

Meson production dynamics and properties of excited nucleon states

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- Introduction

- Why N^* 's
- Current status of N^* 's
- Methodology for N^* 's study

- Dynamical coupled-channels model

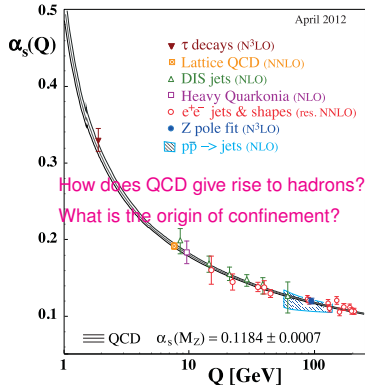
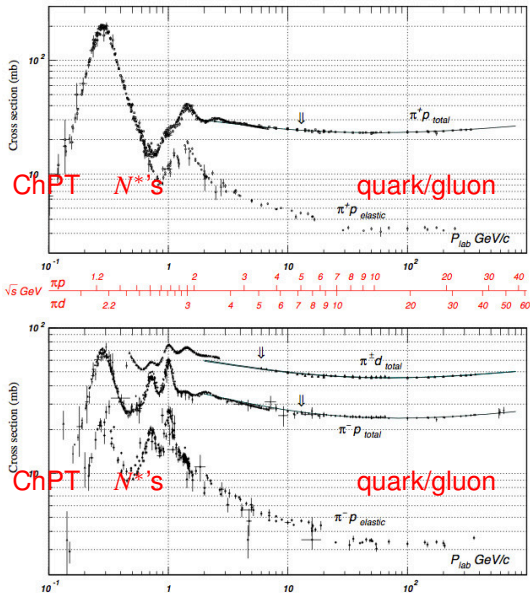
- Jülich hadron-exchange model: ingredients & main features
- Prescription for gauge invariance

- Recent progress: $\pi N \rightarrow \pi N$, ηN , ΛK , ΣK & $\gamma N \rightarrow \pi N$

- $\pi N \rightarrow \pi N$ partial wave amplitudes ($J = 1/2, 3/2, 5/2, 7/2, 9/2$)
- $\pi N \rightarrow \eta N$, ΛK , ΣK cross sections & polarizations
- $\gamma N \rightarrow \pi N$ cross sections, single & double polarizations
- Resonances extracted
- Pole positions & residues

- Summary & perspectives

Why N^* 's



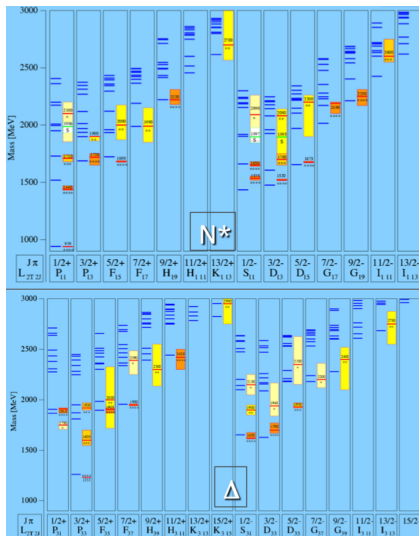
Explaining the excitation spectrum of nucleons is central to our understanding of non-perturbative QCD.

N. Isgur, NSTAR 2000, Newport News, 2000, USA:

- **Nucleons are the stuff of which our world is made.** As such they must be at the center of any discussion of why the world we actually experience has the character it does.
- **Nucleons are the simplest system in which the quintessentially nonabelian character of QCD is manifest.** There are, after all, N_c quarks in a proton because there are N_c colors.
- **While relatively simple, baryons are sufficiently complex to reveal physics hidden from us in the mesons.** One famous example: Gell-Mann and Zweig were forced to the quarks by $3 \times 3 \times 3$ giving the octet and decuplet, while mesons admitted of many possible solutions.

"I am convinced that completing this chapter in the history of science will be one of the most interesting and fruitful areas of physics for at least the next thirty years."

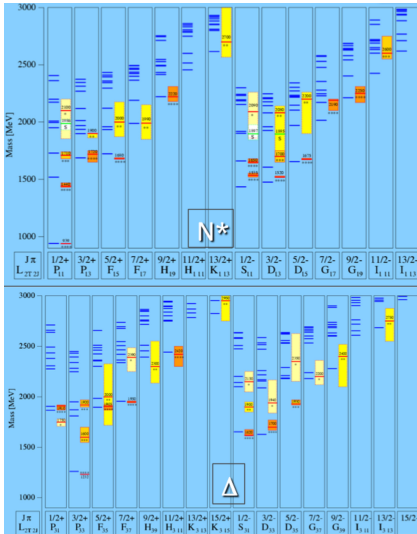
Current status of N^* 's



S. Capstick & N. Isgur, Phys. Rev. D 34 (1986) 2809.

"Missing resonances" problem

Current status of N^* 's



S. Capstick & N. Isgur, Phys. Rev. D 34 (1986) 2809.

"Missing resonances" problem

Particle	J^P	overall	πN	γN	$N\eta$	$N\sigma$	$N\omega$	ΛK	ΣK	$N\rho$	$\Delta\pi$						
N	$1/2^+$	****															
$N(1440)$	$1/2^+$	****	****	****		***				*	***						
$N(1520)$	$3/2^-$	****	****	****	***					***	***						
$N(1535)$	$1/2^-$	****	****	****	****					**	*						
$N(1650)$	$1/2^-$	****	****	****	***			***	**	***	***						
$N(1675)$	$5/2^-$	****	****	***	*		*			*	***						
$N(1680)$	$5/2^+$	****	****	****	*	**				***	***						
$N(1685)$??	*															
$N(1700)$	$3/2^-$	***	***	**	*			*	*	*	***						
$N(1710)$	$1/2^+$	***	***	***	***		**	***	**	*	**						
$N(1720)$	$3/2^+$	****	****	***	***			**	**	**	*						
$N(1860)$	$5/2^+$	**	**							*	*						
$N(1875)$	$3/2^-$	***	*Particle	J^P	overall	πN	γN	$N\eta$	$N\sigma$	$N\omega$	ΛK	ΣK	$N\rho$	$\Delta\pi$			
$N(1880)$	$1/2^+$	**	*	$\Delta(1232)$	$3/2^+$	****	****	****		F							
$N(1895)$	$1/2^-$	**	*	$\Delta(1600)$	$3/2^+$	***	***	***					o	*	***		
$N(1900)$	$3/2^+$	***	*	$\Delta(1620)$	$1/2^-$	****	****	***					r		***	***	
$N(1990)$	$7/2^+$	**	*	$\Delta(1700)$	$3/2^-$	****	****	****							***	***	
$N(2000)$	$5/2^+$	**	*	$\Delta(1750)$	$1/2^+$	*	*						b		**	***	
$N(2040)$	$3/2^+$	*		$\Delta(1900)$	$1/2^-$	**	**	**					i				
$N(2060)$	$5/2^-$	**	*	$\Delta(1905)$	$5/2^+$	****	****	****					d		**	**	**
$N(2100)$	$1/2^+$	*		$\Delta(1910)$	$1/2^+$	****	****	****						d	***	**	**
$N(2150)$	$3/2^-$	**	*	$\Delta(1920)$	$3/2^+$	***	***	**					e		*	*	**
$N(2190)$	$7/2^-$	****	*	$\Delta(1930)$	$5/2^-$	****	****						n	***		**	
$N(2220)$	$9/2^+$	****	*	$\Delta(1940)$	$3/2^-$	**	*	*	**				F				(seen in $\Delta\eta$)
$N(2250)$	$9/2^-$	****	*	$\Delta(1950)$	$7/2^+$	****	****	****					o		***	*	***
$N(2600)$	$11/2^-$	***	*	$\Delta(2000)$	$5/2^+$	**							r				**
$N(2700)$	$13/2^+$	**	*	$\Delta(2150)$	$1/2^-$	*	*							b			
				$\Delta(2200)$	$7/2^-$	*	*							i			
				$\Delta(2300)$	$9/2^+$	**	**							d			
				$\Delta(2350)$	$5/2^-$	*	*							d			
				$\Delta(2390)$	$7/2^+$	*	*							e			
				$\Delta(2400)$	$9/2^-$	**	**							n			
				$\Delta(2420)$	$11/2^+$	****	****	*									
				$\Delta(2750)$	$13/2^-$	**	**										
				$\Delta(2950)$	$15/2^+$	**	**										

