Problems related to gauge invariance, Lorentz covariance, and canonical quantization applied in nucleon structure study

Fan Wang,

Dept. of Phys. And CPNPC (Center for Particle Nuclear Physics and Cosmology),

Nanjing Univ. and Purple Mountain Observatory of CAS (Chinese Academy of Science)

fgwang@chenwang.nju.edu.cn

Outline

I. Introduction
II. A consistent decomposition of the momentum and angular momentum operators of a gauge field system
III. Debates on the decomposition
IV. Summary

I. Introduction

- It is still a popular idea that the polarized deep Inelastic scattering (DIS) measured quark spin invalidates the constituent quark model (CQM), leads to the so-called proton spin crisis.

  We have shown that there is no proton spin crisis but a misidentification of the relativistic quark spin to the non-relativistic quark spin. Using a Fock space extension quark model with 5q components to calculate the relativistic quark spin one can describe the measured quark spin contents well.

- There is another misidentification, to compare the socalled measured quark orbital angular momentum to the non-relativistic canonical orbital angular momentum used in quark model calculation and leads to the socalled second proton spin crisis.

  We have proved the sum of non-relativistic quark spin and orbital angular momentum equals to the relativistic sum exactly.

  PRD58,114032(1998)

- The gluon spin case is even worse, it is a centenary problem that there is no spin operator for massless particles. This contradicts to the photon and gluon spin measurements and the widely used multipole radiation analysis from atomic spectroscopy to hadron spectroscopy.

- All of these call for a critical study of what is the proper spin and orbital angular momentum of a gauge system, the U(1) atom and the SU(3) nucleon.

II.A consistent decomposition of the momentum and angular momentum of a gauge system

Jaffe-Manohar decomposition:

$$\vec{J} = \vec{S}_q + \vec{L}_q + \vec{S}_G + \vec{L}_G$$

$$\vec{S}_q = \frac{1}{2}\int d^3x \psi^\dagger \vec{\Sigma} \psi$$

$$\vec{L}_q = \int d^3x \psi^\dagger \vec{r} \times \frac{\nabla}{i} \psi$$

$$\vec{S}_G = 2\int d^3x T_r\{\vec{E} \times \vec{A}\}$$

$$\vec{L}_G = 2\int d^3x T_r\{E_i \vec{r} \times \nabla A_i\}$$

R.L.Jaffe and A. Manohar,Nucl.Phys.B337,509(1990).

- Each term in this decomposition satisfies the canonical commutation relation of angular momentum operator, so they are qualified to be called quark spin, orbital angular momentum, gluon spin and orbital angular momentum operators.
- However they are not gauge invariant except the quark spin.

Gauge invariant decomposition

$$\vec{J} = \vec{S}_q + \vec{L'}_q + \vec{J'}_G$$

$$\vec{S}_q = \frac{1}{2}\int d^3x\psi^\dagger\vec{\Sigma}\psi$$

$$\vec{L'}_q = \int d^3x\psi^\dagger\vec{r}\times\frac{\vec{D}}{i}\psi$$

$$\vec{J'}_G = 2\int d^3x T_r\{\vec{r}\times(\vec{E}\times\vec{B})\}$$

X.S.Chen and F.Wang, Commun.Theor.Phys. 27,212(1997).

X.Ji, Phys.Rev.Lett.,78,610(1997).

- Each term in this decomposition is gauge invariant.
- However each term no longer satisfies the canonical commutation relation of angular momentum operator except the quark spin, in this sense the second and third terms are not the real quark orbital and gluon angular momentum operators.
- There is no gauge invariant gluon spin and orbital angular momentum operator separately, the only gauge invariant one is the total angular momentum of gluon.
- This means there is no photon (gluon) spin and orbital angular momentum! This contradicts the widely used multipole radiation analysis, the photon spin and gluon spin measurements

Standard definition of momentum and orbital angular momentum

$$p_i = \partial_{\dot{q}_i} L(q_i, \dot{q}_i),$$

$$L_i = q_i \times p_i.$$

This has been used from classical mechanics, quantum mechanics and quantum field theory!

