Where does the proton spin come from?

- Quark and glue spins -- status
- Gauge field tensor operator
- Momentum and angular momentum sum rules and renormalization
- Lattice calculation

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KITPC – July 17, 2012

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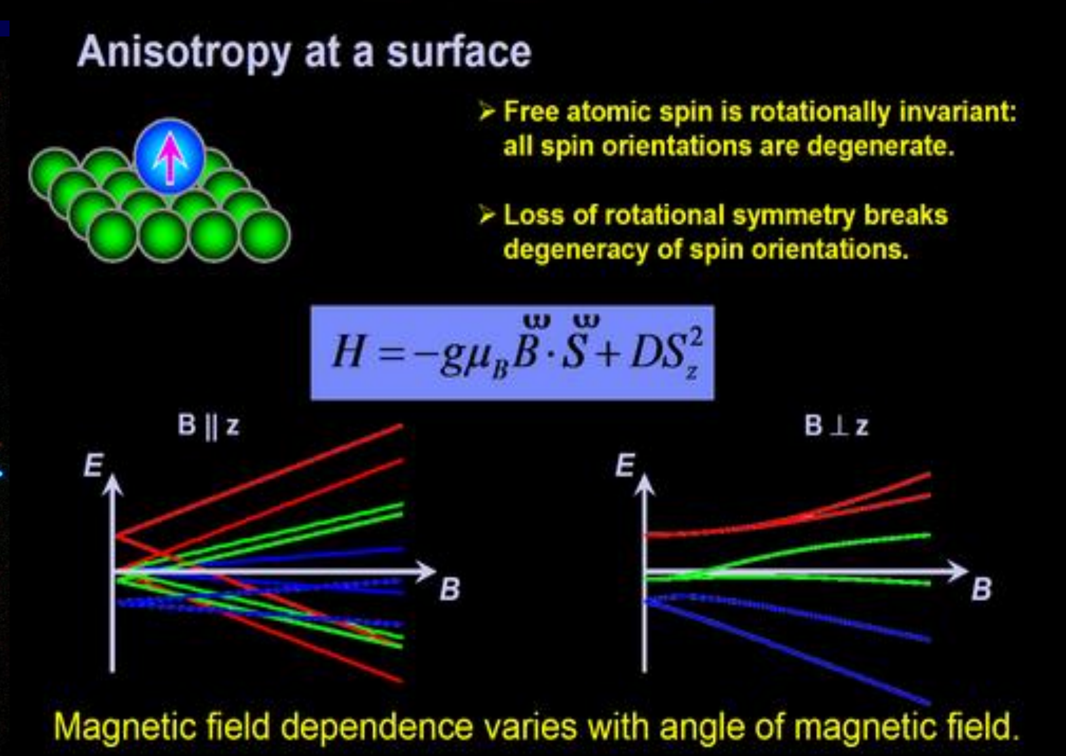
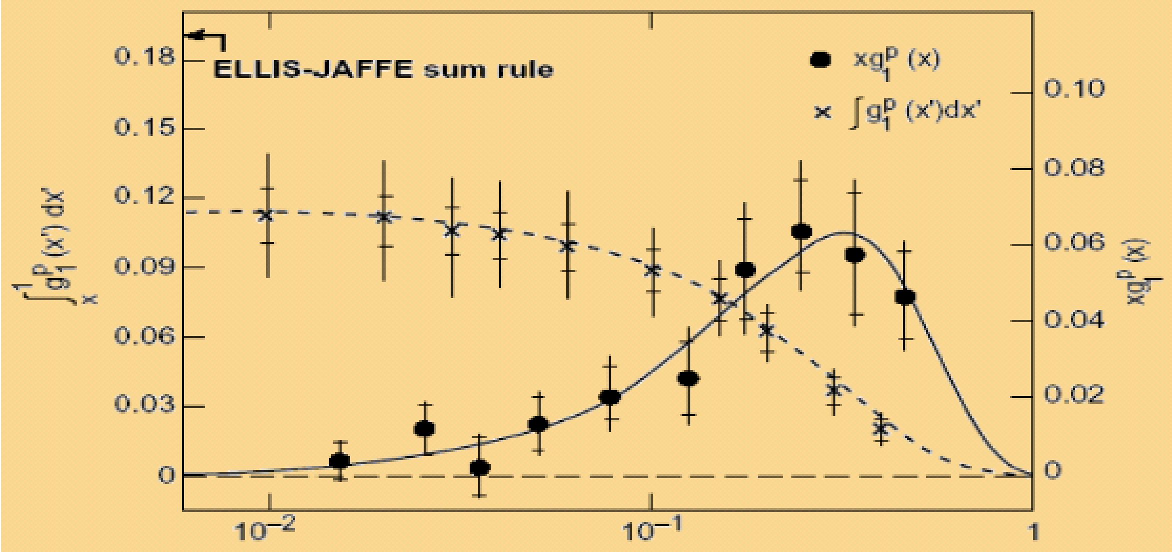


Twenty 4 years since the "spin crisis"

- EMC experiment in 1988/1989 – "the plot":



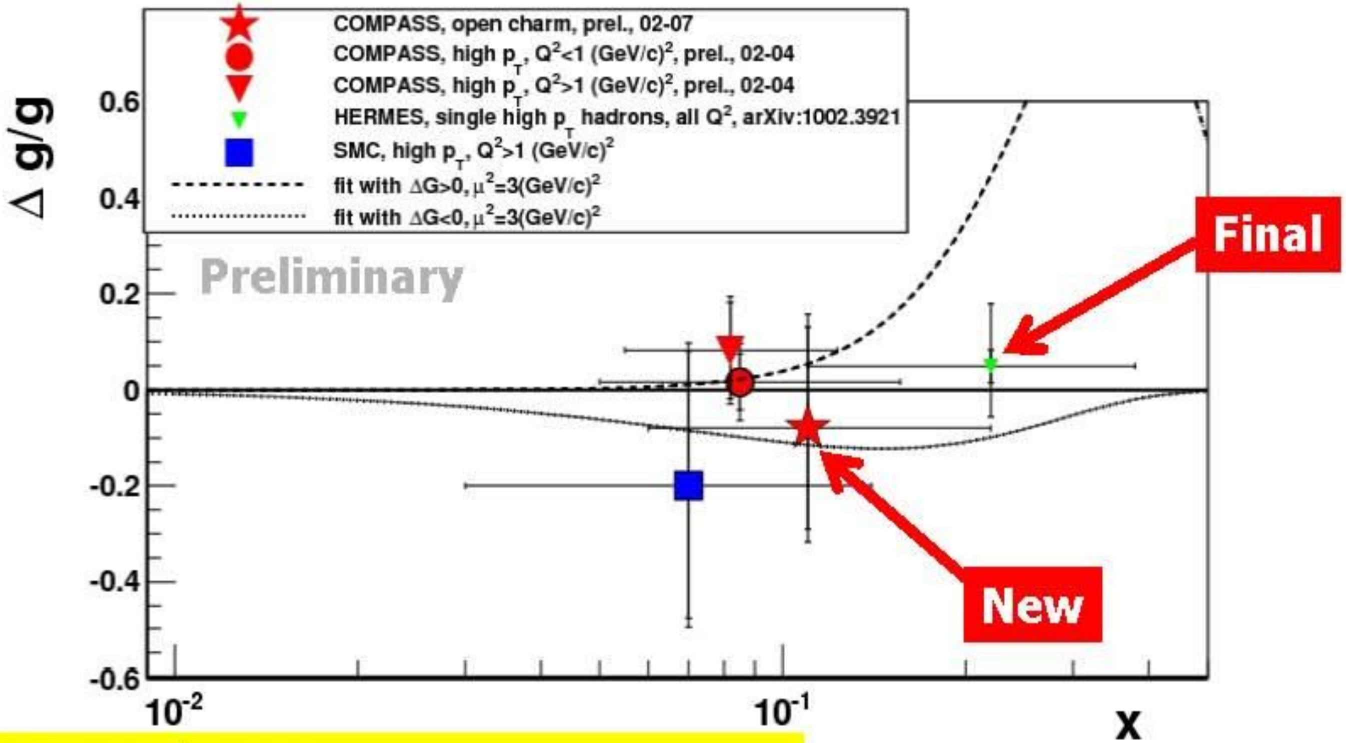
$$g_1(x) = \frac{1}{2}\sum_q e_q^2\left[\Delta q(x) + \Delta\bar{q}(x)\right] + \mathcal{O}(\alpha_s) + \mathcal{O}(1/Q)$$

