Understanding observables at the EIC

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Electron Ion Collider (EIC)

- Scientific highlights
  - Nucleon spin and 3D structure
    - Longitudinal spin of nucleon
    - Confined motion of partons inside the nucleon
    - Spatial imaging of gluons and sea quarks
  - Nucleus
    - QCD at extreme parton densities
    - Quarks and gluons in the nucleus
  - Intensity/precision frontier
    - Electroweak studies
    - Physics beyond the SM

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- Scientific highlights
  - Nucleon spin and 3D structure
    - Longitudinal spin of nucleon
    - (Generalized) parton distribution functions
  - Nucleus
    - QCD at extreme parton densities
    - Quarks and gluons in the nucleus
  - Intensity/precision frontier
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Proton spin structure

- Spin sum rule

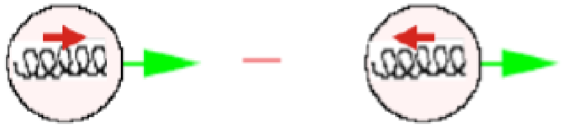
$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

quark spin

gluon helicity

quark/gluon OAM

$$\Delta G = \int dx \Delta g(x) \qquad \Delta g(x) =$$



Proton spin structure

- Spin sum rule

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

  - Jaffe-Manohar decomposition

$$\vec{J} = \int d^3\xi\ \psi^\dagger \frac{\vec{\Sigma}}{2}\psi + \int d^3\xi\ \psi^\dagger \vec{\xi} \times (-i\vec{\nabla})\psi + \int d^3\xi\ \vec{E}_a \times \vec{A}^a + \int d^3\xi\ E_a^i\ \vec{\xi} \times \vec{\nabla} A^{i,a}$$

    - Complete decomposition into quark/gluon spin & orbital
    - Gauge-dependent, but with a clear partonic picture
  - Ji decomposition
    - $J_G = \Delta G + L_G$, gauge- and frame-independent, but not in parton language

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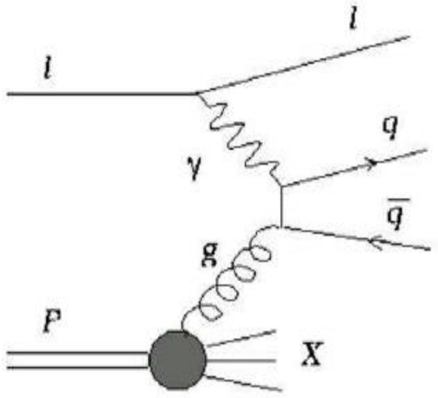
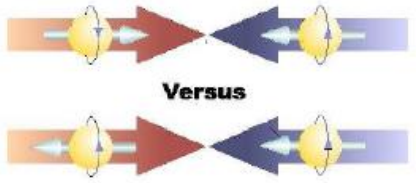
Gluon helicity

