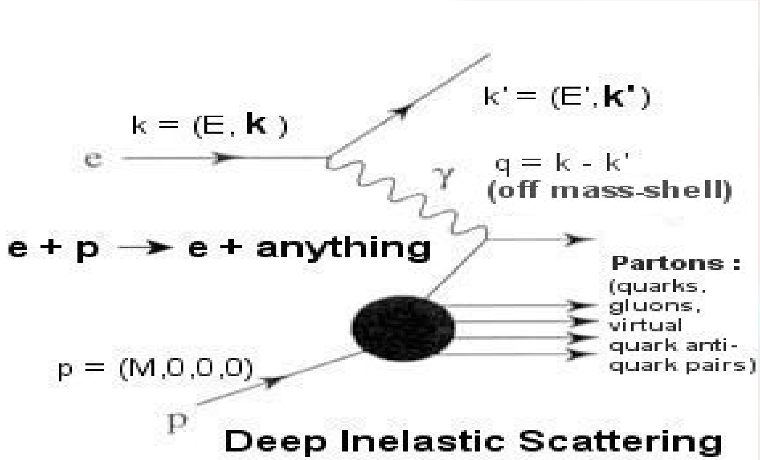
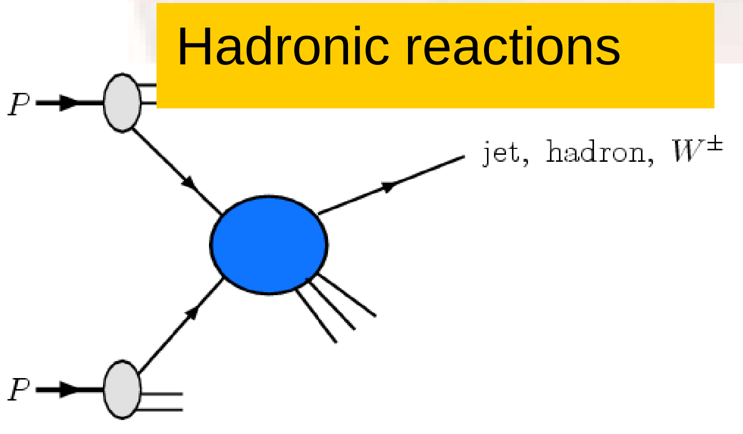
Transverse Momentum Dependent Parton Distributions

Feng Yuan

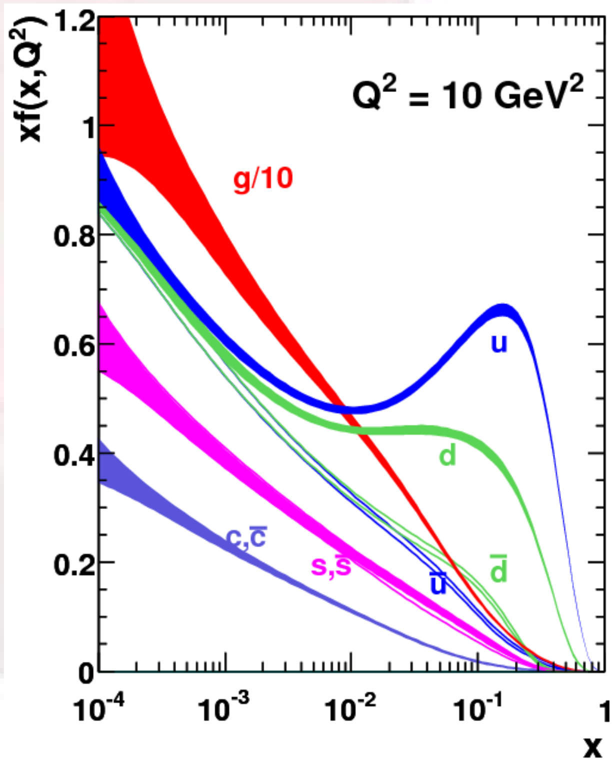
Lawrence Berkeley National Laboratory

Feynman Parton: one-dimension





- Inclusive cross sections probe the momentum (longitudinal) distributions of partons inside nucleon



Extension to transverse direction…

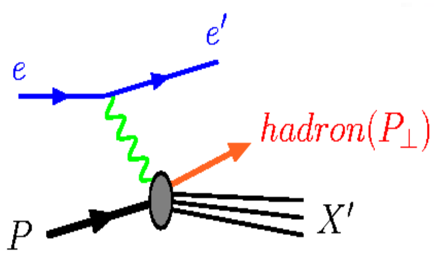
- Semi-inclusive measurements
  - Transverse momentum dependent (TMD) parton distributions
- Deeply Virtual Compton Scattering and Exclusive processes
  - Generalized parton distributions (GPD)
- Recently, there have been very exciting developments in both fields

Wigner Distribution

$W(x,r,k_t)$

↓ d^3r (to the left)

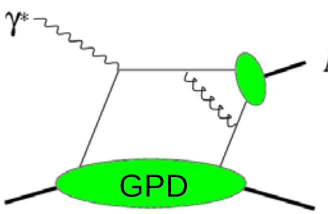
↓ $d^2k_t dz$, F.T. (to the right)



Transverse Momentum Dependent PDF $f(x,k_t)$

↓ d^2k_t

PDF $f(x)$

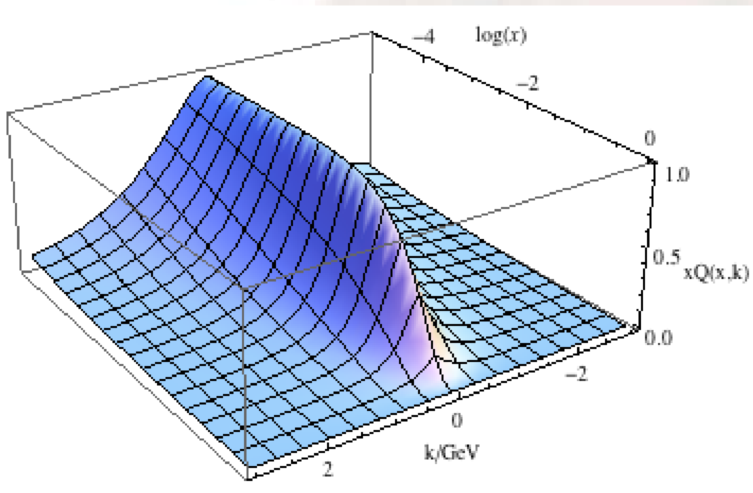


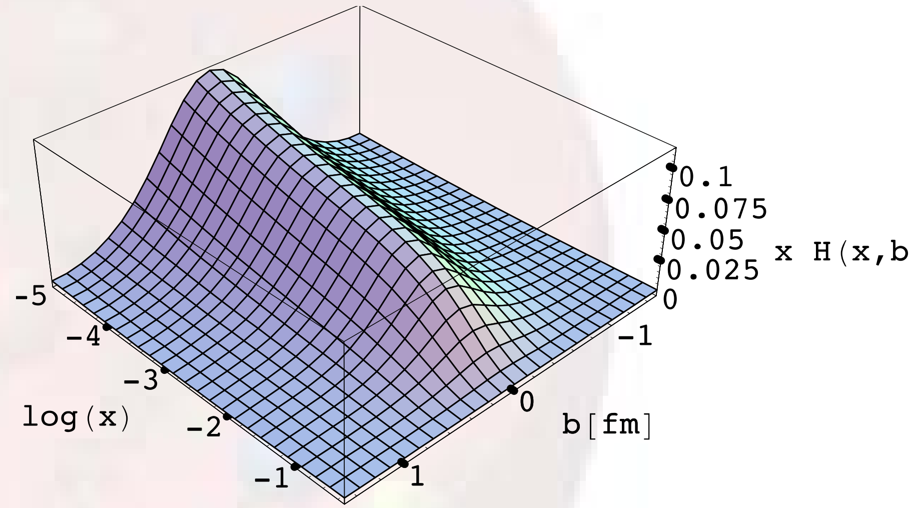
Generalized Parton Distr. $H(x,\xi,t)$

↓ dx

Form Factors $F_1(Q),F_2(Q)$

Transverse profile for the quark distribution: k_t vs b_t

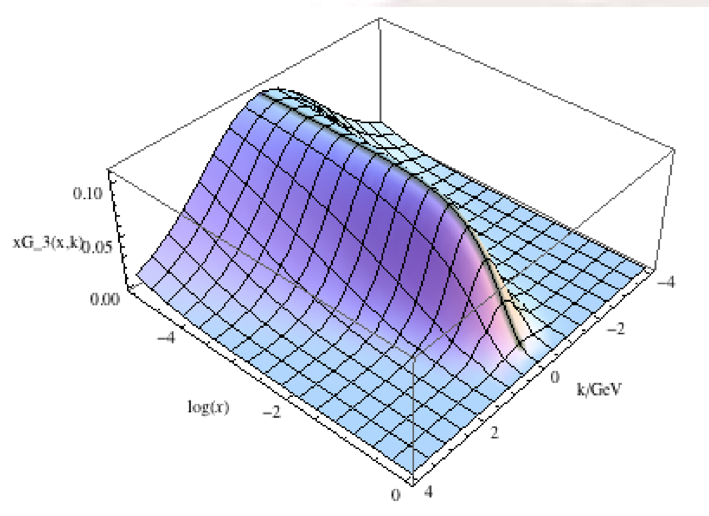




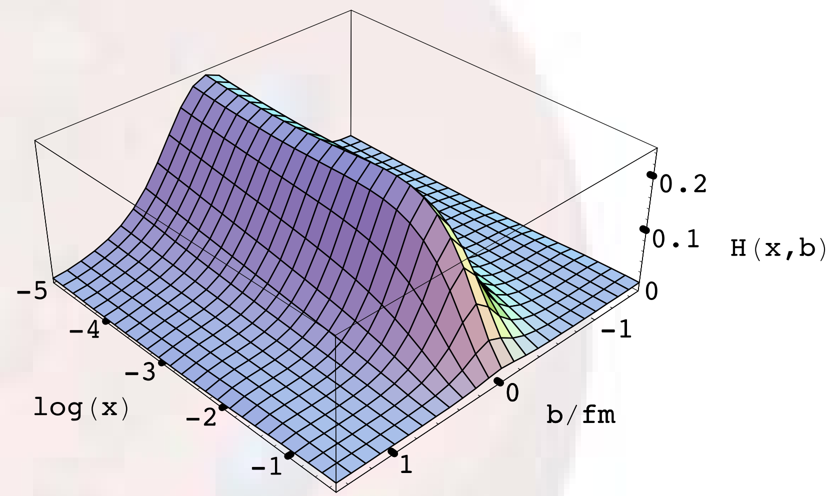
Quark distribution calculated from
a saturation-inspired model
A.Mueller 99, McLerran-Venugopalan 99

