

# Precision Hadron Structure: from electron scattering to atomic physics

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5th Workshop on Hadron Physics in China and Opportunities in US  
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# Unraveling the structure of mesons and baryons

- **Basic question: unraveling strong QCD**

Origin of mass, spin, imaging of hadrons

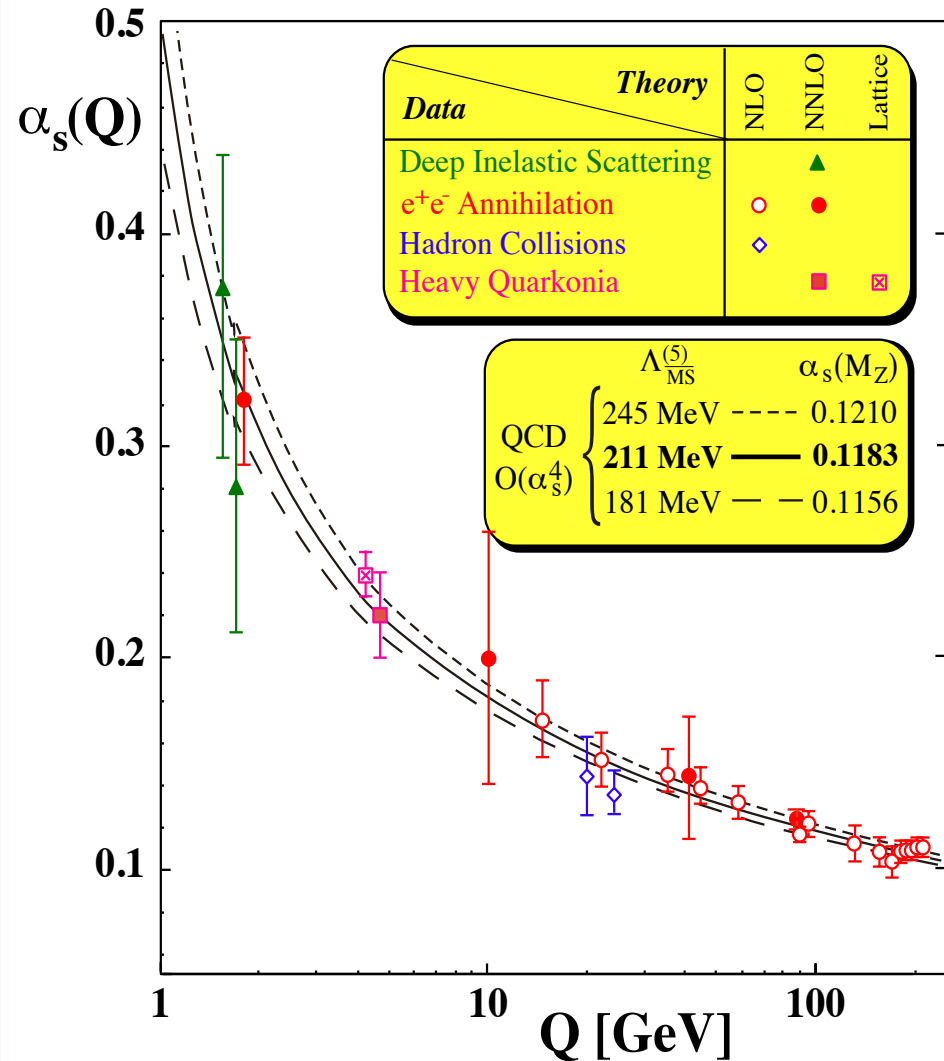
- **Precision hadron physics**

Impact on new physics searches:  $(g-2)_\mu$ , dark photon search, proton radius puzzle, weak mixing angle

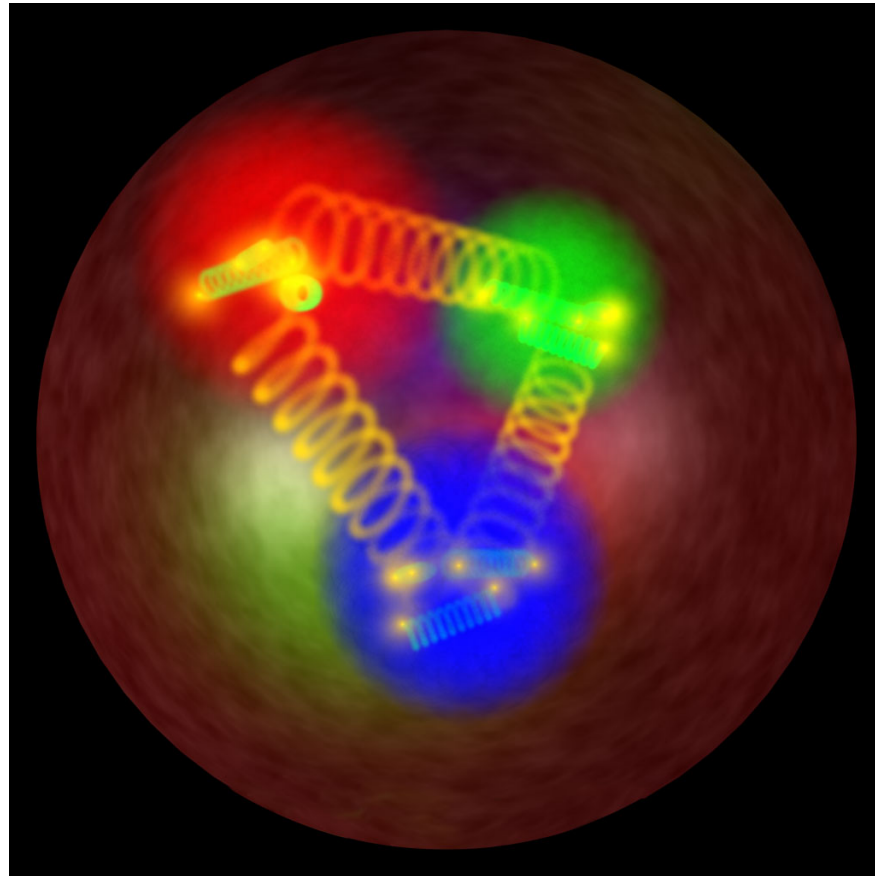
- **Theory tools**

lattice QCD: ab initio

EFT/phenomenology: interplay with precision hadron data

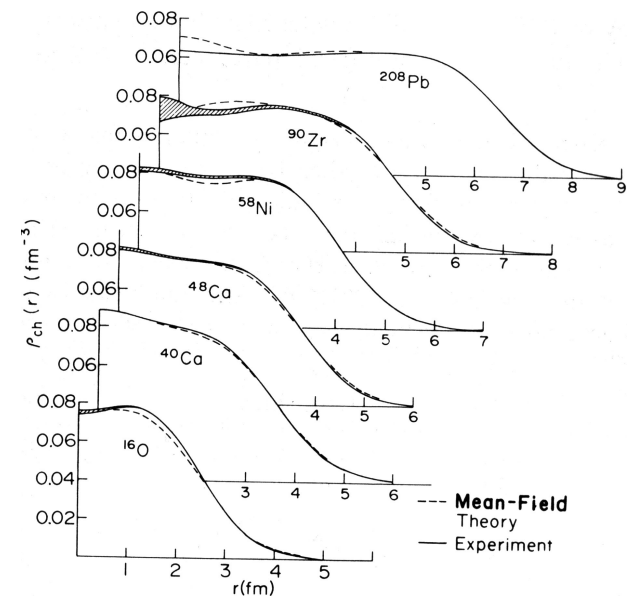
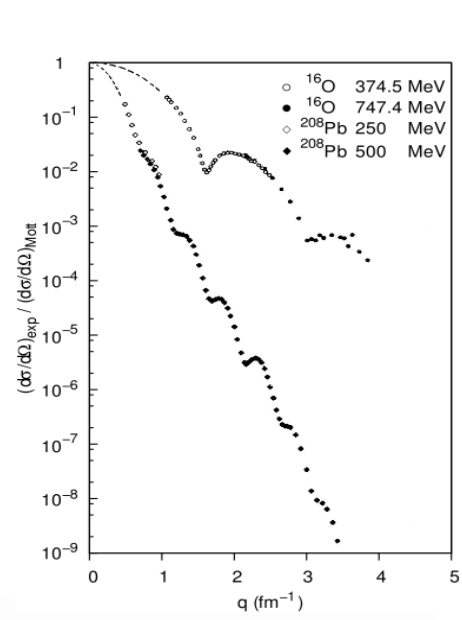


**What is the size of the proton?**

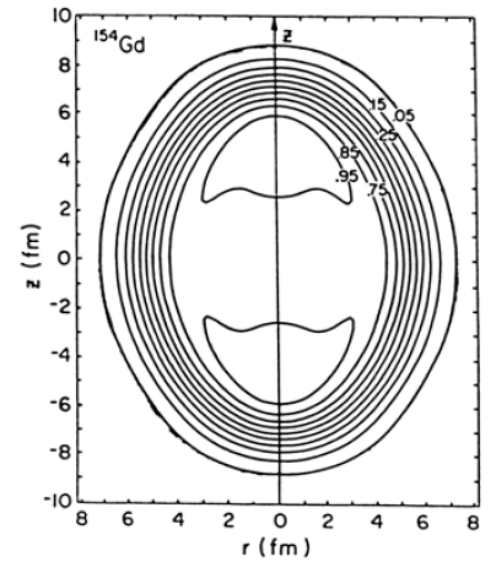
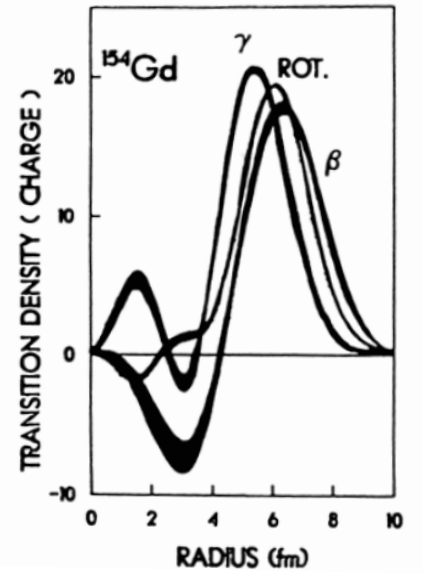