4-star 17
3-star 8
2-star 15
1-star 8

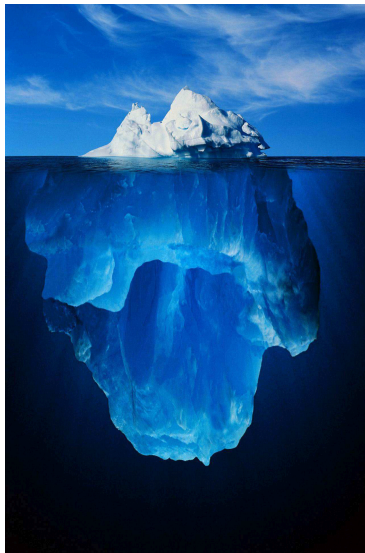
PDG 2012

Problems in N^* study

- Where are the missing resonances?
- Do 1-star & 2-star resonances exist?
- How many resonances in total?
- Resonance structures?
- Resonance parameters?
-

Volker Credé:

**N^* 's: only the tip of the iceberg
has been discovered?**



Experimental efforts

- JLab
- ELSA
- MAMI
- GRALL
- BES
- COSY
- Spring-8
- LEGS
- BATES
- ...

Status of meson photoproduction @ JLab

	σ	Σ	T	P	E	F	G	H	T_x	T_z	L_x	L_z	O_x	O_z	C_x	C_z
Proton target																
$p\pi^0$	✓	✓	✓	/	✓	✓	✓	✓								
$n\pi^+$	✓	✓	✓	/	✓	✓	✓	✓								
$p\eta$	✓	✓	✓	/	✓	✓	✓	✓								
$p\eta'$	✓	✓	✓	/	✓	✓	✓	✓								
$p\omega$	✓	✓	✓		✓	✓	✓	✓								
$K^+\Lambda$	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
$K^+\Sigma^0$	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
$K^0\Sigma^+$	✓	✓	✓	/	✓	✓	✓	✓								
"Neutron" target																
$p\pi^-$	✓	✓	✓		✓	✓	✓	✓								
$p\rho^-$	✓	✓	✓		✓	✓	✓	✓								
$K^+\Sigma^-$	✓	✓	✓		✓	✓	✓	✓								
$K^0\Lambda$	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
$K^0\Sigma^0$	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
$K^0\Sigma^0$	✓	✓														

✓ - published ✓ - acquired

Courtesy of E. Pasyuk, MESON 2012, Poland

Theory: N^* 's unstable & couple strongly to baryon-meson states

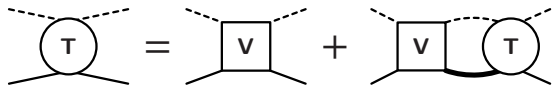


Build coupled-channels meson-baryon reaction models:

- meson production **data analysis**
- understand the **reaction mechanisms**
- extract N^* **parameters**
- understand N^* **structures and dynamical origins**

Most widely used models (hadron level): **K matrix approximation**, **chiral unitary approach**, **dynamical coupled-channels model**, et al.

Theoretical approaches



$$\mathbf{T} = \mathbf{V} + \mathbf{V}\mathbf{G}_0\mathbf{T}$$

T: amplitude

V: pseudo-potential

G₀: free propagator

Sum over all channels and partial waves, and **integrate** over intermediate momenta.

- **K matrix approximation** $G_0 = \frac{1}{E-H_0+i\epsilon} = \frac{\mathcal{P}}{E-H_0} - i\pi\delta(E-H_0) \approx -i\pi\delta(E-H_0)$
 - Principle-value part of the loop integral neglected \Rightarrow off-shell intermediate states omitted \Rightarrow integral equation reduced to algebraic equation
 - Unitarity satisfied, analyticity violated
- **Chiral unitary approach** $V \propto C\bar{u}(p')\gamma^\mu u(p)(k_\mu + k'_\mu) \approx C(k_0 + k'_0)$
 - V: contact interaction, momentum-dependent terms neglected \Rightarrow S-wave
 - Resonances are dynamically generated
- **Dynamical coupled-channels model**
 - On-shell & off-shell effects considered
 - Resonance dynamical generation allowed
 - Unitarity & analyticity respected
 - Examples: Jülich model, ANL-Osaka model

Jülich hadron-exchange model ingredients

(a)

(b)

(d)

(c)

(e)

(a) $T = |F\rangle S \langle F| + X$

T : full amplitude

S : dressed res. propagator

(b) $T = V + V G_0 T$

X : non-pole amplitude

S_0 : bare res. propagator

(c) $V = |f\rangle S_0 \langle f| + U$

U : driving term of X

$|F\rangle$: dressed res. vertex

(d) $X = U + U G_0 X$

V : driving term of T

$|f\rangle$: bare res. vertex

(a)

(a) $S = S_0 + S \underbrace{\langle F| G_0 |f\rangle}_{\text{"self energy"} \Sigma} S_0$

(b)

(b) $|F\rangle = |f\rangle + X G_0 |f\rangle$

Main features of Jülich hadron-exchange model

- Two-body **unitarity** guaranteed by scattering equation
- **Analyticity** (no on-shell factorization); resonance dynamical generation allowed
- **Channels & partial waves correlated** through t - & u -channel interactions \Rightarrow dynamical generation not easy; no need to model background with resonances
- Effective interaction based on **chiral Lagrangians** of Wess & Zumino, supplemented by additional terms for including Δ , ω , η , a_0 , σ
- **GAUGE INVARIANCE**

Generalized Ward-Takahashi Identity (GWTI) for π production current

$$k_\mu M^\mu = - |F_s \tau\rangle S_{p+k} Q_i S_p^{-1} + S_{p'}^{-1} Q_f S_{p'-k} |F_u \tau\rangle + \Delta_{p-p'+k}^{-1} Q_\pi \Delta_{p-p'} |F_t \tau\rangle$$

\Downarrow on shell

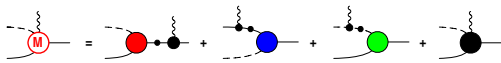
Current conservation

$$k_\mu M^\mu = 0$$

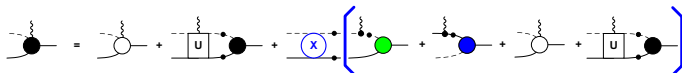
Current conservation necessary but not sufficient!

