The spin of the proton

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It has been 30 years
of the proton "spin crisis" or "spin puzzle"

- **Spin Structure: experimentally**

$$\Sigma = \Delta u + \Delta d + \Delta s \approx 0.020$$

$$\Sigma = \Delta u + \Delta d + \Delta s \approx 0.3$$

spin "crisis" or "puzzle": where is the proton's missing spin?

The Proton "Spin Crisis"

$$\Sigma = \Delta u + \Delta d + \Delta s \approx 0.3$$

In contradiction with the naïve quark model expectation:

Naive Quark Model:

$$\Delta u = \tfrac{4}{3}; \qquad \Delta d = -\tfrac{1}{3}; \qquad \Delta s = 0$$

$$\Sigma = \Delta u + \Delta d + \Delta s = 1$$

Why there is the proton spin puzzle/crisis?

- The quark model is very successful for the classification of baryons and mesons

- The quark model is good to explain the magnetic moments of octet baryons

- The quark model gave the birth of QCD as a theory for strong interaction

So why there is serious problem with spin of the proton in the quark model?

The parton model (Feynman 1969)

- photon scatters incoherently off massless, pointlike, spin-1/2 quarks

- probability that a quark carries fraction ξ of parent proton's momentum is $q(\xi)$, $(0 < \xi < 1)$
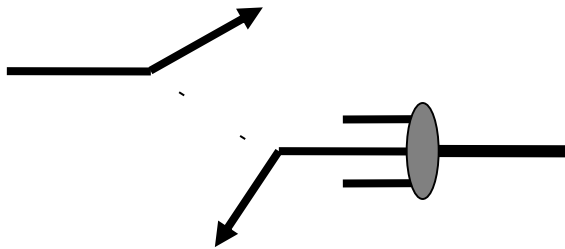
$$F_2(x) = \sum_{q,\bar{q}} \int_0^1 d\xi \ e_q^2 \, \xi \, q(\xi) \, \delta(x - \xi) = \sum_{q,\bar{q}} e_q^2 \, x \, q(x)$$

$$= \frac{4}{9} \, x \, u(x) + \frac{1}{9} \, x \, d(x) + \frac{1}{9} \, x \, s(x) + \dots$$

- the functions u(x), d(x), s(x), ... are called parton distribution functions (pdfs) - they encode information about the proton's deep structure

- **Parton model is established under the collinear approxiamtion:** The transversal motion of partons is neglected or integrated over.

How to get a clear picture of nucleon?

- PDFs are physically defined in the IMF (infinite-momentum frame) or with space-time on the light-cone.

- Whether the physical picture of a nucleon is the same in different frames?

A physical quantity defined by matrix element is frame-independent, but its physical picture is frame-dependent.

The improvement to the parton model?

- What would be the consequence by taking into account the transversal motions of partons?

- **It might be trivial in unpolarized situation. However it brings significant influences to spin dependent quantities (helicity and transversity distributions) and transversal momentum dependent quantities (TMDs or 3dPDFs).**

The Notion of Spin

- Related to the space-time symmetry of the Poincaré group

- Generators $P^\mu = (H, \vec{P})$, space-time translator

  $J^{\mu\nu}$ infinitesimal Lorentz transformation

  $\vec{J}$ $\quad J^k = \dfrac{1}{2}\varepsilon_{ijk}J^{ij}$ angular momentum

  $\vec{K}$ $\quad K^k = J^{k0}$ boost generator

  Pauli-Lubanski vertor $w_\mu = \dfrac{1}{2}J^{\rho\sigma}P^\nu \varepsilon_{\nu\rho\sigma\mu}$

  Casimir operators: $P^2 = P^\mu P_\mu = m^2$ mass

  $\qquad\qquad\qquad w^2 = w^\mu w_\mu = s^2$ spin

The Wigner Rotation

for a rest particle $(m, \vec{0}) = p^{\mu}$ $(0, \vec{s}) = \mathrm{w}^{\mu}$

for a moving particle $\mathrm{L}(p)p = (m, \vec{0})$ $(0, \vec{s}) = \mathrm{L}(p)w/m$

$\mathrm{L}(p) =$ ratationless Lorentz boost

Wigner Rotation

$$\vec{s}, p_{\mu} \rightarrow \vec{s'}, p'_{\mu}$$

$$\vec{s'} = R_w(\Lambda, p)\vec{s} \qquad p' = \Lambda p$$

$$R_w(\Lambda, p) = \mathrm{L}(p')\Lambda \mathrm{L}^{-1}(p) \qquad \text{a pure rotation}$$

Melosh Rotation for Spin-1/2 Particle

The connection between spin states in the rest frame and infinite momentum frame
Or   between spin states in the conventional equal time dynamics and the light-front dynamics

$$\chi^{\uparrow}(T) = w[(q^- + m)\chi^{\uparrow}(F) - q^R\chi^{\downarrow}(F)];$$

$$\chi^{\downarrow}(T) = w[(q^- + m)\chi^{\downarrow}(F) + q^L\chi^{\uparrow}(F)].$$

What is Δq measured in DIS

- Δq is defined by $$\Delta q\ s_\mu = \langle p,s \,|\, \bar{q}\gamma_\mu\gamma_5 q \,|\, p,s\rangle$$

$$\Delta q = \langle p,s \,|\, \bar{q}\gamma^+\gamma_5 q \,|\, p,s\rangle$$

- Using light-cone Dirac spinors

$$\Delta q = \int_0^1 dx \left[q^\uparrow(x) - q^\downarrow(x) \right]$$

- Using conventional Dirac spinors

$$\Delta q = \int d^3\vec{p}\, M_q \left[q^\uparrow(\vec{p}) - q^\downarrow(\vec{p}) \right]$$

$$M_q = \frac{(p_0 + p_3 + m)^2 - \vec{p}_\perp^{\,2}}{2(p_0 + p_3)(p_0 + m)}$$

**Thus Δq is the light-cone quark spin
 or quark spin in the infinite momentum frame,
not that in the rest frame of the proton**

The proton spin crisis

& the Melosh-Wigner rotation

- It is shown that the proton "spin crisis" or "spin puzzle" can be understood by the relativistic effect of quark transversal motions due to the Melosh-Wigner rotation.

