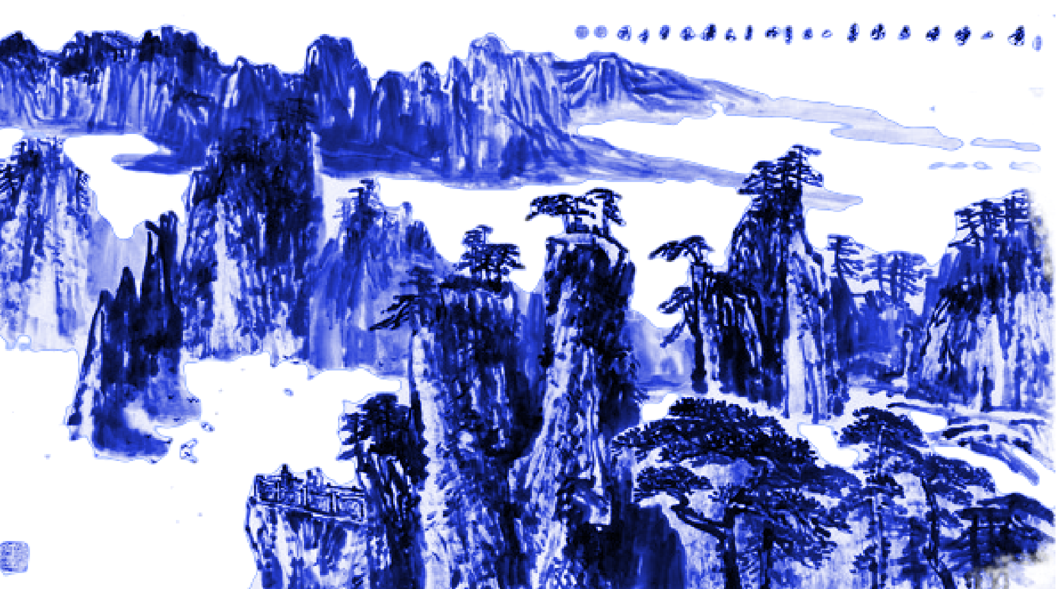
Sum rules in gamma-gamma physics
and its impact on
the hadronic LbL contribution to the $(g\text{-}2)_\mu$



Vladyslav Pauk

Johannes Gutenberg University

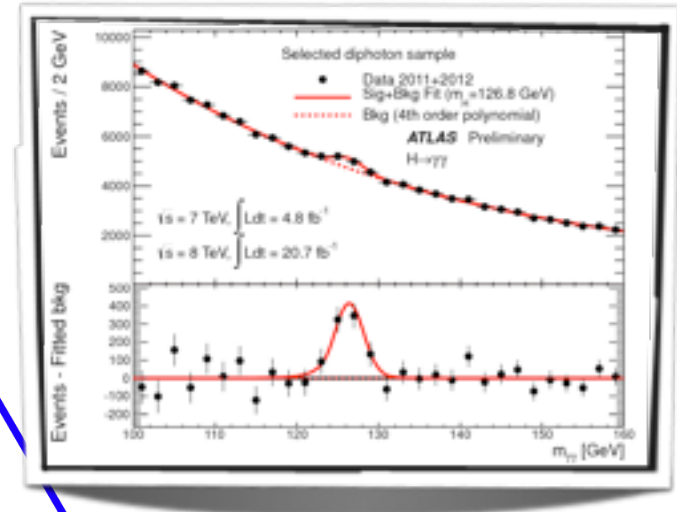
Mainz, Germany

July 3, 2013,
Huangshan

On the horizons..

On the horizons..

High-energy frontier



Low-energy frontier

Precision frontier

LHC
Higgs discovery
Production of new physics particles

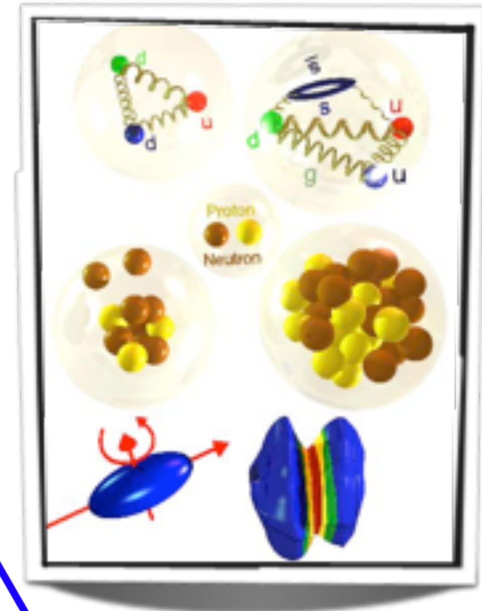
On the horizons..

Hadron physics: Strong interactions, Complex systems
How do quarks/gluons merge into hadrons?
How does hadron structure emerge from basic constituents?
Mass and spin of hadrons?

Low-energy frontier

Precision frontier

High-energy frontier



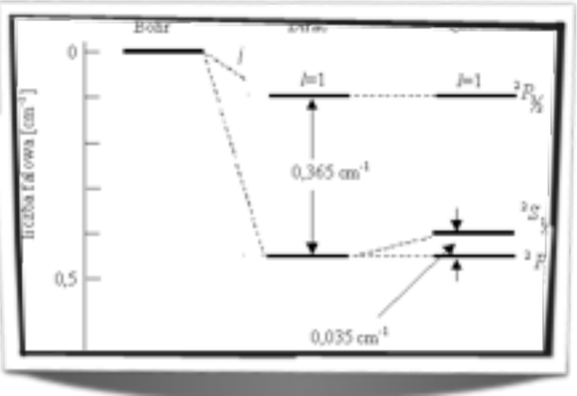
On the horizons..

Precision frontier

High-energy frontier

Low-energy frontier

Testing SM
at low energies,
quantum loop corrections
lamb shift, $(g-2)_\mu$, EDM
Flavor physics
Atomic physics



Outline

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- $(g-2)_\mu$: the crystal ball of particle physics

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- hadronic light-by-light scattering & sum rules

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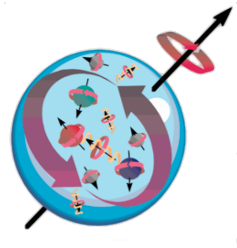
- hadronic light-by-light scattering & sum rules
- hadronic contribution to the $(g-2)_\mu$

The muon's anomalous magnetic moment

The muon's anomalous magnetic moment

The anomalous magnetic moment



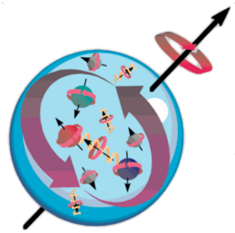
magnetic
dipole moment

$$\vec{\mu} = g\, Q\, \mu_0 \frac{\vec{\sigma}}{2}$$

μ_0: Bohr magneton

Q: charge

The anomalous magnetic moment



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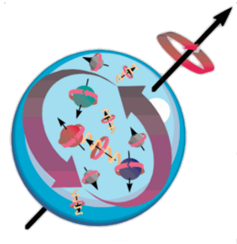
gyromagnetic factor g

The anomalous magnetic moment



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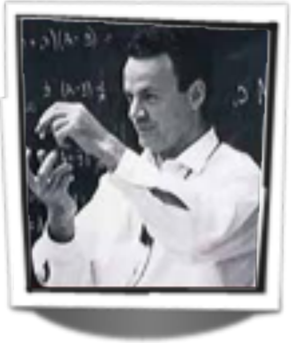
gyromagnetic factor

$$g = 2$$

Dirac theory (1928)

free electron



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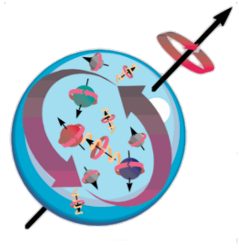

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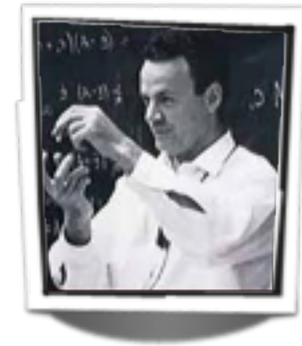

gyromagnetic factor

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quantum
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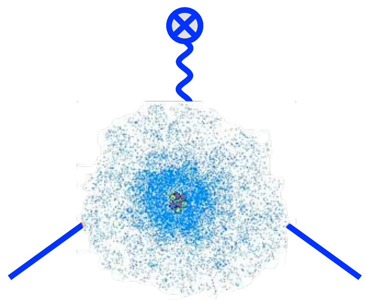
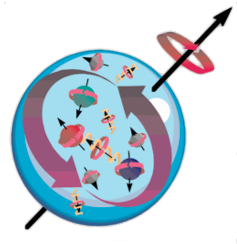


$$a_l \equiv \frac{g-2}{2}$$

anomalous
moment

The anomalous magnetic moment



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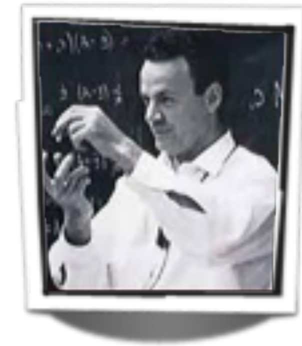

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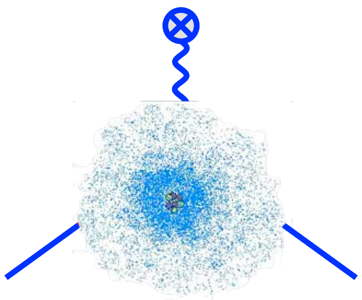

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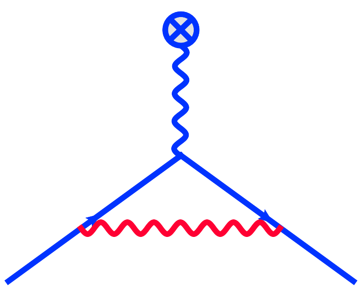
Schwinger (1948)



$a_\mu^{QED(1)} = \alpha_{em} \,/\, 2\pi = 0.001161$

The anomalous magnetic moment

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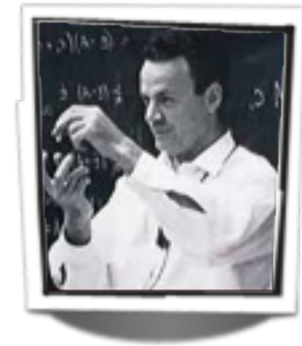
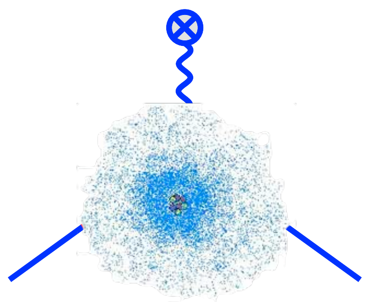


quantum corrections



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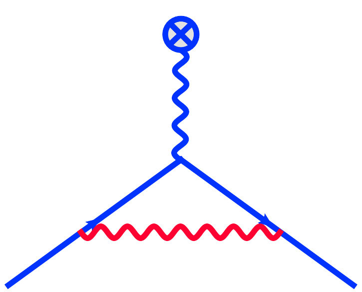
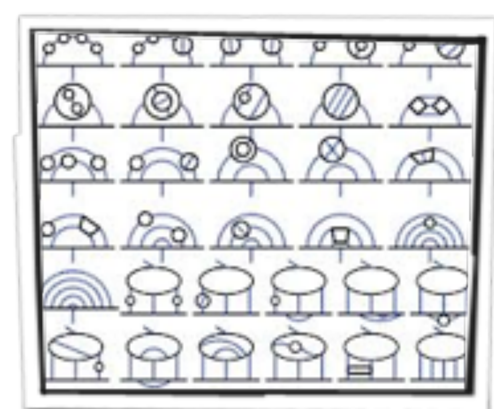


Kinoshita (2012)



$a_\mu^{QED(1)} = \alpha_{em} / 2\pi = 0.001161$

$a_\mu^{QED(5)} = (11\ 658\ 471.896 \pm 0.008)\cdot 10^{-10}$

up to α_{em}^5 !

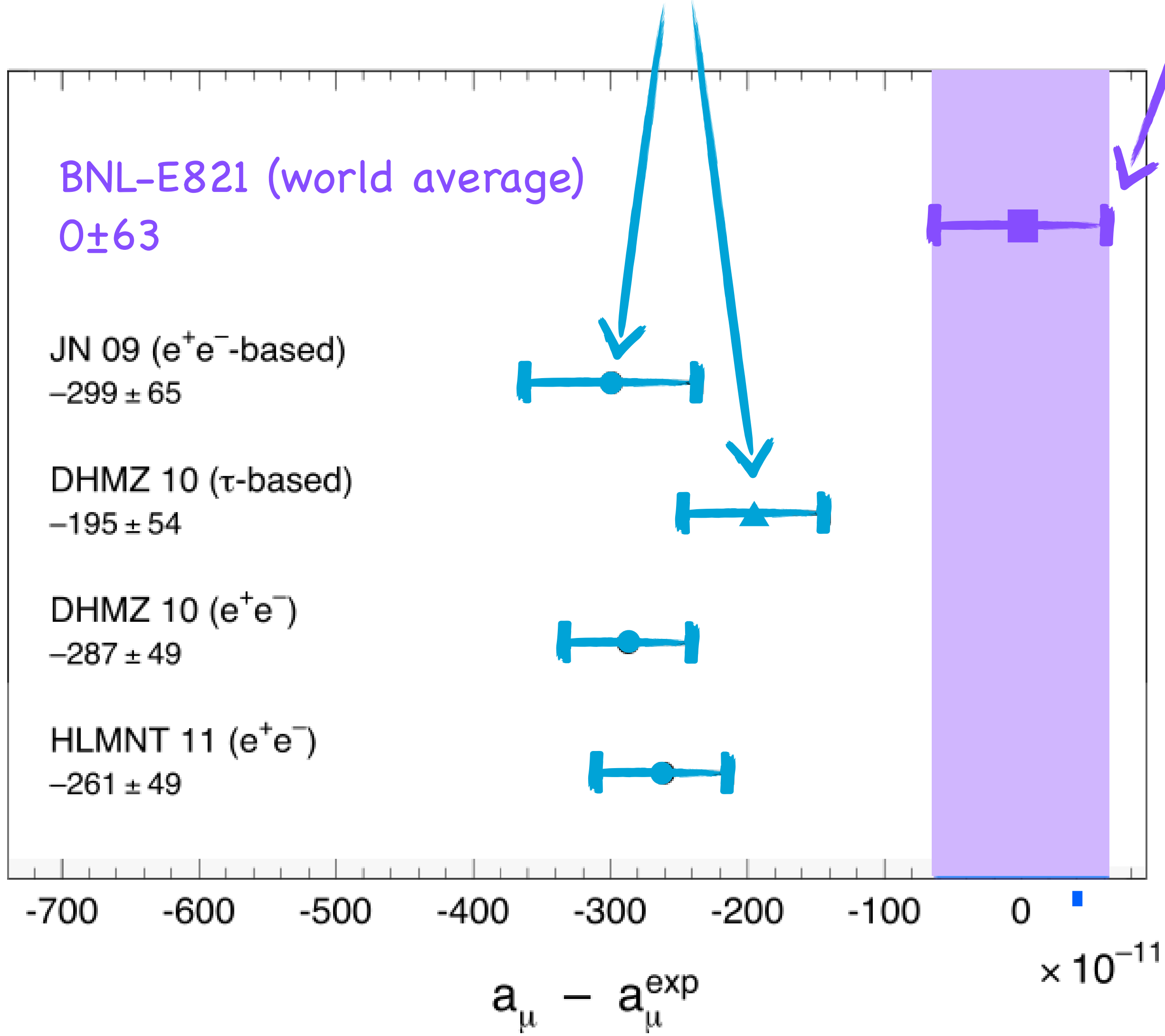
$(g-2)_\mu$: theory vs experiment

present theoretical SM value

$a_\mu^{SM} = (11\ 659\ 184.0 \pm 5.9) \times 10^{-10}$

E821 measurement of $(g-2)_\mu$ (2009)