GPD fit to the DVCS data from HERA,
Kumerick-D.Mueller, 09,10

Gluon distribution

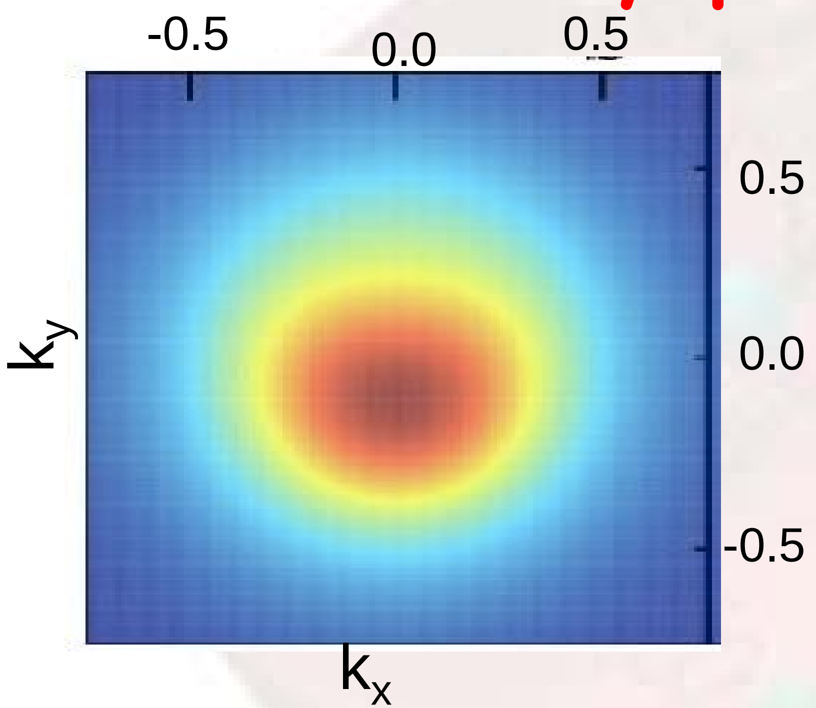


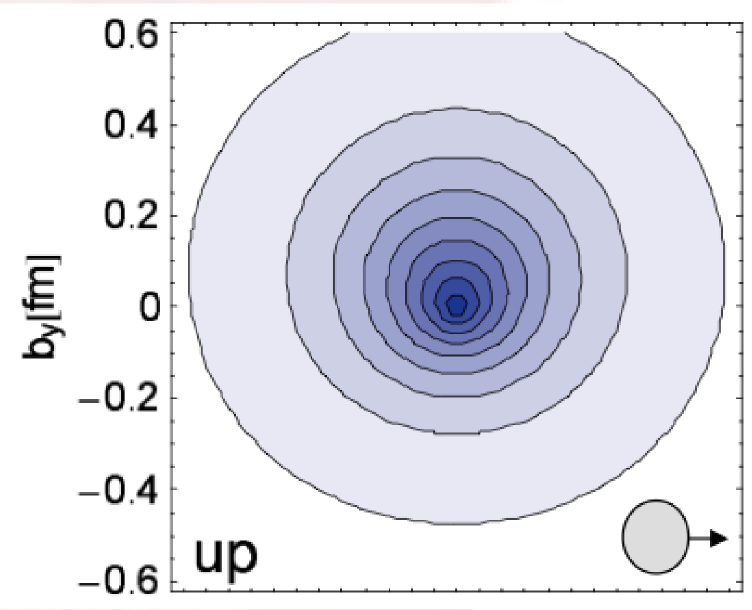
One of the TMD gluon distributions at small-x



GPD fit to the DVCS data from HERA, Kumerick-Mueller, 09,10

Deformation when nucleon is transversely polarized





Quark Sivers function fit to the SIDIS Data, Anselmino, et al. 20009

Lattice Calculation of the IP density of Up quark, QCDSF/UKQCD Coll., 2006

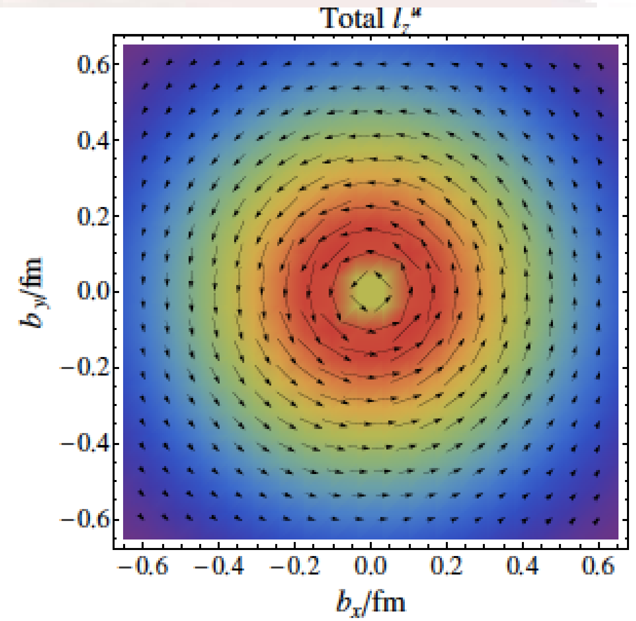
Orbital Angular Momentum from Wigner Distributions

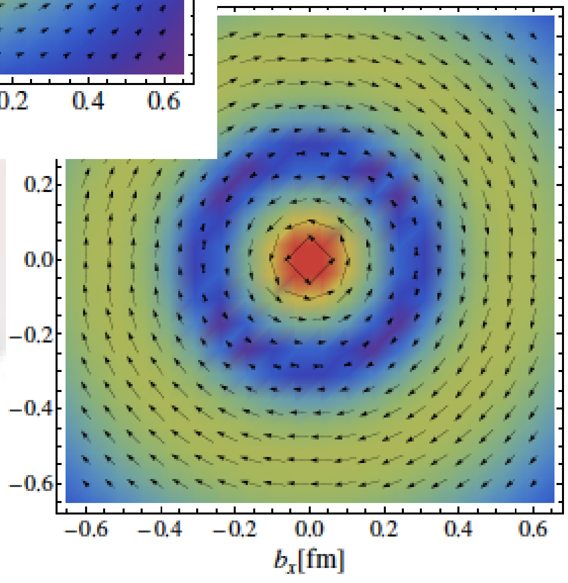
Define the net momentum projection

$$\mathcal{K}(\vec{r}_\perp) = \int d^2k_\perp \vec{k}_\perp \mathcal{H}(\vec{r}_\perp, \vec{k}_\perp)$$

Quark oribital angular momentum

$$L_q = \int d^2r_\perp d^2k_\perp \vec{r}_\perp \times \vec{k}_\perp \mathcal{H}(\vec{r}_\perp, \vec{k}_\perp)$$

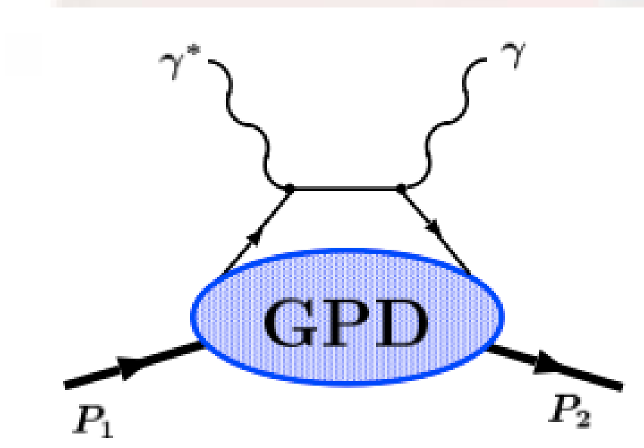


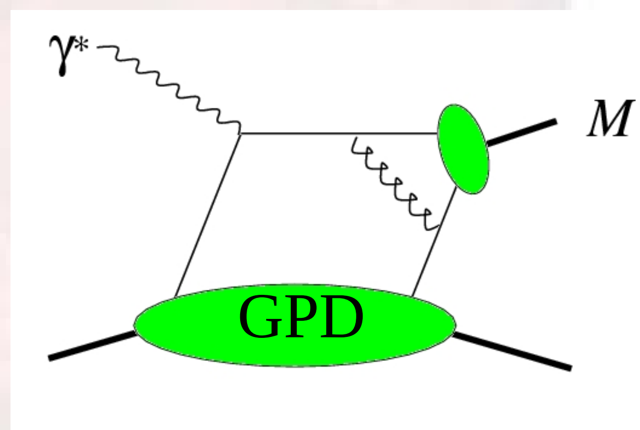


Lorce, Pasquini, Xiong, Yuan, arXiv:1111.482

Access the GPDs

- Deeply virtual Compton Scattering (DVCS) and deeply virtual exclusive meson production (DVEM)



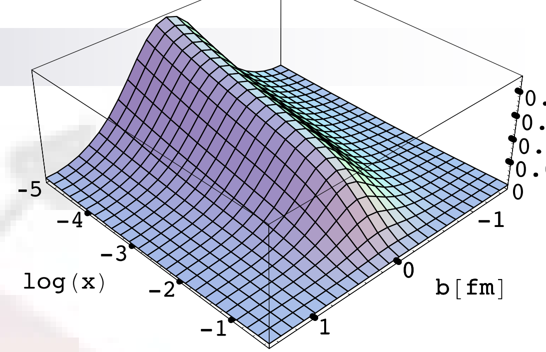


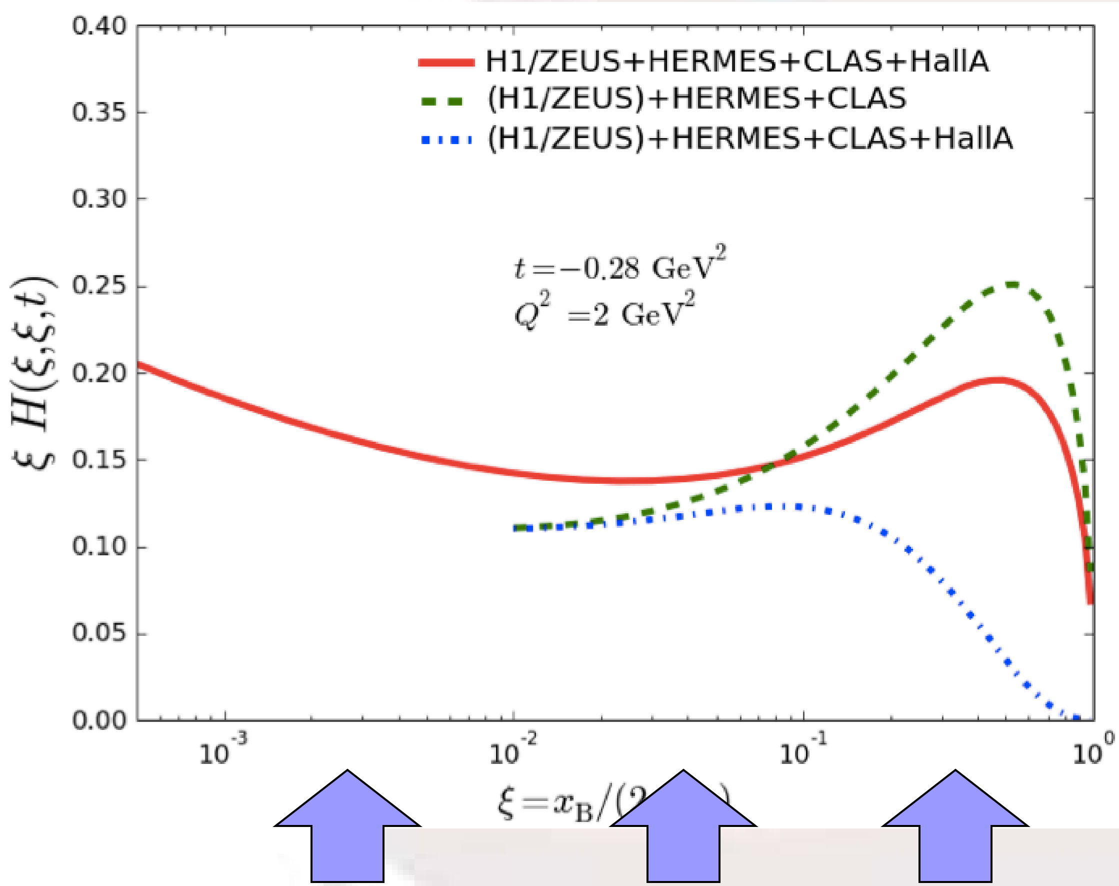
In the Bjorken limit: $Q^2 >> (-t), \Lambda^2_{QCD}, M^2$

