

The g_2 Spin Structure Function

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On Behalf of the E08-027 Collaboration

Electron Scattering

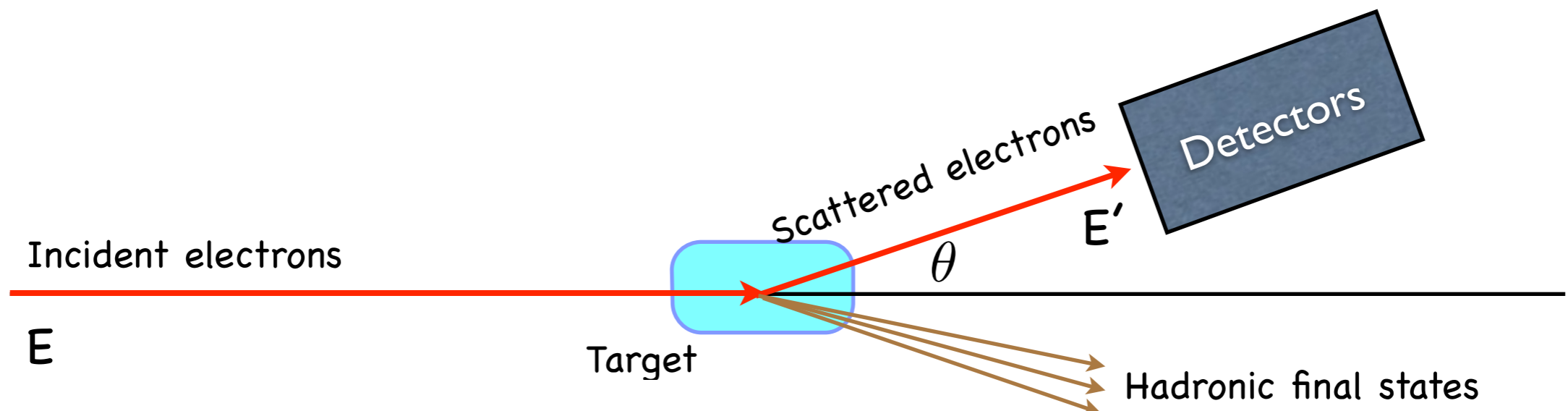
- Inclusive **unpolarized** cross section:

$$\frac{d^2\sigma}{d\Omega dE'} = \sigma_{\text{Mott}} \left[\frac{1}{\nu} F_2(x, Q^2) + \frac{2}{M} F_1(x, Q^2) \tan^2 \frac{\theta}{2} \right]$$

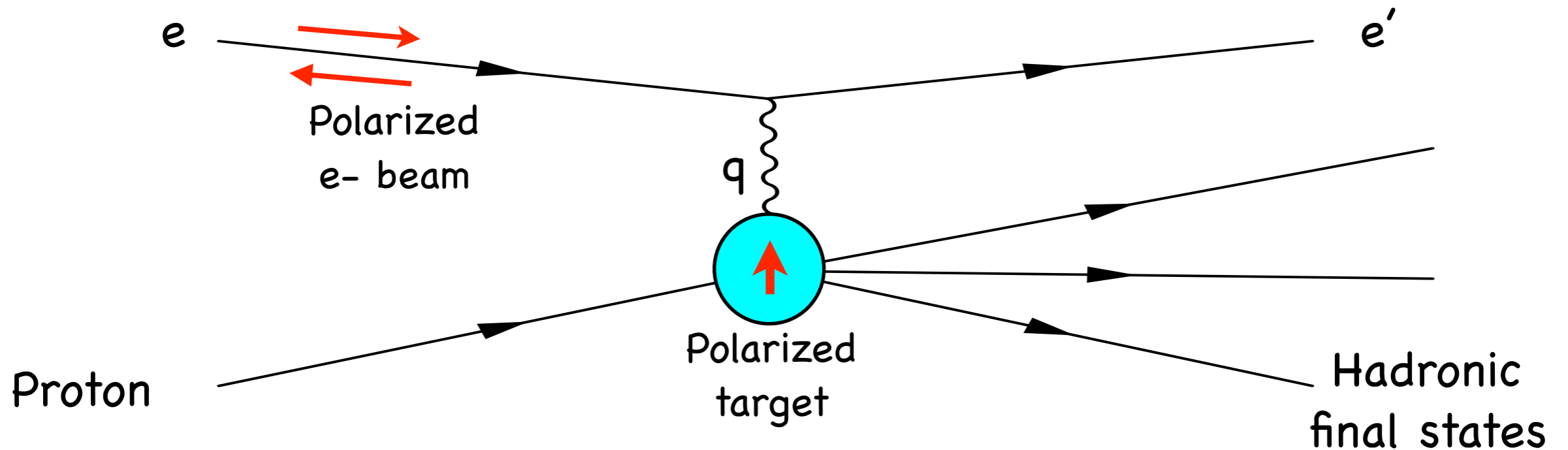
Structure Function which indicates the parton distribution

- At Bjorken Limit $Q^2 \rightarrow \infty$:

$$F_1 = \frac{1}{2} \sum_i e_i^2 q_i(x) \qquad F_2 = 2x F_1$$



Electron Scattering



- If the beam and target are polarized, the asymmetric part of the lepton and hadron tensor will not vanish, which leads to 2 additional structure functions g_1 and g_2

$$\frac{d^2\sigma}{d\Omega dE'} = \sigma_{\text{Mott}} \left[\frac{1}{\nu} F_2(x, Q^2) + \frac{2}{M} F_1(x, Q^2) \tan^2 \frac{\theta}{2} + \gamma g_1(x, Q^2) + \delta g_2(x, Q^2) \right]$$

2 addition structure functions which are related to the polarized parton distributions

Structure Function

- At Bjorken limit, g_1 related to the polarized parton distribution functions

$$g_1 = \frac{1}{2} \sum_i e_i^2 \Delta q_i(x) \quad \Delta q_i(x) = q_i^\uparrow(x) - q_i^\downarrow(x)$$

- However g_2 does not show a simple relation with parton distribution functions at Bjorken limit
- g_2^{WW} is the leading twist part of the g_2 :

$$g_2(x, Q^2) = g_2^{\text{WW}}(x, Q^2) + \bar{g}_2(x, Q^2)$$

which can be calculated from g_1 with the Wandzura-Wilczek relation

$$g_2^{\text{WW}} = -g_1(x, Q^2) + \int_x^1 \frac{dy}{y} g_1(y, Q^2)$$

Structure Function

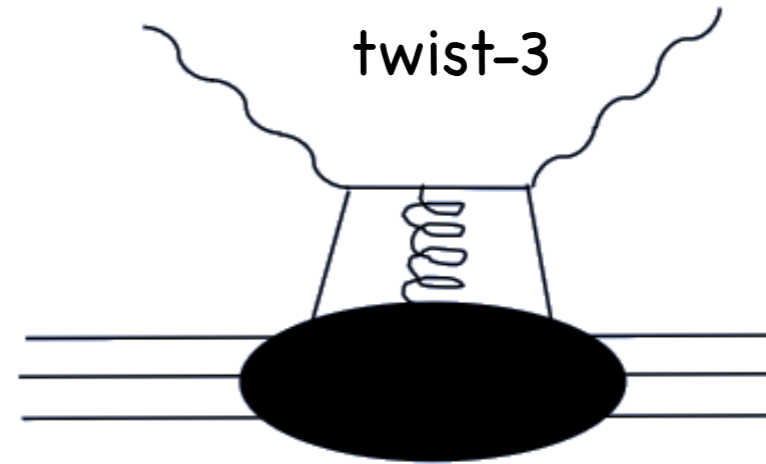
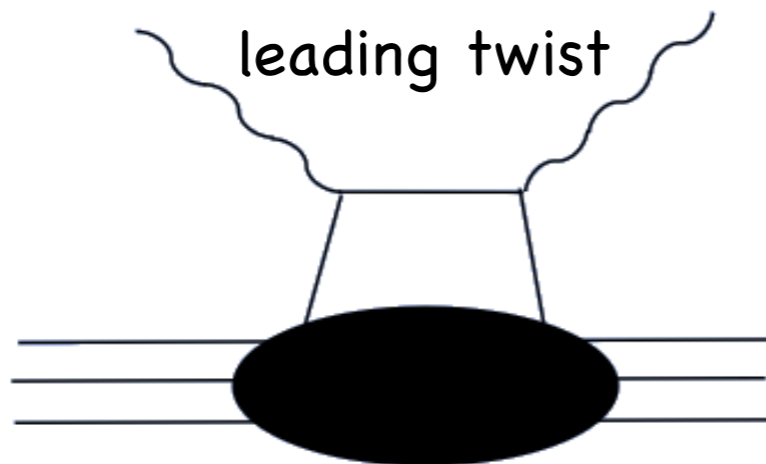
- Higher twist components can be expressed as:

$$\bar{g}_2(x, Q^2) = - \int_x^1 \frac{\partial}{\partial y} \left[\frac{m_q}{M} h_T(y, Q^2) + \zeta(y, Q^2) \right] \frac{dy}{y}$$

quark transverse momentum
contribution

twist-3 part which arises from
quark-gluon interactions

- Will get information about higher twist effect when measuring g_2



Measurements of g_2 and its Moments

- Measurements of g_2 need transversely polarized targets, more difficult experimentally

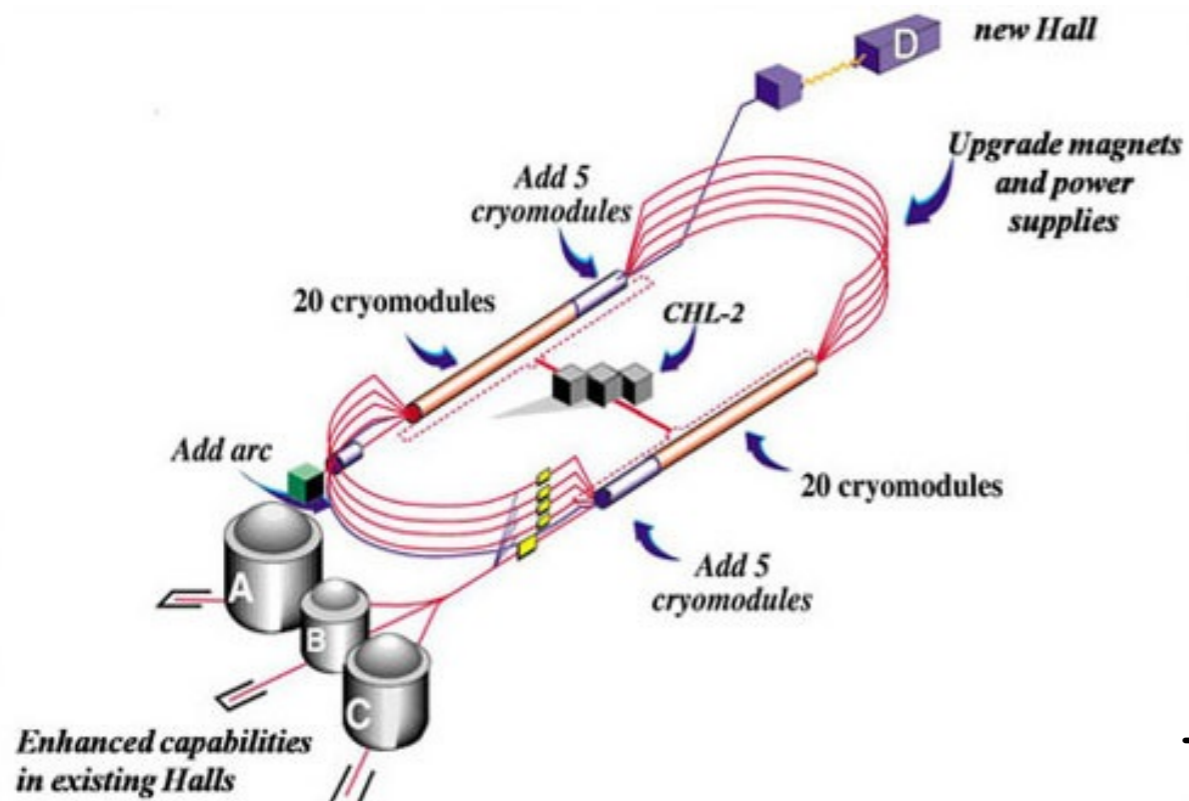
- 0th moment (no x -weighting): Burkhardt-Cottingham (BC) Sum Rule

$$\int_0^1 g_2(x, Q^2) dx = 0$$

- Valid at all Q^2
- 2nd moment (x^2 weighting):
 - High Q^2 – d_2 , twist-3 color polarizability, test of lattice QCD
 - Low Q^2 – spin polarizabilities, test of Chiral Perturbation Theory (χ PT)

Measurements of g_2 and its Moments

- High-intensity electron accelerator
- $E_{\text{max}} = 6 \text{ GeV}$
- $I_{\text{max}} = 200 \text{ uA}$
- $\text{Pol}_{\text{max}} = 90\%$
- Upgrading to 12 GeV

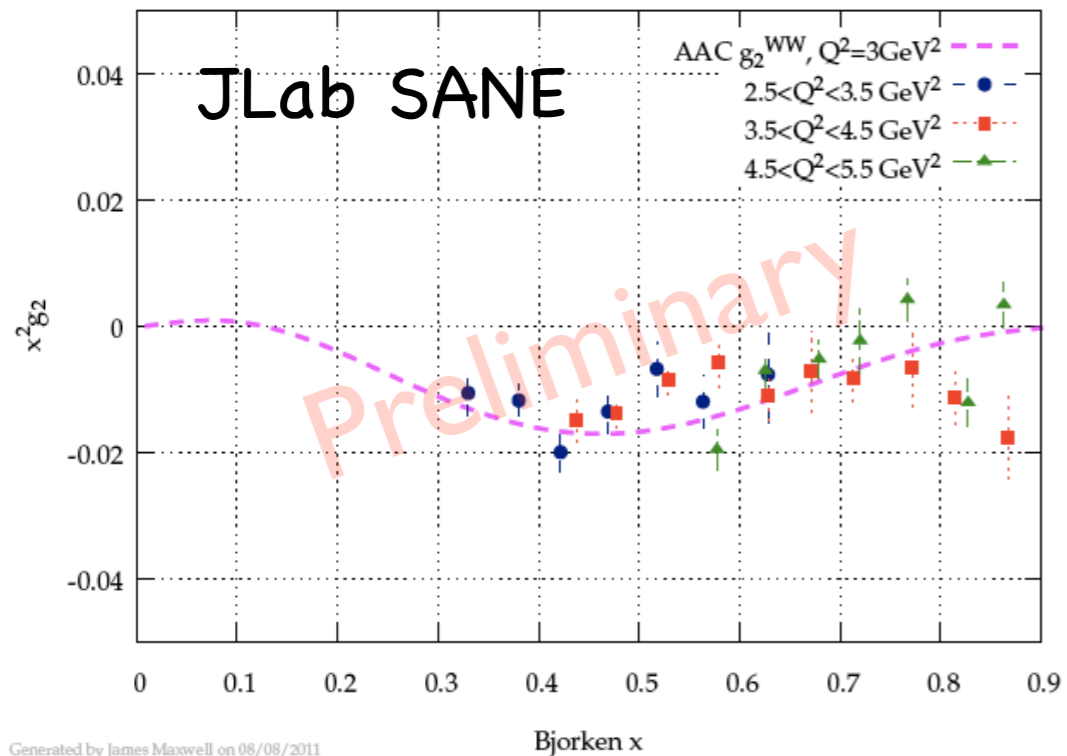
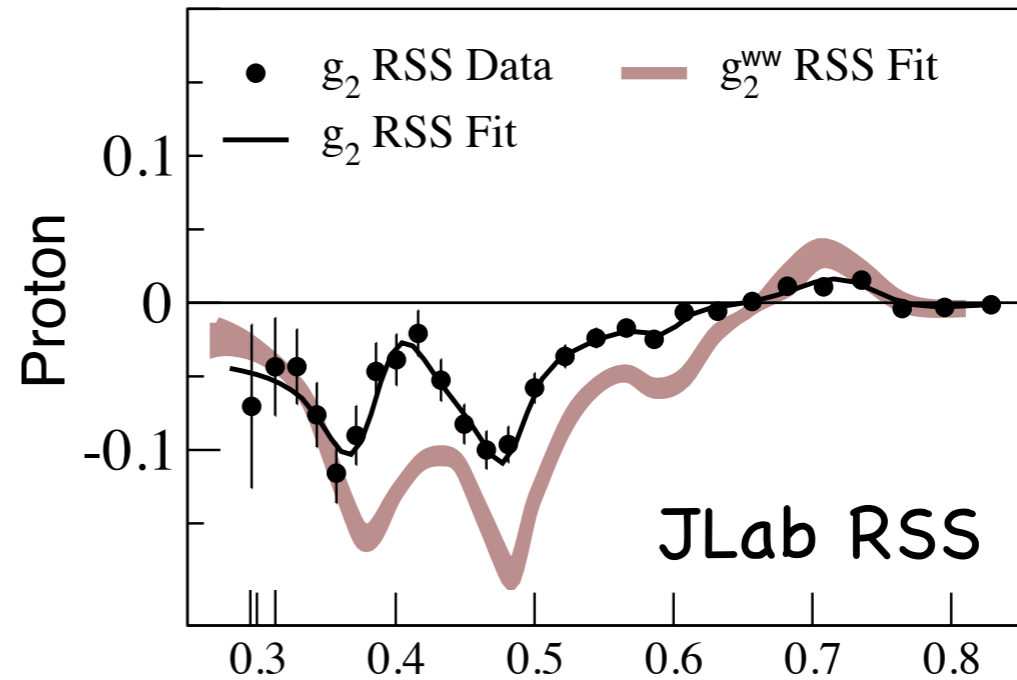
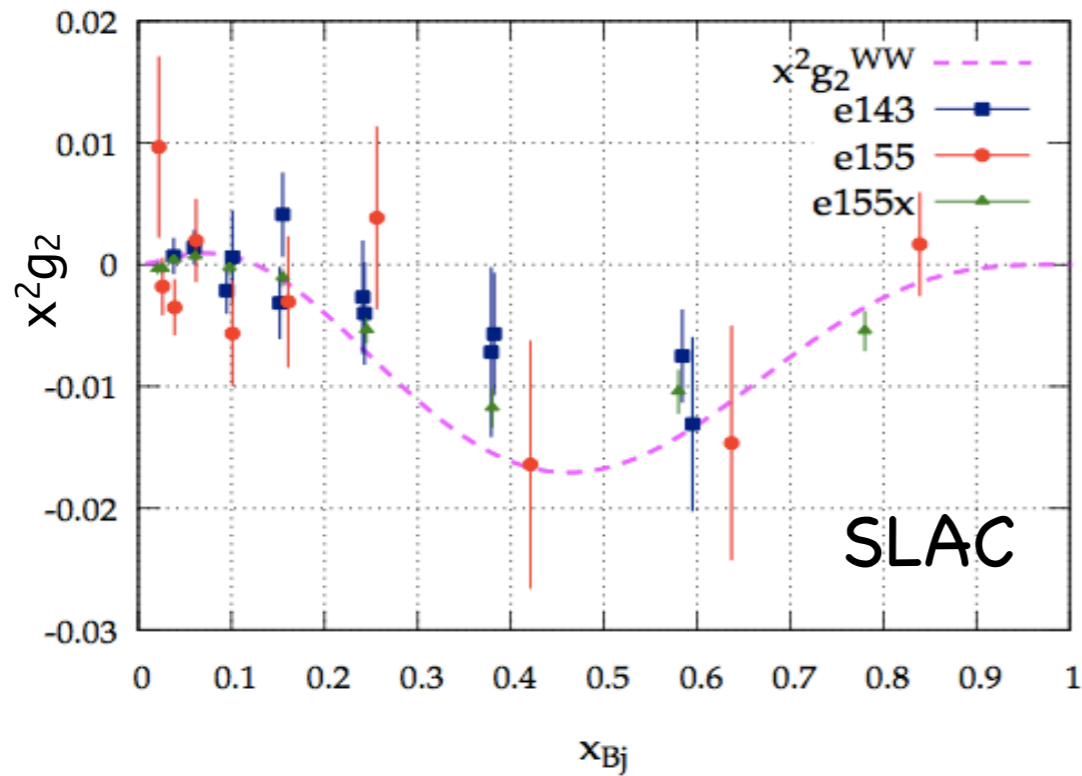


Thomas Jefferson National Accelerator Facility

Measurements of g_2 and its Moments

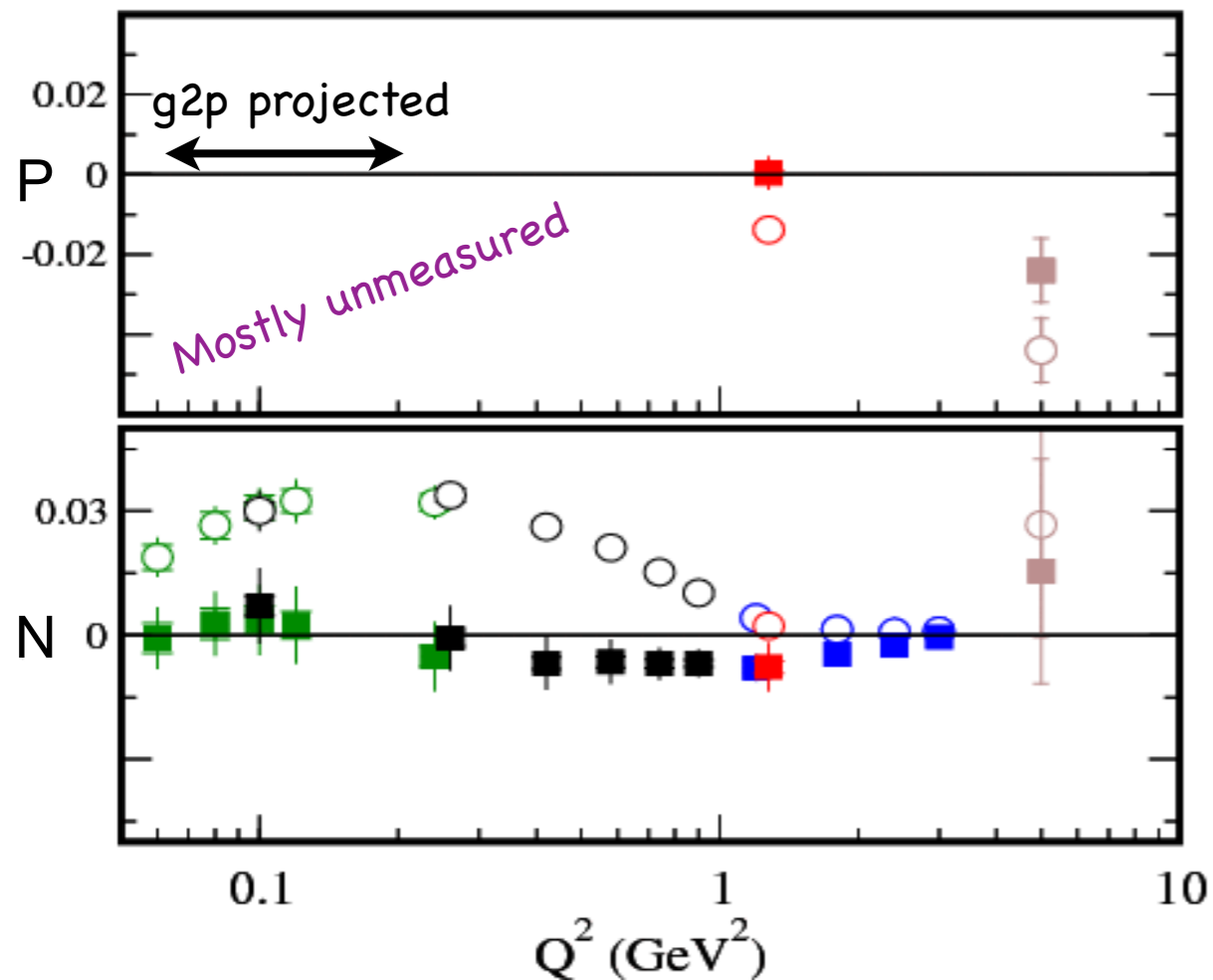
- SLAC E155x: Only dedicated measurement before JLab, not high precision, wider range of Q^2 for moment
- g_2 Measurements on the neutron at JLab:
 - E97-103: $W > 2$ GeV, $Q^2 \approx 1 \text{ GeV}^2$, $x \approx 0.2$, study higher twist (published)
 - E99-117: $W > 2$ GeV, high Q^2 (3-5 GeV^2) (published)
 - E94-010: moments at low Q^2 (0.1-1 GeV^2) (published)
 - E97-110: moments at very low Q^2 (0.02-0.3 GeV^2) (analysis)
 - E01-012: moments at intermediate Q^2 (1-4 GeV^2) (submitted)
 - E06-014: moments at high Q^2 (2-6 GeV^2) (published)
- g_2 Measurements on the proton at JLab:
 - RSS: moments at intermediate Q^2 (1-2 GeV^2) (published)
 - SANE: moments at high Q^2 (2-6 GeV^2) (analysis)
 - E08-027 (g_{2p}): moments at very low Q^2 (0.02-0.2 GeV^2) (analysis)