- Experimental access to gluon helicity



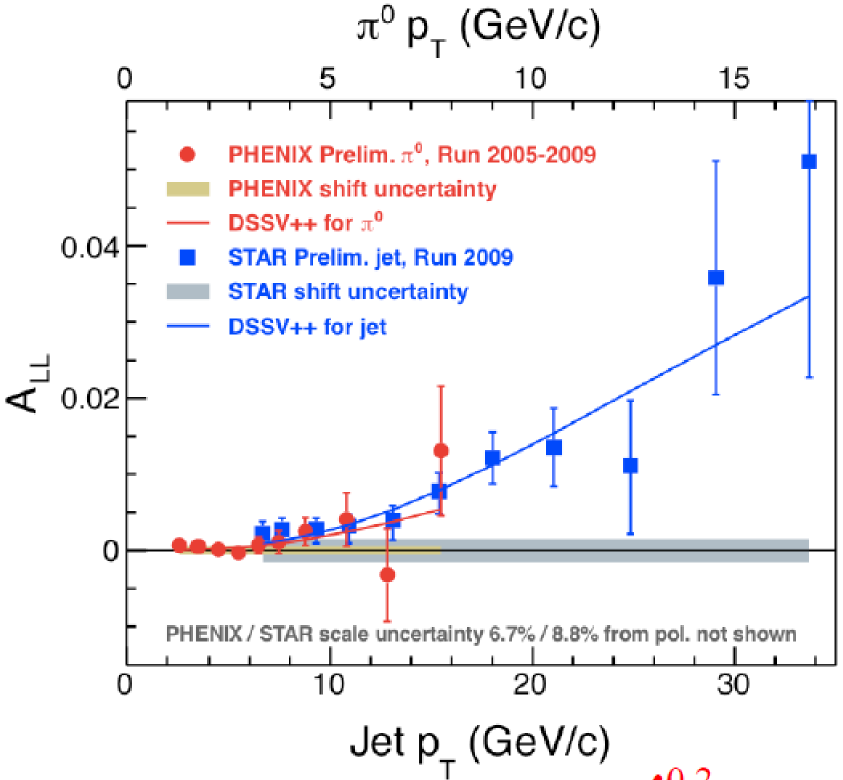


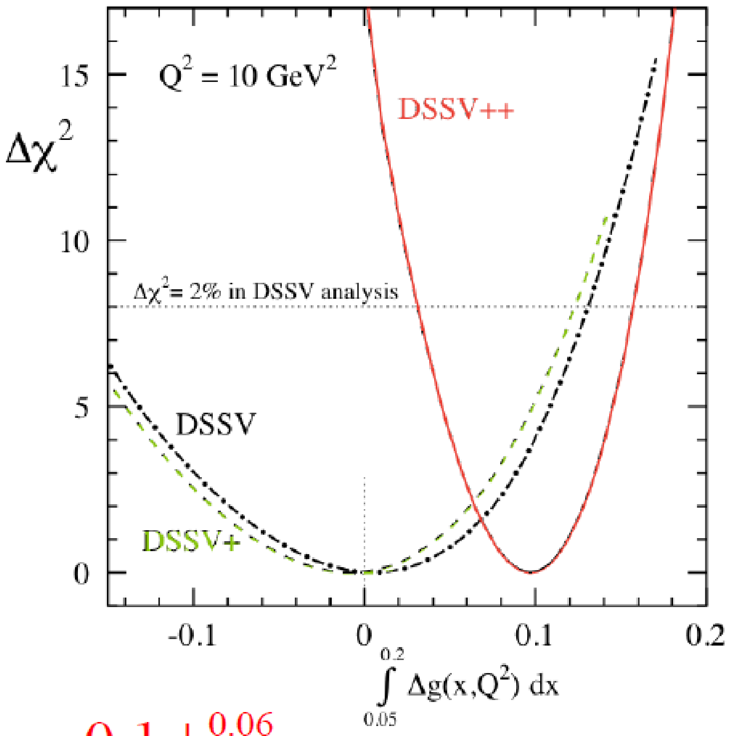
Spin asymmetry

$$A_{LL} \equiv \frac{\sigma_{++} - \sigma_{+-}}{\sigma_{++} + \sigma_{+-}} \equiv \frac{\Delta\sigma}{\sigma}$$

Gluon helicity

- Experimental access to gluon helicity





$$\int_{0.05}^{0.2} \Delta g(x)\,dx = 0.1 \pm^{0.06}_{0.07}$$

Gluon helicity

- Experimental access to gluon helicity
  - RHIC results indicate that gluon helicity contribution to proton spin in the currently explored kinematic region is nonzero, but not yet sufficient to account for the missing 70% (~30% from quark spin)
  - EIC will offer the most powerful tool to precisely quantify how the spin of gluons as well as quarks of various flavors contribute

Gluon helicity

- Confront theory with experiment
  - Light-cone correlation [Manohar PRL, 90']

$$\boxed{\Delta G = \int dx \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle PS| F_a^{+\alpha}(\xi^-) \mathcal{L}^{ab}(\xi^-,0) \tilde{F}_{\alpha,b}^{\ +}(0) |PS\rangle}$$

    - Gauge invariant, but physical meaning unclear
    - In the light-cone gauge $A^+ = 0$, it reduces to the usual notion of gauge boson spin $\vec{E} \times \vec{A}$
    - Difficult to access on the lattice, no way to make a comparison with experiment

