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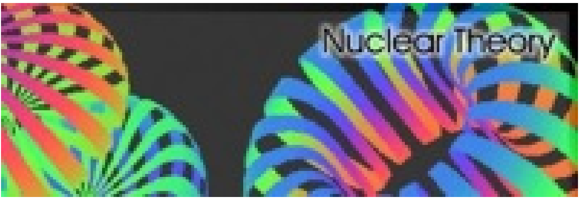
# The Issue of Substance in Hadron Physics

Craig Roberts



Physics Division

Office of Nuclear Physics
Exploring Nuclear Matter - Quarks to Stars

National Science Foundation
WHERE DISCOVERIES BEGIN

***Published collaborations: 2010-present***

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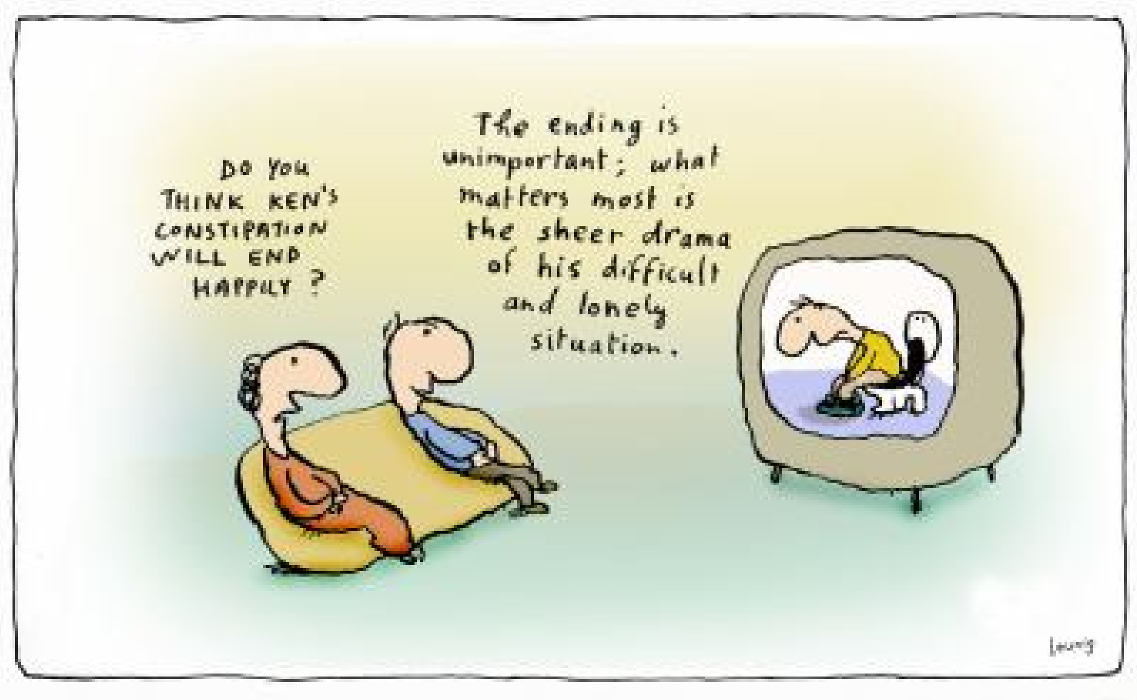
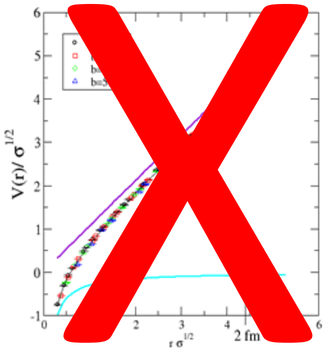
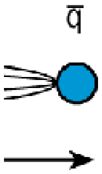
U.S. DEPARTMENT OF ENERGY