Extract the GPDs

- The theoretical framework has been well established
  - Perturbative QCD corrections at NLO, some at NNLO
- However, GPDs depend on x, ξ, t, it is much more difficult than PDFs (only depends on x)
  - There will be model dependence at the beginning

One example: H(x,x,t)





HERA HERMES JLab

D. Mueller, et al, 09, 10

Small-x range constrained by HERA, uncertainties at large-x shall be very much reduced with Jlab 12 GeV COMPASS, and the planed EIC

Of course, there are also other GPDs, in particular, the GPD E

Transverse momentum dependent parton distribution

Straightforward extension

- Spin average, helicity, and transversity distributions

P_T-spin correlations

- Nontrivial distributions, $S_T X P_T$
- In quark model, depends on S- and P-wave interference

Leading Twist TMDs

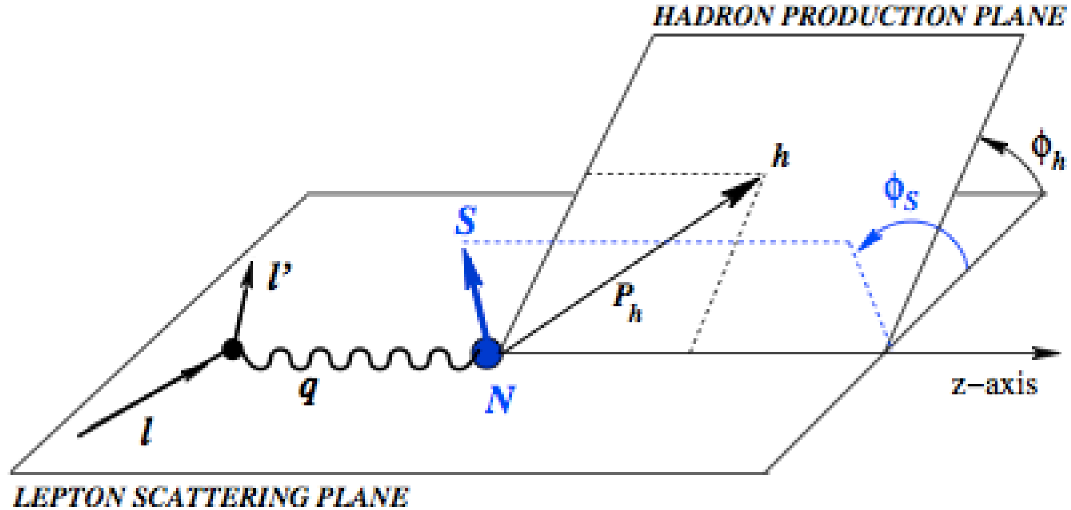
: Nucleon Spin   : Quark Spin

		Quark polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	f_1 =		$h_1^\perp$ = − Boer-Mulder
	L		g_1 = − Helicity	$h_{1L}^\perp$ = −
	T	$f_{1T}^\perp$ = − Sivers	$g_{1T}^\perp$ = −	h_{1T} = − Transversity $h_{1T}^\perp$ = −

Where can we learn TMDs

- Semi-inclusive hadron production in deep inelastic scattering (SIDIS)
- Drell-Yan lepton pair, photon pair productions in pp scattering
- Dijet correlation in DIS
- Relevant e+e- annihilation processes
- …

Semi-inclusive DIS



- Novel Single Spin Asymmetries

$$A_{UT}^{\sin(\phi+\phi_S)} \propto S_\perp \frac{\sum_{q,\bar{q}} e_q^2 \delta q(x)\, H_1^\perp(z)}{\sum_{q,\bar{q}} e_q^2 q(x)\, D_1(z)}$$

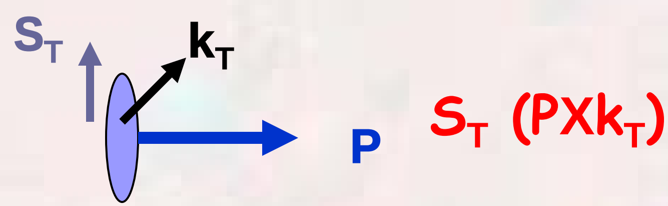
$$z \overset{lab}{=} \frac{E_h}{\nu}$$

$$A_{UT}^{\sin(\phi-\phi_S)} \propto S_\perp \frac{\sum_{q,\bar{q}} e_q^2 f_{1T}^{\perp,q}(x) \cdot D_1(z)}{\sum_{q,\bar{q}} e_q^2 q(x)\, D_1(z)}$$

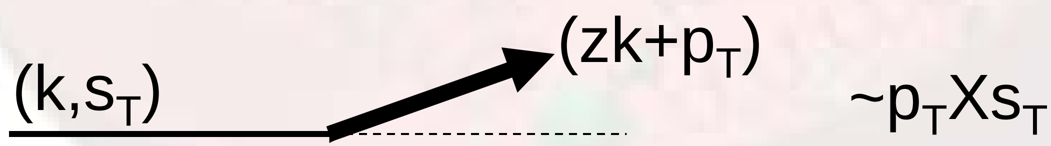
U: unpolarized beam
T: transversely polarized target

Two major contributions

- Sivers effect in the distribution

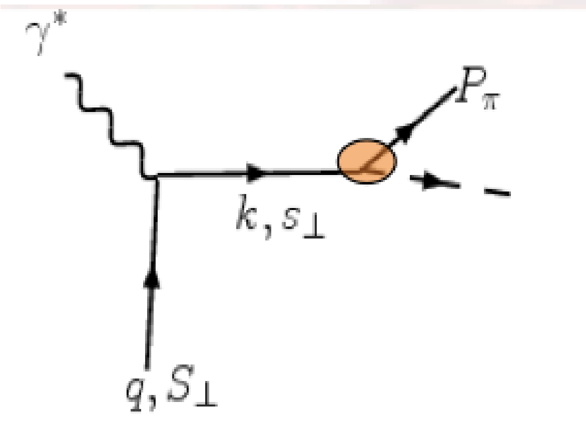


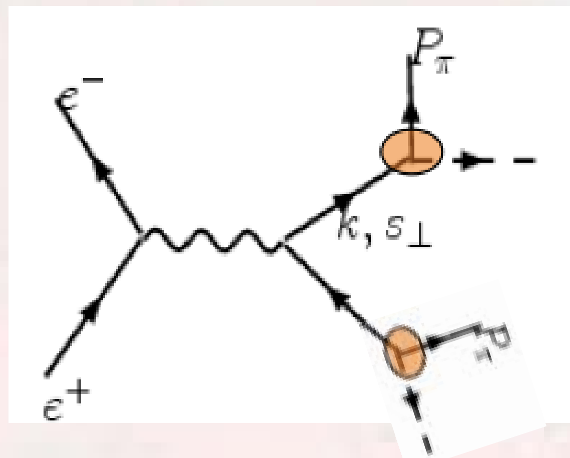
- Collins effect in the fragmentation

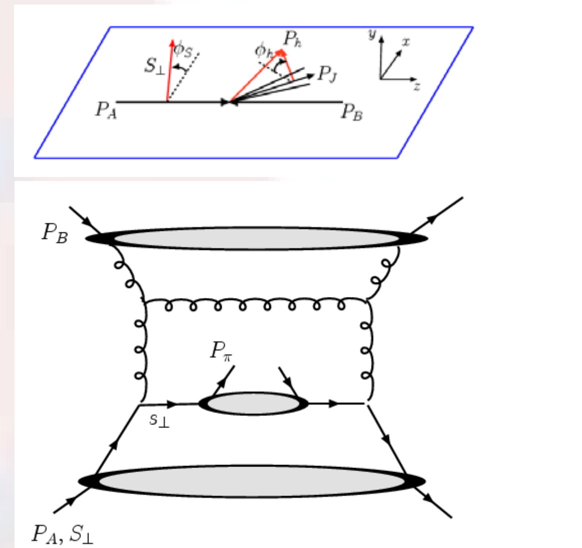


- Other contributions…

Universality of the Collins Fragmentation







ep--> e Pi X

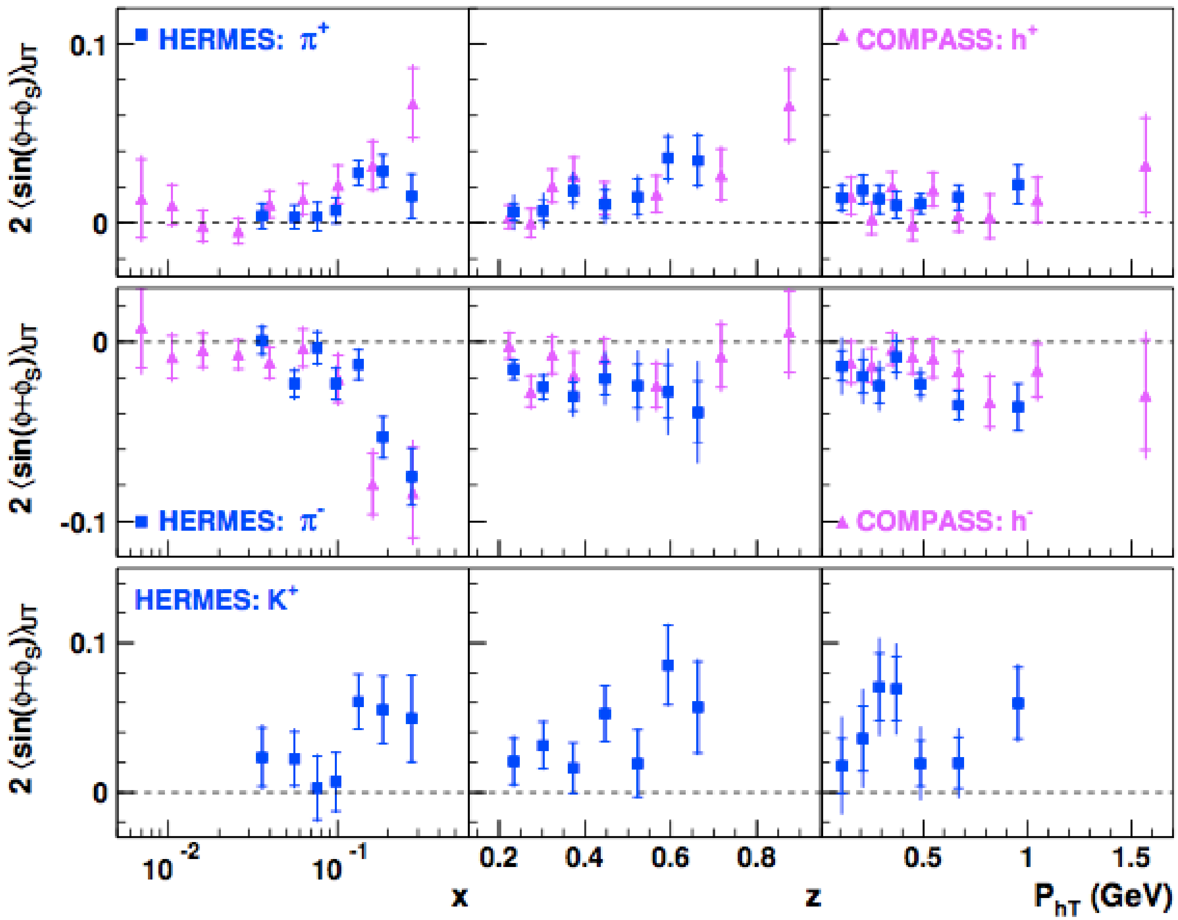
e^+e^---> Pi Pi X

pp--> jet(->Pi) X

Metz 02, Collins-Metz 02,
Yuan 07,
Gamberg-Mukherjee-Mulders 08,10
Meissner-Metz 0812.3783
Yuan-Zhou, 0903.4680