# Sizes and shapes of non-relativistic many-body systems

**Sizes of nuclei**  
as revealed through  
**elastic electron scattering**



**Shapes of deformed nuclei**  
as revealed through  
**inelastic electron scattering**



# Electron scattering facilities MAMI, Jlab:

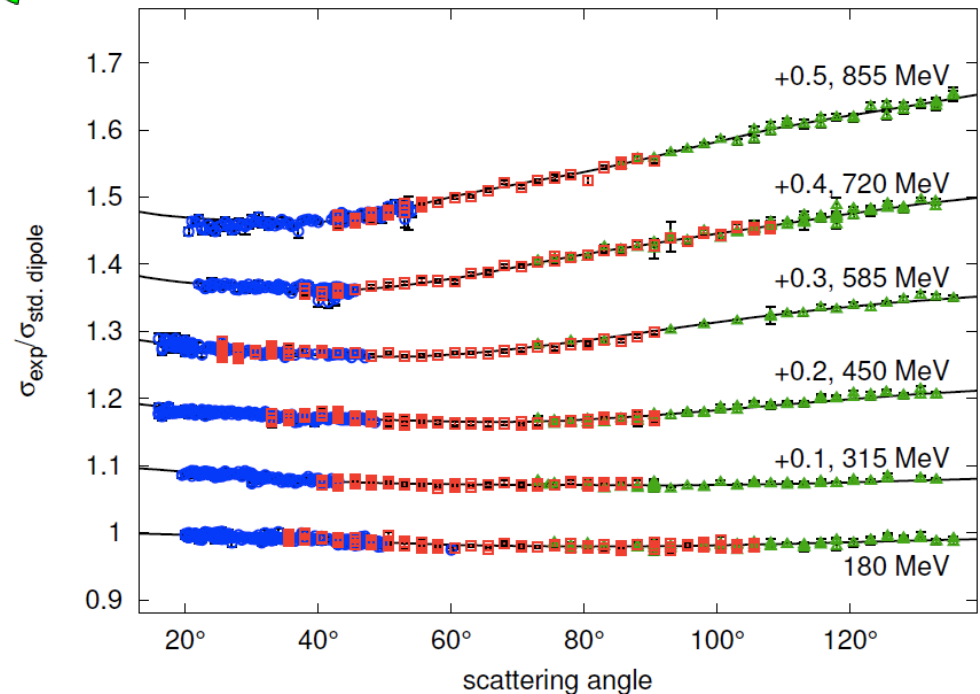
uniquely positioned to deliver high-precision hadron data

recent cross section data **A1@MAMI**

High momentum resolution  $\sim 10^{-4}$

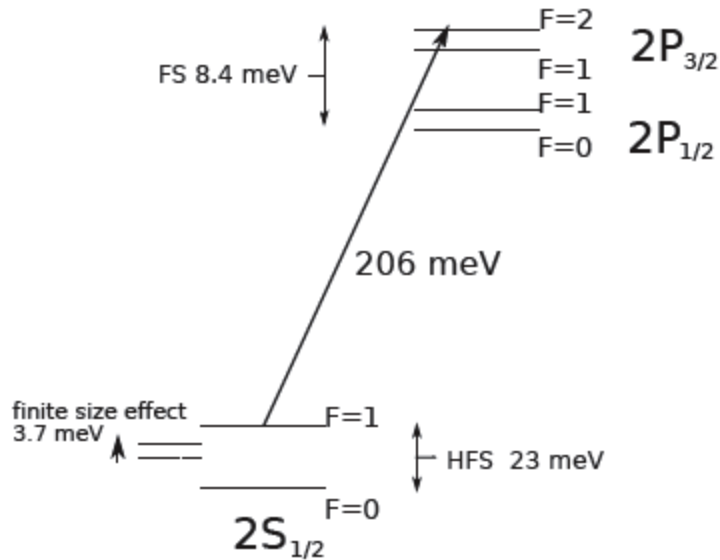


MAMI achieved 1% measurement of  $R_E$



Bernauer et al. (2010)

# extraction of $R_E$ from $\mu\text{H}$ Lamb shift



- Lamb shift is dominated by vacuum polarization : drops 2S state by a lot
  - Experiment measures 2S  $F=1$  to 2P $_{3/2}$   $F=2$  state (  $F$  is total angular momentum )
  - Finite size effect on s-wave states ( $l=0$ )
- Non-relativistic  $1\gamma$ -exchange calculation

$$\Delta E = \frac{2\pi\alpha}{3} R_E^2 \phi_n^2(0)$$

Karplus, Klein, Schwinger (1952)

- Leading term of order  $O(\alpha^4)$  :  $\phi_n^2(0) = m_r^3 \alpha^3 / (\pi n^3)$

## Lamb Shift

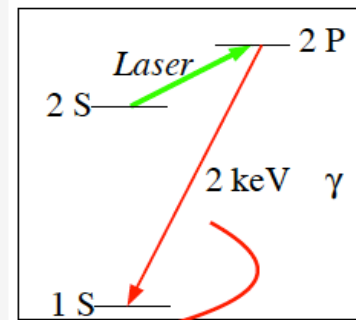
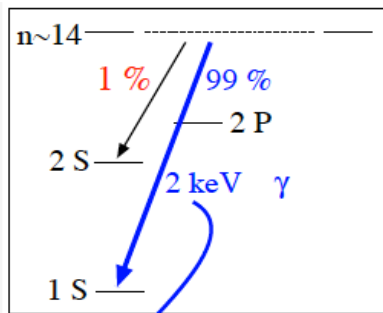
$$\Delta E_{LS} = 209.9779 (49) - 5.2262 R_E^2 + 0.00913 R_{(2)}^3 \text{ meV}$$

3.70 meV

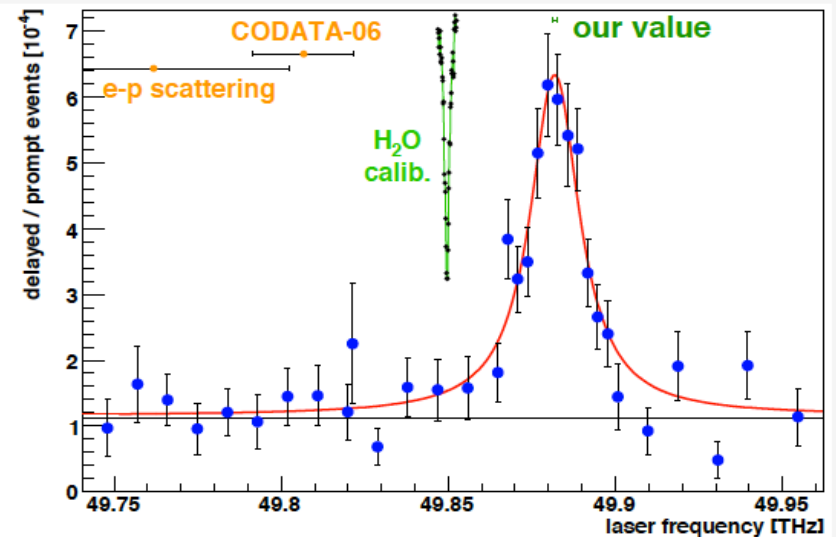
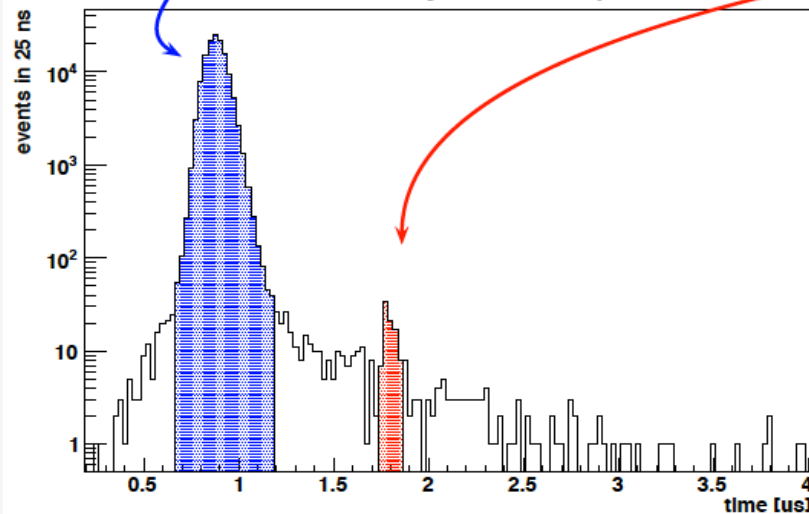
0.026 meV

$R_{(2)}^3$  :  $O(\alpha^5)$  correction term

# Principle of muonic Lamb shift experiment@PSI



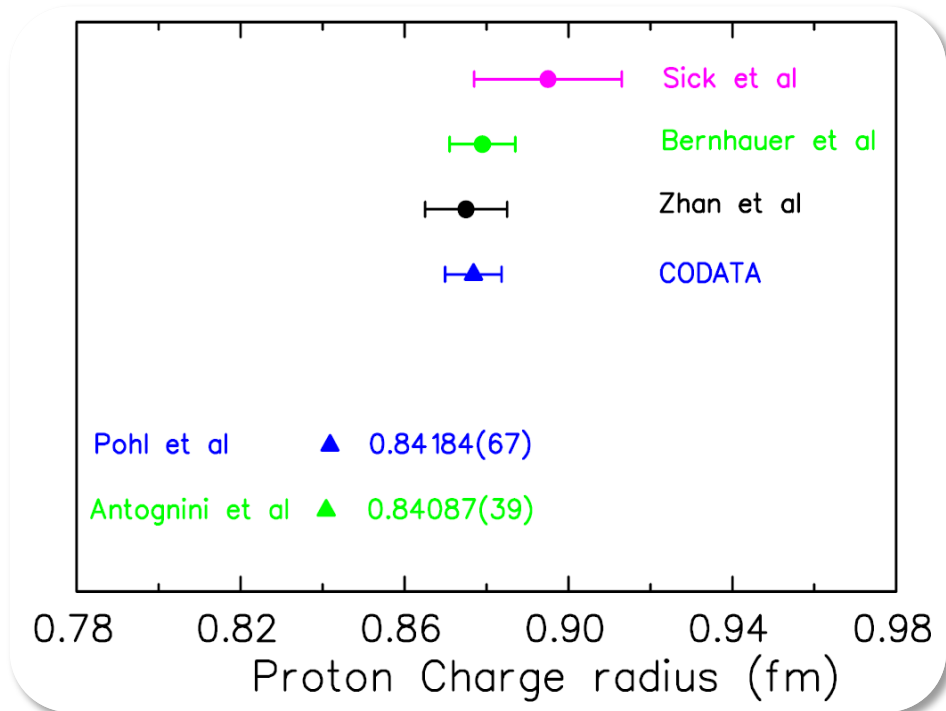
2 keV X-rays time spectrum



**Experimental precision  $\approx 2 \mu\text{eV}$**

**Energy shift ascribed to finite proton size is  $310 \mu\text{eV}$  less than expected !!!**

# Proton radius puzzle ?



**$\mu\text{H}$  data:**

$$R_E = 0.8409 \pm 0.0004 \text{ fm}$$

Pohl et al. (2010)