$$\Delta q = \int_0^1 dx\Delta q(x) = \langle P, s_{||}|\overline{\psi}_q(0)\gamma^+\gamma_5\psi_q(0)|P, s_{||}\rangle$$

- "Spin crisis" or puzzle: $\Delta\Sigma = \sum_q \Delta q + \Delta\bar{q} = 0.2 - 0.3$

Summary Gluon Polarization

Presently all Analysis in LO only



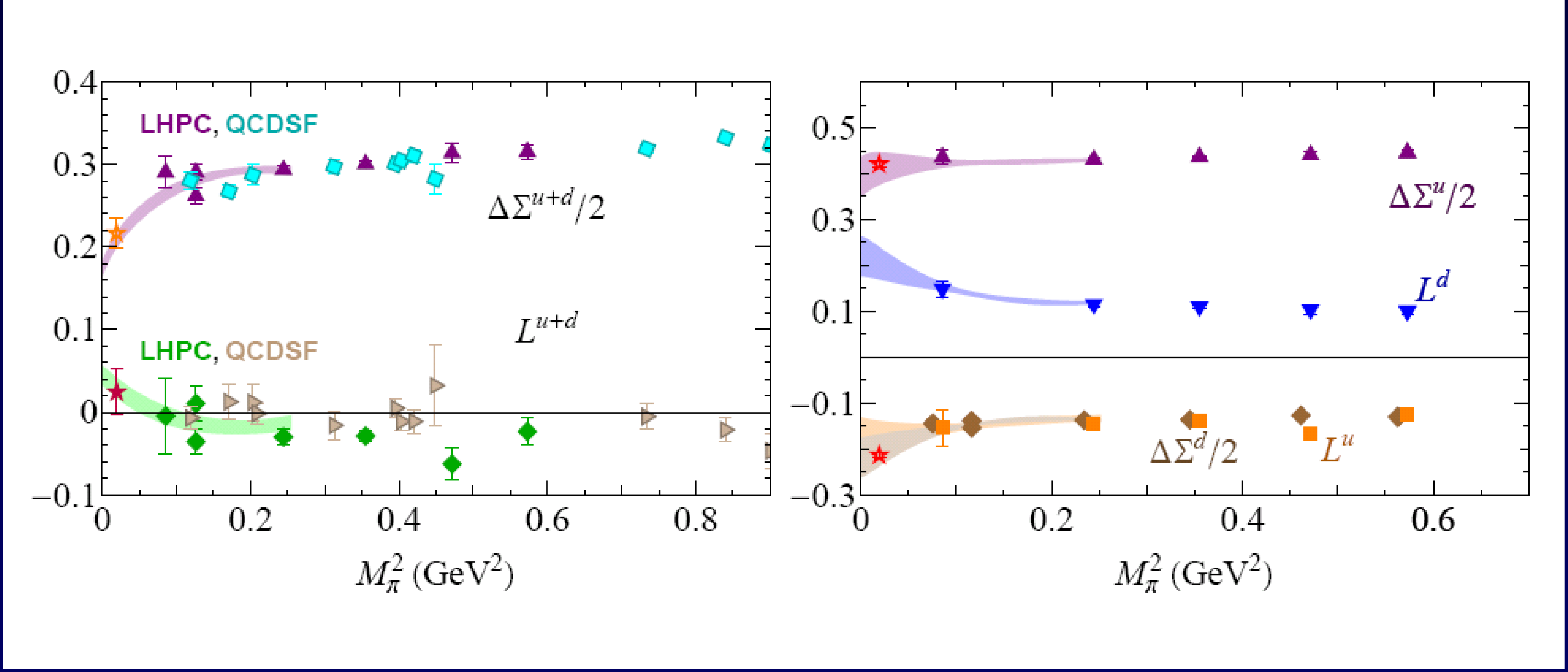
COMPASS Open Charm:

$\Delta G/G$=-0.08+- 0.21(stat) +- 0.11(sys.)
(Systematic error still under investigations)

(Value supersedes previous publication)

C.Franco

Quark Orbital Angular Momentum (connected insertion)



Status of Proton Spin

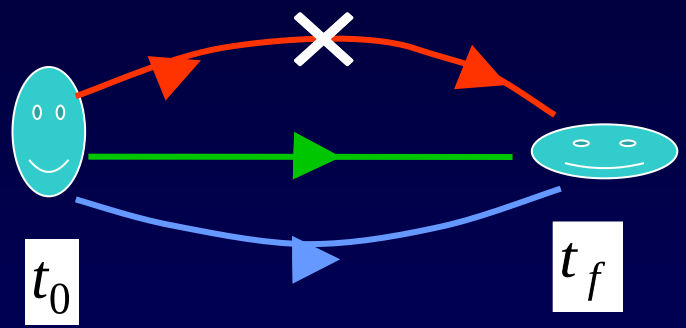
- Quark spin $\Delta\Sigma \sim 20 - 30\%$ of proton spin (DIS, Lattice)
- Quark orbital angular momentum? (lattice calculation (LHPC,QCDSF)→ ~ 0)
- Glue spin $\Delta G/G$ small (COMPASS, STAR) ?
- Glue orbital angular momentum is zero (Brodsky and Gardner) ?