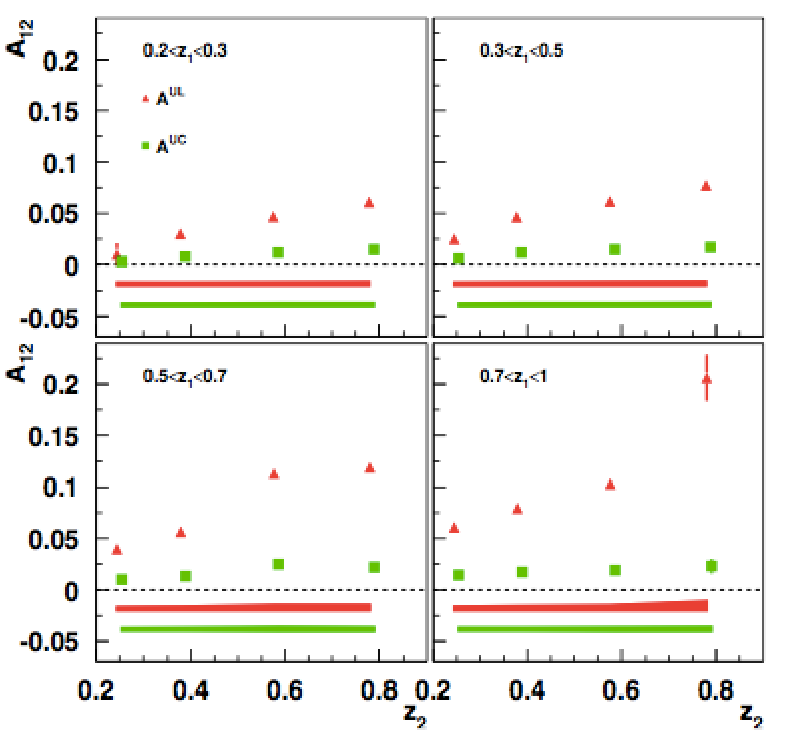
Exps: BELLE, HERMES, STAR, COMPASS

Collins asymmetries in SIDIS

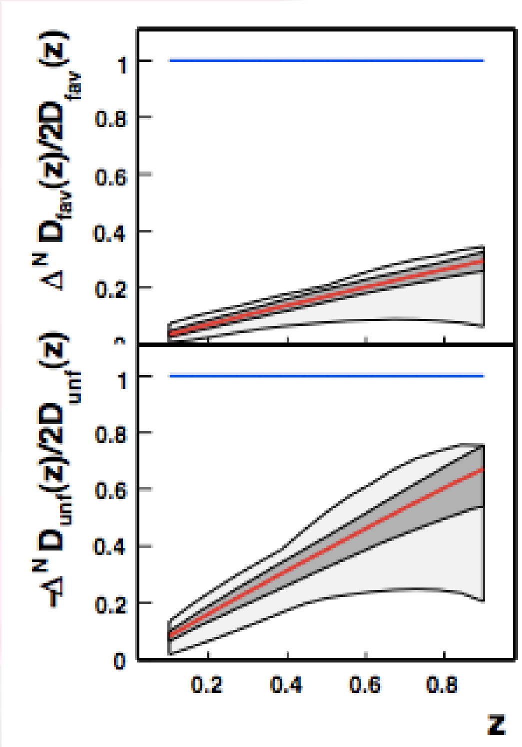


Summarized in the EIC Write-up arXiv:1108.1713

Collins effects in e^+e^-



BELLE Coll., 2008

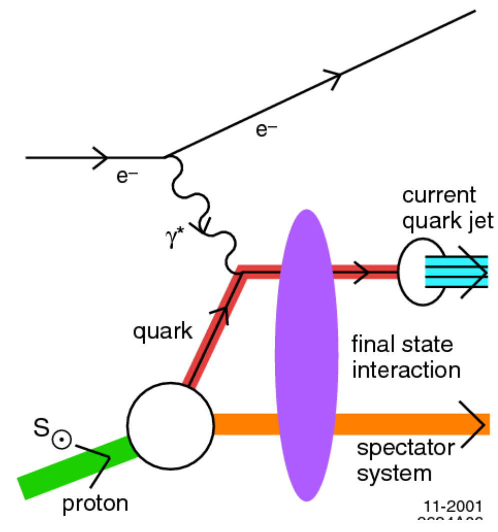


Collins functions extracted from the Data, Anselmino, et al., 2009

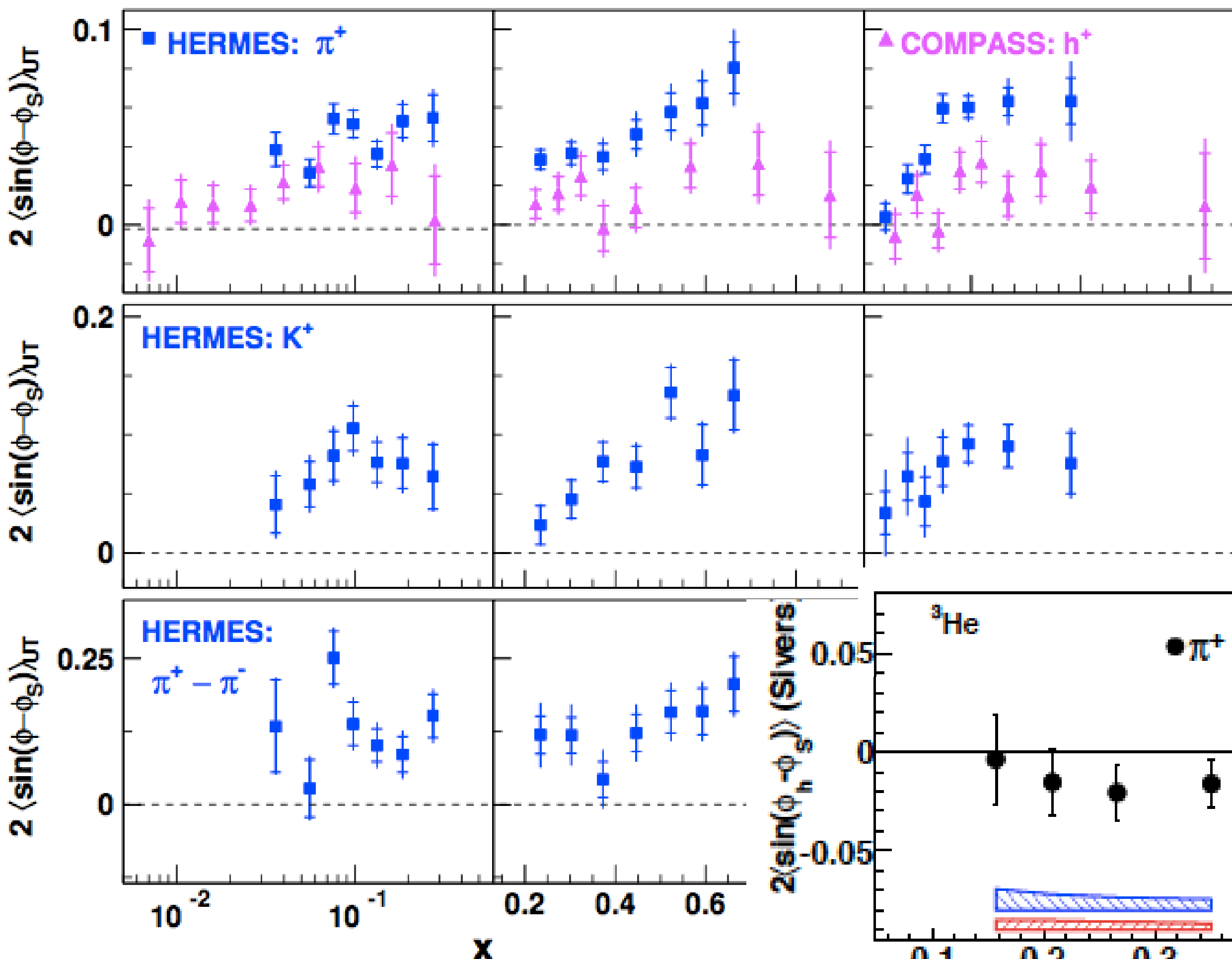
Sivers effect

- It is the final state interaction providing the phase to a nonzero SSA
- Non-universality in general
- Only in special case, we have "Special Universality"

Brodsky,Hwang,Schmidt 02
Collins, 02;
Ji,Yuan,02;
Belitsky,Ji,Yuan,02

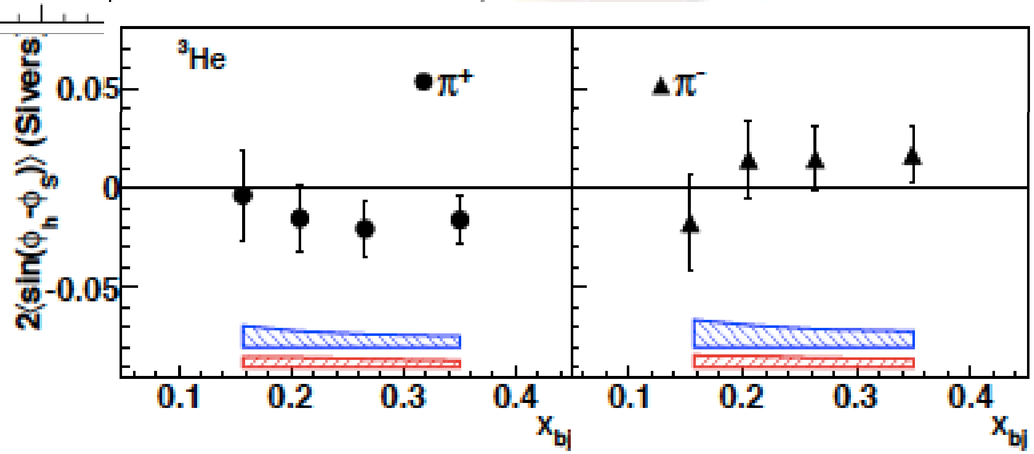


Sivers asymmetries in SIDIS



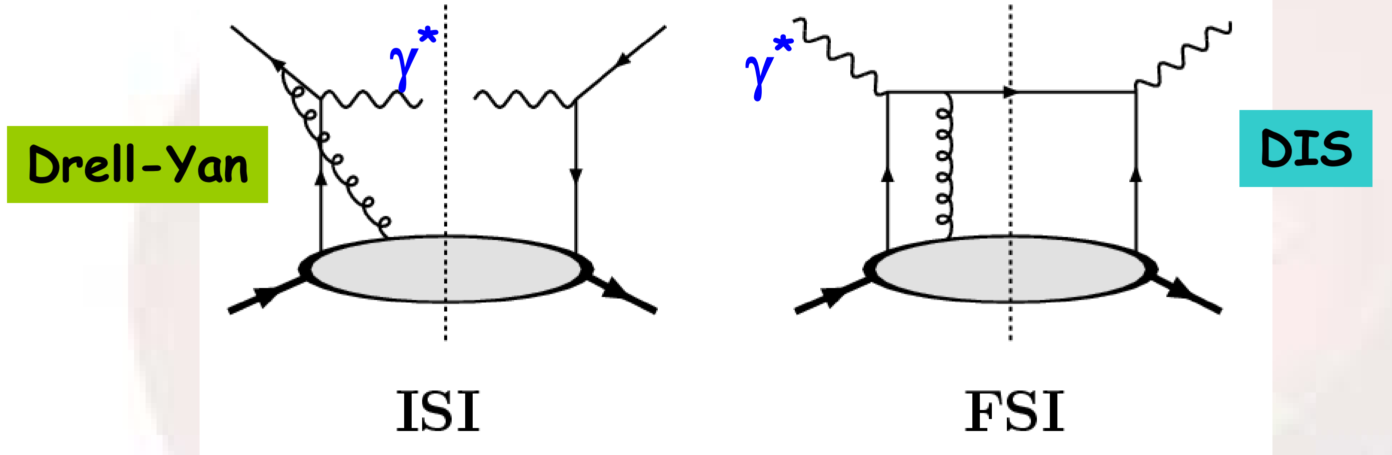
Non-zero Sivers effects Observed in SIDIS