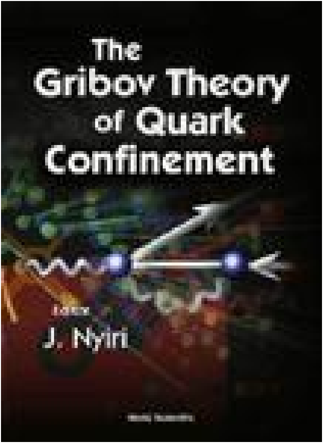
# Confinement





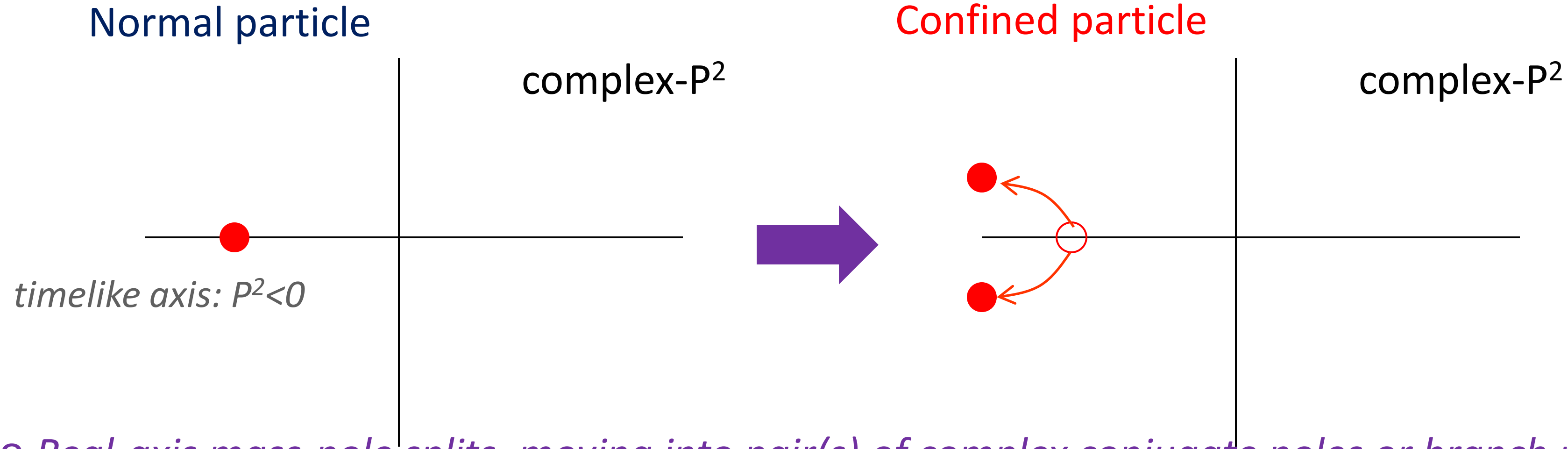
# Confinement

- Gluon and Quark Confinement
  - Empirical Fact: No coloured states have yet been observed to reach a detector
- However
  - There is no agreed, theoretical definition of light-quark confinement
  - Static-quark confinement is irrelevant to real-world QCD
    - *There are no long-lived, very-massive quarks*
    - *But light-quarks are ubiquitous*
- Flux tubes, linear potentials and string tensions play no role in relativistic quantum field theory with light degrees of freedom.
- To suggest otherwise is to misapprehend the core challenge of real-world QCD.



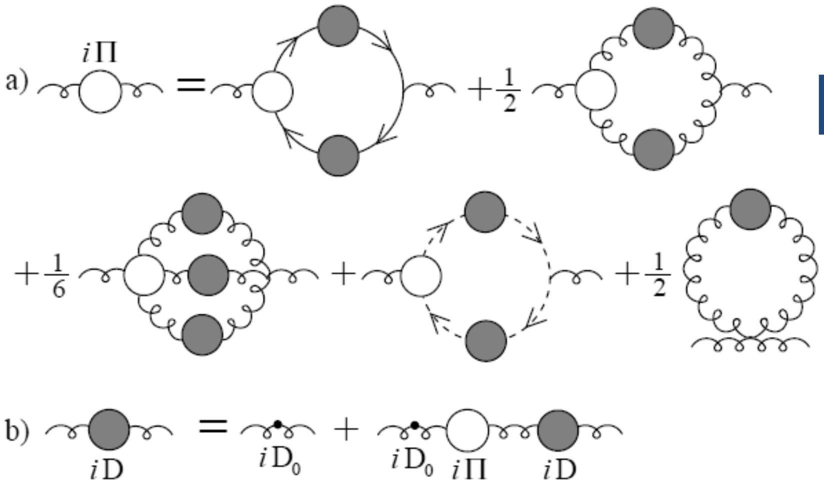
# Confinement

- QFT Paradigm: Confinement is expressed through a *dramatic* change in the analytic structure of propagators for coloured particles & can almost be read from a plot of a states' dressed-propagator
  - Gribov (1978); Munczek (1983); Stingl (1984); Cahill (1989); Roberts, Williams & Krein (1992); Tandy (1994); …



o *Real-axis mass-pole splits, moving into pair(s) of complex conjugate poles or branch points, or more complicated nonanalyticities …*

