The repulsive core of the nucleon potential from QCD

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- Introduction: NN potential and repulsive core
- NN potential from QCD
- Explaining the repulsive core from OPE and RG
- More flavours and/or nucleons

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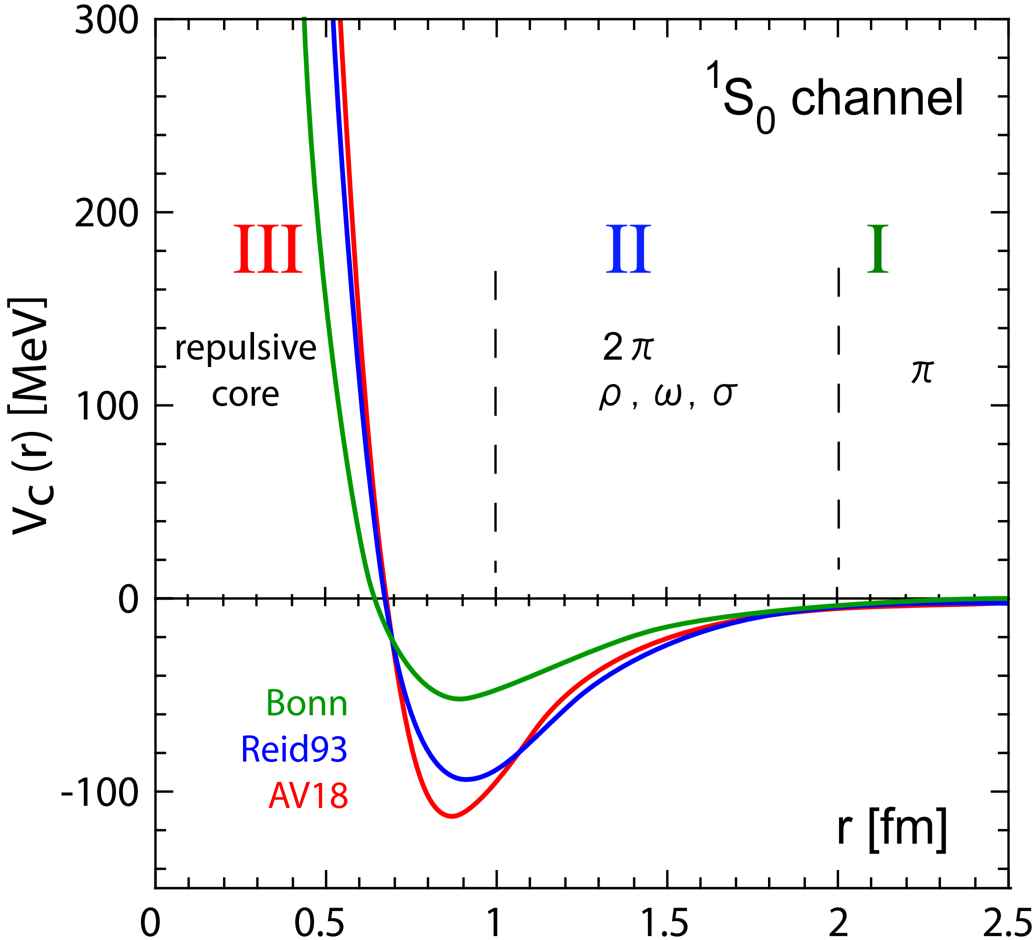
Nucleon potential and repulsive core

Modern nucleon-nucleon potential



I Long range part

one pion exchange potential (OPEP)

II Medium range part

σ, ρ, ω exchange

2π exchange

III Short range part

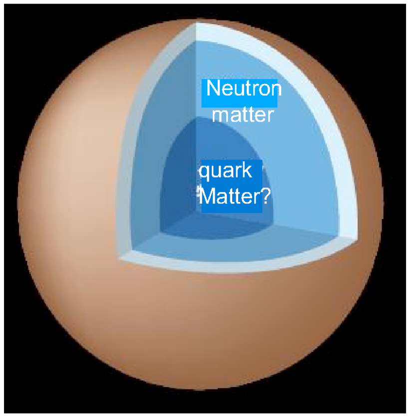
repulsive core (RC)

R. Jastrow(1951)

quark ?

