Physics with the energy-momentum tensor

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Contents:

- T^{ab} and nucleon structure
- T^{ab} and spin
- T^{ab} and quantum measurement
- T^{ab} and “far” beyond the Standard Model

T^{ab} and nucleon structure:
more components, more interesting

$$\boxed{\vec{P}_{\text{total}}} = \int d^3x \psi^+ \frac{1}{i} \vec{D} \psi + \int d^3x \vec{E} \times \vec{B}$$

T^{0z} : momentum, ~ 50% by the gluon

T^{zz} : momentum flow, = ???

T^{z0} : energy flow, = ???

Even the momentum alone can be tricky!

$$\begin{cases} \vec{P}_q = \int d^3x \psi^+ \frac{1}{i} \vec{D} \psi \\ \vec{P}_g = \int d^3x \vec{E} \times \vec{B} \end{cases} \qquad Q^2 \frac{d}{dQ^2} \begin{pmatrix} P_q \\ P_g \end{pmatrix} = \frac{\alpha_s}{2\pi} \begin{pmatrix} -\frac{2n_g}{9} & \frac{n_f}{3} \\ \frac{2n_g}{9} & -\frac{n_f}{3} \end{pmatrix} \begin{pmatrix} P_q \\ P_g \end{pmatrix}$$

$$\begin{cases} \hat{\vec{P}}_q = \int d^3x \psi^+ \frac{1}{i} \vec{D}_{\text{pure}} \psi \\ \vec{P}_g = \int d^3x E^i \vec{D}_{\text{pure}} A^i_{\text{phys}} \end{cases} \qquad Q^2 \frac{d}{dQ^2} \begin{pmatrix} P_q \\ P_g \end{pmatrix} = \frac{\alpha_s}{2\pi} \begin{pmatrix} -\frac{n_g}{18} & \frac{n_f}{3} \\ \frac{n_g}{18} & -\frac{n_f}{3} \end{pmatrix} \begin{pmatrix} P_q \\ P_g \end{pmatrix}$$

$$P_g \to \frac{2n_g}{2n_g + 3n_f} P_N \sim \frac{1}{2} P_N (n_f = 5) \text{ vs } P_g \to \frac{n_g}{n_g + 6n_f} P_N \sim \frac{1}{5} P_N (n_f = 5)$$

New T^{ab} can give new pictures!

T^{ab} and Spin: $S_q + L_q + S_g + L_g = 1/2$

Jaffe-Manohar [NPB337:509 (1990)]

$$\vec{J}_{\text{total}} = \int d^3x \psi^+ \frac{1}{2} \vec{\Sigma} \psi + \int d^3x \vec{x} \times \psi^+ \frac{1}{i} \vec{\nabla} \psi + \int d^3x \vec{E} \times \vec{A} + \int d^3x \vec{x} \times E^i \vec{\nabla} A^i$$

Ji [PRL78:610 (1997)], Chen-Wang [CTP27:212 (1997)]

$$\vec{J}_{\text{total}} = \int d^3x \psi^+ \frac{1}{2} \vec{\Sigma} \psi + \int d^3x \vec{x} \times \psi^+ \frac{1}{i} \vec{D} \psi + \int d^3x \vec{x} \times \left(\vec{E} \times \vec{B} \right)$$

Chen-Lü-Sun-Wang-Goldman [PRL100:232002 (2008); 103:062001 (2009)]

$$\vec{J}_{\text{total}} = \int d^3x \psi^+ \frac{1}{2} \vec{\Sigma} \psi + \int d^3x \vec{x} \times \psi^+ \frac{1}{i} \vec{D}_{\text{pure}} \psi + \int d^3x \vec{E} \times \vec{A}_{\text{phys}} + \int d^3x \vec{x} \times E^i \vec{\mathrm{D}}_{\text{pure}} A^i_{\text{phys}}$$