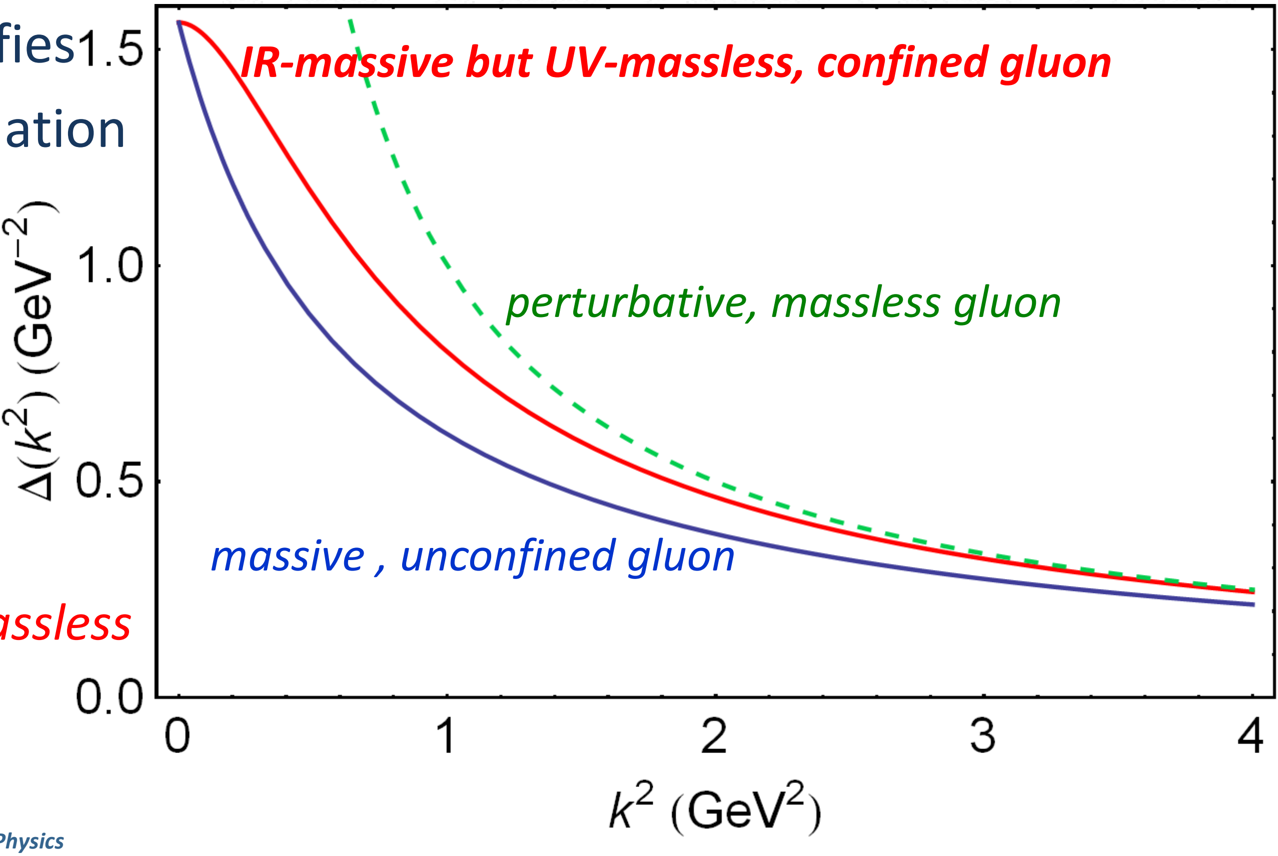
o *Spectral density no longer positive semidefinite & hence state cannot exist in observable spectrum*



# Dressed-gluon propagator

*A.C. Aguilar* et al., Phys.Rev. D80 (2009) 085018

- Gluon propagator satisfies a Dyson-Schwinger Equation
- Plausible possibilities for the solution
- DSE and lattice-QCD agree on the result
  - *Confined gluon*
  - *IR-massive but UV-massless*
  - $m_G \approx 2\text{-}4\ \Lambda_{QCD}$

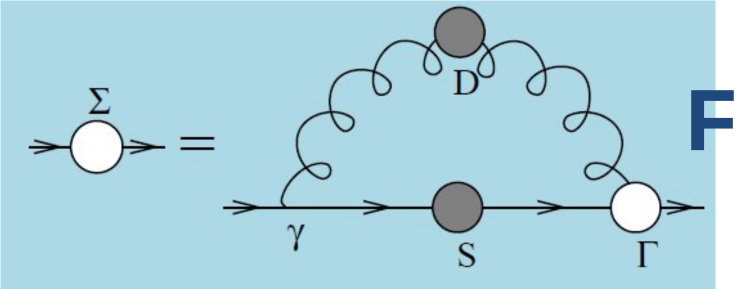


# Dynamical Chiral Symmetry Breaking

# Dynamical Chiral Symmetry Breaking

- Whilst confinement is contentious …
- *DCSB is a fact in QCD*
  - – It is the most important mass generating mechanism for visible matter in the Universe.
    - Responsible for approximately 98% of the proton's mass.
    - Higgs mechanism is (*almost*) irrelevant to light-quarks.

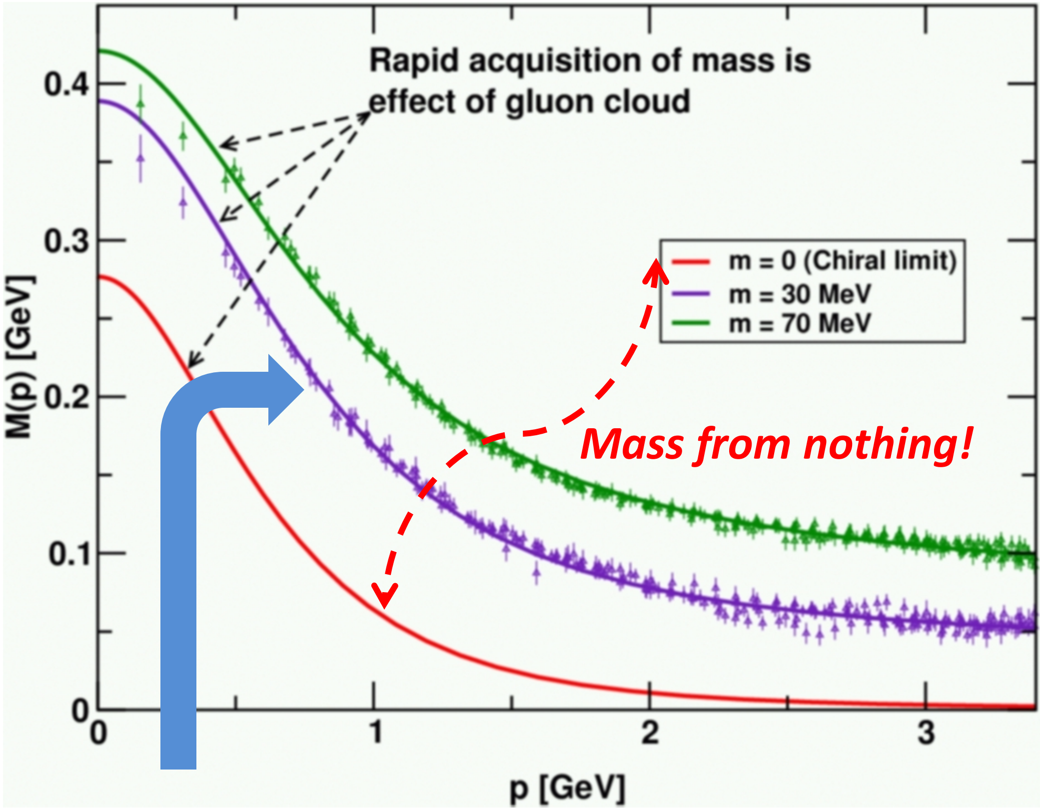
# Frontiers of Nuclear Science: Theoretical Advances