$a_\mu^{exp} = (11\ 659\ 208.9 \pm 6.3) \times 10^{-10}$



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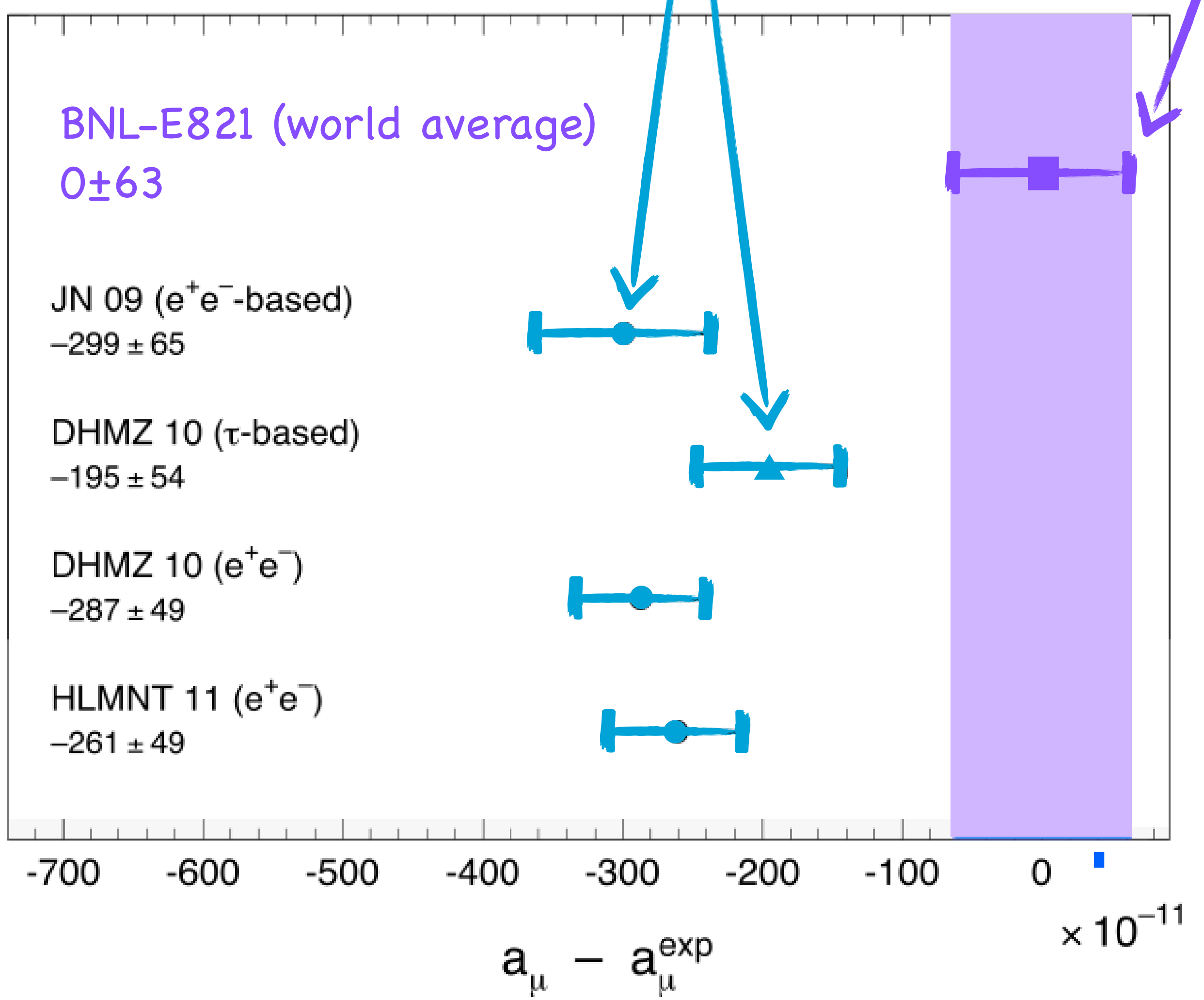

discrepancy between theory and experiment

$a_\mu^{exp} - a_\mu^{th} = (24.9 \pm 8.7) \times 10^{-10}$

(2.9σ)

error(s) or New Physics?

→ clarify situation!

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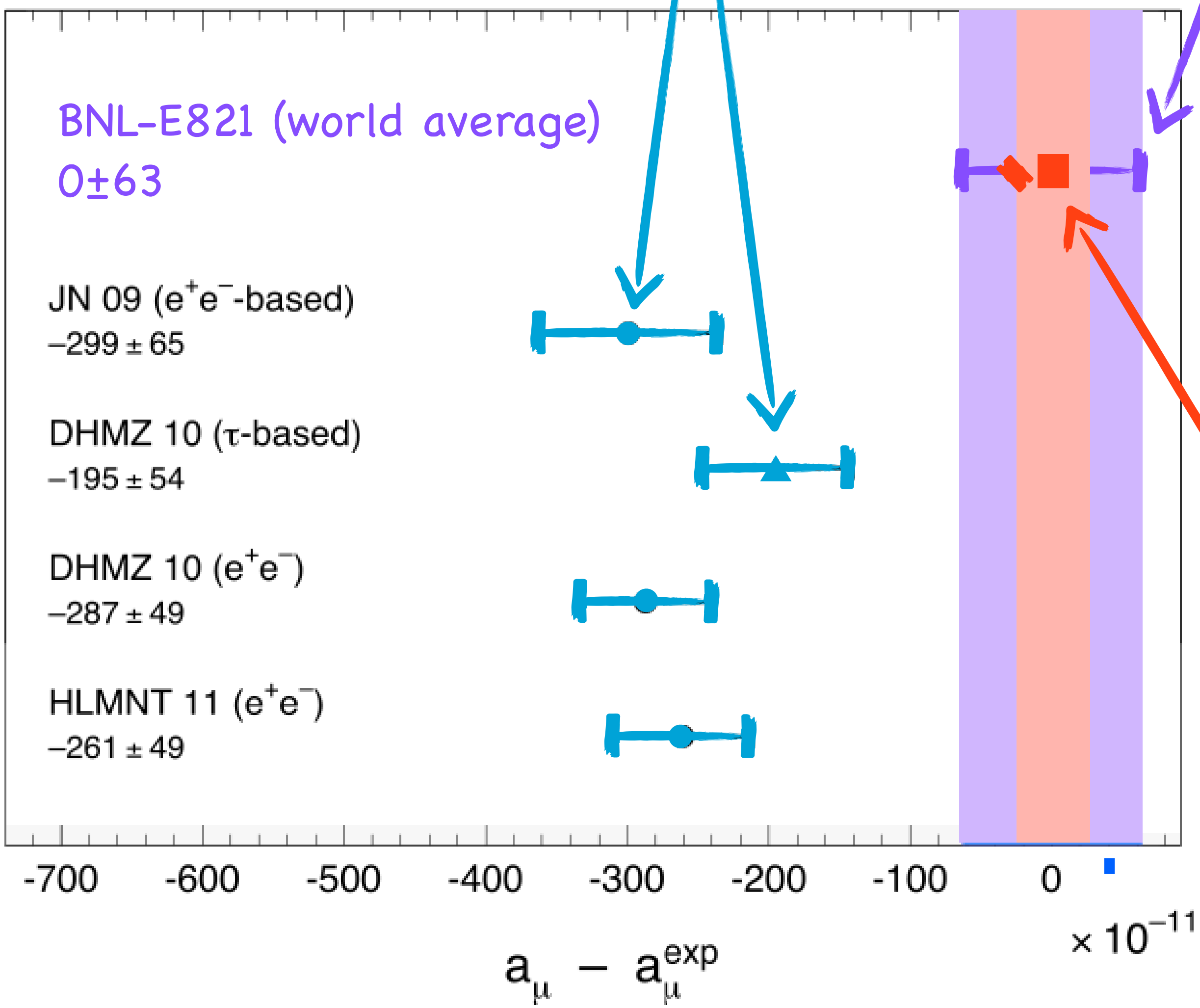

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error(s) or New Physics?
→ clarify situation!

new FNAL $(g-2)_\mu$ measurement (2015):

factor 4 improvement in experimental error
→ improve theory!

$\pm 1.6 \cdot 10^{-10}$

$(g\text{-}2)_\mu$ - the crystal ball of particle physics

testing SM with unprecedented accuracy:

a stringent constraints on new physics

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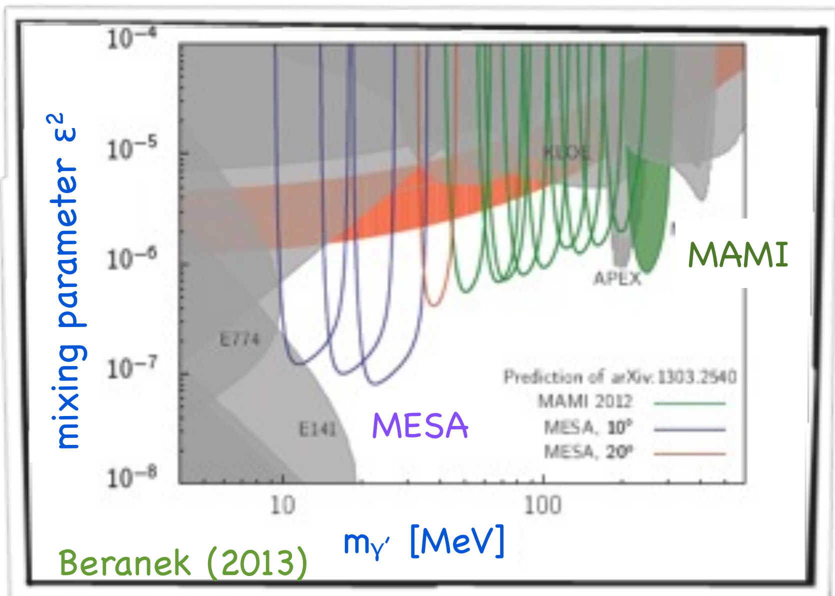
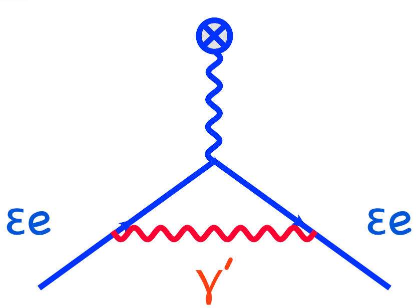


U(1) extension of the SM

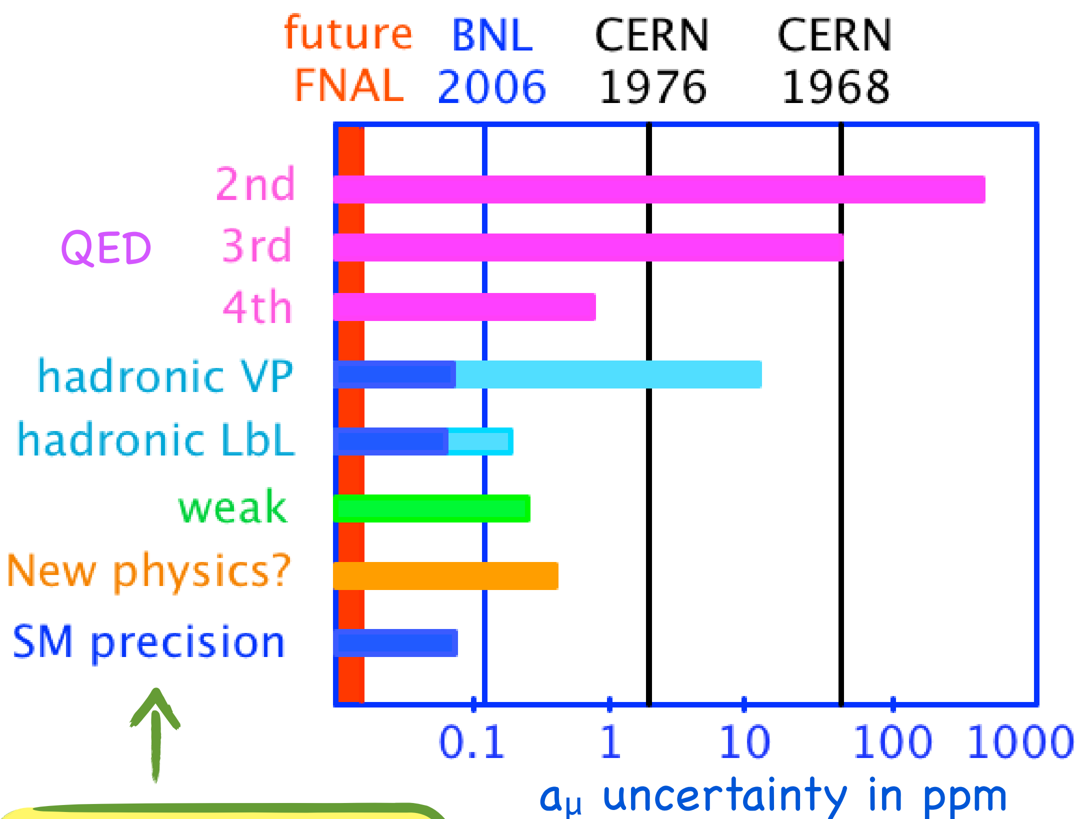


dark photon

$(g-2)_\mu$: SM predictions & uncertainties

sensitivity of $(g-2)_\mu$ experiments
to various corrections

experiments



theory corrections

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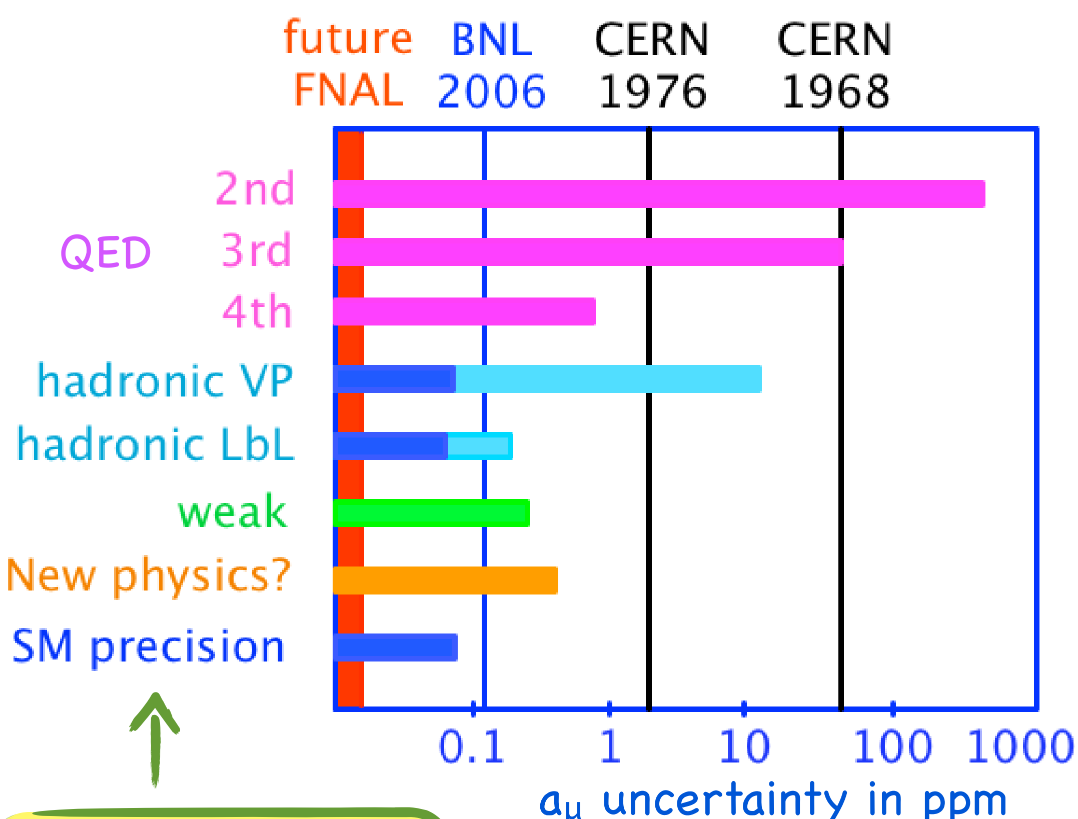
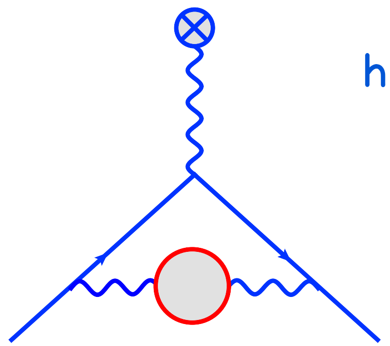


theory corrections

hadronic vacuum polarization (VP)



hadronic VP determined by cross section measurements of $e^+e^- \to$ hadrons

$a_\mu^{had, VP} = (692.3 \pm 4.2) \times 10^{-10}$

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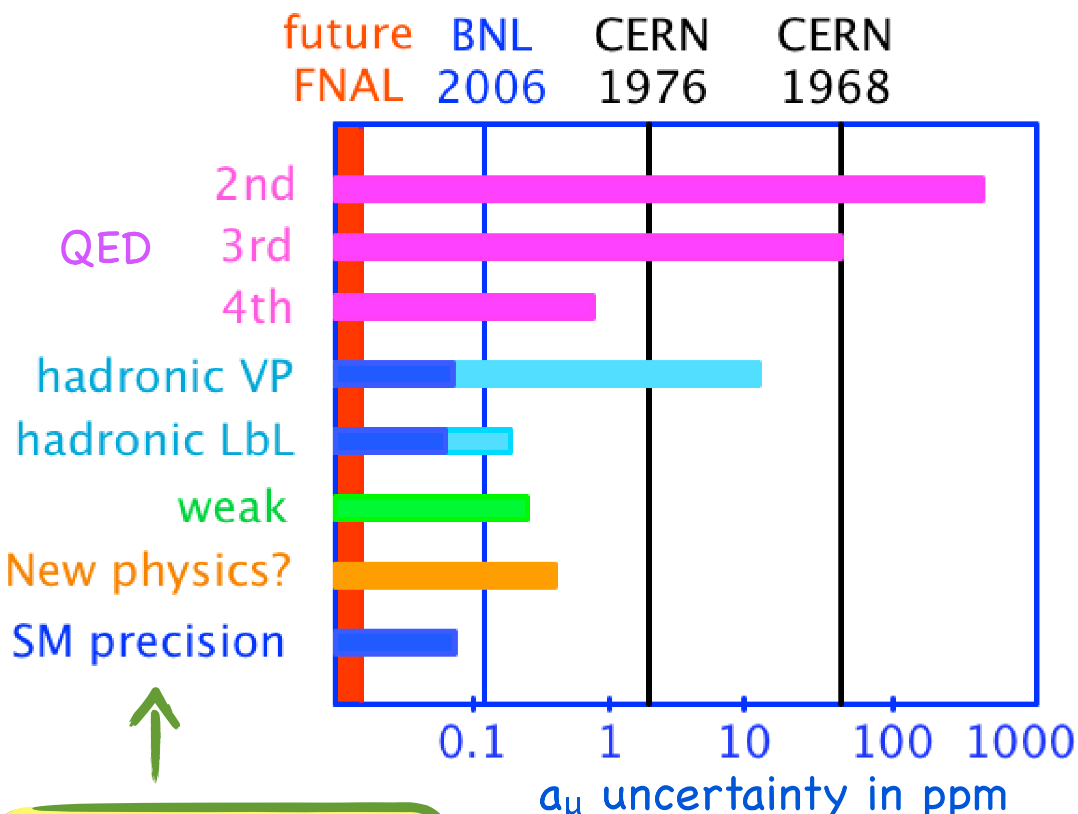

