Gauge-invariance of QCD and the mechanism for generating quark confinement

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OUTLINE

1.Basic properties of QCD

2. Qurrk confinement phenomenon

*** Confinement potential from lattice QCD**

3. Exploring confinement mechanism ---

basic approaches

4. Nonperturbative QCD approach

5. Gauge invariance and interacting verties in QCD

*** BRST symmetry and Slavnov-Taylor identity**

***Transverse symmetry transformation and transverse STI**

6. Gauge-invariant constraint and the mechanism for generating a linear confinement potential

1. Basic properties of QCD

- Quarks and gluons----

  *fundamental particles of costituting hadrons

  *fundamental fields of QCD (Quantum Chromodynamics 量子色动力学)

满足定域规范不变性的经典的色动力学拉氏密度为:

$$\mathscr{L}^{(0)}_{\mathrm{QCD}} = -\frac{1}{4}F^{\mu\nu}_{a}F^{a}_{\mu\nu} + \sum_{f}\bar{\psi}^{(f)}_{\alpha}(\mathrm{i}\gamma_{\mu}D^{\mu}_{\alpha\beta} - m^{(f)}\delta_{\alpha\beta})\psi^{(f)}_{\beta}$$

$$\left.\begin{aligned} F^{a}_{\mu\nu} &= \partial_{\mu}A^{a}_{\nu} - \partial_{\nu}A^{a}_{\mu} + gf^{abc}A^{b}_{\mu}A^{c}_{\nu} \\ D_{\mu} &= \partial_{\mu} - \mathrm{i}gA^{a}_{\mu}\frac{\lambda^{a}}{2} \end{aligned}\right\}$$

Basic properties of QCD

------Asymptotical freedom, dynamical chiral symmetry breaking, quark confinement

- QCD running coupling

$$\alpha_S(\mu^2) = \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda^2_{QCD})}$$

（1）在高能标度或大动量转移时的渐近自由。这已在理论上得到证明。这可以从 QCD 跑动耦合（见式（1.166））直观地理解这一图像：当 $Q^2 \gg \Lambda^2_{QCD}$，QCD 耦合 $\alpha_s \ll 1$，意味着夸克间的相互作用变得很弱。

（2）色禁闭。从跑动耦合 $\alpha_s(Q^2)$ 可看到，当 $Q^2 \approx \Lambda^2_{QCD}$，$\alpha_s(Q^2) \to \infty$。这就是通常说的红外奴役的表现，它与自然界中未观测到带色的客体相联系。当然，这不是色禁闭的证明。色禁闭意味着所观测到的强相互作用物质如强子态、原子核都是夸克、胶子组成的色单态。色禁闭机制的研究是当今 QCD 理论中最具挑战性的突出问题。