Momentum and angular momentum operators for charged particle moving in electro-magnetic field

$$\mathcal{L} = \frac{1}{2}\mathrm{m}v^2 - q(A^0 - \vec{v}\cdot\vec{A})$$

$$\vec{p}=\mathrm{m}\vec{v}+\mathrm{q}\vec{A},$$

$$\vec{L}=\vec{r}\times\vec{p}.$$

$$H = \frac{1}{2m}\left(\vec{p} - q\vec{A}\right)\text{^}2 + \mathrm{q}A^0$$

For a charged particle moving in em field,
the canonical momentum is,

$$\vec{p} = m\dot{\vec{r}} + q\vec{A}$$

- It is gauge dependent, so classically it is Not measurable.
- In QM, we quantize it as $\vec{p} = \frac{\vec{\nabla}}{i}$, no matter what gauge is. Feynman had an explanation on why we quantize it as the canonical momentum.
- It appears to be gauge invariant, but in fact Not!

Under a gauge transformation

$$\psi \to \psi^{'} = e^{-iq\omega(x)}\psi,$$

$$\vec{A} \to \vec{A}^{'} = \vec{A} - \vec{\nabla}\omega, \qquad \varphi \to \varphi^{'} = \varphi + \partial_t \omega,$$

The matrix elements transform as

$$\langle \psi | p | \psi \rangle \to \langle \psi | p | \psi \rangle - \langle \psi | q\nabla\omega | \psi \rangle,$$

$$\langle \psi | L | \psi \rangle \to \langle \psi | L | \psi \rangle - \langle \psi | q\vec{r} \times \nabla\omega | \psi \rangle,$$

$$\langle \psi | H | \psi \rangle \to \langle \psi | H | \psi \rangle + \langle \psi | q\partial_t \omega | \psi \rangle,$$

New momentum operator

- Old generalized momentum operator for a charged particle moving in em field,

$$\vec{p} = m\dot{\vec{r}} + \mathrm{q}A_{\perp} + \mathrm{q}A_{\parallel} = \frac{\nabla}{i}$$

$$\nabla \cdot \overrightarrow{A_{\perp}} = 0, \quad \nabla \times \vec{A}_{\parallel} = 0, \qquad \vec{\nabla} \times \vec{A}_{\perp} = \vec{\nabla} \times \vec{A}.$$

It satisfies the canonical momentum commutation relation, but its matrix elements are not gauge invariant.

- New momentum operator we proposed,

$$\vec{p}_{phys} = \vec{p} - \mathrm{q}\vec{A}_{\parallel}$$

It is both gauge invariant and canonical commutation relation satisfied.

We call

$$\frac{\vec{D}_{pure}}{i} = \vec{p} - q\vec{A}_{//} = \frac{1}{i}\vec{\nabla} - q\vec{A}_{//}$$

physical momentum.

It reduces to the canonical momentum

$$\vec{p} = m\dot{\vec{r}} + q\vec{A} = \frac{1}{i}\vec{\nabla}$$

in Coulomb gauge; the mechanical momentum

$$\vec{p} - q\vec{A} = m\dot{\vec{r}} = \frac{1}{i}\vec{D}$$

never can reduce to the canonical one, do not satisfy the canonical momentum commutation relation and so it is not the right momentum operator!

Gauge invariance and canonical quantization both satisfied decomposition

- Gauge invariance is not sufficient to fix the decomposition of the angular momentum of a gauge system.
- Canonical quantization rule of the angular momentum operator must be respected. It is also an additional condition to fix the decomposition.
- Measurable one must be physical, does not include unphysical pure gauge potential.