The Key issue： $T^{ab} = T^{ba}$ or $T^{ab} \neq T^{ba}$

Symmetric

$T^{ab} = T^{ba}$:

$$\vec{r} \times \vec{P} = \vec{J}$$

$$M^{\lambda\mu\nu} = x^{\mu} T^{\nu\lambda} - x^{\nu} T^{\mu\lambda}, \quad \partial_{\lambda} M^{\lambda\mu\nu} = 0.$$

$$Example: \vec{J} = \vec{r} \times (\vec{E} \times \vec{B})?$$

Canonical

$T^{ab} \neq T^{ba}$:

$$\vec{r} \times \vec{P} = \vec{L}, \quad \vec{J} = \vec{L} + \vec{S}$$

Primitive physics with T^{ab}

$$\vec{F} = G\frac{m_{\rm G} M_{\rm G}}{r^2}\hat{\vec{r}} \;\Rightarrow\; R^{\mu\nu} - \frac{1}{2}Rg^{\mu\nu} = -8\pi G T_{\rm G}^{\mu\nu}$$

$$\vec{F} = m_{\rm I}\vec{a} \;\Rightarrow\; f^{\mu} = \begin{cases} \dfrac{dp_{\rm I}^{\mu}}{d\tau} & \text{for a particle} \\ \partial_{\nu} T_{\rm I}^{\nu\mu} & \text{for a field} \end{cases}$$

Awkwardness： Symmetry and gauge variance

Canonical: $p_i = \dfrac{\partial L(q_i, \dot{q}_i)}{\partial \dot{q}_i}, \quad H = p_i \dot{q}_i - L$

By Nöther's theorem: $T_C^{\mu\nu}(x) = \dfrac{\partial L(\phi_i, \partial_\mu \phi_i)}{\partial(\partial_\mu \phi_i)} \partial^\nu \phi_i - g^{\mu\nu} L$

Symmetric: $\Theta^{\mu\nu}(x) = \dfrac{1}{\sqrt{-g}} \dfrac{\delta I_M}{\delta g_{\mu\nu}(x)}$ (needed by Einstein)

$$T^{\mu\nu}_{symm} = F^{\mu\rho} F_\rho{}^\nu + \frac{1}{4} g^{\mu\nu} F^2 \longrightarrow \vec{P} = \vec{K} = \vec{E} \times \vec{B}$$

$$T^{\mu\nu}_{cano} = -F^{\mu\rho} \partial^\nu A_\rho + \frac{1}{4} g^{\mu\nu} F^2 \longrightarrow \vec{P} = E^i \vec{\partial} A^i, \vec{K} \to \vec{E} \times \vec{B}$$

Conserved T^{ab} and its "arbitrariness"

$$\partial_\mu T^{\mu\nu} = 0 \Rightarrow P^\nu \equiv \int T^{0\nu}\, dV$$

$$\tilde{T}^{\mu\nu} = T^{\mu\nu} + \partial_\rho B^{[\rho\mu]\nu} \Rightarrow \partial_\mu \tilde{T}^{\mu\nu} = 0$$

$$\tilde{P}^\nu \equiv \int \tilde{T}^{0\nu}\, dV = \int (T^{0\nu} + \partial_\rho B^{[\rho 0]\nu})\, dV = P^\nu$$

1. **Different T^{ab}, same conservation laws and conserved 4-momentum**
2. **Inertial T^{ab} is unfixed:** $f^\mu = \partial_\nu T_{\mathrm{I}}^{\nu\mu} = \partial_\nu \tilde{T}_{\mathrm{I}}^{\nu\mu}$

Constrains on T^{ab} from quantum measurement (new!)

If a quantum wave is in mutual eigenstates of more than one observables, then the associated currents must be proportional to each other

$$E.g.: \vec{j}_E \propto \vec{j}_{p_i} \propto \vec{j}_{s_i} \propto \vec{j}_q$$

$$\hat{H}\psi = E\psi, \hat{P}_i\psi = p_i\psi, \hat{S}_i\psi = s_i\psi, \hat{Q}\psi = q\psi$$

The Symmetric T^{ab} stands no chance!