- The quark helicity Δq measured in polarized deep inelastic scattering is actually the quark spin in the infinite momentum frame or in the light-cone formalism, and it is different from the quark spin in the nucleon rest frame or in the quark model.

B.-Q. Ma, J.Phys. G 17 (1991) L53

B.-Q. Ma, Q.-R. Zhang,   Z.Phys.C 58 (1993) 479-482

Quark spin sum is not a Lorentz invariant quantity

Thus the quark spin sum equals to the proton in the rest frame does not mean that it equals to the proton spin in the infinite momentum frame
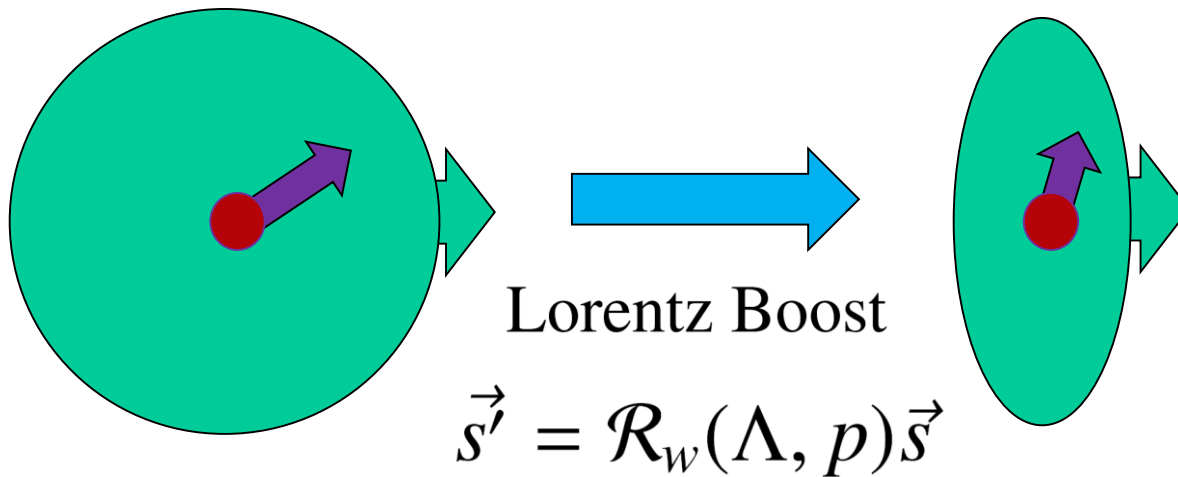
$$\sum_q \vec{s}_q = \vec{S}_p \quad \text{in the rest frame}$$

does not mean that

$$\sum_q \vec{s}_q = \vec{S}_p \quad \text{in the infinite momentum frame}$$

Therefore it is not a surprise that the quark spin sum measured in DIS does not equal to the proton spin

An intuitive picture to understand the spin puzzle

Lorentz Boost

$$\vec{s'} = \mathcal{R}_w(\Lambda, p)\vec{s}$$

Rest Frame

Infinite Momentum Frame

$$\Sigma \vec{s} = \vec{S}_p$$

$$\Sigma \vec{s'} \neq \vec{S}_p$$

A general consensus

The quark helicity Δq defined in the infinite momentum frame is generally not the same as the constituent quark spin component in the proton rest frame, just like that it is not sensible to compare apple with orange.

H.-Y.Cheng, hep-ph/0002157，
Chin.J.Phys.38:753,2000

Other approaches with same conclusion

Contribution from the lower component of Dirac spinors in the rest frame:

B.-Q. Ma, Q.-R. Zhang,   Z.Phys.C 58 (1993) 479-482

D.Qing, X.-S.Chen, F.Wang, **Phys.Rev.D58:114032,1998**.

P.Zavada, **Phys.Rev.D65:054040,2002**.

The Spin Distributions in Quark Model

The spin distribution probabilities in the quark-diquark model

$$u_\uparrow^\uparrow = \tfrac{1}{18}; \quad u_\uparrow^\downarrow = \tfrac{2}{18}; \quad d_\uparrow^\uparrow = \tfrac{2}{18}; \quad d_\uparrow^\downarrow = \tfrac{4}{18};$$

$$u_S^\uparrow = \tfrac{1}{2}; \quad u_S^\downarrow = 0; \quad d_S^\uparrow = 0; \quad d_S^\downarrow = 0. \qquad (7)$$

Naive Quark Model:

$$\Delta u = \tfrac{4}{3}; \qquad \Delta d = -\tfrac{1}{3}; \qquad \Delta s = 0$$

$$\Sigma = \Delta u + \Delta d + \Delta s = 1$$

Relativistic Effect due to Melosh-Rotation

$$\Delta u_v(x) = u_v^\uparrow(x) - u_v^\downarrow(x) = -\tfrac{1}{18}a_\uparrow\cdot(x)W_\uparrow\cdot(x) + \tfrac{1}{2}a_S(x)W_S(x);$$

$$\Delta d_v(x) = d_v^\uparrow(x) - d_v^\downarrow(x) = -\tfrac{1}{9}a_\uparrow\cdot(x)W_\uparrow\cdot(x).$$

from

$$a_S(x) = 2u_v(x) - d_v(x);$$

$$a_\uparrow\cdot(x) = 3d_v(x).$$

We obtain

$$\Delta u_v(x) = [u_v(x) - \tfrac{1}{2}d_v(x)]W_S(x) - \tfrac{1}{6}d_v(x)W_\uparrow\cdot(x);$$

$$\Delta d_v(x) = -\tfrac{1}{3}d_v(x)W_\uparrow\cdot(x).$$

Relativistic SU(6) Quark Model

Flavor Symmetric Case

Relativistic Correction: $M_q = 0.75$

$\Delta u = \frac{4}{3} M_q = 1;$ $\Delta d = -\frac{1}{3} M_q = -0.25;$ $\Delta s = 0$