2.Quark Confinement Phenomenon

- Quark and gluon---

  fundamental particle of constituting hadron

  Proton: u u d

  Meson: quark-antiquark

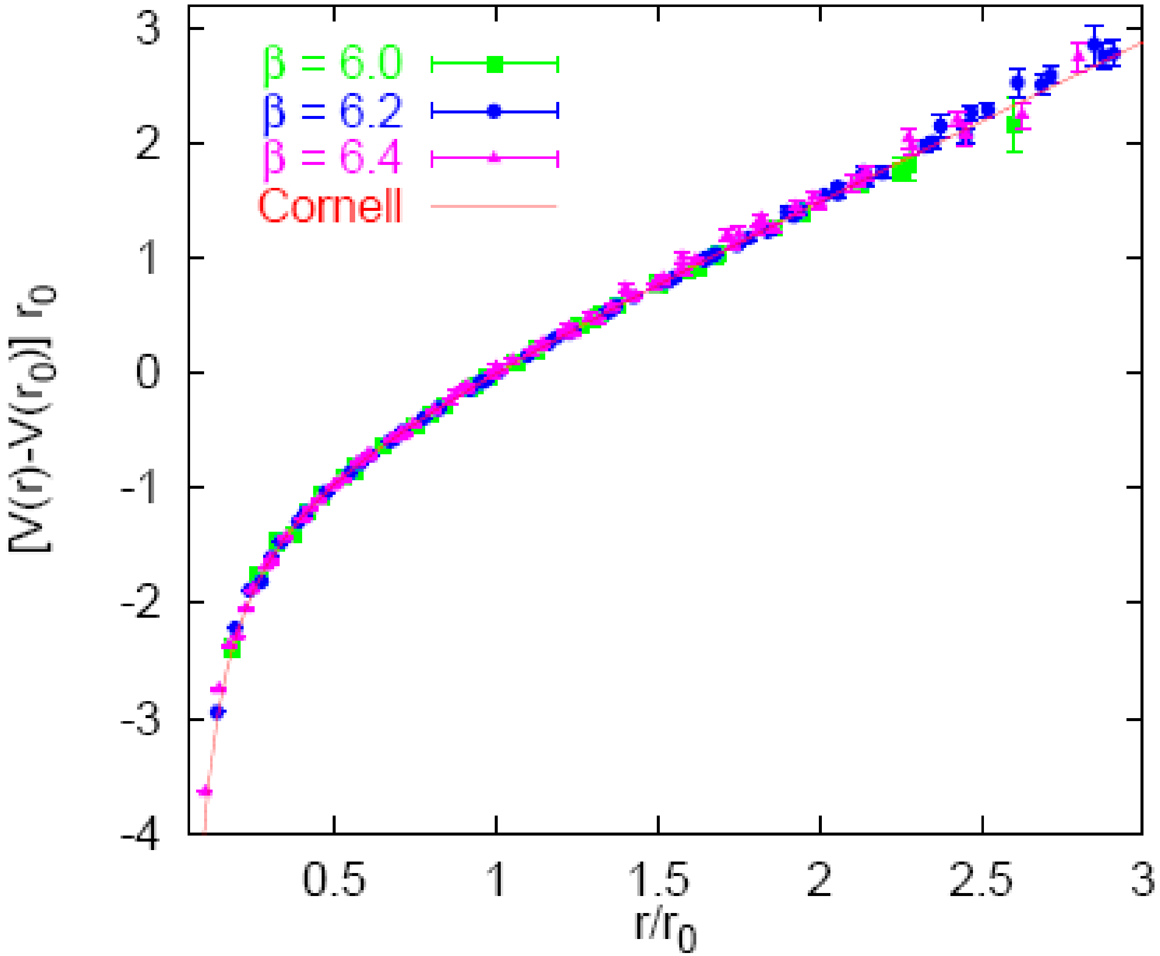
- Quark confinement phenomenon---

  * All hadron states and physical observables are color-singlets

  * Isolated quarks (gluons) have not been observed in nature

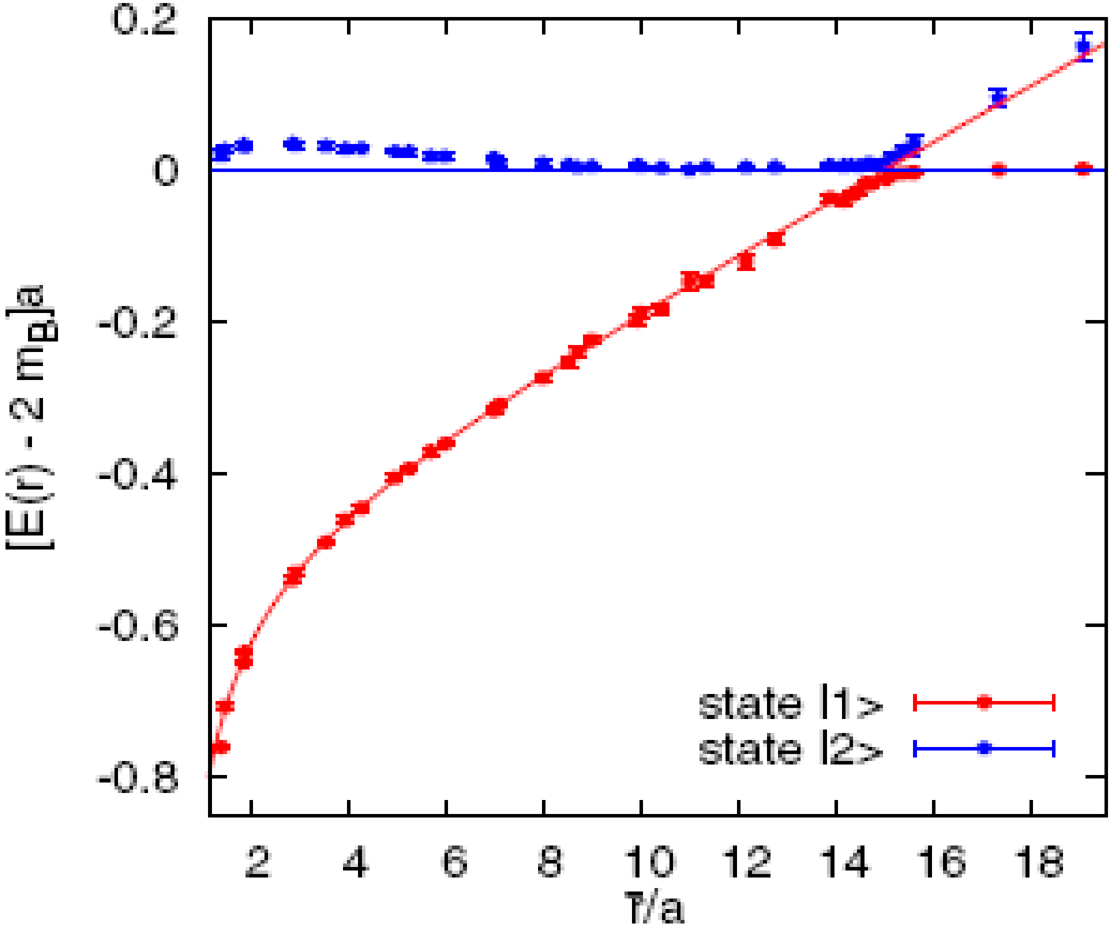
  Confinement mechanism—one of top challenge problems in 20 & also 21 century

Confinement potential from Lattice QCD 1



Confinement potential from Lattice QCD 2

PHYSICAL REVIEW D 71, 114513 (2005)



Confinement potential from Lattice QCD 3

这里要求 T 足够大($T \geqslant R$).计算得到类库仑型势与线性型禁闭势的叠加[141a]

$$V(R) = \frac{4}{3}\frac{\alpha_s}{R} + bR \qquad (6.20)$$

在足够大的分离距离 R_c(~1.2 fm)发生弦破裂,与此相伴产生一海夸克-反夸克对.由拟合 Monte Carlo 数值计算的结果得到一屏蔽势[141b]

$$V(R) = \left(-\frac{a}{R} + \sigma R\right)\frac{1 - e^{\mu R}}{\mu R} \qquad (6.21)$$

其中 $\sigma = (400\ \mathrm{MeV})^2$, $a = 0.21 \pm 0.01$, $\mu^{-1} = 0.9 \pm 0.2$ fm.