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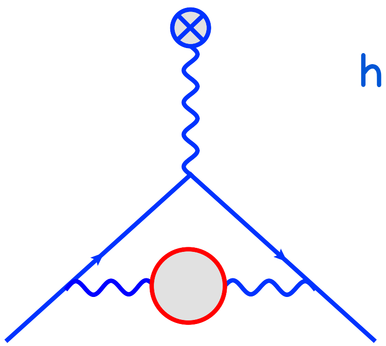

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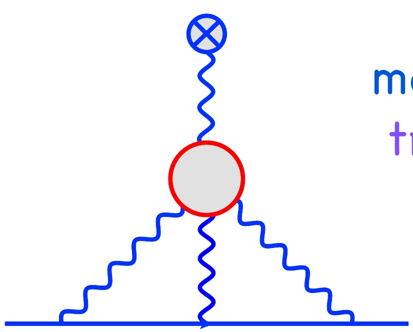
hadronic light-by-light scattering (LbL)

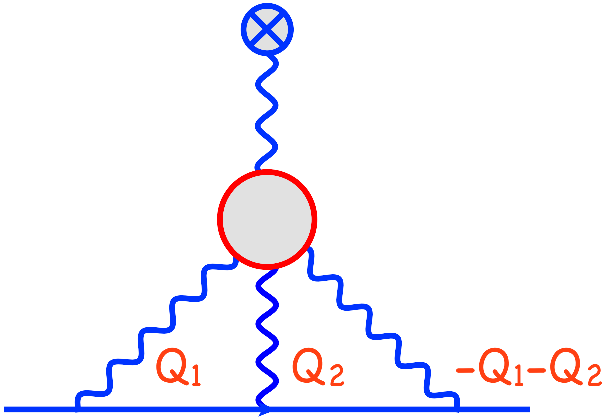


measurements of meson transition form factors required as input to reduce uncertainty

$$a_\mu^{\text{had, LbL}} = (11.6 \pm 4.0) \times 10^{-10}$$

Models of hadronic LbL scattering

hadronic LbL correction



Models of hadronic LbL scattering

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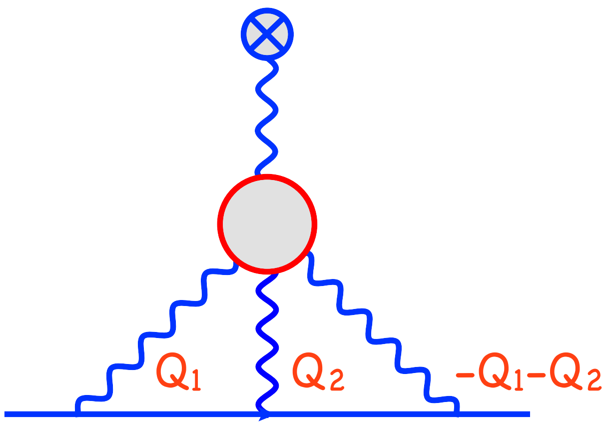
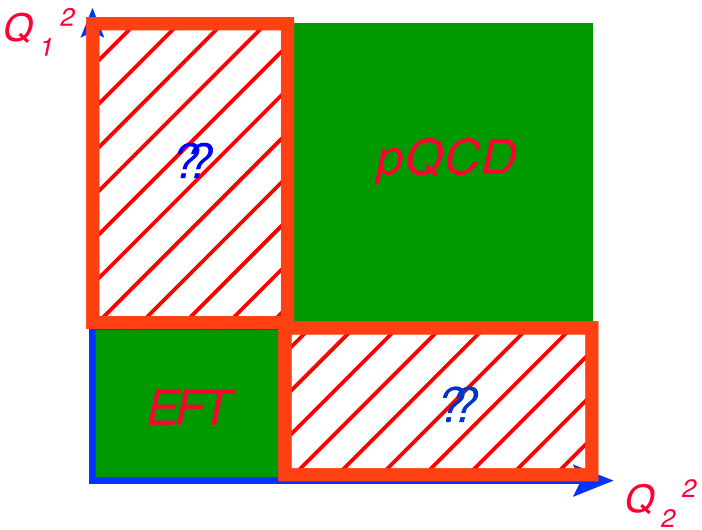
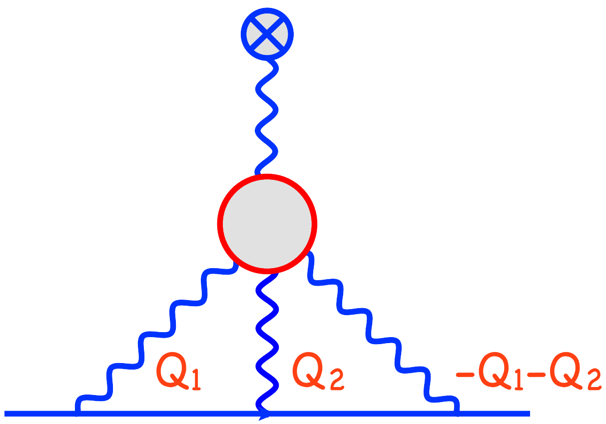


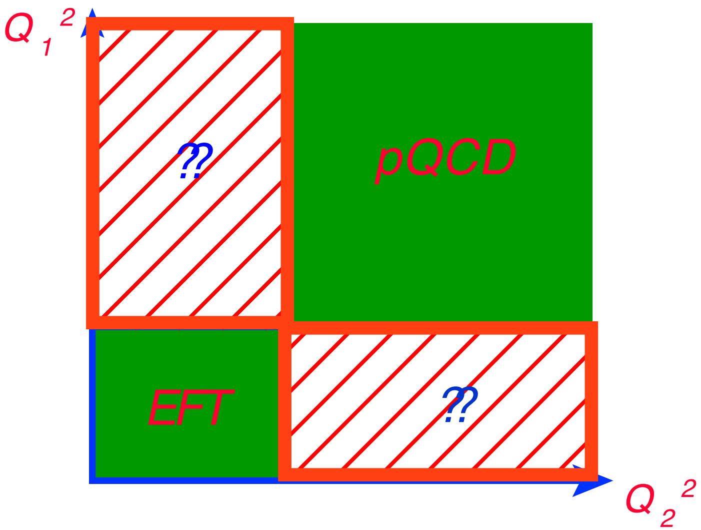


multi-scale problem -
mixed soft - hard regions

Models of hadronic LbL scattering

hadronic LbL correction





multi-scale problem –
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no answer is available in general, unless we have data!

Models of hadronic LbL scattering

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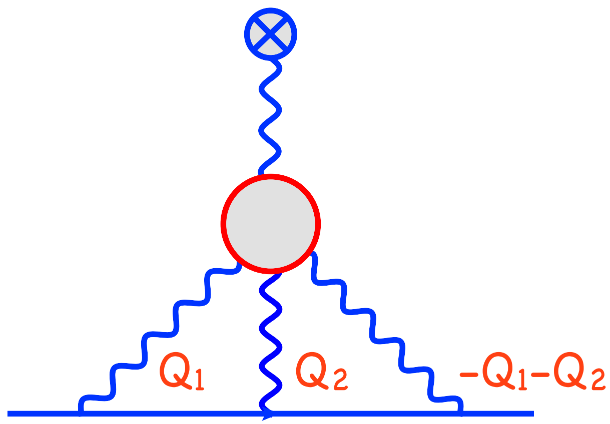

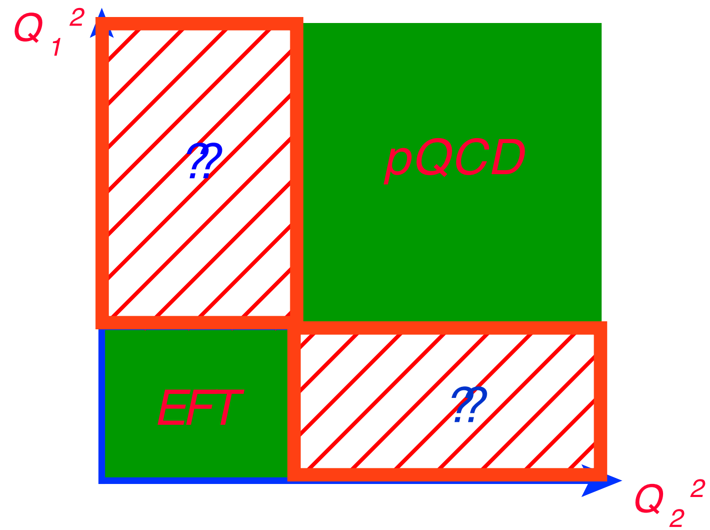

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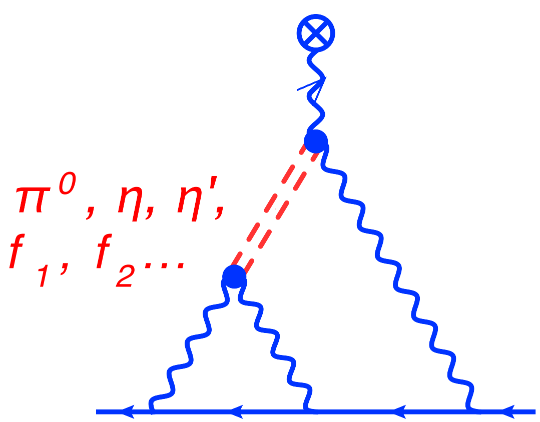

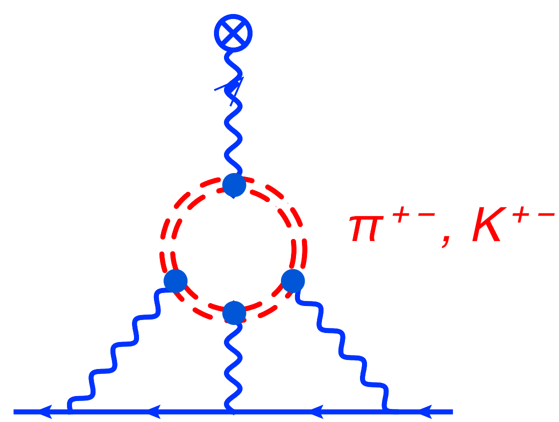

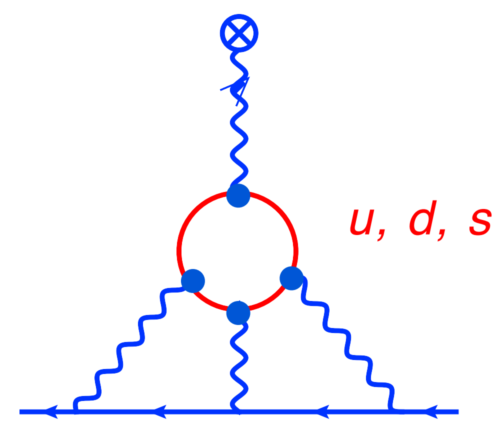

Models of hadronic LbL scattering

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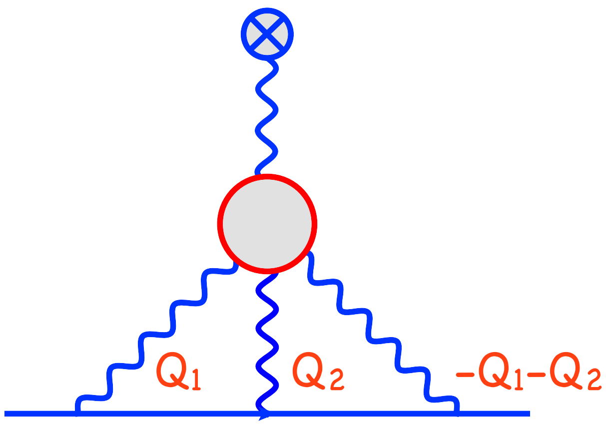

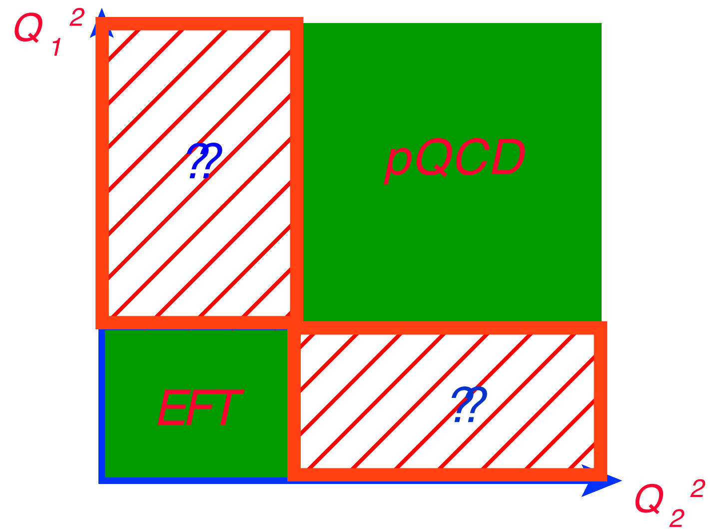

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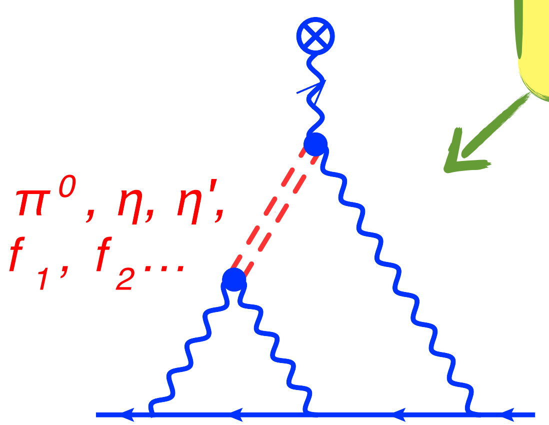
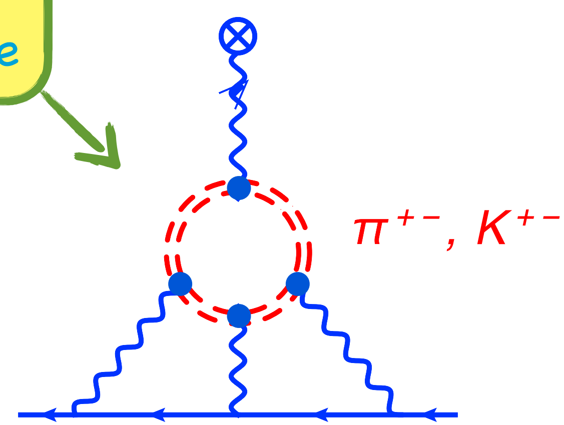
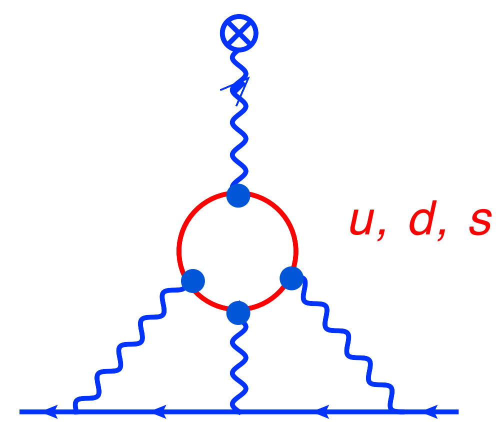
long distance