Gluon helicity

- Confront theory with experiment
  - Recently another gauge invariant decomposition of QCD angular momentum [Chen, Lü, Sun, Wang and Goldman PRL, 08']

$$\vec{J}_{\rm QCD} = \int d^3x\, \psi^\dagger \frac{\vec{\Sigma}}{2} \psi + \int d^3x\, \psi^\dagger \vec{x} \times (-i\vec{\nabla} - g\vec{A}_{||})\psi + \int d^3x\, \vec{E}_a \times \vec{A}_\perp^a + \int d^3x\, E_a^i\, (\vec{x} \times \vec{\nabla}) A_\perp^{i,a} \, .$$

$$\vec{A} = \vec{A}_\perp + \vec{A}_{||}$$

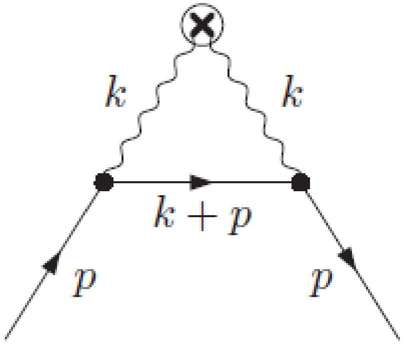
  - Decomposition of vector potential frame-dependent

Gluon helicity

- Confront theory with experiment
  - However [Ji, Zhang and Zhao PRL, 13' and to appear]
    - It becomes physically meaningful for a proton with infinite momentum
      - In particular, ΔG is proven to be equivalent to the infinite momentum limit of gauge invariant gluon spin $\vec{E} \times \vec{A}_{\perp}$
      - Clear physical meaning, static and can be calculated on the lattice
    - Jaffe-Manohar sum rule can be justified to be physical in that all terms in it naturally arise from the infinite momentum limit of gauge invariant operators
    - Offers a practical possibility to calculate parton angular momentum

Proton spin structure

- Confront theory with experiment
  - In practice, the light-cone matrix element can be obtained from the matrix element of gauge invariant static operator at finite momentum and then boost to infinite momentum
  - Caution: the infinite momentum limit is not a smooth limit



$$\langle p, s| \left(\vec{E} \times \vec{A}_\perp\right)^3 |p, s\rangle = \frac{\alpha_S C_F}{4\pi} \left[\frac{5}{3\epsilon} + \frac{5}{3} \ln \frac{\mu^2}{m^2} + \frac{4}{3} \ln \frac{4\vec{p}^2}{m^2} - \frac{1}{9}\right] u^\dagger \Sigma^3 u$$

$$\langle p, s| \left(\vec{E} \times \vec{A}_\perp\right)^3 |p, s\rangle = \frac{\alpha_S C_F}{4\pi} \left(\frac{3}{\epsilon} + 3 \ln \frac{\mu^2}{m^2} + 7\right) u^\dagger \Sigma^3 u$$

finite mom.

infinite mom.

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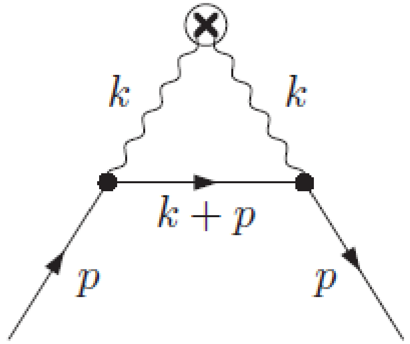

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IR physics remains the same for finite and infinite momentum, only UV is different, therefore a matching is required, which is perturbatively calculable

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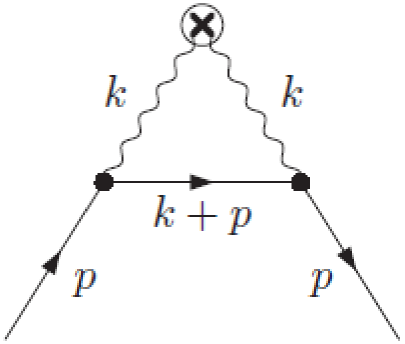

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$$z_{qg}=\frac{\alpha_S C_F}{4\pi}\left(\frac{4}{3}ln\frac{\vec{p}^2}{\mu^2}+const.\right)$$

IR physics remains the same for finite and infinite momentum, only UV is different, therefore a matching is required, which is perturbatively calculable

Lesson from gluon helicity

- Light-cone correlation can be obtained by investigating the (nontrivial) infinite momentum limit of matrix element of time-independent operator at finite momentum
- Gluon helicity is the integral of gluon helicity distribution, is a similar strategy also applicable at the level of parton distribution?