Antognini et al. (2013)



**$7\sigma$   
difference !?**

**ep-data :**

CODATA

$$R_E = 0.8772 \pm 0.0046 \text{ fm}$$

Bernauer et al. (2010)

Zhan et al. (2011)





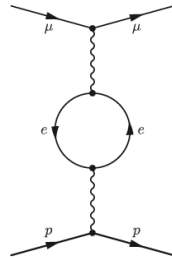
# Lamb shift: QED corrections

- Calculated by several groups

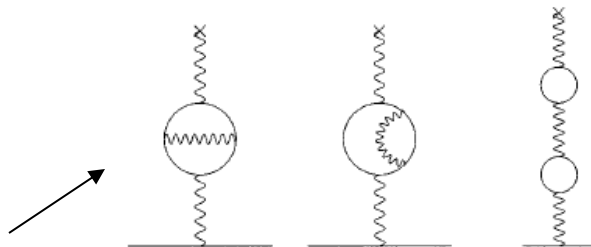
Pachucki (1996, 1999)

Borie (1976, 2005)

- 1 loop electron



$$\Delta E = 205.0282 \text{ meV}$$



$$\Delta E = 1.5081 \text{ meV}$$

- 2 loop electron



$$\Delta E = 0.1509 \text{ meV}$$

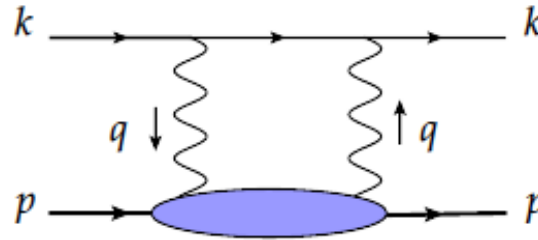
- Muon self-energy, vacuum polarization  $\Delta E = -0.6677 \text{ meV}$

- other QED corrections calculated : all of size 0.005 meV or smaller  $\ll 0.3 \text{ meV}$

# Lamb shift: hadronic corrections (I)

- **Finite-size** correction:

$\gamma\gamma$  box diagram



- “3rd Zemach moment”

non-rel. calculation

**Friar (1979)**

$$R_{(2)}^3 = \int d^3\vec{r}_1 d^3\vec{r}_2 |\vec{r}_1 - \vec{r}_2|^3 \rho_E(r_1) \rho_E(r_2)$$

$$= \frac{48}{\pi} \int_0^\infty \frac{dQ}{Q^4} \left[ G_E^2(Q) - 1 - 2Q^2 G_E(0) \frac{dG_E}{dQ^2}(0) \right]$$

recent evaluation

**Distler, Bernauer,  
Walcher (2011)**

$$R_{(2)}^3 = 2.85 (8) \text{ fm}^3 \longrightarrow \Delta E \approx -0.026 \text{ meV}$$

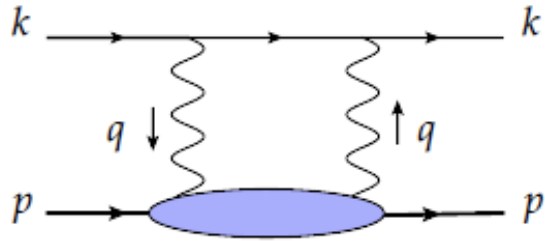
- What do we know model independently ?

Lower blob contains both elastic (nucleon) and in-elastic states

Information is contained in **forward, double virtual Compton scattering**

- For model estimates, see e.g. recent work of **Miller, Thomas, Carroll, Rafelski (2011)**

## Lamb shift: hadronic corrections (II)



$$\begin{aligned}
 T^{\mu\nu}(p, q) &= \frac{i}{8\pi M} \int d^4x e^{iqx} \langle p | T j^\mu(x) j^\nu(0) | p \rangle \\
 &= \left( -g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) T_1(\nu, Q^2) \\
 &+ \frac{1}{M^2} \left( p^\mu - \frac{p \cdot q}{q^2} q^\mu \right) \left( p^\nu - \frac{p \cdot q}{q^2} q^\nu \right) T_2(\nu, Q^2)
 \end{aligned}$$

- Lower blob contains both elastic (nucleon) and in-elastic states



**Hadron physics  
input required**

Information contained in **forward, double virtual Compton scattering**

- Described by two amplitudes **T1** and **T2**: function of energy  $\nu$  and virtuality  $Q^2$

$$\text{Im } T_1(\nu, Q^2) = \frac{1}{4M} F_1(\nu, Q^2)$$

- Imaginary parts of **T1**, **T2**: unpolarized structure functions of proton

$$\text{Im } T_2(\nu, Q^2) = \frac{1}{4\nu} F_2(\nu, Q^2)$$

- $\Delta E$  evaluated through an integral over  $Q^2$  and  $\nu$

$$\begin{aligned}
 \Delta E &= \Delta E^{el} \quad \rightarrow \text{Elastic state: involves } \mathbf{nucleon \ form \ factors} \\
 &+ \Delta E^{subtr} \quad \rightarrow \text{Subtraction: involves } \mathbf{nucleon \ polarizabilities} \\
 &+ \Delta E^{inel} \quad \rightarrow \text{Inelastic, dispersion integrals: involves } \mathbf{structure \ functions \ F1, \ F2}
 \end{aligned}$$

# Lamb shift: hadronic corrections (III)

- Low-energy expansion of forward, doubly virtual Compton scattering constrains subtraction term  $T_1(0, Q^2)$

effective Hamiltonian : 
$$\mathcal{H} = -\frac{1}{2}4\pi\alpha_E\vec{E}^2 - \frac{1}{2}4\pi\beta_M\vec{B}^2$$

↓ electric      ↓ magnetic      polarizabilities

$$\lim_{\nu^2, Q^2 \rightarrow 0} T_1^{\text{non-Born}}(\nu, Q^2) = \frac{\nu^2}{e^2}(\alpha_E + \beta_M) + \frac{Q^2}{e^2}\beta_M$$

$$\lim_{\nu^2, Q^2 \rightarrow 0} T_2^{\text{non-Born}}(\nu, Q^2) = \frac{Q^2}{e^2}(\alpha_E + \beta_M)$$

subtraction term for  $T_1$

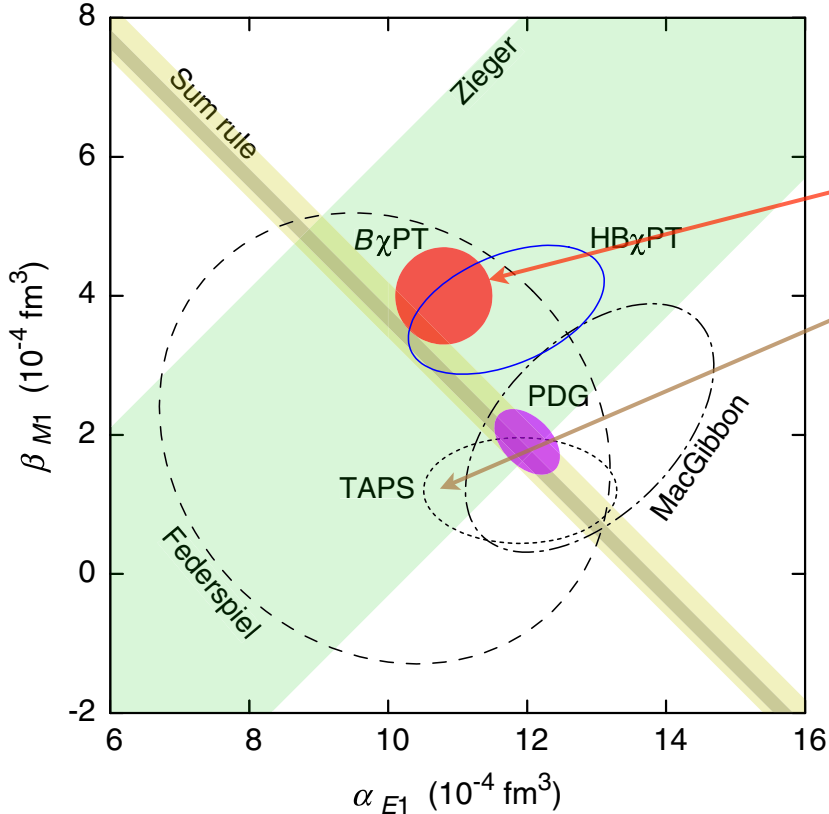
- Numerical evaluations :

( $\mu\text{eV}$ )	Carlson, Vdh (2011)	Pachucki (1999)	Martyntenko (2006)
$\Delta E^{\text{subt}}$	$5.3 \pm 1.9$	1.8	2.3
$\Delta E^{\text{inel}}$	$-12.7 \pm 0.5$	-13.9	-13.8
$\Delta E^{\text{cl}}$	$-29.5 \pm 1.3$	-23.0	-23.0
$\Delta E$	$-36.9 \pm 2.4$	-35.1	-34.5

$\Delta E = (-36.9 \pm 2.4) \mu\text{eV}$  or about 12% of the needed correction ...  
 present experimental precision: 2  $\mu\text{eV}$