$1/N_c$ - expansion



chiral expansion



Models of hadronic LbL scattering

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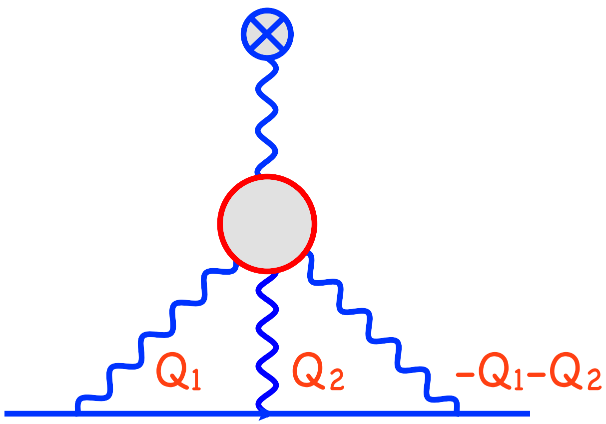

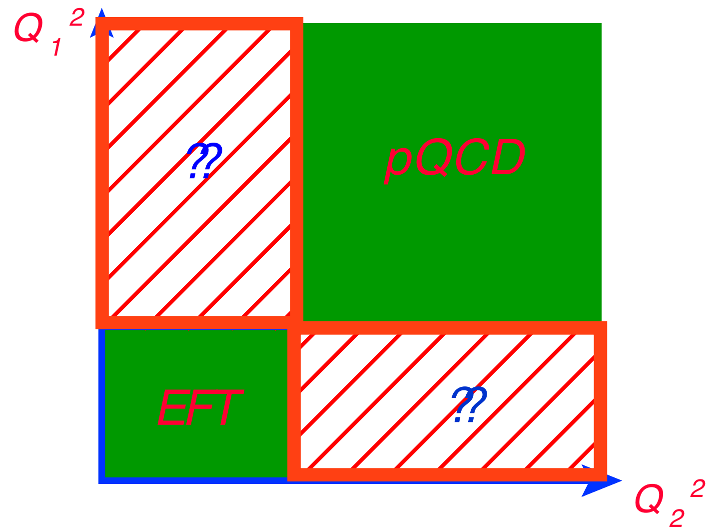

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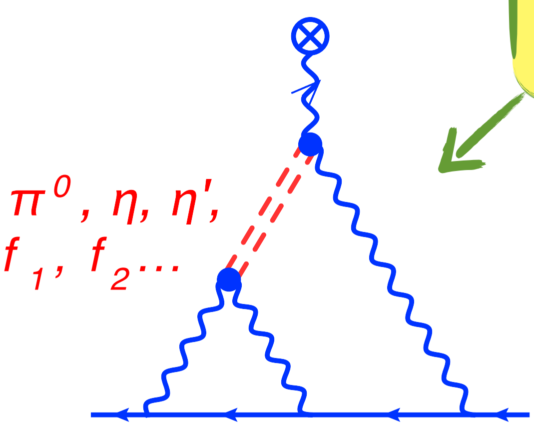
multi-scale problem -
mixed soft - hard regions

long distance

short distance



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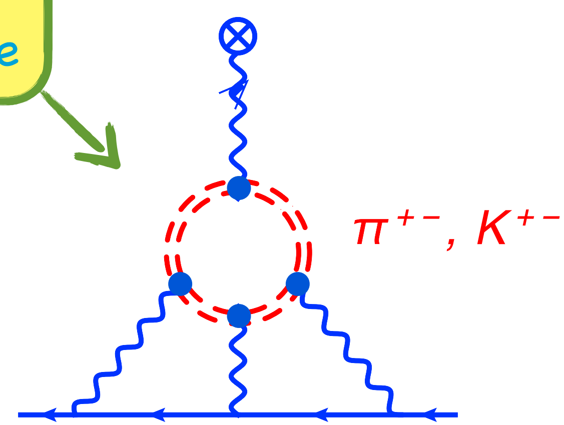

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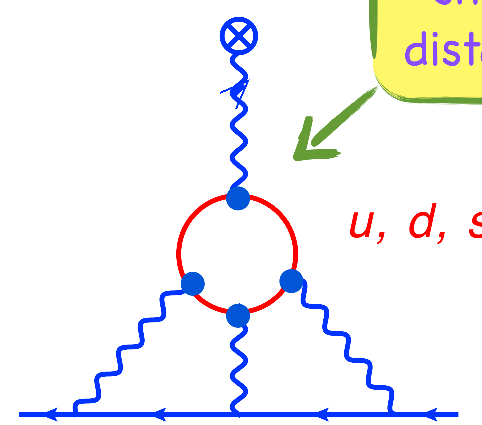
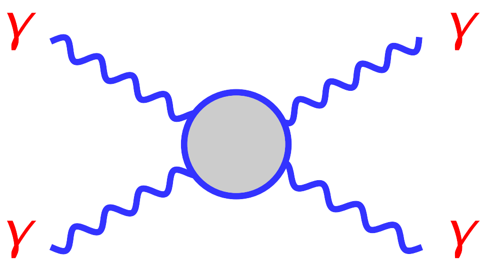


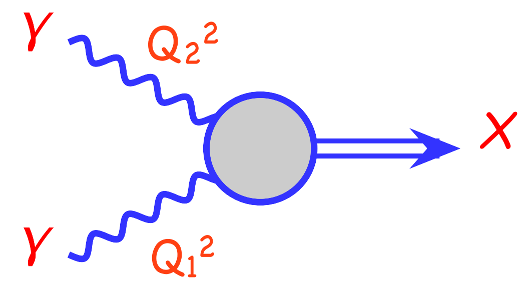
perturbative regime

Hadronic LbL scattering

elastic light-by-light scattering

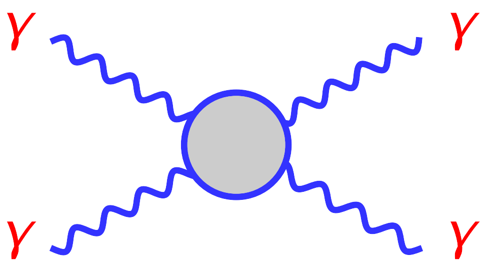


inelastic light-by-light scattering



Hadronic LbL scattering

elastic light-by-light scattering

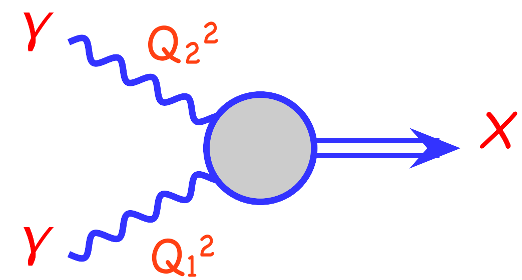


unitarity

⟷

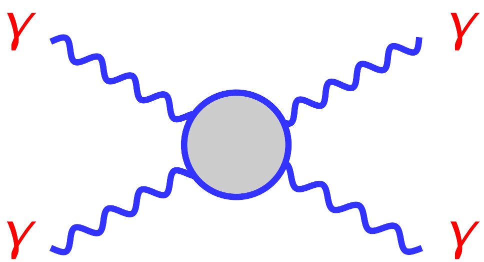
dispersion relations

inelastic light-by-light scattering



Hadronic LbL scattering

elastic light-by-light scattering



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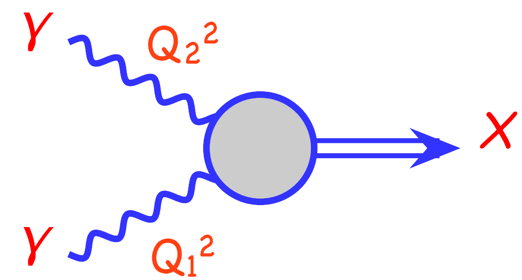
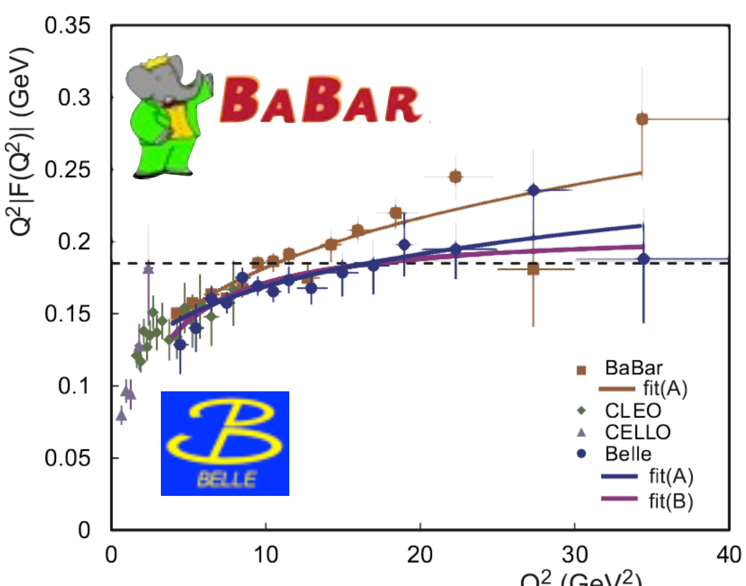
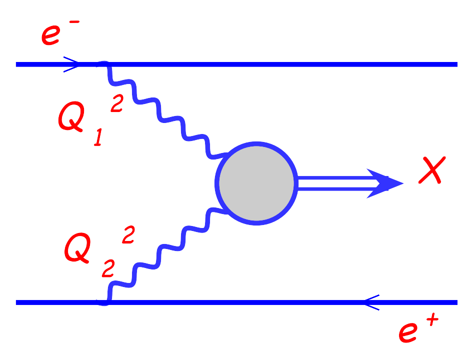


$\pi_0\gamma\gamma$ transition FF
in the space-like region



e^+e^- colliders

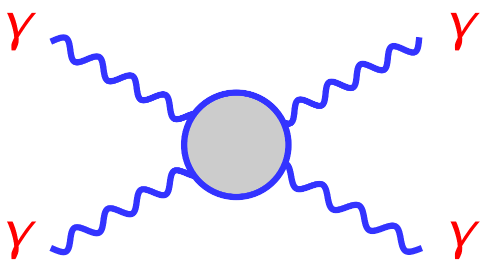


space-like region
1.5 GeV^2 < Q_2^2 < 40 GeV^2
for Q_1^2=0

strong involvement of Mainz groups

Hadronic LbL scattering

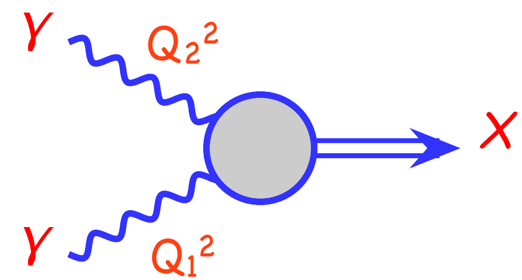
elastic light-by-light scattering



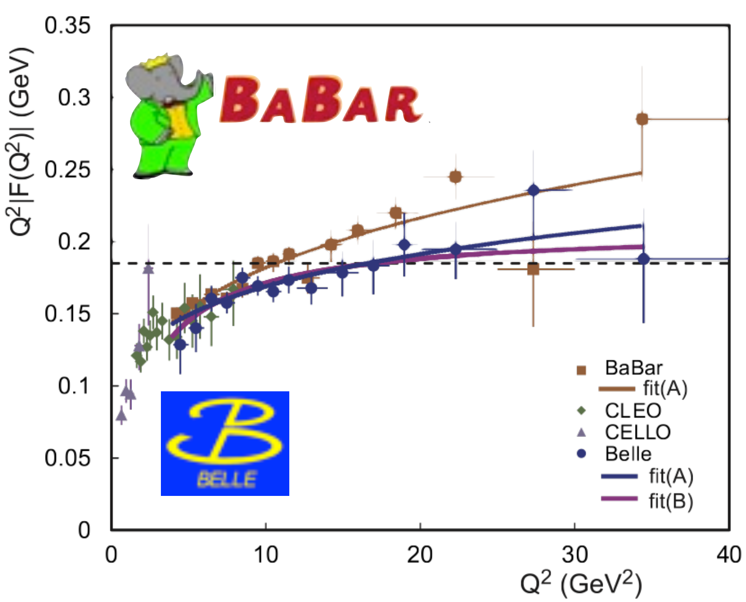
unitarity

dispersion relations

inelastic light-by-light scattering



$\pi_0\gamma\gamma$ transition FF
in the space-like region



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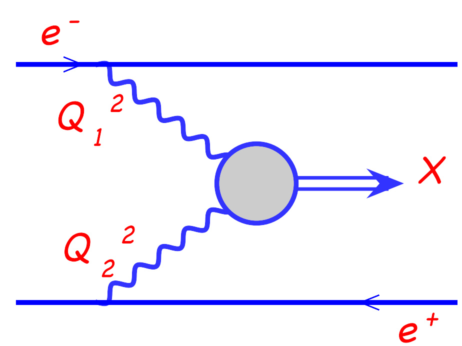

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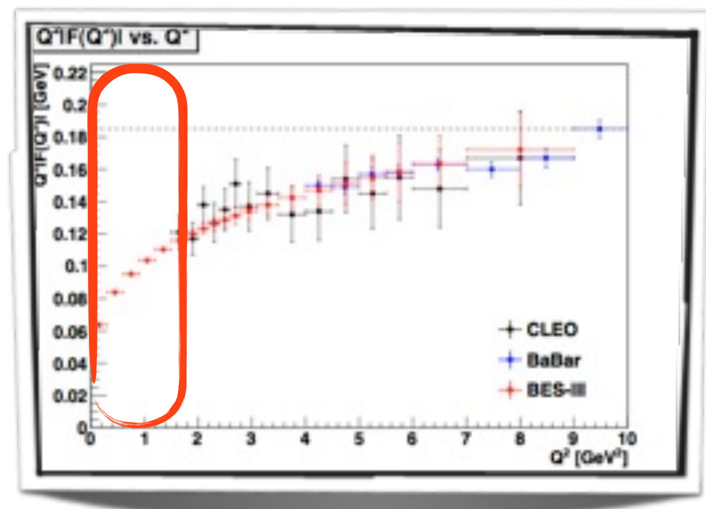
simulations for running at
BES-III (10 fb^{-1})

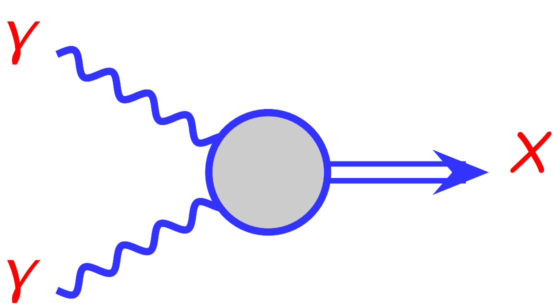


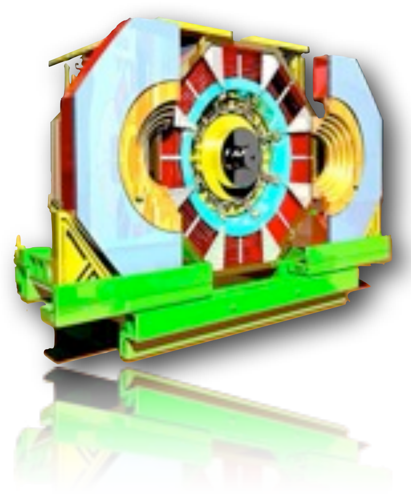
SFB 1044: Kloss (2011)