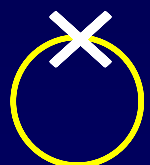
→ Dark Spin Crisis

Hadron Structure with Quarks and Glue

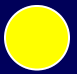
- Quark and Glue Momentum and Angular Momentum in the Nucleon

$$(\bar{u}\gamma_\mu D_\nu u + \bar{d}\gamma_\mu D_\nu d)(t)$$

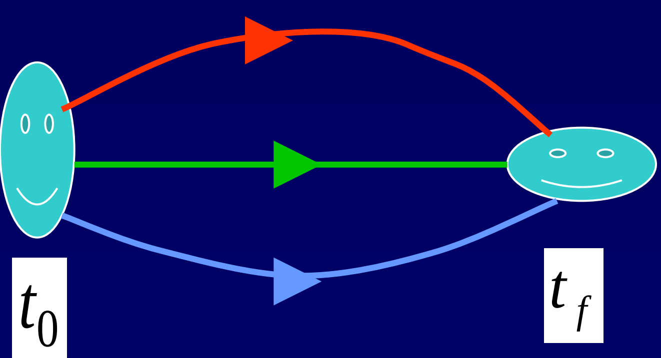


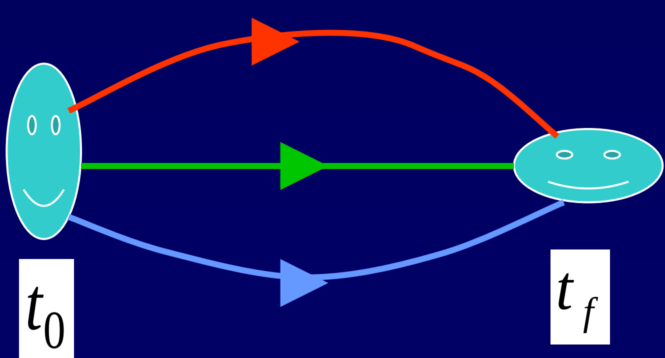


$$\bar{\Psi}\gamma_\mu D_\nu \Psi(t)(u,d,s)$$



$$F_{\mu\alpha}F_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}F^2$$





Momenta and Angular Momenta of Quarks and Glue

- Energy momentum tensor operators decomposed in quark and glue parts gauge invariantly --- Xiangdong Ji (1997)

$$T^{q}_{\mu\nu} = \frac{i}{4}\left[\bar{\psi}\gamma_{\mu}\overleftrightarrow{D}_{\nu}\psi + (\mu \leftrightarrow \nu)\right] \rightarrow \vec{\mathrm{J}}_{q} = \int d^{3}x\left[\frac{1}{2}\bar{\psi}\vec{\gamma}\gamma_{5}\psi + \vec{x}\times\bar{\psi}\gamma_{4}(-i\vec{D})\psi\right]$$

$$T^{g}_{\mu\nu} = F_{\mu\lambda}F_{\lambda\nu} - \frac{1}{4}\delta_{\mu\nu}F^{2} \rightarrow \vec{\mathrm{J}}_{g} = \int d^{3}x\left[\vec{x}\times(\vec{E}\times\vec{B})\right]$$

- Nucleon form factors

$$\langle p,s\,|\,T_{\mu\nu}\,|\,p's'\rangle = \bar{u}(p,s)[T_{1}(q^{2})\gamma_{\mu}\bar{p}_{\nu} - T_{2}(q^{2})\bar{p}_{\mu}\sigma_{\nu\alpha}q_{\alpha}/2m$$

$$-\mathrm{i}T_{3}(q^{2})(q_{\mu}q_{\nu} - \delta_{\mu\nu}q^{2})/m + T_{4}(q^{2})\delta_{\mu\nu}m/2]u(p's')$$

- Momentum and Angular Momentum

$$Z_{q,g}T_{1}(0)_{q,g}\ \ [\mathrm{OPE}] \rightarrow \langle x\rangle_{q/g}(\mu,\overline{\mathrm{MS}}),\quad Z_{q,g}\left[\frac{T_{1}(0)+T_{2}(0)}{2}\right]_{q,g} \rightarrow J_{q/g}(\mu,\overline{\mathrm{MS}})$$

$T_1(q^2)$ and $T_2(q^2)$

- 3-pt to 2-pt function ratios

$$G^{3pt}_{\mu\nu}(\vec{p},t_2;\vec{q},t_1)=\sum_{\vec{x}_1,\vec{x}_2} e^{-i\vec{p}\cdot\vec{x}_2+i\vec{q}\cdot\vec{x}_1}\left\langle 0\,|\,T\left[\chi_N(\vec{x}_2,t_2)T_{\mu\nu}(t_1)\bar{\chi}_N(0)\right]\right\rangle$$

$$\mathrm{Tr}\left[\Gamma_m G^{3pt}_{\mu\nu}(\vec{p}=0,t_2;\vec{q},t_1)\right]=We^{-m(t_2-t_1)}e^{-Et_1}\left[T_1(q^2)+T_2(q^2)\right]$$

- Need both polarized and unpolarized nucleon and different kinematics (p_i, q_j, s) to separate out $T_1(q^2)$, $T_2(q^2)$ and $T_3(q^2)$

Renormalization and Quark-Glue Mixing

Momentum and Angular Momentum Sum Rules

$$\langle x\rangle_q^R = Z_q \langle x\rangle_q^L, \quad \langle x\rangle_g^R = Z_g \langle x\rangle_g^L,$$

$$J_q^R = Z_q J_q^L, \quad J_g^R = Z_g J_g^L,$$

$$\left.\begin{array}{l} Z_q \langle x\rangle_q^L + Z_g \langle x\rangle_g^L = 1, \\ Z_q J_q^L + Z_g J_g^L = \frac{1}{2} \end{array}\right. \Rightarrow \begin{cases} Z_q T_1^q(0) + Z_g T_1^g(0) = 1, \\ Z_q (T_1^q + T_2^q)(0) + Z_g (T_1^g + T_2^g)(0) = 1, \\ Z_q T_2^q(0) + Z_g T_2^g(0) = 0 \end{cases}$$

Mixing

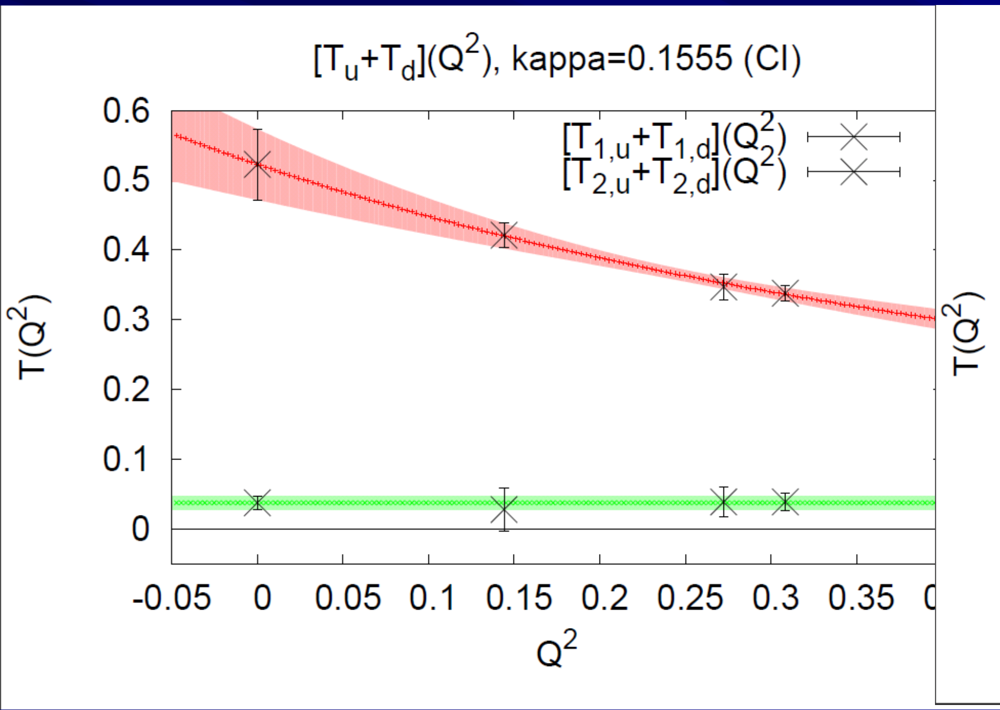
$$\begin{bmatrix} \langle x\rangle_q^{\overline{MS}}(\mu) \\ \langle x\rangle_g^{\overline{MS}}(\mu) \end{bmatrix} = \begin{bmatrix} C_{qq}(\mu) & C_{qg}(\mu) \\ C_{gq}(\mu) & C_{gg}(\mu) \end{bmatrix} \begin{bmatrix} \langle x\rangle_q^R \\ \langle x\rangle_g^R \end{bmatrix}$$