Jlab Hall A ^{3}He



DIS and Drell-Yan

- Initial state vs. final state interactions

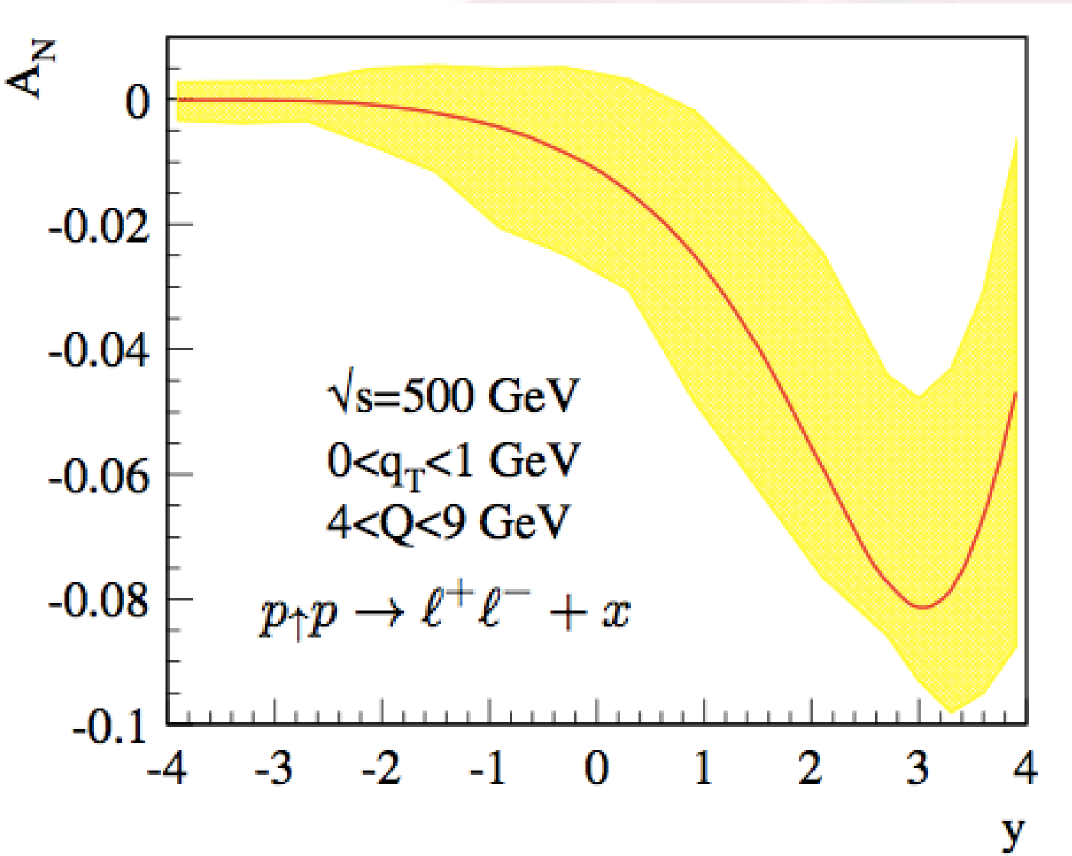


$$\text{Sivers}|_{DY} = -\text{Sivers}|_{DIS}$$

HERMES

- "Universality": QCD prediction

RHIC predictions



Kang, Qiu, 2008

There have been proposals to Do this measurement at RHIC

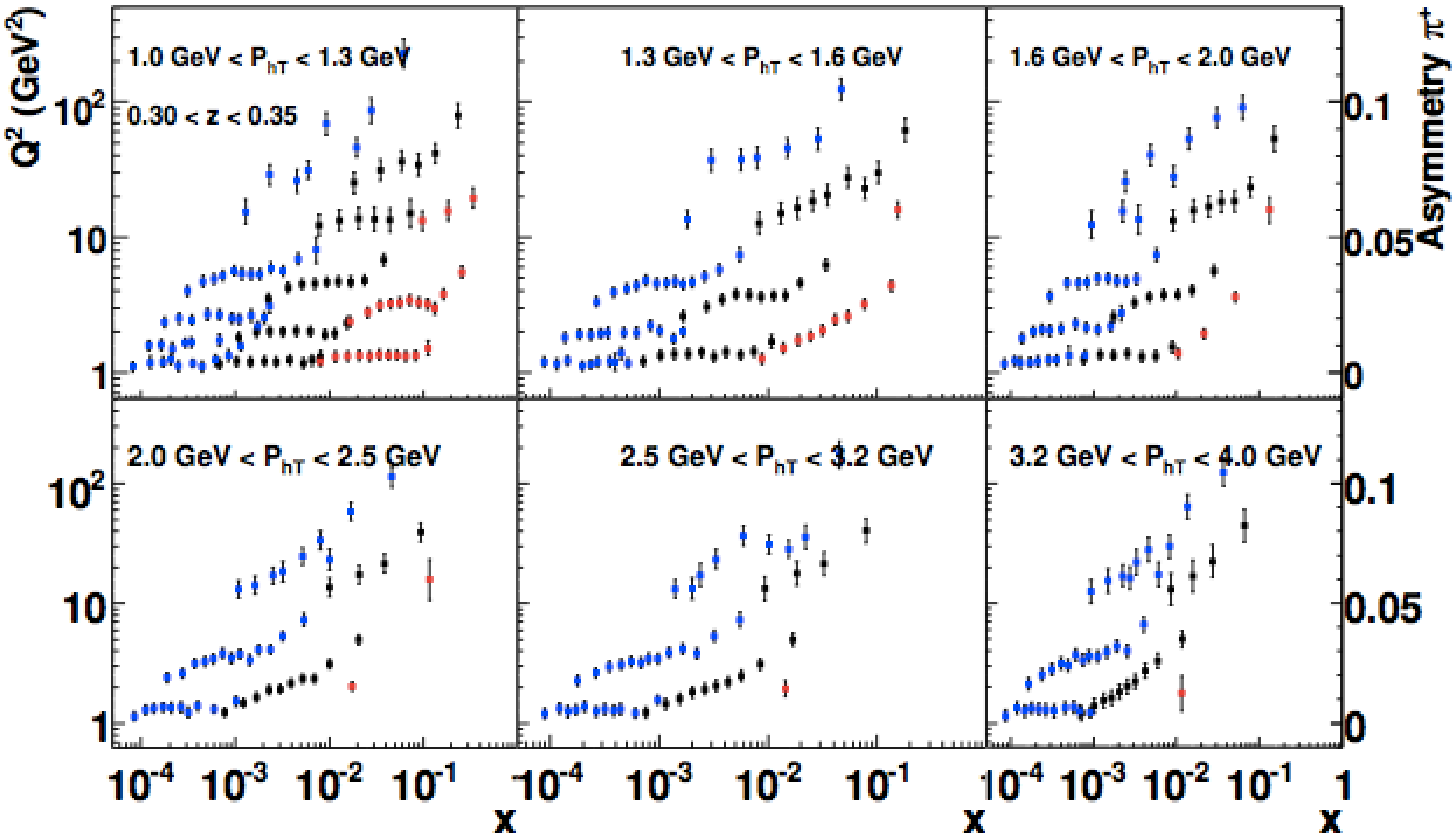
http://spin.riken.bnl.gov/rsc/

Collider or fixed target modes

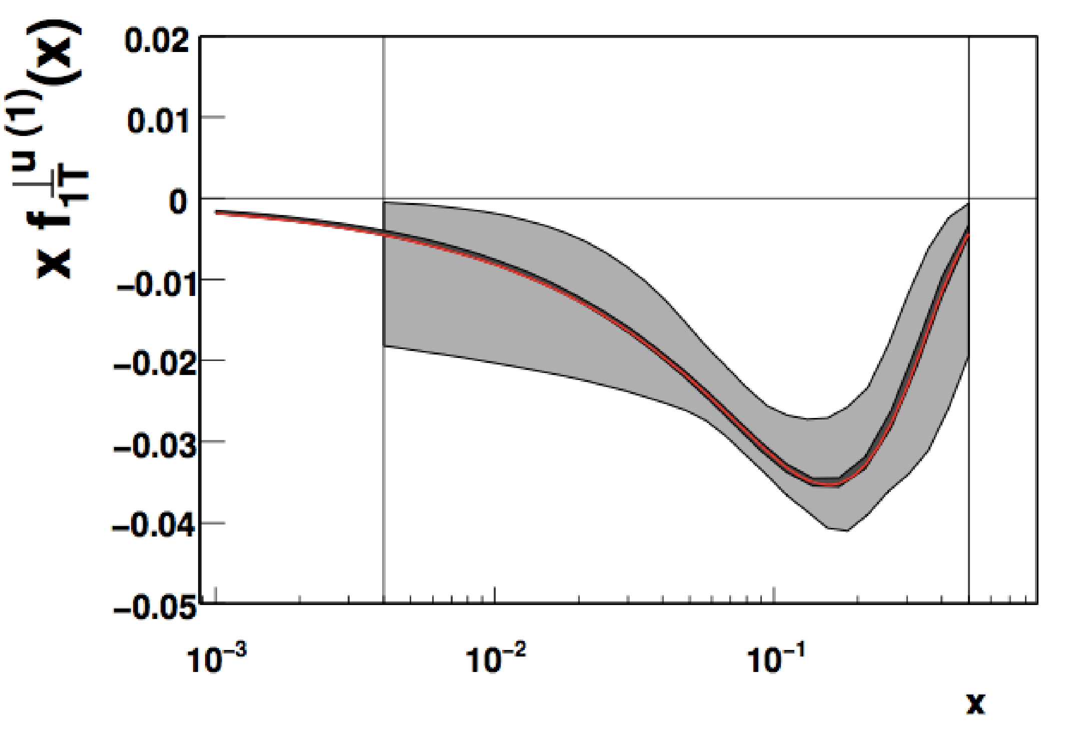
There is also a COMPASS Proposal in the near future