Lesson from gluon helicity

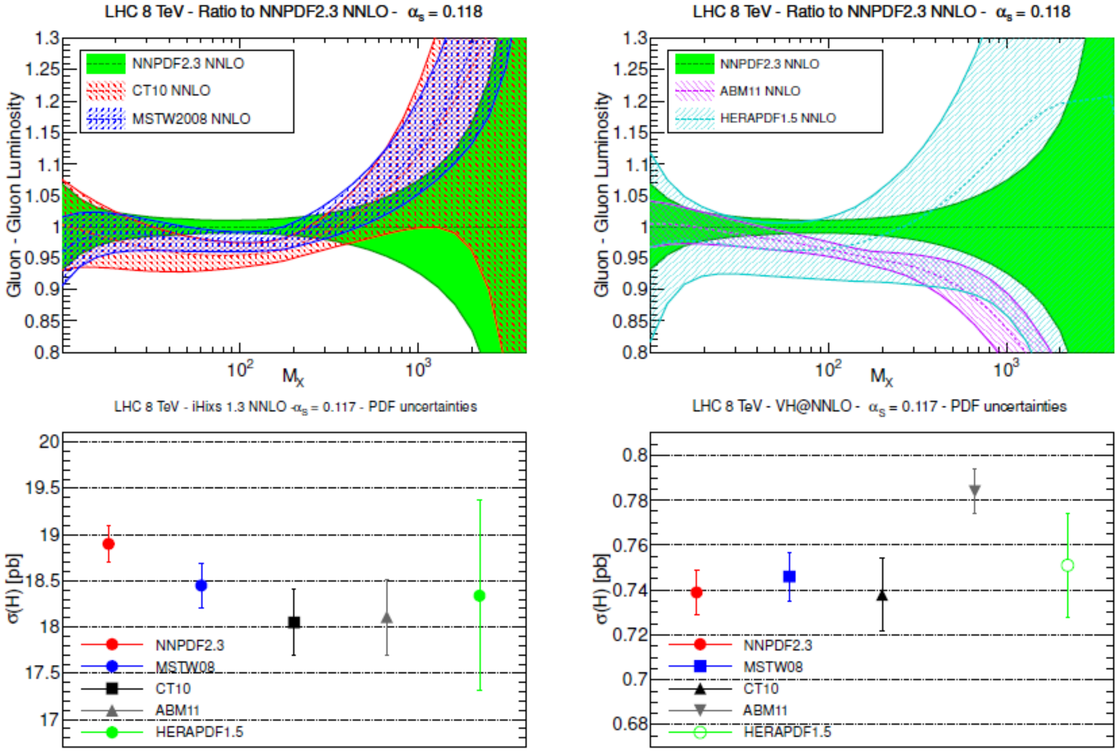
- Light-cone correlation can be obtained by investigating the (nontrivial) infinite momentum limit of matrix element of time-independent operator at finite momentum
- Gluon helicity is the integral of gluon helicity distribution, is a similar strategy also applicable at the level of parton distribution?

YES!