Measurements of g_2 and its Moments



- g_2 Measurements on the proton:
 - SLAC: 1 ~ 10 GeV^2
 - SANE: 2 ~ 6 GeV^2
 - RSS: 1 ~ 2 GeV^2

BC Sum Rule: 0th Moment



- SLAC E155x
- Hall C RSS
- Hall A E94-010
- Hall A E97-110 (preliminary)
- Hall A E01-012 (preliminary)

- BC Sum Rule:

$$\int_0^1 g_2(x, Q^2) dx = 0$$

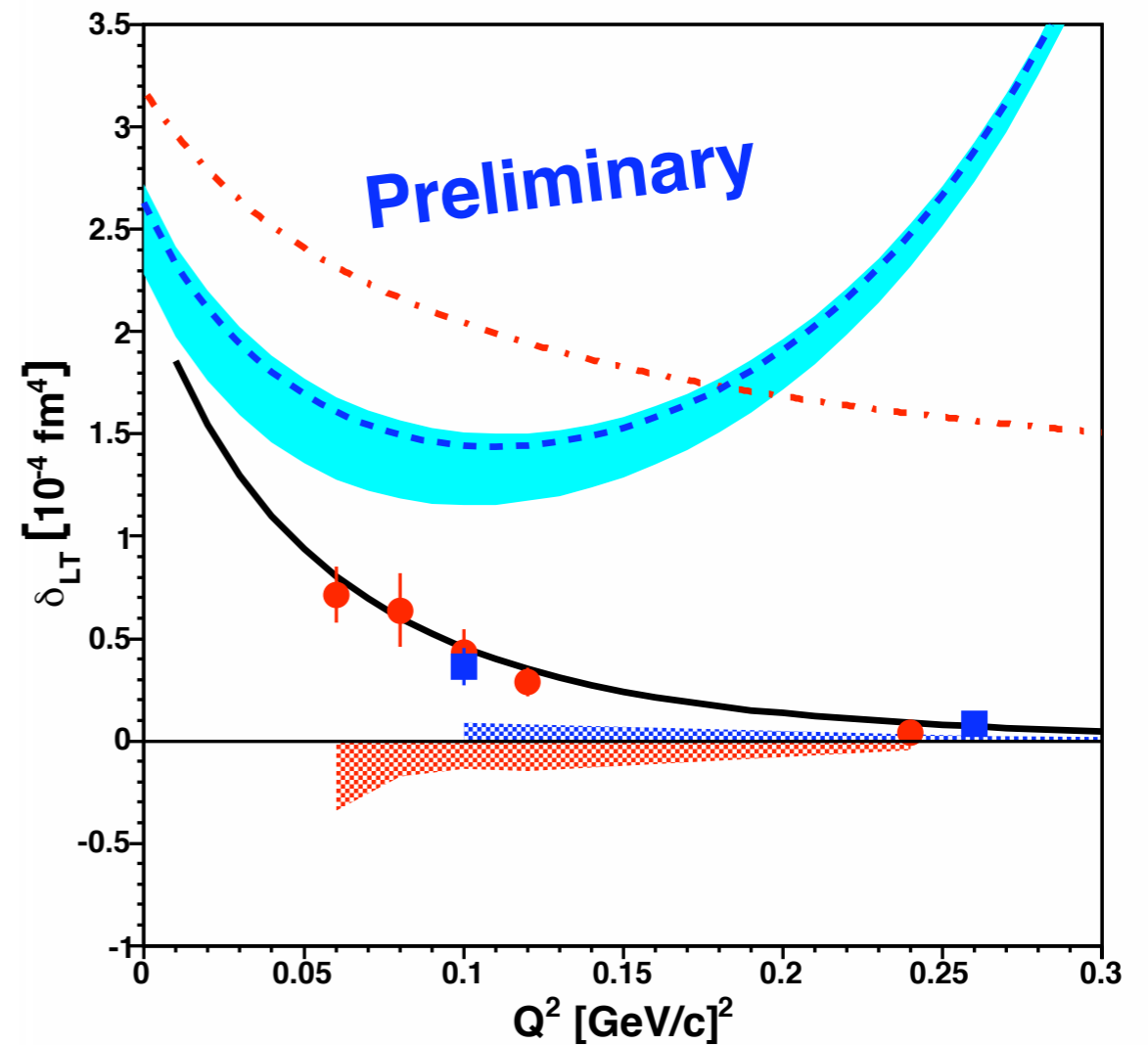
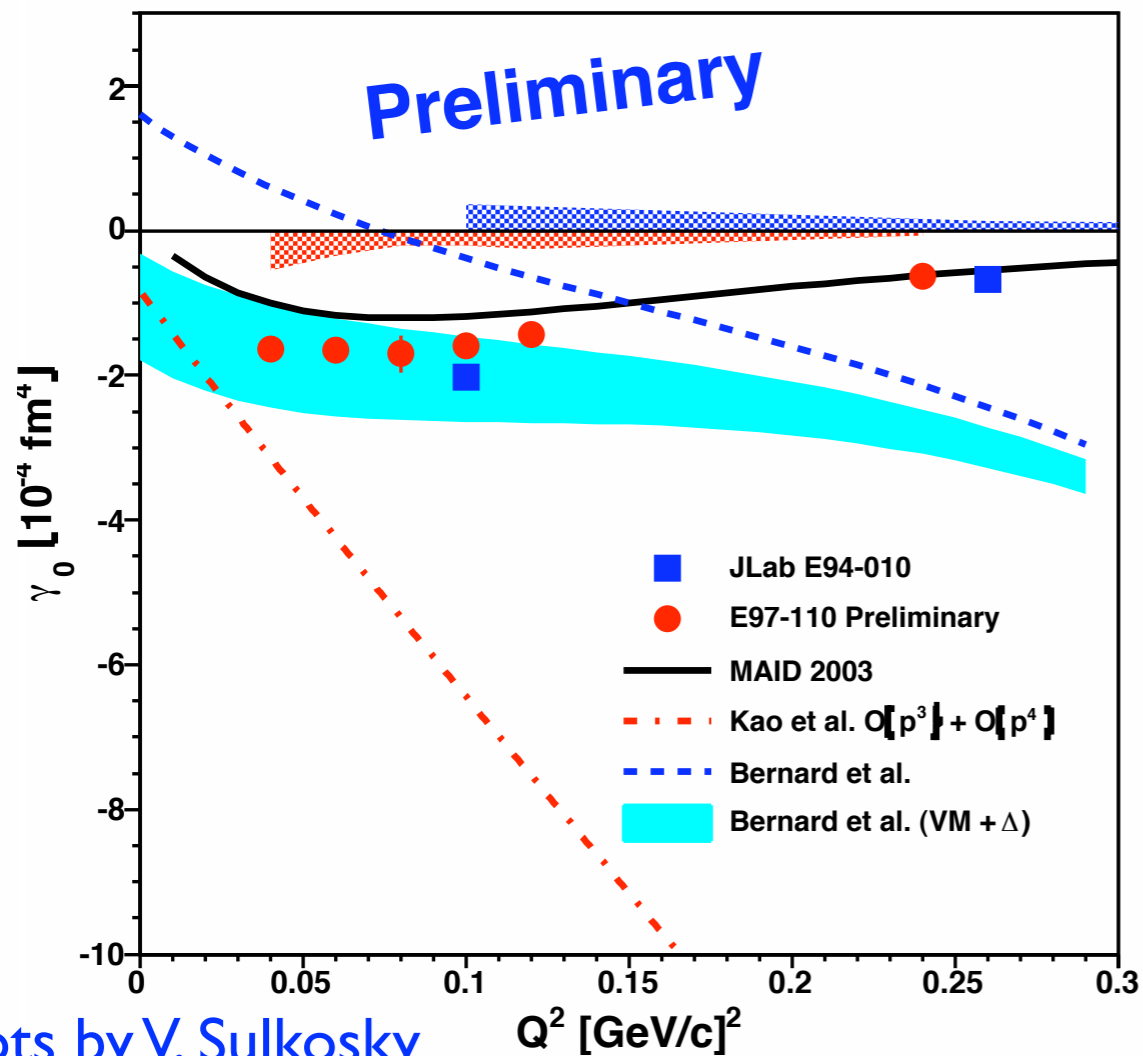
- Violation suggested for proton at large Q^2
- BC Sum = Meas + Low x + Elastic
 - “Meas”: measured x range (open circle)
 - “Low x”: unmeasured low-x part of the integral – assume leading twist behavior
 - “Elastic”: from well known Form Factors (<5%)

Spin Polarizability: 2nd Moment

- Generalized spin polarizabilities γ_0 and δ_{LT} are a benchmark test of χ PT
- One difficulty is how to include the nucleon resonance contributions
 - γ_0 is sensitive to resonances, δ_{LT} is not

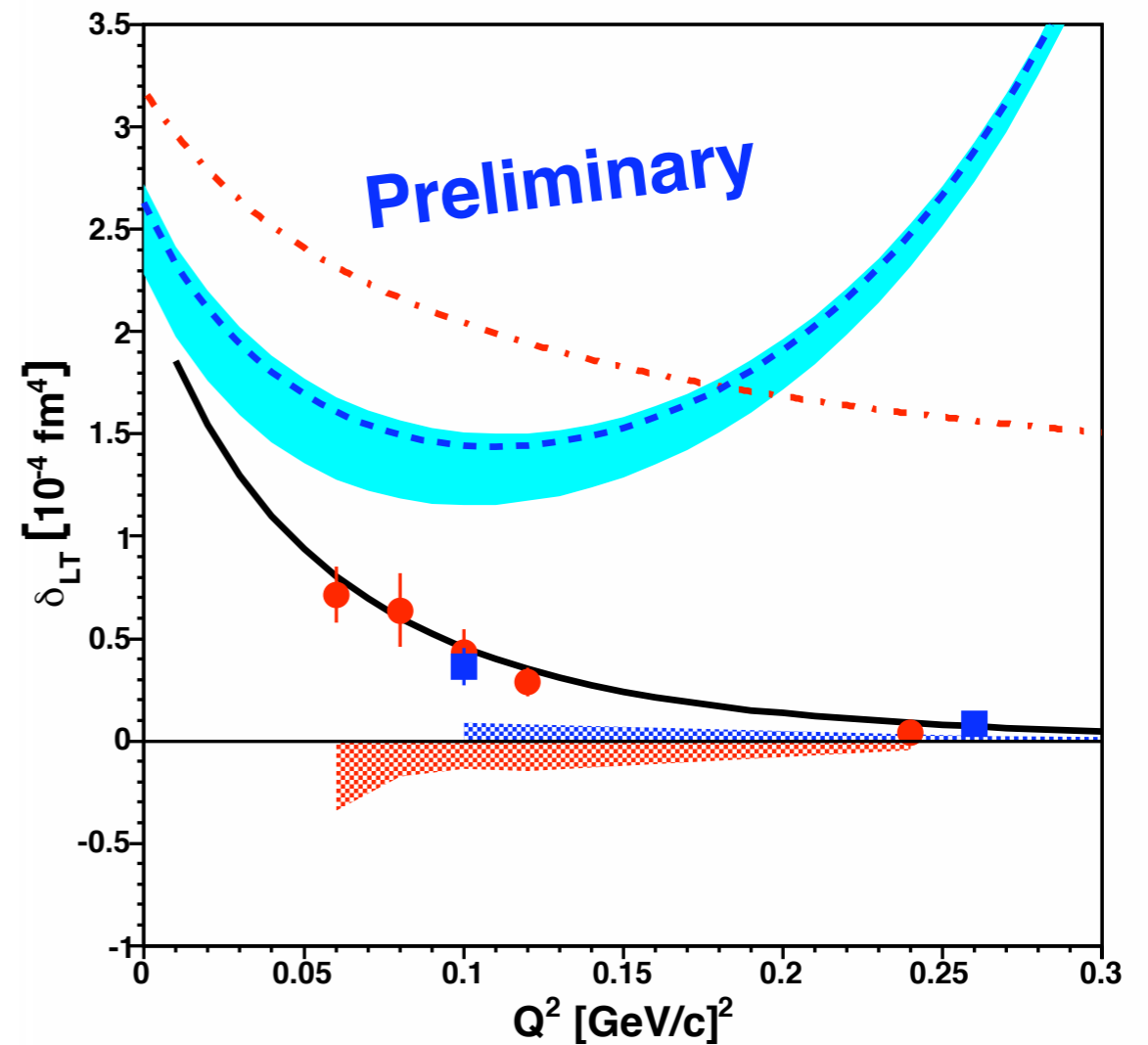
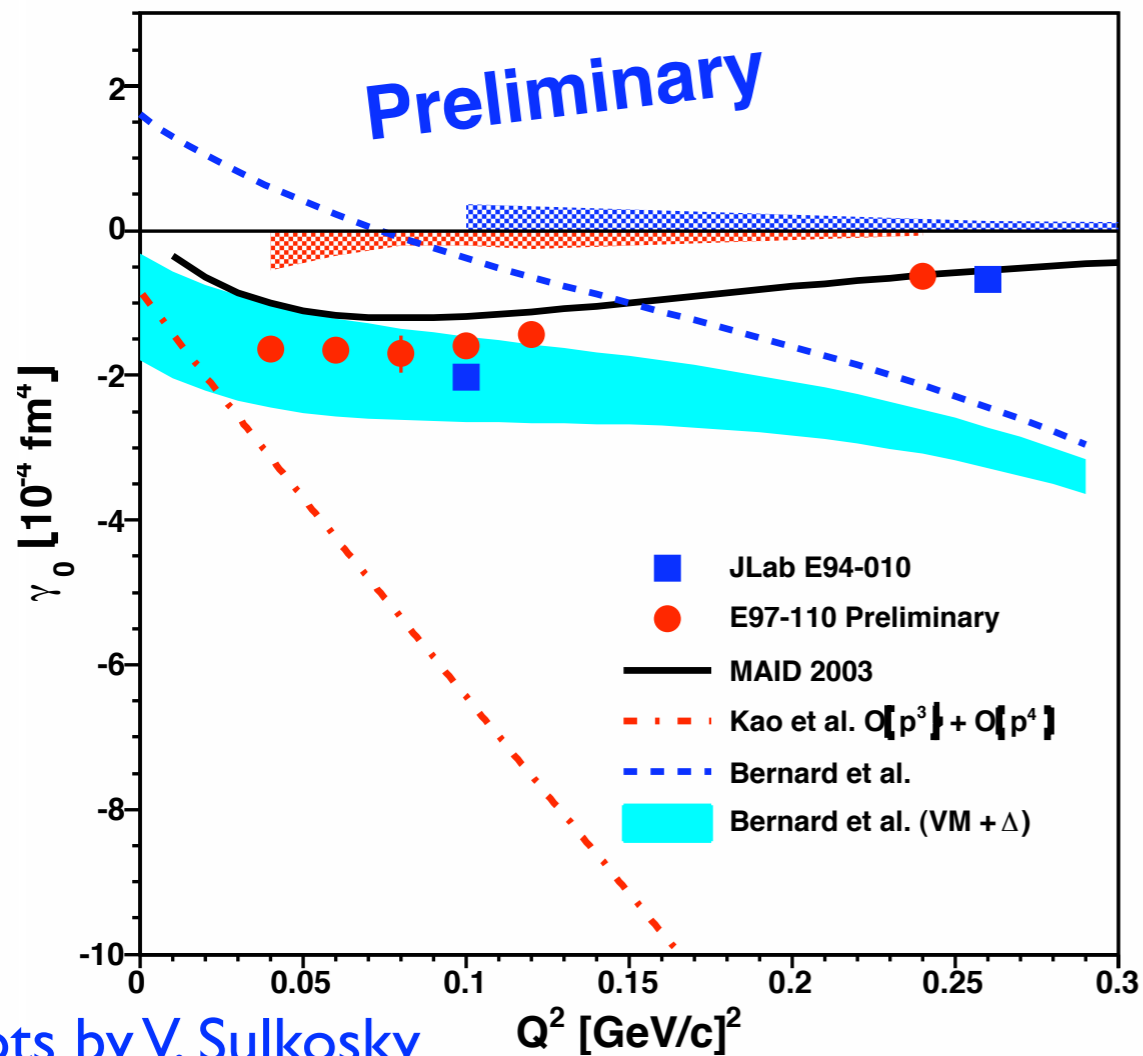
$$\gamma_0(Q^2) = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 [g_1 - \frac{4M^2}{Q^2} x^2 g_2] dx$$

$$\delta_{LT}(Q^2) = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 [g_1 + g_2] dx$$



Spin Polarizability: 2nd Moment

- δ_{LT} is seen as a more suitable testing ground of χ PT – insensitive to Δ resonance
- Significant disagreement between data and both χ PT calculations
- No proton data yet

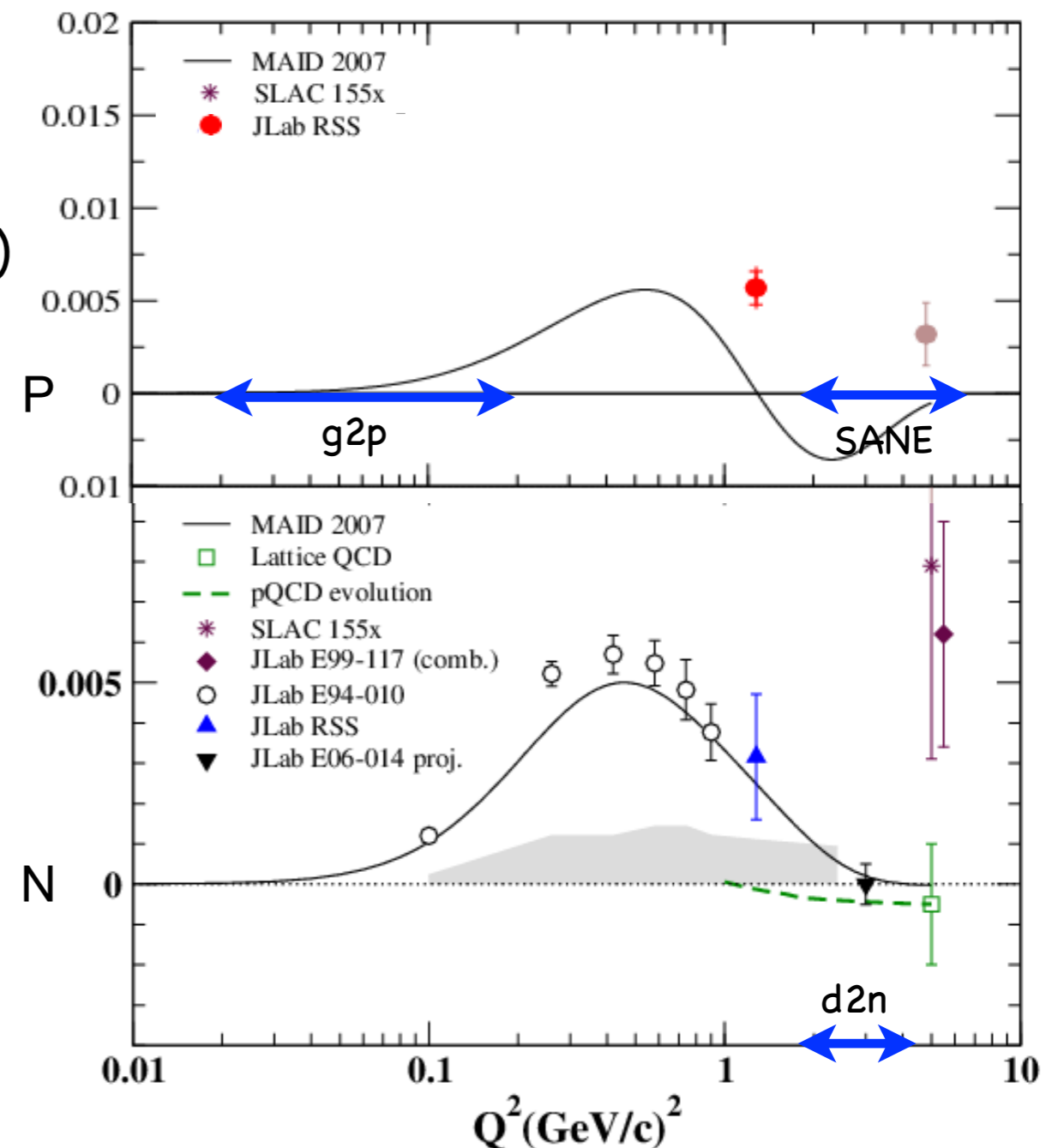


d_2 and Higher Twist

$$d_2(Q^2) = \int_0^1 x^2 [2g_1(x, Q^2) + 3g_2(x, Q^2)] dx$$

$$= 3 \int_0^1 x^2 [g_2(x, Q^2) - g_2^{WW}(x, Q^2)] dx$$

- Clean access of higher twist (twist-3) effect
- Only contributions from measured region
 - Elastic not included, only important for $Q^2 < 2\text{GeV}^2$
 - Contributions from unmeasured low x region usually not significant due to x^2 weighting.
- A benchmark test of Lattice QCD predictions at high Q^2



g₂p Experiment at JLab

- First Measurement of the proton structure function g_2 in the low Q^2 region (0.02–0.2 GeV²)
 - Extract spin polarizability δ_{LT} as a test of χ PT calculations
 - Test BC Sum Rule
 - Finite size effects:
 - Hydrogen hyperfine splitting: proton structure contributes to uncertainty
 - Proton charge radius: proton polarizability contributes to uncertainty
- Data were taken in Jefferson Lab Hall A in 2012
- Analysis is currently underway

g2p Collaboration

Spokespeople

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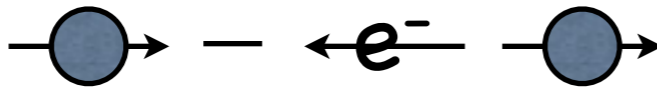
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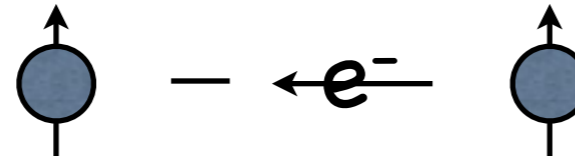
How to get g_2

$$\Delta\sigma_{\parallel} = \text{---} \overrightarrow{e^-} \text{---} \text{---} \text{---} \overleftarrow{e^-} \text{---}$$


$$= \frac{d^2\sigma^{\uparrow\uparrow}}{d\Omega dE'} - \frac{d^2\sigma^{\downarrow\uparrow}}{d\Omega dE'}$$

JLab Hall B experiment EG4
measured this quantity

$$= \frac{4\alpha^2 E'}{M\nu Q^2 E} [(E + E' \cos \theta)g_1 - 2Mxg_2]$$

$$\Delta\sigma_{\perp} = \text{---} \overrightarrow{e^-} \text{---} \text{---} \overleftarrow{e^-} \text{---}$$