It is very important to test the sign change of the quark Sivers function

Electron-Ion Collider Projections: Impressive coverage on Q_2, x, z, and P_T



Quark Sivers function extracted from the data

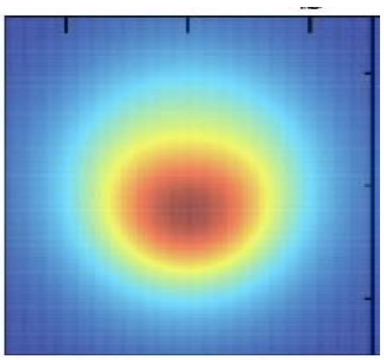


Leading order fit, simple Gaussian assumption for the Sivers function

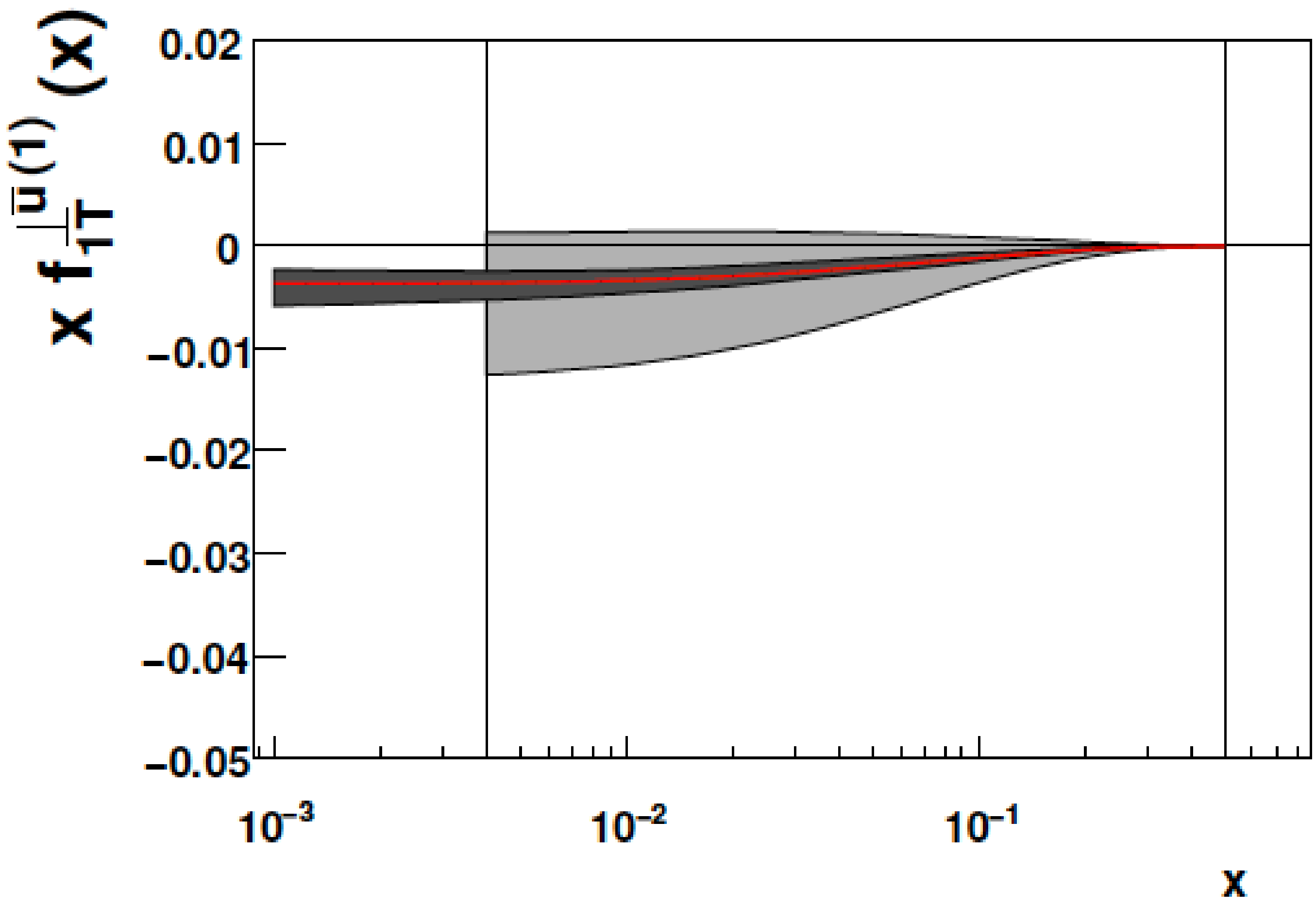
There are still theoretical uncertainti
In the fit: scale dependence, high order corrections, …

Inner band is the impact from the planed EIC kiematics

Alexei Prokudin, et al.



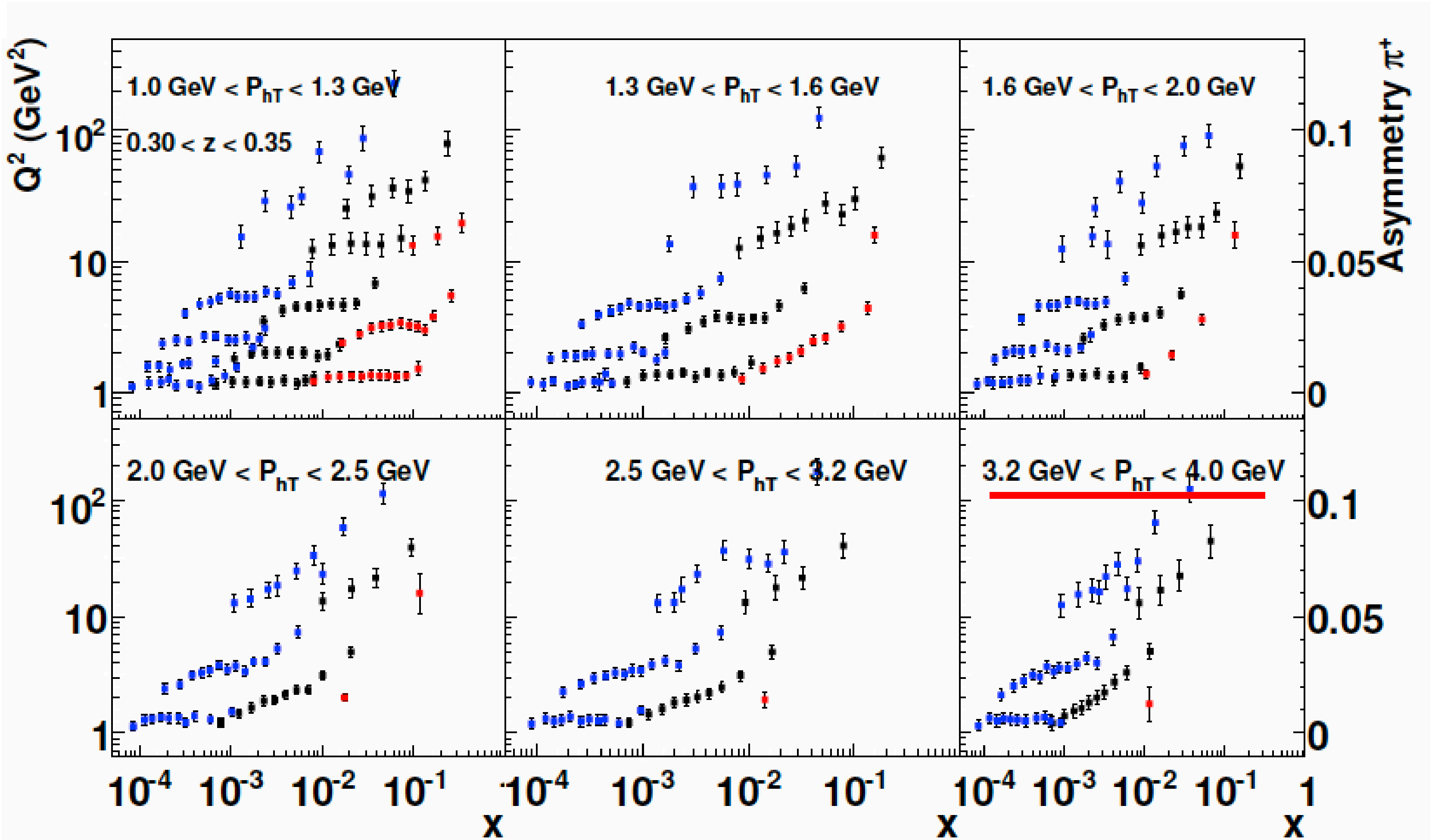
Sea quark:



Large transverse momentum

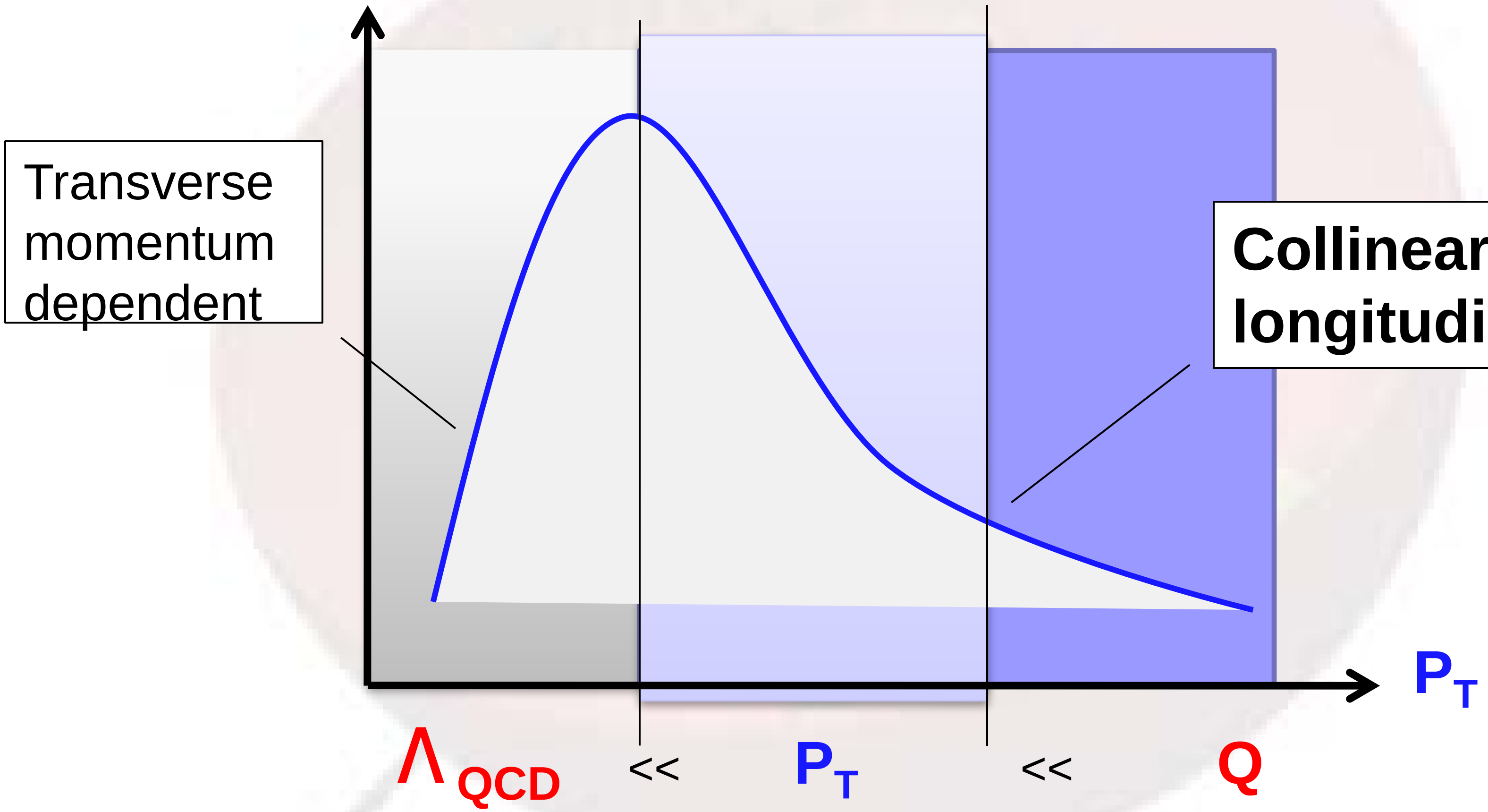
- Possible with the EIC
- QCD dynamics, evolution effects
- Q^2 dependence
- Pt dependence
- Twist-three mechanism

EIC coverage



120fb^{-1}

A unified picture (leading pt/Q)