$$j^\mu = e\bar{\psi}\gamma^\mu\psi$$

$$T^{\mu\nu}_{symm} = \frac{i}{4}\bar{\psi}(\gamma^\mu\partial^\nu + \gamma^\nu\partial^\mu)\psi + h.c. \quad T^{\mu\nu}_{cano} = \frac{i}{2}\bar{\psi}\gamma^\mu\partial^\nu\psi + h.c.$$

$$P^i = T^{0i}_{cano} = \frac{i}{2}\bar{\psi}\gamma^0\partial^i\psi + h.c. \quad K^i = T^{i0}_{cano} = \frac{i}{2}\bar{\psi}\gamma^i\partial^0\psi + h.c. \approx \frac{\mathcal{E}}{e}j^i$$

But the canonical T^{ab} is not fully OK

$$T_C^{\mu\nu}(x) = \frac{\partial L(\phi_a, \partial_\mu\phi_a)}{\partial(\partial_\mu\phi_a)}\partial^\nu\phi_a - g^{\mu\nu}L$$

$$T_C^{i0} = \frac{\partial L(\phi_a, \partial_\mu\phi_a)}{\partial(\partial_i\phi_a)}\partial^0\phi_a, \quad T_C^{ii}(x) = \frac{\partial L(\phi_a, \partial_\mu\phi_a)}{\partial(\partial_i\phi_a)}\partial^i\phi_a + L$$

An improved T^{ab}: free field

$$T^{\mu\nu}_{revised} = \frac{\partial L(\phi_a, \partial_\mu \phi_a)}{\partial(\partial_\mu \phi_a)} \overleftrightarrow{\partial}^\nu \phi_a, \quad \overleftrightarrow{\partial}^\nu \equiv \frac{1}{2}\left(\overrightarrow{\partial}^\nu - \overleftarrow{\partial}^\nu\right)$$

$$T^{\mu\nu}_{C} = \frac{\partial L(\phi_a, \partial_\mu \phi_a)}{\partial(\partial_\mu \phi_a)} \partial^\nu \phi_a - g^{\mu\nu} L$$

E.g. photon

$$T^{\mu\nu}_{revised} = -F^{\mu\rho} \overleftrightarrow{\partial}^\nu A_\rho$$

$$T^{\mu\nu}_{C} = -F^{\mu\rho} \partial^\nu A_\rho + \frac{1}{4} g^{\mu\nu} F^2$$

$$T^{\mu\nu}_{symm} = -F^{\mu\rho} F^\nu{}_\rho + \frac{1}{4} g^{\mu\nu} F^2$$

Proof of validness (general free fields)

$L(\phi_a, \partial_\mu \phi_a)$ is quadratic in and $\partial_\mu \phi$ (ϕ, ϕ^* independent)

$$= \sum_a \frac{1}{2}\left(\frac{\partial L}{\partial \phi_a}\phi_a + \frac{\partial L}{\partial(\partial_\rho \phi_a)}\partial_\rho \phi_a \right) = \sum_a \frac{1}{2}\partial_\rho \left(\frac{\partial L}{\partial(\partial_\rho \phi_a)}\phi_a \right)$$

$$T_C^{\mu\nu} = \frac{\partial L}{\partial(\partial_\mu \phi_a)}\partial^\nu \phi_a - g^{\mu\nu} L, \quad T_{revised}^{\mu\nu} = T_C^{\mu\nu} + \partial_\rho B^{[\rho\mu]\nu}$$

$$B^{[\rho\mu]\nu} = \sum_a \frac{1}{2}\partial_\rho \left(g^{\mu\nu} \frac{\partial L}{\partial(\partial_\rho \phi_a)}\phi_a - g^{\rho\nu} \frac{\partial L}{\partial(\partial_\mu \phi_a)}\phi_a \right)$$