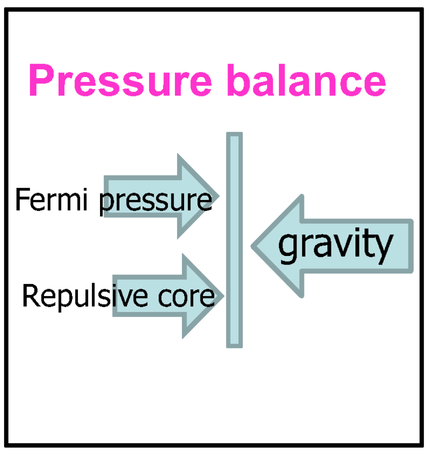
Bonn: Machleidt, Phys.Rev. C63('01)024001
Reid93: Stoks et al., Phys. Rev. C49('94)2950.
AV18: Wiringa et al., Phys.Rev. C51('95) 38.

- Structure of neutron star

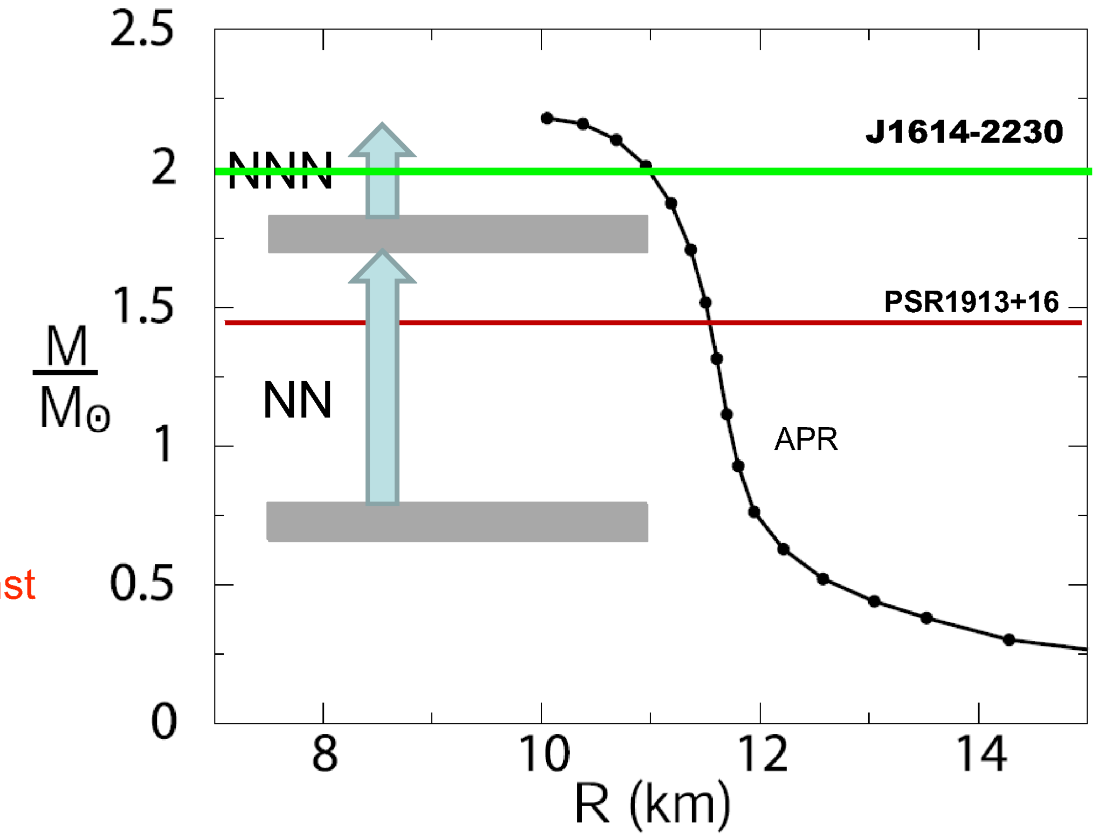


($\rho_{max} \sim 6\rho_0$)

sustains neutron stars against gravitational collapse



Maximum mass of neutron stars



(courtesy of Sinya Aoki)