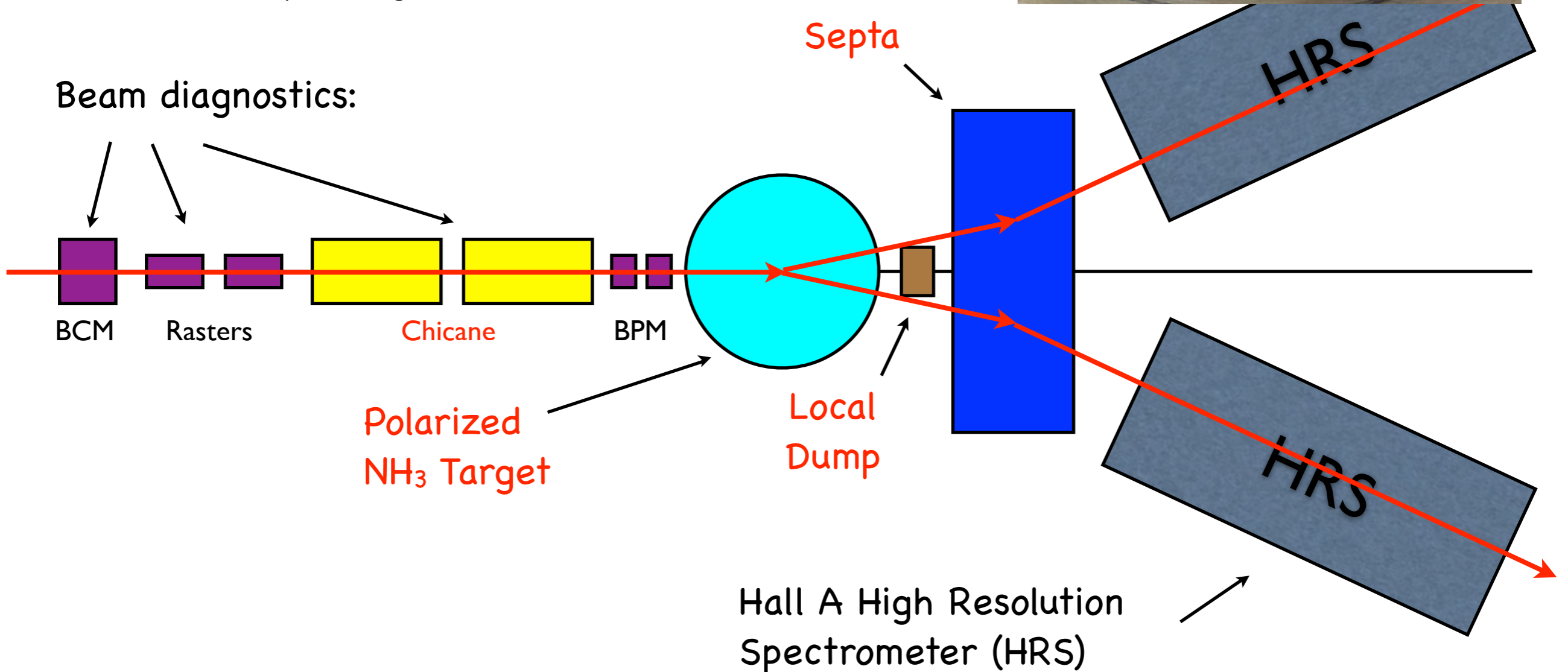
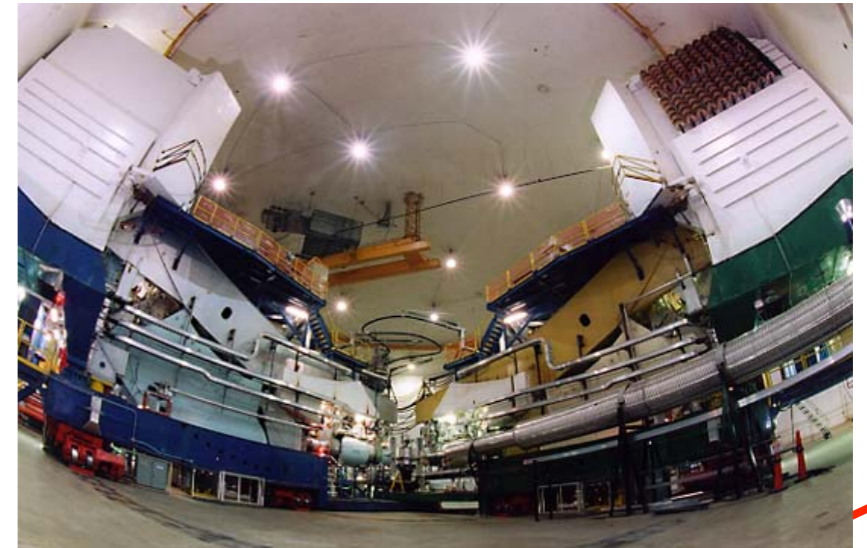
$$= \frac{d^2\sigma^{\uparrow\Rightarrow}}{d\Omega dE'} - \frac{d^2\sigma^{\downarrow\Rightarrow}}{d\Omega dE'}$$

g_2^P experiment will measure
this, combining the EG4 data
to get g_2^P at low Q^2

$$= \frac{4\alpha^2 E'^2}{M\nu Q^2 E} \sin \theta [g_1 + \frac{2E}{\nu} g_2]$$

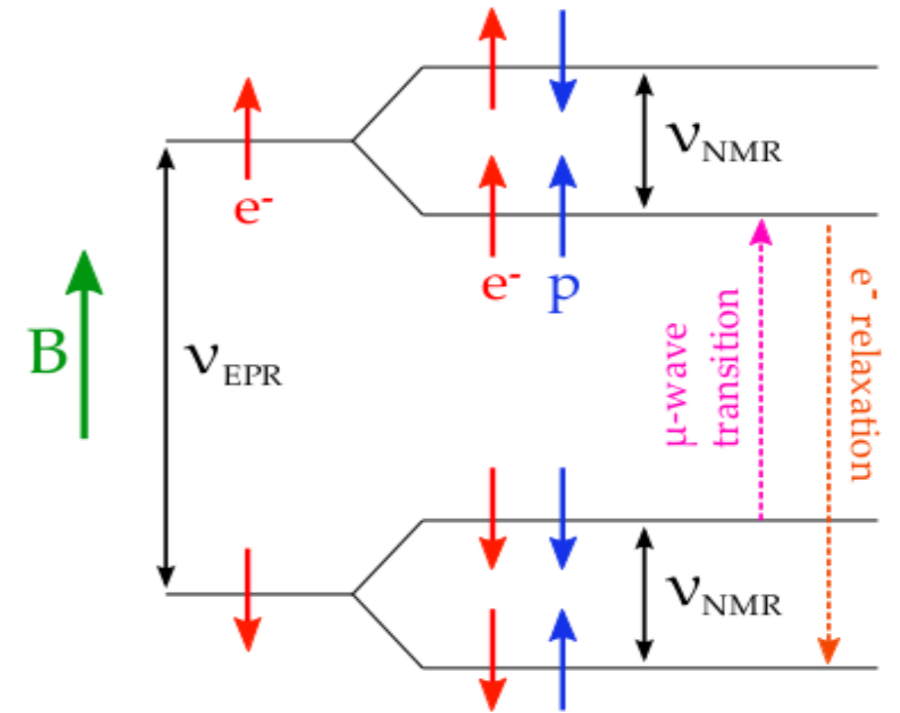
Experiment Setup

- Major New Installation in Hall A
 - Polarized NH_3 Target with 2.5/5T magnetic field
 - Low current ($<100\text{nA}$) beam line diagnostics
 - Septa magnets

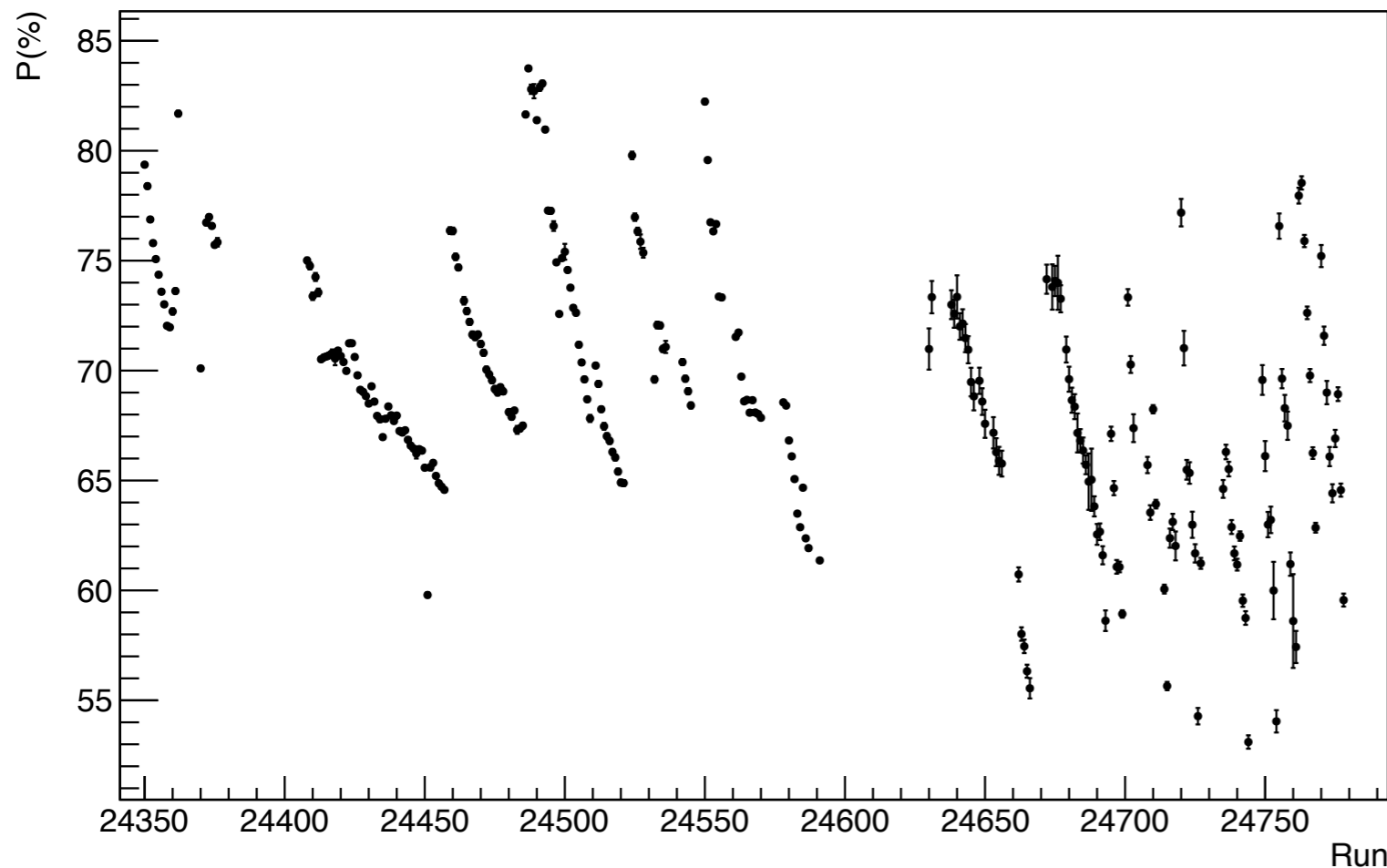


Experiment Setup

- Polarized NH_3 Target
- Dynamic nuclear polarization
- Target polarization measured via NMR



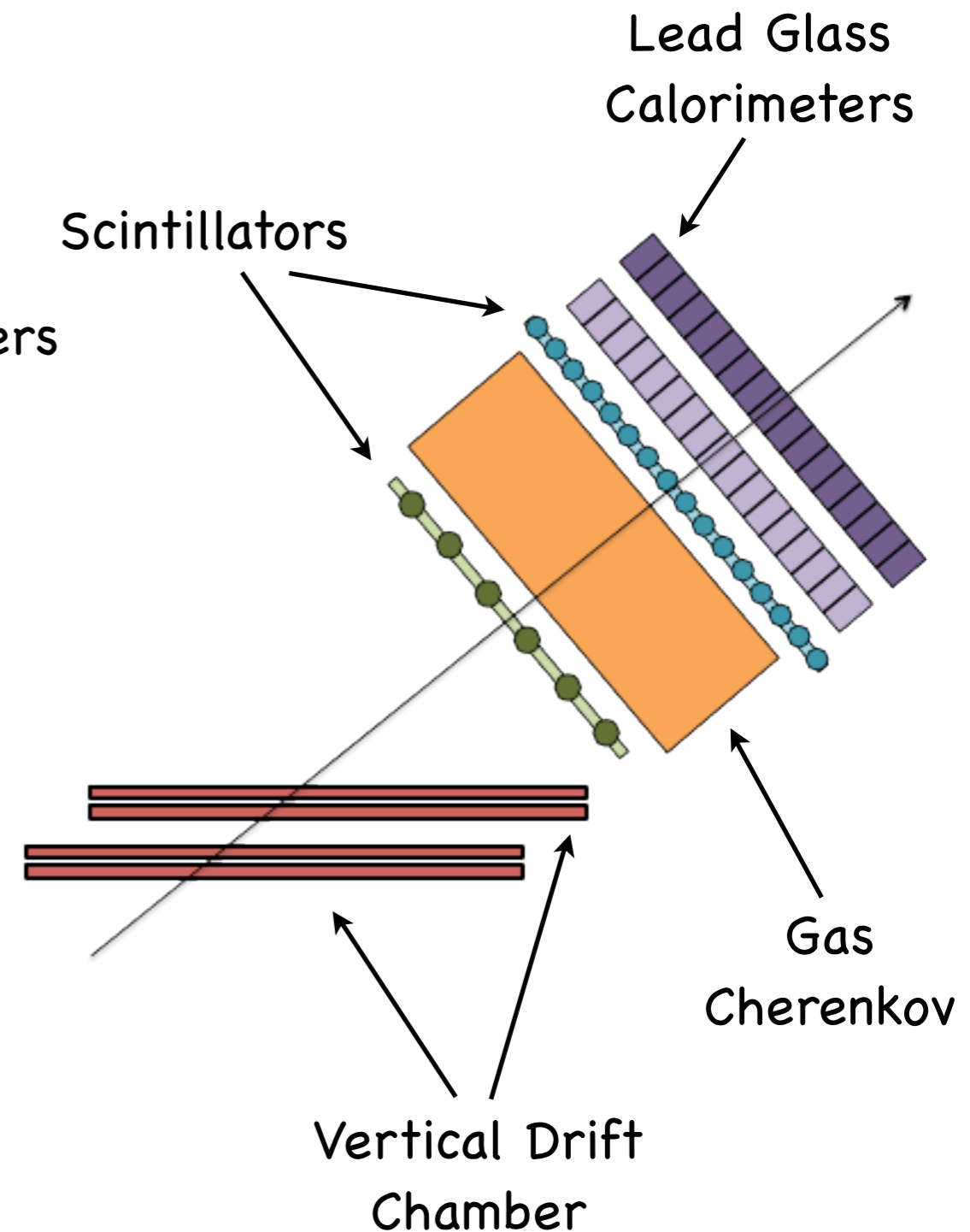
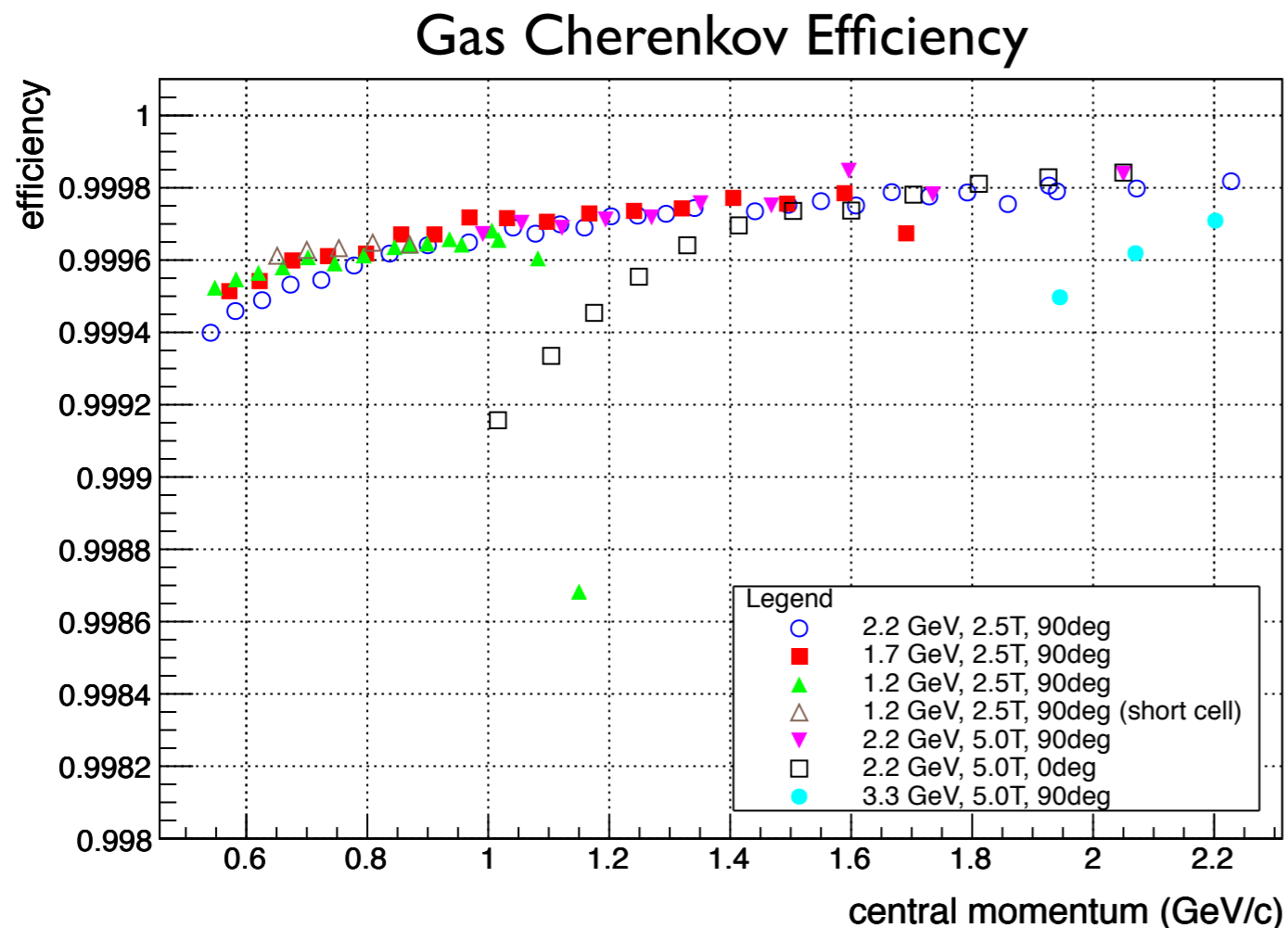
Target Polarization Results for 5T Field Setting



- Average Polarization:
 - 2.5 T: $\sim 15\%$
 - 5.0 T: $\sim 70\%$

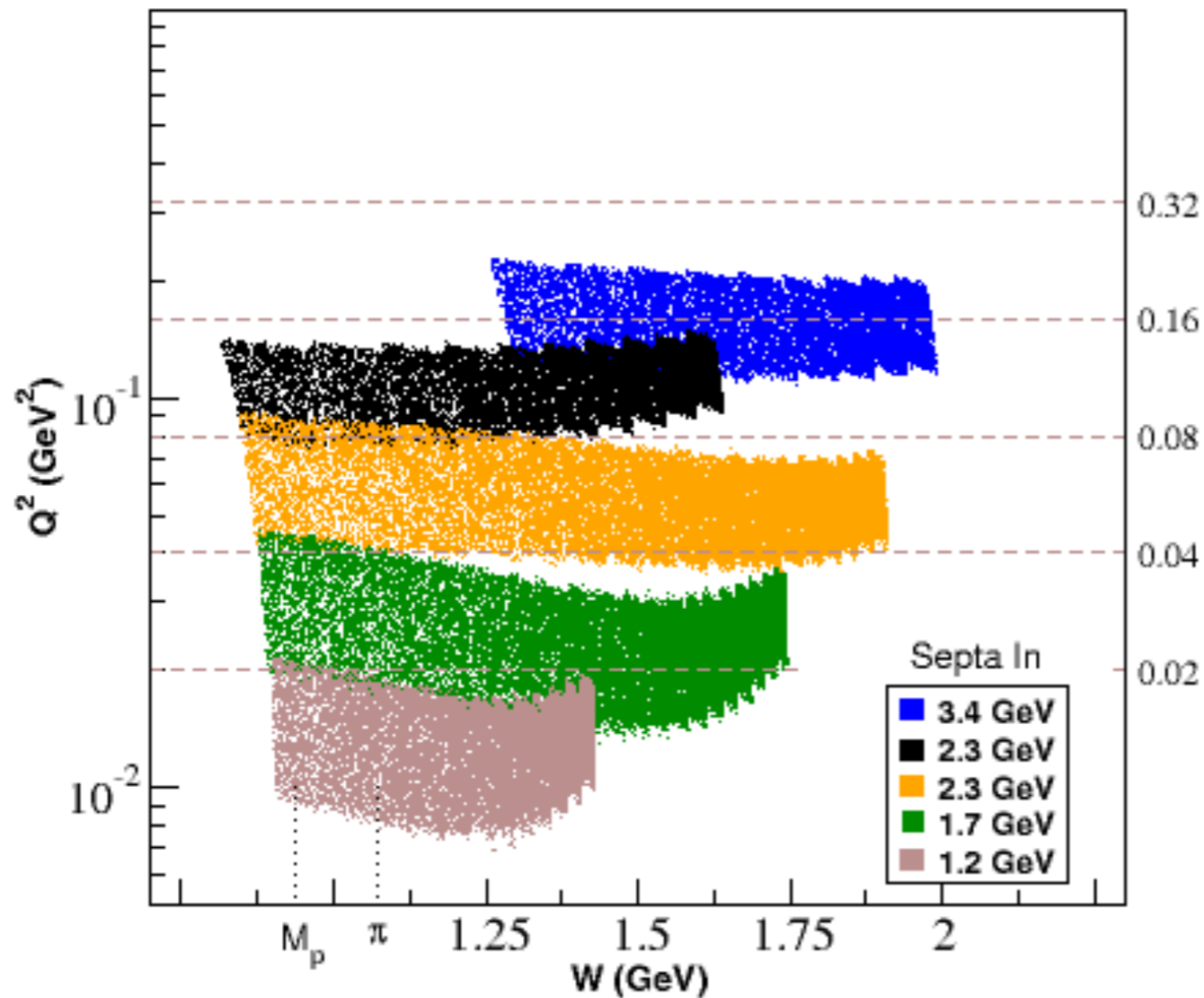
Experiment Setup

- HRS Detector package
 - Vertical Drift Chamber (VDC)
 - Particle identification (PID) Detectors
 - High Efficiency (>99%) for gas Cherenkov and lead glass calorimeters



Kinematics Coverage

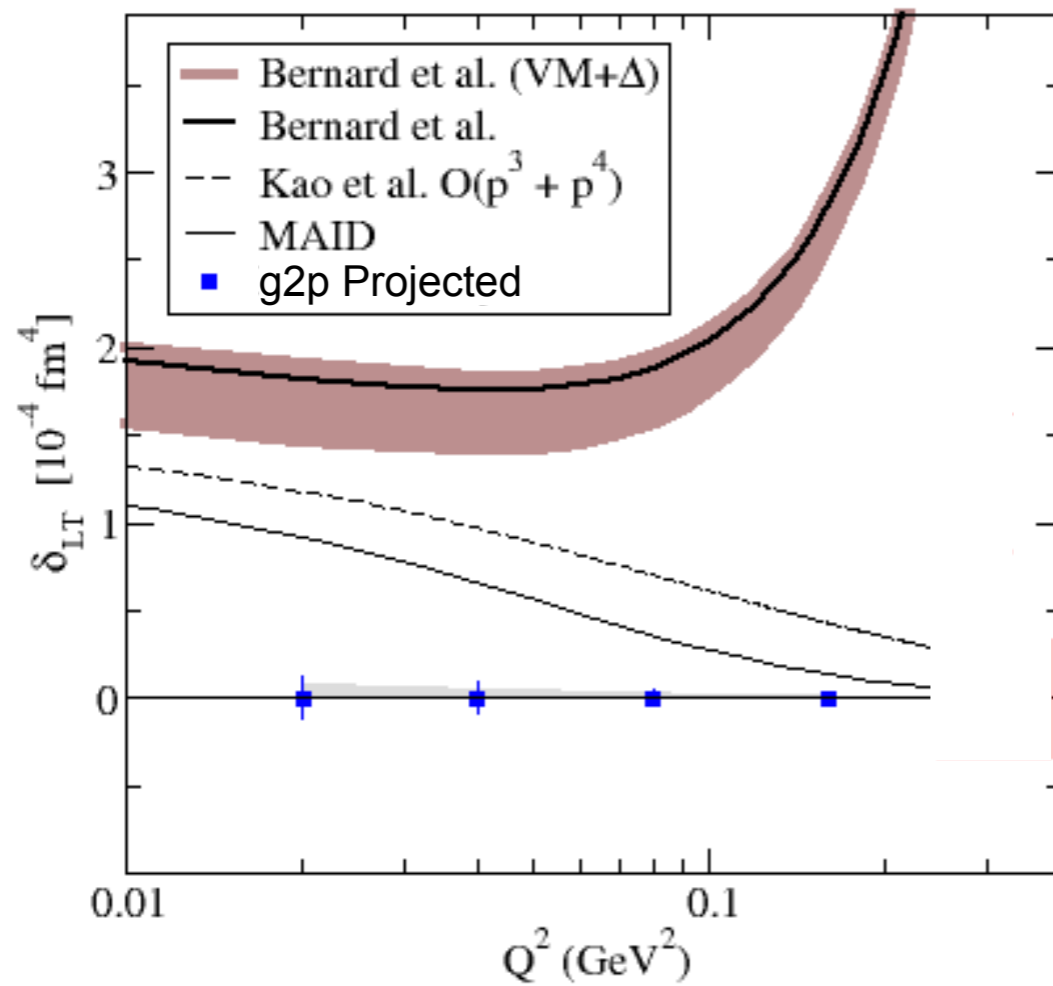
$$M_p < W < 2 \text{ GeV}$$
$$0.02 < Q^2 < 0.2 \text{ GeV}^2$$