Parton distribution

$$\sigma(P_1, P_2) = \sum_{i,j} \int dx_1 dx_2 f_i(x_1) f_j(x_2) \hat{\sigma}_{ij}(p_1, p_2)$$

- PDFs
  - Extracted from experimental data
  - pQCD evolution
  - Different pdf sets
  - Uncertainty in theoretical predictions



Parton distribution

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- PDFs
  - on the lattice
    - Defined as light-cone correlations, PDFs are intrinsically Minkowskian, they cannot be directly computed using lattice QCD, which is a Euclidean approach
    - Parameterization and parameters determined from lattice computed moments
    - Number of calculable moments limited
  - Lesson from gluon helicity provides another possibility to directly access PDFs

(Quasi) parton distribution

- Operator definition of parton distribution

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P|\overline{\psi}(\xi^-)\gamma^+ \exp\left(-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)\right)\psi(0)|P\rangle$$

  - $P^\mu = (P^0, 0, 0, P^z), \xi^\pm = (t \pm z)/\sqrt{2}$
  - Light-cone correlation, expectation of light-front number operator
- Look instead at an off-light-cone quasi parton distribution [Ji PRL, 13′]

$$\tilde{q}(x,\Lambda,P^z) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{izk^z} \langle P|\overline{\psi}(0,0_\perp,z)\gamma^z \exp\left(-ig\int_0^z dz' A^z(0,0_\perp,z')\right)\psi(0)|P\rangle$$

  - Quark fields separated along z-direction, no time dependence, $x = k^z/P^z$
  - Light-cone distribution can be approached by this up to power suppressed corrections in large momentum limit

(Quasi) parton distribution

- As in gluon helicity, this is plagued by existence of divergences
- How to recover parton distribution from the quasi one? [Ji, Xiong, Zhang and Zhao, 13′]

$$\boxed{\tilde{q}_{\rm NS}(x,\Lambda,P^z) = \int dy\, Z\left(\frac{x}{y},\frac{\Lambda}{P^z},\frac{\mu}{P^z}\right) q_{\rm NS}(y,\mu)}$$

  - Can be viewed as a factorization theorem
  - Soft divergences cancel on both sides
  - Collinear divergences captured by quasi distribution
  - Z factor sensitive to UV physics only, and thus perturbatively calculable

One-loop example

- LO simply a delta function
- NLO (in axial gauge $A^z = 0$)

- Remark:
  - In infinite momentum frame, support of quark density comes from requirement of positivity of cut external legs, or from integration over k^-
    - On-shell partons cannot have negative plus-momentum fraction, 0<x<1
  - In finite momentum frame, support of quasi quark density comes from integration over k^0, and the momentum fraction $-\infty < x < \infty$

One-loop example

- One-loop quasi distribution

$$\tilde{q}(x,\Lambda,P^z) = (1+\tilde{Z}_F^{(1)}(\Lambda,P^z))\delta(x-1) + \tilde{q}^{(1)}(x,\Lambda,P^z) + \dots$$

$$\tilde{q}^{(1)}(x,\Lambda,P^z) = \frac{\alpha_S C_F}{2\pi}\begin{cases} \frac{1+x^2}{1-x}\ln\frac{x}{x-1} + 1 + \frac{\Lambda}{(1-x)^2P^z}\,, & x>1\,, \\ \frac{1+x^2}{1-x}\ln\frac{(P^z)^2}{m^2} + \frac{1+x^2}{1-x}\ln\frac{4x}{1-x} - \frac{4x}{1-x} + 1 + \frac{\Lambda}{(1-x)^2P^z}\,, & 0<x<1\,, \\ \frac{1+x^2}{1-x}\ln\frac{x-1}{x} - 1 + \frac{\Lambda}{(1-x)^2P^z}\,, & x<0\,, \end{cases}$$

$$\tilde{Z}_F^{(1)}(\Lambda,P^z) = \frac{\alpha_S C_F}{2\pi}\int dy \begin{cases} -\frac{1+y^2}{1-y}\ln\frac{y}{y-1} - 1 - \frac{\Lambda}{(1-y)^2P^z}\,, & y>1\,, \\ -\frac{1+y^2}{1-y}\ln\frac{(P^z)^2}{m^2} - \frac{1+y^2}{1-y}\ln\frac{4y}{1-y} + \frac{4y^2}{1-y} + 1 - \frac{\Lambda}{(1-y)^2P^z}\,, & 0<y<1\,, \\ -\frac{1+y^2}{1-y}\ln\frac{y-1}{y} + 1 - \frac{\Lambda}{(1-y)^2P^z}\,, & y<0\,. \end{cases}$$

No logarithmic UV divergence, but ln P^z instead in 0<x<1 region
Momentum fraction not restricted to [0,1]