Nucleon potential from lattice QCD

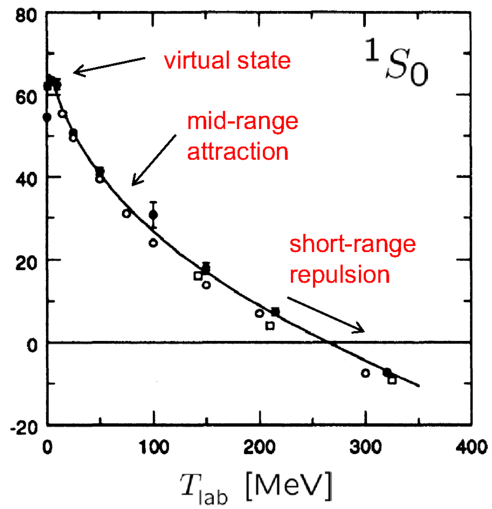
Potentials in QCD ?

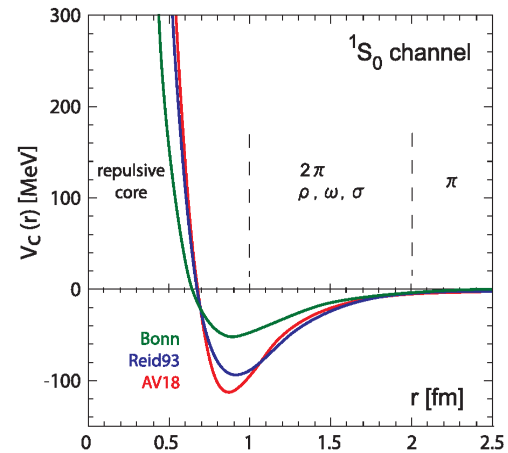
What are “potentials” in quantum field theories such as QCD ?

“Potentials” themselves can NOT be directly measured. analogy: running coupling in QCD

scheme dependent, Unitary transformation

experimental data of scattering phase shifts → potentials, but not unique





useful to “understand” physics

analogy: asymptotic freedom

“Potentials” are useful tools to extract observables such as scattering phase shift. → One may adopt a convenient definition of potentials as long as they reproduce correct physics of QCD.

Our strategy in lattice QCD

Full details: Aoki, Hatsuda & Ishii, PTP123(2010)89.

Consider "elastic scattering"

$$NN \to NN \qquad \cancel{NN \to NN + \text{ others}} \quad \cancel{(NN \to NN + \pi, NN + \bar{N}N, \cdots)}$$

energy $\quad W_k = 2\sqrt{\mathbf{k}^2 + m_N^2} < W_{\rm th} = 2m_N + m_\pi \quad$ Elastic threshold

Quantum Field Theoretical consideration

- S-matrix below inelastic threshold. Unitarity gives $\quad S = e^{2i\delta}$

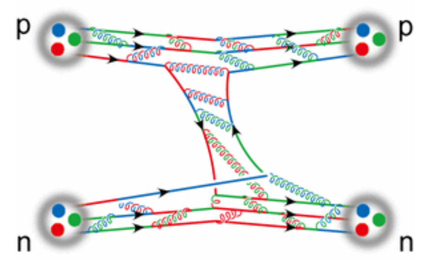
Step 1

define (Equal-time) Nambu-Bethe-Salpeter (NBS) Wave function

Spin model: Balog et al., 1999/2001

$$\varphi_{\mathbf{k}}(\mathbf{r}) = \langle 0 | N(\mathbf{x}+\mathbf{r}, 0) N(\mathbf{x}, 0) | NN, W_k \rangle$$