Proof of validness (particular free fields)

$$L=\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-\frac{1}{2}m^2\phi^2, \quad (\partial_\mu\partial^\mu+m^2)\phi=0$$

$$L=\frac{1}{2}\partial_\mu(\phi\partial^\mu\phi)-\frac{1}{2}\phi\partial_\mu\partial^\mu\phi\frac{1}{2}m^2\phi^2=\frac{1}{2}\partial_\mu(\phi\partial^\mu\phi)$$

$$L=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \quad \partial_\mu F^{\mu\nu}=0$$

$$L=-\frac{1}{4}(\partial_\mu A_\nu-\partial_\nu A_\mu)F^{\mu\nu}$$

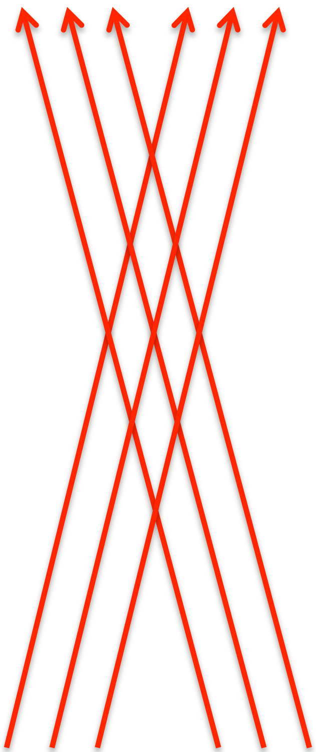
$$=-\frac{1}{4}\left[\partial_\mu(A_\nu F^{\mu\nu})-\partial_\nu(A_\mu F^{\mu\nu})\right]+\frac{1}{4}\left[A_\nu\partial_\mu F^{\mu\nu}-A_\mu\partial_\nu F^{\mu\nu}\right]$$

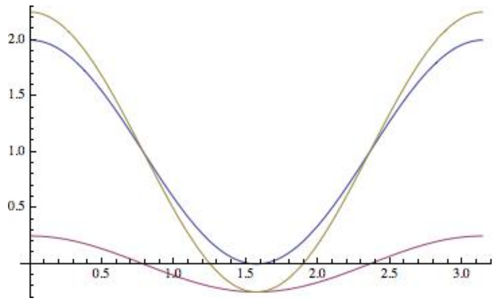
$$=-\frac{1}{2}\partial_\mu(A_\nu F^{\mu\nu})$$

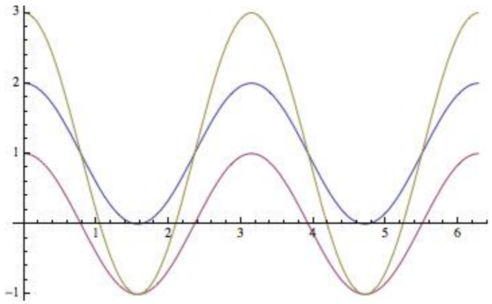
Experimental test -I： momentum flux (radiation pressure!)

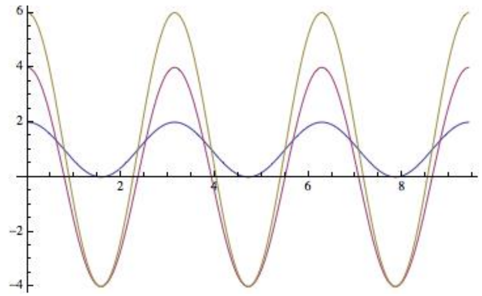
$$T^{\mu\nu}_{revised} = -F^{\mu\rho}\overleftrightarrow{\partial}^{\nu} A_{\rho}, T^{\mu\nu}_{C} = -F^{\mu\rho}\partial^{\nu} A_{\rho} + \frac{1}{4} g^{\mu\nu} F^2, T^{\mu\nu}_{symm} = -F^{\mu\rho} F^{\nu}{}_{\rho} + \frac{1}{4} g^{\mu\nu} F^2$$

$$\vec{A} = \vec{e}_y \left[\sin(k_x x + k_z z - \omega t) + \sin(-k_x x + k_z z - \omega t)\right] = \vec{e}_y \cos(k_x x)\sin(k_z z - \omega t)$$









$$\overline{T}^{z0}_{revised} = \overline{T}^{z0}_{C} = \overline{T}^{z0}_{symm} = k_z \omega [1 + \cos(2k_x x)]$$

$$\overline{T}^{zz}_{revised} = k_z^2 [1 + \cos(2k_x x)]$$

$$\overline{T}^{zz}_{C} = \overline{T}^{zz}_{symm} = k_z^2 [1 + \cos(2k_x x)] + k_x^2 \cos(2k_x x)$$

Observable Effects!