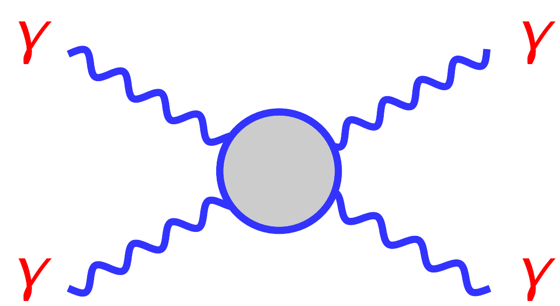
strong involvement of Mainz groups

Hadronic light-by-light scattering
and
sum rules

BES-III



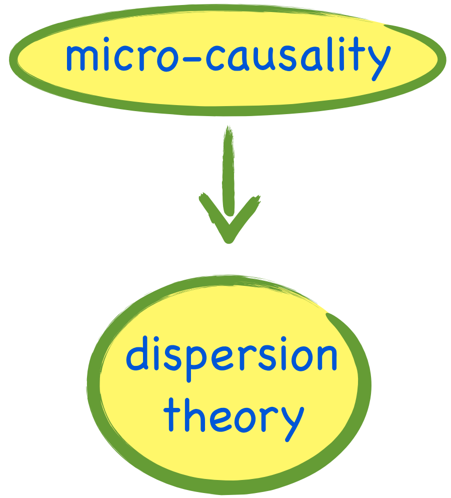




Sum rules

micro-causality

Sum rules



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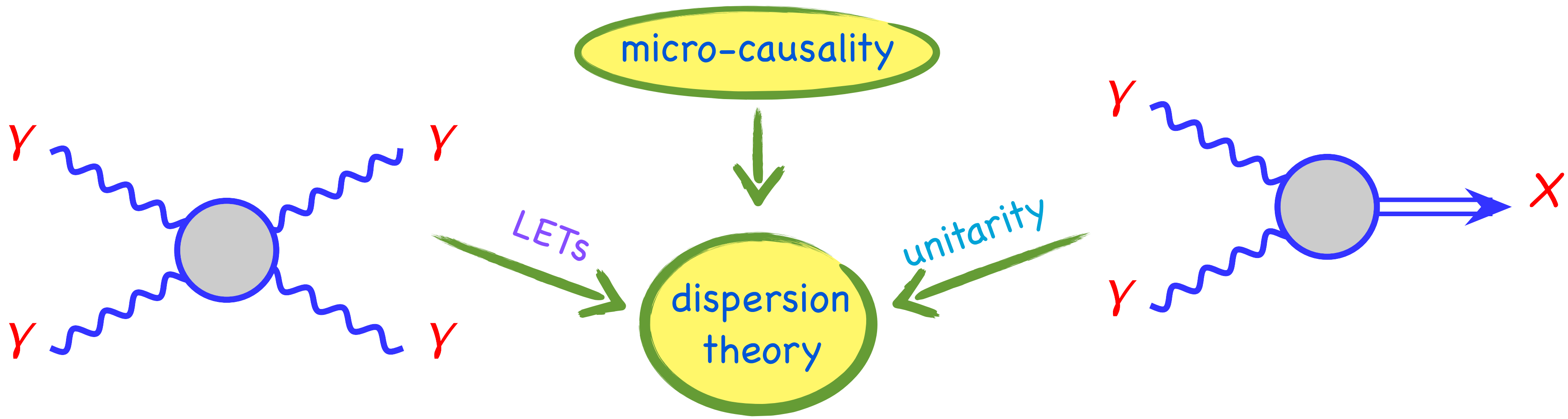
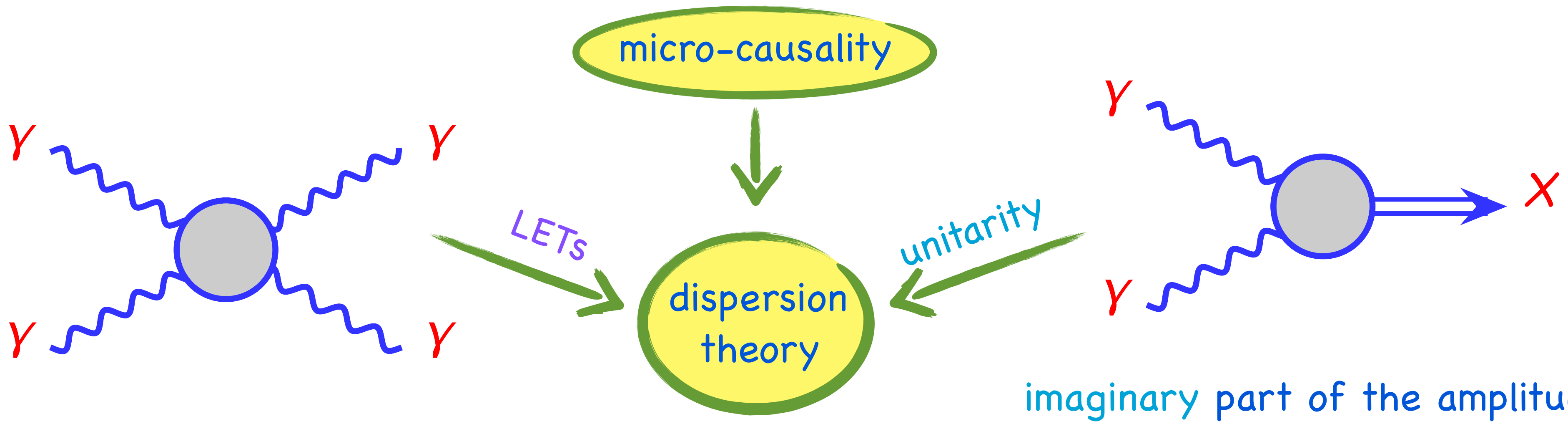

micro-causality
γ
γ
γ
γ
LETs
dispersion theory
unitarity
γ
γ
X

Sum rules



imaginary part of the amplitude

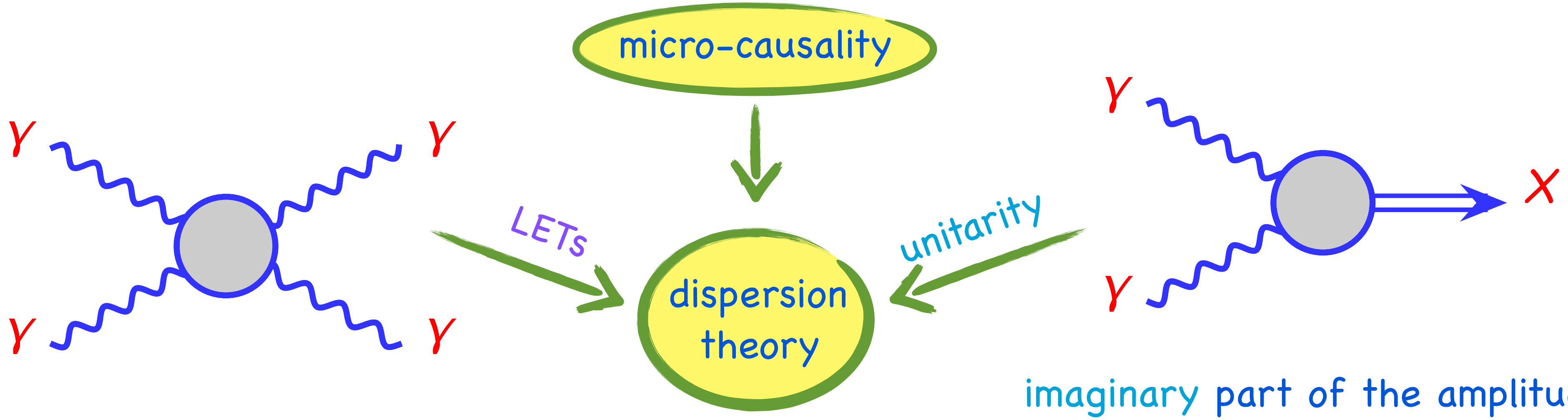
–

photon-photon fusion into leptons and hadrons:

$$\mathrm{Im} f^{(-)}(s) = -\frac{s}{8}[\sigma_2(s) - \sigma_0(s)]$$

$$\mathrm{Im} f^{(+)}(s) = -\frac{s}{8}[\sigma_{tot}(s)]$$

Sum rules



real part of the amplitude – low-energy structure of the elastic LbL scattering:

$$\mathcal{L}^{(8)} = c_1 (F_{\mu\nu} F^{\mu\nu})^2 + c_2 (F_{\mu\nu} \tilde{F}^{\mu\nu})^2$$

Euler, Heisenberg (1936)

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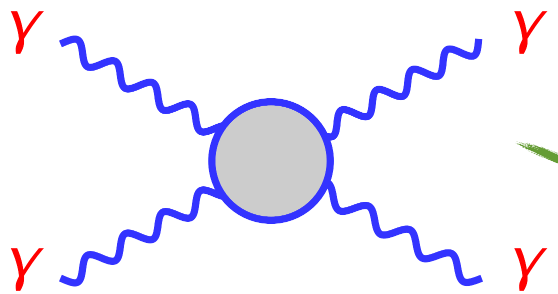
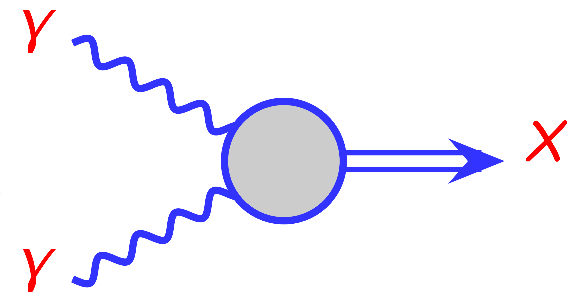


micro-causality

LETs → dispersion theory ← unitarity



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Sum rules

$$\int_{s_0}^{\infty} \frac{ds}{s} [\sigma_2(s) - \sigma_0(s)] = 0$$

V. Pascalutsa,
V.P., M. Vdh (2012)

$$c_1 \pm c_2 = \frac{1}{8\pi} \int_{s_0}^{\infty} \frac{ds}{s^2} [\sigma_\parallel(s) \pm \sigma_\perp(s)]$$

Sum rules

3 superconvergent relations:

helicity difference sum rule →

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)} [\sigma_0 - \sigma_2]_{Q_2^2=0}$$

sum rules involving longitudinal photons →

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)^2} \left[\sigma_\parallel + \sigma_{LT} + \frac{(s+Q_1^2)}{Q_1 Q_2} \tau_{TL}^a \right]_{Q_2^2=0}$$

$$0 = \int_{s_0}^{\infty} ds \left[\frac{\tau_{TL}}{Q_1 Q_2}\right]_{Q_2^2=0}$$

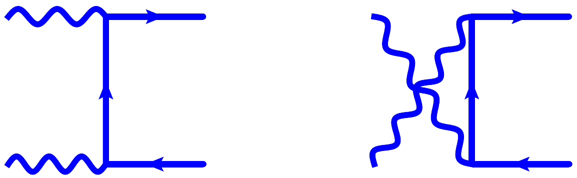
SRs involving LbL low-energy constants:

V. Pascalutsa, V.P., M. Vdh (2012)

$$c_1 \pm c_2 = \frac{1}{8\pi} \int_{s_0}^{\infty} \frac{ds}{s^2} [\sigma_\parallel(s) \pm \sigma_\perp(s)]$$

...

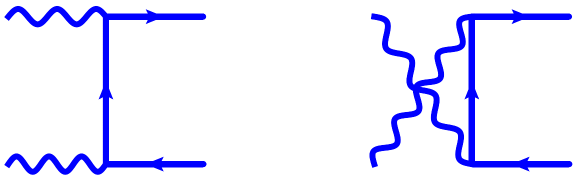
Pair production in QED



tree level QED

Pair production in QED

$$\sigma_2 - \sigma_0 = -\alpha^2 \frac{8\pi s}{(s+Q_1^2)^2}\left[\left(3 - \frac{Q_1^2}{s}\right)\sqrt{1-\frac{4m^2}{s}} - \left(1-\frac{Q_1^2}{s}\right)\text{arctanh}\sqrt{1-\frac{4m^2}{s}}\right]$$



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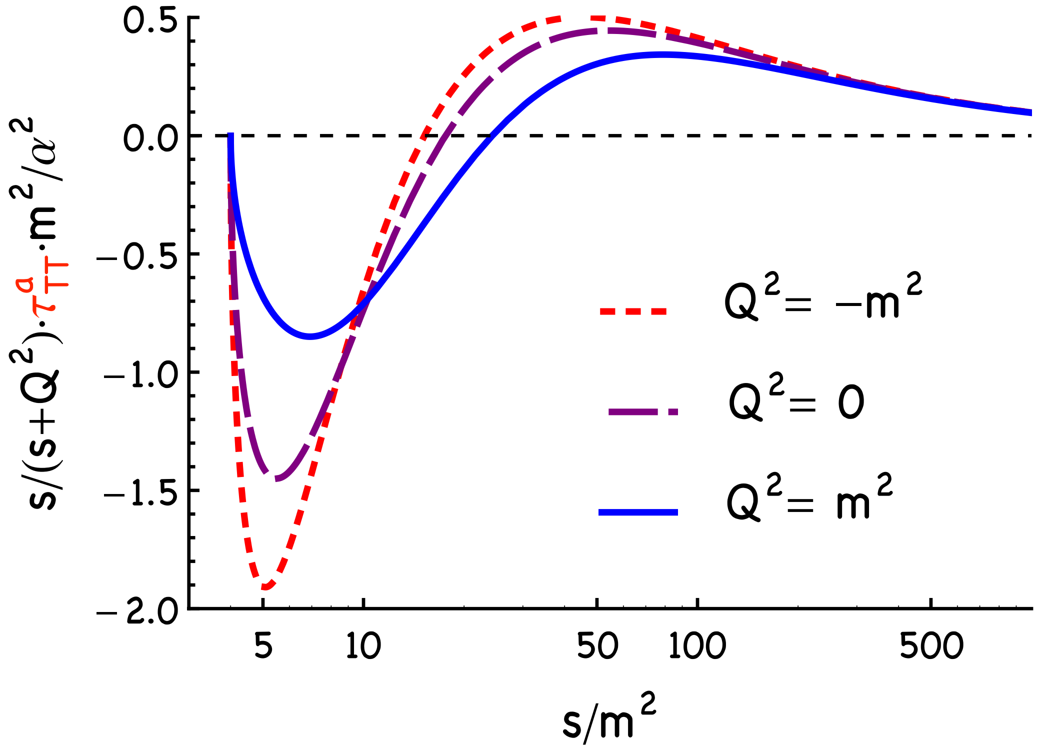

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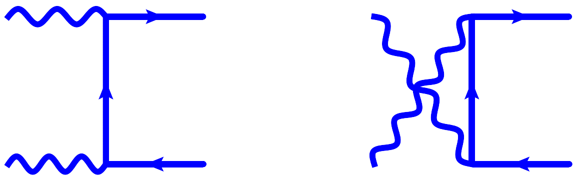

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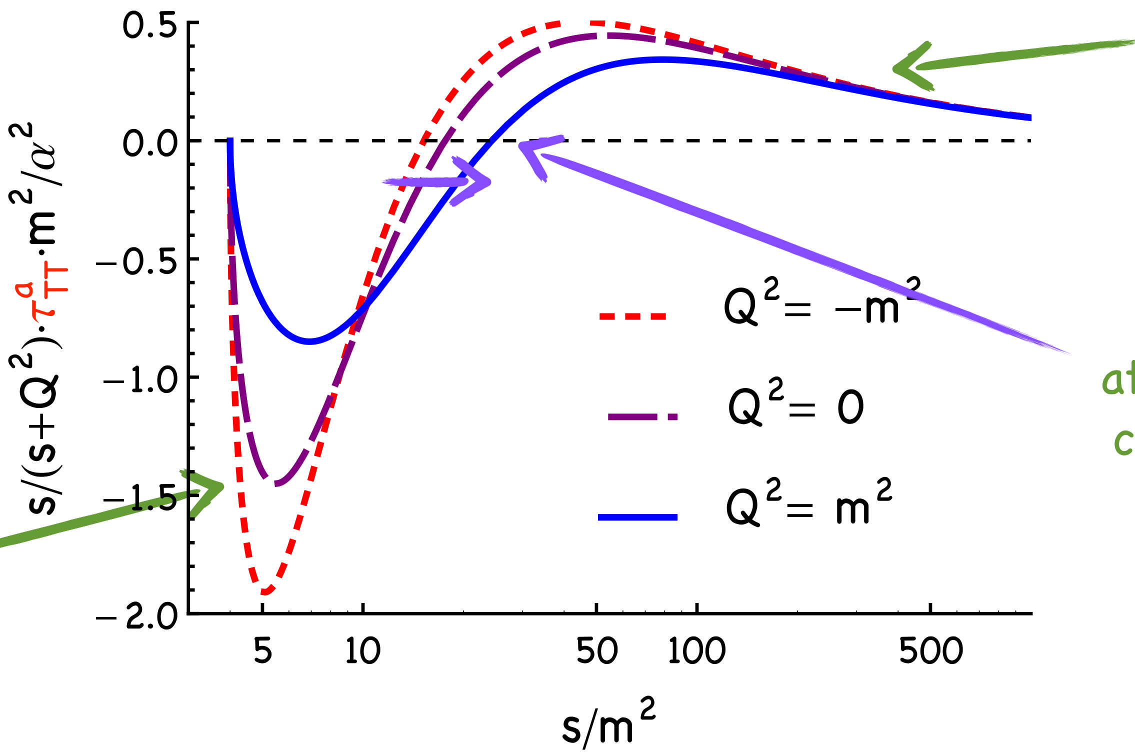
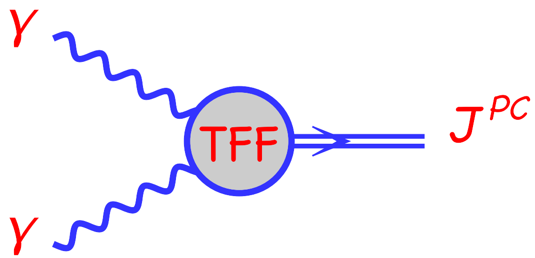


σ_2 dominates at higher energies

at larger Q^2 higher energy contributions are required

σ_0 dominates at lower energies

Meson production in γγ collision: $c\bar{c}$ mesons



input for the SRs:

determined by

γγ→M transition form factors

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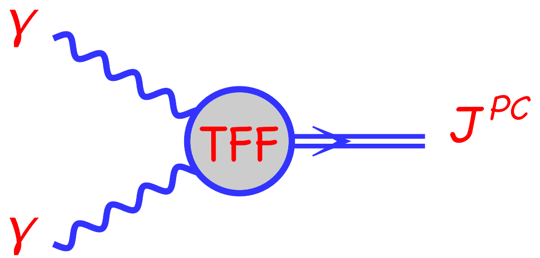