$\Sigma = \Delta u + \Delta d + \Delta s = 0.75$

$F_2^n(x)/F_2^p(x) \geq \frac{2}{3}$ for all x

Relativistic SU(6) Quark Model
Flavor Asymmetric Case

Relativistic Correction: $\quad M_u \approx 0.6; \quad M_d \approx 0.9$

$\Delta u = \frac{4}{3} M_u = 0.8; \quad \Delta d = -\frac{1}{3} M_d = -0.3; \quad \Delta s = 0$

$\Sigma = \Delta u + \Delta d + \Delta s \approx 0.5$

$F_2^n(x) / F_2^p(x) \rightarrow \frac{1}{4}$ at large x

B.-Q.Ma, Phys. Lett. **B 375** (1996) 320.

Relativistic SU(6) Quark Model

Flavor Asymmetric Case + Intrinsic Sea

For Intrinsic $d\bar{d}$ Sea ($\sim 15\%$): $\Delta d_{sea} \approx -0.07$

For Intrinsic $s\bar{s}$ Sea ($\sim 5\%$): $\Delta s_{sea} \approx -0.03$

Thus: $\Sigma = \Delta u + \Delta d + \Delta s + \Delta d_{sea} + \Delta s_{sea} \approx 0.4$

S. J. Brodsky and B.-Q.Ma, Phys. Lett. **B 381** (1996) 317.

More detailed discussions, see, B.-Q.Ma, J.-J.Yang, I.Schmidt,
Eur.Phys.J.A12(2001)353
Understanding the Proton Spin "Puzzle" with a New "Minimal" Quark Model
Three quark valence component could be as large as 70% to account for the data

A relativistic quark-diquark model
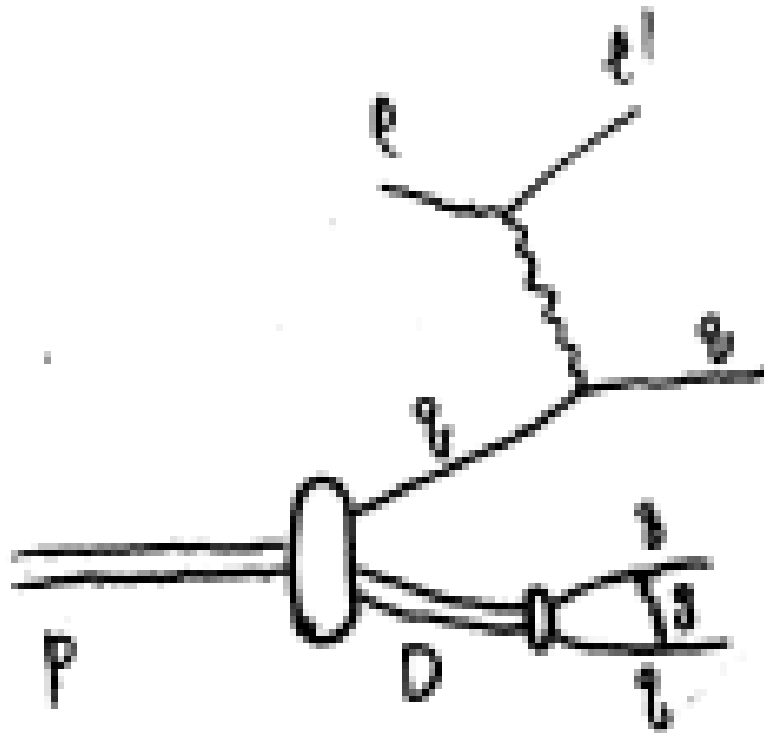
A relativistic quark-diquark model

- The unpolarized distribution of quark q in hadron h can be written as

$$q(x) = c_q^S a_S(x) + c_q^V a_V(x),$$

where $a_D(x)$ is

$$a_D(x) \propto \int [\mathrm{d}^2 \mathbf{k}_\perp] |\phi(x, \mathbf{k}_\perp)|^2 \quad (D = S \text{ or } V),$$

- BHL prescription of the light-cone momentum space wave function for quark-diquark

$$\phi(x, \mathbf{k}_\perp) = A_D \exp\left\{ -\frac{1}{8\alpha_D^2} \left[\frac{m_q^2 + \mathbf{k}_\perp^2}{x} + \frac{m_D^2 + \mathbf{k}_\perp^2}{1 - x} \right] \right\},$$

A relativistic quark-diquark model

- longitudinally polarized quark distribution

$$\Delta q(x) = \tilde{c}_q^S \tilde{a}_S(x) + \tilde{c}_q^V \tilde{a}_V(x)$$

where

$$\tilde{a}_D(x) = \int [\mathrm{d}^2\, \mathbf{k}_\perp] W_D(x, \mathbf{k}_\perp) |\phi(x, \mathbf{k}_\perp)|^2 \quad (D = S \text{ or } V)$$

- Melosh-Winger rotation factor

Longitudinally polarized

$$W_D(x, \mathbf{k}_\perp) = \frac{(k^+ + m_q)^2 - \mathbf{k}_\perp^2}{(k^+ + m_q)^2 + \mathbf{k}_\perp^2}$$

where $k^+ = x\mathcal{M}$, $\mathcal{M}^2 = \frac{m_q^2 + \mathbf{k}_\perp^2}{x} + \frac{m_D^2 + \mathbf{k}_\perp^2}{1-x}$.

