

- The classical potential depends on only one Casimir invariant $\vec{H}_3^2 + \vec{H}_8^2$, but the effective potential depends on three Casimir invariants $H_1,\ H_2,\ H_3$ (or equivalently $|\vec{H}_3|$, $|\vec{H}_8|$, and the angle θ between $\vec{H}_3$ and $\vec{H}_8$).

- Define the renormalized coupling $\bar{g}$ by

$$\forall_p \quad \frac{\partial^2 V}{\partial H_p^2}\Big|_{H_1=H_2=H_3=\bar{\mu}^2} = \frac{g^2}{\bar{g}^2},$$

and find

$$\frac{1}{\bar{g}^2} = \frac{1}{g^2} + \frac{11}{16\pi^2}\Big(\ln\frac{\bar{\mu}^2}{\mu^2} - c + \frac{5}{4}\Big),$$

$$\beta(\bar{\mu}) = \bar{\mu}\frac{\partial \bar{g}}{\partial \bar{\mu}} = -\frac{11\bar{g}^3}{16\pi^2}.$$

Asymptotic Freedom!

- Find the renormalized potential

$$V_{ren} = \sum_p \Big(\frac{3}{4}H_p^2 + \frac{11\bar{g}^2}{48\pi^2}H_p^2 \ln\big(\frac{\bar{g}H_p}{\bar{\mu}^2} - \frac{5}{4}\big)\Big).$$

which has the minimum $V_{min} = -\dfrac{11\bar{\mu}^4}{32\pi^2}\exp\big(-\dfrac{32\pi^2}{11\bar{g}^2} + \dfrac{3}{2}\big)$ at

$$\langle H_1\rangle = \langle H_2\rangle = \langle H_3\rangle = \frac{\bar{\mu}^2}{\bar{g}}\exp\big(-\frac{16\pi^2}{11\bar{g}^2} + \frac{3}{4}\big).$$

Notice that the effective potential breaks the original SO(2) invariance (of $H^2 + H'^2$) of the classical Lagrangian.

Mass Gap!

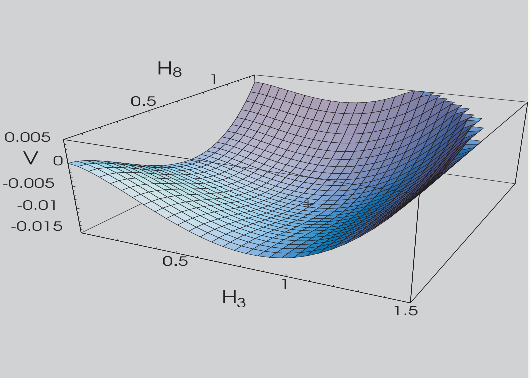

Figure : The effective potential with $\cos\theta = 0$, which has a unique minimum at $H = H' = H_0$ (or $H_1 = H_2 = H_3 = H_0$).

Monopole Condensation vs Dual Meissner Effect

- The confinement is NOT the dual Meissner effect.

1. In SC the Cooper pair has charge, but in QCD the monopole-antimonopole pair has no chromo-magnetic charge.
2. In SC the magnetic field comes from the electric current, but in QCD the chromo-electric field comes from color charge.
3. In SC the supercurrent screens magnetic field, but in QCD the monopole condensation confines the chromo-electric field.
4. In SC the vector potential describes the magnetic field, but in QCD the Coulomb (i.e., scalar) potential describes the color.
5. In SC Higgs mechanism works, but in QCD it is the dynamical symmetry breaking.

Observable Consequences: Glueballs and Hybrids

- ECD has two types of gauge covariant gluons, two color neutral neurons and six colored chromons, and justifies "the constituent gluon model".

- ECD generalizes the quark model to the "quark and gluon" model, and predicts the chromoballs made of chromons and the hybrid hadrons made of quarks and gluons.

- Moreover, the monopole condensation generates the monoball, the 0^{++} vacuum fluctuation mode.

Where are they???

A. Chromoball

- ECD implies the existence of the chromoballs (and in principle neuroballs). However, finding them may not be easy.

  1. The chromoballs are intrinsically unstable because the chromo-electric background (creates the quark pairs but) annihilates them.
  2. The neuroballs can hardly exist, because neurons have no color.

- So only the low-lying chromoballs could actually be detected.