3.Explore confinement mechanism
----basic approaches

- QCD vacuum property and colour confinement

— A dual Meissner effect in a condensate of magnetic monopoles

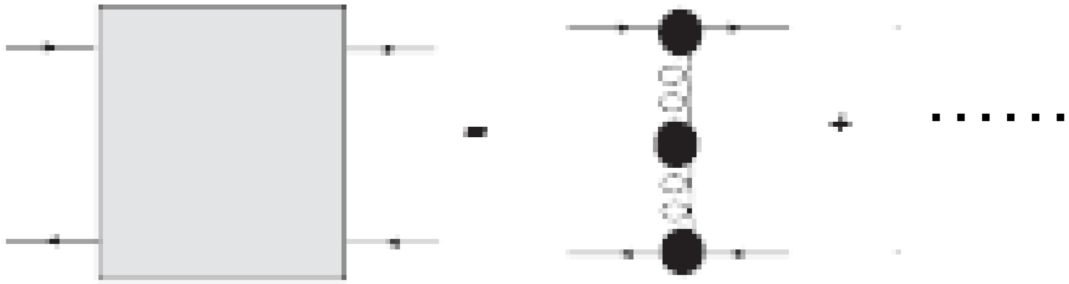
— Hard to prove from QCD

- QCD nonperturbative interactions and colour confinement

---Infrared behavior of QCD Green's functions

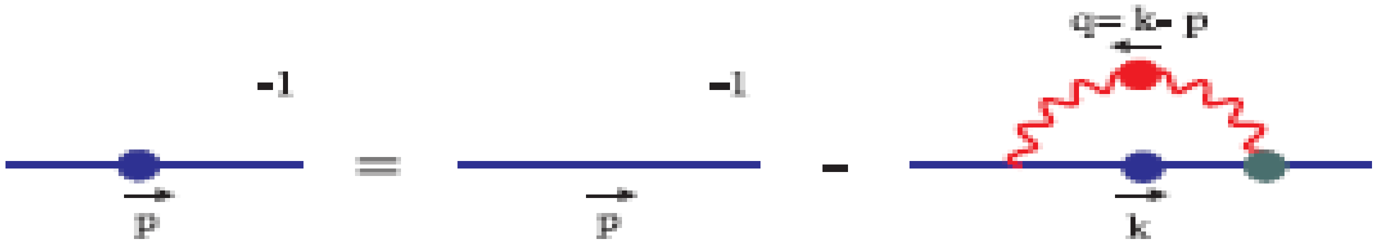
---Infrared slavery： confinement is caused by infrared singularity

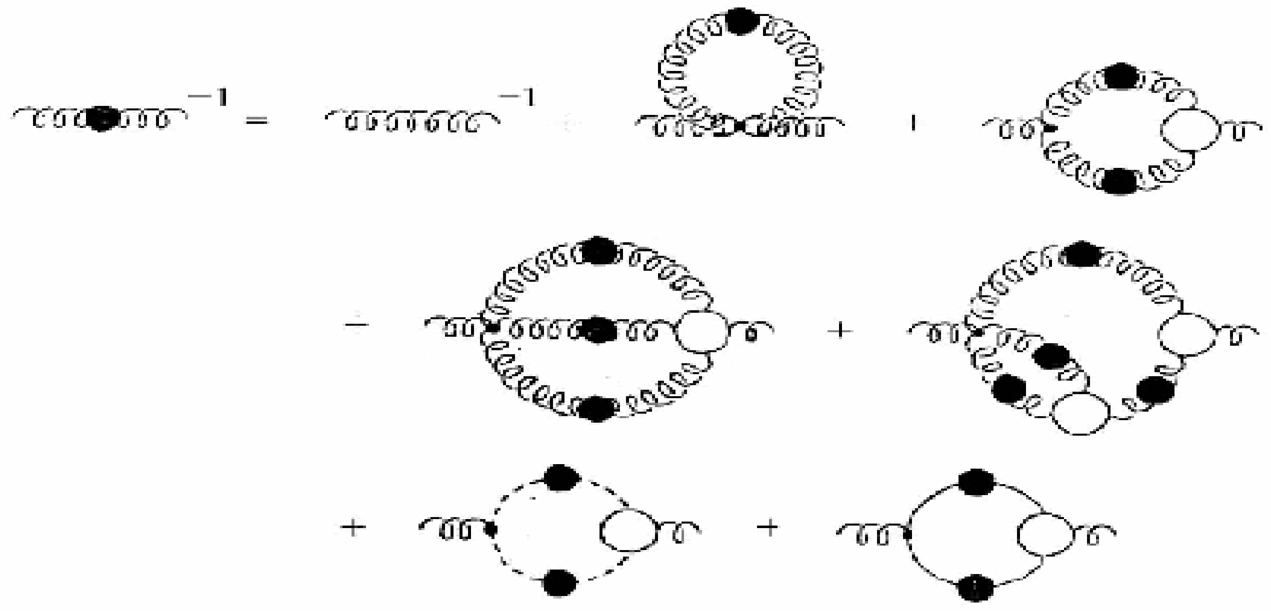
4. Nonperturbative QCD approach to exploring quark confinement dynamics