The Melosh-Wigner rotation
in pQCD based parametrization of quark helicity distributions

"**The helicity distributions measured on the light-cone are related by a Wigner rotation (Melosh transformation) to the ordinary spin S_i^z of the quarks in an equal-time rest-frame wavefunction description. Thus, due to the non-collinearity of the quarks, one cannot expect that the quark helicities will sum simply to the proton spin.**"

S.J.Brodsky, M.Burkardt, and I.Schmidt,

Nucl.Phys.B441 (1995) 197-214, p.202

pQCD counting rule

$$q_{\mathrm{h}}^{\pm} \propto (1 - x)^{p}$$

$$p = 2n - 1 + 2\,|\,\Delta s_z\,| \qquad \Delta s_z = s_q - s_N$$

- **Based on the minimum connected tree graph of hard gluon exchanges.**
- **"Helicity retention" is predicted -- The helicity of a valence quark will match that of the parent nucleon.**

Parameters in pQCD counting rule analysis

In leading term

$$q_i^+ = \frac{\tilde{A}_{q_i}}{B_3} x^{-\frac{1}{2}} (1-x)^3$$

$$q_i^- = \frac{\tilde{C}_{q_i}}{B_5} x^{-\frac{1}{2}} (1-x)^5$$

Baryon	q_1	q_2	$\tilde{A}_{q_1}$	$\tilde{C}_{q_1}$	$\tilde{A}_{q_2}$	$\tilde{C}_{q_2}$
p	u	d	1.375	0.625	0.275	0.725

B.-Q. Ma, I. Schmidt, J.-J. Yang, Phys.Rev.D63(2001) 037501.

New Development: H. Avakian, S.J.Brodsky, D.Boer, F.Yuan, Phys.Rev.Lett.99:082001,2007.

Two different sets of parton distributions

- SU(6) quark-diquark model

$$\Delta u_v(x) = [u_v(x) - \frac{1}{2}d_v(x)]W_S(x) - \frac{1}{6}d_v(x)W_V(x),$$
$$\Delta d_v(x) = -\frac{1}{3}d_v(x)W_V(x).$$

- pQCD based counting rule analysis

$$u_v^{\text{pQCD}}(x) = u_v^{\text{para}}(x),$$
$$d_v^{\text{pQCD}}(x) = \frac{d_v^{\text{th}}(x)}{u_v^{\text{th}}(x)}u_v^{\text{para}}(x),$$
$$\Delta u_v^{\text{pQCD}}(x) = \frac{\Delta u_v^{\text{th}}(x)}{u_v^{\text{th}}(x)}u_v^{\text{para}}(x),$$
$$\Delta d_v^{\text{pQCD}}(x) = \frac{\Delta d_v^{\text{th}}(x)}{u_v^{\text{th}}(x)}u_v^{\text{para}}(x),$$

- CTEQ5 set 3 as input.

Different predictions in two models

Helicity distribution

- SU(6) quark-diquark model:
  $\Delta u(x)/u(x) \to 1$ as $x \to 1$.
  $\Delta d(x)/d(x) \to -\frac{1}{3}$ as $x \to 1$.