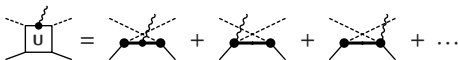
The vast majority of existing models does not satisfy gauge invariance!

Prescription for gauge invariance



$$M^\mu = M_s^\mu + M_u^\mu + M_t^\mu + M_{\text{int}}^\mu$$





$$M_{\text{int}}^\mu = M_c^\mu + X G_0 (M_u^\mu + M_t^\mu + M_c^\mu)_T$$

$$M_c^\mu = \Gamma_{NN\pi}(q) C^\mu + M_{KRf}^\mu$$

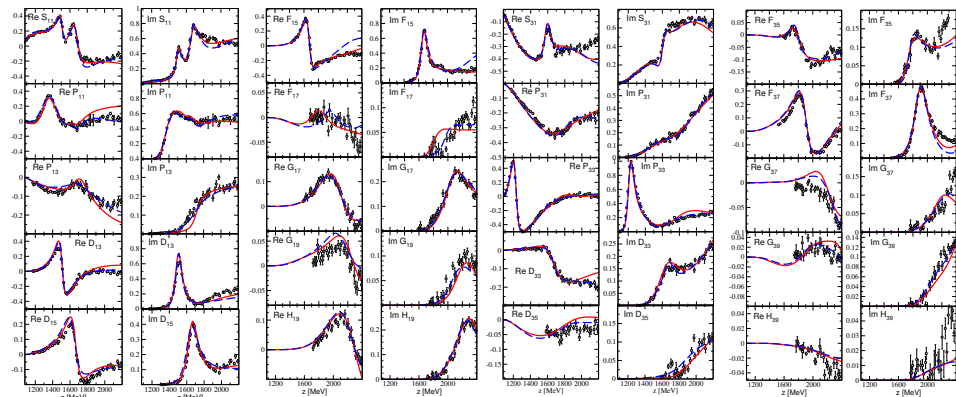
$$C^\mu = e_\pi \frac{(2q - k)^\mu}{t - q^2} (f_t - \hat{F}) + e_f \frac{(2p' - k)^\mu}{u - p^2} (f_u - \hat{F}) + e_i \frac{(2p + k)^\mu}{s - p^2} (f_s - \hat{F})$$

$$\hat{F} = 1 - \hat{h} (1 - \delta_s f_s) (1 - \delta_u f_u) (1 - \delta_t f_t)$$

Recent progress: $\pi N \rightarrow \pi N, \eta N, \Lambda K, \Sigma K$ & $\gamma N \rightarrow \pi N$

- Channel space: $\gamma N \oplus \pi N \oplus \eta N \oplus \pi \Delta \oplus \rho N \oplus \sigma N \oplus K \Lambda \oplus K \Sigma$
- Resonances considered: $J = 1/2, 3/2, 5/2, 7/2, 9/2$
- Reactions and observables investigated:
 - $\pi N \rightarrow \pi N$: partial wave amplitudes up to $J = 9/2$
 - $\pi^- p \rightarrow \eta n, K^0 \Lambda, K^0 \Sigma^0, K^+ \Sigma^-$: $d\sigma/d\Omega, P, \sigma$ (up to ~ 2.4 GeV)
 - $\pi^+ p \rightarrow K^+ \Sigma^+$: $d\sigma/d\Omega, P, \sigma$ (up to ~ 2.4 GeV)
 - $\gamma p \rightarrow \pi^0 p, \pi^+ n$: $d\sigma/d\Omega, \Sigma, \sigma$ (up to ~ 1.65 GeV)
 - $\gamma n \rightarrow \pi^- p, \pi^0 n$: $d\sigma/d\Omega, \Sigma, \sigma$ (up to ~ 1.65 GeV)
 - $\gamma p \rightarrow \pi^0 p, \pi^+ n$: $d\sigma/d\Omega, \Sigma, T, P, G, H, E, C_{x'_L}, C_{z'_L}$
(phenomenological parametrization of photo-coupling vertices, up to ~ 2.4 GeV)
- Quantities extracted for N^{*} 's: pole positions, residues, photo-couplings

$\pi N \rightarrow \pi N$ partial wave amplitudes



S_{11} : 1535, 1650

P_{11} : 1440, 1710, 1750

P_{13} : 1720

D_{13} : 1520

D_{15} : 1675

F_{15} : 1680

F_{17} : 1990

G_{17} : 2190

G_{19} :

H_{19} :

S_{31} : 1620

P_{31} : 1910

P_{33} : 1232, 1600

D_{33} : 1700

D_{35} :

F_{35} : 1905

F_{37} : 1950

G_{37} : 2200

G_{39} :

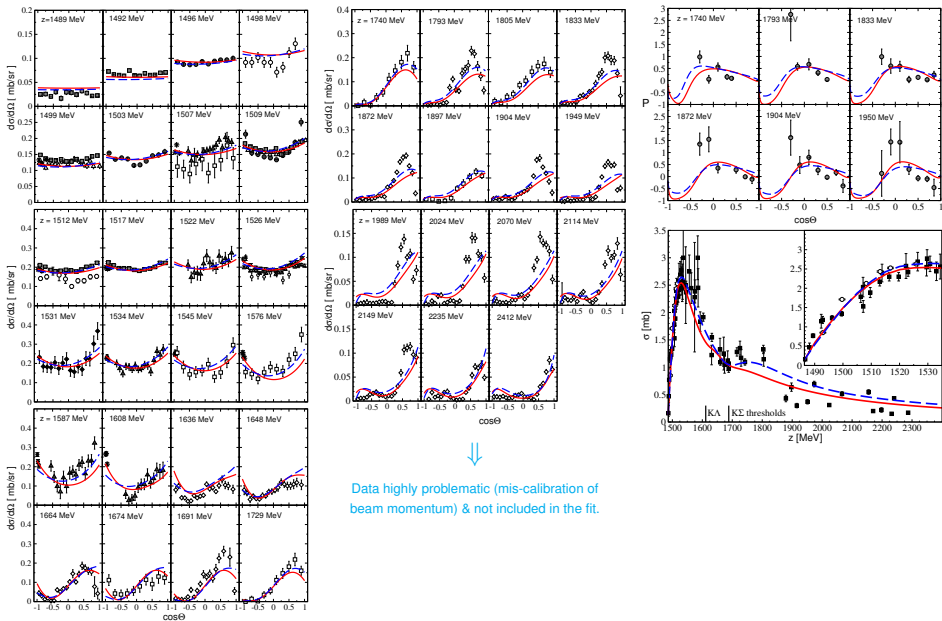
H_{39} :