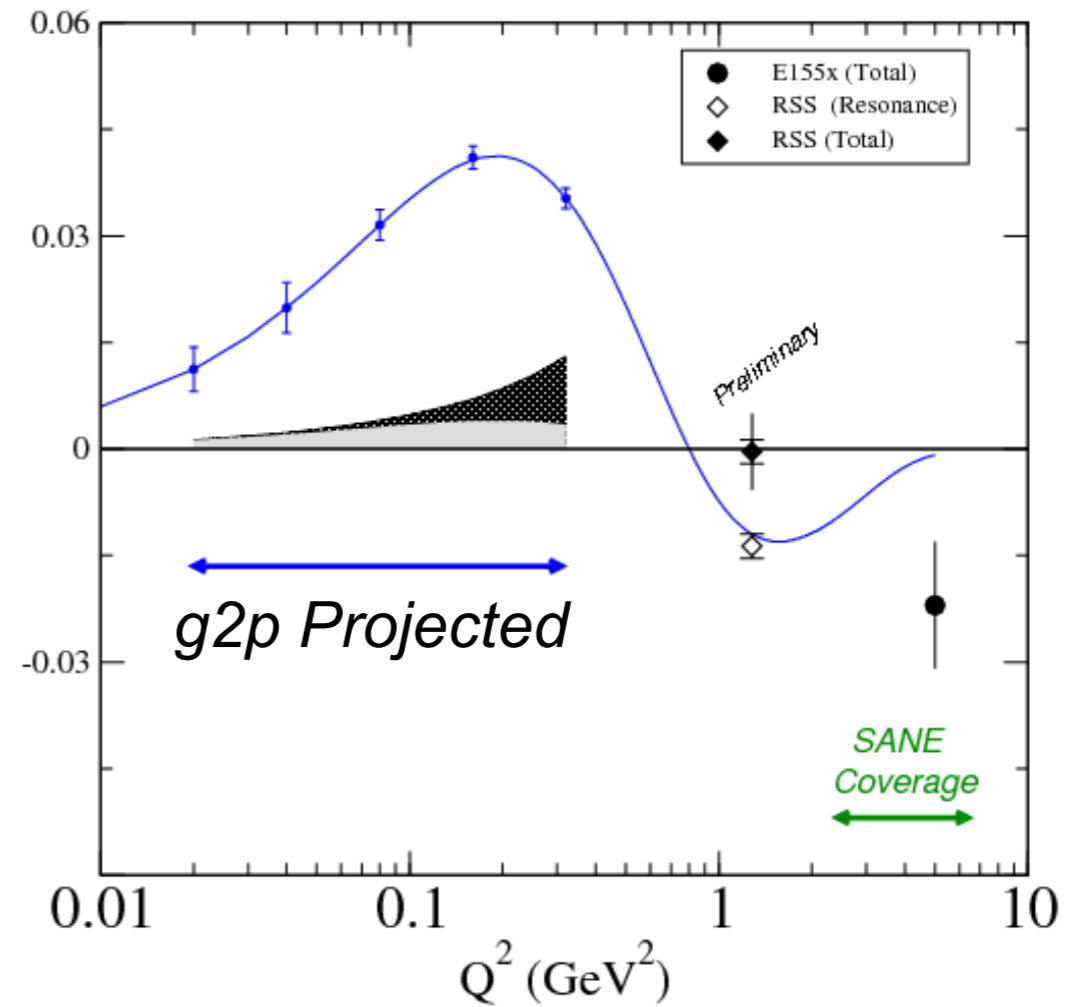
Beam Energy (GeV)	Target Field (T)
2.254	2.5
1.706	2.5
1.158	2.5
2.254	5
3.352	5

Projections

LT Spin Polarizability



BC Sum Integral



$$\delta_{LT}(Q^2) = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 [g_1 + g_2] dx$$

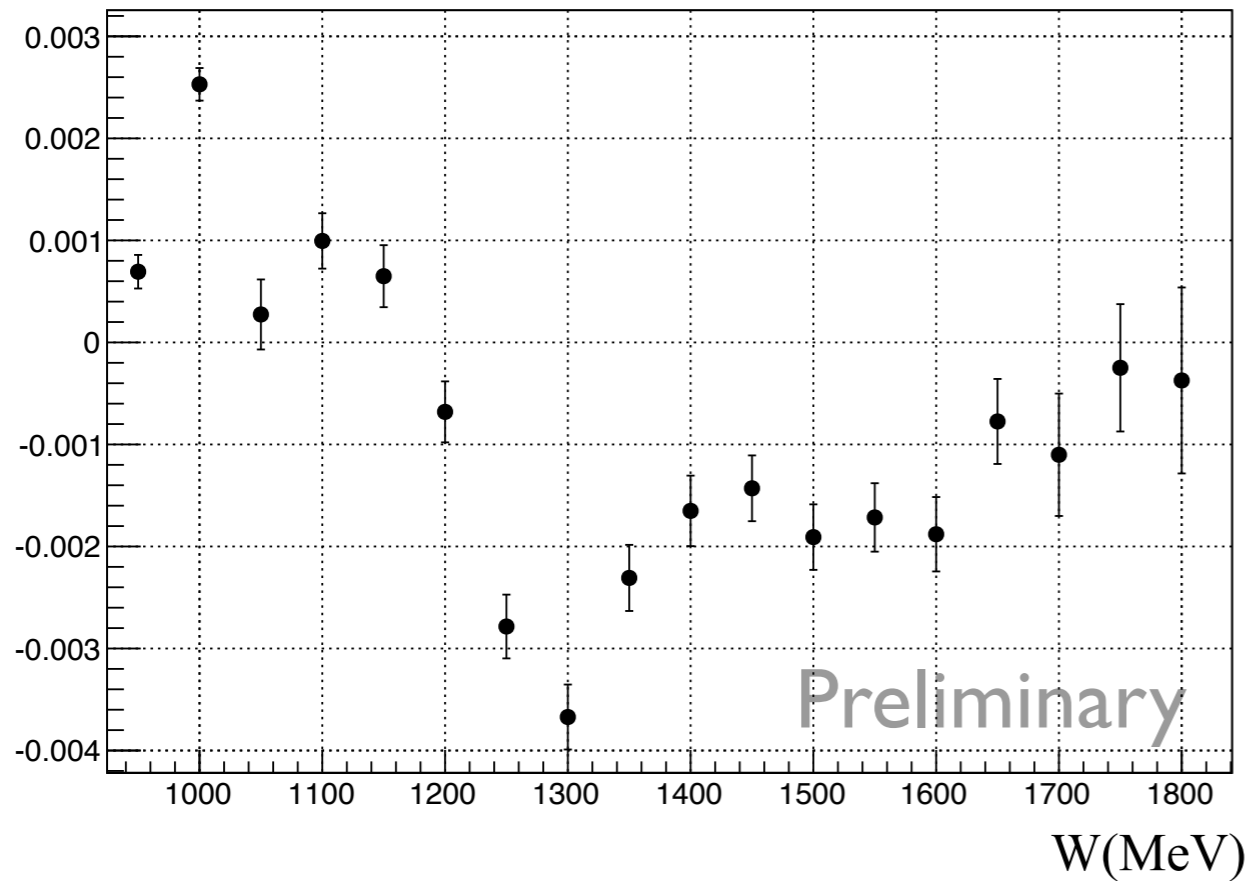
$$\int_0^1 g_2(x, Q^2) dx = 0$$

Analysis Status

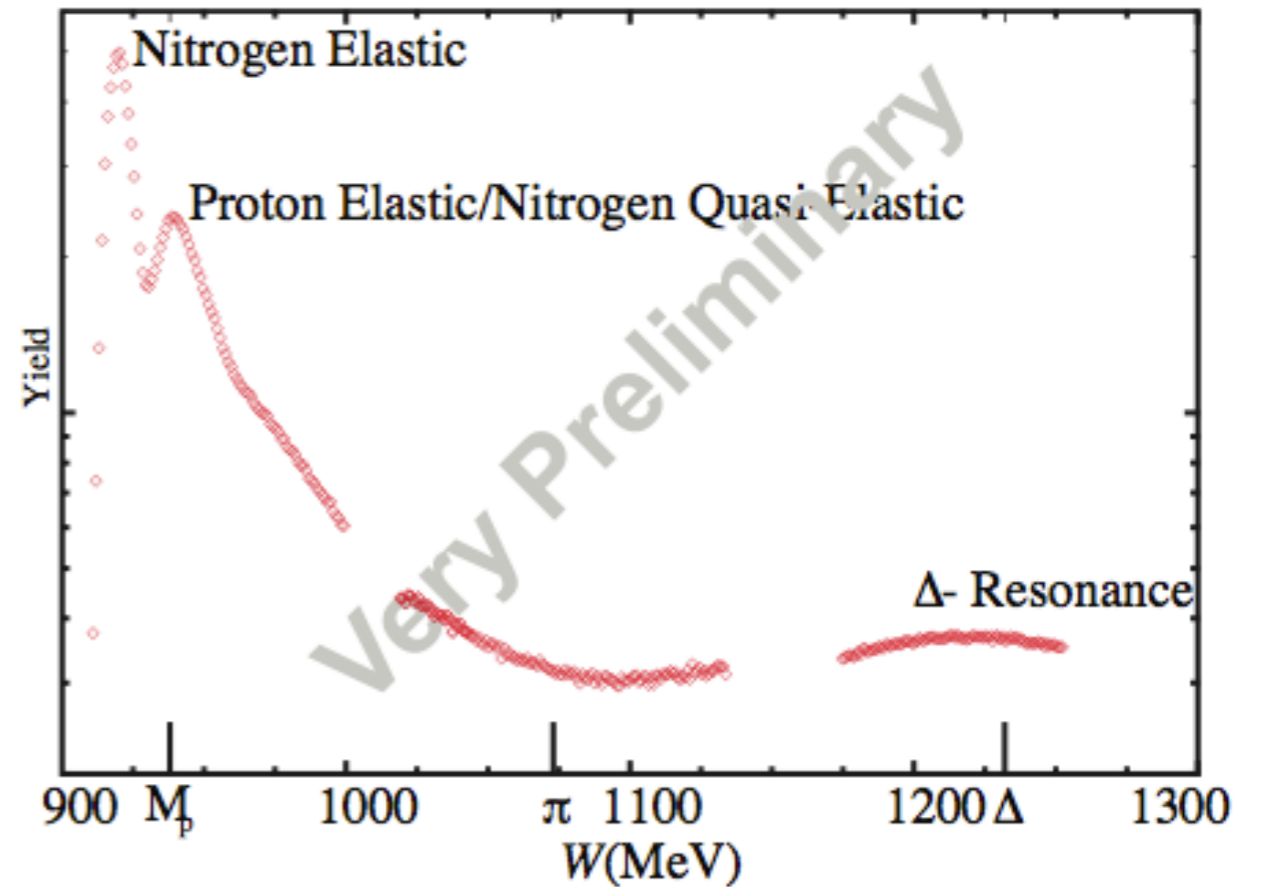
- **Completed**
 - Run Database
 - HRS Optics
 - Field measurement analysis
 - VDC t_0 calibration
 - Simulation package
 - Optics with target field (LHRS)
 - Detector Calibrations/ Efficiency Studies
 - Gas Cherenkov
 - Lead Glass Calorimeters
 - Scintillator trigger efficiencies
 - Scalers
- BCM calibration
- Helicity decoding
- Dead time calculations
- Target Polarization Analysis
- BPM Calibrations
- **In Progress**
 - Raster Size Calibrations
 - Packing Fraction/Dilution Analysis
 - Elastic Analysis
 - Yields/Radiative Corrections

Preliminary Results

Asymmetry



Yield



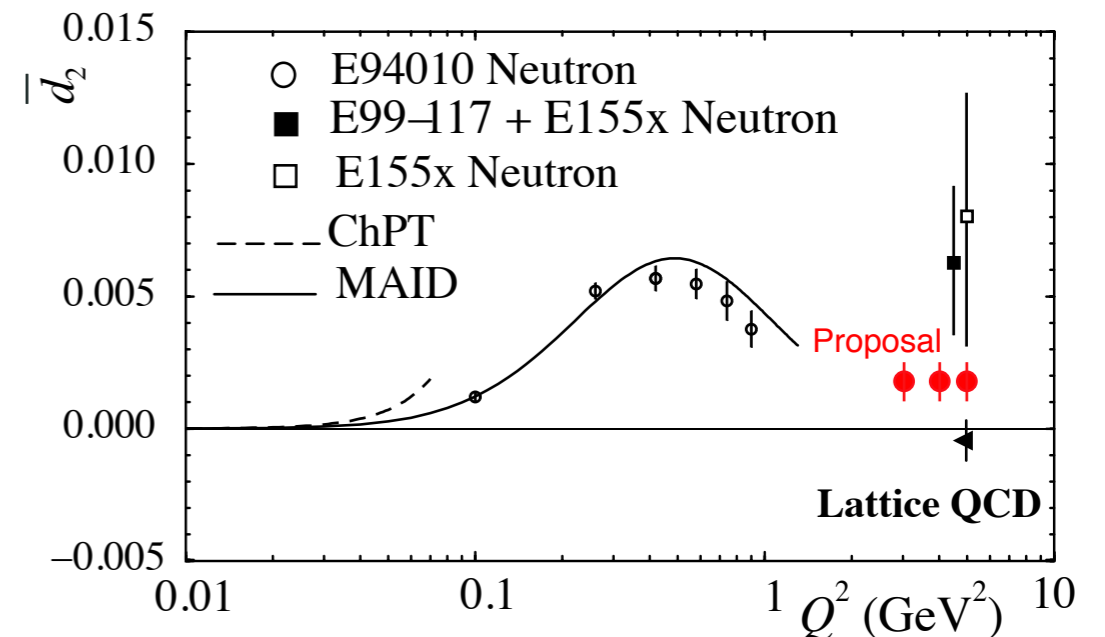
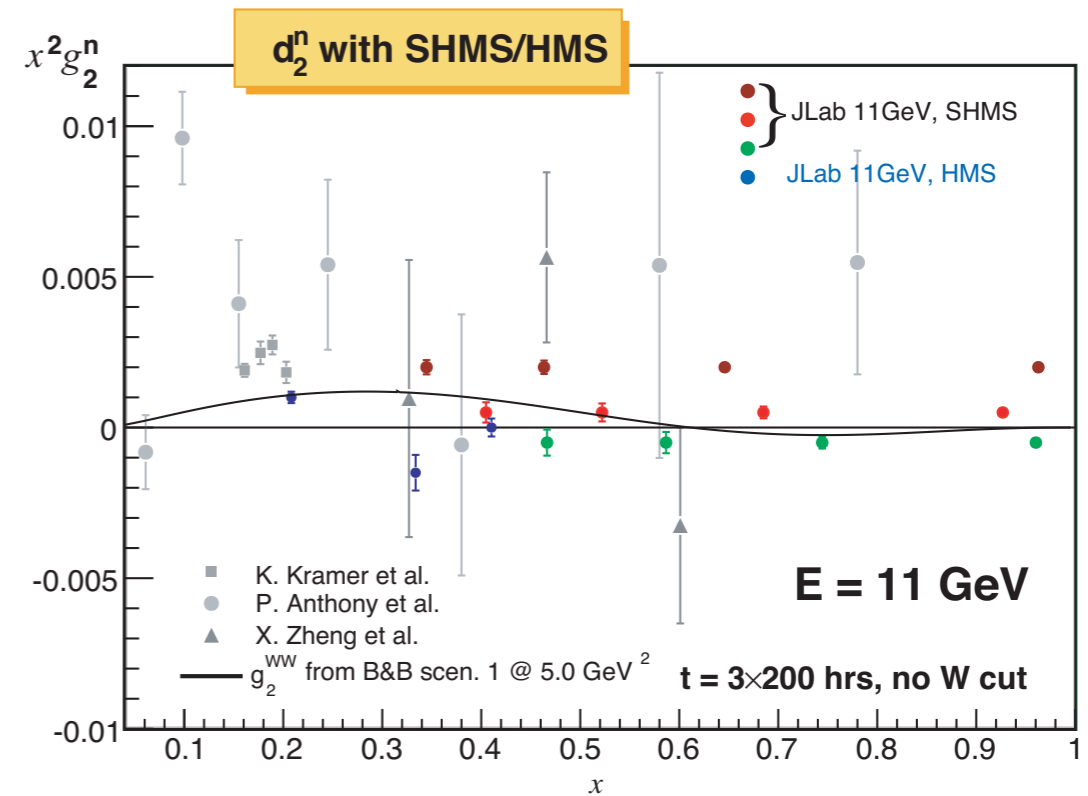
$$\Delta\sigma_{\perp} = \sigma_{\text{total}} \cdot A_{\perp}$$

Conclusion of g2p

- g2p experiment will provide first measurement of the proton structure function g_2 in the low Q^2 region (0.02–0.2 GeV²)
- The result will provide insight on several outstanding physics puzzles:
 - Spin polarizability δ_{LT} discrepancy seen for neutron data
 - BC Sum Rule violation suggested for proton at large Q^2
 - Contribute to the uncertainty of some finite size effects like hydrogen hyperfine splitting and proton charge radius puzzle

Future Experiments

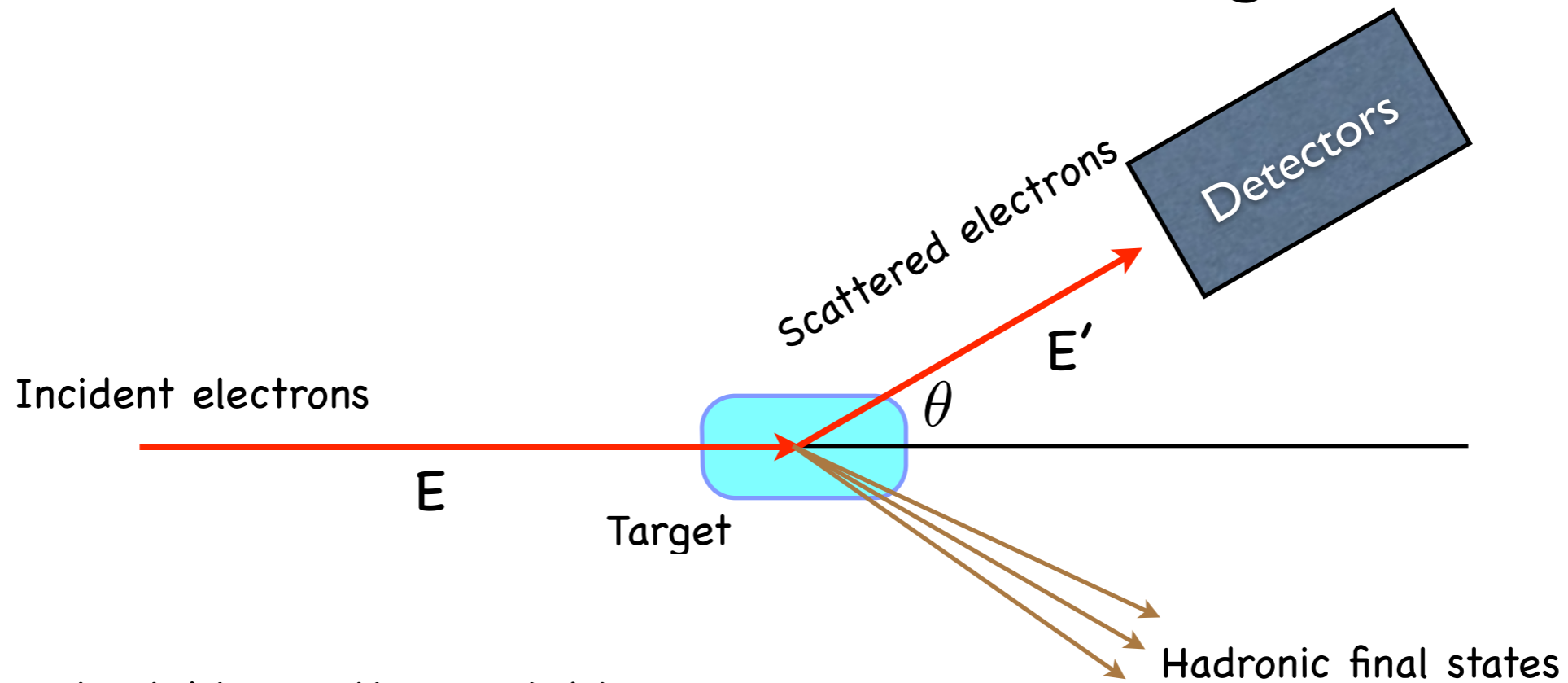
- JLab at 12 GeV
- Hall A
 - E12-06-122: A1n in valence quark region (8.8 and 6.6 GeV)
- Hall B
 - E12-06-109: longitudinal spin structure of the nucleon
- Hall C
 - E12-06-110: A1n in valence quark region (11 GeV)
 - E12-06-121: g_2^n and d_2^n at high Q^2



Thanks

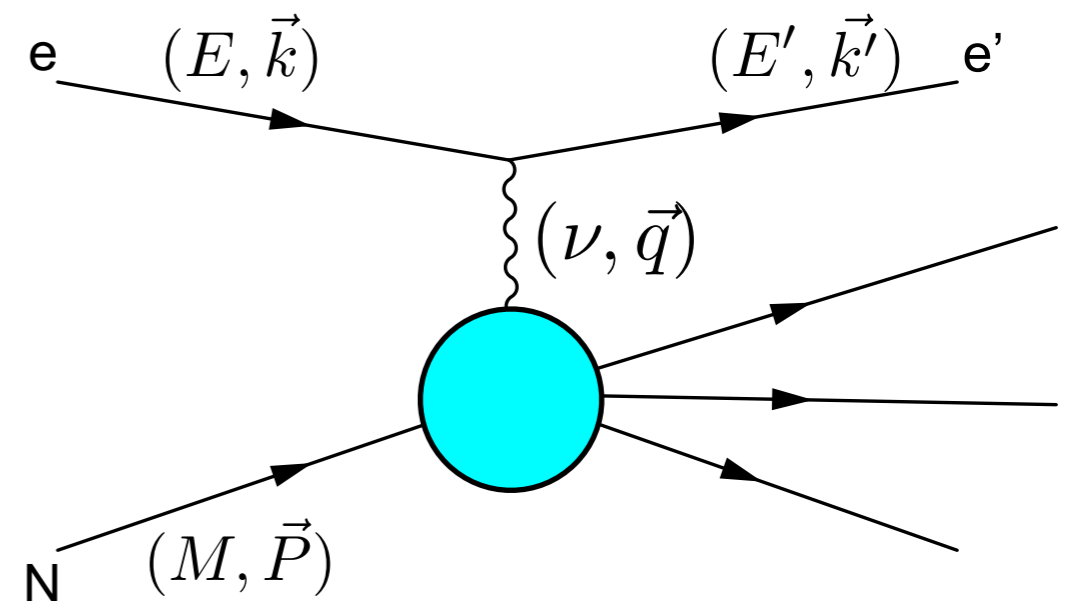
Backups

Electron Scattering



- Important kinematics variables:

- $\nu = E - E'$
- Q^2 : Momentum transfer squared
- W : Invariant mass of residual hadronic system
- $x = \frac{Q^2}{2M\nu}$, Bjorken variable: fraction momentum of struck quark



Structure Function

- “twist” in Operator Production Expansion

$$\begin{aligned}
 T_{\mu\nu}(P, q) &= i \int d^4 z \exp(iq \cdot z) \langle N(P) | \mathcal{T} (j_\mu(z) j_\nu(0)) | N(P) \rangle \\
 &= \sum_{n=\text{even}} \langle N(P) | O_n^{\mu_1 \dots \mu_n} | N(P) \rangle \frac{2^n}{(Q^2)^n} \left(P_{\mu\nu}^{(L)} C_n^{(L)}(Q^2) q_{\mu_1} \dots q_{\mu_n} \right. \\
 &\quad \left. + \left[-q^2 g_{\mu\mu_1} g_{\mu_2\nu} + [g_{\mu\mu_1} q_{\mu_2} q_\nu + g_{\mu_2\nu} q_\mu q_{\mu_1}] - g_{\mu\nu} q_{\mu_1} q_{\mu_2} \right] \right. \\
 &\quad \left. \times C_n^{(2)}(Q^2) q_{\mu_3} \dots q_{\mu_n} \right), \tag{5.125}
 \end{aligned}$$