Ji-Qiu-Vogelsang-Yuan, 2006
Yuan-Zhou, 2009

Final Results

$$A(P_A, S_\perp) + B(P_B) \to \gamma^*(q) + X \to \ell^+ + \ell^- + X,$$

- P_T dependence

$$\frac{d\triangle\sigma}{d^2q_\perp dy} = \int q_T(z_1,k_\perp)\bar{q}(z_2,k_\perp) + \left(\frac{d\triangle\sigma^{QS}}{d^2q_\perp dy} - \frac{d\triangle\sigma^{QS}}{d^2q_\perp dy}|_{aspt.}\right)$$

Sivers function at low P_T

Qiu-Sterman Twist-three

- Which is valid for all P_T range

CSS Resummation

$$\frac{d\Delta\sigma(S_\perp)}{dydQ^2d^2q_\perp} = \sigma_0\epsilon^{\alpha\beta}S_\perp^\alpha W_{UT}^\beta(Q;q_\perp)$$

- Separate the singular and regular parts

$$W_{UT}^\alpha(Q;q_\perp) = \int \frac{d^2b}{(2\pi)^2}e^{i\vec{q}_\perp\cdot\vec{b}}\widetilde{W}_{UT}^\alpha(Q;b) + Y_{UT}^\alpha(Q;q_\perp)$$

- TMD factorization in b-space

$$\widetilde{W}_{UT}^\alpha(Q;b) = \tilde{f}_{1T}^{(\perp\alpha)}(z_1,b,\zeta_1)\bar{q}(z_2,b,\zeta_2) \times H_{UT}(Q)\left(S(b,\rho)\right)^{-1} ,$$

Kang-Xiao-Yuan, 2011

Final resum form

$$\widetilde{W}_{UT}^{\alpha}(Q;b) = e^{-\mathcal{S}_{UT}(Q^2,b)}\widetilde{W}_{UT}^{\alpha}(C_1/b,b)$$
$$= (-ib_{\perp}^{\alpha}/2)\, e^{-\mathcal{S}_{UT}(Q^2,b)}\Sigma_{i,j}$$
$$\times \Delta C_{qi}^{T} \otimes f_{i/A}^{(3)}(z_1) C_{\bar{q}j} \otimes f_{j/B}(z_2)$$

- Sudakov the same

$$\mathcal{S}_{UT}(Q^2,b) = \int_{C_1^2/b^2}^{C_2^2Q^2} \frac{d\mu^2}{\mu^2}\left[\ln\left(\frac{C_2^2Q^2}{\mu^2}\right) A_{UT}(C_1;g(\mu)) + B_{UT}(C_1,C_2;g(\mu))\right] ,$$

Coefficients at one-loop order

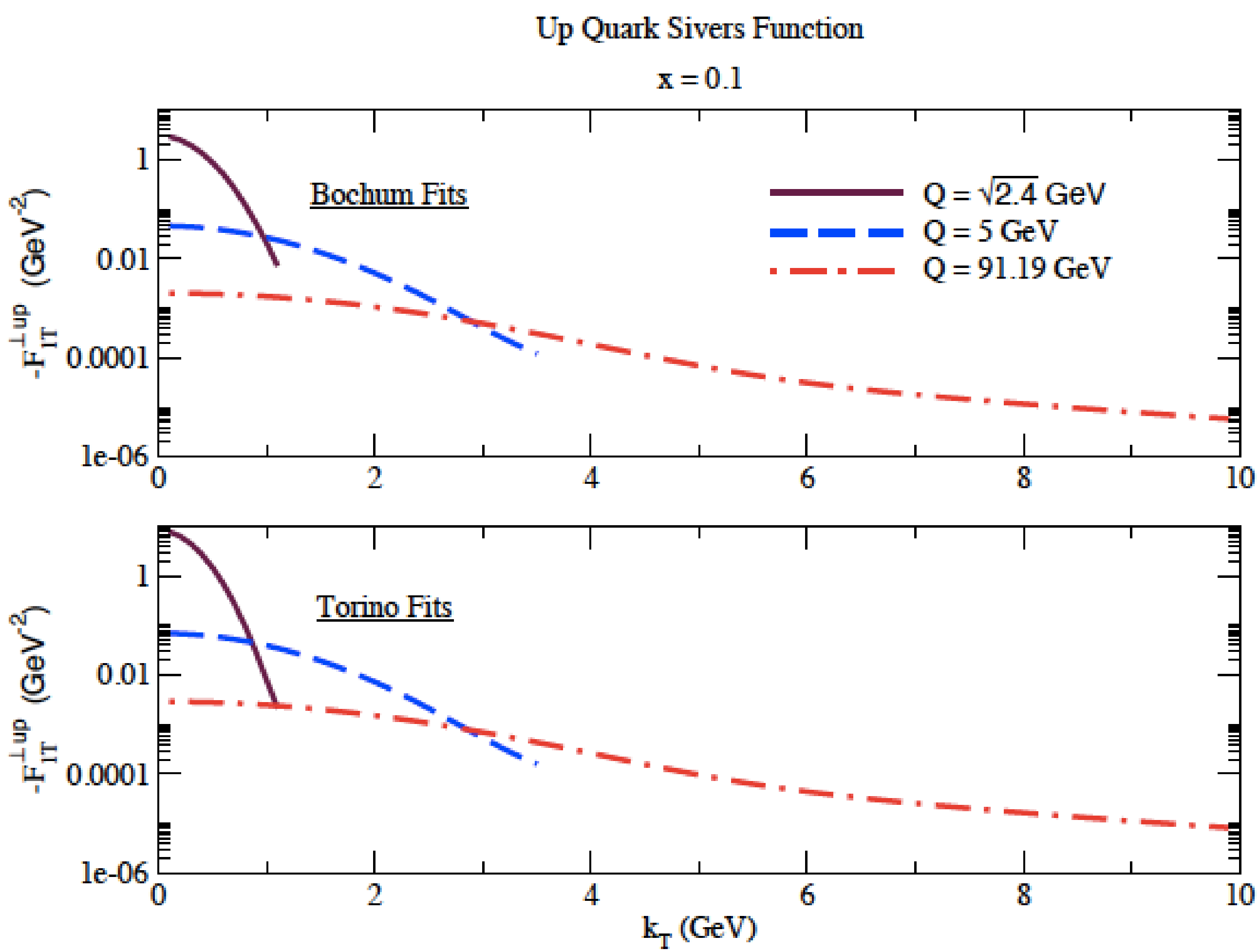
$$A_{UT}^{(1)} = C_F,\ B_{UT}^{(1)} = -3/2C_F,\ \Delta C_{qq}^{T(0)} = \delta(1-x)\ ,$$

$$\Delta C_{qq}^{T(1)} = -\frac{1}{4N_c}(1-x) + \frac{C_F}{2}\delta(x-1)\left[\frac{\pi^2}{2} - 4\right]\ ,$$

- Together with $A^{(2)}$, this resum at Next-to-leading-logarithmic level
- Phenomenological implementation is underway

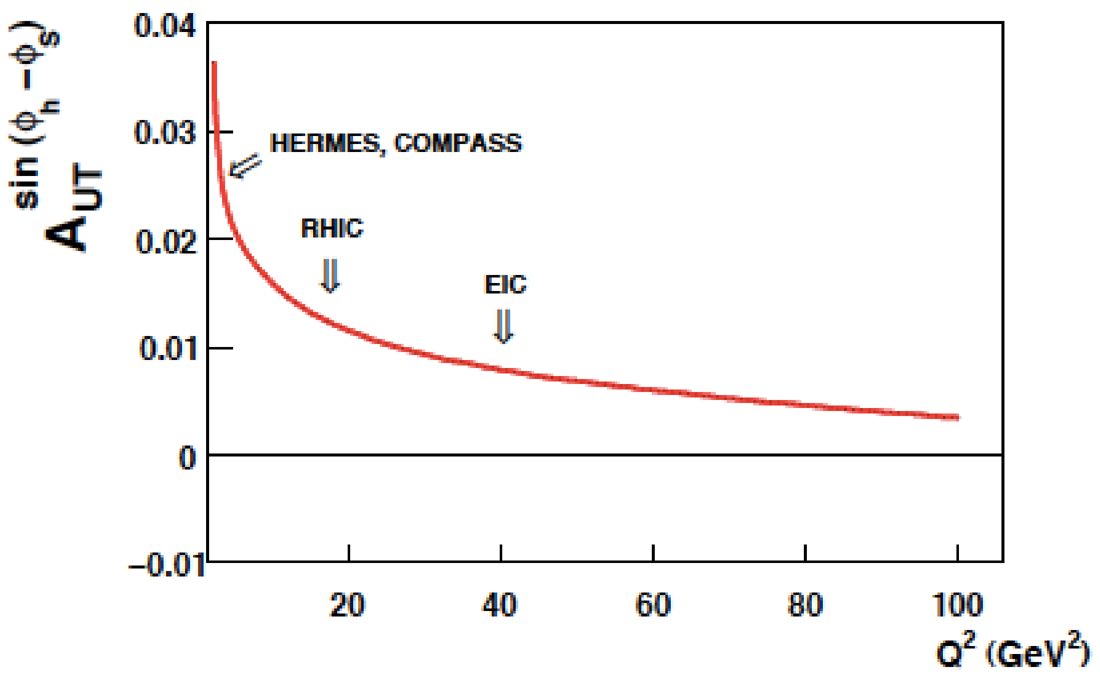
Directly working on TMDs

Aybat-Collins-Qiu-Rogers, 2011



Q^2-dependence

- Aybat-Prokudin-Rogers, 2011

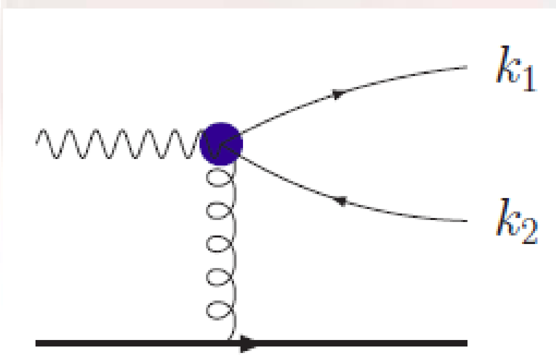


Needs a cross check!