QCD eigen-state

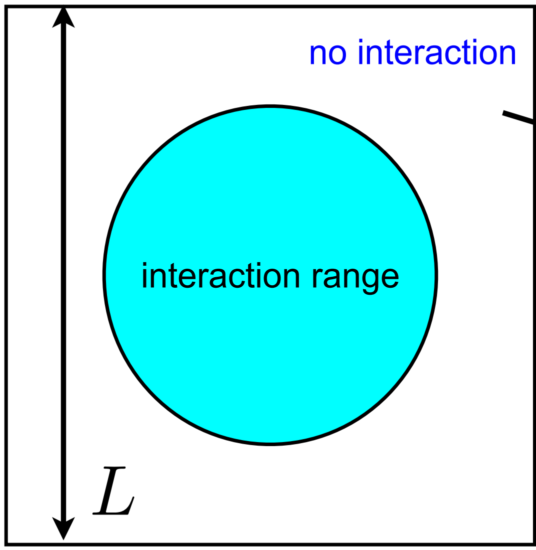


$N(x) = \varepsilon_{abc} q^a(x) q^b(x) q^c(x)$: local operator

"scheme"

Asymptotic behavior of NBS wave function

Lin et al., 2001; CP-PACS, 2004/2005



$$r = |\mathbf{r}| \to \infty$$

$$\varphi_{\mathbf{k}}^{l} \to A_l \frac{\sin(kr - l\pi/2 + \delta_l(k))}{kr}$$

partial wave

scattering phase shift (phase of the S-matrix by unitarity) in QCD !

NBS wave function ⇔ scattering wave function in quantum mechanics

cf. Luescher's finite volume method   allowed k at L ⇒ $\delta_l(k_n)$

Step 2

- Define a **potential** through application of a Schrödinger operator:

$$V_{\mathbf{k}}(\mathbf{r}) = \frac{\left[E_{\mathbf{k}} + \frac{1}{\mu}\nabla^2\right]\varphi_{\mathbf{k}}(\mathbf{r})}{\varphi_{\mathbf{k}}(\mathbf{r})}$$

where $E_{\mathrm{k}} = \frac{\mathrm{k}^2}{\mu}$ is the kinetic energy. Note the energy dependence of potential!

Step 3

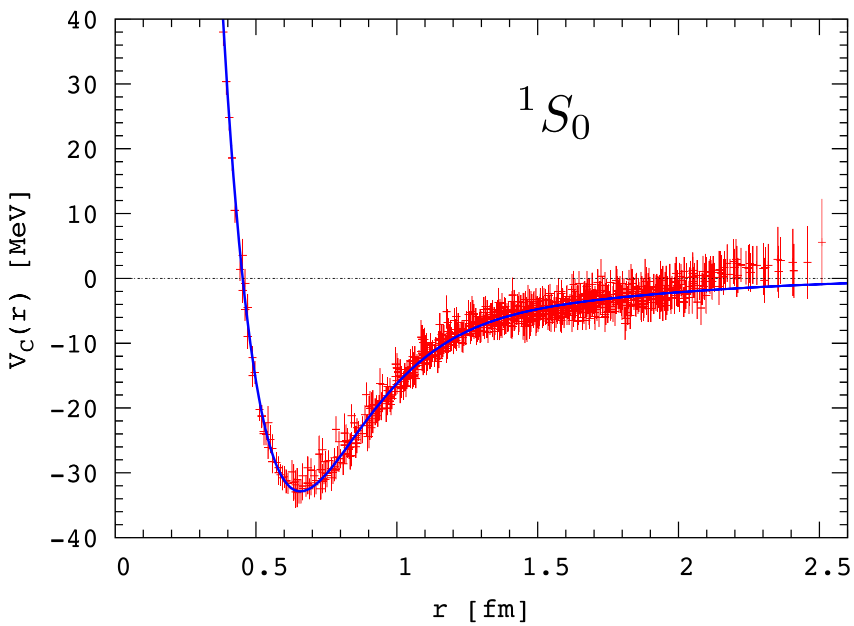
- Solve the Schrödinger equation with this (zero energy) potential in infinite volume to find phase shifts and possible bound states below the inelastic threshold.

Qualitative features of NN potential reproduced!