**

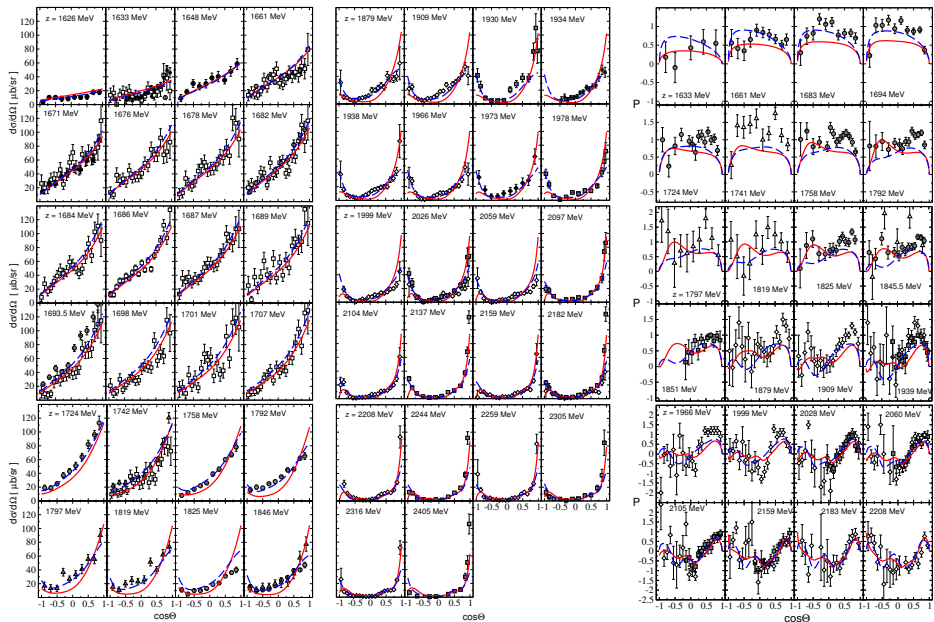
*

no star

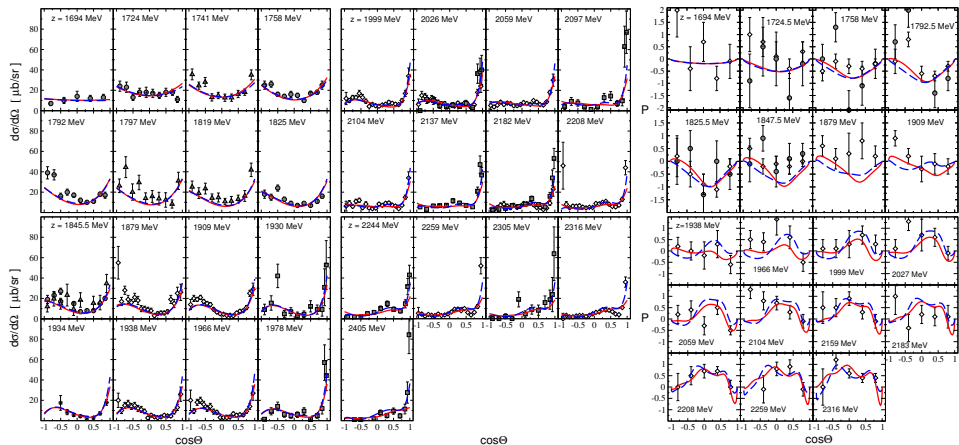
$d\sigma/d\Omega$, P & σ for $\pi^-p \rightarrow \eta n$



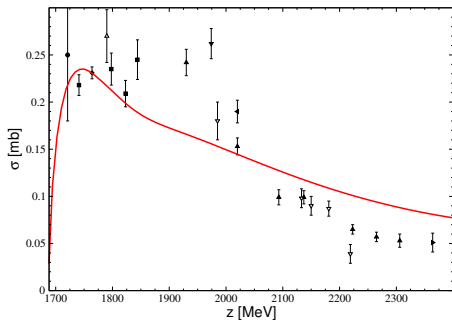
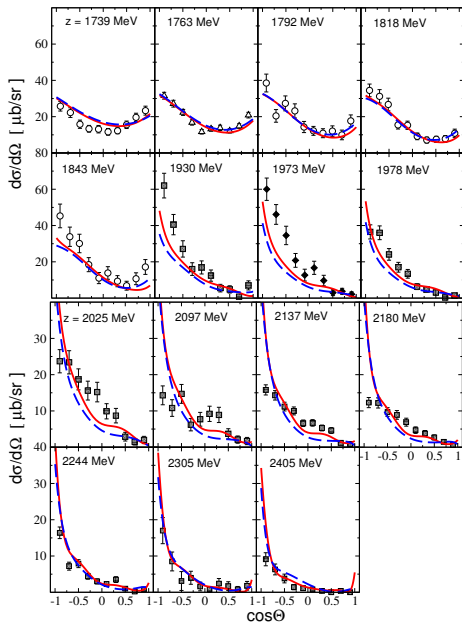
$d\sigma/d\Omega$ & P for $\pi^-p \rightarrow K^0\Lambda$



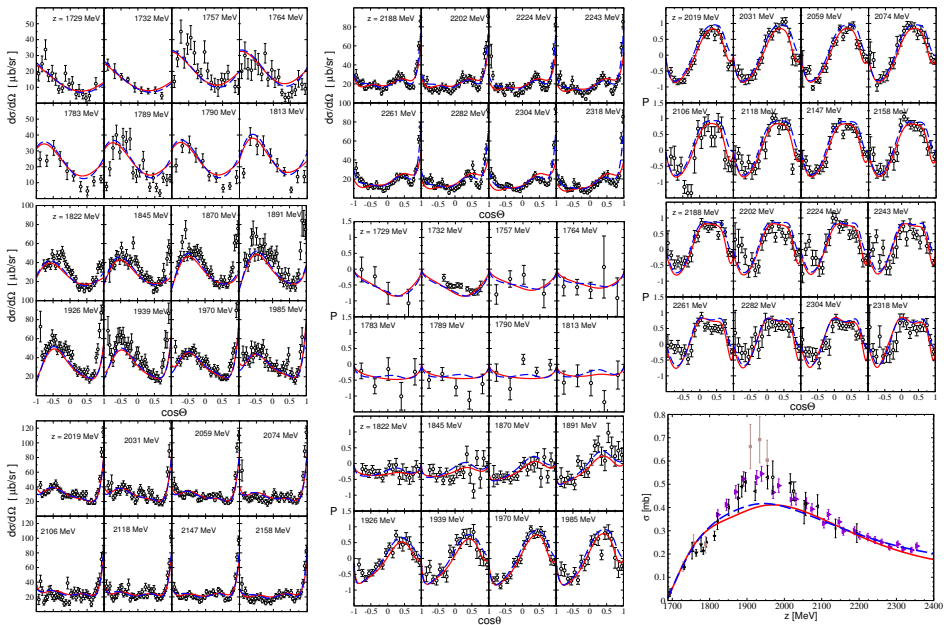
$d\sigma/d\Omega$ & P for $\pi^-p \rightarrow K^0\Sigma^0$