X.S.Chen, X.F.Lu, W.M.Sun, F.Wang and T.Goldman, Phys.Rev.Lett. 100(2008) 232002.

arXiv:0806.3166; 0807.3083; 0812.4366[hep-ph];

0909.0798[hep-ph]

$$\vec{J}_{QED} = \vec{S}_e + \vec{L}''_e + \vec{S}''_\gamma + \vec{L}''_\gamma$$

$$\vec{S}_e = \int d^3x \psi^+ \frac{\vec{\Sigma}}{2} \psi$$

$$\vec{L}''_e = \int d^3x \psi^+ \vec{x} \times \frac{\vec{D}_{pure}}{i} \psi$$

$$\vec{S}''_\gamma = \int d^3x \vec{E} \times \vec{A}_{phy}$$

$$\vec{L}''_\gamma = \int d^3x E^i \vec{x} \times \vec{D}_{pure} A^i_{phys}$$

$$\vec{A} = \vec{A}_{phys} + \vec{A}_{pure}, \qquad \nabla \cdot \vec{A}_{phys} = 0, \nabla \times \vec{A}_{phys} = \nabla \times \vec{A}.$$

$$\vec{A}_{phy} = \nabla \times \frac{1}{4\pi} \int d^3x' \frac{\vec{B}(x')}{|\vec{x} - \vec{x}'|}, \qquad \vec{D}_{pure} = \vec{\nabla} - ie\vec{A}_{pure}.$$

It provides the theoretical basis of the multipole radiation analysis and the photon spin measurements.

QCD

$$\vec{J}_{QCD} = \vec{S}_q + \vec{L}_q^{\,''} + \vec{S}_g^{\,''} + \vec{L}_g^{\,''}$$

$$\vec{S}_q = \int d^3x \psi^+ \frac{\vec{\Sigma}}{2} \psi$$

$$\vec{L}_q^{\,''} = \int d^3x \psi^+ \vec{x} \times \frac{\vec{D}_{pure}}{i} \psi$$

$$\vec{S}_g^{\,''} = \int d^3x \vec{E}^a \times \vec{A}^a_{phys}$$

$$\vec{L}_g^{\,''} = \int d^3x E^{ia} \vec{x} \times \vec{D}^{adj}_{pure} A^{ia}_{phys}$$

Non Abelian complication

$$\vec{A} = \vec{A}_{pure} + \vec{A}_{phy} \qquad \vec{A}_{pure} = T^a \vec{A}^a_{pure}$$

$$\vec{D}_{pure} \times \vec{A}_{pure} = \vec{\nabla} \times \vec{A}_{pure} - ig \vec{A}_{pure} \times \vec{A}_{pure} = 0$$

$$\vec{D}_{pure} = \vec{\nabla} - ig \vec{A}_{pure}$$

$$\vec{D}^{adj}_{pure} \cdot \vec{A}_{phys} = \nabla \cdot \vec{A}_{phys} - ig[A^i_{pure,}, A^i_{phys}] = 0$$

$$\vec{D}^{adj}_{pure} = \nabla - ig[\vec{A}_{pure},]$$

Consistent separation of nucleon momentum and angular momentum

Sq Lq Sg Lg

$$\vec{J}_{\rm total} = \int d^3x \psi^\dagger \frac{1}{2}\vec{\Sigma}\psi + \int d^3x \vec{x} \times \psi^\dagger \frac{1}{i}\vec{D}_{\rm pure}\psi + \int d^3x \vec{E} \times \vec{A}_{\rm phys} + \int d^3x \vec{x} \times E^i \vec{\mathcal{D}}_{\rm pure} A^i_{\rm phys}$$

$$\vec{P}_{\rm total} = \int d^3x \psi^\dagger \frac{1}{i}\vec{D}_{\rm pure}\psi + \int d^3x E^i \vec{\mathcal{D}}_{\rm pure} A^i_{\rm phys}$$

Pq Pg

Standard construction of orbital angular momentum $\vec{L} = \int d^3x \, \vec{x} \times \vec{P}$

- Each term is gauge invariant and so in principle measurable.
- Each term satisfies angular momentum commutation relation.
- In Coulomb gauge it reduces to Jaffe-Manohar decomposition, the usual one used in QM and QFT.
- Jaffe-Manohar's quark, gluon orbital angular momentum and gluon spin are gauge dependent. Only in Coulomb gauge it will give the right rsults.

III. Debates on the decomposition

- Canonical commutation relation:

  Renormalization ruins the canonical commutation relation. However there is no way to avoid the canonical quantization rule.

  hep-ph:

0807.3083,0812.4336,0911.0248,1205.6983

- Non-locality: Non-local operators are popular in gauge field theory.

  The A-B effect is a non-local effect.