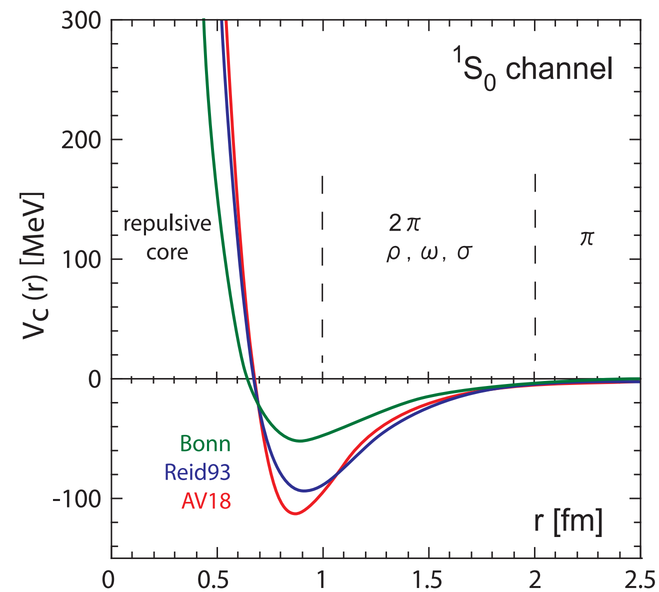
NN potential

2+1 flavor QCD, spin-singlet potential (PLB712(2012)437)

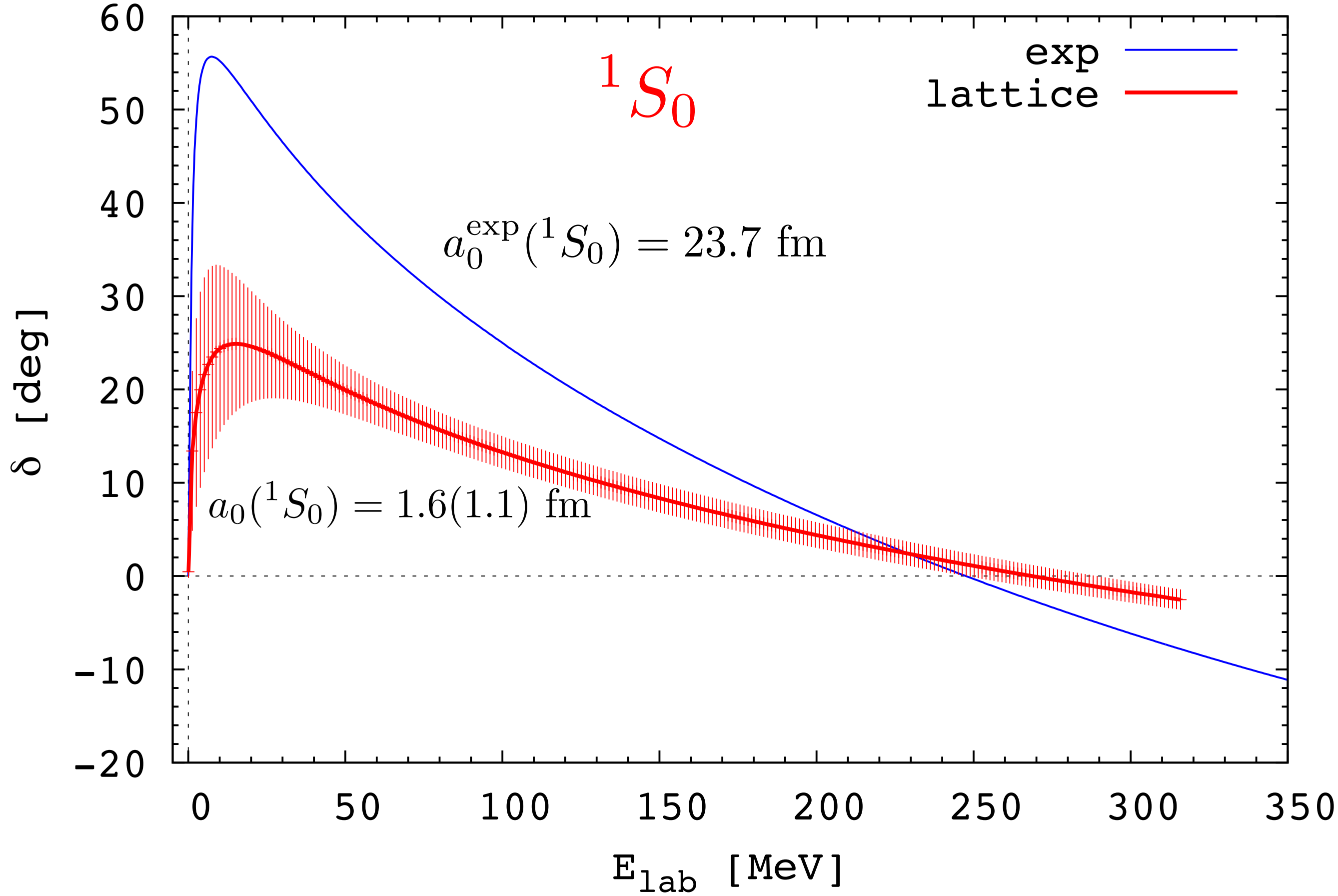
a=0.09fm, L=2.9fm $m_\pi \simeq 700$ MeV