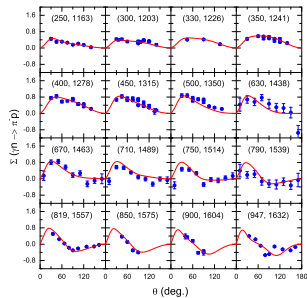
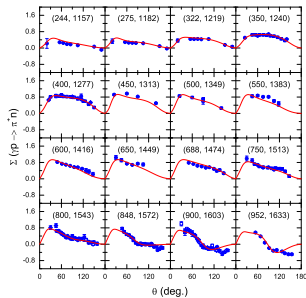
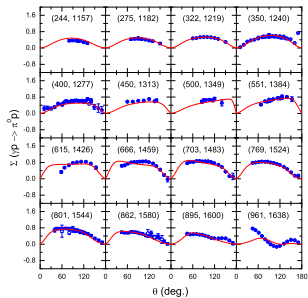
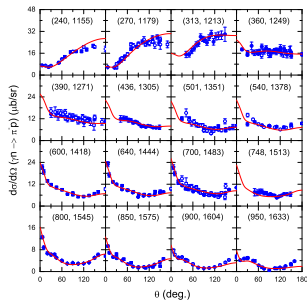
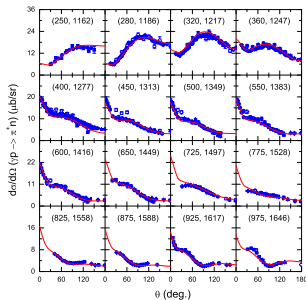
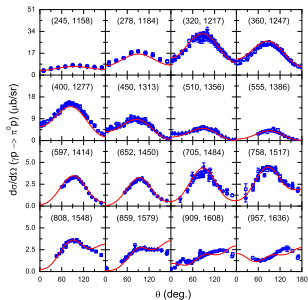
$d\sigma/d\Omega$ & σ for $\pi^-p \rightarrow K^+\Sigma^-$



$d\sigma/d\Omega$, P & σ for $\pi^+p \rightarrow K^+\Sigma^+$

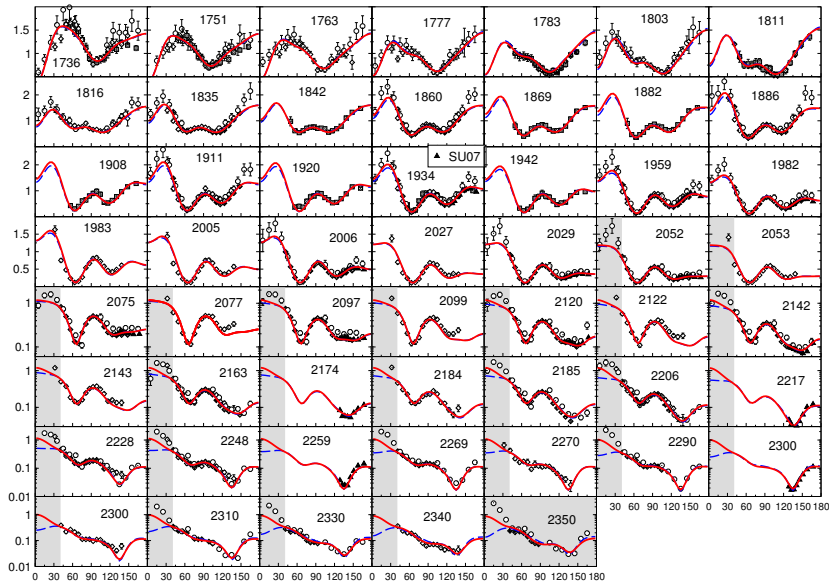


$d\sigma/d\Omega$ & Σ_γ for $\gamma p \rightarrow \pi^0 p$, $\gamma p \rightarrow \pi^+ n$ & $\gamma n \rightarrow \pi^- p$



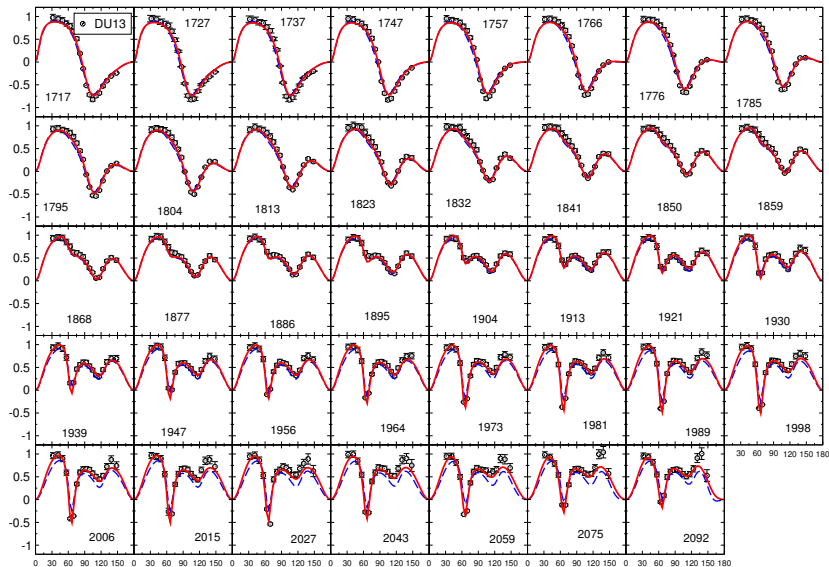
Selected results for $\gamma p \rightarrow \pi^0 p$ (phenomenological parametrization of Γ_{γ})

$d\sigma/d\Omega$ [$\mu\text{b}/\text{sr}$] as a function of θ [deg]



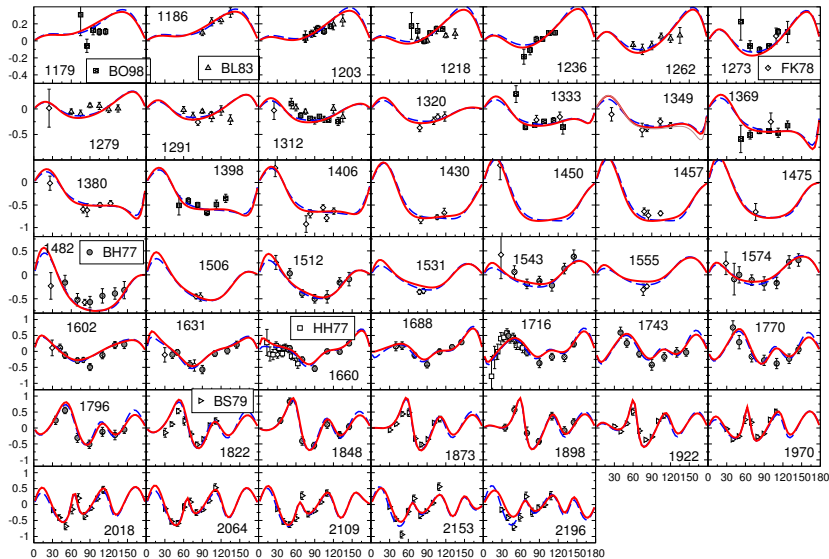
Selected results for $\gamma p \rightarrow \pi^0 p$ (phenomenological parametrization of Γ_{γ})

Σ as a function of θ [deg]