- Calculate interacting potential between quark and quark(anti-quark)
- Use Dyson-Schwinger equations for QCD Green functions

DSE quations for QCD propagators





* How to determine QCD Green's functions

- Equations of motion for Green's functions

  --- Dyson-Schwinger (DS) equations

  (a set of integral equations)

- Constraint relations among Green's functions

  --- the Slavnov-Taylor (Ward-Takahashi) identities for the QCD vertices

  (a set of differential equations)

5. The Gauge invariance of QCD & the Slavnov-Taylor identities

- BRST symmetry

$$\delta\psi = \mathrm{i}g\omega c^a t^a \psi, \ \delta\bar{\psi} = -\mathrm{i}g\,\bar{\psi}\, t^a \omega c^a, \ \delta A^a_\mu = \omega D^{ab}_\mu c^b$$

$$\delta c^a = -\frac{1}{2} g f^{abc} \omega c_b c_c, \ \delta\,\bar{c}^a = \frac{\omega}{\xi}\partial^\mu A^a_\mu$$

- Slavnov-Taylor identity for quark-gluon vertex

$$q^\mu \Gamma^a_{V\mu}(p_1, p_2, q) G^{-1}(q^2) = S_F^{-1}(p_1)\left[t^a - B^a_4(p_1, p_2)\right] - \left[t^a - B^a_4(p_1, p_2)\right] S_F^{-1}(p_2) \qquad (2.31)$$

- which constrains longitudinal part of vertex

*Transverse Symmetry Transformations in Gauge Theories

H.X.He,Phys.Rev.D80,016004(2009)

Consider an infinitesimal symmetry transformation

$$\phi^a(x) \longrightarrow \phi^a(x) + \delta\phi^a(x), \tag{6}$$

we call this transformation a symmetry if it leaves the equation of motion invariant. We introduce corresponding transverse symmetry transformation

$$\phi^a(x) \longrightarrow \phi^a(x) + \delta_T\phi^a(x), \tag{7}$$

where $\delta_T\phi^a(x)$ is defined by the infinitesimal Lorentz transformation for such symmetry transformation : $\delta_T\phi^a(x) = \delta_{Lorentz}\delta\phi^a(x)$, which reads

$$\delta_T\phi^a(x) = -\frac{i}{2}\epsilon^{\mu\nu}S^{(\delta\phi^a)}_{\mu\nu}\delta\phi^a(x). \tag{8}$$

Here $S^{(\delta\phi^a)}_{\mu\nu}$ denotes the generator of the intrinsic part for the infinitesimal Lorentz transformation, that is, the intrinsic spin operator of the field involved in $\delta\phi^a(x)$, where the field

Transverse Symmetry Transformation in QCD

*BRST transformation

$$\delta\psi = ig\omega t^a c_a \psi$$

$$\delta\bar{\psi} = -ig\bar{\psi} t^a \omega c_a$$

$$\delta A^a_\mu = \omega D^{ab}_\mu c_b$$

$$\delta c^a = -\frac{1}{2} g\omega f^{abc} c_b c_c$$

$$\delta\bar{c}^a = \frac{\omega}{\xi}\partial^\mu A^a_\mu,$$

- Transverse symmetry transformation for BRST symmetry (H.X.He, PR.D80,016004(2009))

$$\delta_T\psi = \frac{1}{4} g\omega\epsilon^{\mu\nu} c_a t^a \sigma_{\mu\nu}\psi$$

$$\delta_T\bar{\psi} = \frac{1}{4} g\epsilon^{\mu\nu}\bar{\psi}\sigma_{\mu\nu} t^a \omega c_a$$

$$\delta_T A^a_\mu = \omega\epsilon^{\mu\nu} D^{ab}_\nu c_b$$

$$\delta_T c^a = \delta_T \bar{c}^a = 0$$

*Transverse STI for the quark-gluon vertex

H.X.He, Phys.Rev.D80,016004(2009)