*C.D. Roberts, Prog. Part. Nucl. Phys. 61 (2008) 50*
*M. Bhagwat & P.C. Tandy, AIP Conf.Proc. 842 (2006) 225-227*

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

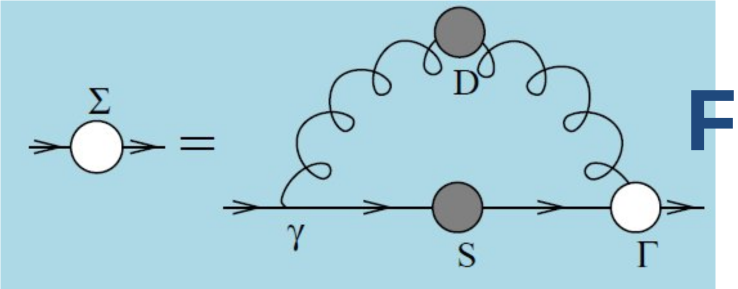
**In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies (m =0, red curve) acquires a large constituent mass at low energies.**



Rapid acquisition of mass is effect of gluon cloud

*Mass from nothing!*

## DSE prediction of DCSB confirmed



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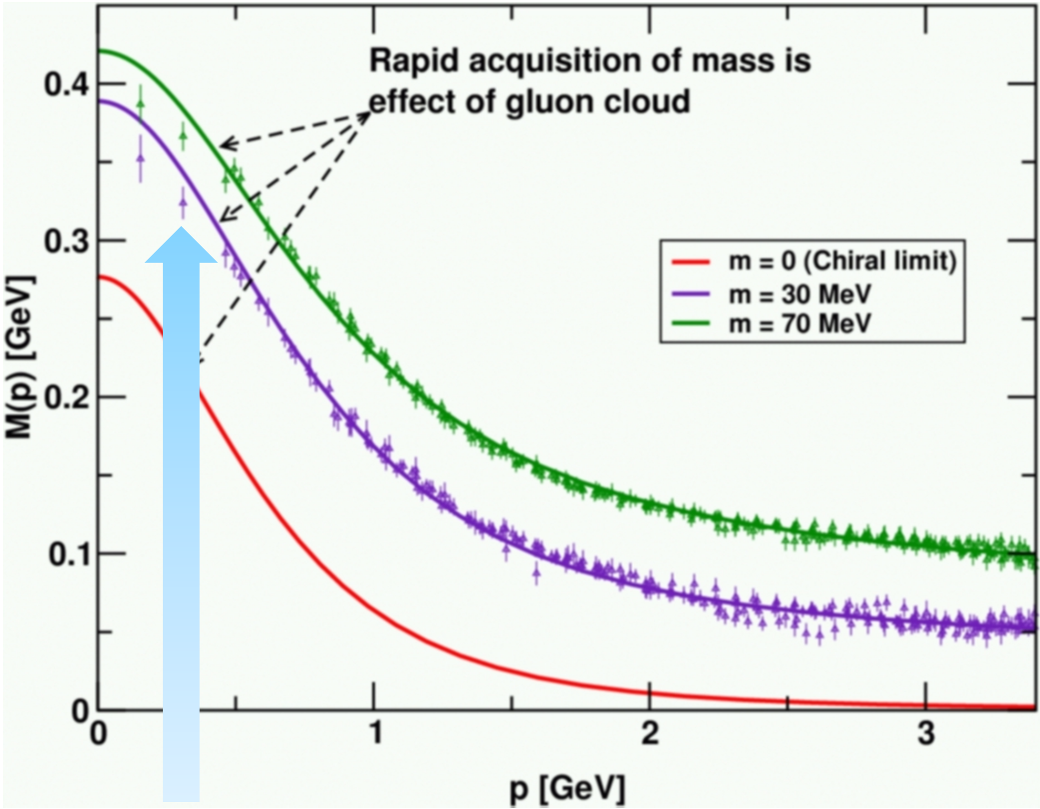
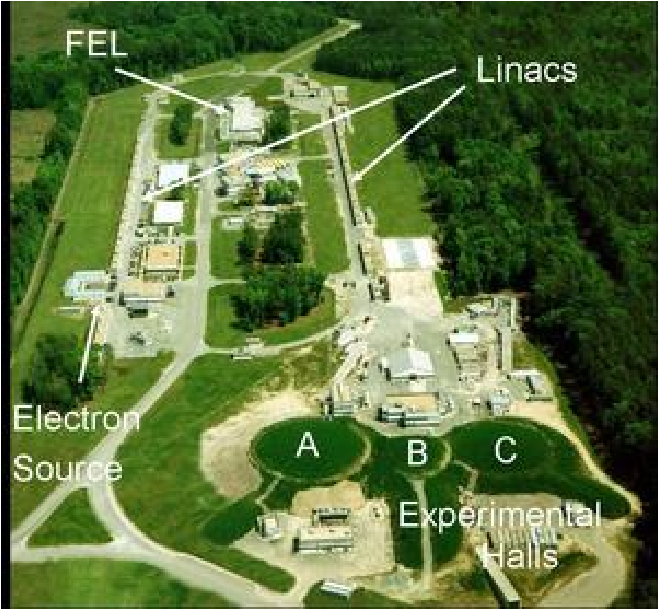