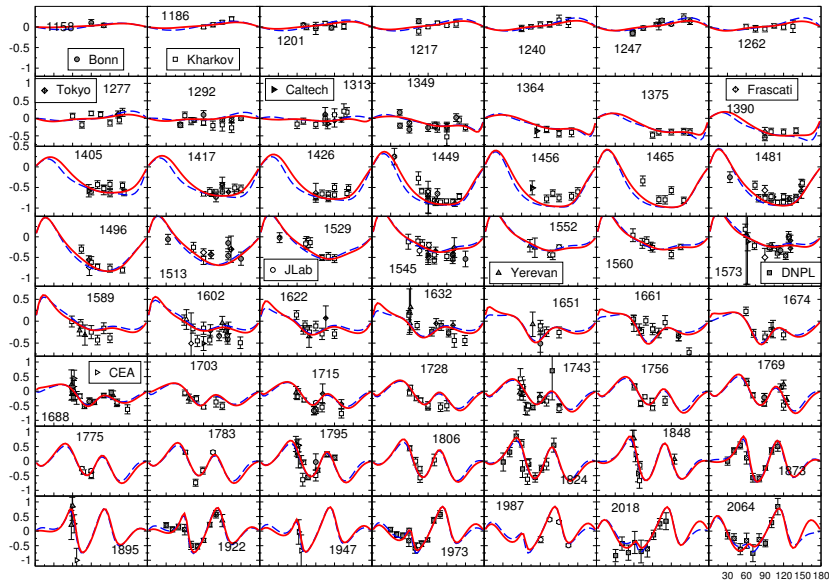
Selected results for $\gamma p \rightarrow \pi^0 p$ (phenomenological parametrization of Γ_{γ})

T as a function of θ [deg]



Selected results for $\gamma p \rightarrow \pi^0 p$ (phenomenological parametrization of Γ_{γ})

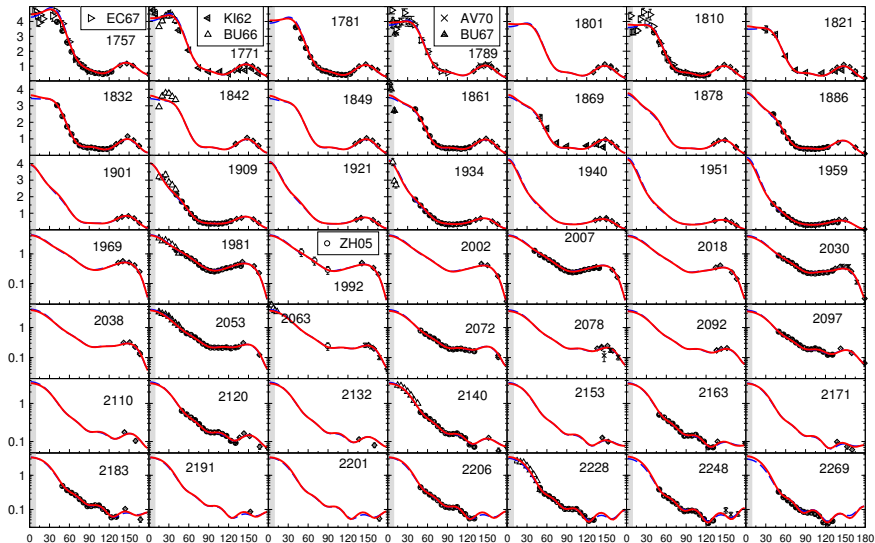
P as a function of θ [deg]



30 60 90 120 150 180

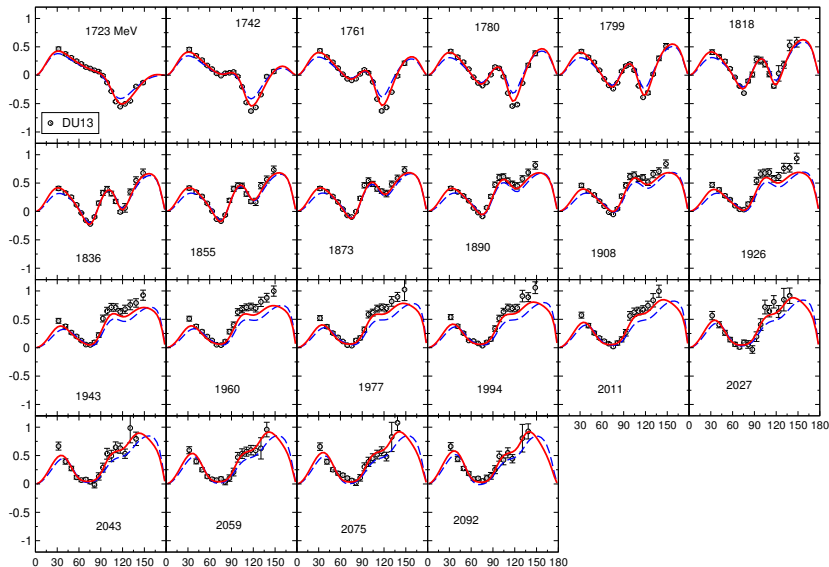
Selected results for $\gamma p \rightarrow \pi^+ n$ (phenomenological parametrization of Γ_{-})

$d\sigma/d\Omega$ [$\mu\text{b/sr}$] as a function of θ [deg]



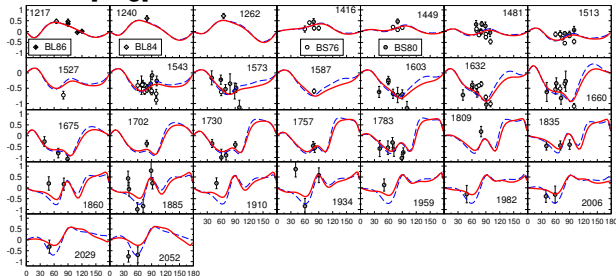
Selected results for $\gamma p \rightarrow \pi^+ n$ (phenomenological parametrization of Γ_{-})

Σ as a function of θ [deg]

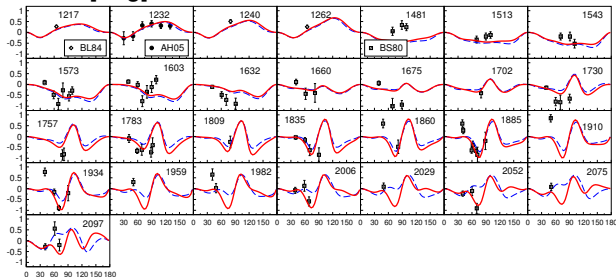


Selected results for $\gamma p \rightarrow \pi^+ n$ (phenomenological parametrization of Γ_{-})

H as a function of θ [deg]



G as a function of θ [deg]



Resonances and their structures

Resonances found:

- 4-star resonances: 14 (17 in PDG):

$P_{33}(1232)$, $P_{11}(1440)$, $D_{13}(1520)$, $S_{11}(1535)$, $S_{31}(1620)$, $S_{11}(1650)$, $D_{15}(1675)$,
 $F_{15}(1680)$, $D_{33}(1700)$, $P_{13}(1720)$, $F_{35}(1905)$, $P_{31}(1910)$, $F_{37}(1950)$, $G_{17}(2190)$

- 3-star resonances: 2 (8 in PDG):

$P_{11}(1710)$, $P_{33}(1600)$

- 2-star resonances: 1 (15 in PDG):

$F_{17}(1990)$

- 1-star resonance: 1 (8 in PDG):