One-loop example

- One-loop light-cone distribution

$$q(x,\Lambda) = (1 + Z_F^{(1)}(\Lambda) + \dots)\delta(x-1) + q^{(1)}(x,\Lambda) + \dots$$

$$q^{(1)}(x,\Lambda) = \frac{\alpha_S C_F}{2\pi} \begin{cases} 0\,, & x > 1 \text{ or } x < 0\,, \\ \frac{1+x^2}{1-x} \ln \frac{\Lambda^2}{m^2} - \frac{1+x^2}{1-x} \ln (1-x)^2 - \frac{2x}{1-x}\,, & 0 < x < 1\,, \end{cases}$$

$$Z_F^{(1)}(\Lambda) = \frac{\alpha_S C_F}{2\pi} \int dy \begin{cases} 0\,, & y > 1 \text{ or } y < 0\,, \\ -\frac{1+y^2}{1-y} \ln \frac{\Lambda^2}{m^2} + \frac{1+y^2}{1-y} \ln (1-y)^2 + \frac{2y}{1-y}\,, & 0 < y < 1\,, \end{cases}$$

Logarithmic UV divergence in 0<x<1 region
Momentum fraction restricted to [0,1]
Same mass singularity as the quasi distribution

One-loop example

- Matching between quasi and light-cone quark distribution

$$\tilde{q}_{\rm NS}(x, \Lambda, P^z) = \int dy\, Z\left(\frac{x}{y}, \frac{\Lambda}{P^z}, \frac{\mu}{P^z}\right) q_{\rm NS}(y, \mu)$$

- Z factor $Z\left(\xi, \frac{\Lambda}{P^z}, \frac{\mu}{P^z}\right) = \delta(\xi - 1) + \frac{\alpha_s}{2\pi} Z^{(1)}\left(\xi, \frac{\Lambda}{P^z}, \frac{\mu}{P^z}\right) + \ldots$

$$Z^{(1)}(\xi)/C_F = \left(\frac{1+\xi^2}{1-\xi}\right) \ln \frac{\xi}{\xi - 1} + 1 + \frac{1}{(1-\xi)^2} \frac{\Lambda}{P^z}\,, \qquad \xi > 1$$

$$Z^{(1)}(\xi)/C_F = \left(\frac{1+\xi^2}{1-\xi}\right) \ln \frac{(P^z)^2}{\mu^2} + \left(\frac{1+\xi^2}{1-\xi}\right) \ln\left[4\xi(1-\xi)\right] - \frac{2\xi}{1-\xi} + 1 + \frac{\Lambda}{(1-\xi)^2 P^z}\,, \qquad 0 < \xi < 1$$

$$Z^{(1)}(\xi)/C_F = \left(\frac{1+\xi^2}{1-\xi}\right) \ln \frac{\xi - 1}{\xi} - 1 + \frac{\Lambda}{(1-\xi)^2 P^z}\,. \qquad \xi < 0$$

One-loop example

- Matching between quasi and light-cone quark distribution

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- Time-independent quasi distribution and perturbatively calculable matching factor allow a direct computation of parton distribution

Generalized parton distribution

- GPDs defined as off-forward matrix element, e.g.

$$F_q(x,\xi,t) = \int \frac{dz^-}{4\pi} e^{ixp^+z^-} \langle p''|\bar{\psi}(-\frac{z}{2})\gamma^+ L(-\frac{z}{2},\frac{z}{2})\psi(\frac{z}{2})|p'\rangle_{z^+=0,\vec{z}_\perp=0}$$

$$= \frac{1}{2p^+}[H(x,\xi,t)\bar{u}(p'')\gamma^+ u(p') + E(x,\xi,t)\bar{u}(p'')\frac{i\sigma^{+\nu}\Delta_\nu}{2m}u(p')]$$

$$p^\mu = \frac{p''^\mu + p'^\mu}{2}, \qquad \Delta^\mu = p''^\mu - p'^\mu, \qquad t = \Delta^2, \qquad \xi = \frac{p''^+ - p'^+}{p''^+ + p'^+}$$