# Static Polarizability Status

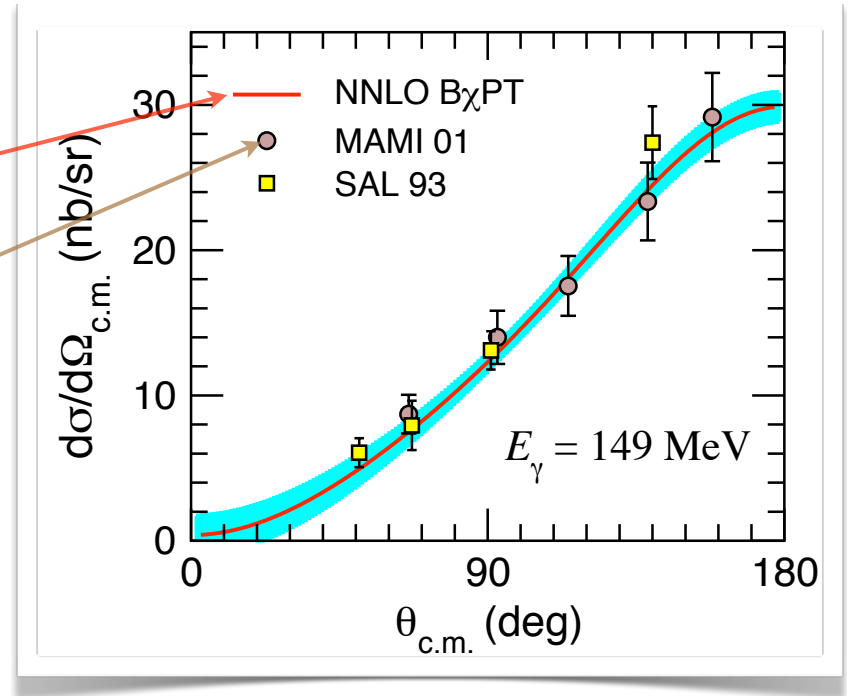
Krupina, Pascalutsa (2013)



Theory:

BChPT - Lensky & V.P., EPJC(2010)

HBChPT - Griesshammer, McGovern, Phillips, EPJA (2013)



$$\beta_{M1} = (1.9 \pm 0.5) \times 10^{-4} \text{ fm}^3 \text{ [PDG]}$$

$$\beta_{M1} = (4.0 \pm 0.7) \times 10^{-4} \text{ fm}^3 \text{ [BChPT@NNLO]}$$

**3 $\sigma$  difference**

**More precise measurement of  $\beta_M$  underway at A2@MAMI using linearly polarized photons**

# Proton radius puzzle: what could it mean ?

- unknown correction ? ...after known constraints have been built in !

- Change in Rydberg constant ?

In absence of further (sizeable) corrections, use of muonic extraction of  $R_E$  plugged into electron H Lamb shift yields  $R_\infty$  which is  $4.9\sigma$  away from CODATA value (and factor 4.6 more precise)

Pohl et al. (2010)

- New physics ?

- explain  $3\sigma$   $(g-2)_\mu$  discrepancy AND  $7\sigma$   $R_E$  discrepancy from  $\mu$ H Lamb shift simultaneously invoking a correction by a hypothetical light boson ?

- $(g-2)_e$  puts strong limit on coupling to e  $\rightarrow$  much smaller,

Non-universality e  $-\mu$  ?

- New parity violating muonic forces ?

- Can rare Kaon decay data help ?

Tucker, Smith (2010)

Barger, Chiang, Keung, Marfatia (2011)

Batell, McKeen, Pospelov (2011)

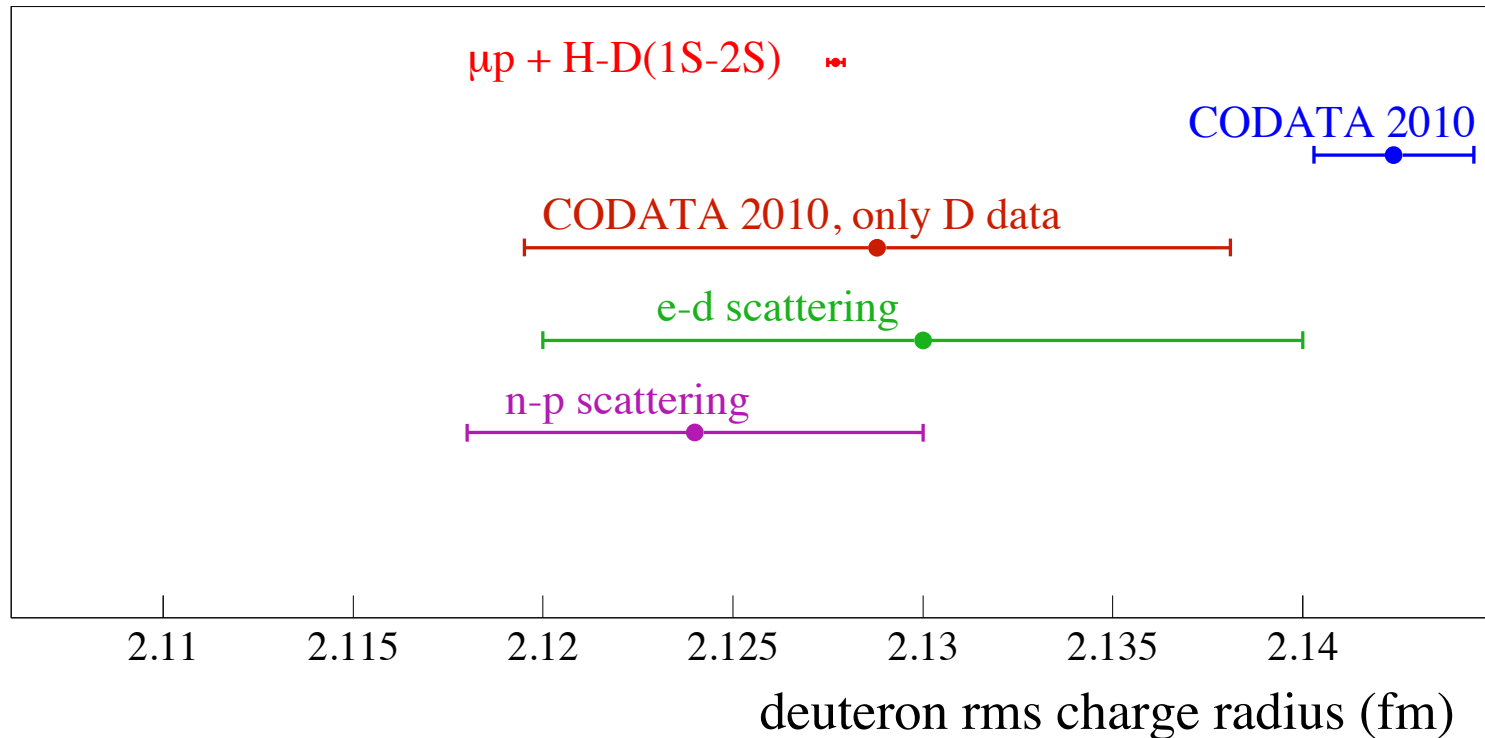
Brax, Burrage (2011)

Rislow, Carlson (2012)

# Proton radius puzzle: what's next ?

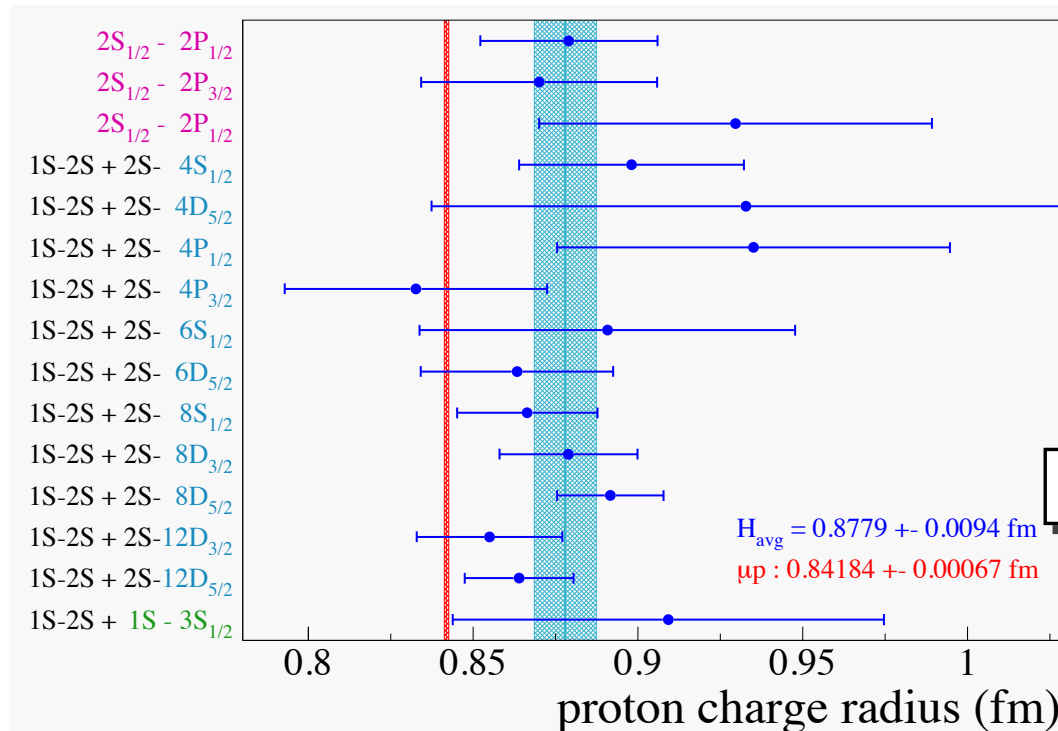
- **Muonic** Lamb shift : muonic D, muonic  $^3\text{He}$  measurements planned

H/D isotope shift (1S-2S):  $r_d^2 - r_p^2 = 3.82007 \pm 0.00065 \text{ fm}^2$  Parthey et al. (2010)



# Proton radius puzzle: what's next ?

- **Muonic** Lamb shift : muonic D, muonic  $^3\text{He}$  measurements planned
  - **Electronic** H Lamb shift : higher accuracy measurement very timely
- New proposal (York Univ, Canada) :  $R_E$  to 0.7%