Structure of Nucleon, eq 5.125

- quark-quark and quark-gluon correlation



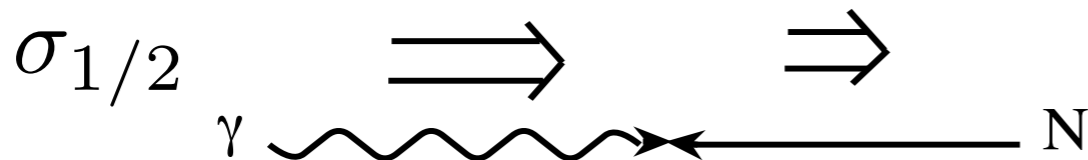
Proton Polarizability

- Proton electric and magnetic polarizabilities: response to low-frequency, long-wavelength electromagnetic fields
- From the dispersion relation of the real Compton scattering (RCS) amplitude, one could derive electric and magnetic polarizability and forward spin polarizability

$$\alpha + \beta = \frac{1}{2\pi^2} \int_{\nu_0}^{\infty} \frac{\sigma_T}{\nu'^2} d\nu'$$

electric and magnetic polarizability

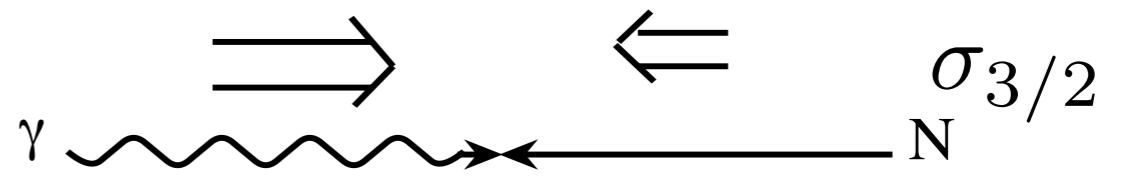
$$\sigma_T = \frac{1}{2} (\sigma_{1/2} + \sigma_{3/2})$$



$$\gamma_0 = -\frac{1}{4\pi^2} \int_{\nu_0}^{\infty} \frac{\sigma_{TT}}{\nu'^3} d\nu'$$

forward spin polarizability

$$\sigma_{TT} = \frac{1}{2} (\sigma_{1/2} - \sigma_{3/2})$$



Generalized Longitudinal-Transverse Polarizability

- Start from forward spin-flip doubly-virtual Compton scattering (VVCS) amplitude g_{TT} and g_{LT}

$$\text{Re}[g_{TT}^{\text{non-pole}}(\nu, Q^2)] = \frac{\nu}{2\pi^2} \mathcal{P} \int_{\nu_\pi}^{\infty} \frac{d\nu' K}{\nu'^2 - \nu^2} \sigma_{TT}(\nu', Q^2)$$

$$\text{Re}[g_{LT}^{\text{non-pole}}(\nu, Q^2)] = \frac{1}{2\pi^2} \mathcal{P} \int_{\nu_\pi}^{\infty} \frac{d\nu' \nu' K}{\nu'^2 - \nu^2} \sigma_{LT}(\nu', Q^2)$$

- g_{TT} and g_{LT} can be expanded in power series of ν

$O(\nu^3)$ term of g_{TT} leads to the generalized forward spin polarizability γ_0

$$\begin{aligned} \gamma_0(Q^2) &= \frac{1}{2\pi^2} \int_{\nu_\pi}^{\infty} \frac{K(\nu, Q^2)}{\nu} \frac{\sigma_{TT}(\nu, Q^2)}{\nu^3} d\nu \\ &= \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 [g_1 - \frac{4M^2}{Q^2} x^2 g_2] dx \end{aligned}$$

$O(\nu^2)$ term of g_{LT} leads to the generalized longitudinal-transverse polarizability δ_{LT}

$$\begin{aligned} \delta_{LT}(Q^2) &= \frac{1}{2\pi^2} \int_{\nu_\pi}^{\infty} \frac{K(\nu, Q^2)}{\nu} \frac{\sigma_{LT}(\nu, Q^2)}{Q\nu^2} d\nu \\ &= \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 [g_1 + g_2] dx \end{aligned}$$

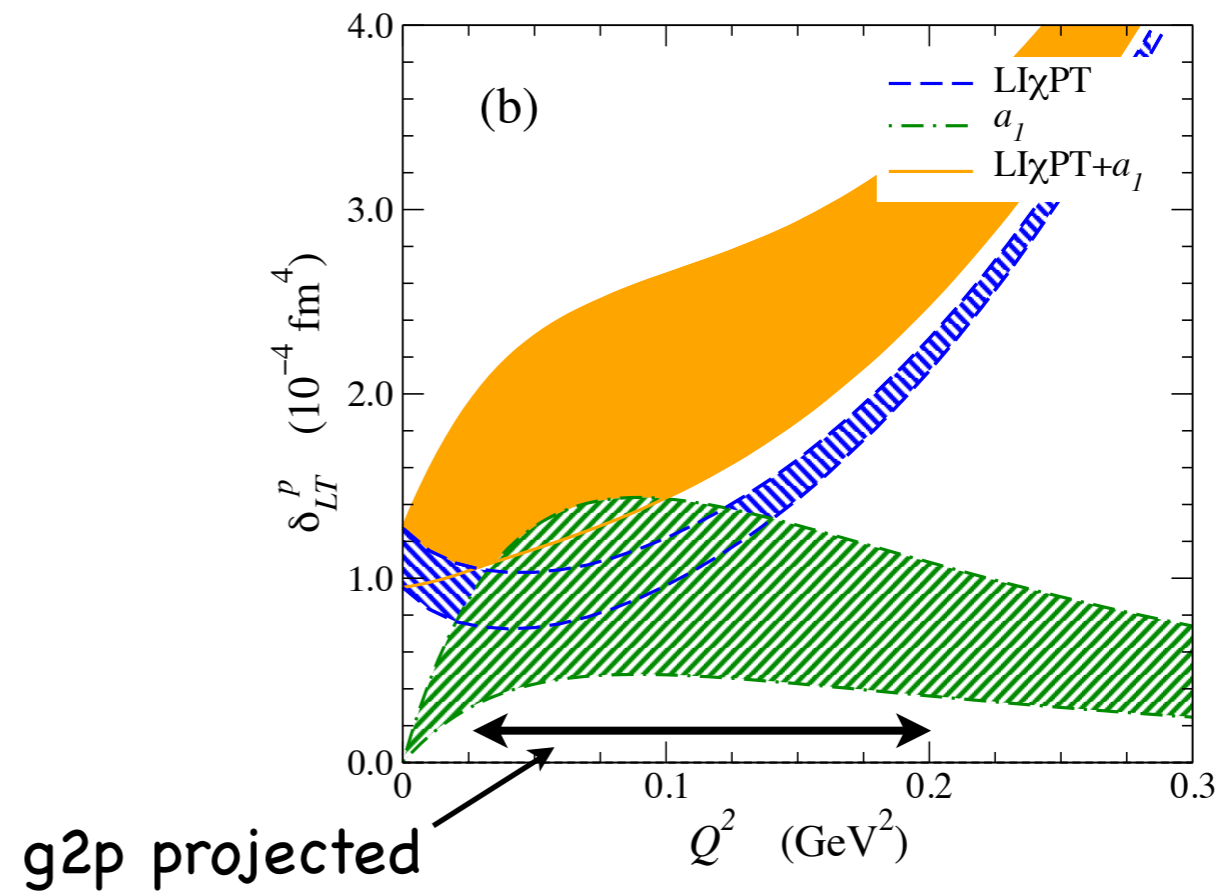
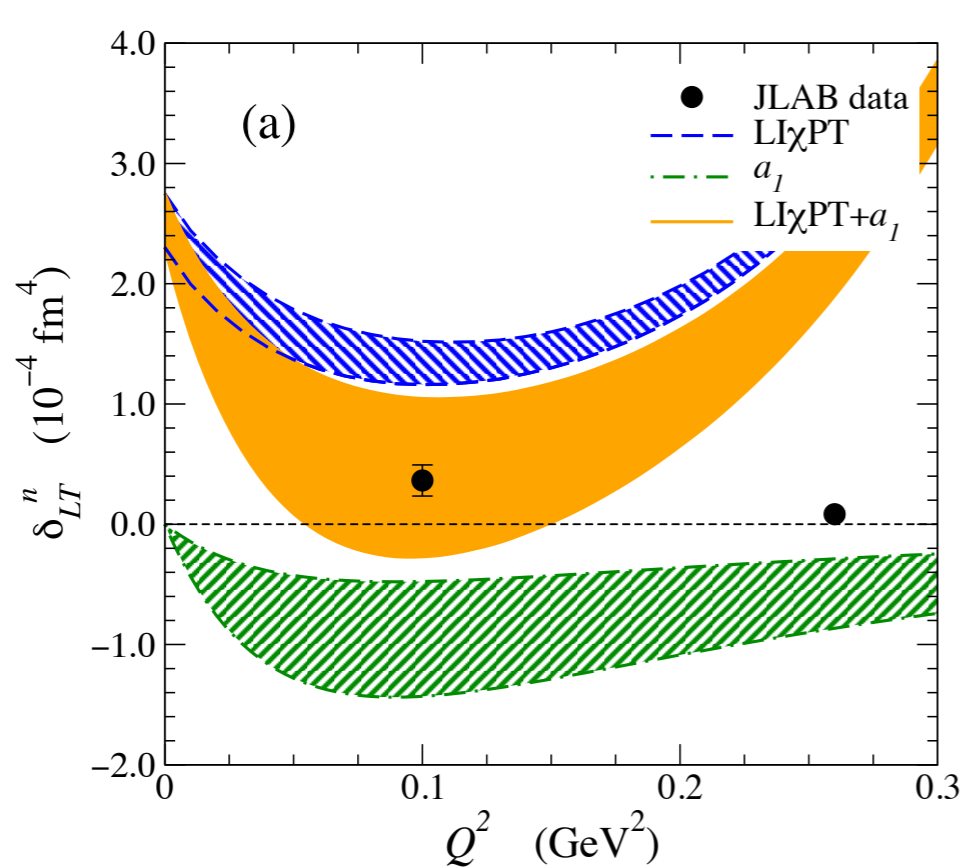
δ_{LT} puzzle

- At low Q^2 , the generalized polarizabilities have been evaluated with NLO χ PT calculations:
 - Relativistic Baryon χ PT (V. Bernard, T. Hemmert and Ulf-G. Meissner, [Phys. Rev. D, 67\(2003\)076008](#))
 - Heavy Baryon χ PT (C.W. Kao, T. Spitzenberg and M. Vanderhaeghen, [Phys. Rev. D, 67\(2003\)016001](#))
- One issue in the calculation is how to properly include the nucleon resonance contributions, especially the Δ resonance
 - γ_0 is sensitive to resonances
 - δ_{LT} is insensitive to the Δ resonance
- δ_{LT} should be more suitable than γ_0 to serve as a testing ground for the chiral dynamics of QCD

δ_{LT} puzzle

Kochelev's new calculation result:

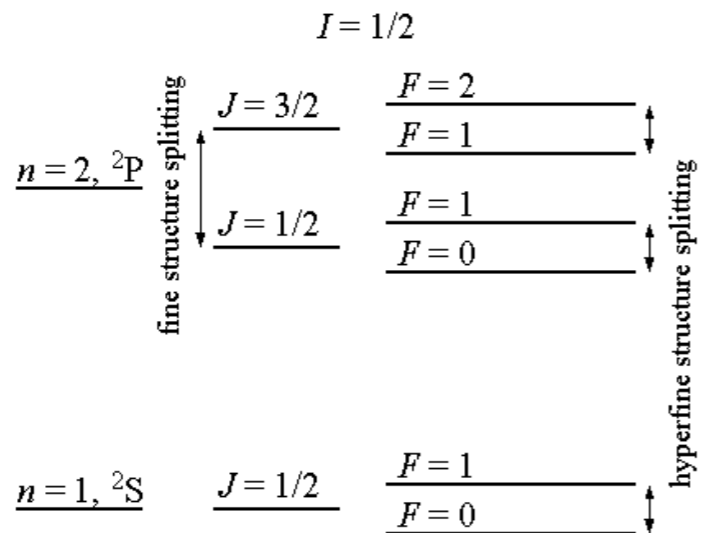
- Include the axial-anomaly $a_1(1260)$ meson contribution
- Improves agreement with neutron



Still need Proton δ_{LT} Data

Hydrogen Hyperfine Structure

- Hydrogen hyperfine splitting in the ground state has been measured to a relative high accuracy of 10



$$\Delta E = 1420.4057517667(9)\text{MHz}$$

$$= (1 + \delta)E_F$$

$$\delta = (\delta_{\text{QED}} + \delta_R + \delta_{\text{small}}) + \Delta_S$$

- Δ_S is the proton structure correction and has the largest uncertainty

$$\Delta_S = \Delta_Z + \Delta_{\text{pol}}$$

- Δ_Z can be determined from elastic scattering, which is $-41.0 \pm 0.5 \times 10$
- Δ_{pol} involves contributions of the inelastic part (excited state), and can be extracted to 2 terms corresponding to 2 different spin-dependent structure function of proton

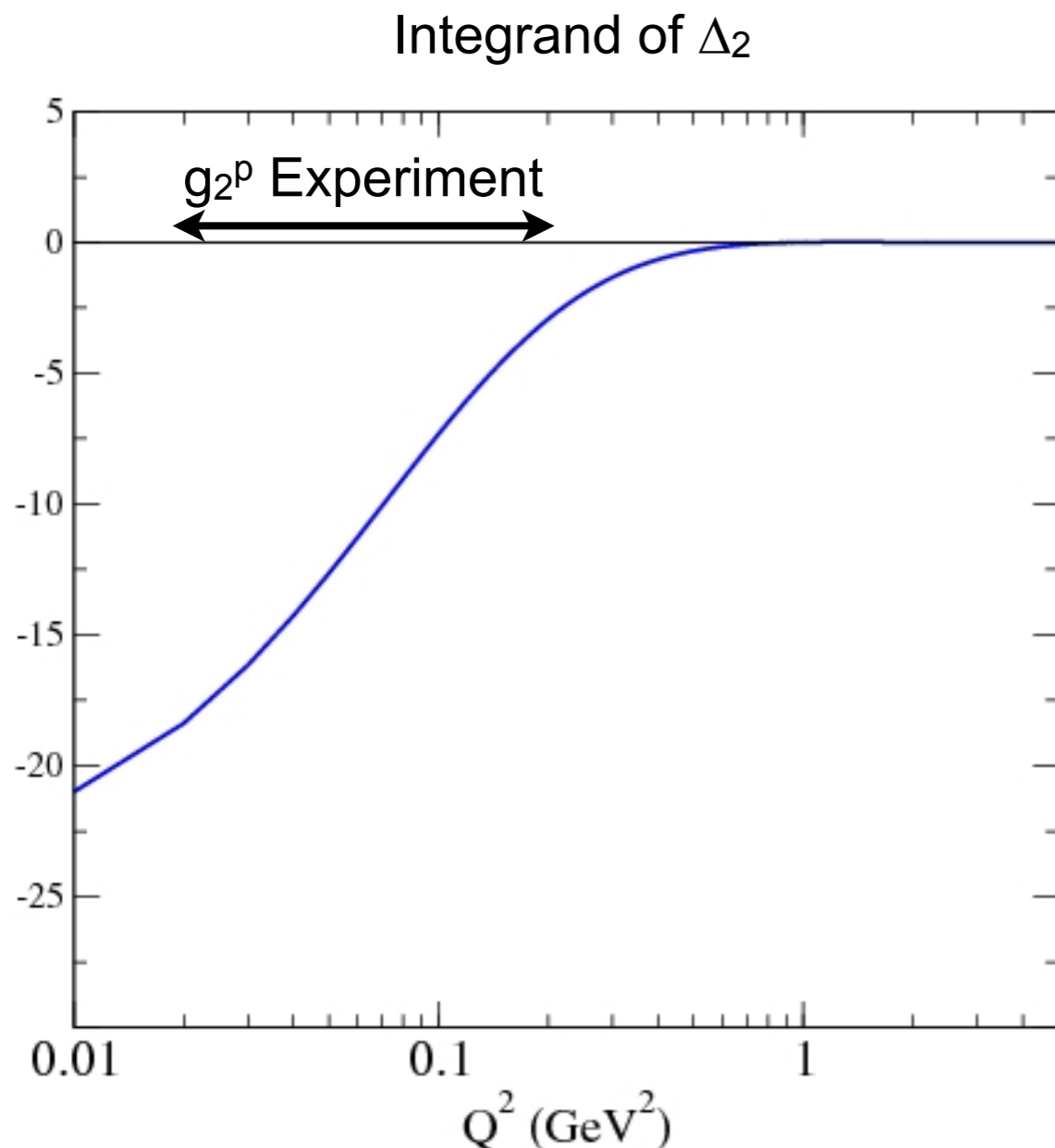
Hydrogen Hyperfine Structure

$$\Delta_{\text{pol}} = \frac{\alpha m_e}{\pi g_p m_p} (\Delta_1 + \Delta_2)$$

$$\Delta_2 = -24m_p^2 \int_0^\infty \frac{dQ^2}{Q^4} B_2(Q^2)$$

$$B_2(Q^2) = \int_0^{x_{th}} dx \beta_2(\tau) g_2(x, Q^2)$$

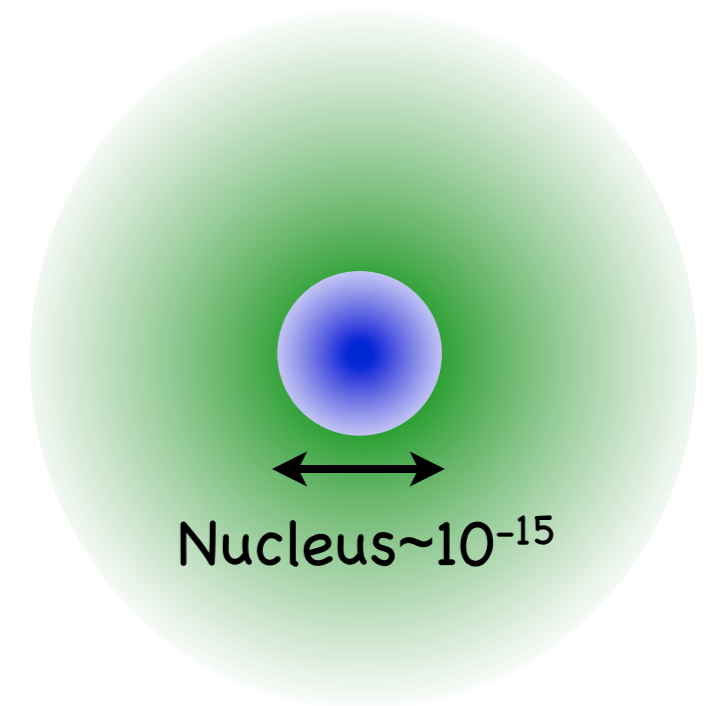
$$\beta_2(\tau) = 1 + 2\tau - 2\sqrt{\tau(\tau + 1)}$$



- B_2 is dominated by low Q^2 part
- g_2^p is unknown in this region, so there may be huge error when calculating Δ_2
- This experiment will provide a constraint

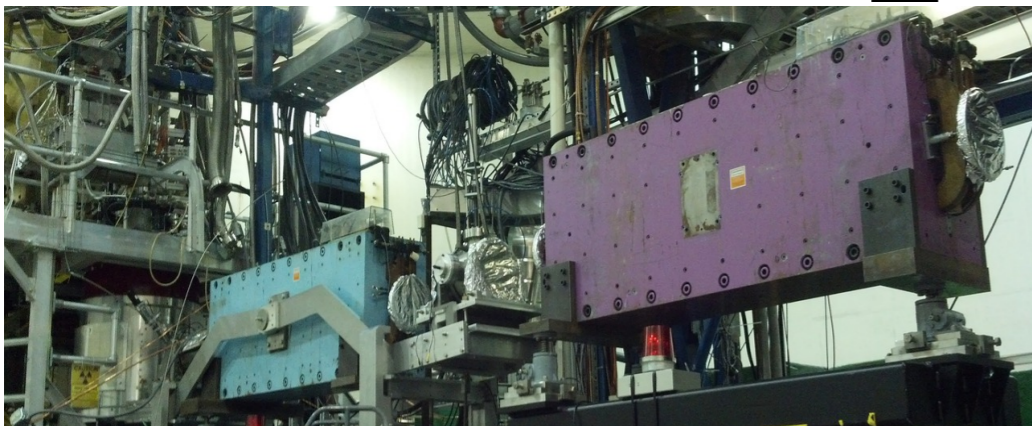
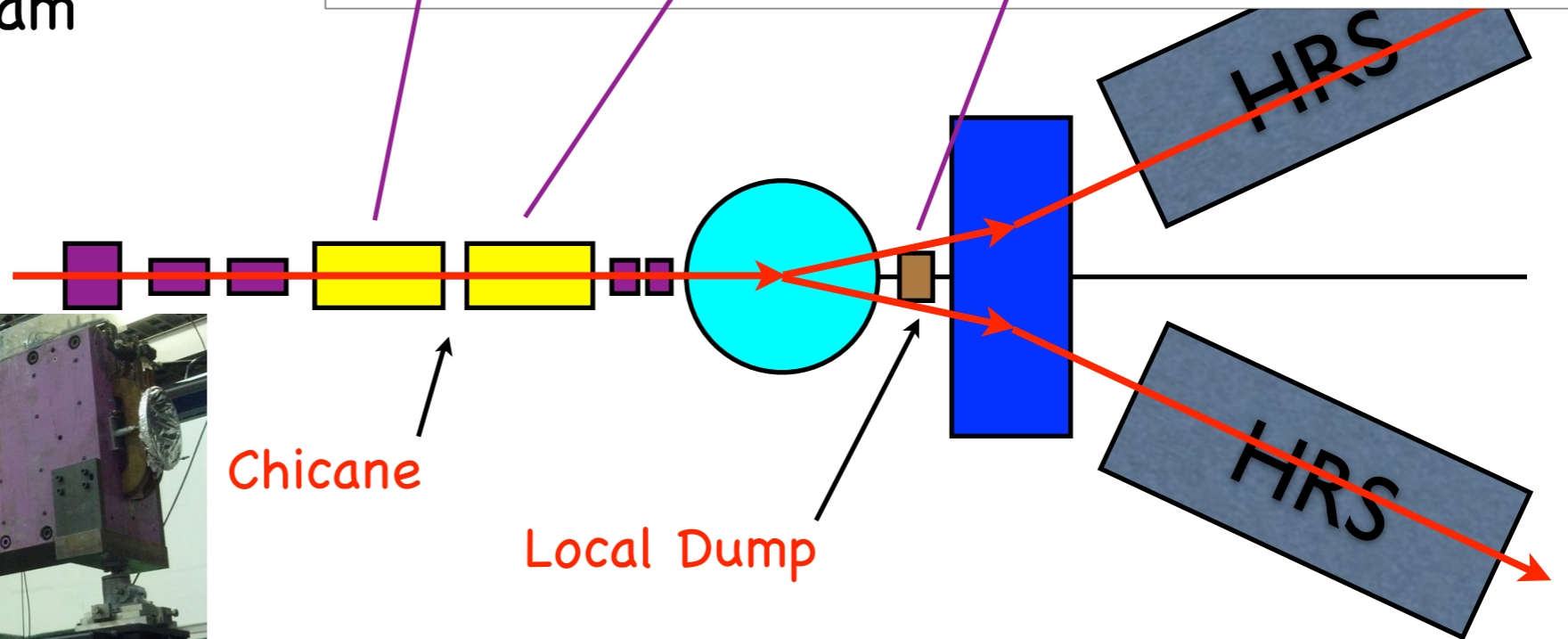
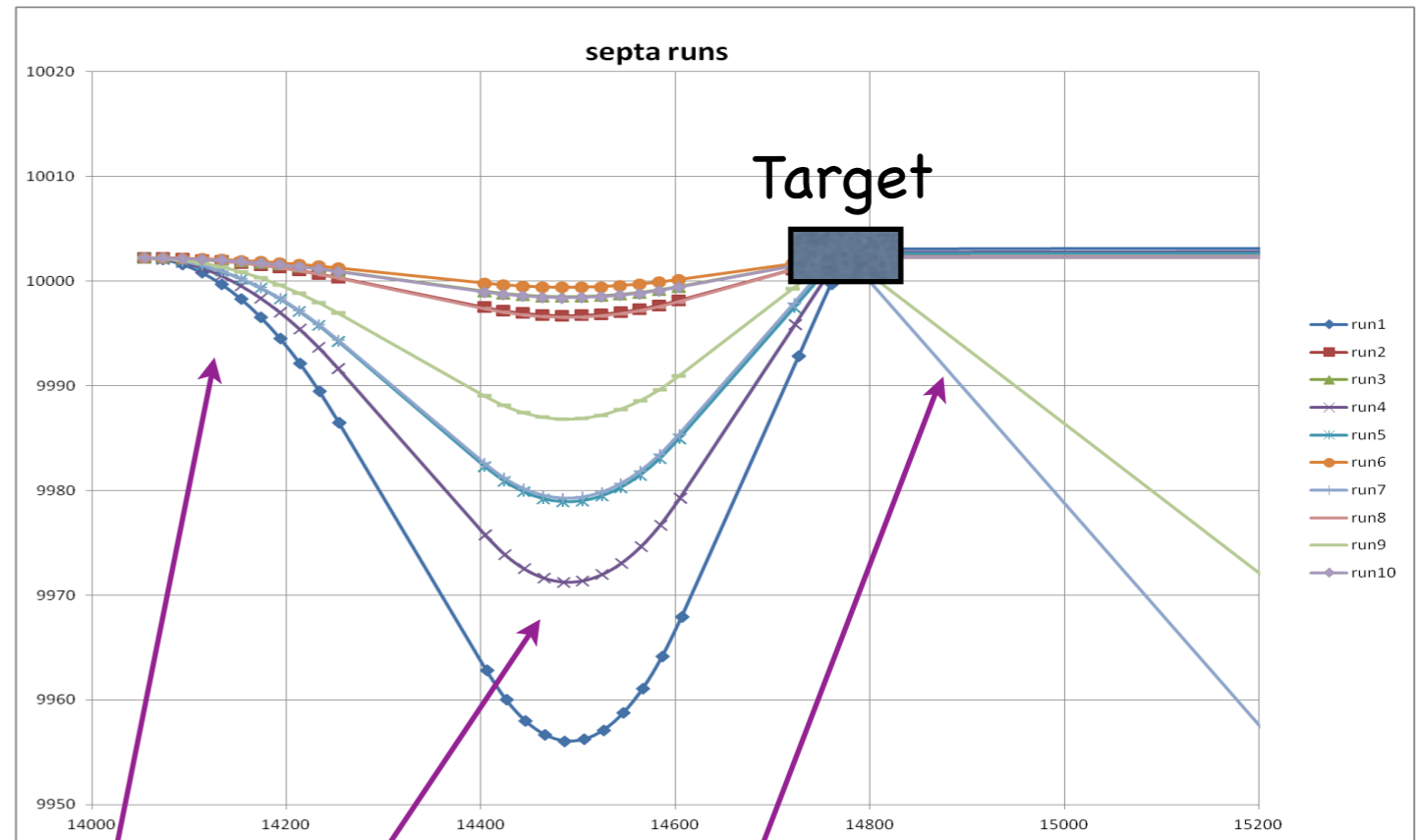
Proton Size Radius