$G_{37}(2200)$

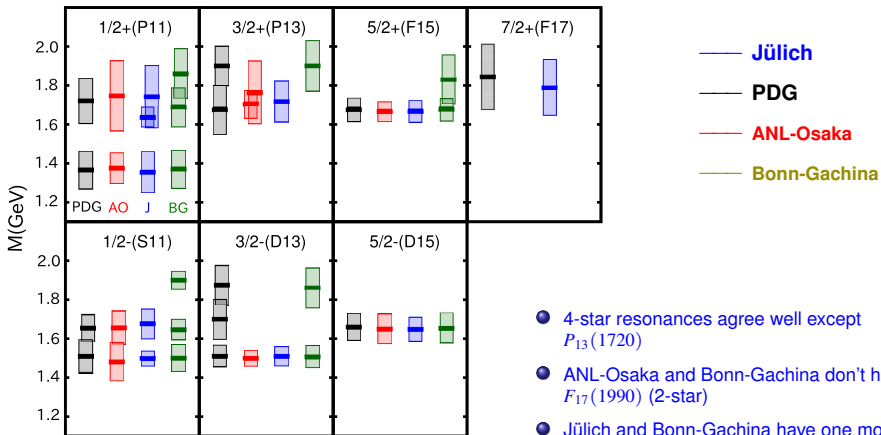
- resonance not listed in PDG:

$P_{11}(1750)$

Resonance structures:

- $P_{11}(1440)$, $P_{33}(1600)$, $P_{11}(1750)$: dynamically generated
- Others: genuine resonances

Pole positions ($I = 1/2$)



Courtesy of T. Sato, Nstar 2013, Spain

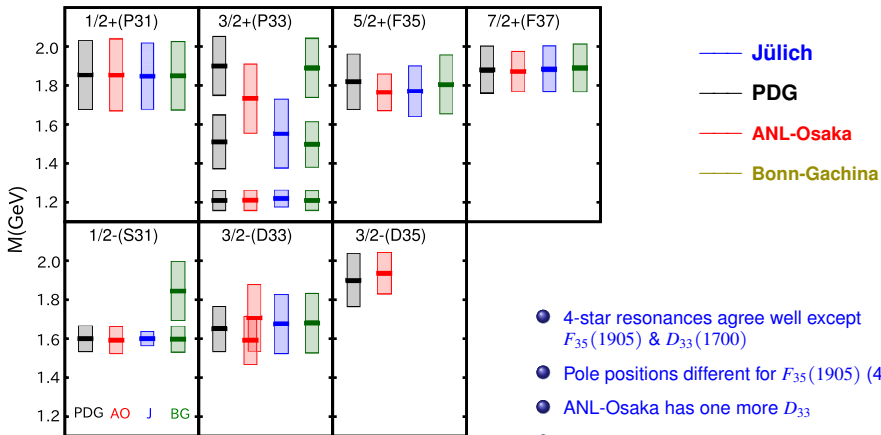
4-star resonances compared:

P_{11} (1440), P_{13} (1720), F_{15} (1680)

S_{11} (1535), S_{11} (1650), D_{13} (1520), D_{15} (1675)

- 4-star resonances agree well except P_{13} (1720)
- ANL-Osaka and Bonn-Gachina don't have F_{17} (1990) (2-star)
- Jülich and Bonn-Gachina have one more P_{11} and one less P_{13}
- Bonn-Gachina has one more F_{15} and one more S_{11}
- Bonn-Gachina and PDG have more D_{13} 's

Pole positions ($I = 3/2$)



Courtesy of T. Sato, Nstar 2013, Spain

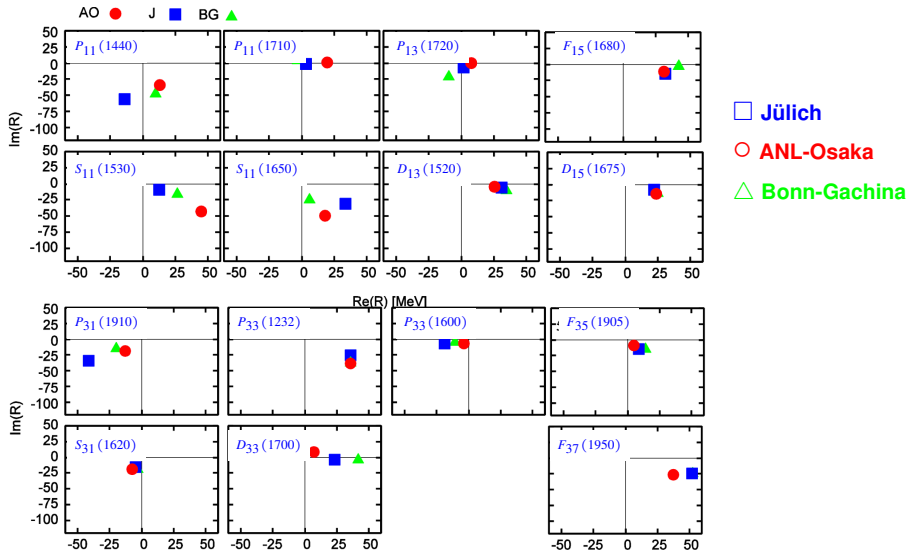
4-star resonances compared:

$P_{31}(1910)$, $P_{33}(1232)$, $F_{35}(1905)$, $F_{37}(1950)$

$S_{31}(1620)$, $D_{33}(1700)$

- 4-star resonances agree well except $F_{35}(1905)$ & $D_{33}(1700)$
- Pole positions different for $F_{35}(1905)$ (4-star)
- ANL-Osaka has one more D_{33}
- Jülich and ANL-Osaka have one less P_{33}
- Bonn-Gachina has one more S_{31}
- Jülich and Bonn-Gachina don't have D_{35}