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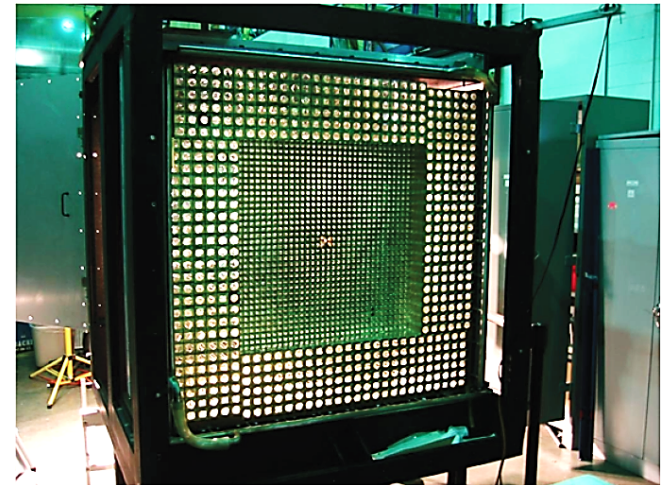
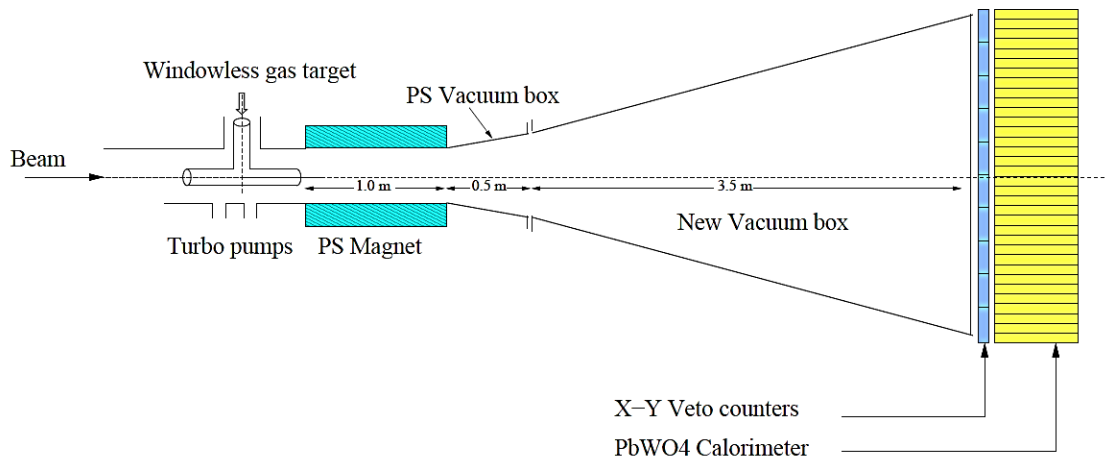
New proposal (York Univ, Canada) :  $R_E$  to 0.7%

- new  $G_{Ep}$  measurements at very low  $Q^2$  down to  $Q^2 \approx 2 \times 10^{-4} \text{ GeV}^2$

**JLAB/Hall B** approved expt : magnetic-spectrometer-free experiment (HyCal)

$$Q^2 = 2 \times 10^{-4} - 2 \times 10^{-2} \text{ GeV}^2$$

ep  $\rightarrow$  ep cross sections normalized to Moller scattering



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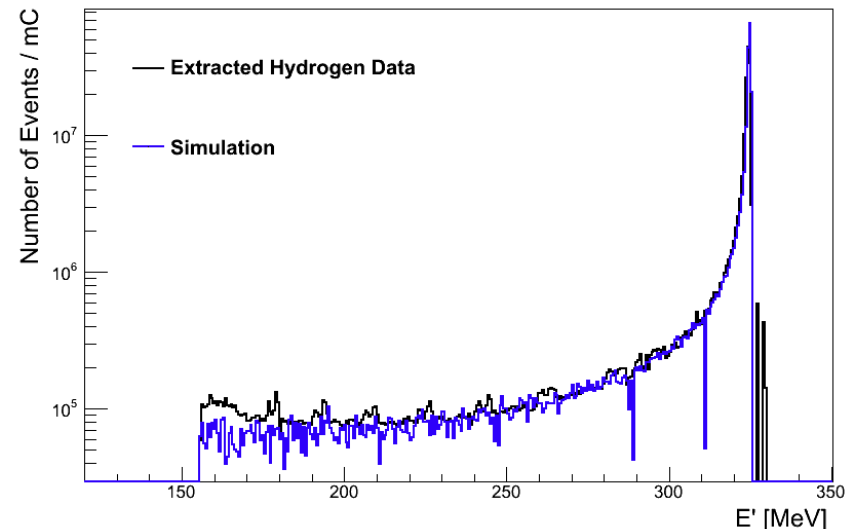
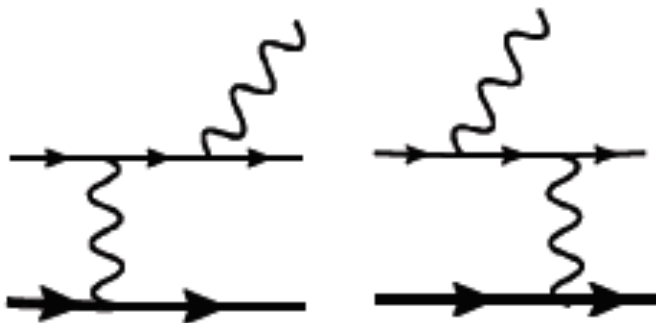
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**MAMI/A1** : using initial state radiation  
(2013)



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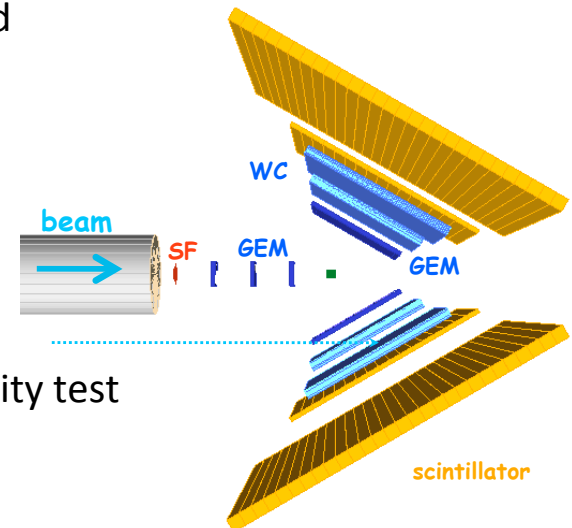
(2013)

- $\mu - p$  scattering (MUSE) at low  $Q^2$  at PSI: (2015 – 2017)

simultaneous measurement of  $\mu^\pm p$  and  $e^\pm p$ : lepton universality test

$$0.002 \text{ GeV}^2 < Q^2 < 0.07 \text{ GeV}^2$$

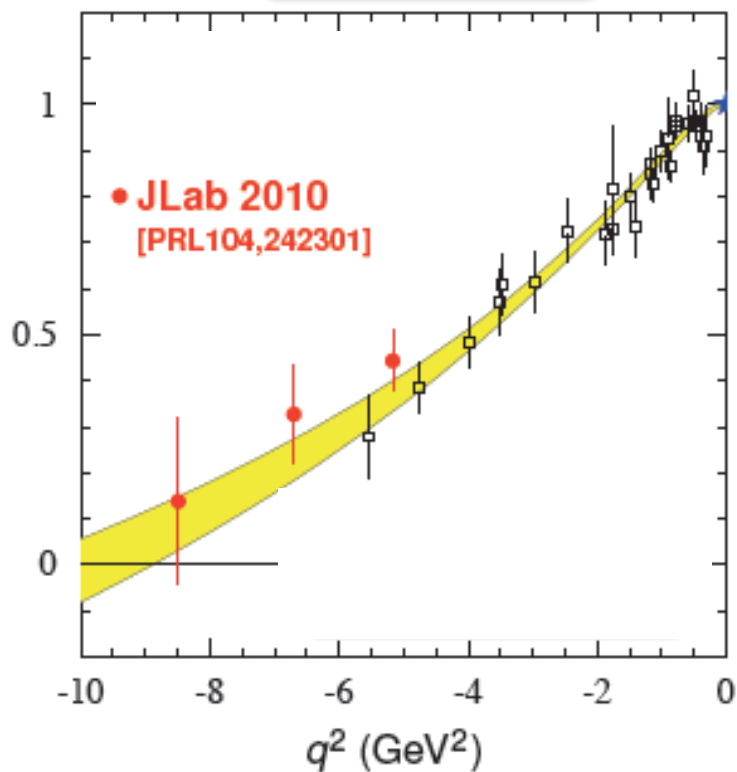
2 beam polarities give  $2\gamma$  exchange test



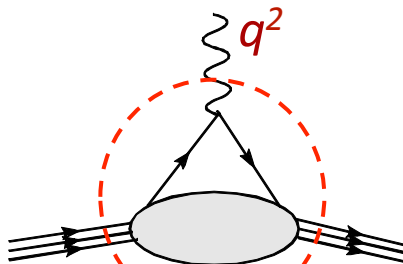
# Complementing hadron structure in space- and timelike regions