He et al, Phys.Lett.B 480, 222 (2000)

$$
\begin{aligned}
& iq^{\mu}\Gamma_{V}^{a\nu}(p_1,p_2) - iq^{\nu}\Gamma_{V}^{a\mu}(p_1,p_2) \\
&= [S_F^{-1}(p_1)\sigma^{\mu\nu}(t^a - B_4^a(p_1,p_2)) \\
&\quad + (t^a - B_4^a(p_1,p_2))\sigma^{\mu\nu}S_F^{-1}(p_2)]G(q^2) \\
&\quad + 2m\Gamma_T^{a\mu\nu}(p_1,p_2) + (p_{1\lambda} + p_{2\lambda})\varepsilon^{\lambda\mu\nu\rho}\Gamma_{A\rho}^{a}(p_1,p_2) \\
&\quad - \int \frac{d^4k}{(2\pi)^4} 2k_{\lambda}\varepsilon^{\lambda\mu\nu\rho}\Gamma_{A\rho}^{a}(p_1,p_2;k). \qquad (52)
\end{aligned}
$$

*The Quark-Gluon Vertex Function in QCD--- longitudinal part

$$\Gamma_V^{a\mu}(p_1, p_2) = \Gamma_{V(L)}^{a\mu}(p_1, p_2) + \Gamma_{V(T)}^{a\mu}(p_1, p_2), \qquad (66)$$

$$\begin{aligned}\Gamma_{V(L)}^{a\mu}(p_1, p_2) &= q^{-2}q^\mu[q_\nu\Gamma_V^{a\nu}(p_1, p_2)] \\ &= q^\mu[S_F^{-1}(p_1)(t^a - B_4^a(p_1, p_2)) \\ &\quad - (t^a - B_4^a(p_1, p_2))S_F^{-1}(p_2)]G(q^2)/q^2,\end{aligned} \qquad (67)$$

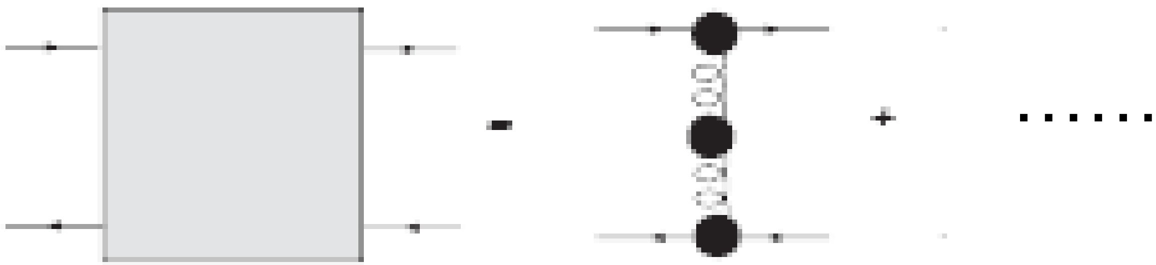
The Quark-Gluon Vertex Function in QCD--- transverse part

H.X.He, Phys.Rev.D80,016004(2009)

He, Phys.Rew.C63, 025207 (2001)