**Hint of lattice-QCD support for DSE prediction of violation of reflection positivity**

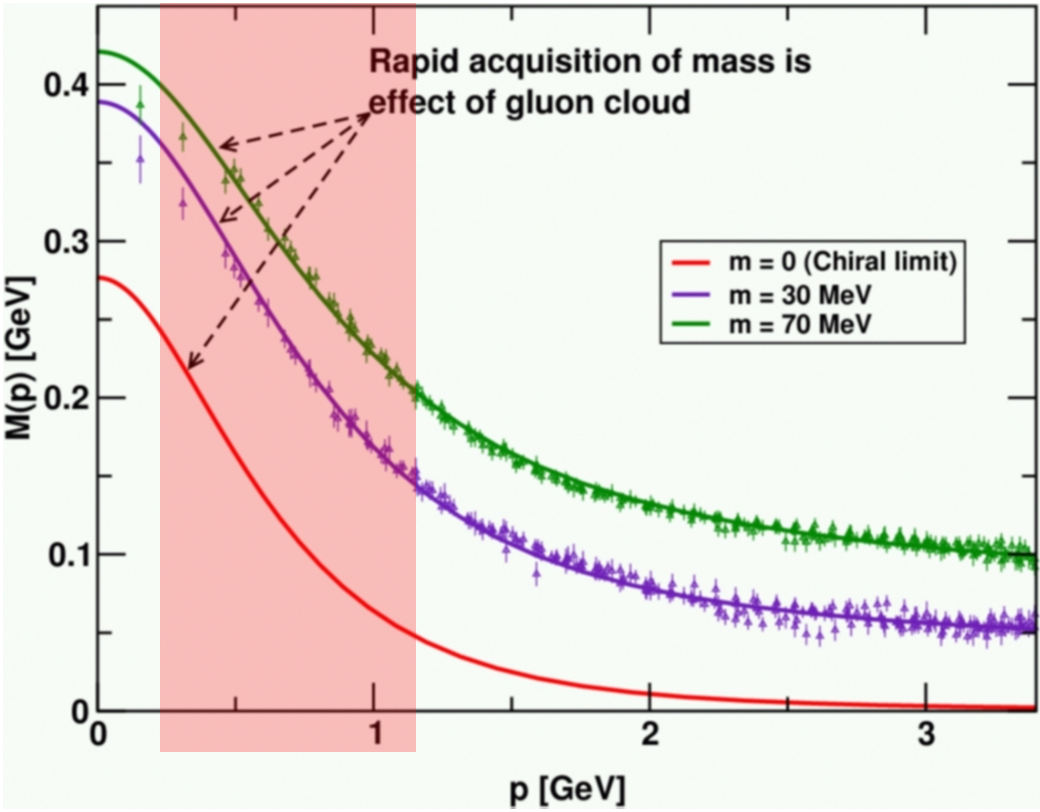
*Craig Roberts: The Issue of Substance in Hadron Physics*

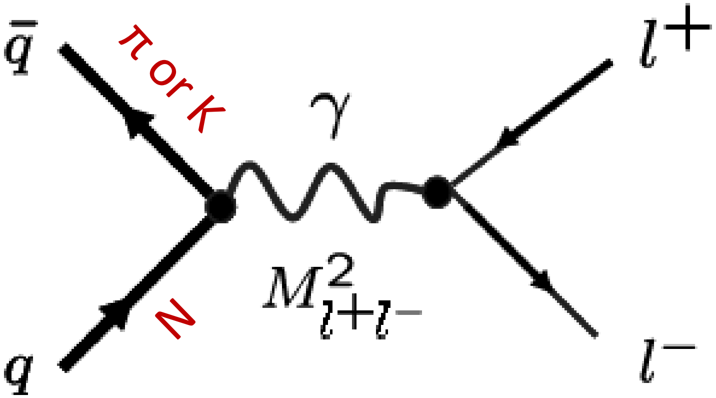


# 12GeV
# The Future of JLab

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

- **Jlab 12GeV: This region scanned by $2<Q^2<9$ GeV$^2$ elastic & transition form factors.**