Generalized parton distribution

- GPDs defined as off-forward matrix element
  - Encode more information about nucleon structure than parton distributions
  - Hybrid of parton distributions, form factors and distribution amplitudes
  - Provide three-dimensional spatial picture of nucleons
  - Reveal spin structure of nucleons
  - Accessible in exclusive processes, e.g. DVCS, meson production

Generalized parton distribution

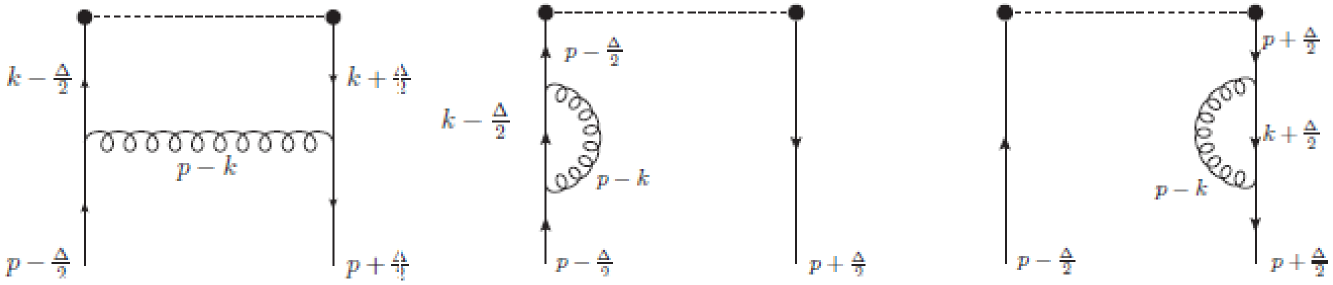
- GPDs defined as off-forward matrix element
  - Light-cone GPDs can be studied from large momentum limit of quasi ones

$$\begin{aligned}\mathcal{F}_q(x,\xi,t,p^z) &= \int \frac{dz}{4\pi} e^{-izk^z} \langle p''|\bar{\psi}(-\frac{z}{2})\gamma^z L(-\frac{z}{2},\frac{z}{2})\psi(\frac{z}{2})|p'\rangle \\ &= \frac{1}{2p^z}[\mathcal{H}(x,\xi,t,p^z)\bar{u}(p'')\gamma^z u(p') + \mathcal{E}(x,\xi,t,p^z)\bar{u}(p'')\frac{i\sigma^{z\nu}\Delta_\nu}{2m}u(p')]\end{aligned}$$

$$p^\mu = (p^0,0,0,p^z), \qquad \xi = \frac{p''^z - p'^z}{p''^z + p'^z} = \frac{\Delta^z}{2p^z}$$

Generalized parton distribution

- One-loop matching
  - LO H($\mathcal{H}$) simply a delta function, E=$\mathcal{E}$=0
  - NLO (in axial gauge $A^z = 0$)



Generalized parton distribution

- One-loop matching

$$\mathcal{H}^{(1)}(x,\xi,t,\mu,p^z) =$$

$$\frac{\alpha_S C_F}{2\pi}\begin{cases} \frac{(\xi^2+x)\ln\frac{\xi+x}{x-\xi}}{2\xi(\xi^2-1)} + \frac{(-2\xi^2+x^2+1)\ln\frac{(x-1)^2}{x^2-\xi^2}}{2(\xi^2-1)(x-1)} + \frac{\mu}{p^z(1-x)^2} & x<-\xi \\ \frac{x+\xi}{2\xi(1+\xi)}\left(1+\frac{2\xi}{1-x}\right)\ln\frac{p_z^2}{-t} + \frac{1+x^2-2\xi^2}{(1-x)(1-\xi^2)}\ln[2(\xi+1)] - \frac{x+\xi^2}{\xi(\xi^2-1)}\ln 4\xi & \\ +\frac{x+\xi}{(1+\xi)(x-1)} + \frac{\mu}{p^z(1-x)^2} & -\xi<x<\xi \\ \frac{1+x^2-2\xi^2}{(1-x)(1-\xi^2)}\ln\frac{p_z^2}{-t} + \frac{1+x^2-2\xi^2}{2(1-x)(1-\xi^2)}\left(\ln[16(x^2-\xi^2)] - 2\ln\frac{1-x}{1-\xi^2}\right) & \\ -\frac{x+\xi^2}{2\xi(1-\xi^2)}\ln\frac{x-\xi}{x+\xi} - \frac{2(x-\xi^2)}{(1-x)(1-\xi^2)} + \frac{\mu}{p^z(1-x)^2} & \xi<x<1 \\ -\frac{(\xi^2+x)\ln\frac{\xi+x}{x-\xi}}{2\xi(\xi^2-1)} - \frac{(-2\xi^2+x^2+1)\ln\frac{(x-1)^2}{x^2-\xi^2}}{2(\xi^2-1)(x-1)} + \frac{\mu}{p^z(1-x)^2} & x>1, \end{cases}$$