- pQCD based counting rule analysis:
  $\Delta u(x)/u(x) \to 1$ as $x \to 1$.
  $\Delta d(x)/d(x) \to 1$ as $x \to 1$.
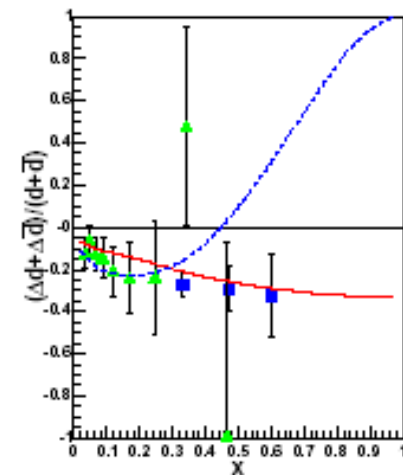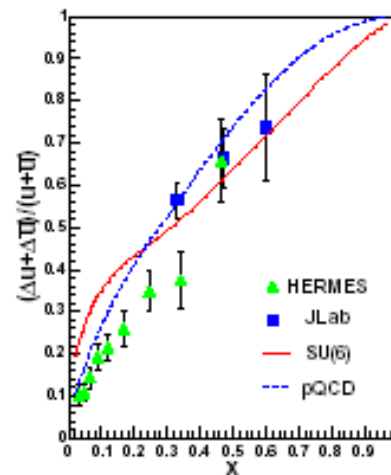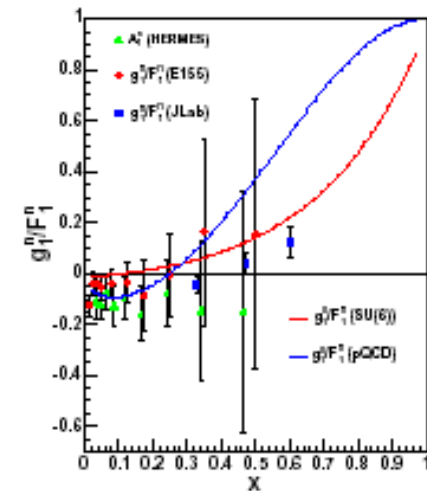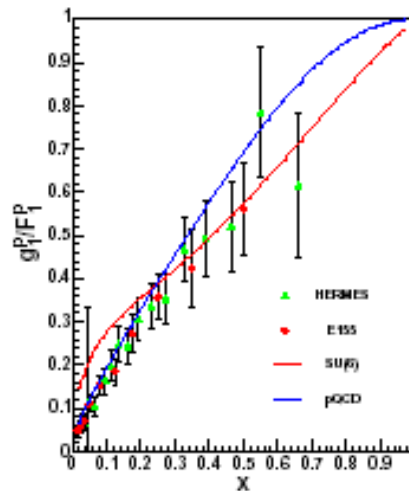
The proton spin in a light-cone chiral quark model
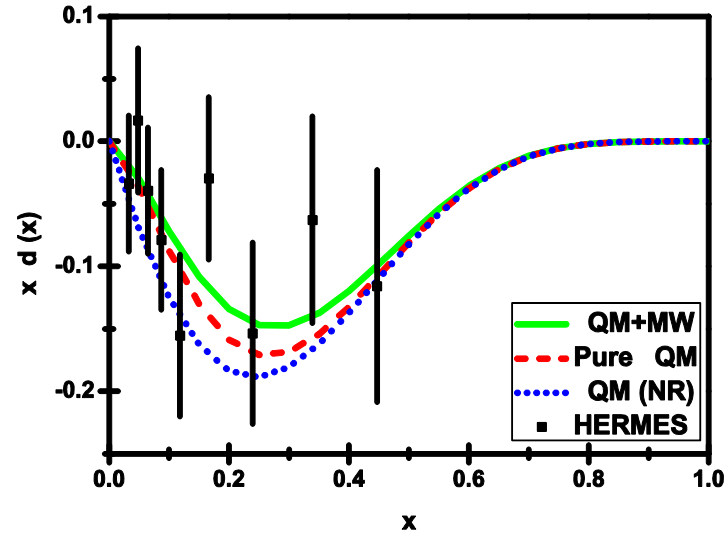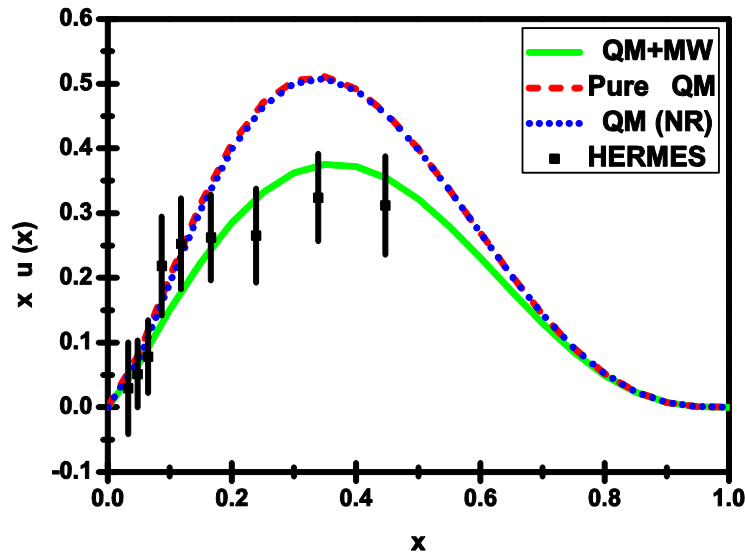
An upgrade of previous work by including Melosh-Wigner rotation: T. P. Cheng and L. F. Li, PRL 74 (1995) 2872

Chances：New Research Directions

- New quantities：Transversity, Generalized Parton Distributions, Collins Functions, Silver Functions, Boer-Mulders Functions, Pretzelosity, Wigner Distributions

- Hyperon Physics：The spin structure of Lambda and Sigma hyperons

B.-Q. Ma, I. Schmidt, J.-J. Yang, PLB 477 (2000) 107, PRD 61 (2000) 034017
B.-Q. Ma, J. Soffer, PRL 82 (1999) 2250

The Melosh-Wigner Rotation in Transversity

$$2\,\delta q = \langle\, p, \uparrow \,|\, \overline{q}_\lambda \gamma^\perp \gamma^+ q_{-\lambda} \,|\, p, \downarrow \,\rangle$$

$$\delta q(x) = \int \left[\mathrm{d}^2 k_\perp \right] \tilde{M}_q(x, k_\perp) \, \Delta q_{\mathrm{RF}}(x, k_\perp)$$

$$\tilde{M}_q(x, k_\perp) = \frac{(k^+ + m)^2}{(k^+ + m)^2 + k_\perp^2}$$

I.Schmidt&J.Soffer, Phys.Lett.B 407 (1997) 331

B.-Q. Ma, I. Schmidt, J. Soffer, Phys.Lett. B 441 (1998) 461–467.

The Melosh-Wigner Rotation in Quark Orbital Angular Moment

$$\hat{L}_q = -i\left(k_1 \frac{\partial}{\partial k_2} - k_2 \frac{\partial}{\partial k_1} \right).$$

$$L_q(x) = \int [d^2 k_\perp] M_L(x, k_\perp) \Delta q_{QM}(x, k_\perp)$$

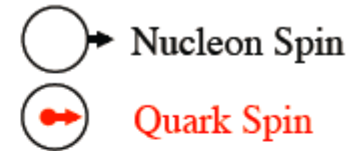$$M_L(x, k_\perp) = \frac{k_\perp^2}{(k^+ + m)^2 + k_\perp^2}$$

Spin and orbital sum in light-cone formalism

$$\frac{1}{2} M_q + M_L = \frac{1}{2}$$

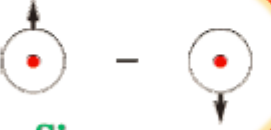$$M_q(x,k_\perp) = \frac{(k^+ + m)^2 - k_\perp^2}{(k^+ + m)^2 + k_\perp^2} \qquad M_L(x,k_\perp) = \frac{k_\perp^2}{(k^+ + m)^2 + k_\perp^2}$$

$$\frac{1}{2} \Delta q(x) + L_q(x) = \frac{1}{2} \Delta q_{QM}(x)$$

Ma&Schmidt, Phys.Rev.D 58 (1998) 096008

Leading-Twist TMD PDFs

Nucleon Spin
Quark Spin

		Quark polarization		
		Unpolarized (U)	**Longitudinally Polarized (L)**	**Transversely Polarized (T)**
Nucleon Polarization	U	f_1		$h_1^\perp$ **Boer-Mulders**
	L		g_1 **Helicity**	$h_{1L}^\perp$ **Long-Transversity**
	T	$f_{1T}^\perp$ **Sivers**	g_{1T} **Trans-Helicity**	h_1 **Transversity** $h_{1T}^\perp$ **Pretzelosity**