phenomenological potential



NN potential → phase shift



It has a reasonable shape. The strength is weaker due to the heavier quark mass.

Need calculations at physical quark mass.

(courtesy of Sinya Aoki)

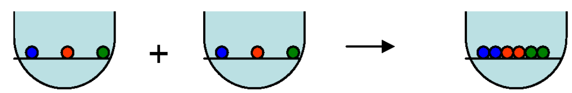
Explaining the repulsive core from OPE and RG

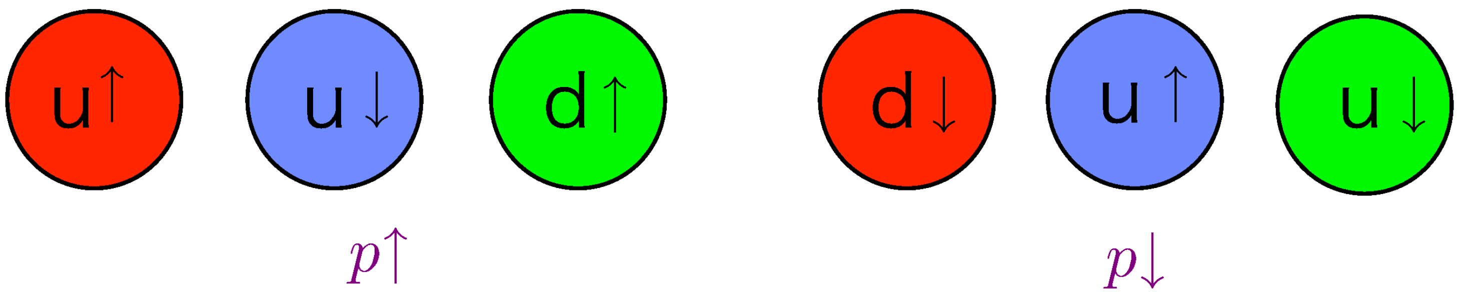
Origin of the repulsive core ?

quarks are "fermion" two can not occupy the same position. ("Pauli principle")

they have 3 colors(red,blue,green), 2 spin($\uparrow \downarrow$), 2 flavors(up,down)

6 quark can occupy the same position





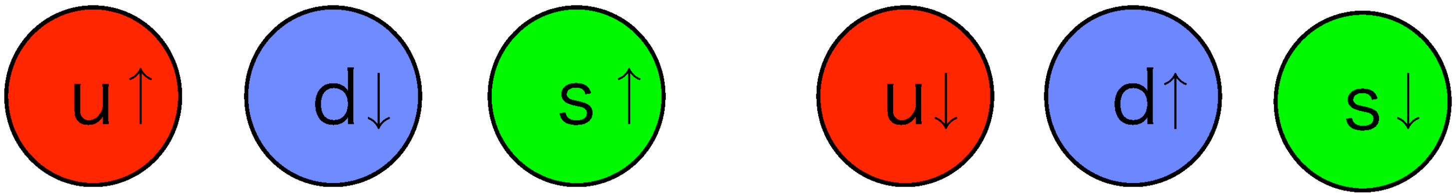
but allowed color combinations are limited + interaction among quarks

?

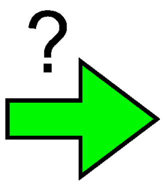
repulsive core ?

What happen if strange quarks are added ?