  All of the parton distribution operators are non-local, only in light-cone gauge becomes local. The new one is local in Coulomb gauge.

Lorentz covariance

- The Lorentz transformation (LT) property of 4-coordinate x^{μ}, momentum p^{μ}, and field tensor $F^{\mu\nu}$ are fixed by measurement. They transform with the usual homogeneous LT.
- The LT property of 4-vector potential A^{μ} is gauge dependent, because there is gauge degree of freedom:

Lorentz gauge, usually they are chosen to transform with homogeneous LT, but it can also be chosen to transform with inhomogeneous LT, i.e.,

$$A'^{\mu}(x') = \Lambda^{\mu}_{\nu} A^{\nu}(x) + \partial'^{\mu} \varpi(x')$$

Because there are residual gauge degree of freedom.

- Coulomb gauge, vector potential must be transformed with the inhomogeneous LT to make the transformed potential still satisfy the Coulomb gauge condition, i.e,.

$$A^{\mu'}(x') = \Lambda^{\mu}_{\nu} A^{\nu}(x) + \partial^{\mu'}\omega(x'), \sim\sim \nabla' \cdot A'(x') = 0$$

because homogeneous LT mixing the unphysical components to the vector potential and one must do additional gauge transformation to eliminate the unphysical components.

- Light-cone gauge vector potential must also be transformed by inhomogeneous LT to make the

$$\boldsymbol{A}^{+'} = \mathbf{0}$$

in the new Lorentz frame. Only for limited boosting along the infinite momentum direction light-cone gauge fixing can be preserved automatically!

- The Lorentz frame independence of a theory must be independent of the gauge fixing, no matter the Lorentz gauge, Coulomb gauge or light-cone gauge is used. All of them must be Lorentz invariant.
- The Lorentz transformation form of the gauge potentials are gauge dependent accordingly!

A systematic analysis had been given by C. Lorce in

arXiv:1205.6483[hep-ph],PRD87(2013)034031.

Uniqueness of the decomposition and the gauge invariant extension

Gauge invariance is the necessary condition for the measurabilily but not sufficient one.

Different gauge invariant extensions are not all physical equivalent and usually result in physically different ones.

QED case, to keep the physical content intact, our additional physical condition is unique ,

$$\vec{\nabla} \cdot \vec{A}_{phys} = 0$$

$$\vec{\nabla} \times \vec{A}_{pure} = 0 \quad .$$

QCD case the generalization of this condition is more complicated and under hot debate. We believe the physical one is unique and our physical condition is a generalization of Coulomb gauge , i.e., only two helicity gluon components are physical.

- To obtain the gauge invariant momentum, orbital angular momentum, gluon spin, etc., is to **discover** the gauge invariant one through the decomposition of gauge potential and does not change the physical meaning of these standard operators.
- The gauge invariant extension through a gauge link or other methods usually mixes the gluon and quark part and change the physical meaning of these operators.

Centenary question: Spin and orbital angular momentum of massless photon and gluon

- For a long time it is believed that one can not decompose the total angular momentum of a massless particle, the photon and gluon, into spin and orbital ones.
- Now there seems to be a consensus that this conclusion should be modified as: there are no local gauge invariant spin and orbital angular momentum operators but there are *nonlocal ones*.

- The measured photon spin should be

$$\vec{E} \times \vec{A}_{phys}$$

- The measured gluon spin is the matrix elements of the above gluon spin operator boosted to the infinite momentum frame.
- The complicating of the boosting is not due to the use of physical component

$$A^{\mu}_{phys}$$

but due to the spin operator itself . There is the well-known Wigner rotation.