The Melosh-Wigner Rotation in "Pretzelosity"

$$g_1^q(x, k_\perp) - h_1^q(x, k_\perp) = h_{1T}^{\perp (1) q}(x, k_\perp) \ .$$

$$\frac{(k^+ + m)^2 - \mathbf{k}_\perp^2}{(k^+ + m)^2 + \mathbf{k}_\perp^2} - \frac{(k^+ + m)^2}{(k^+ + m)^2 + \mathbf{k}_\perp^2} = -\frac{\mathbf{k}_\perp^2}{(k^+ + m)^2 + \mathbf{k}_\perp^2}$$

$$\text{Pretzelosity} = \Delta q - \delta q = -L_q$$

$$\text{Pretzelosity} = -\int [d^2 \mathbf{k}_\perp] \frac{\mathbf{k}_\perp^2}{(k^+ + m)^2 + \mathbf{k}_\perp^2} \Delta q_{QM}(x, \mathbf{k}_\perp)$$

J.She, J.Zhu, B.-Q.Ma, Phys.Rev.D79 (2009) 054008

New Sum Rule of Physical Observables

$$g_1^q(x, k_\perp) - h_1^q(x, k_\perp) = h_{1T}^{\perp (1) q}(x, k_\perp) \; .$$

$$\frac{(k^+ + m)^2 - \mathbf{k}_\perp^2}{(k^+ + m)^2 + \mathbf{k}_\perp^2} - \frac{(k^+ + m)^2}{(k^+ + m)^2 + \mathbf{k}_\perp^2} = -\frac{\mathbf{k}_\perp^2}{(k^+ + m)^2 + \mathbf{k}_\perp^2}$$

$$\text{Pretzelosity} = \Delta q - \delta q = -L_q$$

$$\text{Pretzelosity} = -\int [d^2 \mathbf{k}_\perp] \frac{\mathbf{k}_\perp^2}{(k^+ + m)^2 + \mathbf{k}_\perp^2} \Delta q_{QM}(x, \mathbf{k}_\perp)$$

The Melosh-Wigner Rotation in five 3dPDFs

分布函数	Melosh转动因子 $(W_D(D = V, S))$
g_{1L}	$[(x\mathscr{M}_D + m_q)^2 - p_\perp^2]/[(x\mathscr{M}_D + m_q)^2 + p_\perp^2]$
g_{1T}	$2M_N(x\mathscr{M}_D + m_q)/[(x\mathscr{M}_D + m_q)^2 + p_\perp^2]$
h_1	$(x\mathscr{M}_D + m_q)^2/[(x\mathscr{M}_D + m_q)^2 + p_\perp^2]$
$h_{1L}^\perp$	$-2M_N(x\mathscr{M}_D + m_q)/[(x\mathscr{M}_D + m_q)^2 + p_\perp^2]$
$h_{1T}^\perp$	$-2M_N^2/[(x\mathscr{M}_D + m_q)^2 + p_\perp^2]$

$\mathscr{M}_D^2 = \frac{m_q^2 + p_\perp^2}{x} + \frac{m_D^2 + p_\perp^2}{1-x}$ 是旁观双夸克的不变质量。

Names for New (tmd) PDF: g_{1T} and $h_{1L}^{\perp}$

g_{1T} trans-helicity   横纵度

$h_{1L}^{\perp}$ longi-transversity / heli-transversity   纵横度

Proposal for measuring new transverse momentum dependent parton distributions g_{1T} and $h_{1L}^{\perp}$ through semi-inclusive deep inelastic scattering

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The Necessity of Polarized p pbar Collider

The polarized proton antiproton Drell-Yan process

is ideal to measure

the pretzelosity distributions of the nucleon.

Probing the leading-twist transverse-momentum-dependent parton distribution function $h_{1T}^{\perp}$ via the polarized proton-antiproton Drell-Yan process

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Probing Pretzelosity in pion p Drell-Yan Process

COMPASS pion p Drell-Yan process

can also measure

the pretzelosity distributions of the nucleon.

Single spin asymmetry in πp Drell–Yan process

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THE EUROPEAN PHYSICAL JOURNAL C

Azimuthal asymmetries in lepton-pair production at a fixed-target experiment using the LHC beams (AFTER)

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unpolarized and single polarized pp and pd processes. We conclude that it is feasible to measure these azimuthal asymmetries, consequently the three-dimensional or transverse momentum dependent parton distribution functions (3dPDFs or TMDs), at this new AFTER facility.

Quark Wigner distributions in a light-cone spectator model

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We investigate the quark Wigner distributions in a light-cone spectator model. The Wigner distribution, as a quasidistribution function, provides the most general one-parton information in a hadron. Combining the polarization configurations, unpolarized, longitudinal polarized, or transversal polarized, of the quark and the proton, we can define 16 independent Wigner distributions at leading twist. We calculate all these Wigner distributions for the u quark and the d quark, respectively. In our calculation, both the scalar and the axial-vector spectators are included, and the Melosh–Wigner rotation effects for both the quark and the axial-vector spectator are taken into account. The results provide us a very rich picture of the quark structure in the proton.