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$c\bar{c}$ states:

bound states

	$\int \frac{ds}{s}(\sigma_2 - \sigma_0)$ [nb]
$\eta_c(1S)$	-15.6 ± 2.1
$\chi_{c0}(1P)$	-3.6 ± 0.2
$\chi_{c2}(1P)$	3.4 ± 0.4
Sum resonances	-15.8 ± 2.1
duality estimate continuum ($\sqrt{s} \geq 2m_D$)	15.1
resonances + continuum	-0.7 ± 2.1

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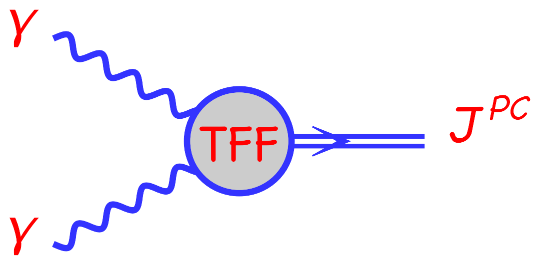

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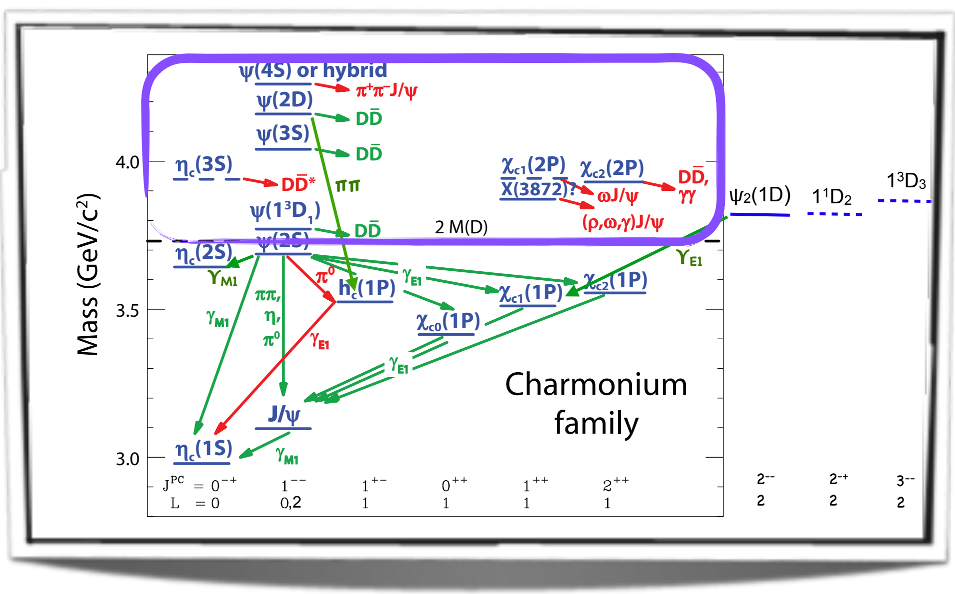
states above the nearby

$\bar{D}D$ threshold s_D≃14GeV^2!



the charmonium spectrum

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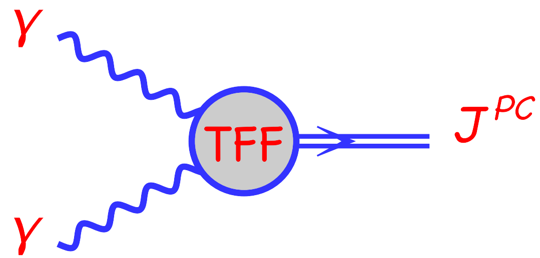

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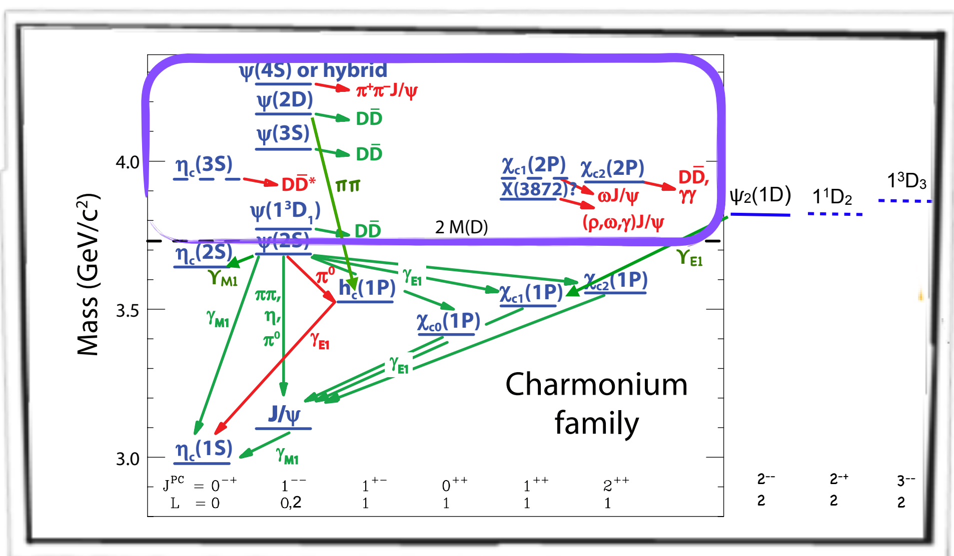
quark-hadron duality



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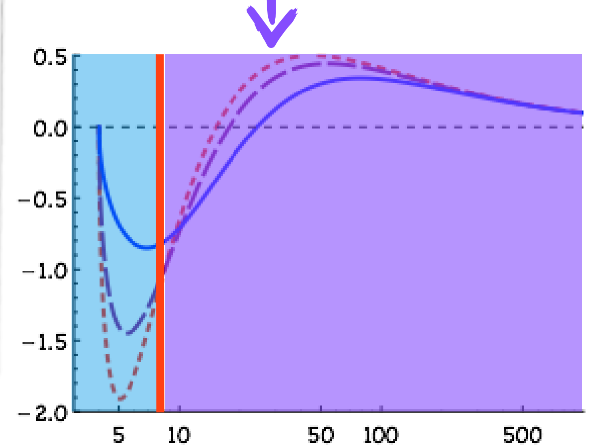

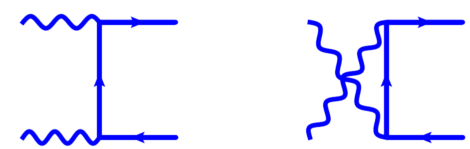

perturbative
γγ→$c\bar{c}$ process

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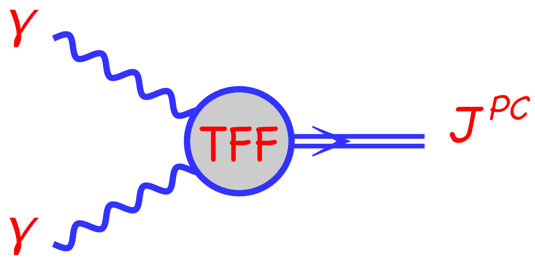

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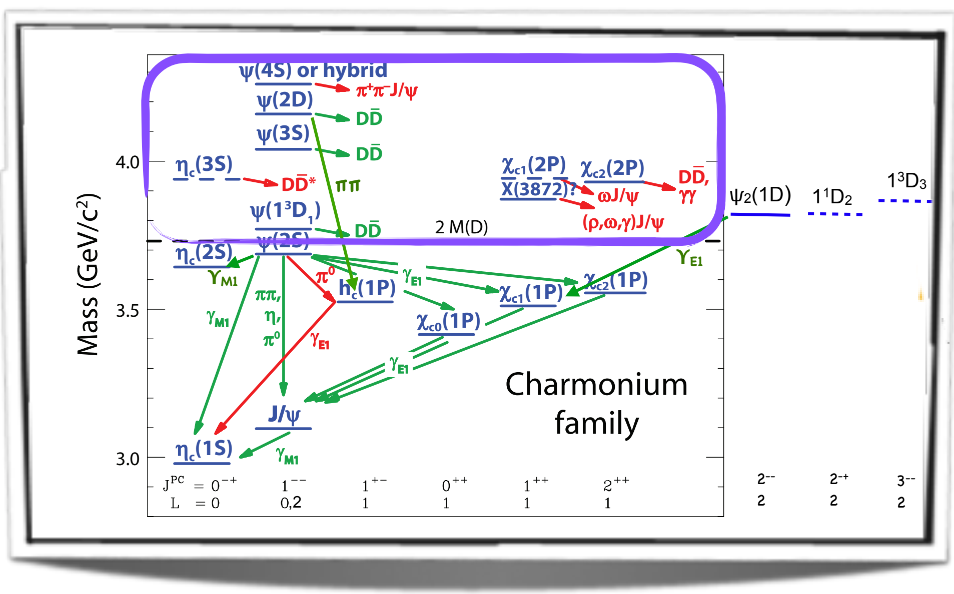

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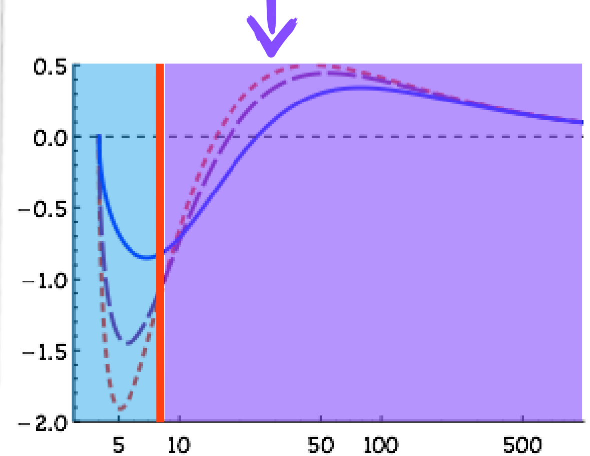

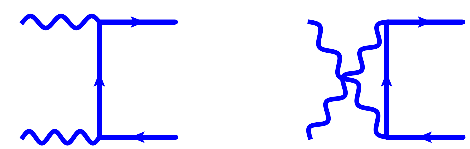

perturbative
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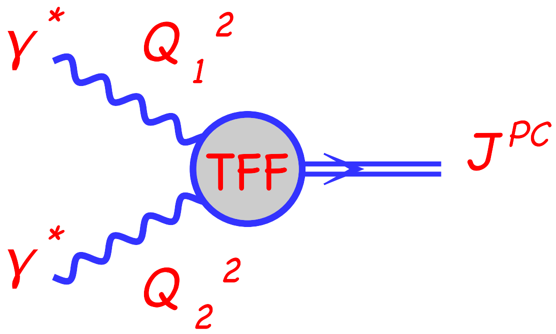
Meson production in $\gamma^*\gamma$ collision

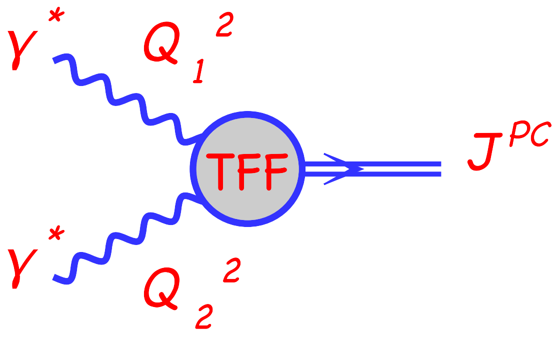


one photon is virtual Q_1^2, second is quasi-real $Q_2^2 \simeq 0$:

axial-vector mesons 1^{++} are allowed

$\gamma^*\gamma^* \rightarrow f_1(1285) \,/\, f_1(1420)$

Meson production in $\gamma^*\gamma$ collision



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$\gamma^*\gamma^* \to f_1(1285)$ / $f_1(1420)$

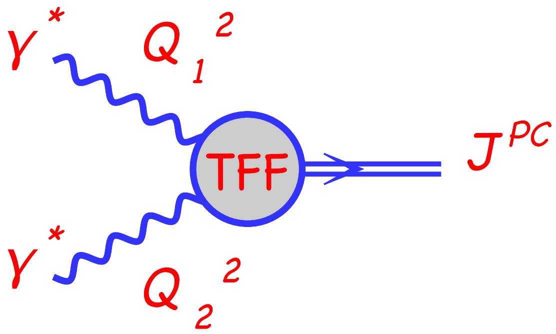
$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)^2} \left[\sigma_\parallel + \sigma_{LT} + \frac{(s+Q_1^2)}{Q_1 Q_2} \tau_{TL}^a \right]_{Q_2^2=0}$$

cancelation mechanism between scalar, axial-vector and tensor mesons

	m_M [MeV]	$\Gamma_{\gamma\gamma}$ [keV]	$\int \frac{ds}{s^2} \sigma_\parallel(s)$ [nb / GeV^2]	$\int ds \left[\frac{1}{s} \frac{\tau_{TL}^a}{Q_1 Q_2} \right]_{Q_i^2=0}$ [nb / GeV^2]	$\int ds \left[\frac{1}{s^2} \sigma_\parallel + \frac{1}{s} \frac{\tau_{TL}^a}{Q_1 Q_2} \right]_{Q_i^2=0}$ [nb / GeV^2]
$f_1(1285)$	1281.8 ± 0.6	3.5 ± 0.8	0	-93 ± 21	-93 ± 21
$f_1(1420)$	1426.4 ± 0.9	3.2 ± 0.9	0	-50 ± 14	-50 ± 14
$f_0(980)$	980 ± 10	0.29 ± 0.07	20 ± 5	0	20 ± 5
$f_0'(1370)$	$1200 - 1500$	3.8 ± 1.5	48 ± 19	0	48 ± 19
$f_2(1270)$	1275.1 ± 1.2	3.03 ± 0.35	138 ± 16	$\gtrsim 0$	138 ± 16
$f_2'(1525)$	1525 ± 5	0.081 ± 0.009	1.5 ± 0.2	$\gtrsim 0$	1.5 ± 0.2
$f_2(1565)$	1562 ± 13	0.70 ± 0.14	12 ± 2	$\gtrsim 0$	12 ± 2
Sum					76 ± 36

uncertainty: higher mass states or non-resonant contributions with axial-vector quantum numbers

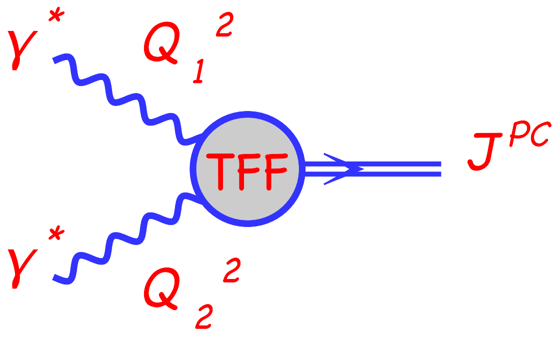
Meson production in $\gamma^*\gamma$ collision: TFF



at finite Q_1^2 the SRs imply information on

meson transition form-factors:

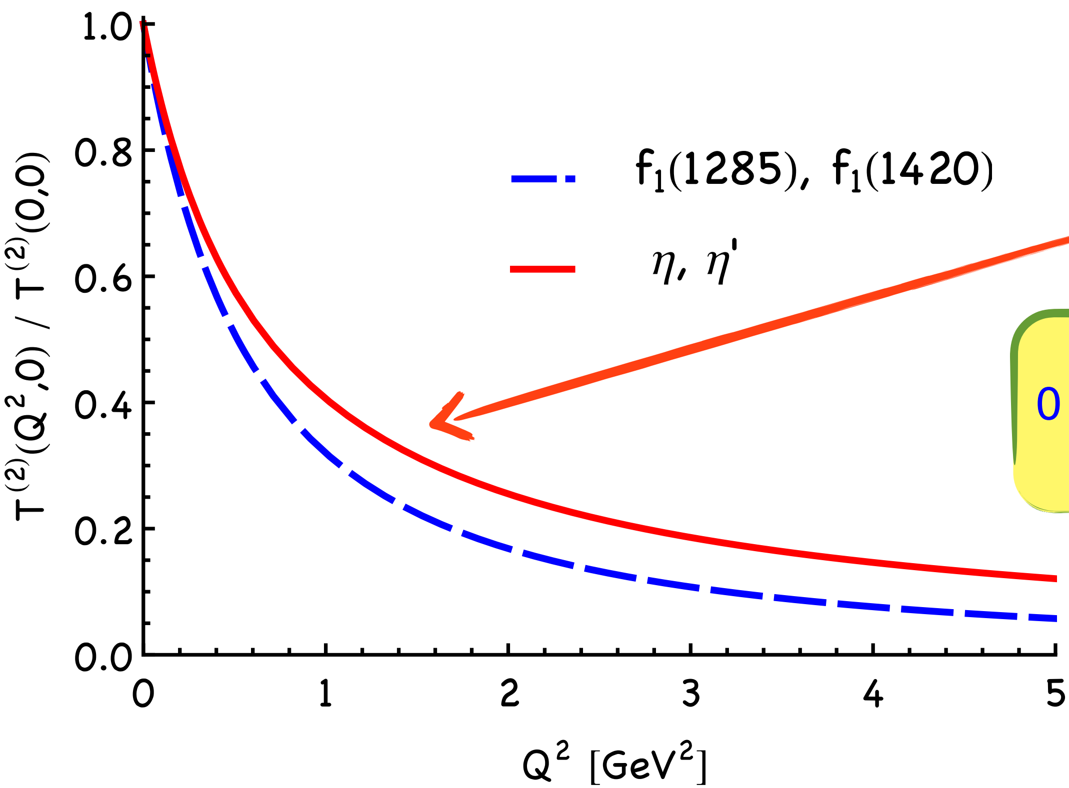
Meson production in $\gamma^*\gamma$ collision: TFF



at finite Q_1^2 the SRs imply information on meson transition form-factors:

estimate for the $f_2(1270)$ tensor FF in terms of the η, η' and f_1 FFs

$f_2(1270)$



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direct measurements → BES III