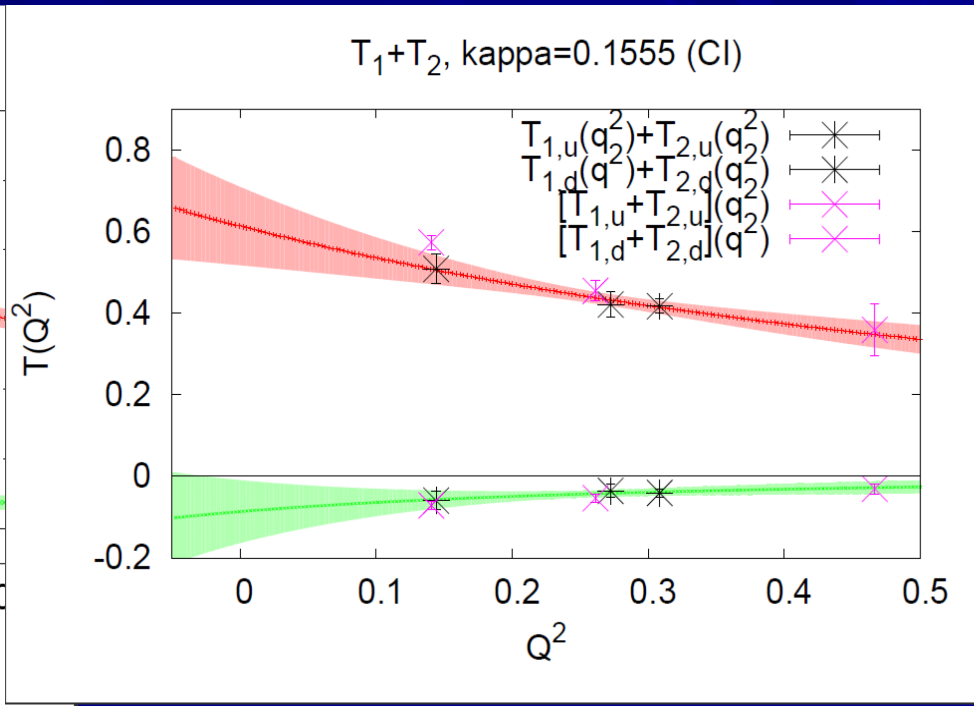
Lattice Parameters

- Quenched 16^3 x 24 lattice with Wilson fermion
- Quark spin and <x> were calculated before for both the C.I. and D.I.
- κ = 0.154, 0.155, 0.1555 (m_π = 650, 538, 478 MeV)
- 500 configurations
- 400 noises (Optimal Z_4 noise with unbiased subtraction) for DI
- 16 nucleon sources

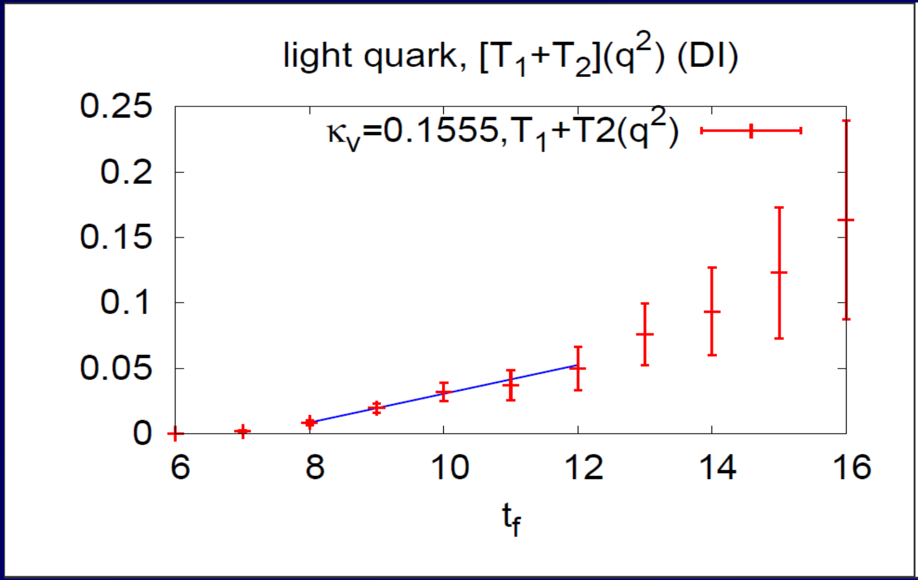
Connected Insertions of $T_1(q^2)$ and $T_2(q^2)$ for u/d Quarks