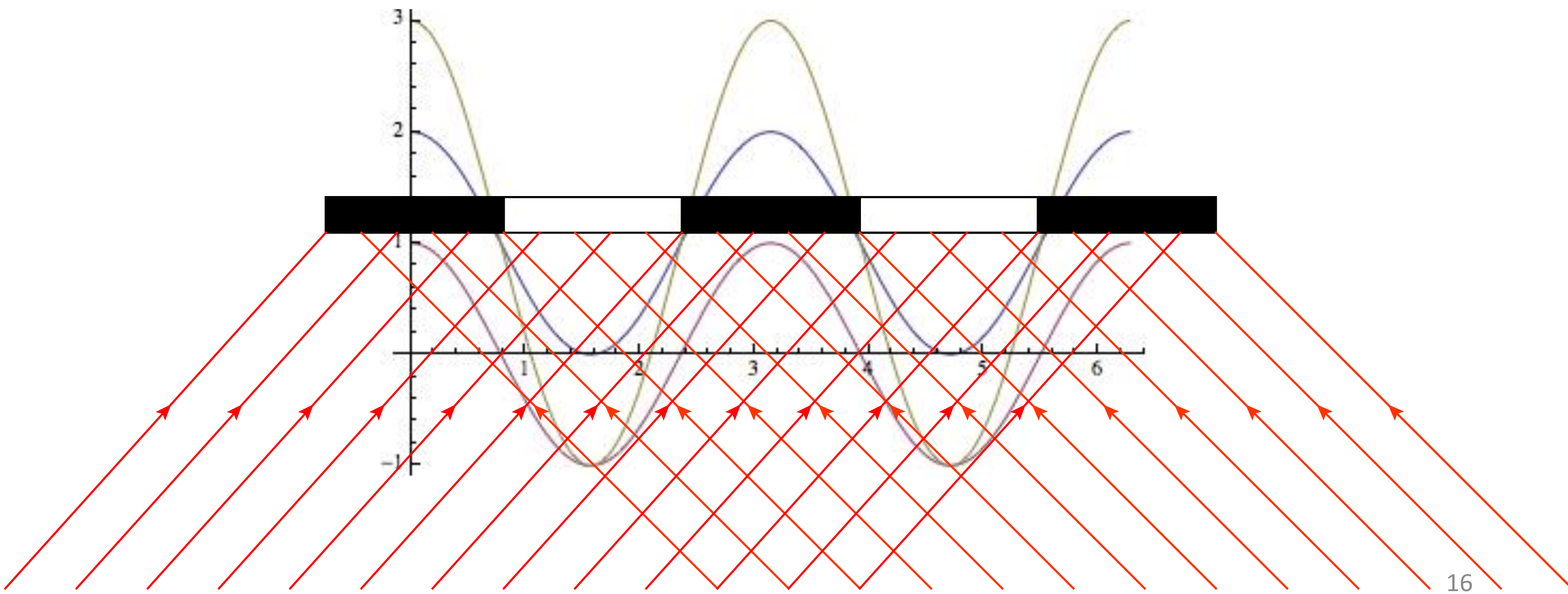
Backward flow

Schematic set up

$$\vec{A} = \vec{e}_y \left[\sin(k_x x + k_z z - \omega t) + \sin(-k_x x + k_z z - \omega t) \right] = \vec{e}_y \cos(k_x x) \sin(k_z z - \omega t)$$

$$\bar{T}^{z0} = k_z \omega [1 + \cos(2k_x x)], \quad \bar{T}^{zz}_{revised} = k_z^2 [1 + \cos(2k_x x)]$$