Meson production in γγ collision: light-quark states

the I=1 channel

	$\int \frac{ds}{s}(\sigma_2-\sigma_0)$ [nb]	c_1 [10^{-4} GeV^{-4}]	c_2 [10^{-4} GeV^{-4}]
π^0	-195 ± 13	0	10.94 ± 0.70
$a_0(980)$	-20 ± 8	0.021 ± 0.007	0
$a_2(1320)$	134 ± 8	0.039 ± 0.002	0.039 ± 0.002
$a_2(1700)$	18 ± 3	0.003 ± 0.001	0.003 ± 0.001
Sum	-63 ± 17	0.06 ± 0.01	10.98 ± 0.70

the I=0 channel

	$\int \frac{ds}{s}(\sigma_2-\sigma_0)$ [nb]	c_1 [10^{-4}GeV^{-4}]	c_2 [10^{-4}GeV^{-4}]
η	-191 ± 10	0	0.65 ± 0.03
η'	-300 ± 10	0	0.33 ± 0.01
$f_0(980)$	-19 ± 5	0.020 ± 0.005	0
$f_0'(1370)$	-91 ± 36	0.049 ± 0.019	0
$f_2(1270)$	449 ± 52	0.141 ± 0.016	0.141 ± 0.016
$f_2'(1525)$	7 ± 1	0.002 ± 0.000	0.002 ± 0.000
$f_2(1565)$	56 ± 11	0.012 ± 0.002	0.012 ± 0.002
Sum	-89 ± 66	0.22 ± 0.03	1.14 ± 0.04

Meson production in γγ collision: light-quark states

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dominant contribution to c_2 comes from π^0

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Sum	-89 ± 66	0.22 ± 0.03	1.14 ± 0.04

dominant contribution to c_2 comes from η, η' and $f_2(1270)$

Meson production in γγ collision: light-quark states

the I=1 channel

	$\int \frac{ds}{s}(\sigma_2 - \sigma_0)$ [nb]	c_1 $[10^{-4}\ \mathrm{GeV}^{-4}]$	c_2 $[10^{-4}\ \mathrm{GeV}^{-4}]$
π^0	-195 ± 13	0	10.94 ± 0.70
$a_0(980)$	-20 ± 8	0.021 ± 0.007	0
$a_2(1320)$	134 ± 8	0.039 ± 0.002	0.039 ± 0.002
$a_2(1700)$	18 ± 3	0.003 ± 0.001	0.003 ± 0.001
Sum	-63 ± 17	0.06 ± 0.01	10.98 ± 0.70

dominant contribution to c_2 comes from π^0

the I=0 channel

	$\int \frac{ds}{s}(\sigma_2 - \sigma_0)$ [nb]	c_1 $[10^{-4}\mathrm{GeV}^{-4}]$	c_2 $[10^{-4}\mathrm{GeV}^{-4}]$
η	-191 ± 10	0	0.65 ± 0.03
η'	-300 ± 10	0	0.33 ± 0.01
$f_0(980)$	-19 ± 5	0.020 ± 0.005	0
$f_0'(1370)$	-91 ± 36	0.049 ± 0.019	0
$f_2(1270)$	449 ± 52	0.141 ± 0.016	0.141 ± 0.016
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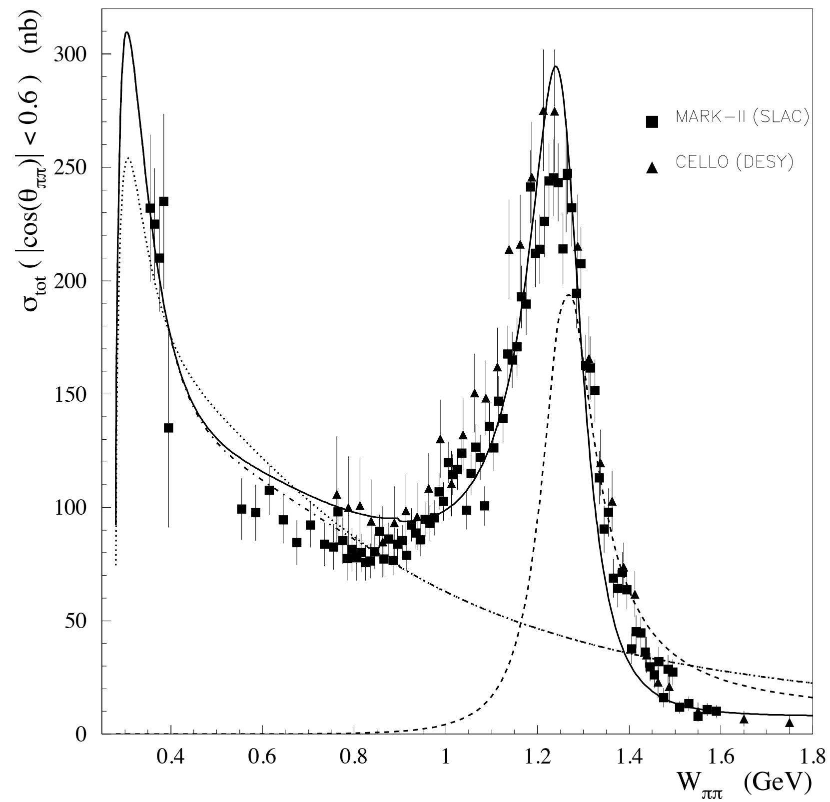

unitarized QED approximation + f_2:

$$[c_1 + c_2]^{Born\ unitary + f_2} = 8.53 \cdot 10^{-4}\mathrm{GeV}^{-4}$$

Meson production in γγ collision: light-quark states

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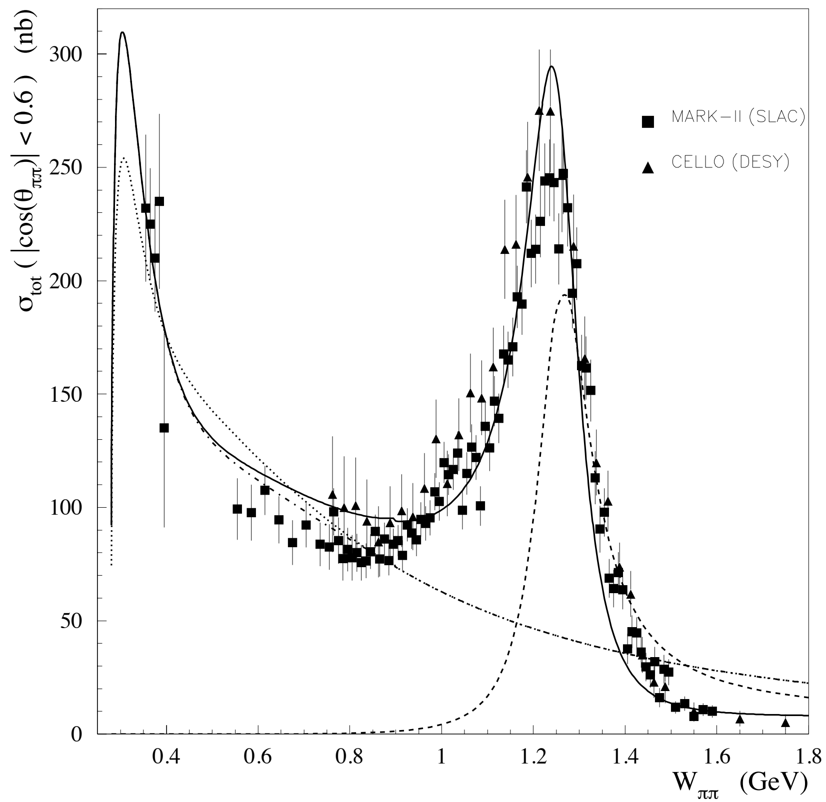

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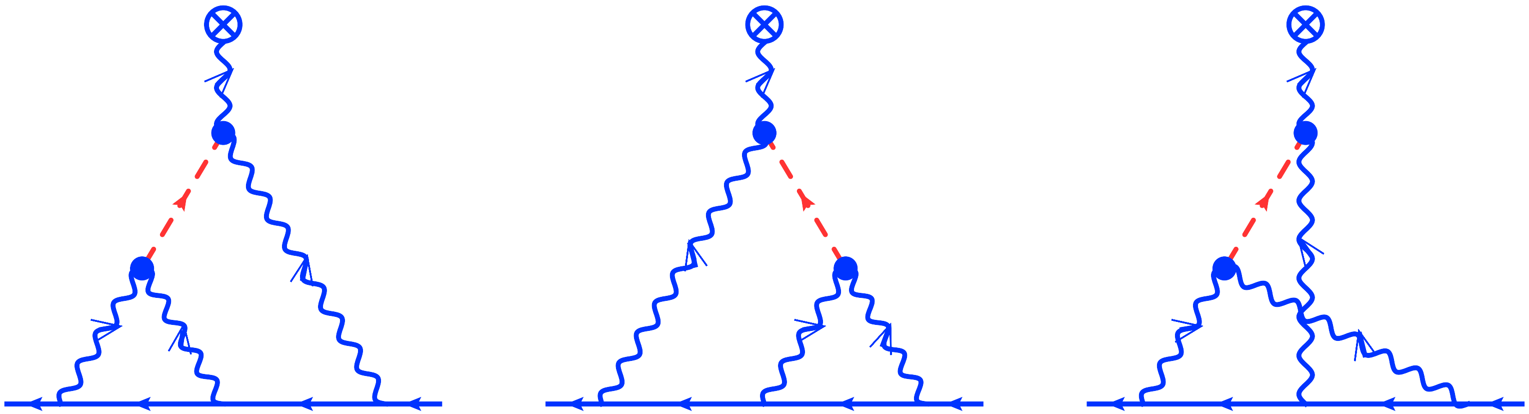
+ contributions from axial-vector states

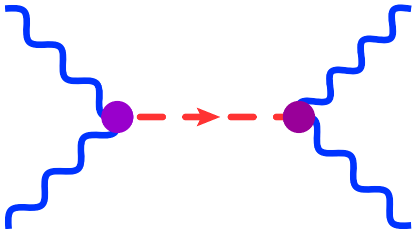
Hadronic contributions to the $(g-2)_\mu$.



The pole contribution

$F(Q_1, Q_2)$

non-perturbative dynamics



exchange of a pole

The pole contribution

$F(Q_1, Q_2)$

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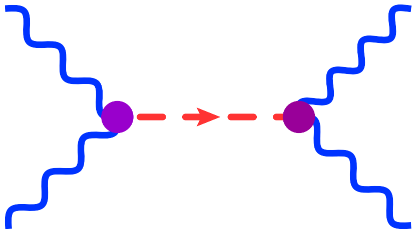

exchange of a pole

pseudoscalar poles: π^0, η, η'

$a_\mu^{LbL,PS} = +8.3\ (1.2) \times 10^{-10}$

Knecht, Nyffeler (2001)

The pole contribution

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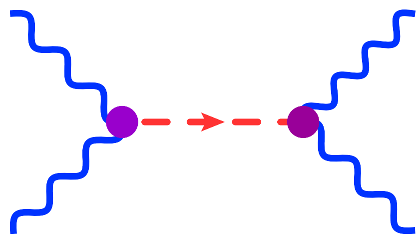

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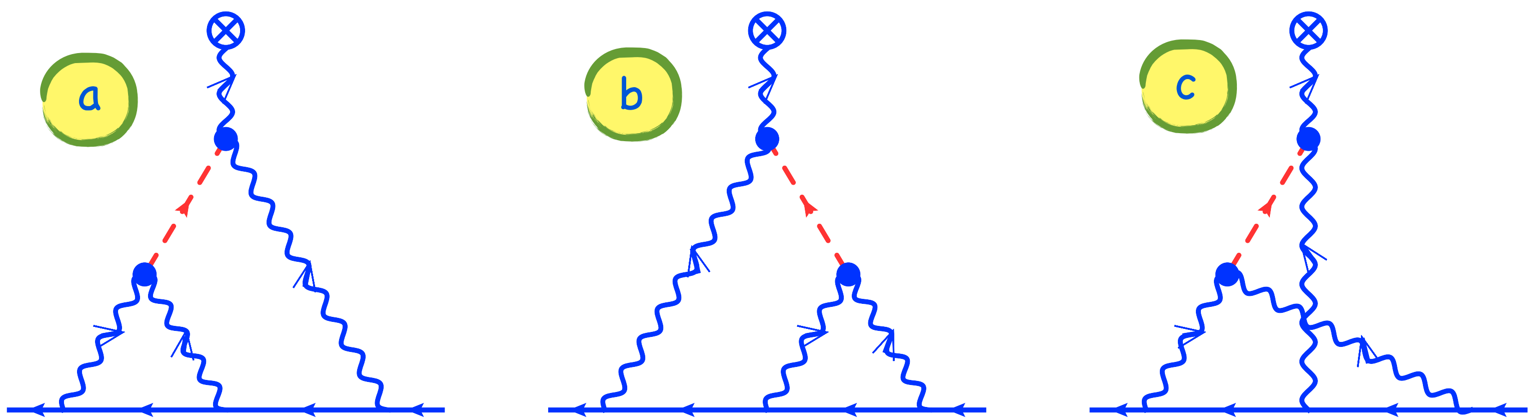
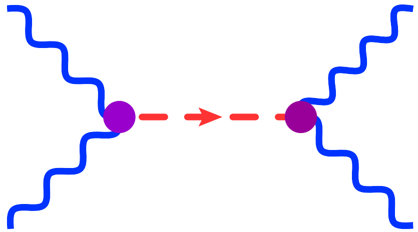
three topologies



The pole contribution

F(Q₁, Q₂)

non-perturbative dynamics



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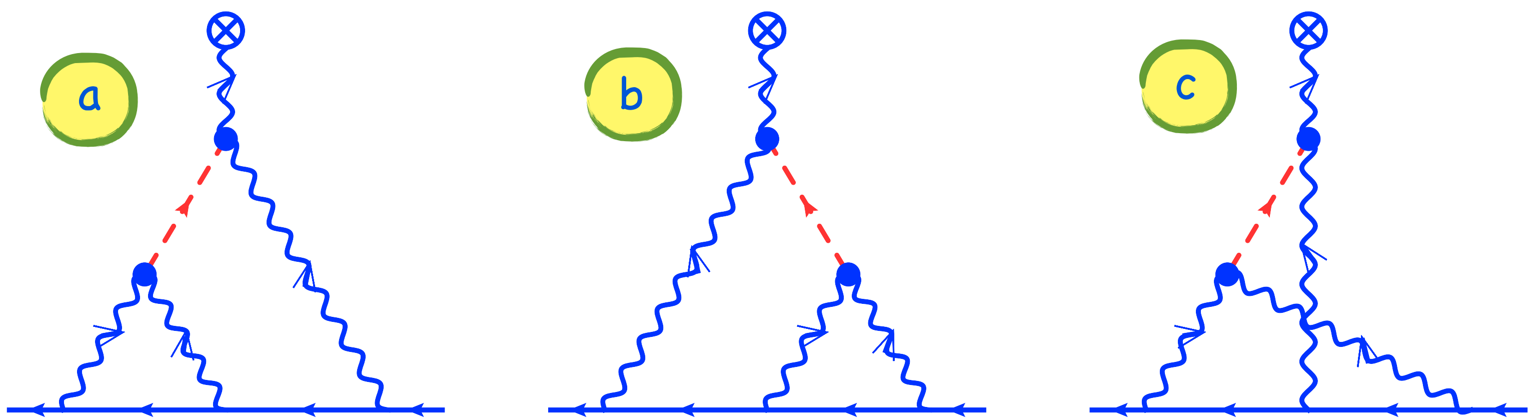

$$a_\mu^{LbL} = \frac{-e^6}{48m} \int \frac{d^4 q_1}{(2\pi)^4} \int \frac{d^4 q_2}{(2\pi)^4} \frac{1}{q_1^2 q_2^2 (q_1+q_2)^2} \frac{1}{(p+q_1)^2 - m^2} \frac{1}{(p-q_2)^2 - m^2}$$

$$\times \left[\frac{F\left(q_1^2, (q_1+q_2)^2\right) F\left(q_2^2, 0\right)}{q_2^2 - m_P^2} T_{ab}(q_1, q_2, p) + \frac{F\left(q_1^2, q_2^2\right) F\left((q_1+q_2)^2, 0\right)}{(q_1+q_2)^2 - m_P^2} T_c(q_1, q_2, p) \right]$$