Residues of πN amplitudes at pole positions

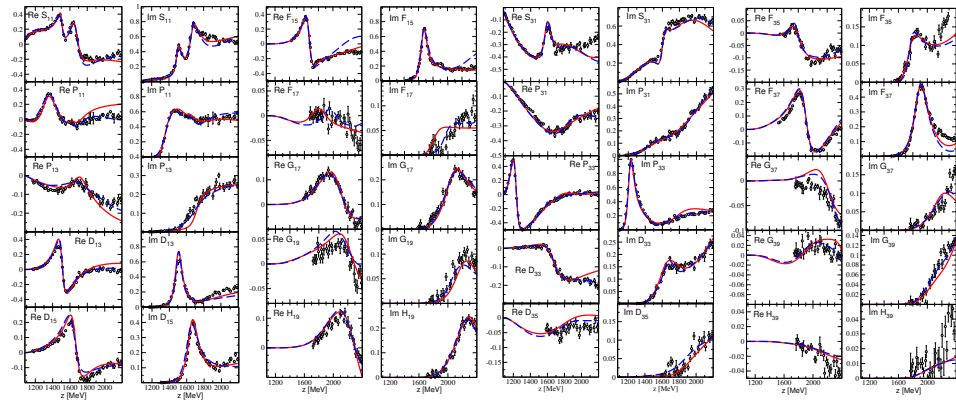


Courtesy of T. Sato, Nstar 2013, Spain

Summary & perspectives

- Channel space: $\pi N \oplus \eta N \oplus \pi \Delta \oplus \rho N \oplus \sigma N \oplus K \Lambda \oplus K \Sigma \oplus \gamma N$
- Unitarity; Analyticity; Gauge invariance
- Channels & partial waves correlated
- Recent progress: $\pi N \rightarrow \pi N$, ηN , ΛK , ΣK & $\gamma N \rightarrow \pi N$
 - $\pi N \rightarrow \pi N$ partial wave amplitudes ($J = 1/2, 3/2, 5/2, 7/2, 9/2$)
 - $\pi N \rightarrow \eta N, \Lambda K, \Sigma K$ cross sections & polarizations
 - $\gamma N \rightarrow \pi N$ cross sections, single & double polarizations
- Resonances extracted
 - 4-star resonances (14/17): $G_{19}(2250)$, $H_{19}(2220)$, $H_{311}(2420)$ not found
 - 3-star resonances (2/8): $P_{11}(1710)$, $P_{33}(1600)$
 - 2-star resonances (1/15): $F_{17}(1990)$
 - 1-star resonance (1/8): $G_{37}(2200)$
 - resonance not listed in PDG: $P_{11}(1750)$
 - Pole positions & residues extracted
- Next step work
 - $\pi N \rightarrow \omega N, \pi \pi N$
 - $\gamma N \rightarrow \eta N, K \Lambda, K \Sigma, \omega N, \pi \pi N$
 - $e N \rightarrow e \pi N$

πN amplitudes: comparison with GWU/SAID



S_{11} : 1535, 1650

P_{11} : 1440, ~~1710~~, 1750

P_{13} : 1720

D_{13} : 1520

D_{15} : 1675

F_{15} : 1680 + 1860

F_{17} : ~~1990~~

G_{17} : 2190

G_{19} :

H_{19} :

S_{31} : 1620

P_{31} : 1910

P_{33} : 1232, 1600

D_{33} : 1700

D_{35} :

F_{35} : 1905

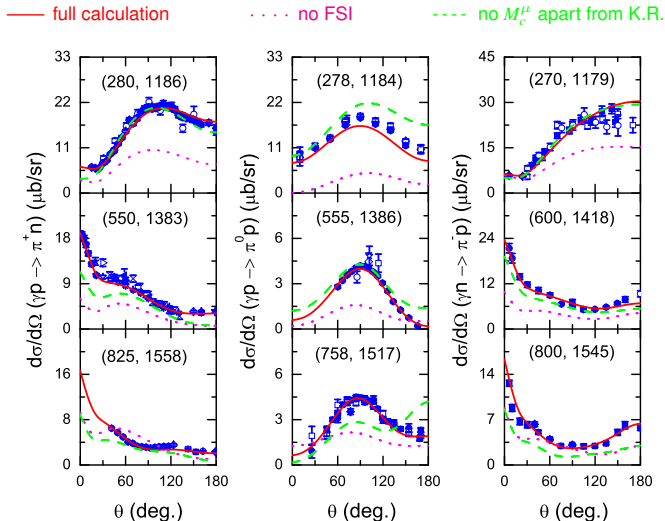
F_{37} : 1950

G_{37} : ~~2200~~

G_{39} :

H_{39} :

Relevance of FSI & gauge invariance

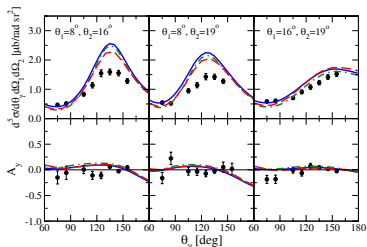
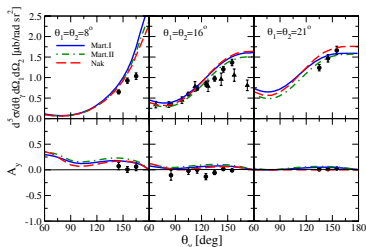


The effects of both FSI & gauge invariance are important!

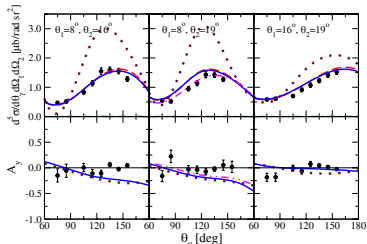
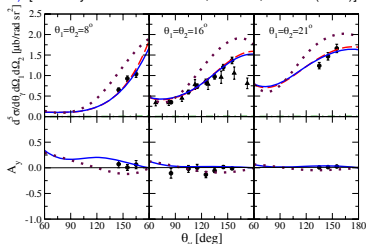
Relevance of gauge invariance in $pp \rightarrow pp\gamma$

None of the existing models can describe high-precision KVI data for coplanar geometries involving small p scattering angles.

Lines: Martinus, Scholten, Tjon, PRC 58, 686 (1998); PRC 56, 2945 (1997); Herrmann, Nakayama, Scholten, Arellano, NPA 582, 568 (1995)



Construct the contact current \rightarrow full amplitude obeys WTI (gauge invariant) [K. Nakayama & H. Haberzettl, PRC 80, 051001 (2009)]



It is important to properly take into account the interaction current for NN bremsstrahlung reaction!

Covariance & 3-D integral equation

- Jülich πN model — TOPT

$$T_{\text{TO}}(\mathbf{p}', \mathbf{p}; \sqrt{s}) = V_{\text{TO}}(\mathbf{p}', \mathbf{p}; \sqrt{s}) + \int d^3 p'' V_{\text{TO}}(\mathbf{p}', \mathbf{p}''; \sqrt{s}) G_{\text{TO}}(\mathbf{p}'', \sqrt{s}) T_{\text{TO}}(\mathbf{p}'', \mathbf{p}; \sqrt{s})$$

$$G_{\text{TO}}(\mathbf{p}'', \sqrt{s}) = \frac{1}{\sqrt{s} - E(\mathbf{p}'') - \omega(\mathbf{p}'') + i0}$$

- Converting to a covariant 3-D reduction like equation

$$V(\mathbf{p}', \mathbf{p}; \sqrt{s}) \equiv (2\pi)^3 \sqrt{2E(\mathbf{p}') 2\omega(\mathbf{p}')} \sqrt{2E(\mathbf{p}) 2\omega(\mathbf{p})} V_{\text{TO}}(\mathbf{p}', \mathbf{p}; \sqrt{s})$$

$$T(\mathbf{p}', \mathbf{p}; \sqrt{s}) \equiv (2\pi)^3 \sqrt{2E(\mathbf{p}') 2\omega(\mathbf{p}')} \sqrt{2E(\mathbf{p}) 2\omega(\mathbf{p})} T_{\text{TO}}(\mathbf{p}', \mathbf{p}; \sqrt{s})$$

$$T(\mathbf{p}', \mathbf{p}; \sqrt{s}) = V(\mathbf{p}', \mathbf{p}; \sqrt{s}) + \int \frac{d^3 p''}{(2\pi)^3} V(\mathbf{p}', \mathbf{p}''; \sqrt{s}) G_0(\mathbf{p}'', \sqrt{s}) T(\mathbf{p}'', \mathbf{p}; \sqrt{s})$$

$$G_0(\mathbf{p}'', \sqrt{s}) \equiv \frac{1}{2E(\mathbf{p}'') 2\omega(\mathbf{p}'')} \frac{1}{\sqrt{s} - E(\mathbf{p}'') - \omega(\mathbf{p}'') + i0}$$

- Similarly, make 3-D reduction of the covariant photoproduction equation