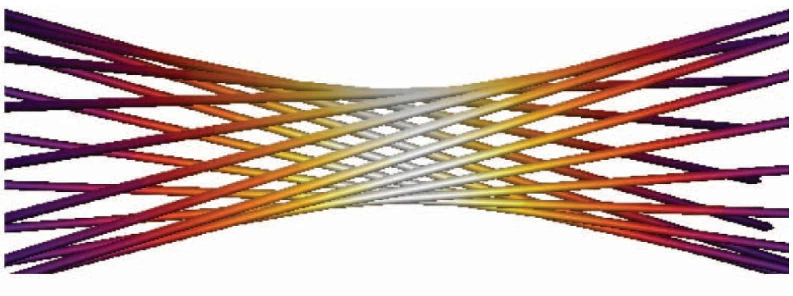
$$\bar{T}^{zz}_C = \bar{T}^{zz}_{symm} = k_z^2 [1 + \cos(2k_x x)] + k_x^2 \cos(2k_x x)$$

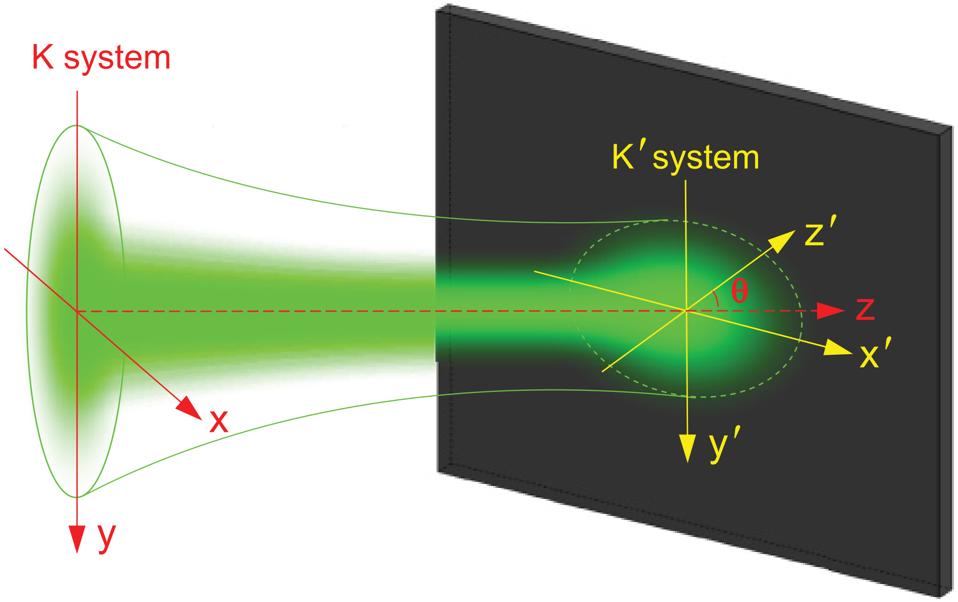


Experimental test -II： Energy flux

Spiral energy flux for spin

Geometric spin hall effect





$$\langle y\rangle|_{z=0} = \lambda(\sigma/2)\tan\theta + O(\theta_0)^2,$$

PRL 103, 100401 (2009); (theory)
PRA 85, 035804 (2012) (experiment)

Photon energy flux does not tell two T^{ab}s

$$T^{\mu\nu}_{symm} = F^{\mu\rho}F_{\rho}{}^{\nu} + \frac{1}{4}g^{\mu\nu}F^2 \longrightarrow \vec{P} = \vec{K} = \vec{E}\times\vec{B}$$

$$T^{\mu\nu}_{cano} = -F^{\mu\rho}\partial^{\nu}A_{\rho} + \frac{1}{4}g^{\mu\nu}F^2 \longrightarrow \vec{P} = E^i\vec{\partial}A^i, \vec{K} \to \vec{E}\times\vec{B}$$