$\Lambda(uds)$ - $\Lambda(uds)$ interaction

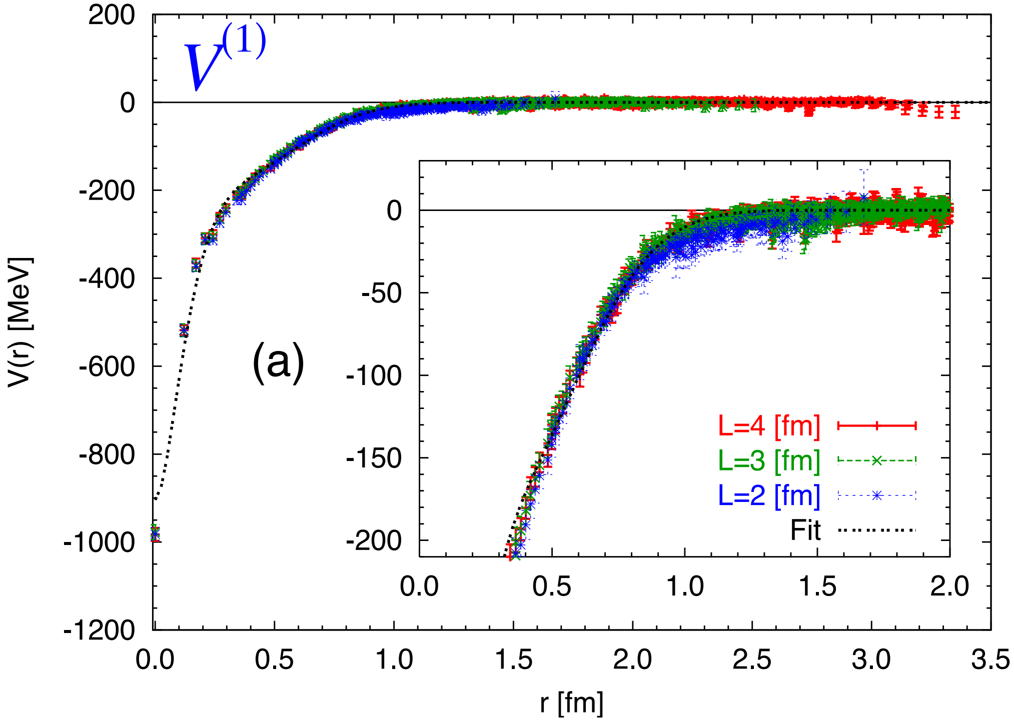


all color combinations are allowed



no repulsive core ?

Inoue *et al.* (HAL QCD Coll.), Progress of Theoretical Physics 124(2010)591



flavor SU(3) limit

$$m_u = m_d = m_s$$

Indeed, attractive instead of repulsive core appears.

This suggests that "Pauli principle" is important for the repulsive core.

Force is attractive at all distances. Bound state ?

(courtesy of Sinya Aoki)

BASIC IDEA

NBS wave function:

$$\varphi^{E}(\mathbf{x}) = \langle 0|N(\mathbf{x},0)N(\mathbf{0},0)|2\mathrm{N},E\rangle$$

short distance operator product expansion:

$$N(\mathbf{x},0)N(\mathbf{0},0) \approx \sum_{A} C_A(\mathbf{x})\mathcal{O}_A(\mathbf{0},0)$$

UV/IR separation:

$$\varphi^{E}(\mathbf{x}) = \sum_{A} C_A(\mathbf{x})\langle 0|\mathcal{O}_A(\mathbf{0},0)|2\mathrm{N},E\rangle$$

solving the renormalization group (asymptotic freedom):

$$C_A(\mathbf{x}) \approx \ln\left(\tfrac{r_o}{r}\right)^{\beta_A} d_A \qquad r = |\mathbf{x}| \qquad d_A = \mathrm{const.}$$

factorization:

$$\varphi^{E}(\mathbf{x}) \approx \sum_{A} \ln\left(\tfrac{r_o}{r}\right)^{\beta_A} D_A \qquad D_A = d_A\langle 0|\mathcal{O}_A(\mathbf{0},0)|2\mathrm{N},E\rangle$$

anomalous dimensions:

$$\beta_A = \frac{\gamma_A^{(1)} - 2\gamma_N^{(1)}}{2\beta_o} \qquad \beta_o = \frac{1}{16\pi^2}\left(11 - \tfrac{2}{3}n_f\right)$$