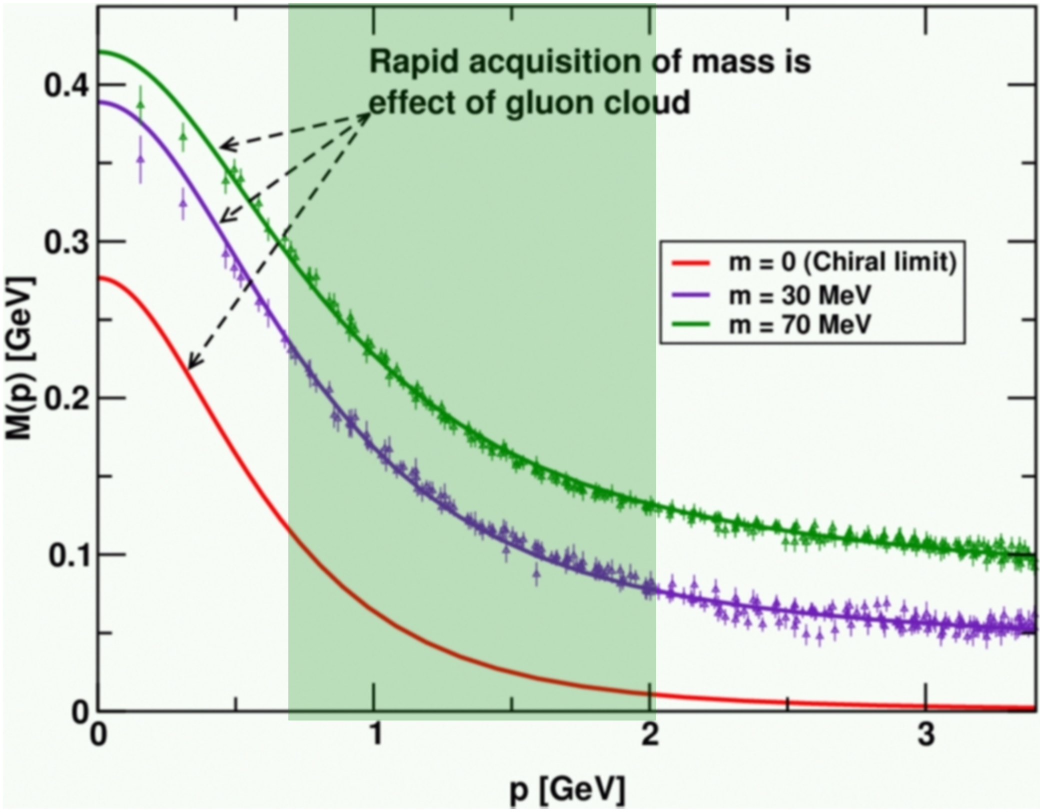




# The Future of Drell-Yan

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

- **Valence-quark PDFs and PDAs probe this critical and complementary region**



# Science Challenges for the coming decade: 2013-2022

- Search for exotic hadrons
- Exploit opportunities provided by new data on nucleon elastic and transition form factors
- Precision experimental study of valence region, and theoretical computation of distribution functions and distribution amplitudes
- Develop QCD as a probe for physics beyond the Standard Model

# Overarching Science Challenges for the coming decade: 2013-2022

- Search for exotic hadrons
- Exploit opportunities provided by new data on nucleon elastic and transition form factors
- Precision experimental study of valence region, and theoretical computation of distribution functions and distribution amplitudes
- Develop QCD as a probe for physics beyond the Standard Model

**Discover meaning of confinement, and its relationship to DCSB – the origin of visible mass**

# Charting the interaction between light-quarks

*This is a well-posed problem whose solution is an elemental goal of modern hadron physics. The answer provides QCD's running coupling.*

- Confinement can be related to the analytic properties of QCD's Schwinger functions.
- Question of light-quark confinement can be translated into the challenge of charting the infrared behavior of QCD's universal *β*-function
- Through QCD's DSEs, the pointwise behaviour of the β-function determines the pattern of chiral symmetry breaking.
- DSEs connect β-function to experimental observables. Hence, comparison between computations and observations of
  - Hadron spectrum, Elastic & transition form factors, Parton distribution fns

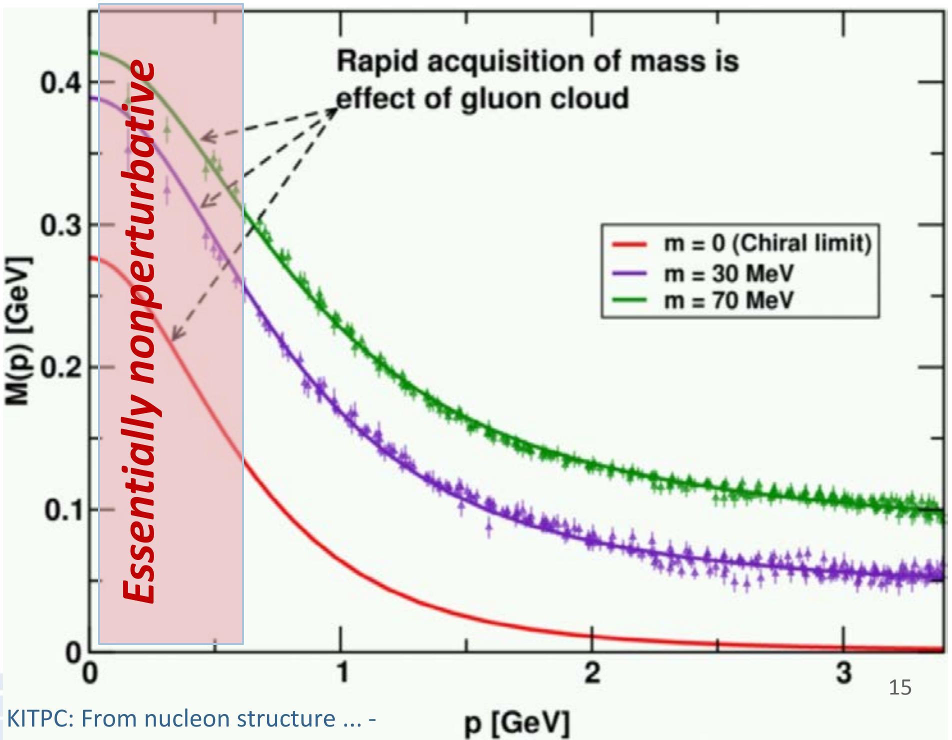
  can be used to chart β-function's long-range behaviour.