## Proton spacelike form factors

$$e^- p \rightarrow e^- p$$



JLab, MAMI

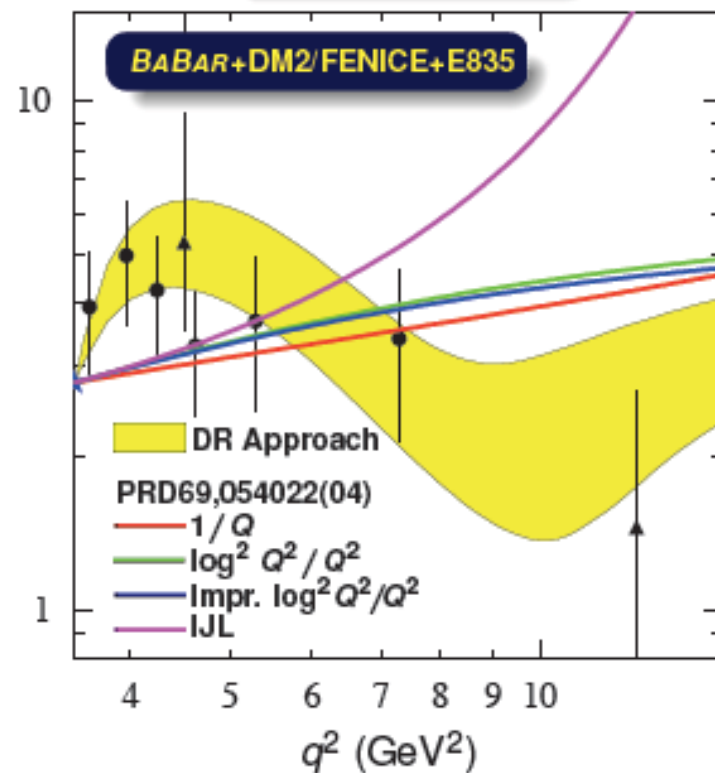


y-axis:

$$\mu_p G_E / G_M$$

## Proton timelike form factors

$$e^+ e^- \rightarrow p \bar{p}$$

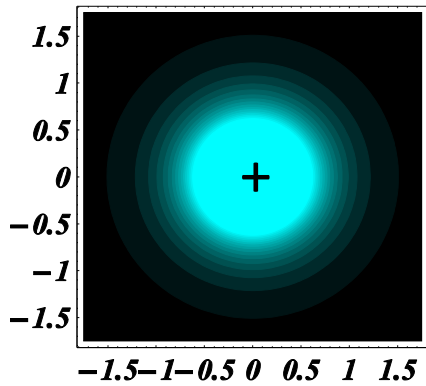


BES-III, PANDA

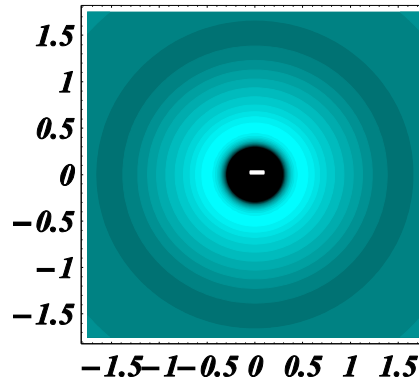
# Spatial imaging of hadrons

**Charge**, mass, spin densities of quarks in a hadron

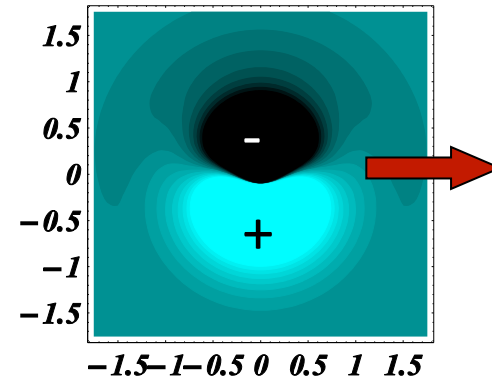
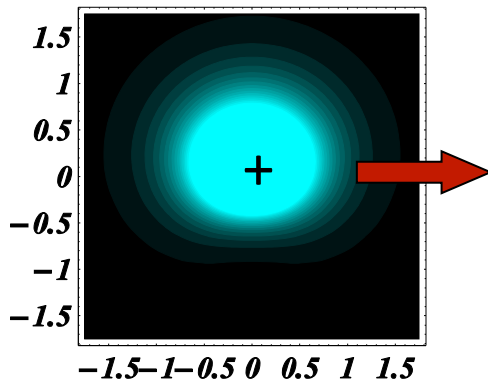
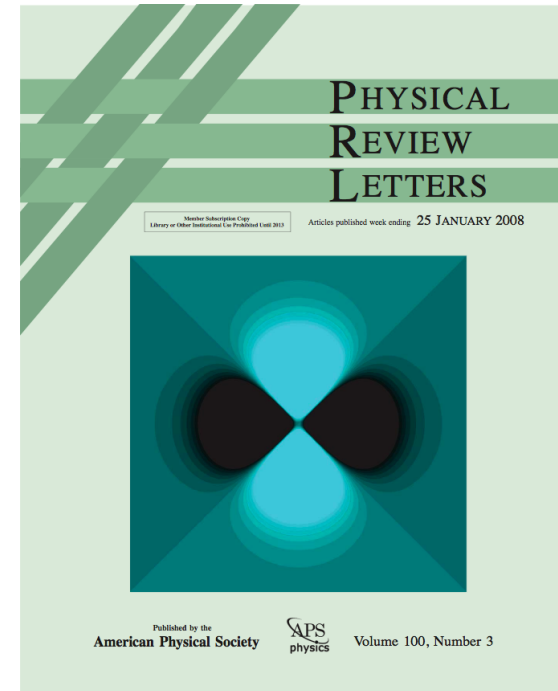
proton



neutron



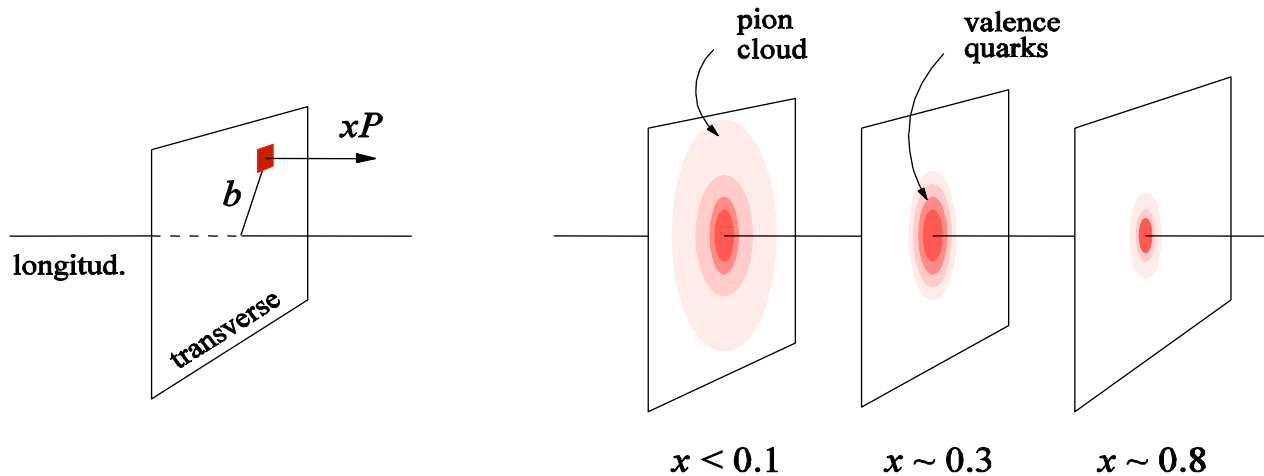
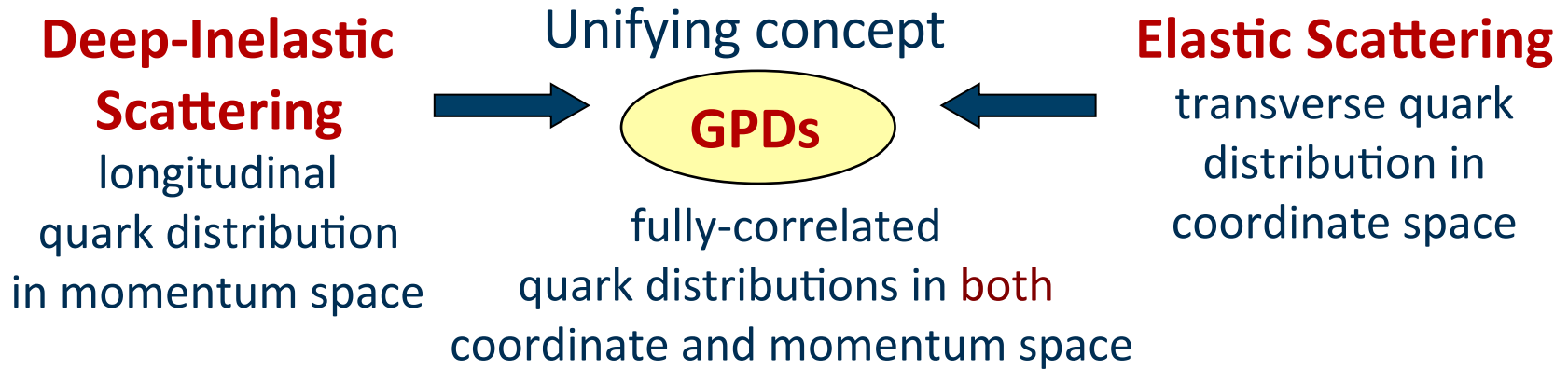
$p \rightarrow \Delta^+ (1232)$



Miller (2007)

Carlson, Vdh (2008)

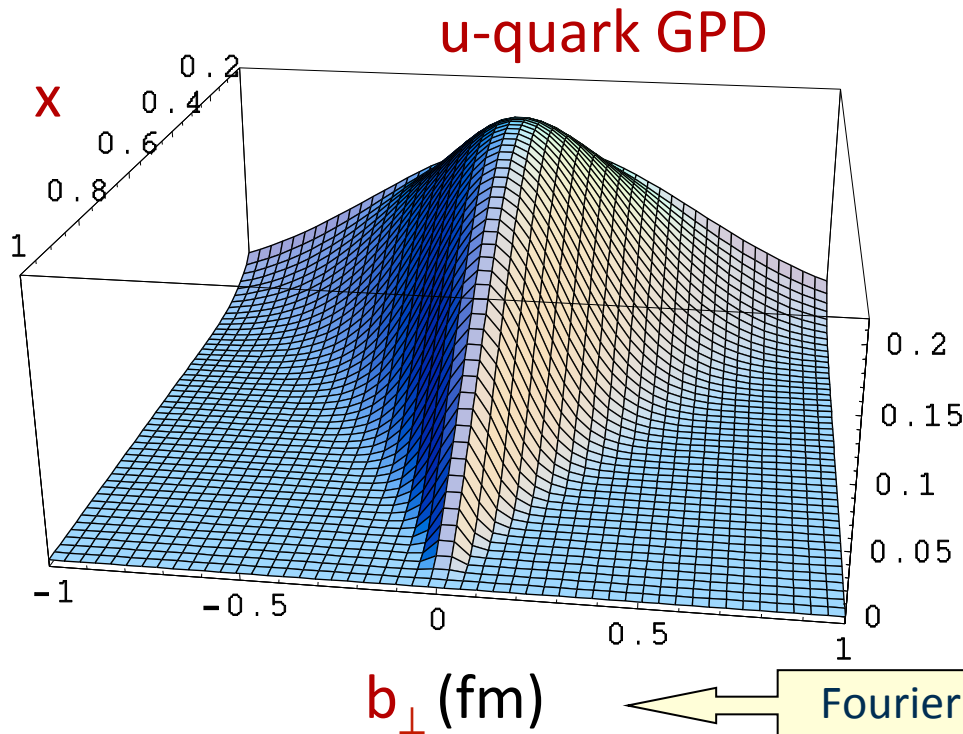
# Generalized Parton Distributions (GPDs): 3D image of hadrons