Generalized parton distribution

- One-loop matching

$$H^{(1)}(x,\xi,t,\mu)=\frac{\alpha_S C_F}{2\pi}\begin{cases}\frac{x+\xi}{2\xi(1+\xi)}(1+\frac{2\xi}{1-x})\ln\frac{\mu^2}{-t}+\frac{x+\xi^2}{2\xi(1-\xi^2)}\ln\frac{4\xi^2}{\xi^2-x^2} & \\ -\frac{1+x^2-2\xi^2}{2(1-x)(1-\xi^2)}(\ln\frac{(1-x)^2}{(1+\xi)^2}-\ln\frac{\xi-x}{x+\xi}) & -\xi<x<\xi \\ \frac{1+x^2-2\xi^2}{(1-x)(1-\xi^2)}\ln\frac{\mu^2}{-t}-\frac{1+x^2-2\xi^2}{(1-x)(1-\xi^2)}\ln\frac{(1-x)^2}{(1-\xi^2)} & \xi<x<1 \\ 0 & \text{otherwise,}\end{cases}$$

  - Quasi GPD and light-cone GPD have the same evolution kernel in DGLAP and ERBL region
  - Matching transforms $\ln p^z$ dependence to $\ln \mu$ dependence

Generalized parton distribution

- One-loop matching

$$\mathcal{E}^{(1)}(x,\xi,t,\mu,p^z)=E_1(x,\xi,t,\mu)=\frac{\alpha_S C_F}{2\pi}\frac{m^2}{-t}\begin{cases}\frac{2(x-\xi)}{1+\xi}\ln\left(\frac{-t}{m^2}\right)+\frac{2\xi(1+x)}{1-\xi^2}\ln\left(\frac{4\xi^2(1-x)^2}{(1+\xi)^2(\xi^2-x^2)}\right) & \\ +\frac{2\left(x+\xi^2\right)}{1-\xi^2}\ln\left(\frac{x+\xi}{\xi-x}\right) & -\xi<x<\xi \\ \frac{4\left(x+\xi^2\right)}{1-\xi^2}\ln\left(\frac{-t}{m^2}\right)+\frac{4\xi(1+x)}{1-\xi^2}\ln\left(\frac{1-\xi}{1+\xi}\right) & \xi<x<1 \\ 0 & \text{otherwise,}\end{cases}$$

  - No matching is required for GPD E up to one-loop and leading power accuracy, therefore the light-cone E can be smoothly approached by the quasi one in the large momentum limit

Connecting lattice to continuum

- Extracting pdf or other light-cone quantities from lattice simulation of quasi distribution requires matching between lattice result at finite momentum and continuum result at infinite momentum
  - This can be divided into two parts:
    - Lattice and continuum matching at finite momentum
    - Continuum matching between finite and infinite momentum
  - The matching (as a illustration) so far is the second matching
  - Same leading logarithm expected for lattice

Summary

- Now it is practically possible to calculate parton angular momentum contribution to proton spin
- Also parton distribution or other light-cone quantities can be studied by investigating a related time-independent quantity at large momentum
  - Space-like correlation for parton distribution
  - Allows calculation of parton distribution and related quantities on the lattice
  - Matching required to connect quasi and light-cone distributions, but it depends on UV physics only, and therefore is perturbatively calculable

BACKUP SLIDES