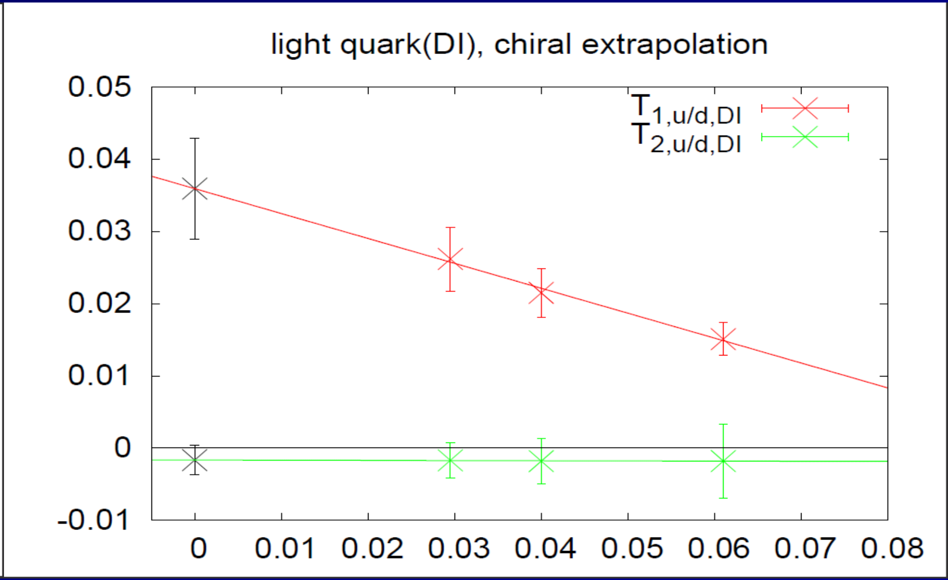
cross check

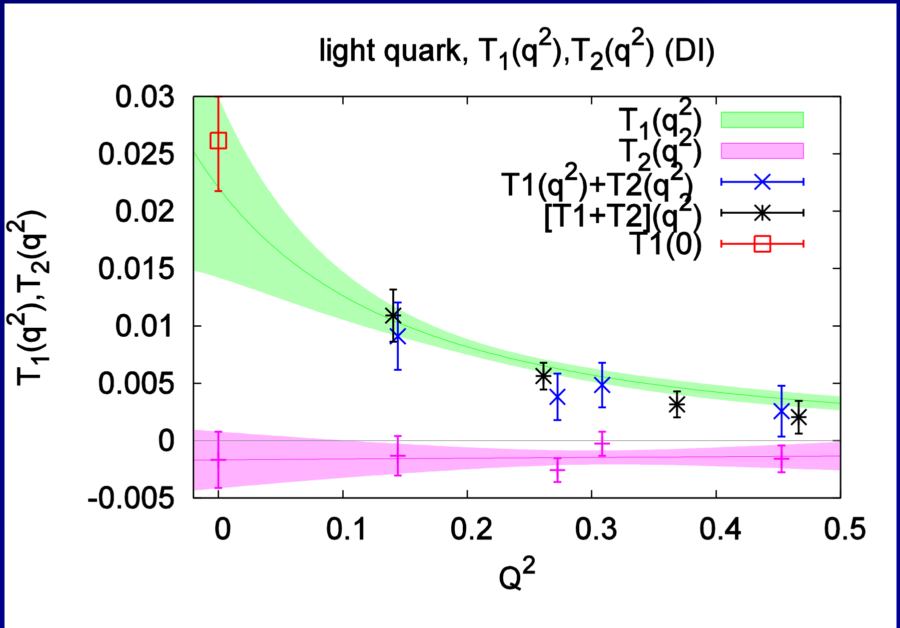




Disconnected Insertions of T_1 (q^2) and T_2 (q^2) for u/d Quarks







Gauge Operators from the Overlap Dirac Operator

- Overlap operator

$$D_{ov} = 1 + \gamma_5 \varepsilon(H); \quad H = \gamma_5 D_W(m_0)$$

- Index theorem on the lattice (Hasenfratz, Laliena, Niedermayer, Lüscher)

$$\text{index } D_{ov} = -Tr\gamma_5 (1 - \frac{a}{2} D_{ov})$$

- Local version (Kikukawa & Yamada, Adams, Fujikawa, Suzuki)

$$q_L(x) = -tr\gamma_5 (1 - \frac{a}{2} D_{ov}(x,x)) \xrightarrow[a \to 0]{} a^4 q(x) + O(a^6)$$

- Study of topological structure of the vacuum
  - Sub-dimensional long range order of coherent charges (Horvàth et al; Thacker talk in Lattice 2006)
  - Negativity of the local topological charge correlator (Horvàth et al)

- We obtain the following result

$$\mathbf{tr}_s \sigma_{\mu\nu} a D_{ov}(x,x) = c^T a^2 F_{\mu\nu} + O(a^3),$$

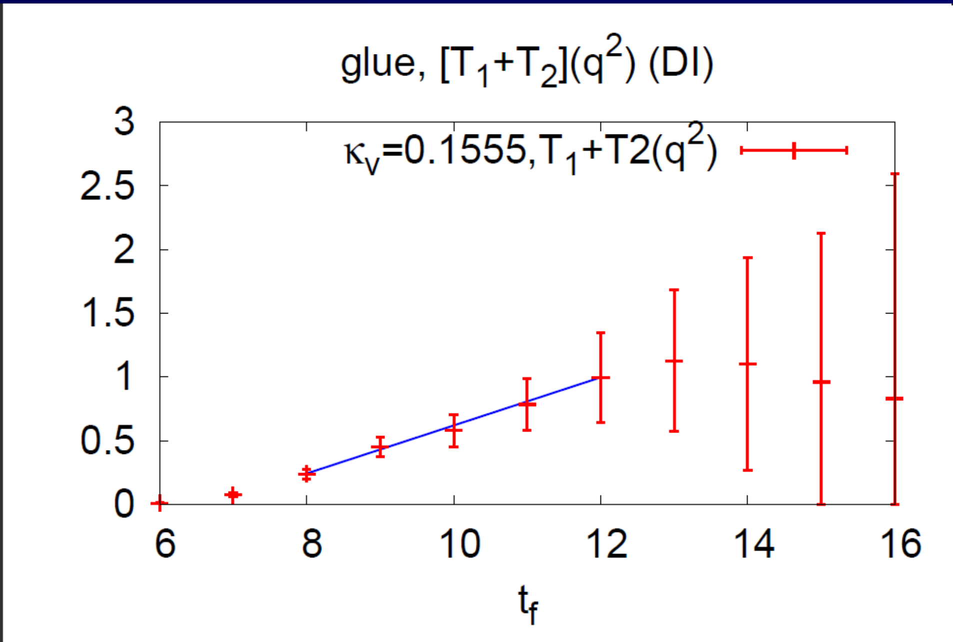
$$c^T = \rho \int_{-\pi}^{\pi} \frac{d^4k}{(2\pi)^4} \frac{2\left[(\rho + r\sum_{\lambda}(c_{\lambda}-1))c_{\mu}c_{\nu} + 2rc_{\mu}s_{\nu}^2\right]}{(\sum_{\mu} s_{\mu}^2 + [\rho + \sum_{\nu}(c_{\nu}-1)]^2)^{3/2}}$$

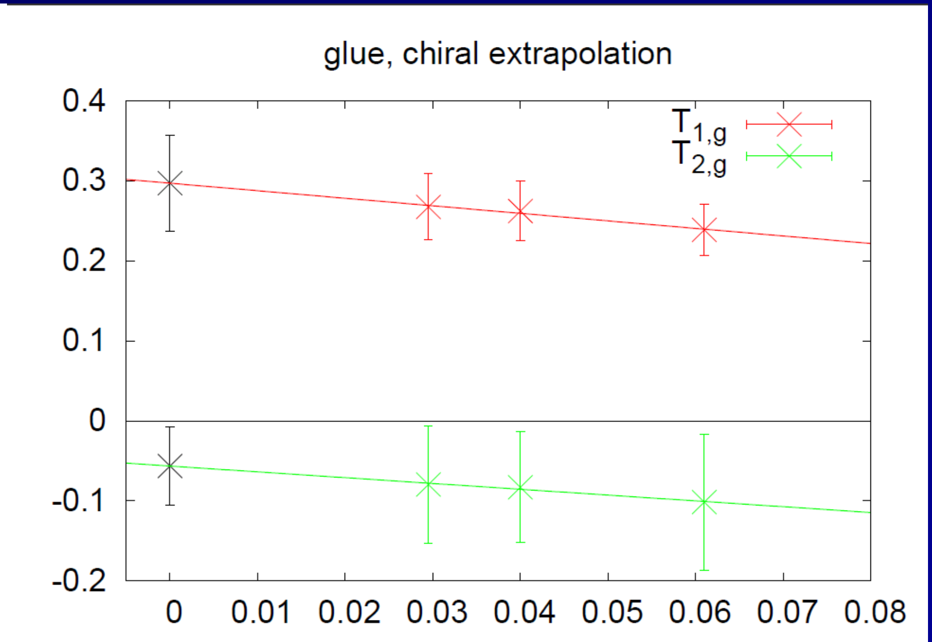
where, $r = 1, \ \rho = 1.368, \quad c^T = 0.11157$