In symmetric T^{ab}, Poynting Vector is both momentum and energy flow

The canonical expression is gauge dependent

In radiation gauge, Poynting vector is energy flow, but not momenum

The Dirac Particle

$$j^\mu = e\bar{\psi}\gamma^\mu\psi$$

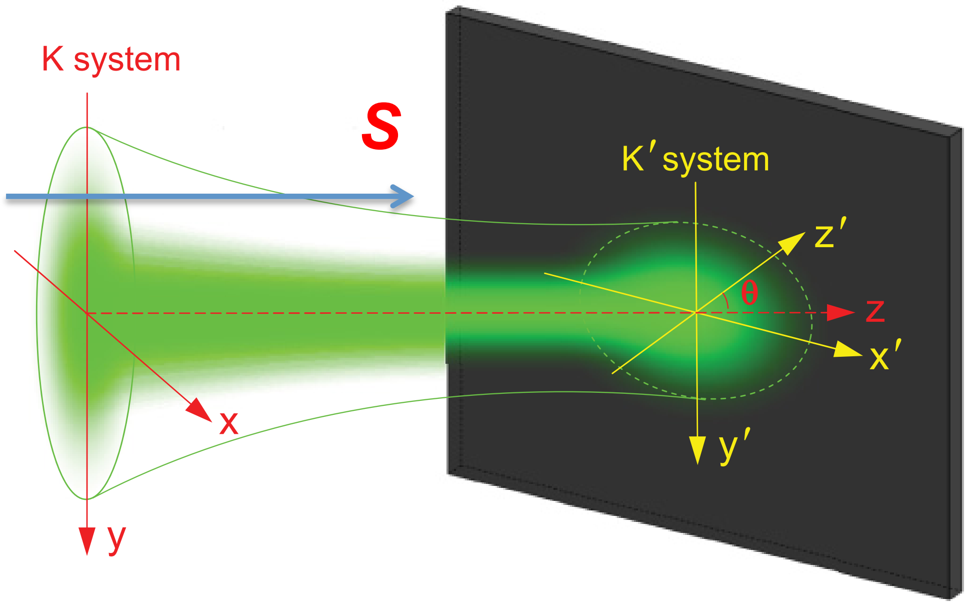
$$T^{\mu\nu}_{symm} = \frac{i}{4}\bar{\psi}(\gamma^\mu\partial^\nu + \gamma^\nu\partial^\mu)\psi + h.c. \quad T^{\mu\nu}_{cano} = \frac{i}{2}\bar{\psi}\gamma^\mu\partial^\nu\psi + h.c.$$

$$P^i = T^{0i}_{cano} = \frac{i}{2}\bar{\psi}\gamma^0\partial^i\psi + h.c. \quad K^i = T^{i0}_{cano} = \frac{i}{2}\bar{\psi}\gamma^i\partial^0\psi + h.c. \approx \frac{\mathcal{E}}{e}j^i$$

$$T^{\mu\nu}_{cano} = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_i)}\partial^\nu\phi_i - g^{\mu\nu}\mathcal{L}$$

The two tensors give totally different energy-flow

Geometric spin hall effect of the electron



Longitudinal polarization, tilted incidence

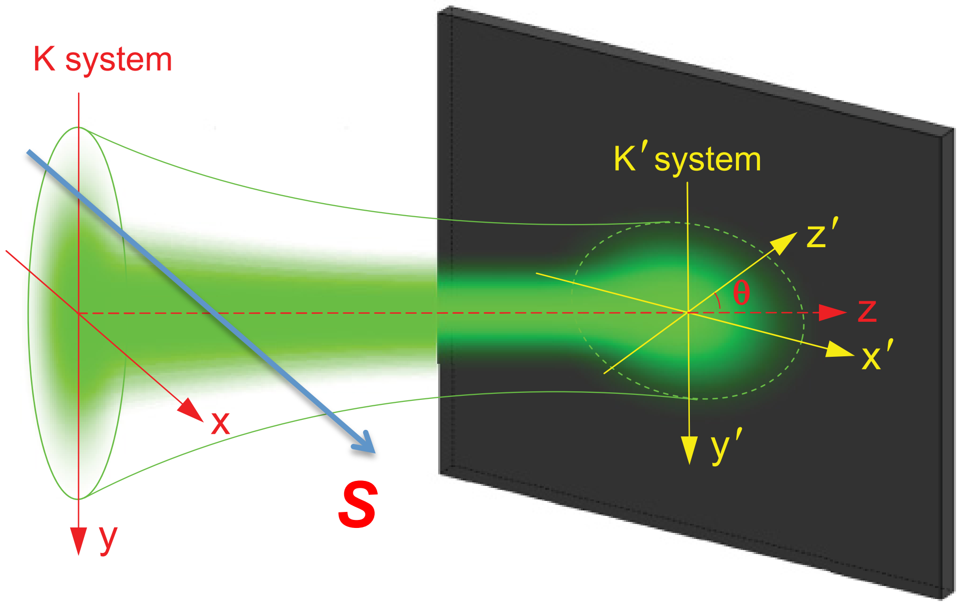
Prediction of the symmetric T^{ab}:

$$y|_{z=0} = \frac{\sigma}{2}\frac{\lambda}{2\pi}\tan\theta$$

Prediction of the canonical T^{ab}:

$$y|_{z=0} = \sigma\frac{\lambda}{2\pi}\tan\theta$$

Geometric spin hall effect of the electron



Non-relativistic, transverse polarization, right incidence

Prediction of the symmetric T^{ab}:

$$y\,\Big|_{z=0} = \frac{\sigma}{2}\,\frac{\lambda}{2\pi}$$

Prediction of the canonical T^{ab}:

$$y\,\Big|_{z=0} = \sigma\,\frac{\lambda}{2\pi}$$

Issue with the Angular momentum tensor

$$M^{\mu\nu\lambda} = x^\mu T^{\lambda\nu}_{symm} - x^\nu T^{\lambda\mu}_{symm}$$

$$M^{\mu\nu\lambda}_C = x^\mu T^{\lambda\nu}_C - x^\nu T^{\lambda\mu}_C + \frac{\partial L}{\partial(\partial_\lambda \phi_a)} \Sigma^{\mu\nu}_{ab} \phi_b$$

$$M^{\mu\nu\lambda}_{revised?} = x^\mu T^{\lambda\nu}_{revised} - x^\nu T^{\lambda\mu}_{revised} + \frac{\partial L}{\partial(\partial_\lambda \phi_a)} \Sigma^{\mu\nu}_{ab} \phi_b$$

$$+\frac{1}{2} g^{\lambda\nu} \frac{\partial L}{\partial(\partial_\mu \phi_a)} \phi_a - \frac{1}{2} g^{\lambda\mu} \frac{\partial L}{\partial(\partial_\nu \phi_a)} \phi_a$$

Our trick applies to longitudinal spin flux only, but not to transverse flux of angular momentum!

The interacting fields: scalar case

$$T^{\mu\nu}(x) = \frac{1}{\sqrt{-g}} \frac{\delta I_M}{\delta g_{\mu\nu}(x)}$$

$$I_\phi = \int d^4x \sqrt{-g} \left(\frac{-1}{2} D_\mu \phi D^\mu \phi + \frac{1}{8} R \phi^2 \right)$$

$$\rightarrow T^{\mu\nu}_{new} = \frac{\partial L(\phi_a, \partial_\mu \phi_a)}{\partial(\partial_\mu \phi_a)} \overleftrightarrow{\partial}^\nu \phi_a$$

This gives a gravitational theory different from Einstein's GR

T^{ab} and fundamental principles of physics

- **Gauge principle: gauge dependence of the Hamiltonian in external gauge and gravitational fields**

$$H_e = \vec{\alpha} \cdot \frac{1}{i}\vec{D}_e + M_e\beta + q_e A^0,$$

$$\vec{J}_e = \frac{1}{2}\vec{\Sigma} + \vec{x} \times \frac{1}{i}\vec{\partial}$$

- **Equivalence principle: source of gravity**

$$I_\phi = \int d^4x\sqrt{-g}\left(\frac{-1}{2}D_\mu\phi D^\mu\phi + \frac{1}{8}R\phi^2\right)$$

- **Reduction of a quantum wave** $|\psi\rangle = \sum C_i|\psi_i\rangle \to |C_i|^2|\psi_i\rangle$

Challenging on any of the above would go far beyond the standard Model

Summary and further studies

- **Expression of T^{ab} is not arbitrary. It can be tested experimentally!**
- **We are not even clear about the free fields**
- **A new expression of T^{ab} derived from quantum conservation laws**
- **Two experimental schemes: momentum flux and energy flux**
- **Interacting fields and M^{abc}**
- **Much work to do with hadron structure**

Thank you!

谢谢!