TMD gluon distributions

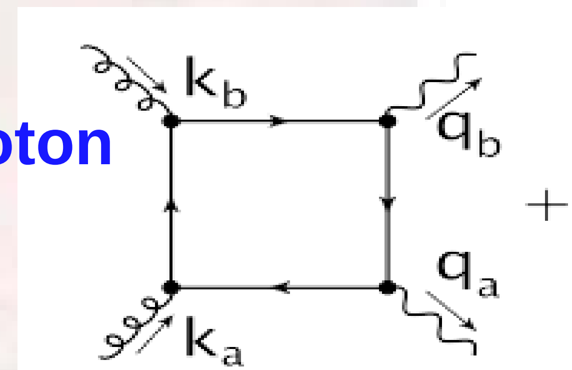
- It is not easy, because gluon does not couple to photon directly
- Can be studied in two-particle processes

Dijet In DIS



Vogelsang-Yuan, 2007
Dominguez-Xiao-Yuan, 2010
Boer-Brodsky-Mulders-Pisano, 2010

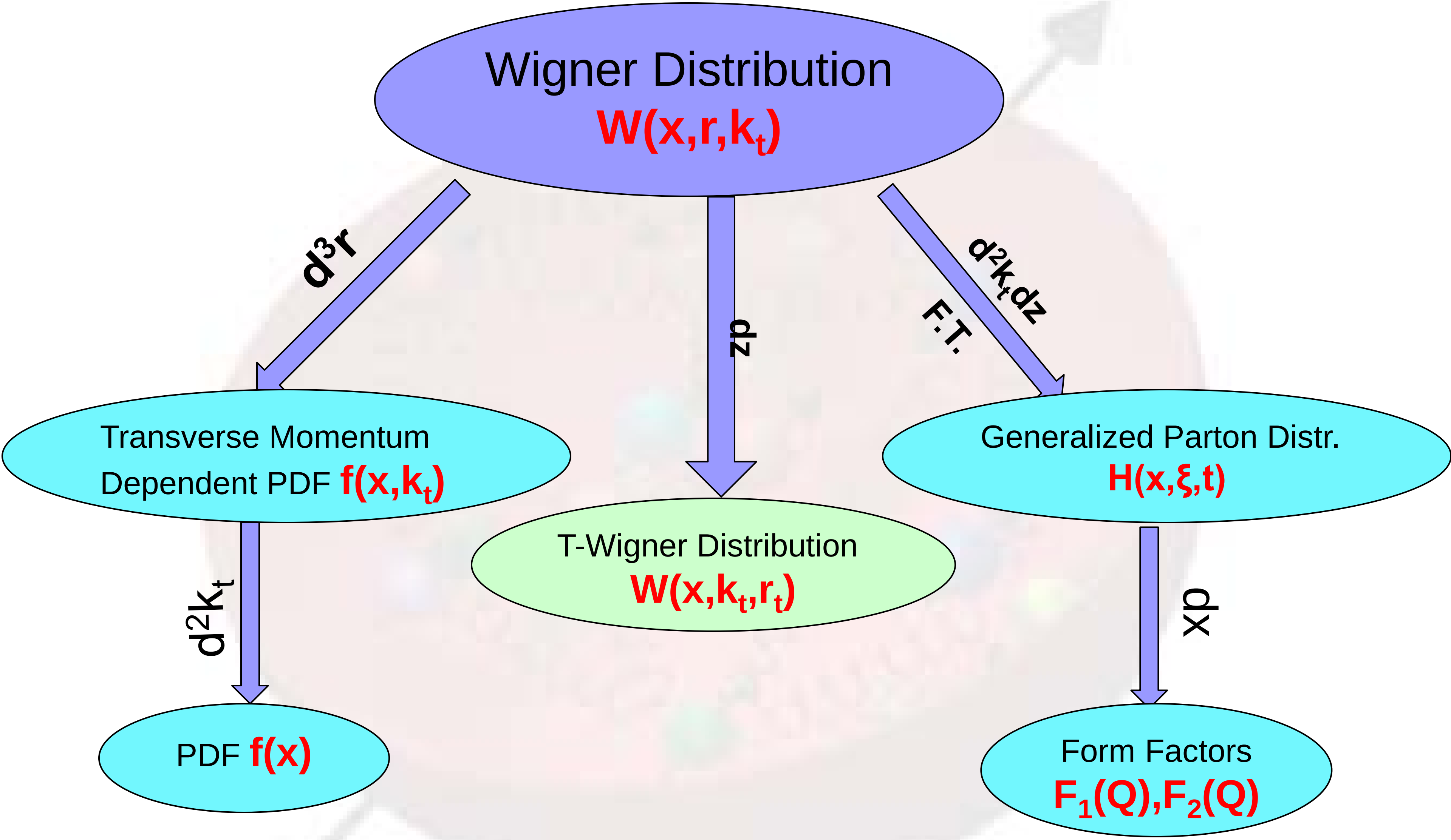
Di-photon In pp



Qiu-Schlegel-Vogelsang, 2011

Summary

- There have been great progresses in both experiment and theory for GPD and TMD physics
- Future Jlab 12 GeV upgrade, COMPASS, RHIC and the planed **EIC** experiments, will lead us a complete 3D tomography of the nucleon



Wigner Distribution
W(x,r,k_t)
d^3r
dz
d^2k_t dz
F.T.
Transverse Momentum
Dependent PDF f(x,k_t)
T-Wigner Distribution
W(x,k_t,r_t)
Generalized Parton Distr.
H(x,ξ,t)
d^2k_t
dx
PDF f(x)
Form Factors
F_1(Q),F_2(Q)

Transverse Wigner Distributions

- Integrate out z in the Wigner function

$$W_T(x, k_\perp, b_\perp = r_\perp) = \int dr_z W(x, \vec{r}, k_\perp)$$

  - Depends on x, k_t, b_t
  - Also referred as GTMD in the literature
    - See for example, Metz, et al., 09; Pasquini, 10,11
- It has close connection to the small-x parton distributions in large nuclei

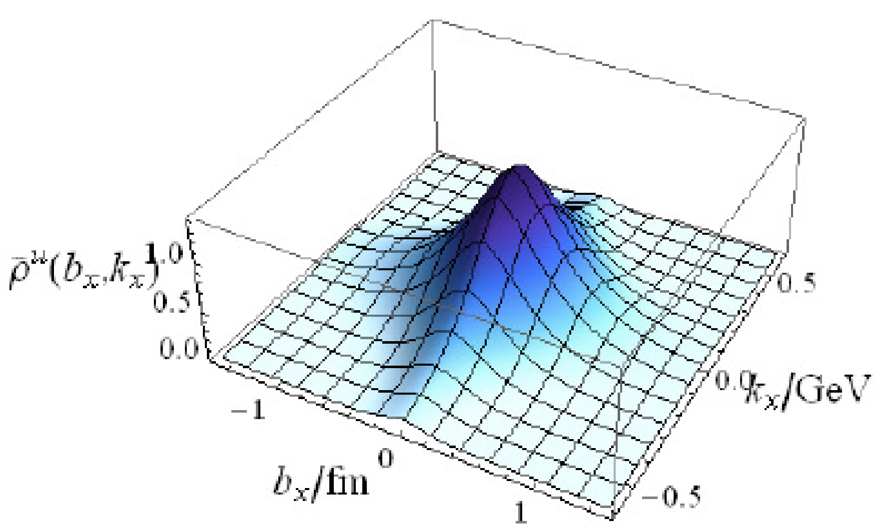
e.g., gluon number distr.
Mueller, NPB 1999

$$\frac{dN}{d^2b d^2\ell} = \frac{N_c^2 - 1}{4\pi^3 \alpha N_c} \int_1^\infty \frac{dt}{t} e^{-t\ell^2/Q_s^2}$$

Further integrate out x

$$\mathcal{H}(\vec{r}_\perp, \vec{k}_\perp) = \int dxdzW(\vec{r}, x, k_\perp)$$

AMO Exp.



Quark model calculation: Xiong, et al.

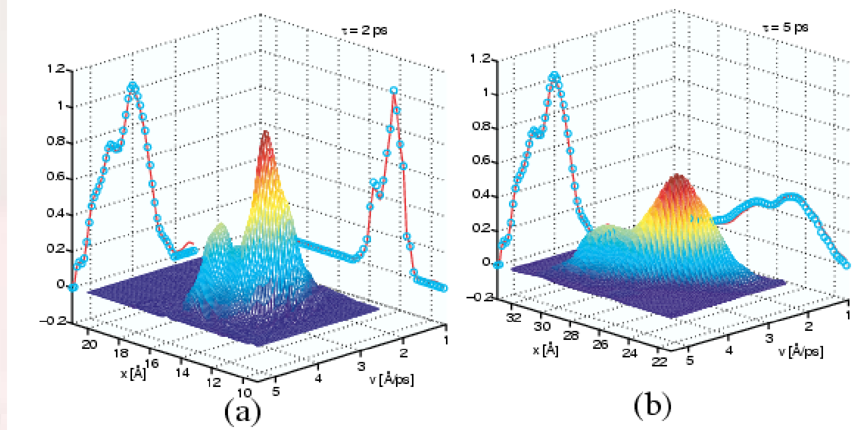


FIG. 2 (color online). Three-dimensional surface plots showing the phase space distribution function $W(x, p)$ (Wigner function) at two different times after the pump pulse dissociates the I_2 molecule [(a) $\tau = 2$ ps, and (b) $\tau = 5$ ps]. The marginal position and momentum distributions of the dissociated molecules are shown on the side panels: The full curves are the reconstructed distributions (integrals over the Wigner function with respect to momentum and position, respectively), and the open circles are the measured distributions. To provide a more intuitive view the momentum variable is presented in velocity units.

Skovsen et al. (Denmark) PRL91, 090604

Wigner function: Phase Space Distributions

- Define as

Wigner 1933

$$W(x,p) = \int \psi^*(x-\eta/2)\psi(x+\eta/2)e^{ip\eta}d\eta \ ,$$

  - When integrated over x (p), one gets the momentum (probability) density
  - Not positive definite in general, but is in classical limit
  - Any dynamical variable can be calculated as

$$\langle O(x,p)\rangle = \int dxdp O(x,p)W(x,p)$$

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Wigner distribution for the quark

- The quark operator

Ji: PRL91,062001(2003)

$$\hat{\mathcal{W}}_\Gamma(\vec{r},k) = \int \overline{\Psi}(\vec{r}-\eta/2)\Gamma\Psi(\vec{r}+\eta/2)e^{ik\cdot\eta}d^4\eta$$

- Wigner distributions

$$W_\Gamma(\vec{r},k) = \frac{1}{2M_N}\int\frac{d^3\vec{q}}{(2\pi)^3}\left\langle \vec{q}/2\left|\hat{\mathcal{W}}_\Gamma(\vec{r},k)\right|-\vec{q}/2\right\rangle$$

$$= \frac{1}{2M_N}\int\frac{d^3\vec{q}}{(2\pi)^3}\mathrm{e}^{-i\vec{q}\cdot\vec{r}}\left\langle \vec{q}/2\left|\hat{\mathcal{W}}_\Gamma(0,k)\right|-\vec{q}/2\right\rangle$$

After integrating over r, one gets TMD

After integrating over k, one gets Fourier transform of GPDs

TMD Parton Distributions

- The definition contains explicitly the gauge links

Collins-Soper 1981, Collins 2002, Belitsky-Ji-Yuan 2002

$$f(x,k_\perp) = \frac{1}{2}\int\frac{d\xi^- d^2\xi_\perp}{(2\pi)^3}e^{-i(\xi^- k^+ - \vec{\xi}_\perp\cdot\vec{k}_\perp)} \times\langle PS|\overline{\psi}(\xi^-,\xi_\perp)L^\dagger_{\xi_\perp}(\xi^-)\gamma^+ L_0(0)\psi(0)|PS\rangle$$

- The polarization and kt dependence provide rich structure in the quark and gluon distributions
  - Mulders-Tangerman 95, Boer-Mulders 98

Generalized Parton Distributions

Mueller, et al. 1994; Ji, 1996, Radyushkin 1996

- Off-diagonal matrix elements of the quark operator (along light-cone)

$$F_q(x,\xi,t) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{i\lambda x} \left\langle P' \left| \overline{\psi}_q\left(-\frac{\lambda}{2}n\right) \not{n} \mathcal{P} e^{-ig\int_{\lambda/2}^{-\lambda/2} d\alpha\, n\cdot A(\alpha n)} \psi_q\left(\frac{\lambda}{2}n\right) \right| P \right\rangle$$

$$= H_q(x,\xi,t)\,\frac{1}{2}\overline{U}(P')\,\not{n}U(P) + E_q(x,\xi,t)\,\frac{1}{2}\overline{U}(P')\frac{i\sigma^{\mu\nu}n_\mu\Delta_\nu}{2M}U(P)\,.$$

- It depends on quark momentum fraction x and skewness ξ, and nucleon momentum transfer t

$$\xi = -n\cdot(P'-P)/2$$
$$t = \Delta^2 = (P-P')^2$$