It remains to calculate the one-loop anomalous dimensions!

potential:

$$V(\mathbf{x}) = E + \frac{1}{\mu}\frac{\nabla^2\varphi^E(\mathbf{x})}{\varphi^E(\mathbf{x})} \qquad \nabla^2 = \frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}r^2\frac{\mathrm{d}}{\mathrm{d}r} - \frac{\hat{L}^2}{r^2}$$

(S-wave case: $\hat{L} = 0$)

β_1: max of all β_C (many operators, mixing) β_2: second largest

(1) $\beta_1 \neq 0$:

$$V(r) \approx -\frac{\beta_1}{r^2(-\log r)}$$

$\beta_1 > 0$: attractive $\beta_1 < 0$: repulsive

(2) $\beta_1 = 0$, second largest: $\beta_2 < 0$

$$V(r) \approx \left(\frac{D_2^E}{D_1^E}\right)\frac{-\beta_2}{r^2(-\log r)^{1-\beta_2}}$$

$\left(\frac{D_2^E}{D_1^E}\right) < 0$: attractive $\left(\frac{D_2^E}{D_1^E}\right) > 0$: repulsive

6-quark operators

baryon creation operators:

$$B^{fgh}_{\alpha\beta\gamma}(\mathbf{x}) = B^{F}_{\Gamma}(\mathbf{x}) = \epsilon^{abc} q^{a,f}_{\alpha}(\mathbf{x}) q^{b,g}_{\beta}(\mathbf{x}) q^{c,h}_{\gamma}(\mathbf{x})$$

$$\begin{array}{rclc} a,b,c & = & 1,\ldots,3 & \text{colour} \\ \alpha,\beta,\gamma & = & 1,\ldots,4 & \text{spinor} \\ f,g,h & = & 1,\ldots,N_f & \text{flavour} \end{array}$$

multi-index:

$$A \sim (F_1, F_2, \Gamma_1, \Gamma_2)$$

6-quark local operator:

$$\mathcal{O}_A(\mathbf{x}) = B^{F_1}_{\Gamma_1}(\mathbf{x}) B^{F_2}_{\Gamma_2}(\mathbf{x})$$

not all independent!

operator mixing: anomalous dimensions = matrix diagonalization problem
matrix: block diagonal (spin, isospin)

$N_f = 2$: nucleon problem   largest block: 30×30 matrix

Anomalous dimensions in QCD

for $^1S_0$ and $^3S_1$ two-nucleon states: $\beta_1 = 0!$

$$\beta_2 = -\frac{3}{33-2n_f} \quad \left(^1S_0\right) \qquad\qquad \beta_2 = -\frac{1}{33-2n_f} \quad \left(^3S_1\right)$$

Chiral effective theory (non-perturbative) $\longrightarrow$

$$\frac{D_2^E}{D_1^E} = \frac{2E}{\mu} \quad \left(^1S_0\right) \qquad\qquad \frac{D_2^E}{D_1^E} = \frac{2\mu}{E} \quad \left(^3S_1\right)$$

$$E = \sqrt{\mathbf{k}^2 + \mu^2}$$

Both cases: $D_2^E / D_1^E \approx 2$ (weak energy dependence): REPULSION!

More flavours and/or nucleons

- More flavours (baryon-baryon potential)
  Most $\beta_C < 0$, but $\beta_1 > 0$ (in the flavour singlet channel).
  Argument based on Pauli principle works!
  Largest block size: 123×123

- More nucleons (three-nucleon forces)
  All $\beta_C < 0$, only repulsive channels $\longrightarrow$ universal repulsion.
  Unambiguous model-independent result.
  Largest block size: 117×117

- Three-baryon forces
  Most demanding computationally, but most inconclusive:
  there are also (a few) attractive channels.
  Largest block size: 1518×1518

PROBLEMS

- Energy dependence of the potential (weak)
  Solved by introducing nonlocal potential $\longrightarrow$ derivative expansion

- Choice of nucleon creating operator $\longrightarrow$ local three-quark colour singlet natural

- Scale at which asymptotic behaviour sets in ?

Thank you!