Burkardt (2000, 2003),  
Belitsky, Ji, Yuan (2004)

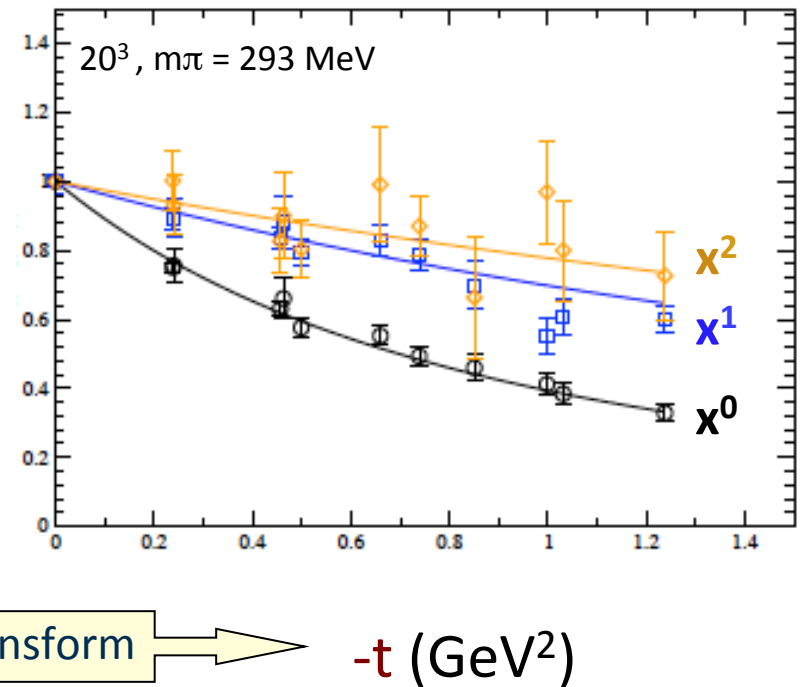
# GPDs: transverse image of hadrons

**GPDs:** quark distributions w.r.t. longitudinal momentum  $x$  and transverse position  $b_{\perp}$



**lattice QCD:**  
moments of GPDs

$x^n$  moment of u-d GPD



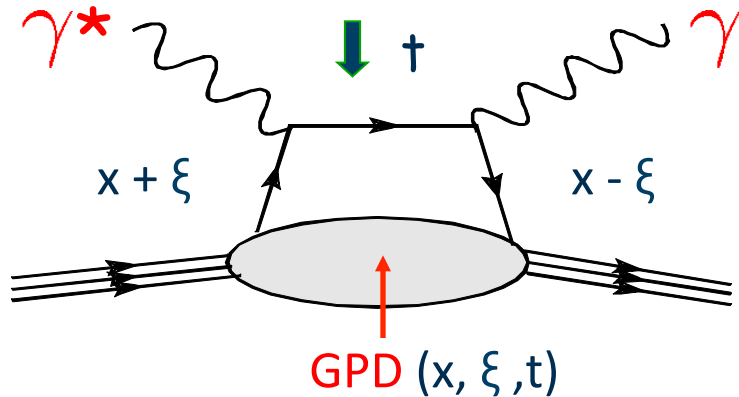
Guidal, Polyakov, Radyushkin, Vdh (2005)

Diehl, Feldmann, Jakob, Kroll (2005)

LHPC Coll.

# QCD factorization: tool to access GPDs

$Q^2 \gg 1 \text{ GeV}^2$

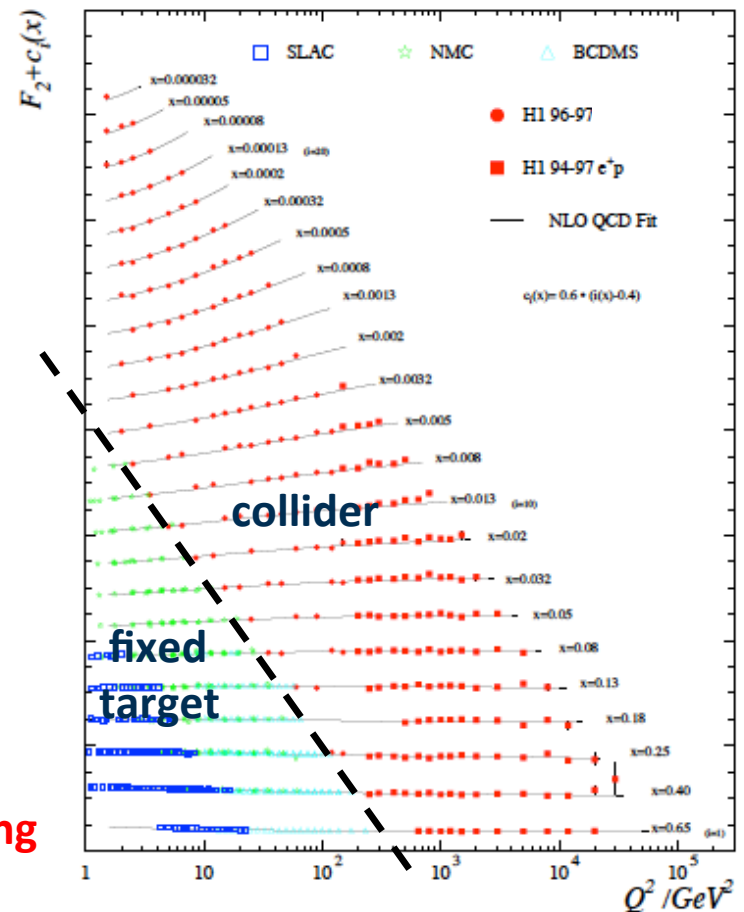


➔ at large  $Q^2$ : **QCD factorization theorem** :  
hard exclusive process described by **GPDs**  
model independent !

Müller et al.(1994), Ji(1995), Radyushkin(1995),  
Collins, Frankfurt, Strikman (1996)

➔ **KEY**  $Q^2$  leverage required to test **QCD scaling**

world data on proton **F2**





# „complete“ picture of nucleon

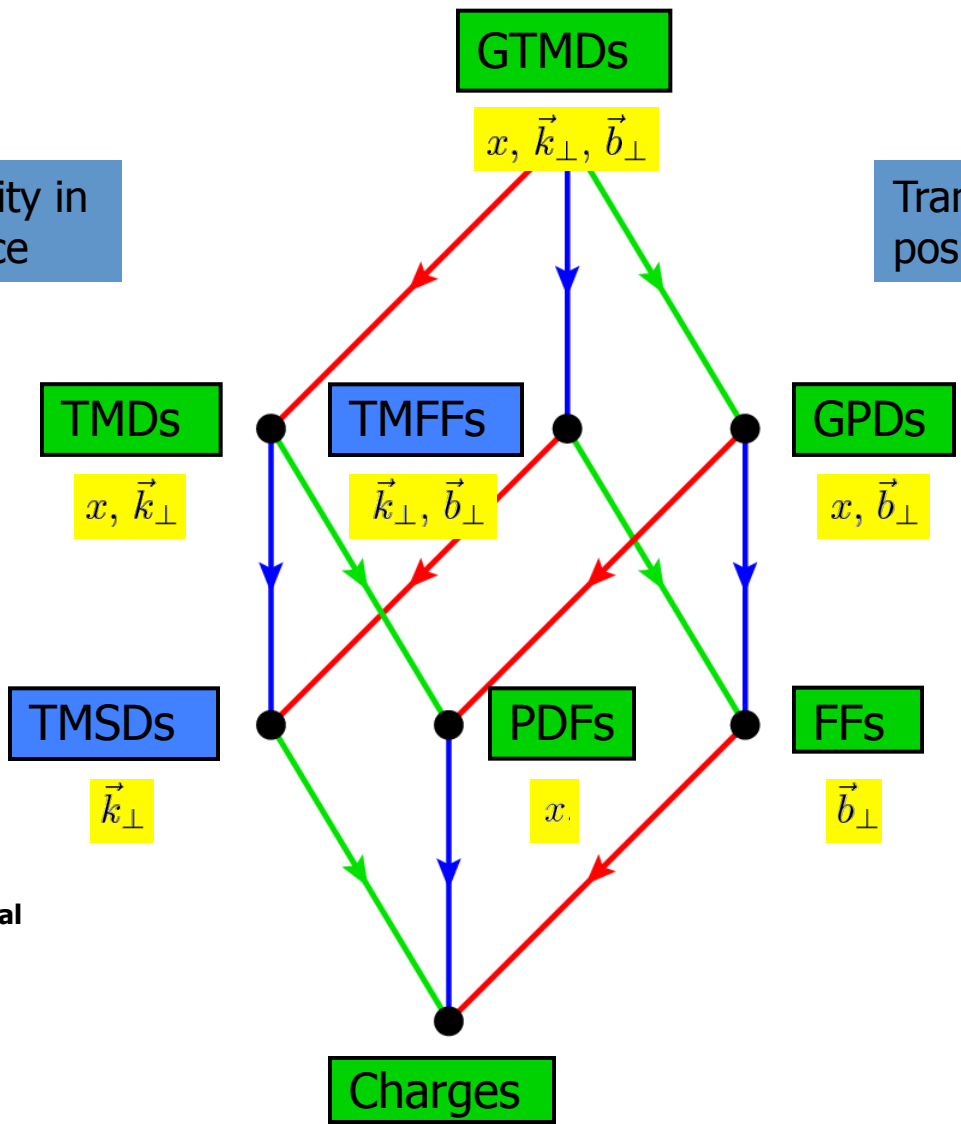
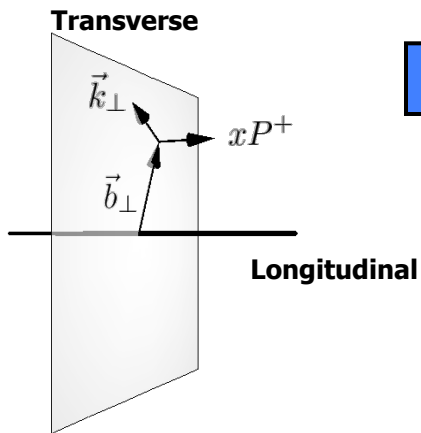
$$\xi = 0$$