Four momentum operators

only $\vec{p}_{phys}$ is physical one

$$\vec{p}_{cano} = m\dot{\vec{r}} + q\vec{A} = \frac{1}{i}\vec{\nabla}$$

$$\vec{p}_{kine} = \vec{p} - q\vec{A} = m\dot{\vec{r}} = \frac{1}{i}\vec{D}$$

$$p_{lc} = p^{+} - qA^{+}$$

$$\vec{p}_{phys} = m\dot{\vec{r}} + q\vec{A} - q\vec{A}_{pure} = m\dot{\vec{r}} + q\vec{A}_{phys} = \frac{\vec{\nabla}}{i} - q\vec{A}_{pure}$$

The kinematic quark momentum $\vec{p} - g\vec{A}$

suggested by X.D. Ji and M. Wakamatsu is not the right quantum mechanical momentum operator:

- Three components do not commute and so cannot consist of a complete set of commuting operators to describe the 3-d quark momentum distribution.
- They can not be reduced to the canonical one in any gauge.
- They are still a mixing of quark and gluon momentum and has never been measured except in classic physics ! In different hadron they have different gluon content and so not universal ones.

Partial gauge invariant and power counting for electron momentum

- X. Ji developed the socalled power counting to relate his gauge invariant kinematical momentum to the canonical one, to assume the vector potential as a perturbative correction.
- Gauge invariance is an exact symmetry, there is only gauge invariance or non-invariance. Partial gauge invariance is nonsense!
- The only gauge invariant physical one is what we defined, the canonical momentum used in non-relativistic quantum mechanics is the Coulomb gauge ones. To use them in other gauge one has to use our gauge invariant version. arXiv:1002.3421[hep-ph]

- The light-cone momentum

$$p_{lc} = p^{+} - qA^{+}$$

is the infinite momentum frame version of the physical momentum,

$$p_{phys} = p - qA_{pure}$$

- Because various arguments show consistently that the A^{+} only includes the longitudinal component $A_{//}$ at least in the infinite momentum frame. And the kinematical momentum cannot be transformed to the light-cone one through the gauge transformation.

- In the naïve parton model the parton distribution f(x) is a distribution of parton canonical momentum

$$x = p^{+} / P^{+}$$

distribution. Taking into account of the gluon interaction, under the collinear approximation, the parton distribution is the light-cone momentum

$$p^{+} - gA^{+}$$

distribution.

- The further gauge transformation can only introduces the pure gauge gluon field A^{μ}_{pure} into the parton momentum. The transverse (physical) components A^{μ}_{phys} will never be involved in the measured parton momentum.

Physical momentum satisfy the canonical commutation relations, reduced to the canonical momentum in Coulomb gauge, and the measured quark momentum distribution should be the matrix elements of the physical momentum boosting to infinite momentum frame.

The measured electron momentum in atomic and molecular structure should be the matrix elements of the physical momentum in the lab frame not the socalled power counting ones.

Quark orbital angular momentum

- The quark kinematical orbital angular momentum

$$\vec{L}_q = \int d^3x \vec{x} \times \psi_q^+ (\vec{p} - g\vec{A}) \psi_q$$

calculated in LQCD and "measured" in DVCS is not the real orbital angular momentum used in quantum mechanics. It does not satisfy the Angular Momentum Algebra,

$$\vec{L} \times \vec{L} = i\vec{L}$$

and the gluon contribution is ENTANGLED in it.

- E. Leader suggested to use the gauge variant canonical momentum and angular momentum operators as the physical one and tried to prove that the matrix elements of physical states of gauge dependent operator are gauge invariant.
- His argument is based on F. Strocchi and A.S. Wightman’s theory and this theory is limited to the extended Lorentz gauge and so at most only true for very limited gauge transformations.
- Our gauge invariant momentum and angular momentum operator reduce to the canonical one in physical gauge, i.e., they are generators for physical field. In general they are the generators of parallel displacement. arXiv:1203.1288[hep-ph]

Evolution of the parton distribution

- Most of the evolutions are based on the free parton picture and perturbative QCD.
- The measured parton distribution is always a mixing of non-perturbative and perturbative one.
- The first moment of polarized structure function

$$\Gamma_1(Q^2) = \int g_1(x, Q^2)dx$$

shows dramatical changes in the low Q^2 region, the simple evolution can not describe the low Q^2 behavior.