Three QCD spin sums for the proton spin

$$\vec{J}_{QCD} = \int d^3x \psi^\dagger \frac{\vec{\Sigma}}{2} \psi + \int d^3x \psi^\dagger \vec{x} \times (-i\nabla)\psi$$
$$+ \int d^3x \vec{E}^a \times \vec{A}^a + \int d^3x E^{ai} \vec{x} \times \nabla A^{ai}$$
$$\equiv \vec{S}_q + \vec{L}_q + \vec{S}_g + \vec{L}_g,$$

$$\vec{J}_{QCD} = \int d^3x \psi^\dagger \frac{\vec{\Sigma}}{2} \psi + \int d^3x \psi^\dagger \vec{x} \times (-i\vec{D})\psi + \int d^3x \vec{x} \times (\vec{E} \times \vec{B})$$
$$\equiv \vec{S}_q + \vec{L}_q + \vec{J}_g,$$

$$\vec{J}_{QCD} = \int d^3x \psi^\dagger \frac{\vec{\Sigma}}{2} \psi + \int d^3x \psi^\dagger \vec{x} \times (-i\vec{D}_{pure})\psi$$
$$+ \int d^3x \vec{E}^a \times \vec{A}^a_{phys} + \int d^3x E^{ai} \vec{x} \times \nabla A^{ai}_{phys}$$
$$\equiv \vec{S}_q + \vec{L}_q + \vec{S}_g + \vec{L}_g,$$

Angular momentum of quarks and gluons from generalized form factors

$$\vec{J}_q = \int d^3x\, \psi^\dagger [\vec{\gamma}\gamma_5 + \vec{x} \times (-i\vec{D})]\psi$$

$$\vec{J}_g = \int d^3x\, [\vec{x} \times (\vec{E} \times \vec{B})]$$

$$\langle P'|T^{\mu\nu}_{q,g}|P\rangle = \overline{U}(P')\,[A_{q,g}(\Delta^2)\gamma^{(\mu}\overline{P}^{\nu)} + B_{q,g}(\Delta^2)\overline{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M + C_{q,g}(\Delta^2)(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2)/M + \overline{C}_{q,g}(\Delta^2)g^{\mu\nu}M]\,U(P)\,,$$
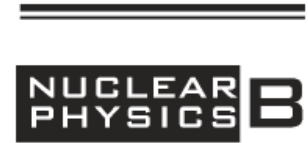
$$J_{q,g} = \frac{1}{2}[A_{q,g}(0) + B_{q,g}(0)]\,.$$

X. Ji, PRL78(1997)611

Angular momentum of quarks and gluons on the light-cone

Light-cone representation of the spin and orbital angular momentum of relativistic composite systems ☆

Stanley J. Brodsky [a,*], Dae Sung Hwang [b], Bo-Qiang Ma [c,d,e], Ivan Schmidt [f]

Angular momenta of quarks and gluons on the light-cone

$$\langle J^i \rangle = \frac{1}{2}\epsilon^{ijk} \int \mathrm{d}^3 x \left\langle T^{0k}x^j - T^{0j}x^k \right\rangle$$

$$= A(0)\langle L^i \rangle + \left[A(0) + B(0) \right] \bar{u}(P)\frac{1}{2}\sigma^i u(P).$$

$$\langle J^z \rangle = \left\langle \frac{1}{2}\sigma^z \right\rangle \left[A(0) + B(0) \right].$$

One can define individual quark and gluon contributions to the total angular momentum from the matrix elements of the energy–momentum tensor [9]. However, this definition is only formal; $A_{q,g}(0)$ can be interpreted as the light-cone momentum fraction carried by the quarks or gluons $\langle x_{q,g} \rangle$. The contributions from $B_{q,g}(0)$ to J_z cancel in the sum. In fact, we shall show that the contributions to $B(0)$ vanish when summed over the constituents of each individual Fock state.

S.J. Brodsky, D.S. Hwang, B.-Q. Ma, I. Schmidt, Nucl. Phys. B 593 (2001) 311

Sum rules of quarks and gluons on the light-cone

$$A_f(0) + A_b(0) = F_1(0) = 1,$$

which corresponds to the momentum sum rule.

$$B(0) = B_f(0) + B_b(0) = 0,$$

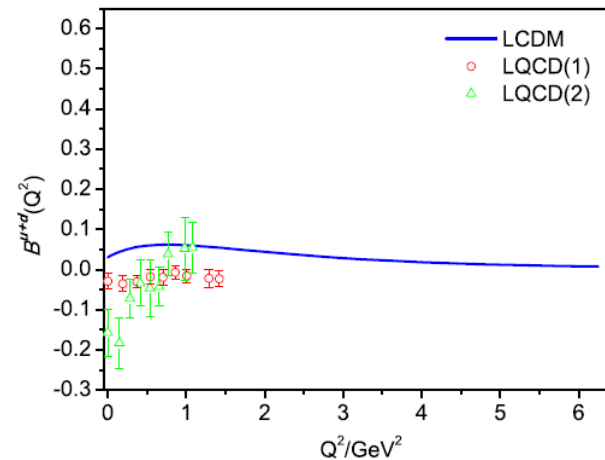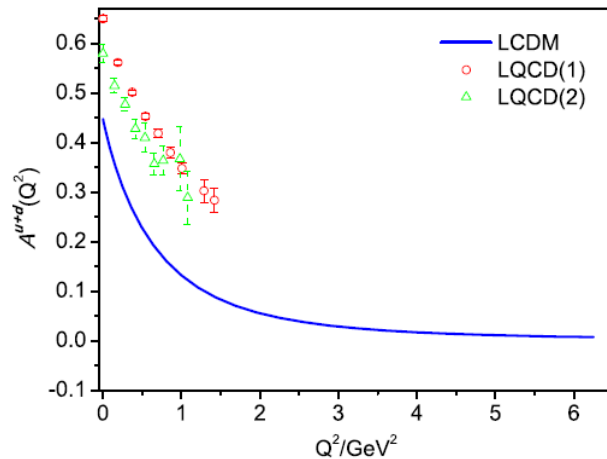which is another example of the vanishing of the anomalous gravitomagnetic moment.