$$
\begin{aligned}
\Gamma^{a\mu}_{V(T)}(p_1,p_2) &= q^{-2} i q_\nu [i q^\mu \Gamma^{a\nu}_V(p_1,p_2) - i q^\nu \Gamma^{a\mu}_V(p_1,p_2)] \\
&= [q^2 + (p_1+p_2)^2 - ((p_1+p_2)\cdot q)^2 q^{-2}]^{-1} G(q^2)/q^2 \times \{ i[S_F^{-1}(p_1)\sigma^{\mu\nu}(t^a - B^a_4(p_1,p_2)) \\
&\quad + (t^a - B^a_4(p_1,p_2))\sigma^{\mu\nu} S_F^{-1}(p_2)] q_\nu q^2 + i[S_F^{-1}(p_1)\sigma^{\mu\lambda}(t^a - B^a_4(p_1,p_2)) - (t^a - B^a_4(p_1,p_2))\sigma^{\mu\lambda} S_F^{-1}(p_2)] \\
&\quad \times (p_{1\lambda} + p_{2\lambda}) q^2 + i[S_F^{-1}(p_1)\sigma^{\lambda\nu}(t^a - B^a_4(p_1,p_2)) - (t^a - B^a_4(p_1,p_2))\sigma^{\lambda\nu} S_F^{-1}(p_2)] q_\nu (p_{1\lambda} + p_{2\lambda}) q^\mu \\
&\quad - i[S_F^{-1}(p_1)\sigma^{\mu\nu}(t^a - B^a_4(p_1,p_2)) - (t^a - B^a_4(p_1,p_2))\sigma^{\mu\nu} S_F^{-1}(p_2)] q_\nu (p_1+p_2)\cdot q + i[S_F^{-1}(p_1)\sigma^{\lambda\nu}(t^a \\
&\quad - B^a_4(p_1,p_2)) + (t^a - B^a_4(p_1,p_2))\sigma^{\lambda\nu} S_F^{-1}(p_2)] q_\nu (p_{1\lambda} + p_{2\lambda}) [p_1^\mu + p_2^\mu - (p_1+p_2)\cdot q q^\mu q^{-2}] \\
&\quad - i q_\nu q^2 \bar{C}^{a\mu\nu}_A(p_1,p_2) + q_\nu q_\alpha (p_{1\lambda} + p_{2\lambda}) \varepsilon^{\lambda\mu\nu\beta} \bar{C}^{a\beta\alpha}_V(p_1,p_2) - i q_\nu (p_{1\lambda} + p_{2\lambda}) \\
&\quad \times [p_1^\mu + p_2^\mu - (p_1+p_2)\cdot q q^\mu q^{-2}] \bar{C}^{a\lambda\nu}_A(p_1,p_2) \}, \qquad (68)
\end{aligned}
$$

6.Gauge-invariant constraint & mechanism for generating quark confinement potential



- Asymptotic freedom in high energy limit by perturbative one-gluon exchange
- Quark confinement in low energy limit should be related to infrared behavior of the gluon propagator & quark-gluon vertex function

* Infrared behavior of gluon & ghost propagators - from DSEs & Lattice QCD

- Scaling solution

$$D^G(p^2) = -\frac{G(p^2)}{p^2}, \ D_{\mu\nu}(p^2) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\frac{Z(p^2)}{p^2}.$$

orresponding power laws in the infrared are

$$G(p^2) \sim (p^2)^{-\kappa}, \qquad Z(p^2) \sim (p^2)^{2\kappa}.$$

κ is positive [48] one obtains an infrared enhanced ghost and an infrared

- Decouping solution (massive gluon solution which is more favour)

$$Z(p^2) \sim p^2; \qquad G(p^2) \sim const.$$

*6.1 Gauge-invariant constraint on infrared structure of the quark-gluon vertex ---Ward-type identity **(He et al, to be published)**

$$\begin{aligned}\Gamma_\mu(p,p) &= [\frac{\partial}{\partial p_\mu}S^{-1}(p)]H(p,p,0)G(0)\\ &+S^{-1}(p)[\frac{\partial}{\partial k_\mu}H(k,p,q)]_{k=p}G(0)\\ &-[\frac{\partial}{\partial k^\mu}\bar{H}(p,k,q)]_{k=p}S^{-1}(p)G(0),\end{aligned}$$

$$\Gamma_\mu(p,p)=\sum_{i=1}^{4}\tilde{\lambda}_i(p^2,p^2,0)\tilde{L}_{i,\mu}$$

$$\tilde{L}_{1,\mu}=\gamma_\mu,\tilde{L}_{2,\mu}=\not{p}p_\mu,\tilde{L}_{3,\mu}=p_\mu,\tilde{L}_{4,\mu}=\not{p}\gamma_\mu,$$

n$_{q\to 0}$ $\tilde{\lambda}_i(k^2,p^2,q^2)$ are Lorentz and Dirac scalar functions:

$$\tilde{\lambda}_1(p^2,p^2;0)=G(0)Z_f^{-1}(p^2)\{\chi_0(p)-2p^2\chi_3(p)+M(p^2)[\chi_2(p)-\chi_1(p)]\}.$$