Momentum space	$\vec{k}_\perp \leftrightarrow \vec{z}_\perp$	Position space
	$\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$	

Transverse density in momentum space

Transverse density in position space

Lorcé (2011)

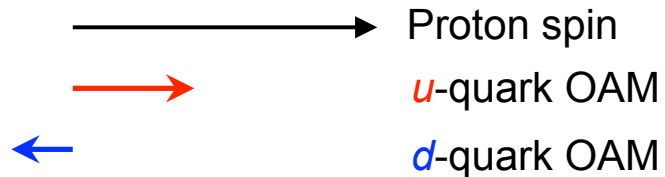
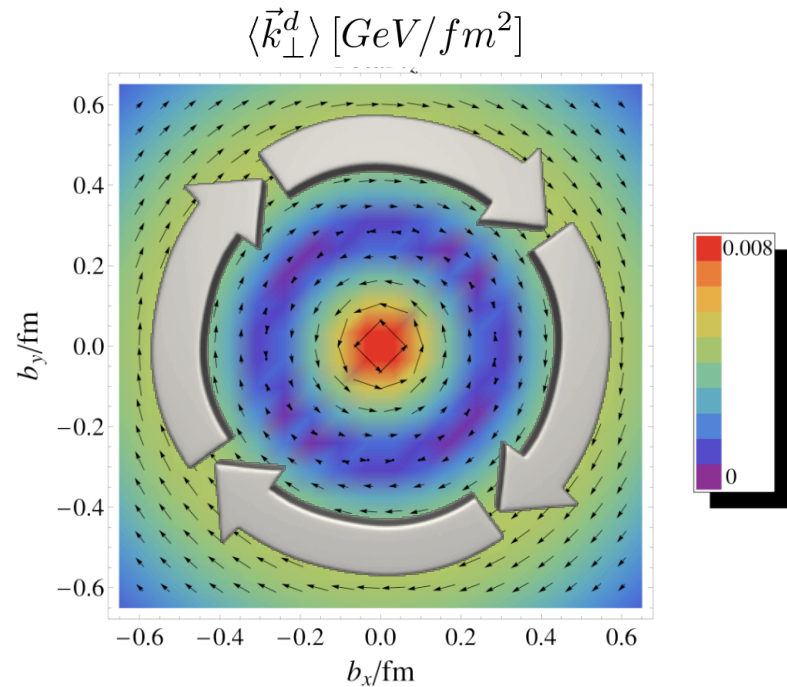
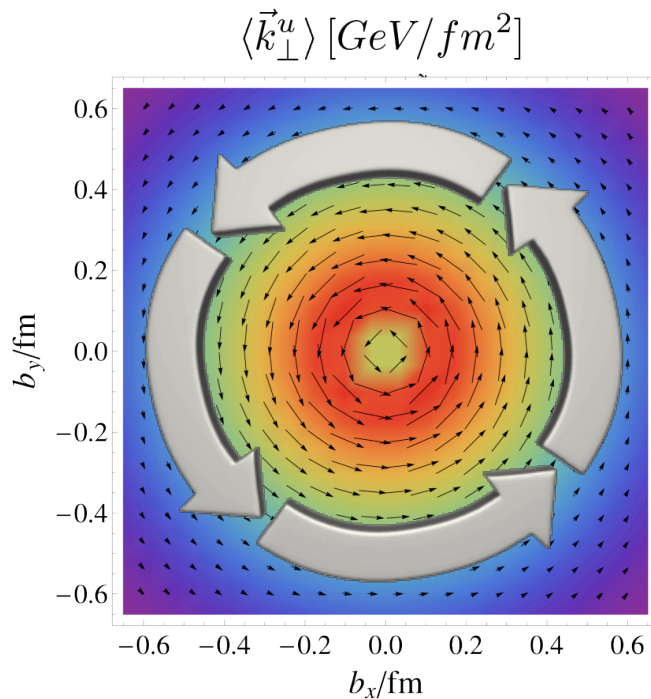


- $\int d^2 b_\perp$
- $\int dx$
- $\int d^2 k_\perp$

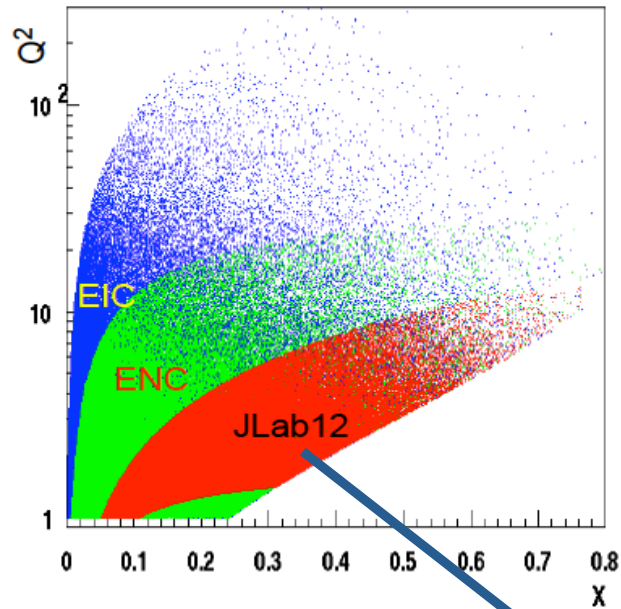
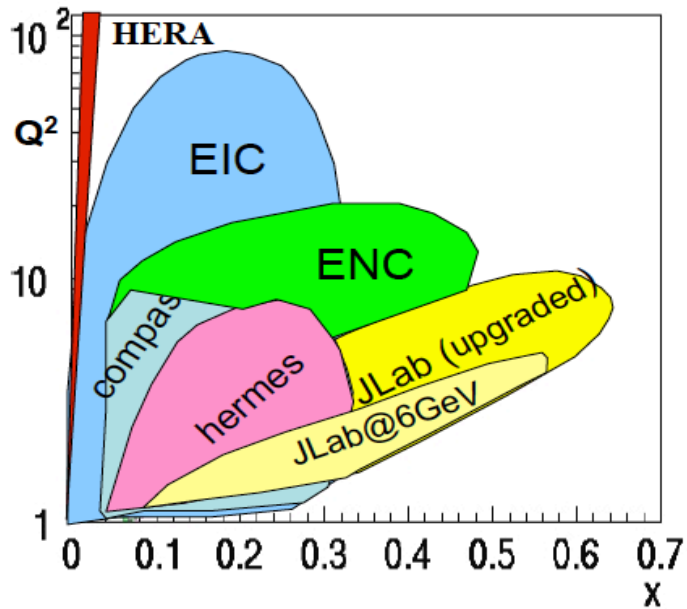
# Quark orbital angular momentum in proton

$$\ell_z^q = \int d^2\vec{b}_\perp (\vec{b}_\perp \times \langle \vec{k}_\perp^q \rangle)_z$$

$$\langle \vec{k}_\perp^q \rangle = \int dx d^2\vec{k}_\perp \vec{k}_\perp \rho_{LU}^q(x, \vec{b}_\perp, \vec{k}_\perp)$$



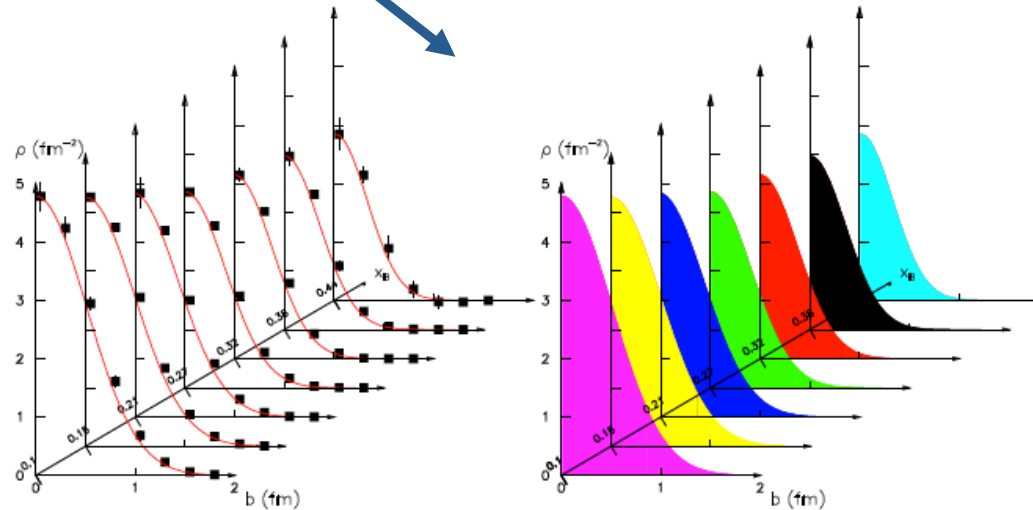
# Energy-luminosity frontier in lepton-nucleon physics



JLab 11 GeV  
projections:

Guidal, Moutarde, Vdh  
(2013)

- **High-energy, high-luminosity facilities:** Compass, JLab@11 GeV, collider projects (EIC, ENC, ...)
- **Global nucleon structure analysis effort required**



# CONCLUSIONS

- **Strong interplay between high-energy ↔ precision ↔ low-energy frontiers**
- **Impact of hadron physics on new physics searches:**  
 $(g-2)_\mu$ ,  $Q_{\text{weak}}$ , new dark photon searches
- **Unraveling hadron structure in strong QCD:**
  - proton radius puzzle has shaken textbook beliefs
  - combination of new experiments + theory opens perspectives for an imaging of hadrons to an unprecedented level of detail