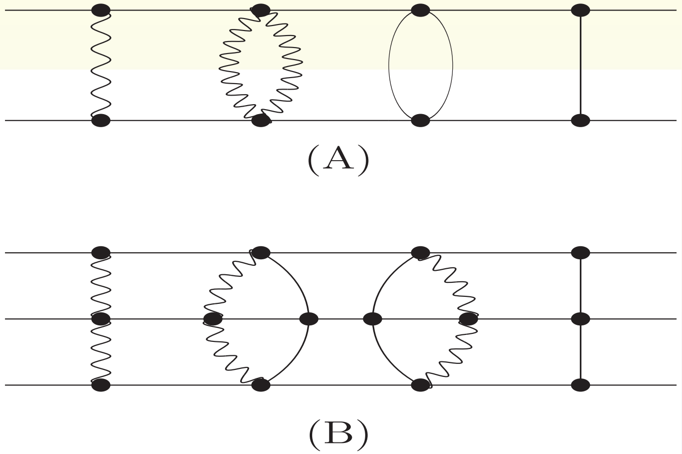

Figure : The possible binding Feynman diagrams for the chromoballs.

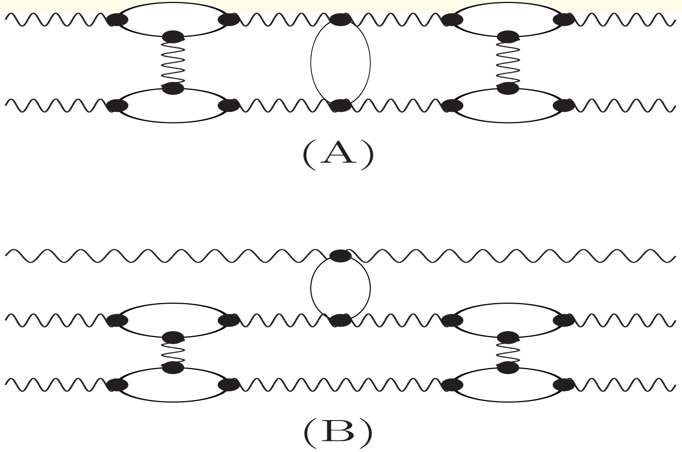

Figure : The possible interactions among the neurons. This is identical to the photon interaction in QED with charged vector.

Table : The possible low-lying gg chromoballs.

$^{(2S+1)}L_J$	J^{PC}	possible candidates
$^1S_0$	0^{++}	$f_0(500), f_0(1370)$
$^5S_2$	2^{++}	$f_2(1950), f_2(2010)$
$^3P_0$	0^{-+}	$\eta(1295), \eta(1405), \eta(1475)$
$^3P_1$	1^{-+}	???
$^3P_2$	2^{-+}	$\eta_2(1645)$
$^1D_2$	2^{++}	Regge recurrence of $^1S_0$
$^5D_0$	0^{++}	$f_0(1500)$
$^5D_1$	1^{-+}	???
$^5D_2$	2^{++}	Regge recurrence of $^5S_2$

The possible low-lying ggg chromoballs.

Config	space	spin	color	L	S	J^{PC}
$(1s)^3$	S	S	d	0	1,3	$1^{--},\ 3^{--}$
	S	A	f	0	0	0^{-+}
$(1s)^2(1p)$	S	S	d	1	1,3	$(0,1,2,3,4)^{+-}$
	S	A	f	1	0	1^{++}
	M	M	d	1	1,2	$(0,1,2,3)^{+-}$
	M	M	f	1	1,2	$(0,1,2,3)^{++}$
$(1s)(1p)^2$	S	S	d	$0,2$	$1,3$	$(1,2,3,4,5)^{--}$
	S	A	f	$0,2$	0	$0^{-+}, 2^{-+}$
	M	M	d	$0,2$	$1,2$	$(0,1,2,3,4)^{--}$
	M	M	f	$0,2$	$1,2$	$(0,1,2,3,4)^{-+}$
	M	M	d	1	$1,2$	$(0,1,2,3)^{--}$
	M	M	f	1	$1,2$	$(0,1,2,3)^{-+}$
	A	S	d	1	$1,3$	$(0,1,2,3,4)^{-+}$
	A	A	f	1	0	1^{--}

- We can estimate the chromoball annihilation rate from the QCD effective action. With $\alpha_s \simeq 0.5$, $\Lambda_{QCD} \simeq 339\ MeV$, and $E_p \simeq (g/\pi)\Lambda_{QCD}^2$, we have

$$\Gamma_A \simeq \sum_p \frac{11g^2}{96\pi}\ E_p^2 \times \frac{4\pi}{3\Lambda_{QCD}^3} \simeq 398\ MeV.$$

  Notice that this instability comes from the asymptotic freedom (anti-screening).