Aγγ transition amplitude

dipole parametrization

A→γγ transition FF:

$$\frac{A(Q_1^2,0)}{A(0,0)} = \frac{1}{\left(1+Q_1^2/\Lambda_A^2\right)^2}$$

$$[A(0,0)]^2 = \frac{12}{\pi\alpha^2}\frac{1}{m_A^2}\Gamma_{\gamma\gamma}$$

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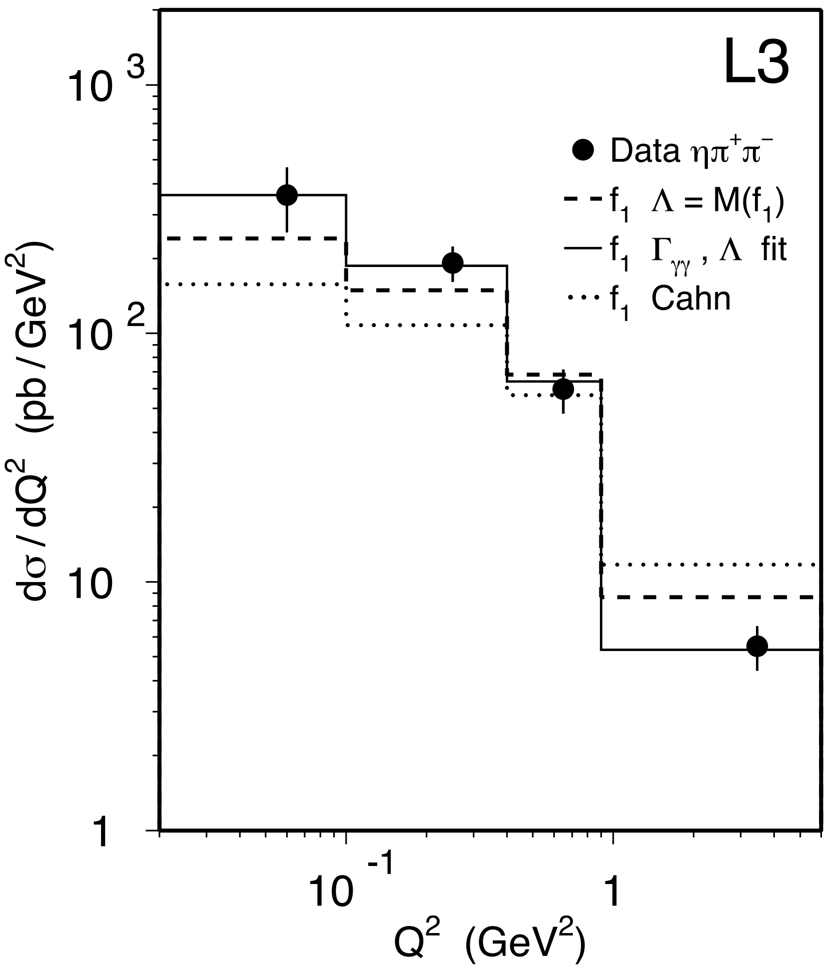
for 2γ decay widths $\Gamma_{\gamma\gamma}$ and dipole masses Λ_A entering the FF, we use the experimental results from the L3 Collaboration.

	m_A [MeV]	$\tilde{\Gamma}_{\gamma\gamma}$ [keV]	Λ_A [MeV]
$f_1(1285)$	1281.8 ± 0.6	3.5 ± 0.8	1040 ± 78
$f_1(1420)$	1426.4 ± 0.9	3.2 ± 0.9	926 ± 78

L3 Collaboration



Two-dimensional representation

$$a_\mu^{LbL} = \frac{\alpha}{(2\pi)^2} \frac{\Lambda_{A1}^6 \Lambda_{A2}^6 \tilde{\Gamma}_{\gamma\gamma}(A)}{m\, m_A^5} \int \mathrm{d}Q_1 \int \mathrm{d}Q_2 \left[2 w_a(Q_1, Q_2) + w_c(Q_1, Q_2)\right]$$

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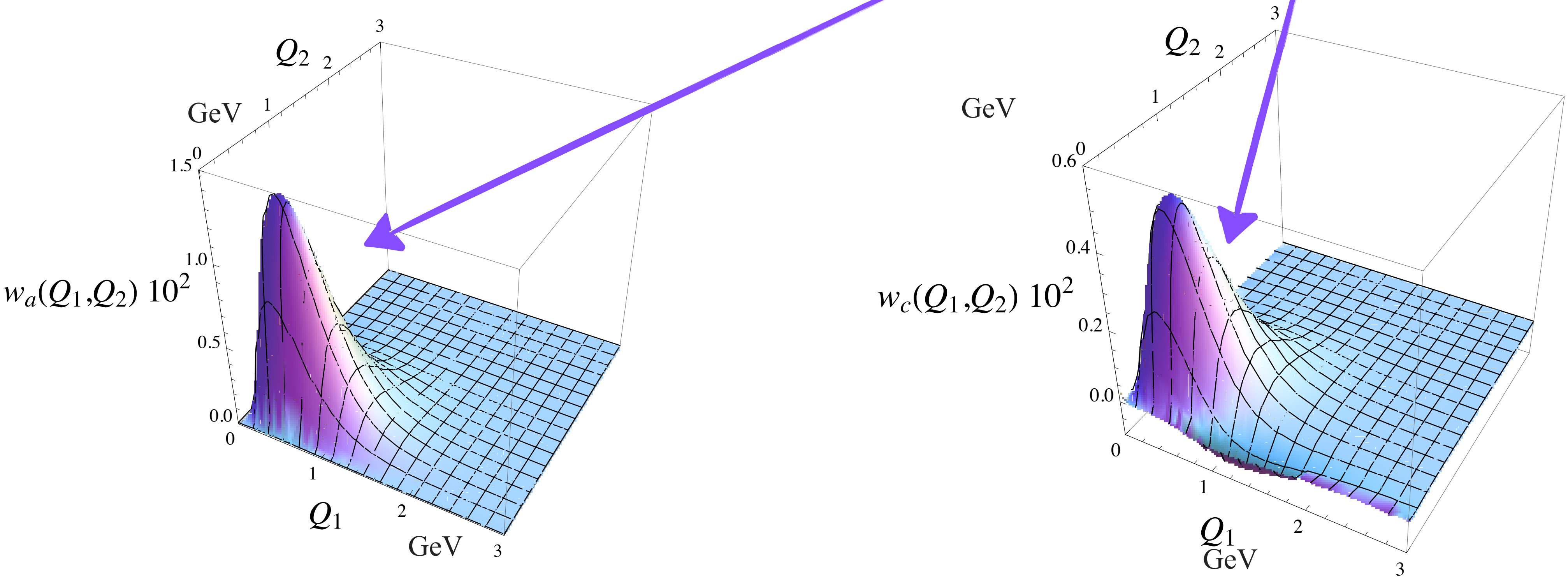

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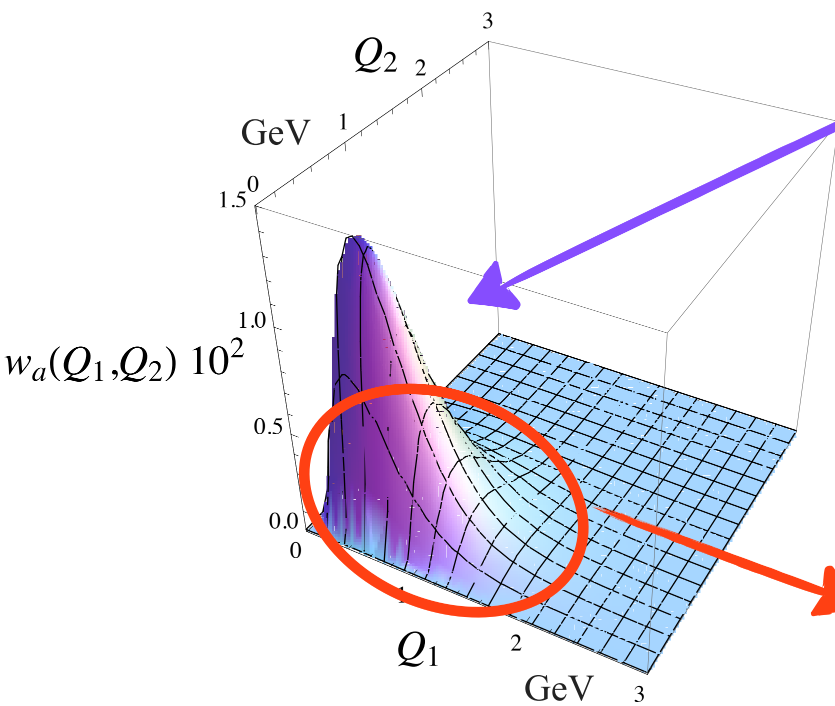
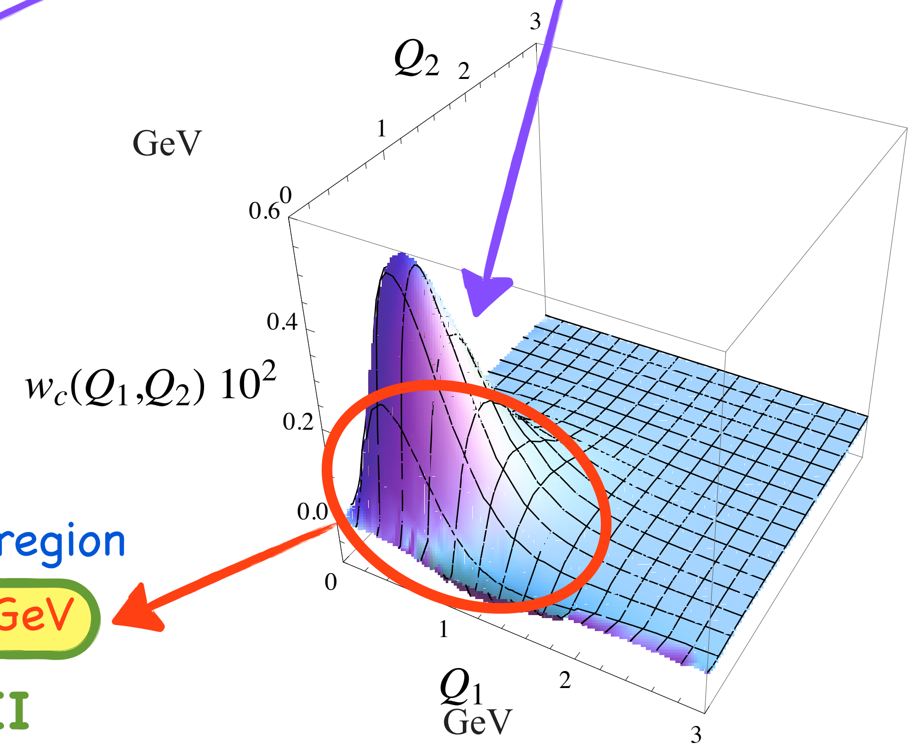


dominating region

$Q_1 \sim Q_2 \sim 1$ GeV

BES III



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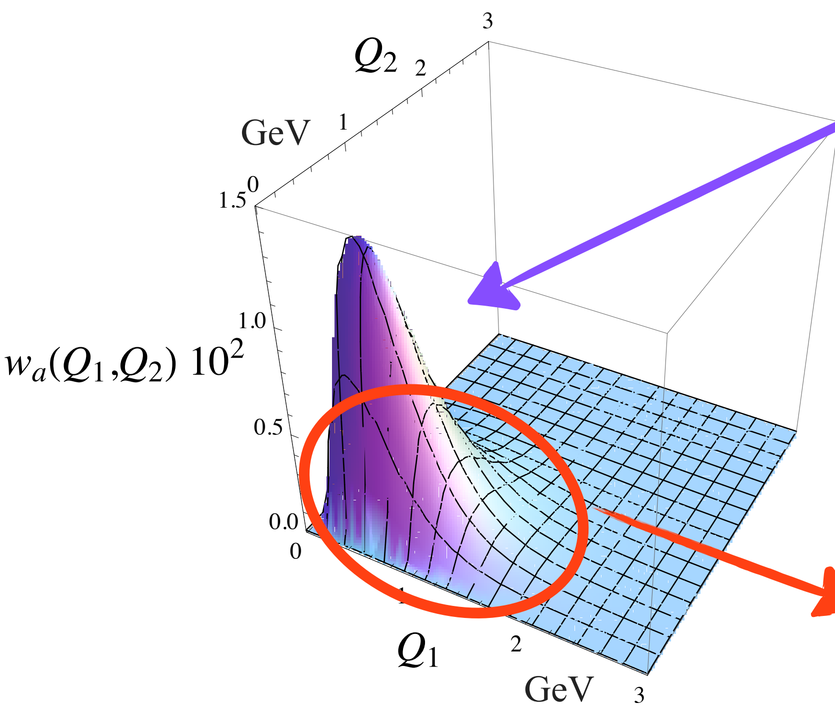

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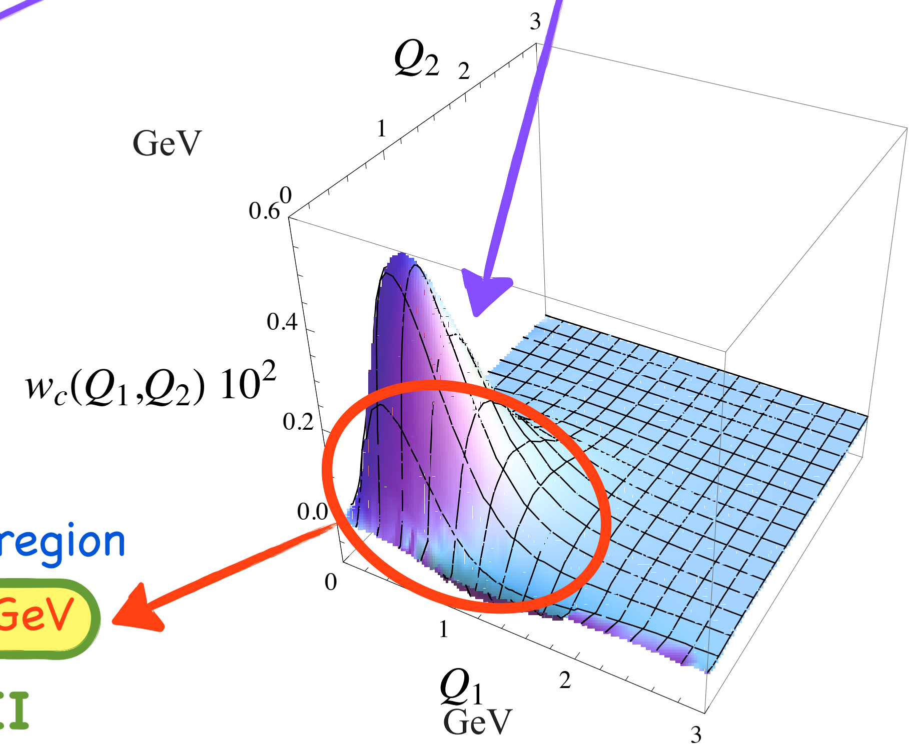

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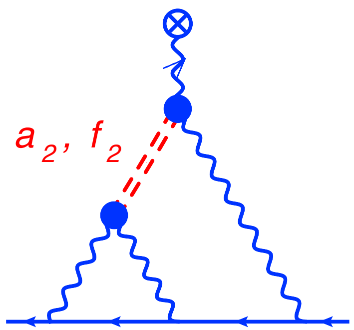
the contribution of the axial-vector pole to the $(g-2)_\mu$

V.P. et al. (2013)

Perspectives & conclusions

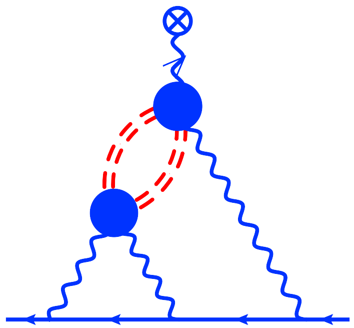
tensor meson contribution:
dominant at higher energies

f2(1270)
f2(1565)
a2(1320)



not measured so far,
experimental
input required

pion-pair: dispersion framework,
helicity amplitudes



measurements for
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Perspectives & conclusions

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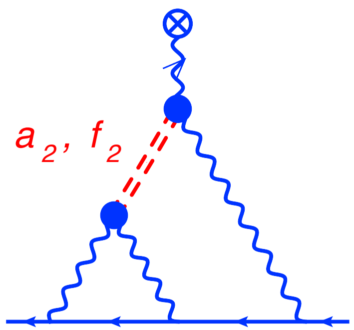

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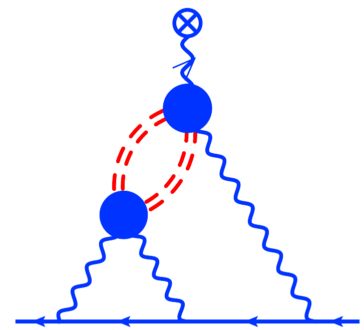

measurements for
the threshold region
required

- testing SM and searches for new physics
- $(g-2)_\mu$ complicated phenomenon involves non-perturbative dynamics
- model independent approach - sum rules
- correlations between different hadronic states - information about low-energy light-by-light scattering
- hadronic contributions to the $(g-2)_\mu$

谢谢