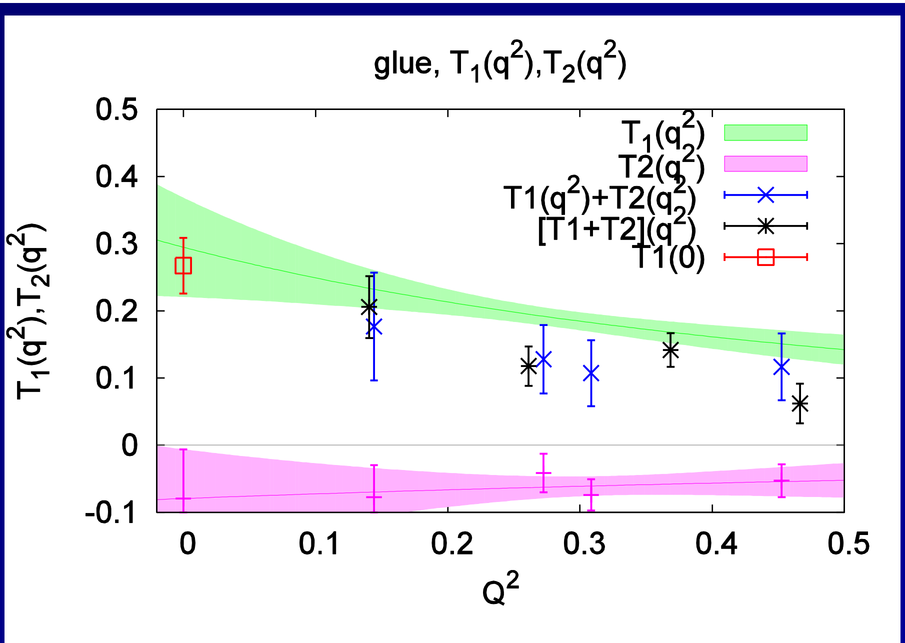
Liu, Alexandru, Horvath – PLB 659, 773 (2007)

- Noise estimation $D_{ov}(x,x) \rightarrow \left\langle \eta_x^{\dagger}(D_{ov}\eta)_x \right\rangle$
with Z_4 noise with color-spin dilution and some dilution in space-time as well.

Glue $T_1(q^2)$ and $T_2(q^2)$







Renormalized results: $Z_q = 1.05$, $Z_g = 1.05$

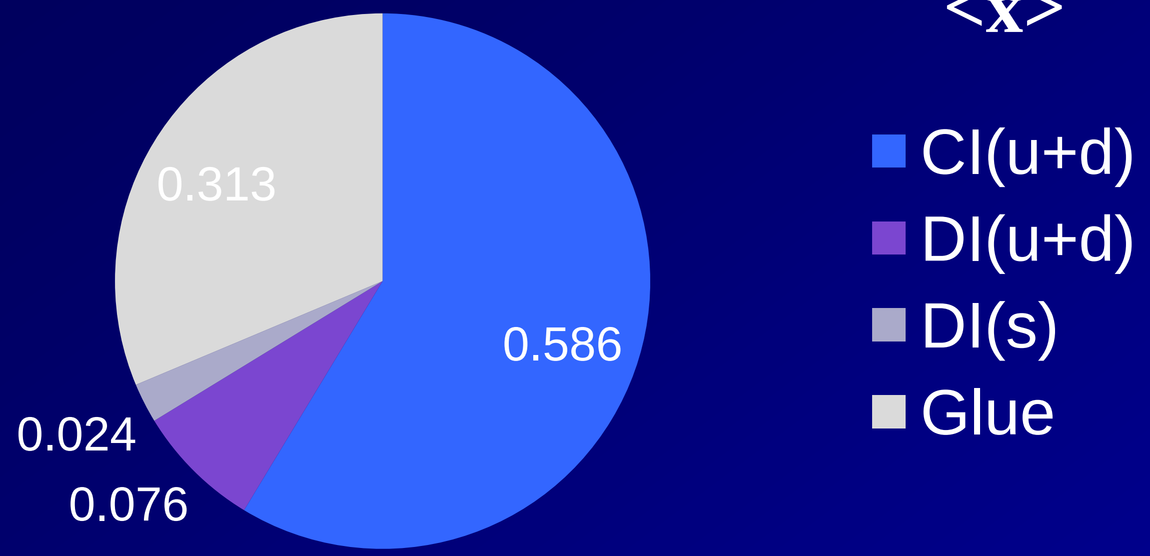
	CI(u)	CI(d)	CI(u+d)	DI(u/d)	DI(s)	Glue
<x>	0.428 (40)	0.156 (20)	0.586 (45)	0.038 (6)	0.024 (6)	0.313 (56)
$T_2(0)$	0.297 (112)	-.218 (80)	0.064 (22)	-0.002 (2)	-.001 (3)	-.059 (52)
2J	0.726 (128)	-.072 (82)	0.651 (51)	0.036 (7)	0.023 (7)	0.254 (76)

$$T_2(0)^R_{CI} + T_2(0)^R_{DI} + T_2(0)^R_g = 0.002(56)$$

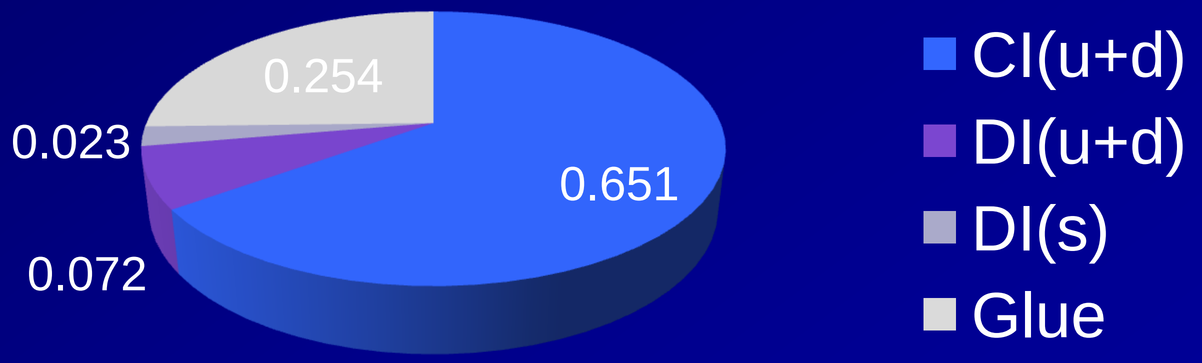
S. Brodsky et al. NPB 593, 311(2001) → no anomalous gravitomagnetic moment