VI. Summary

- There are different quark and gluon momentum and orbital angular momentum operators. Confusions disturbing or even misleading the nucleon spin structure studies.
- There is no debate on what is the proper quark spin, the axial vector current operator

$$\int \psi^{+}(x)\frac{\vec{\Sigma}}{2}\psi(x)d^{3}x$$

- There might be a consensus on what is the proper gluon spin operator,

$$\int \vec{E}(x) \times \vec{A}_{phys}(x) d^3x$$

- There might be still debate on what is the proper quark and gluon orbital angular momentum operator. We show it must be the gauge invariant quark and gluon canonical orbital angular momentum operators,

$$\int \psi^{+}(x) \vec{x} \times (\vec{p} - g\vec{A}_{pure}) \psi(x) d^3x$$

$$\int \vec{x} \times E^{i} (\overrightarrow{\nabla} - ig\vec{A}_{pure}) A^{i}_{phy} d^3x$$

- The well known Poynting vector is only the energy flow operator, but not the momentum density flow in the interacting theory. It had been shown by optical measurement and should be checked further. The energy-momentum tensor of EM cannot be the symmetric one. It will lead to contradictory results.
- To do the quantum mechanics calculation in general gauge, other than the Coulomb gauge, one has to use our gauge invariant version. The gauge dependence of the eigen value of hydrogen atom under the time dependent gauge transformation is a typical example.

Thanks

for

your patient

Poincare covariance

For the whole gauge field system, one has the Poincare covariance:

$$[P^\alpha, P^\beta] = 0,$$

$$i[P^\alpha, J^{\rho\sigma}] = g^{\alpha\rho} P^\sigma - g^{\alpha\sigma} P^\rho,$$

$$i[J^{\alpha\beta}, J^{\rho\sigma}] = g^{\beta\rho} J^{\alpha\sigma} - g^{\alpha\rho} J^{\beta\sigma} - g^{\sigma\alpha} J^{\rho\beta} + g^{\sigma\beta} J^{\rho\alpha}$$

No full Poincare covariance for the individual part

- When one separates a gauge field system into its Fermion and Boson part, one can not have the full Poincare covariance for the Fermion and Boson parts separately.
- For the interacting dependent generators,

$$P^0 = H, \quad K^j = J^{0j},$$

we don't know how to separate them into Fermion and Boson parts to make them to satisfy the Poincare algebra even though we already have the momentum and angular momentum operators as discussed above.

3-dimensional translation and rotation for individual part can be retained

$$[p_i, p_j] = 0,$$

$$[J_i, J_j] = i\epsilon_{ijk} J_k$$

$$[p_i, J_j] = i\varepsilon_{ijk} p_k$$

General Lorentz covariance for individual part are retained

- General Lorentz transformation

$$x' = \Lambda x,$$

$$\psi'(x') = U_\Lambda(x) S[\Lambda] \psi(x),$$

$$A'_\mu(x') = \Lambda_\mu{}^\upsilon U^\Lambda(x)[A_\upsilon(x) + \frac{i}{g}\partial_\upsilon] U_\Lambda{}^{-1}$$

QED

$$A'^{\;phys}_{\mu}(x') = \Lambda_{\mu}{}^{\upsilon}[A_{\upsilon}{}^{phys}(x) + \partial_{\upsilon}\Omega_{\Lambda}{}^{phys}(x)],$$

$$A'^{\;pure}_{\mu}(x') = \Lambda_{\mu}{}^{\upsilon}[A_{\upsilon}{}^{pure}(x) + \partial_{\upsilon}\Omega_{\Lambda}{}^{pure}(x)],$$

$$\Omega_{\Lambda} = \Omega_{\Lambda}{}^{phys} + \Omega_{\Lambda}{}^{pure}.$$

Two special choices:

$$\Omega_{\Lambda} = 0, \qquad \Omega_{\Lambda}{}^{phys} = -\Omega_{\Lambda}{}^{pure};$$

Simple Lorentz transformation law for full A_{μ}, but physical and pure gauge part will be mixed under Lorentz transformation.