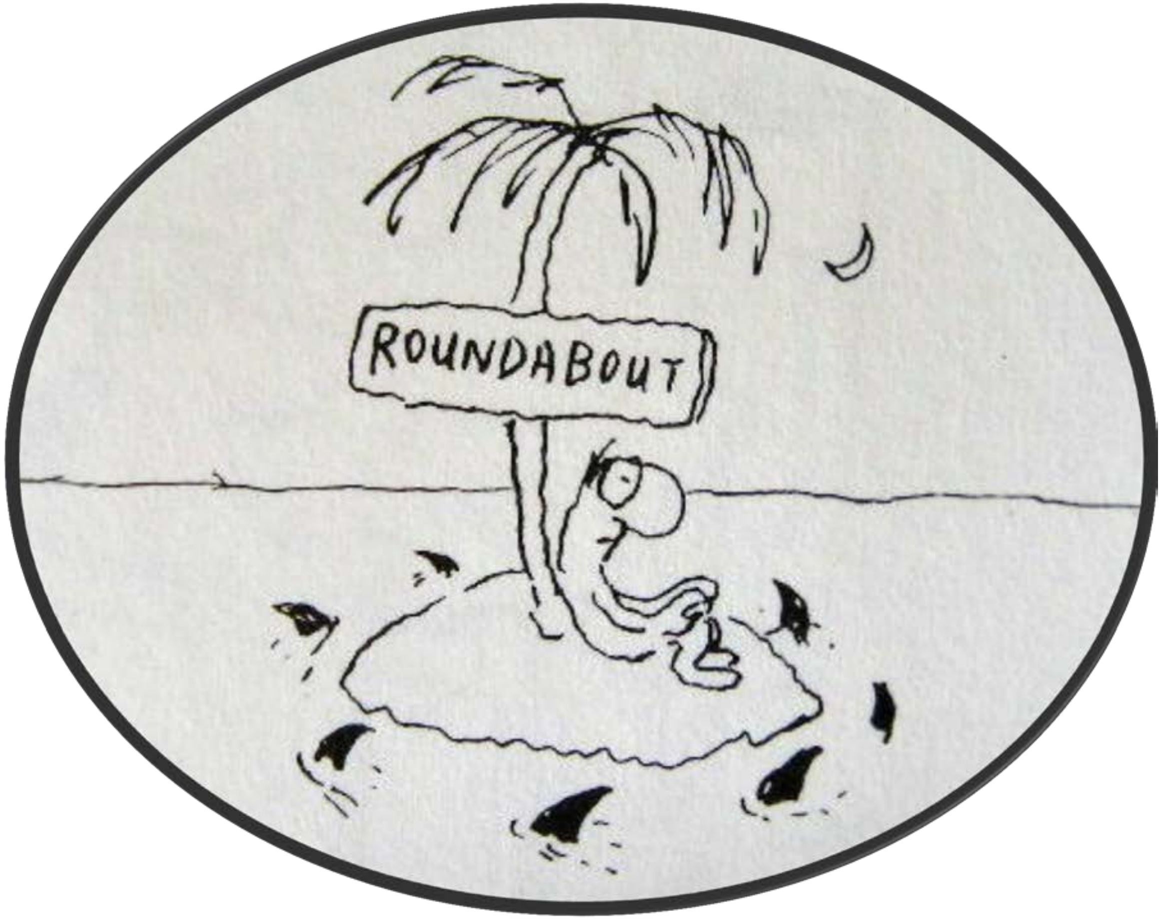
# *Necessary Precondition*

- Experiment ↔ Theory comparison leads to an understanding of long-range behaviour of strong running-coupling
- However, if one wants to draw reliable conclusions about $Q^2$-dependence of QCD's running coupling,
- Then, approach must veraciously express $Q^2$-dependence of QCD's running masseS
- True for ALL observables
  - ✓ From spectrum …
  - ✓ through elastic & transition form factors …
  - ✓ to PDFs and GPDs … etc.

- Mass function exhibits inflexion point at $Q_{IR} \approx m_G \approx 0.6$GeV
- So … pQCD is *definitely invalid* for momenta $Q<Q_{IR}$
- E.g., use of DGLAP equations cannot be justified in QCD at $Q<Q_{IR}=0.6$GeV, irrespective of order.