- This instability could be one of the reasons why there are so few examples of glueballs experimentally.

- Another reason is that they may not exist as pure states because they could mix with quarkoniums. So understanding the mixing is crucial to identify the glueballs experimentally.

- However, glueballs contain the oddballs which can not be viewed as $q\bar{q}$ states (e.g., $0^{+-}, 0^{--}, 1^{-+}, 2^{+-}$ etc.). So we can identify the pure glueballs by searching for these oddballs.

- Fortunately ECD has a glueball candidate, $f_0(500)$ which has a broad width 500 MeV, which has been interpreted as a four quark state (or two quarkonium bound state).

The quantum numbers of oddballs.

State	$q\bar{q}$	$g\bar{g}$	ggg	State	$q\bar{q}$	$g\bar{g}$	ggg
0^{++}	O	O	O	2^{++}	O	O	O
0^{+-}	X	X	O	2^{+-}	X	X	O
0^{-+}	O	O	O	2^{-+}	O	O	O
0^{--}	X	X	O	2^{--}	O	X	O
1^{++}	O	O	O	3^{++}	O	O	O
1^{+-}	O	X	O	3^{+-}	O	X	O
1^{-+}	X	O	O	3^{-+}	X	O	O
1^{--}	O	X	O	3^{--}	O	X	O

B. Mixing

- ECD provides a clear picture of glueball-quarkonium mixing as well as the mixing among chromons and neurons, and gg and ggg glueballs.
- There are three low-lying channels $(0^{++},\ 2^{++},\ \text{and}\ 0^{-+})$ in which the glueball-quarkonium mixing could play important role.
- The mixing could clarify not just glueball-quarkonium mixing but the quarkonium octet-singlet mixing.

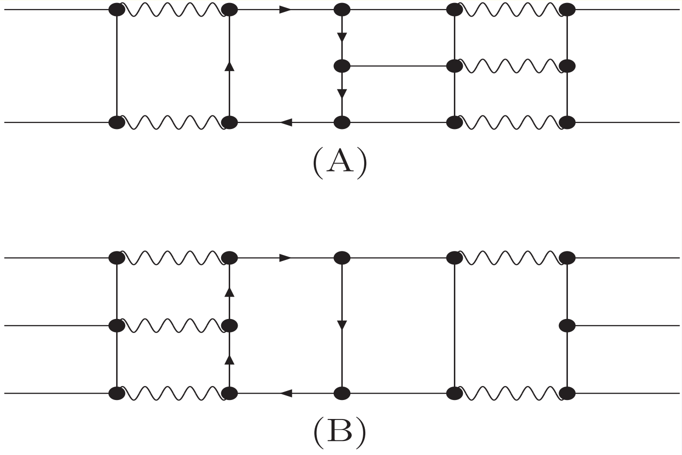

Figure : The possible glueball-quarkonium mixing.

- Let the $q\bar{q}$ octet-singlet mixing matrix be

$$M^2 = \begin{pmatrix} \langle 8|H|8\rangle & \langle 8|H|1\rangle \\ \langle 1|H|8\rangle & \langle 1|H|1\rangle \end{pmatrix}$$

$$= \begin{pmatrix} E + \frac{2}{3}\Delta & -\frac{\sqrt{2}}{3}\Delta \\ -\frac{\sqrt{2}}{3}\Delta & E + \frac{1}{3}\Delta + 3A \end{pmatrix},$$

$$|8\rangle = \frac{|u\bar{u}\rangle + |d\bar{d}\rangle - 2|s\bar{s}\rangle}{\sqrt{6}}, \quad |1\rangle = \frac{|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle}{\sqrt{3}},$$

$$E = \langle u\bar{u}|H|u\bar{u}\rangle_{Ex} = \langle d\bar{d}|H|d\bar{d}\rangle_{Ex}, \quad E + \Delta = \langle s\bar{s}|H|s\bar{s}\rangle_{Ex},$$

$$A = \langle q'\bar{q}'|H|q\bar{q}\rangle_{An}, \quad (\mathrm{for\ all\ q, q'}).$$

- We can generalize it to the glueball-quarkonium mixing mass matrix

$$M^2 = \begin{pmatrix} E + \frac{2}{3}\Delta & -\frac{\sqrt{2}}{3}\Delta & 0 \\ -\frac{\sqrt{2}}{3}\Delta & E + \frac{1}{3}\Delta + 3A & \nu \\ 0 & \nu & \mu \end{pmatrix}.$$

  We can fix E and Δ, assume $3A = \nu^2$, and choose (one or) two mass eigenstates from PDG (and vary μ) to fix the matrix completely.