$$\Omega_{\Lambda}{}^{pure} = 0$$

Complicated Lorentz transformation law for full A_{μ}, but physical part keeps

QCD

$$A'_{\mu}{}^{phys}(x') = \Lambda_{\mu}{}^{\upsilon} U_{\Lambda}(x) A_{\upsilon}{}^{phys}(\mathrm{x}) U_{\Lambda}{}^{-1}(x)$$

$$-\frac{i}{g} \Lambda_{\mu}{}^{\upsilon} U_{\Lambda}(x) U_{pure}(x) U_{\Lambda}{}^{phys,-1}(x) [\partial_{\upsilon} U_{\Lambda}{}^{phys}(x)]$$

$$U_{pure}{}^{-1}(x) U_{\Lambda}{}^{-1}(x)$$

$$A'_{\mu}{}^{pure}(x') = \Lambda_{\mu}{}^{\upsilon} U_{\Lambda}(x) [A_{\upsilon}{}^{pure}(x) + \frac{i}{g} \partial_{\upsilon}] U_{\Lambda}{}^{-1}(x)$$

$$+\frac{i}{g} \Lambda_{\mu}{}^{\upsilon} U_{\Lambda}(x) U_{pure}(x) U_{\Lambda}{}^{phys,-1}(x) \left(\partial_{\upsilon} U_{\Lambda_{phys}}(x)\right)$$

$$U_{pure}{}^{-1}(x) U_{\Lambda}{}^{-1}(x)$$

Gauge Invariant extension

- There are infinite possible decomposition of the gauge potential $A_\mu(x)$ into gauge covariant part ${A_\mu}^{GI}(x)$ and pure gauge part ${A_\mu}^{pure}(x)$,

$$ {A'_\mu}^{GI}(x) = U(x){A_\mu}^{GI}(x)U^{-1}(x) $$

$$ {A'_\mu}^{pure}(x) = U(x)[{A_\mu}^{pure}(x) + \frac{i}{g}\partial_\mu]U^{-1}(x) $$

- Gauge invariant is the necessary condition of an operator to be an observable but not the sufficient condition.

- Among the infinite possible gauge invariant extensions, only one might be used to construct the observable which is the physical part of the gauge potential.
- For QED, it is the Coulomb gauge fixing parts which are physical because there are only two transverse components or helicity components retained. They are observed in Compton scattering.
- For QCD a natural choice is to choose the two transverse or helicity components of the gluon field as physical parts which might be observed in gluon jet.

Hamiltonian of hydrogen atom

Coulomb gauge:

$$\vec{A}^{c}_{//} = 0, \qquad \vec{A}^{c}_{\perp} \neq 0, \qquad A^{c}_{0} = \varphi^{c} \neq 0.$$

Hamiltonian of a non-relativistic charged particle

$$H_c = \frac{(\vec{p} - q\vec{A}^{c}_{\perp})^2}{2m} + q\varphi^{c}.$$

Gauge transformed one

$$\vec{A}_{//} = \vec{A}^{c}_{//} + \vec{\nabla}\omega(x) = \vec{\nabla}\omega(x), \vec{A}_{\perp} = \vec{A}^{c}_{\perp}, \varphi = \varphi^{c} - \partial_t \omega(x)$$

$$H = \frac{(\vec{p} - q\vec{A})^2}{2m} + q\varphi = \frac{(\vec{p} - q\vec{\nabla}\omega - q\vec{A}^{c}_{\perp})^2}{2m} + q\varphi^{c} - q\partial_t \omega.$$

Following the same recipe, we introduce a new Hamiltonian,

$$H_{phy} = H + q\partial_t \omega(x) = \frac{(\vec{p} - q\vec{A}_{//} - q\vec{A}_{\perp}^{c})^2}{2m} + q\varphi^c$$

$$\omega = \nabla^{-2}\nabla \cdot \vec{A}$$

which is gauge invariant, i.e.,

$$\left\langle \psi \,|\, H_{phy} \,|\, \psi \right\rangle = \left\langle \psi^c \,|\, H_c \,|\, \psi^c \right\rangle$$

This means the time displacement operator H and the energy operator H_{phys} are not be the same.

A check

- We derived the Dirac equation and the Hamiltonian of electron in the presence of a massive proton from a em Lagrangian with electron and proton and found that indeed the time translation operator and the Hamiltonian are different, exactly as we obtained phenomenologically before.

W.M. Sun, X.S. Chen, X.F. Lu and F. Wang, arXiv:1002.3421[hep-ph]