- The finite size of the nucleus plays a small but significant role in atomic energy levels
- Simplest: proton
- 2 ways to measure:
 - energy splitting of the $2S_{1/2}-2P_{1/2}$ level (Lamb shift)
 - scattering experiment
- The results do not match when using muonic hydrogen
 - $\langle R_p \rangle = 0.84184 \pm 0.00067 \text{ fm}$ by Lamb shift in muonic hydrogen
 - $\langle R_p \rangle = 0.87680 \pm 0.0069 \text{ fm}$ CODATA world average



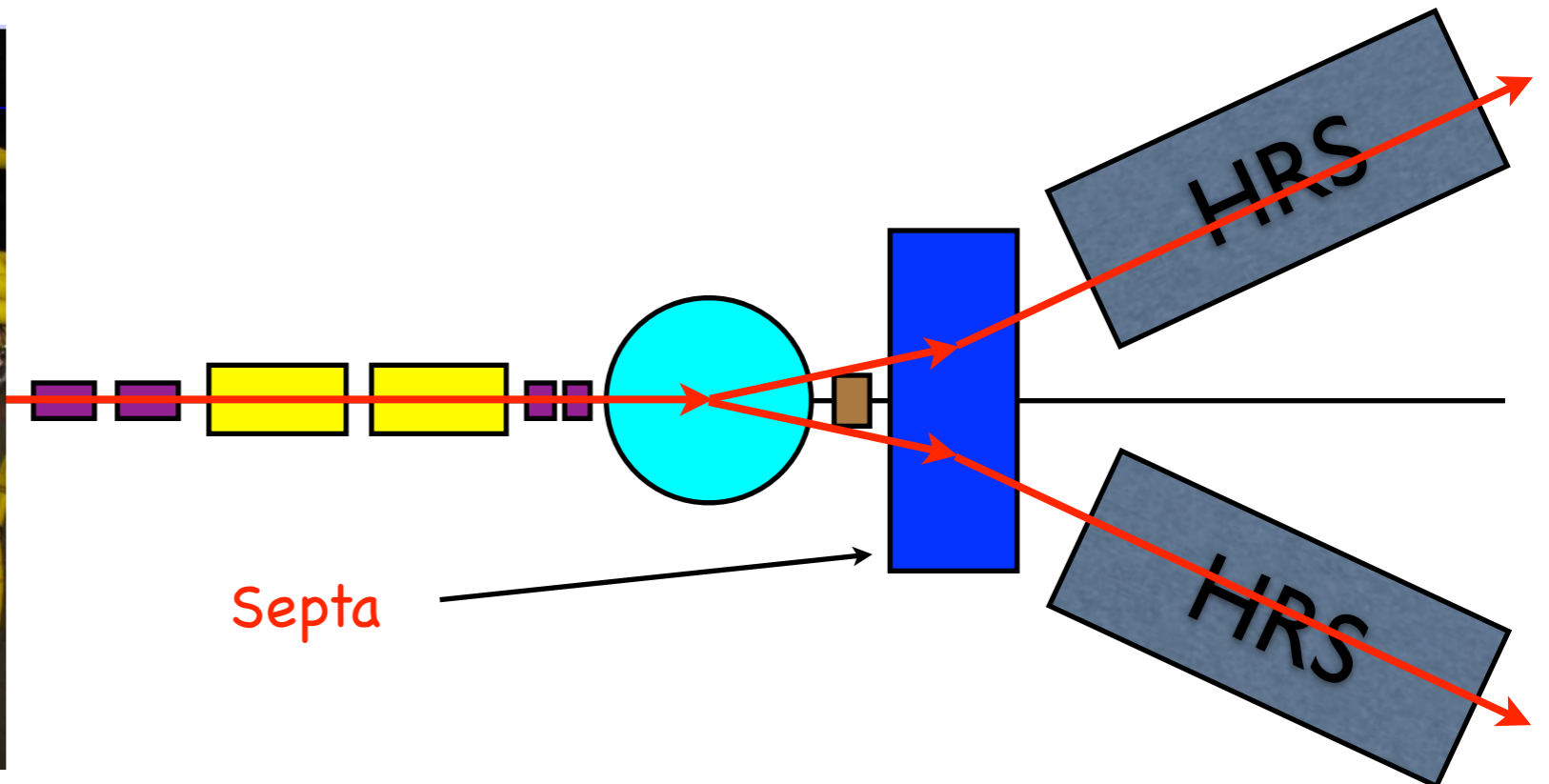
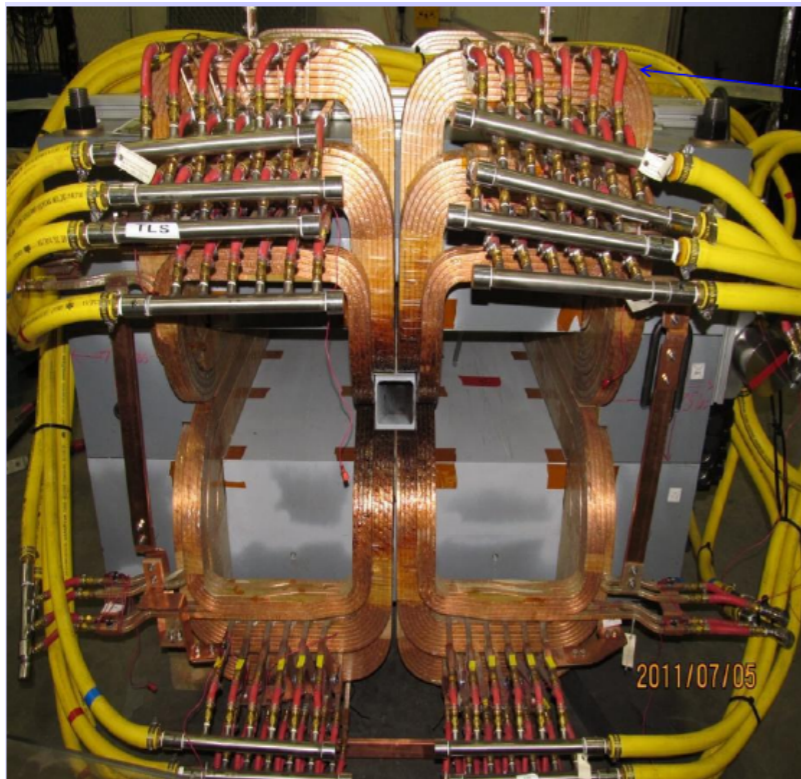
Experiment Setup

- Chicane and Local Dump
- Outgoing beam will be tilted by the large target field
- Use Chicane to provide an incident angle
- Use local dump to stop non-straight beam



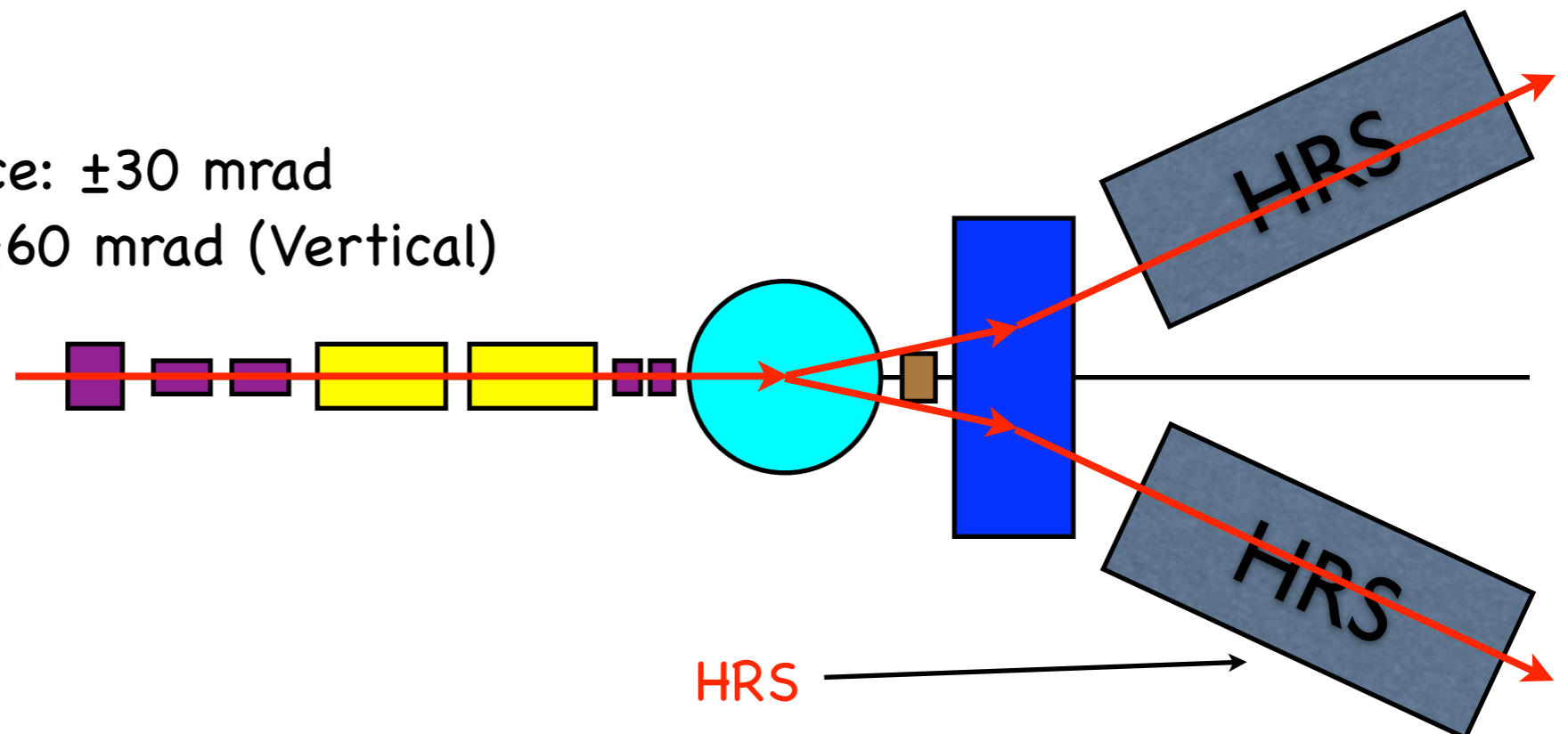
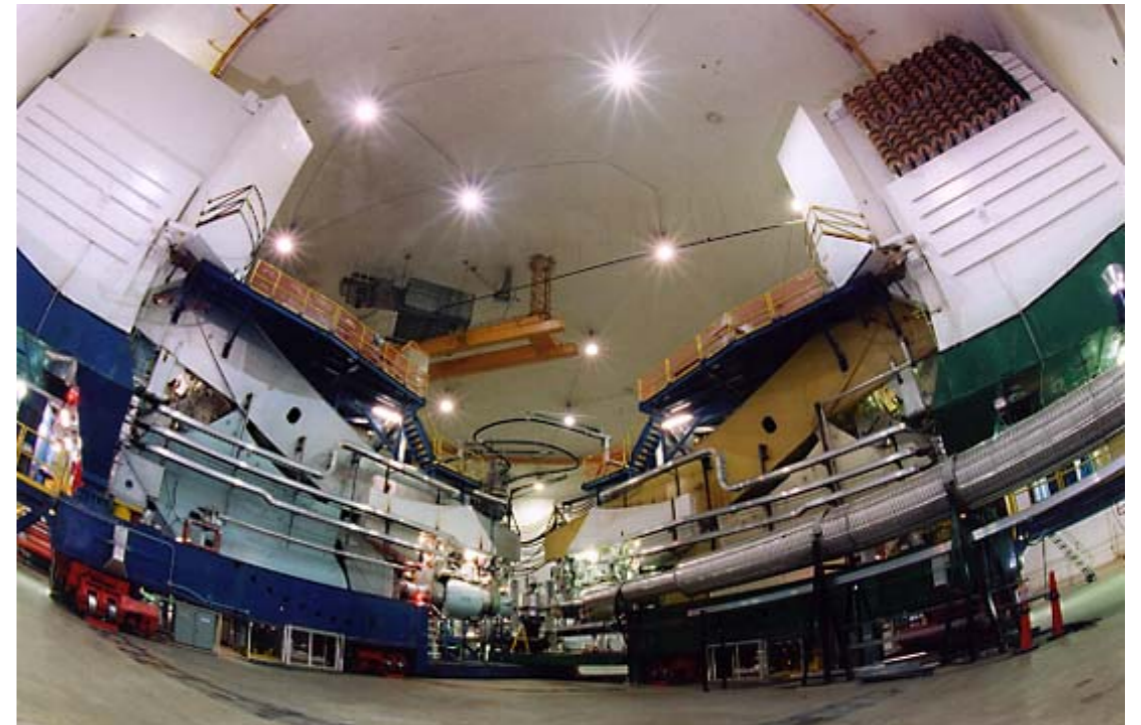
Experiment Setup

- Septa magnets
 - Detector package has a minimum angle limit at 12.5°
 - Use septa magnets to bend 5.6° scattered electrons to 12.5° to allow access to the lowest possible Q^2

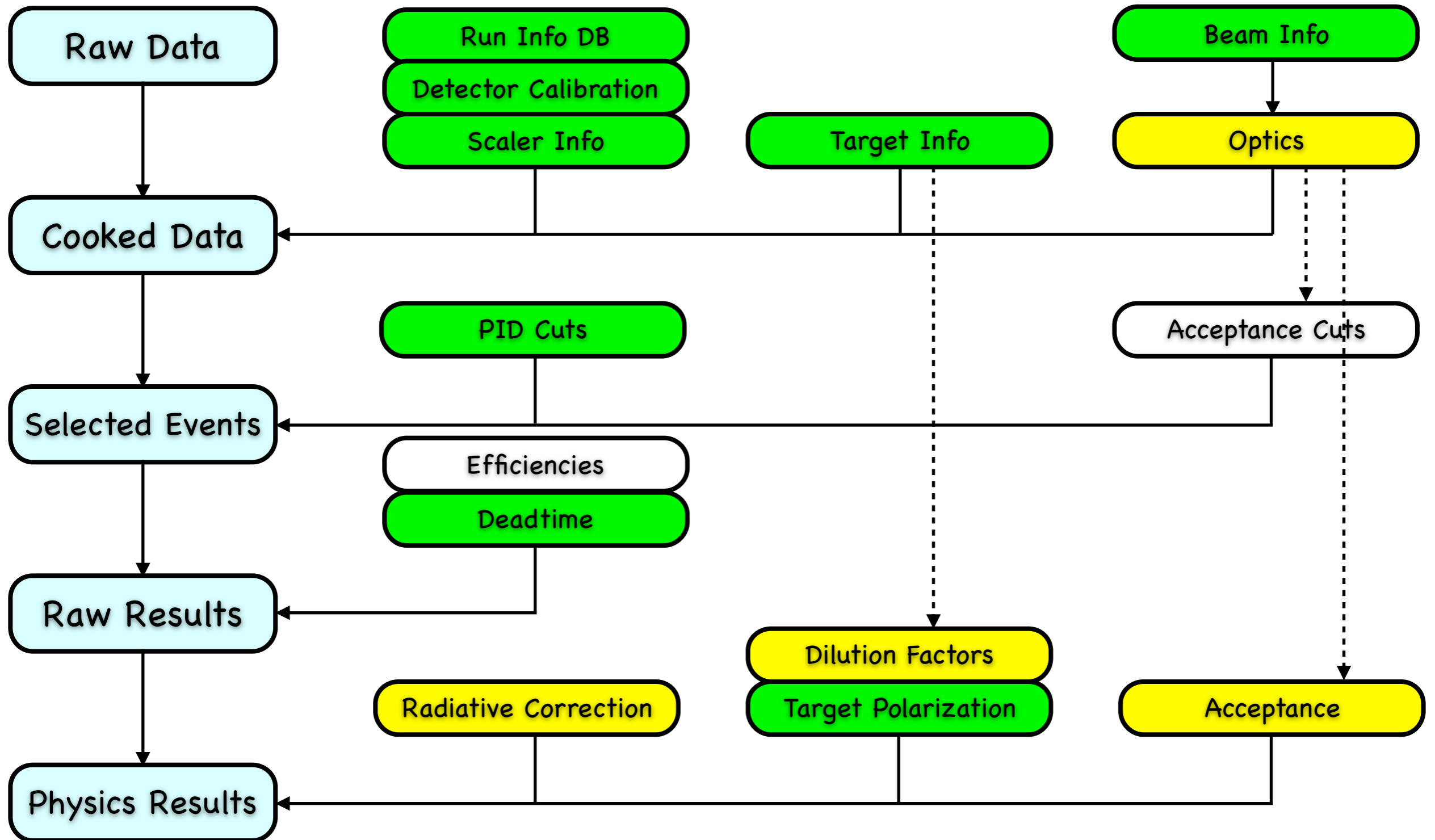


Experiment Setup

- Hall A High Resolution Spectrometer
 - High momentum resolution: 10^{-4} level over a range of 0.8–4.0 GeV/c
 - High momentum acceptance: $|\delta p/p| < 4.5\%$
 - Wide range of angular settings: $12.5^\circ \sim 150^\circ$ for left arm, $12.5^\circ \sim 130^\circ$ for right arm
 - Angular acceptance: ± 30 mrad (Horizontal) and ± 60 mrad (Vertical)

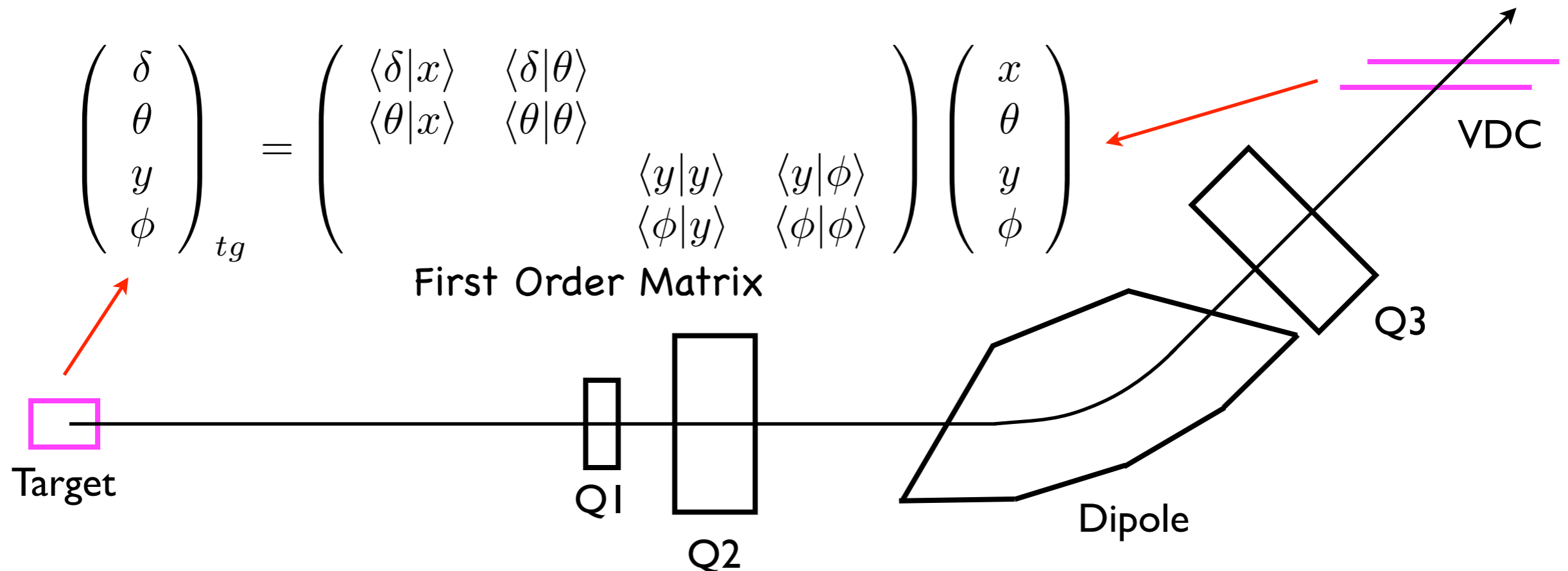


Analysis Status



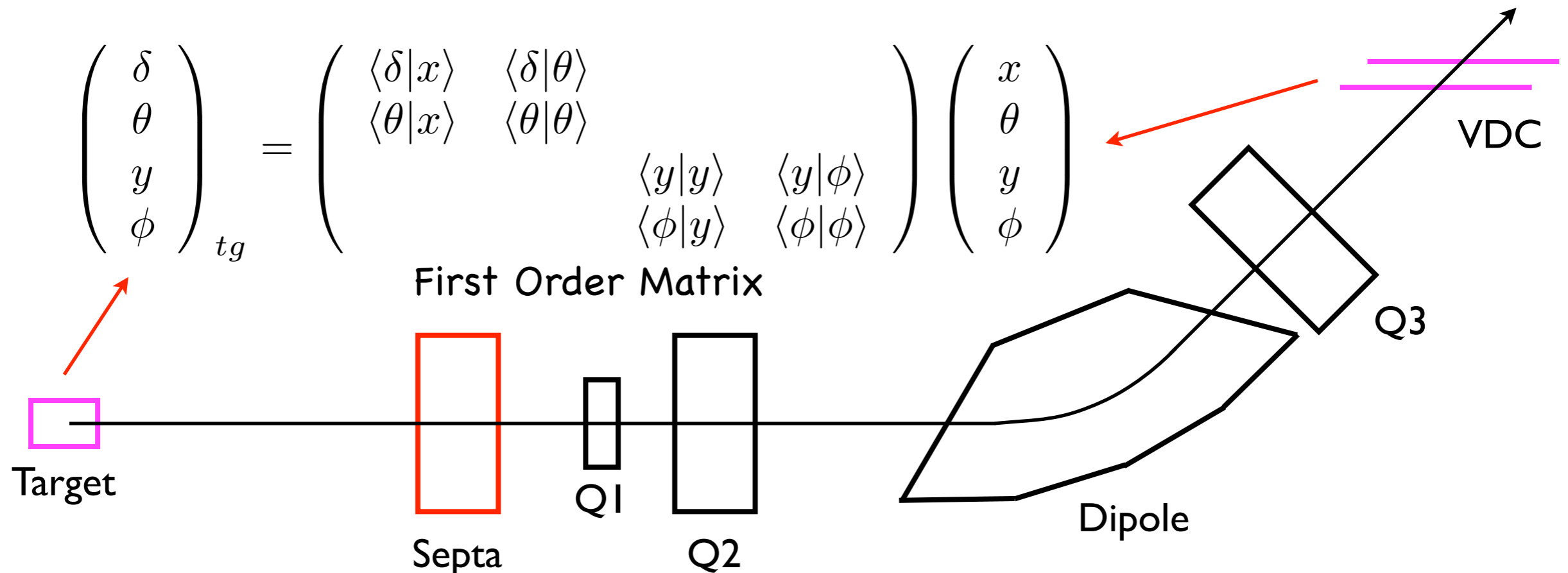
HRS Optics: Overview

- HRS has a series of magnets
- 3 quadrupoles to focus and 1 dipole to disperse on momentums
- Optics study will provide a matrix to transform VDC readouts to kinematics variables which represents the effects of these magnets