*6.2 Gauge-invariant constraint on infrared behavior of the quark-gluon vertex

(He, to be published)

Assuming $\lim_{q\to 0}\bar{\chi}_i = \lim_{q\to 0}\chi_i \sim (q^2)^{\alpha_{\chi_i}} f_{(i)}(p^2)$, the r.h.s of STI has

$$\lim_{q\to 0}[S^{-1}(k)H(k,p,q) - \bar{H}(p,k,q)S^{-1}(p)]$$

$$\sim \sum_{i=0}^{3}(q^2)^{\alpha_{\chi_i}+1/2}\hat{q}^{\mu}\sum_{j=1}^{3} f_{(i,j)}(p^2)\tilde{L}_{\mu}^{(j)}(p), \qquad (4)$$

l.h.s of the STI in the limit of vanishing gluon momentum is easy to write

$$\lim_{q\to 0} q^{\mu}\Gamma_{\mu}(k,p,q)G^{-1}(q^2)$$

$$\sim (q^2)^{\delta_{qg}+1/2-\alpha_G}\hat{q}^{\mu}\sum_{i=1}^{3} f_{(j)}(p^2)\tilde{L}_{\mu}^{(j)}(p). \qquad (6)$$

6.3 The mechanism for generating Infrared singularity in the quark-gluon vertex

-----a qualitative analysis

The r.h.s of $\bar{S}T\bar{I}$ does not disappear if $\alpha_{\chi_i} \leq -1/2$. If χ_i has the IR-singularity with $\alpha_{\chi_i} = -1/2$ and DCSB appears ($M \neq 0$). In this case we have

$$\lim_{q\to 0} \Gamma_\mu(k,p,q) \sim (q^2)^{-1/2+\alpha_G} \tilde{L}_\mu(p),$$

$$\lim_{q\to 0} \alpha_{qg}(q^2) \sim (q^2)^{-1+\delta_{\alpha_{gh}}}, \qquad (14)$$

which generate an IR singular quark-gluon vertex and an

*6.4 The mechanism for generating a linear confinement potential– a qualitative analysis

action quark potential

$$\lim_{q \to 0} V(q) \sim \frac{1}{(q^2)^2}. \tag{15}$$

This quark potential in the coordinate space is normally written

$$V(\vec{r}) = \int \frac{d^3q}{(2\pi)^3} V(q^0 = 0, \vec{q}) e^{i\vec{q}\cdot\vec{r}} \sim |\vec{r}|, \tag{16}$$

generating a linearly rising potential between massive quarks, implying quark confinement at large distances.

*6.5 The mechanism for generating a quark confinement potential–

the more complete calculations(He, to be published)

A more complete calculation gives

$$\lim_{q \to 0} V(q) \sim \frac{C_0^2}{q^2} + \frac{C_1^2 M^2}{(q^2)^2},$$

for very large M. We thus obtain

$$V(\vec{r}) = \int \frac{d^3q}{(2\pi)^3} V(q^0 = 0, \vec{q}) e^{i\vec{q}\cdot\vec{r}} \sim \frac{a}{r} + br,$$

where $a = \frac{1}{4\pi} f_0^2(p^2)$,

Summary

- Gauge invariance of QCD imposes powerful constraint (STI) for the infrared structure and the infrared behavior of the qqg vertex . The results show:
- There exists an IR-singularity in the quark-ghost scattering kernel, which generates the IR-singularity in the qqg vertex and such
- Infrared divergent quark-gluon vertex generates a linear confinement potential between masive quarks, realizing infrared slavery