E. Leader, arXiv:1109.1230 → transverse angular momentum

<x>


0.313
0.586
0.024
0.076
Cl(u+d)
Dl(u+d)
Dl(s)
Glue

2 J


0.254
0.023
0.651
0.072
Cl(u+d)
Dl(u+d)
Dl(s)
Glue

Flavor-singlet g_A

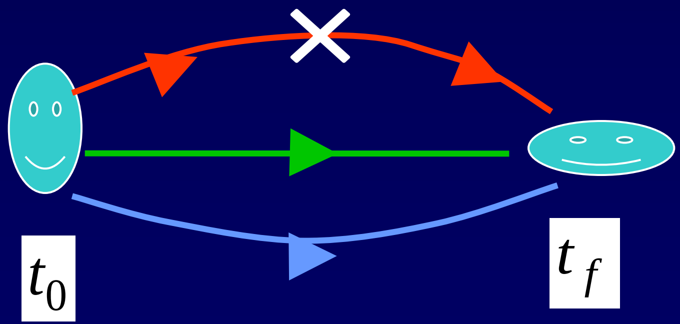
- Quark spin puzzle (dubbed `proton spin crisis')
  -
  -

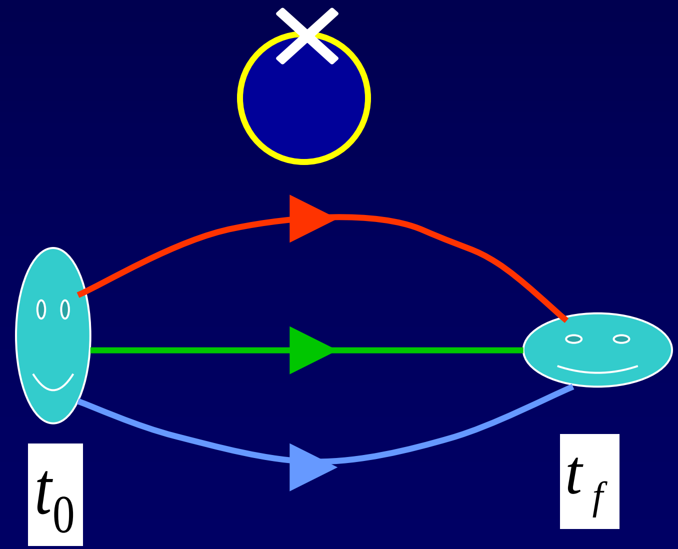
$$g_A^0 = \Delta u + \Delta d + \Delta s = \begin{cases} 1 & \text{NRQM} \\ 0.75 & \text{RQM} \end{cases}$$

  - Experimentally (EMC, SMC, … $\Delta\Sigma = g_A^0 \sim 0.2 - 0.3$

$$\overline{\Psi}\gamma_\mu\gamma_5\Psi(t)(u,d,s)$$

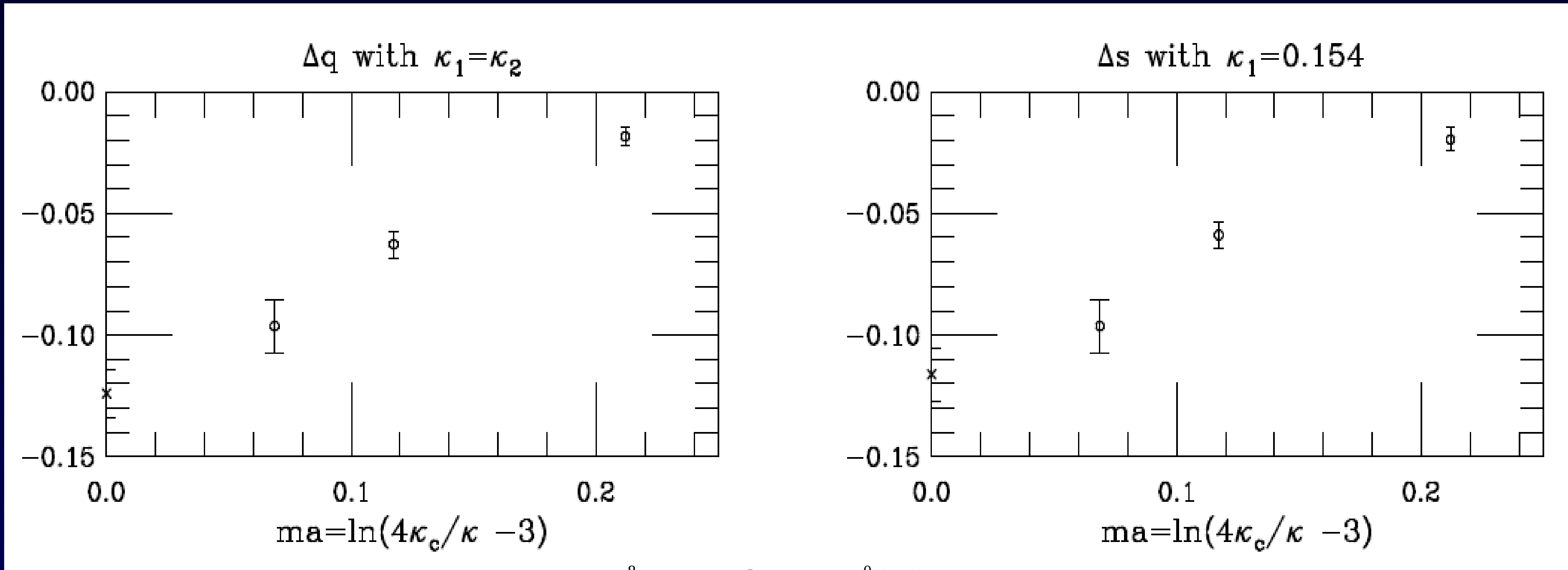
$$(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d)(t)$$





$$g^0_{A,con} = (\Delta u + \Delta d)_{con}$$

$$g^0_{A,dis} = (\Delta u + \Delta d + \Delta s)_{dis}$$



$g_A^8 = \Delta u + \Delta d - 2\Delta s \approx g_A^0(\mathrm{CI})$

S.J. Dong, J.-F. Lagae, and KFL, PRL 75, 2096 (1995)

- DI sea contribution independent of quark mass

$$\Delta u = \Delta d \cong \Delta s$$

- This suggests U(1) anomaly at work.
- $$g_A^8 = \Delta u + \Delta d - 2\Delta s \approx g_A^0(\mathrm{CI})$$

Lattice resolution: U(1) anomaly

$$g_A^0 = (\Delta u + \Delta d)_{con} + (\Delta u + \Delta d + \Delta s)_{dis} = 0.62(9) + 3(-0.12(1)) = 0.25(12)$$