Optics for g2p

- Septa magnet
- Target magnetic field
- Optics matrix will cover septa magnet
- Target magnetic field will break the focusing nature of the spectrometer so more difficult



Optics Goal

- The g2p experiment will measure the proton structure function g_2 in the low Q^2 region (0.02–0.2 GeV²) for the first time
- Goal: 5% systematic uncertainty when measuring cross section
- Optics Goal:
 - <1.0% systematic uncertainty of scattering angle, which will contribute <4.0% to the uncertainty of cross section

$$\sigma \sim 1 / \sin^4(\theta/2)$$

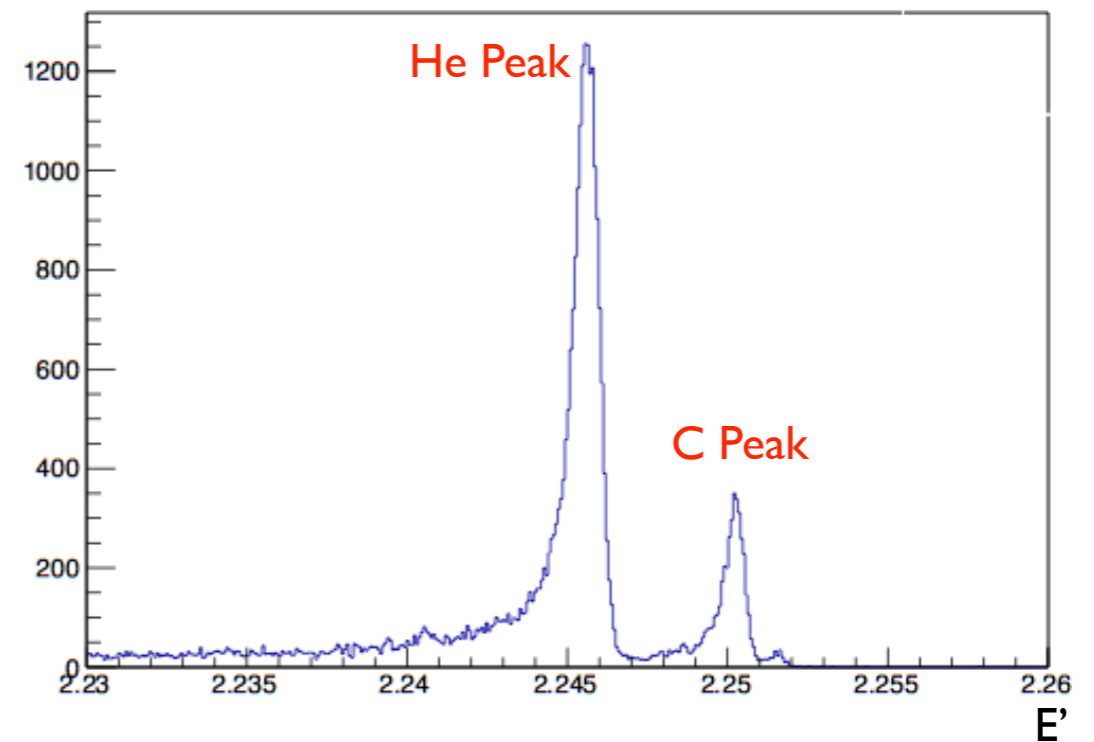
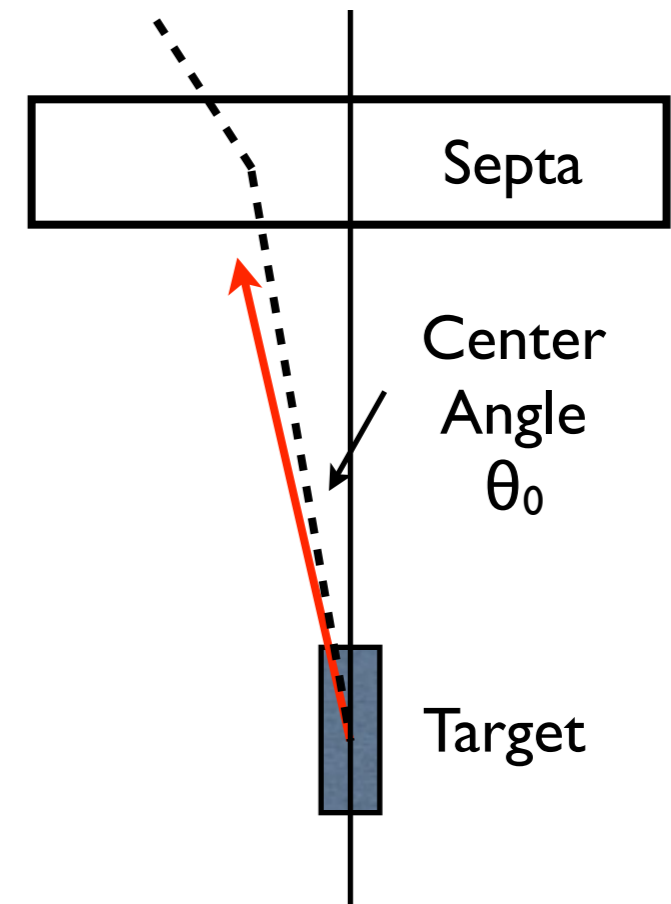
- Momentum uncertainty is not as sensitive, but it is not hard to reach 10⁻⁴ level

Angle Calibration

- Determine the center scattering angle
 - Survey: $\sim 1\text{mrad}$
 - Idea: Use elastic scattering on different target materials

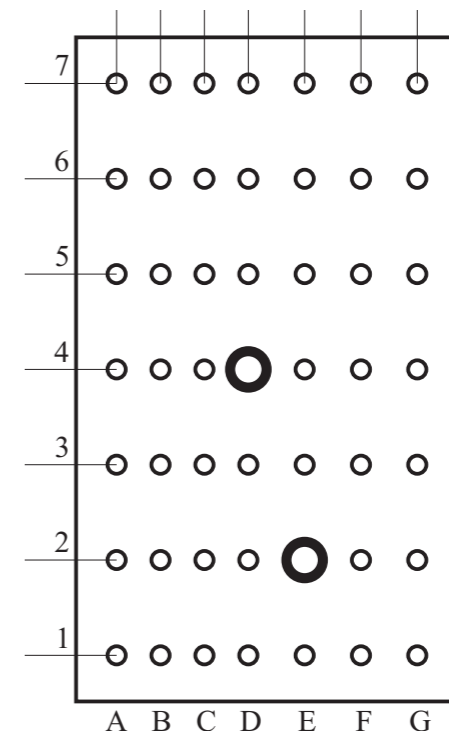
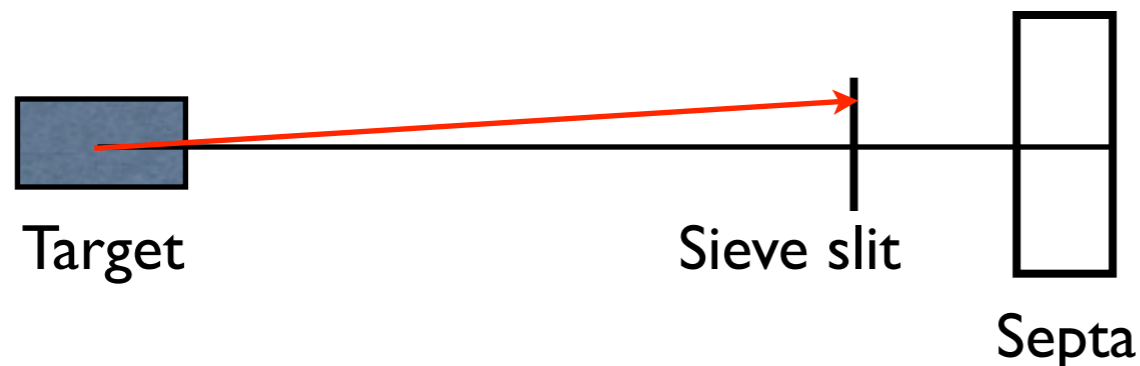
$$\Delta E' = \frac{E}{1 + \frac{E}{M_1}(1 - \cos \theta)} - \frac{E}{1 + \frac{E}{M_2}(1 - \cos \theta)}$$

- Data taking: Carbon foil in LHe, or CH₂ foil
- Two elastic peak took at the same time
- The accuracy to determine this difference is $< 50\text{KeV} \rightarrow < 0.5\text{mrad}$



Matrix Calibration

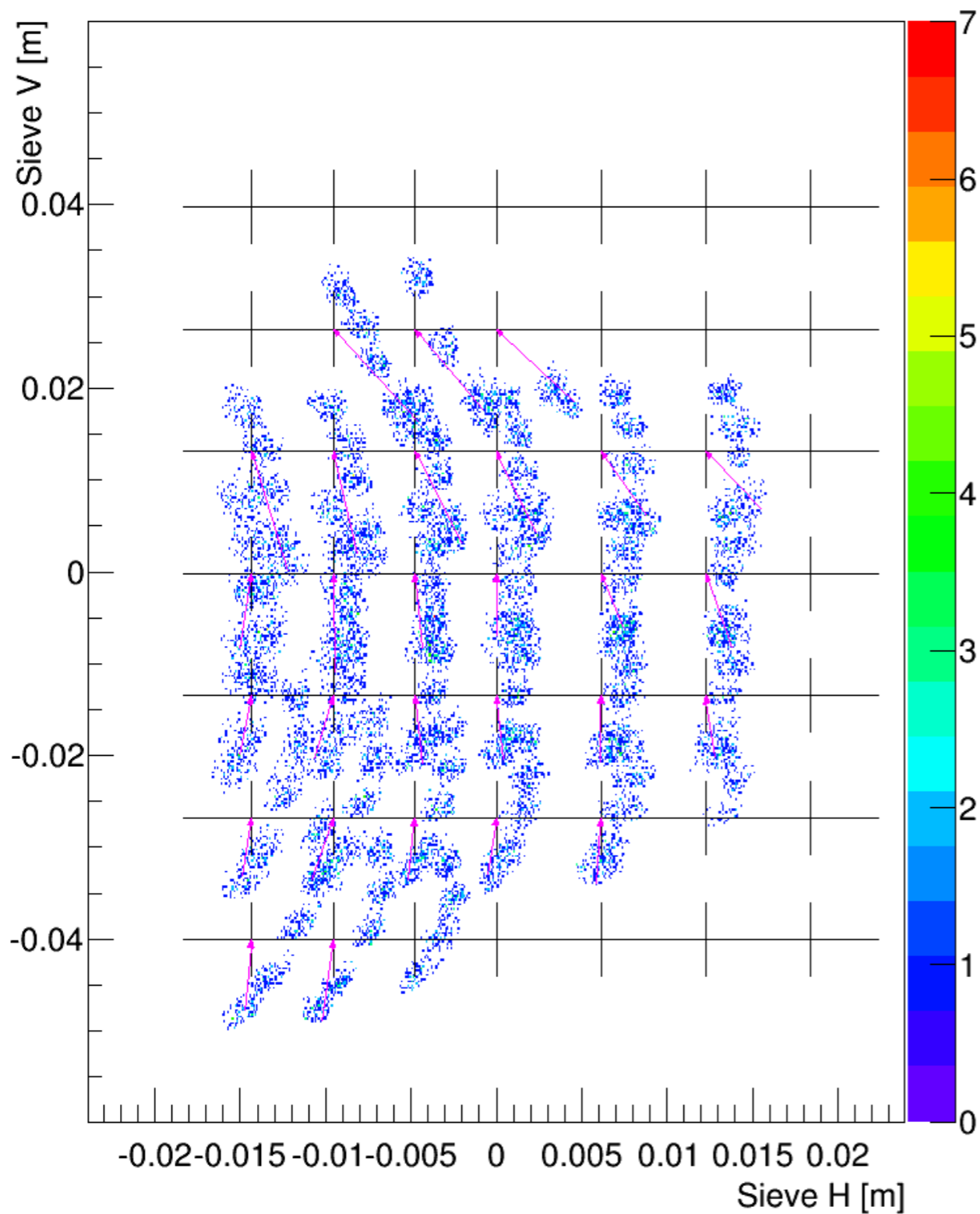
- Calibrate the angle and momentum matrix elements:
 - Use carbon foil target and point beam
 - Use sieve slit to get the real scattering angle from geometry
 - Angle: Fit with data which we already know the real scattering angle
 - Momentum: Use the real scattering angle to calculate elastic scattering momentum of carbon target