- With this we can predict the mass of the third physical state, the quark and gluon contents of the physical states, and the branching ratios of radiative decay of ψ in each channel.

- PDG identifies low-lying physical states in 0^{++} and 2^{++} channels below 2 GeV to be $f_0(980),\ f_0(1370),\ f_0(1500),\ f_0(1710)$ and $f_2(1270),\ f_2'(1525),\ f_2(1950)$. In 0^{-+} channel it has $\eta(548),\ \eta'(958)$ $\eta(1295),\ \eta(1405)$.

- In the 0^{++} channel we may select $f_0(500)$ and/or $f_0(980)$, and put $E = a_0^2,\ \Delta = 2(K^2 - a_0^2),\ a_0 = a_0(980),\ K = K(1500)$.

- In the 2^{++} channel we may choose $f_2(1270),\ f_2'(1525)$, and put $E = a_2^2,\ \Delta = 2(K^{*2} - a_2^2),\ a_2 = a_2(1310),\ K^* = K_2^*(1430)$.

- In the 0^{-+} channel we may need to generalize the mass matrix to the 4×4 matrix to include the lowest $(1s)^3$ ggg state,

$$M^2 = \begin{pmatrix} E + \frac{2}{3}\Delta & -\frac{\sqrt{2}}{3}\Delta & 0 & 0 \\ -\frac{\sqrt{2}}{3}\Delta & E + \frac{1}{3}\Delta + 3A & \nu & \nu' \\ 0 & \nu & \mu & \epsilon \\ 0 & \nu' & \epsilon & \mu' \end{pmatrix}.$$

Here we may choose $\eta(548)$, $\eta'(958)$, $\eta(1295)$ to be the physical states, put $E = \pi^2,\ \Delta = 2(K^2 - \pi^2),\ \pi = \pi(140),\ K = K(498)$, and express $\mu = 4\alpha^2$ and $\mu' = 9\alpha^2$ in terms of the gluon mass α.

C. Hybrid and Knotball

- The existence of the gauge covariant gluons requires us to generalize the quark model to "the quark and gluon model".
- In particular, ECD predicts hybrid $q\bar{q}g$ mesons or $qqqg$ baryons made of one chromon and $q\bar{q}$ octet or qqq octet. This should be contrasted with four-quark or penta-quark hadrons.
- These hybrids have no intrinsic instability, because the chromon in the hybrids is stable.
- Future EICs, in particular J Lab (Hall D) or IMP of CAS, could reveal such hybrid states.

- The color confinement indicates the existence of the knots in QCD, twisted chromo-electric flux rings which has the $\pi_3(S^2)$ topology.
- In spite of the non-trivial topology, however, they may not be stable because the chromo-electric field induces the pair production of quarks.
- Rough estimate suggests that the mass and decay width of the lowest energy knot glueball may be of 59 GeV and 6.9 GeV.

D. Monoball: 0^{++} Vacuum Fluctuation Mode

- ECD predicts another important state, the 0^{++} vacuum fluctuation mode of the monopole condensation. Unlike the glueballs made of the gluons the monoball may not mix with quarkoniums.
- The confirmation of the monoball can be viewed as the experimental verification of the monopole condensation in QCD.
- Candidate: $f_0(1370)$, $f_0(1500)$, $f_0(1710)$,...???

Discussion

- ECD provides a clear picture of the glueballs. But they are intrinsically unstable, and could mix with quarkoniums.
- But the oddballs can exist as pure glueballs, and we should look for the lowest-lying 0^{+-} oddball.
- ECD predicts the hybrid hadrons made of quarks and valence gluons.
- ECD predicts the monoball, and the experimental verification of the monoball becomes a most urgent issue.

- **ECD predicts two types of gluon jets, the neuron-chromon jets and the chromon-chromon jets. We need to re-analize the existing data to confirm this.**

- Challenge: Search for the new states (in particular the monoball and the oddballs) at J Lab and IMP in Lanzhou.

Theoretical and Experimental Challenge Ahead!

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