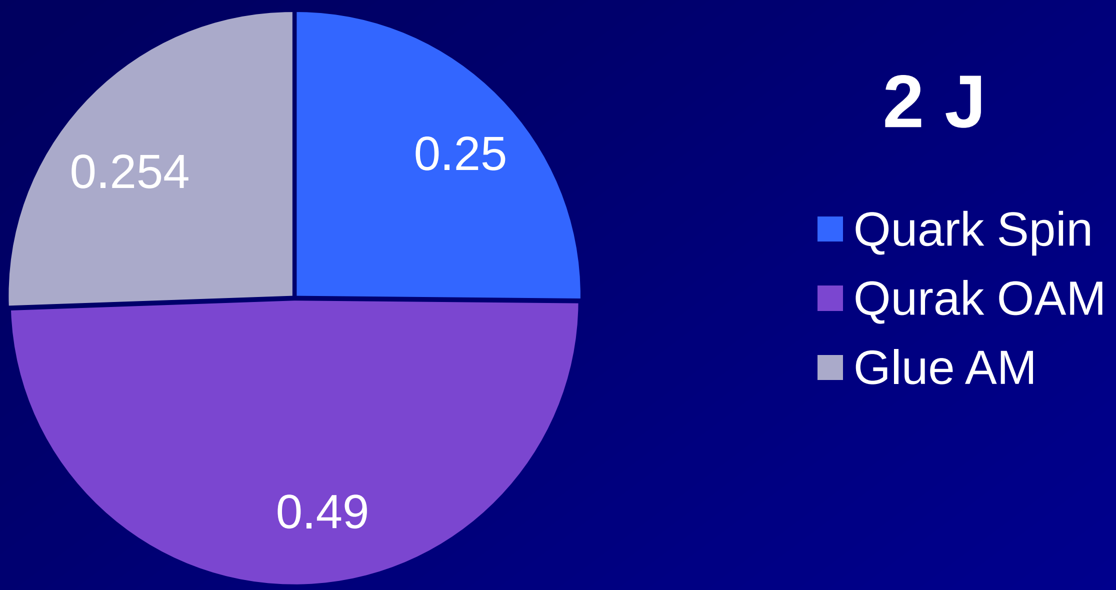
	Lattice	Expt. (SMC)	NRQM	RQM
$g_A^0 = \Delta u + \Delta d + \Delta s$	0.25(12)	0.22(10)	1	0.75
$g_A^3 = \Delta u - \Delta d$	1.20(10)	1.2573(28)	5/3	1.25
$g_A^8 = \Delta u + \Delta d - 2\Delta s$	0.61(13)	0.579(25)	1	0.75
Δu	0.79(11)	0.80(6)	1.33	1
Δd	-.42(11)	-0.46(6)	-0.33	-0.25
Δs	-.12(1)	-0.12(4)	0	0
F_A	0.45(6)	0.459(8)	0.67	0.5
D_A	0.75(11)	0.798(8)	1	0.75
F_A / D_A	0.60(2)	0.575(16)	0.67	0.67

$$F_A = (\Delta u - \Delta s)/2; \quad D_A = (\Delta u - 2\Delta d + \Delta s)/2$$

Renormalized results:

	CI(u)	CI(d)	CI(u+d)	DI(u/d)	DI(s)	Glue
2J	0.726 (128)	-.072 (82)	0.651 (51)	0.036 (7)	0.023 (7)	0.254 (76)
g_A	0.95 (11)	-0.32 (12)	0.65 (8)	-0.12 (1)	-0.12 (1)	
2 L	-0.25 (18)	0.26 (14)	0.00 (10)	0.17 (2)	0.15 (2)	

Quark Spin, Orbital Angular Momentum, and Gule Angular Momentum



$$\Delta q \approx 0.25;$$

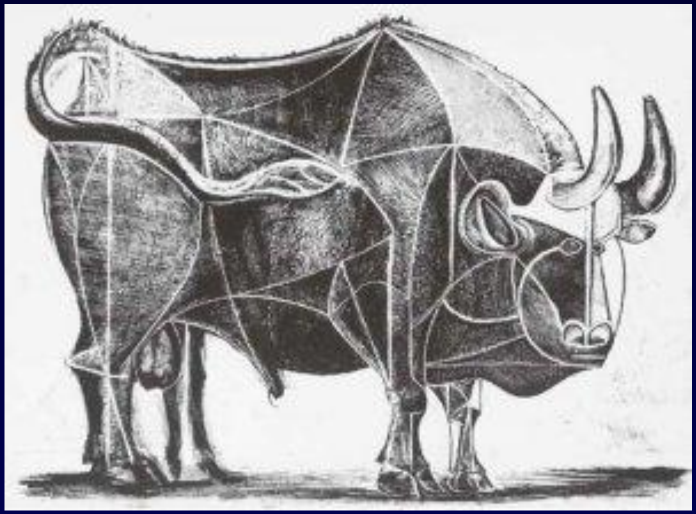
$$2\, L_q \approx 0.49\ (0.0(\mathrm{CI})+0.49(\mathrm{DI}));$$

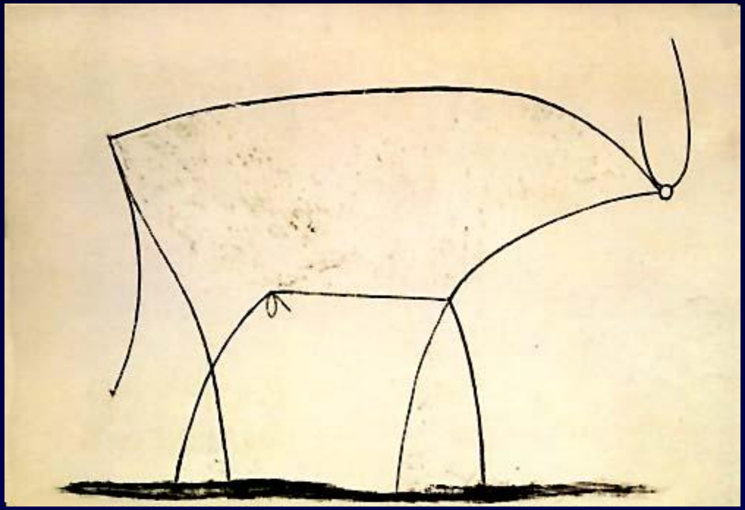
$$2\, \mathrm{J}_g \approx 0.25$$

Summary

- Momentum fraction of quarks (both valence and sea) and glue have been calculated for a quenched lattice:
  - Glue momentum fraction is ~ 31%.
  - g_A^0 ~ 0.25 in agreement with expt.
  - Glue angular momentum is ~ 25%.
  - Quark orbital angular momentum is large for the sea quarks (~ 50%).
- These are quenched results so far.

Le Taureau of Pablo Picasso (1945)





5th stage → 11th stage

Dynamical fermion with chiral symmetry and light quark masses → Quenched approximation

Current project

- Dynamical domain-wall fermion gauge (RBC + UKQCD configurations, lowest pion mass ~ 140 MeV on 5.5 fm box) + overlap fermion for the valence.
- Quark loops with low mode averaging and improved nucleon propagator.