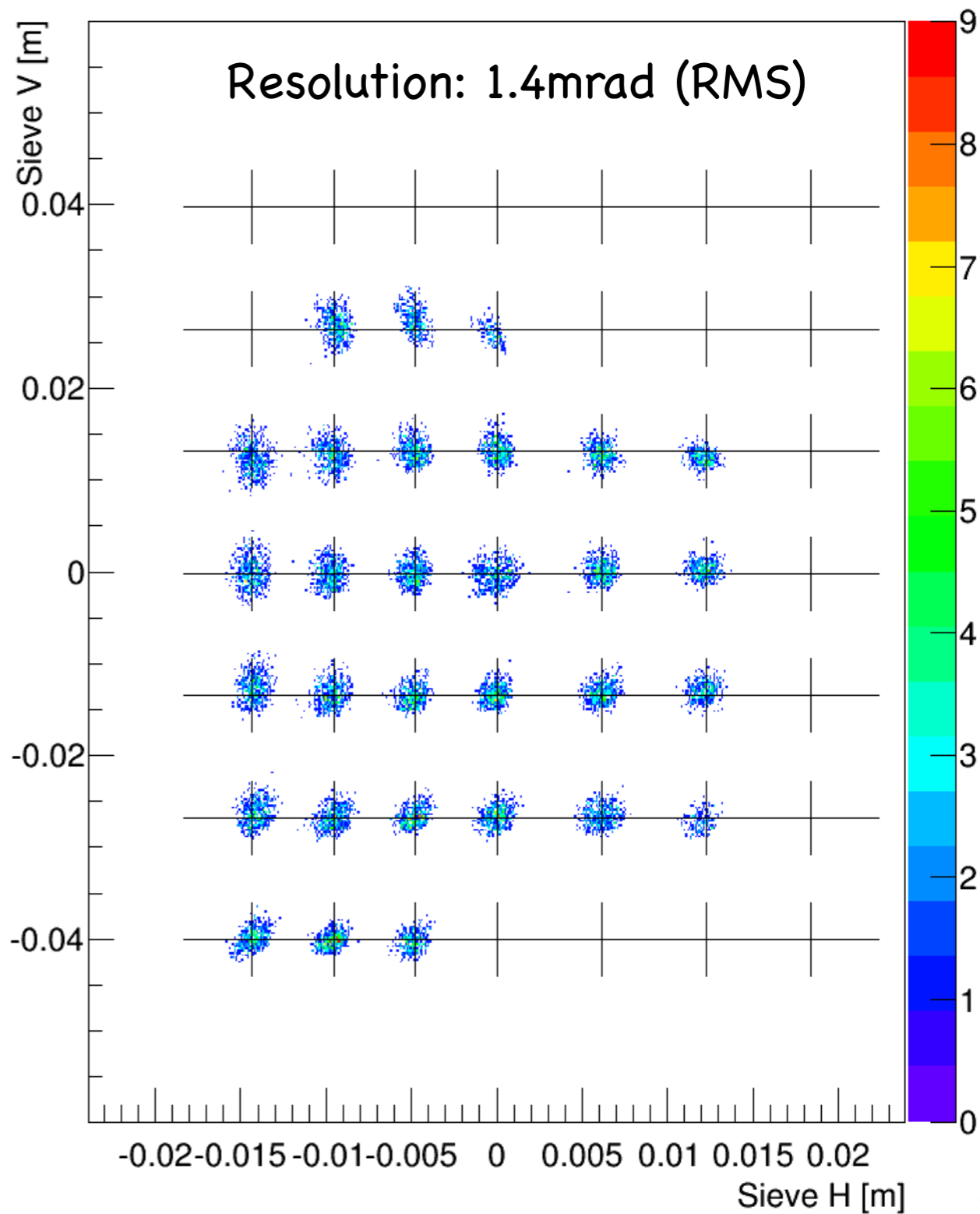
Matrix Calibration: Angle

LHRS

Before Calibration



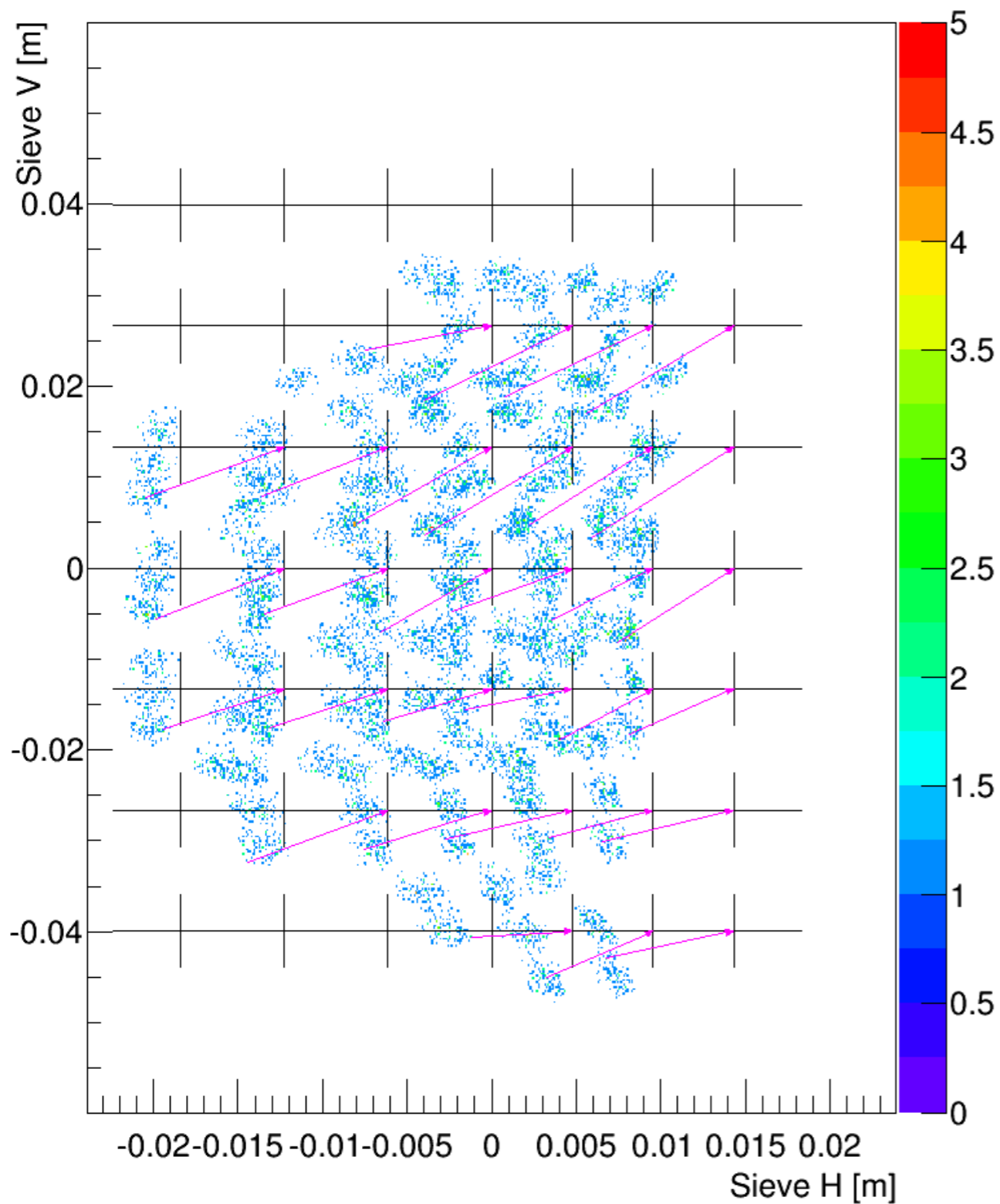
After Calibration



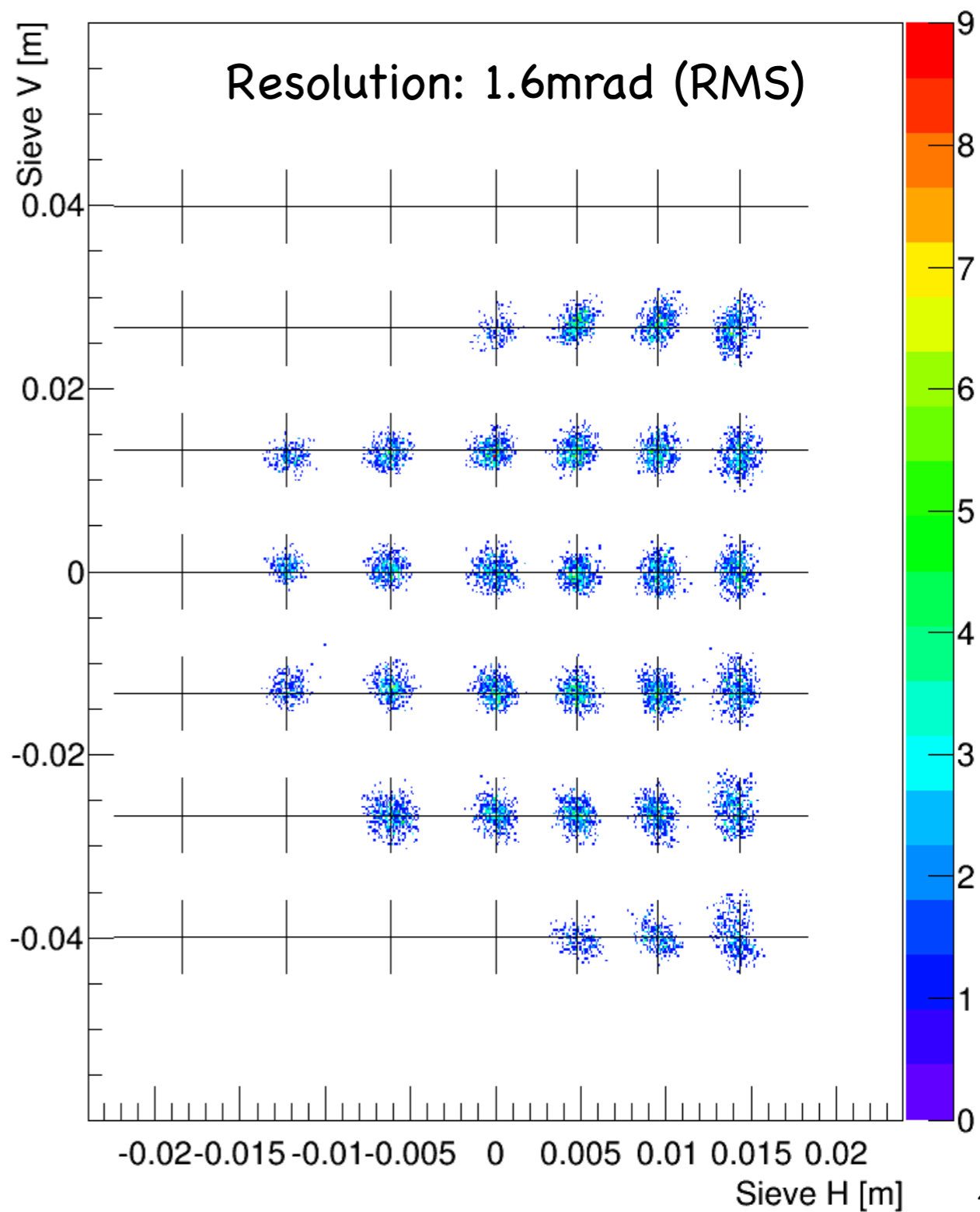
Matrix Calibration: Angle

RHRS

Before Calibration



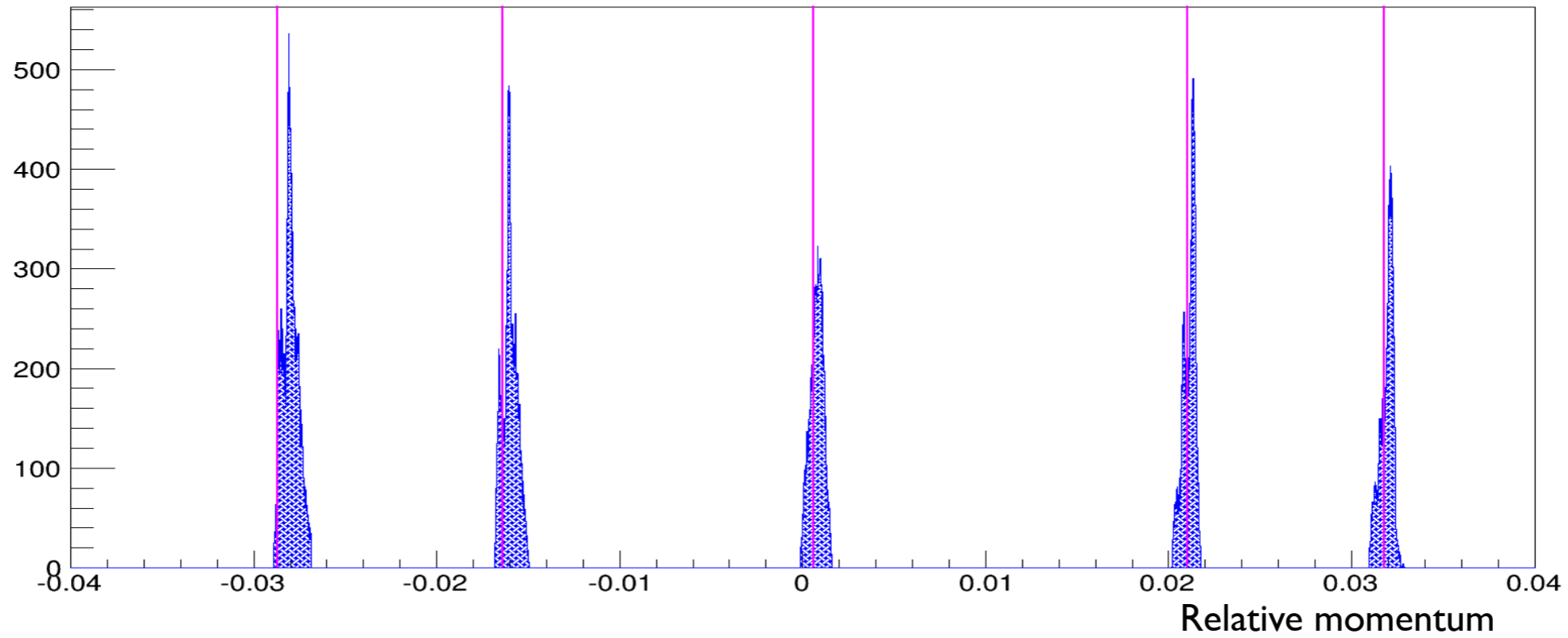
After Calibration



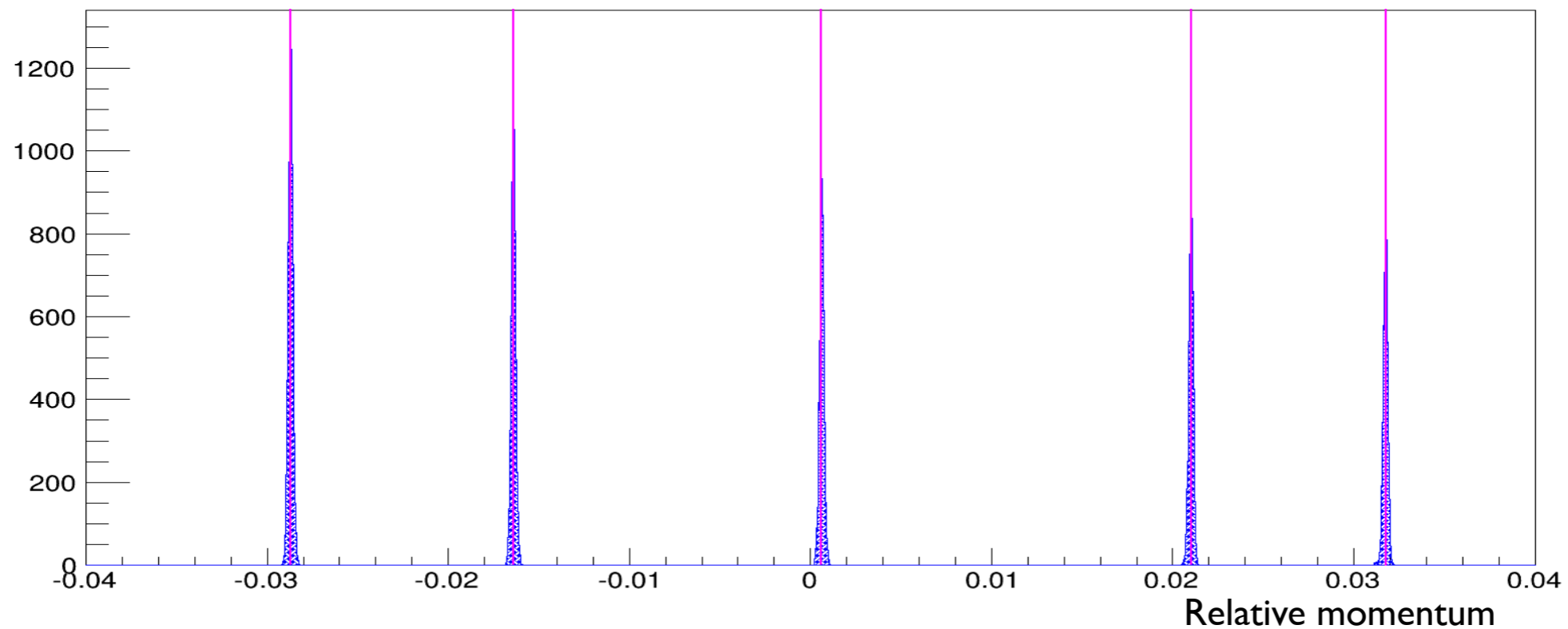
Matrix Calibration: Momentum

LHRS

Before Calibration



After Calibration

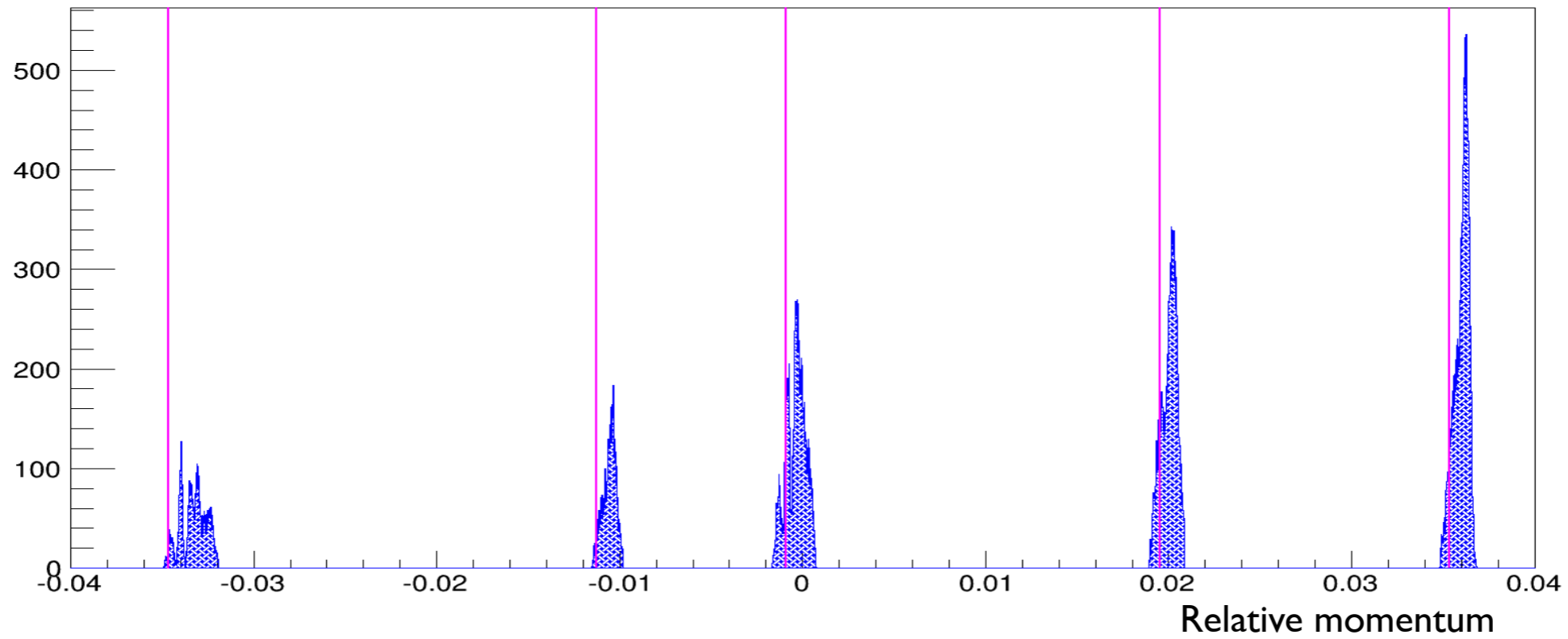


RMS: 1.4×10^{-4}

Matrix Calibration: Momentum

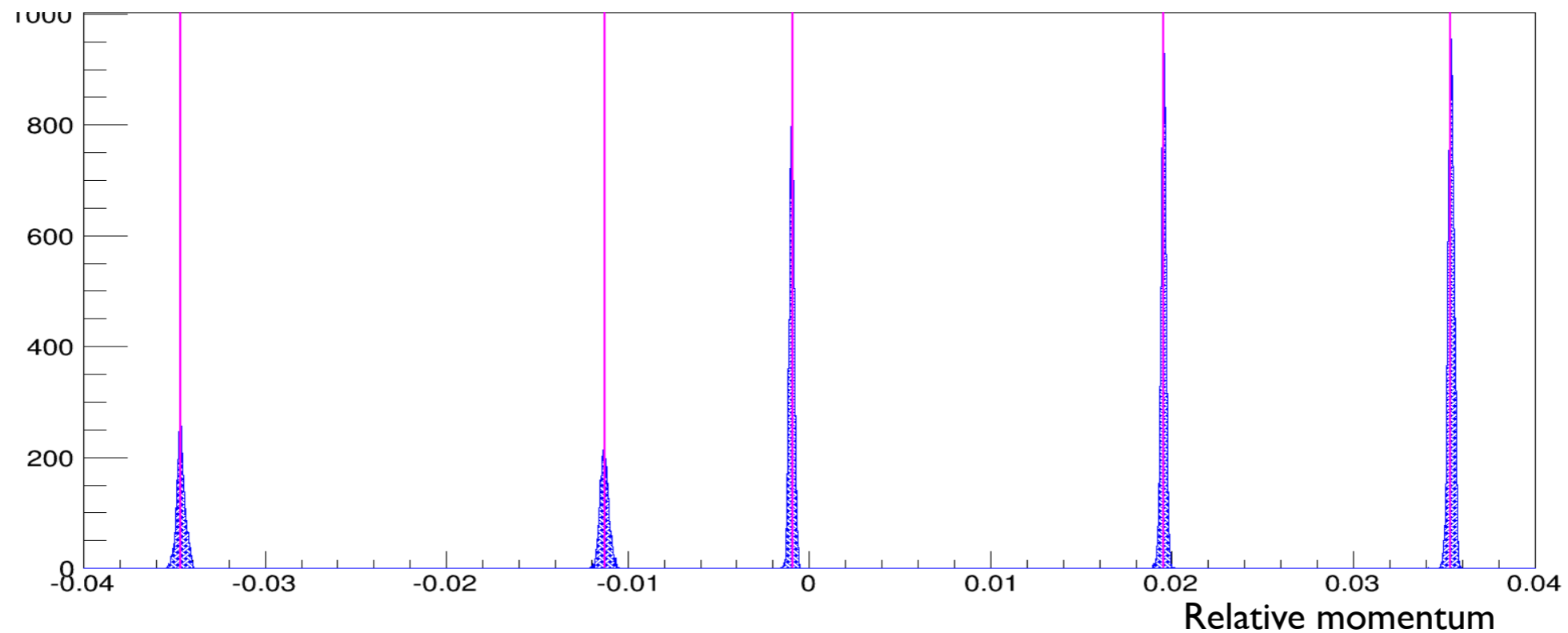
RHRS

Before Calibration



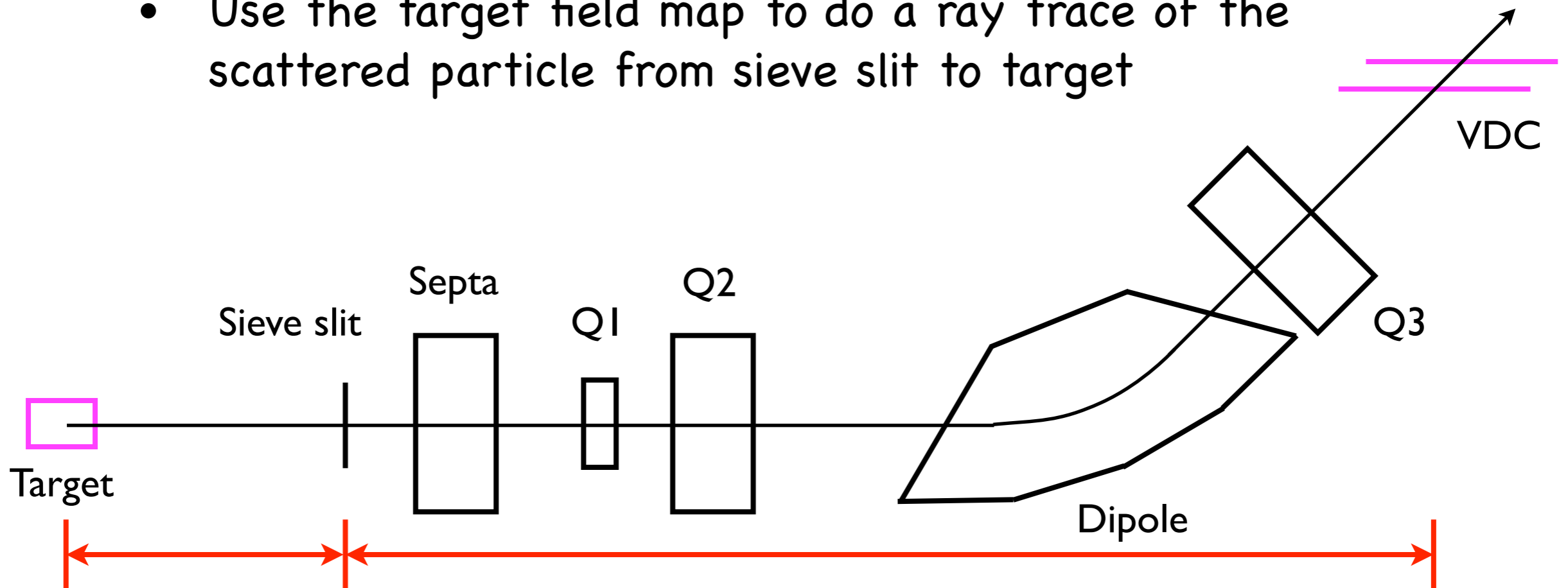
After Calibration

RMS: 1.7×10^{-4}



Optics Study with Target Field

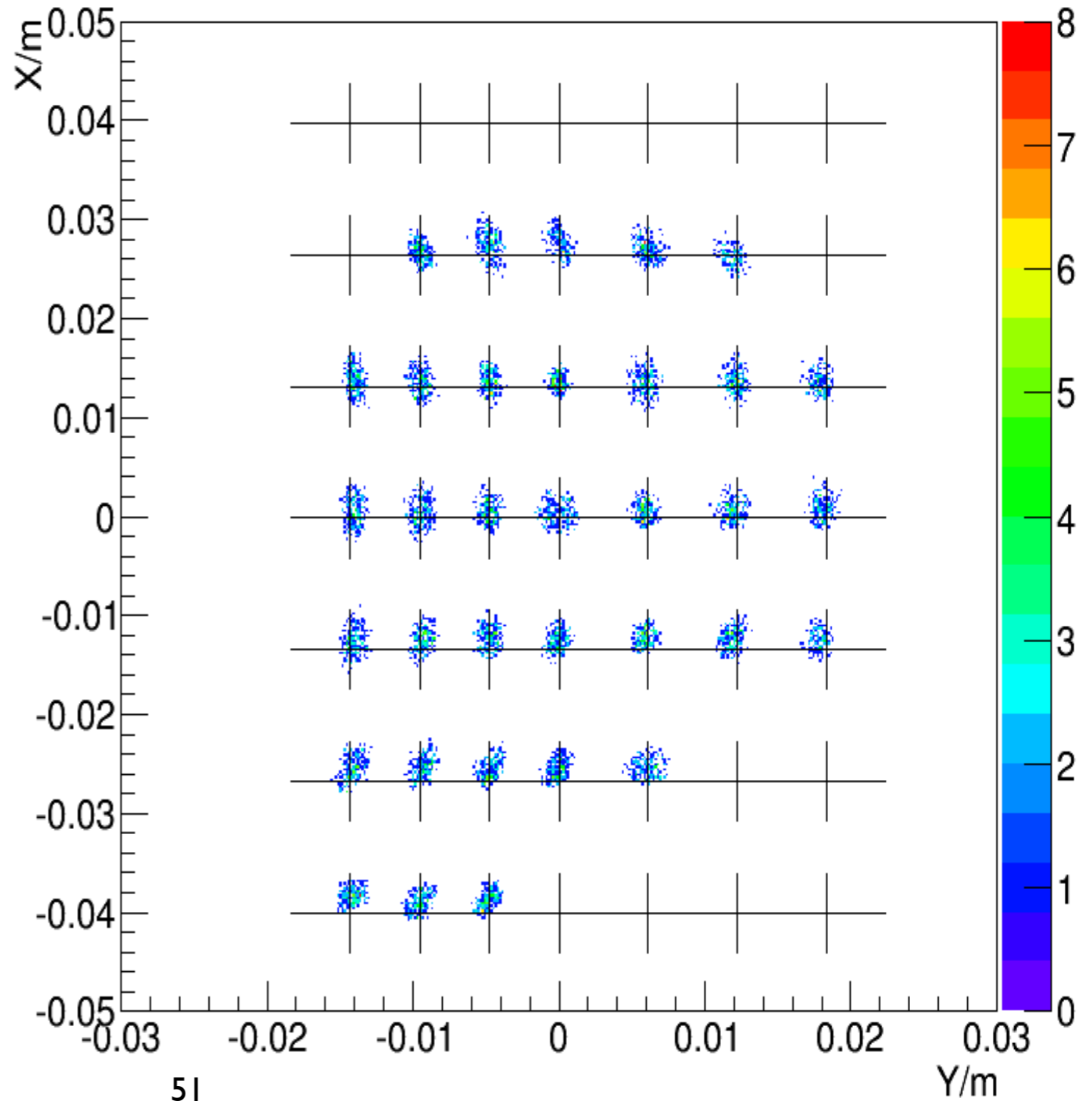
- To include target field
 - Normal sieve slit method is not useful
 - Idea: separate reconstruction process to 2 parts:
 - Use HRS optics matrix to do the reconstruction from VDC to sieve slit
 - Use the target field map to do a ray trace of the scattered particle from sieve slit to target



Optics Study with Target Field

- Use carbon foil target and point beam
- Sieve pattern is decided by both the beam position and the reconstructed angle
- Directly use BPM readout to provide beam position here

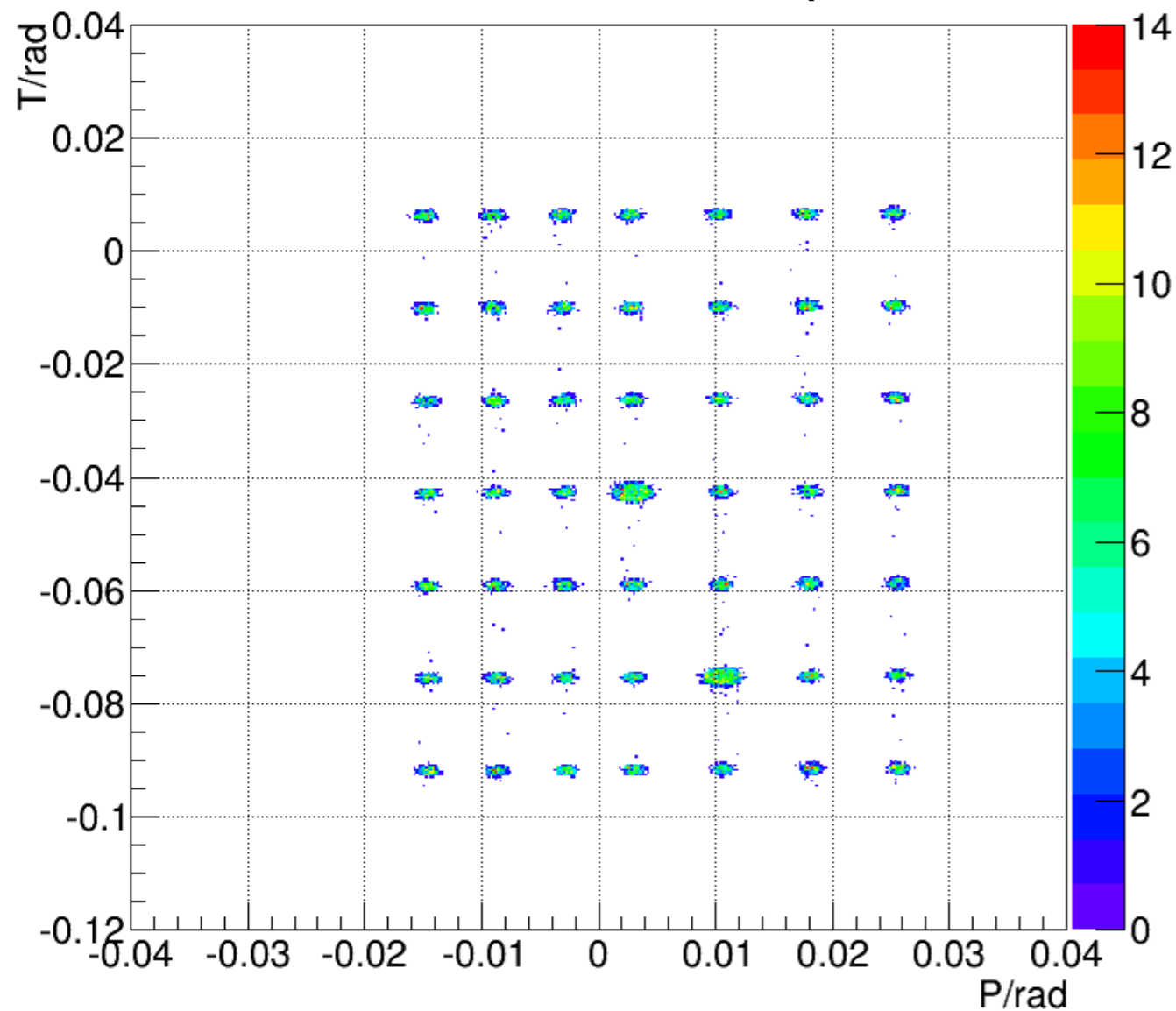
Sieve pattern after calibration



Optics Study with Target Field

- Compare reconstructed target theta and phi angle with the calculated result

Calculated theta and phi



Reconstructed theta and phi