A and B are called gravitational form factors

S.J. Brodsky, D.S. Hwang, B.-Q. Ma, I. Schmidt, Nucl. Phys. B 593 (2001) 311

Generalized form factors of the nucleon in a light-cone spectator-diquark model

Tianbo Liu[1] and Bo-Qiang Ma[1,2,3,*]

- **We start from a quark model with total angular momentum from quarks, but we don't have a correct sum of angular momenta from generalized form factors.**

- **The definition of quark angular momentum as from generalized form factors is artificial.**

Arbitrary in defining angular momenta: what is C?

$$\langle J^z \rangle = \left\langle \frac{1}{2}\sigma^z \right\rangle \left[A(0) + B(0) \right].$$

$$J_{q/g} = \frac{1}{2}[A_{q/g}(0) + B_{q/g}(0)] \pm C$$

as $\quad B(0) = B_{\mathrm{f}}(0) + B_{\mathrm{b}}(0) = 0,$

so $\quad \langle J^z \rangle = \left\langle \frac{1}{2}\sigma^z \right\rangle \left[A(0) + \text{❌} 0) \right].$

so $\quad J_{q,g} = \frac{1}{2}[A_{q,g}(0) + \text{❌}_g(0)].$

But A(0) is the momentum fraction, not angular momentum

A simple QED system as thought experiment

Angular momentum decomposition from a QED example

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We investigate the angular momentum decomposition with a quantum electrodynamics example to clarify the proton spin decomposition debates. We adopt the light-front formalism where the parton model is well defined. We prove that the sum of fermion and boson angular momenta is equal to half the sum of the two gravitational form factors $A(0)$ and $B(0)$, as is well known. However, the suggestion to make a separation of the above relation into the fermion and boson pieces, as a way to measure the orbital angular momentum of fermions or bosons, respectively, is not justified from our explicit calculation.

A simple QED system as thought experiment: an electron

$$\psi^{\uparrow}_{\uparrow +} = \frac{k^1 - ik^2}{x(1-x)} \phi(x, \vec{k}_{\perp}),$$

$$\psi^{\uparrow}_{\uparrow -} = -\frac{k^1 + ik^2}{1-x} \phi(x, \vec{k}_{\perp}),$$

$$\psi^{\uparrow}_{\downarrow +} = \frac{1-x}{x} m \phi(x, \vec{k}_{\perp}),$$

$$\psi^{\uparrow}_{\downarrow -} = 0,$$

$$\phi(x, \vec{k}_{\perp}) = -\frac{\sqrt{2}e}{\sqrt{1-x}} \frac{x(1-x)}{\vec{k}^2_{\perp} + (1-x)^2 m^2 + x\lambda^2}$$

A simple QED system as thought experiment: an electron

$$S_f = \frac{1}{2} - \frac{e^2}{16\pi^2},$$

$$L_f = -\frac{e^2}{12\pi^2\varepsilon} + \frac{e^2}{12\pi^2},$$

$$S_b = \frac{3e^2}{16\pi^2\varepsilon} - \frac{e^2}{8\pi^2},$$

$$L_b = -\frac{5e^2}{48\pi^2\varepsilon} + \frac{5e^2}{48\pi^2},$$

$$L = -\frac{3e^2}{16\pi^2\varepsilon} + \frac{3e^2}{16\pi^2}.$$

$$S_f + S_b + L_f + L_b = \frac{1}{2}.$$

A simple QED system as thought experiment: an electron

$$A_f(0) = 1 - \frac{e^2}{6\pi^2\varepsilon} + \frac{e^2}{8\pi^2}, \qquad\qquad A_b(0) = \frac{e^2}{6\pi^2\varepsilon} - \frac{e^2}{8\pi^2},$$

$$B_f(0) = \frac{e^2}{12\pi^2}, \qquad\qquad\qquad B_b(0) = -\frac{e^2}{12\pi^2}.$$

$$A_f(0) + A_b(0) = 1,$$

$$B_f(0) + B_b(0) = 0,$$

$$\frac{1}{2}[A(0) + B(0)] = S + L = \frac{1}{2},$$

A simple QED system as thought experiment: an electron

$$\frac{1}{2}\left[A_f(0) + B_f(0)\right] \neq S_f + L_f,$$

$$\frac{1}{2}\left[A_b(0) + B_b(0)\right] \neq S_b + L_b.$$

Therefore $J_{q,g} = \frac{1}{2}[A_{q,g}(0) + B_{q,g}(0)]$ is unjustified.

Spectator diquark model calculation

Quark angular momentum in a spectator model

CrossMark

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We cannot identify the canonical angular momentums with half the sum of gravitational form factors:

$$\frac{1}{2}\left[A_q^S(0) + B_q^S(0)\right] = 0.356 \neq J_q^S,$$

$$\frac{1}{2}\left[A_q^V(0) + B_q^V(0)\right] = -0.038 \neq J_q^V.$$

T.Liu and B.-Q. Ma, Phys.Lett.B741 (2015) 256, arXiv:1501.00062

The Melosh-Wigner rotation
is not the whole story

- **The role of sea is not addressed**

- **The role of gluon is not addressed**

  Gluons are hidden in the spectators in our quark-diquark model

  It is important to study the roles played by the sea quarks and gluons. Thus more theoretical and experimental researches can provide us a more completed picture of the nucleon spin structure.

Conclusions

- **The relativistic effect of parton transversal motions plays an significant role in spin-dependent quantities: helicity and transversity, five 3dPDFs or TMDs, GPDs, the Wigner distributions.**

- **It is still challenging to measure the quark orbital angular momentum:**

1. **The pretzelosity with quark transversal motions is an important quantity for the spin-orbital correlation of the nucleon**

2. **The sum rule between helicity and transversity pdfs can serve as an estimate of quark orbital angular momentum.**

- **It is necessary to push forward theoretical explorations and experimental measurements of new quantities of the nucleon.**