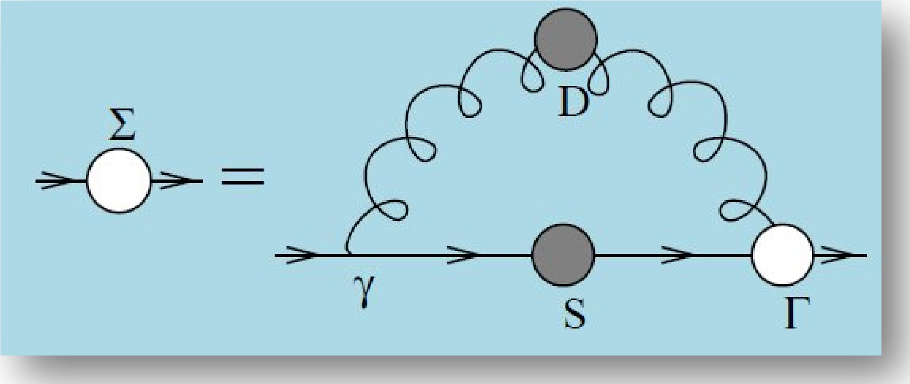
*Distribution Functions of the Nucleon and Pion in the Valence Region*, Roy J. Holt and Craig D. Roberts, arXiv:1002.4666 [nucl-th], Rev. Mod. Phys. **82** (2010) pp. 2991-3044



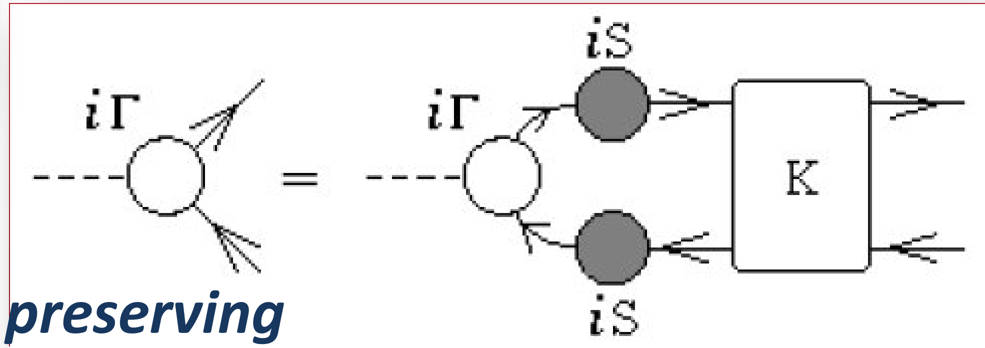


# Persistent Challenge

# Truncation



# Persistent challenge - truncation scheme



- There are now two ***nonperturbative*** & ***symmetry preserving*** truncation schemes
  1. ***1995*** – H.J. Munczek, Phys. Rev. D **52** (1995) 4736, *Dynamical chiral symmetry breaking, Goldstone's theorem and the consistency of the Schwinger-Dyson and Bethe-Salpeter Equations*

     ***1996*** – A. Bender, C.D. Roberts and L. von Smekal, Phys.Lett. B **380** (1996) 7, *Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation*
  2. ***2009*** – Lei Chang and C.D. Roberts, Phys. Rev. Lett. 103 (2009) 081601, 0903.5461 [nucl-th], *Sketching the Bethe-Salpeter kernel*
- Enables proof of numerous exact results

*Maris, Roberts and Tandy*
*nucl-th/9707003, Phys.Lett. B**420** (1998) 267-273*

# Pion's Goldberger -Treiman relation

- Pion's Bethe-Salpeter amplitude

Solution of the Bethe-Salpeter equation

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\Big]$$

***Pseudovector components necessarily nonzero. Cannot be ignored!***

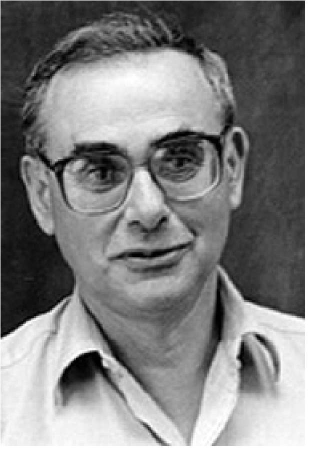
- Dressed-quark propagator $S(p) = \dfrac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$

- Axial-vector Ward-Takahashi identity entails

$$f_\pi E_\pi(k;P=0) = B(p^2)$$

***Exact in Chiral QCD***

***Miracle*: two body problem solved, almost completely, once solution of one body problem is known**



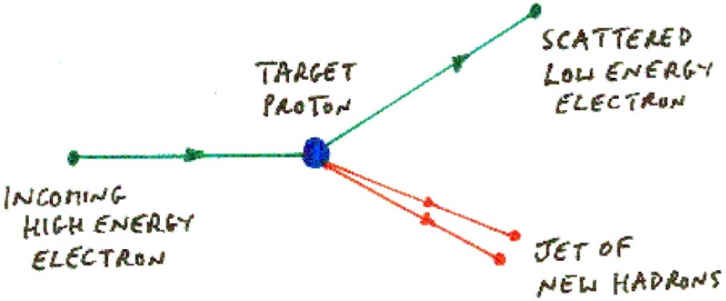
# Dichotomy of the pion
# Goldstone mode and bound-state

$$f_\pi E_\pi(p^2) = B(p^2)$$

- Goldstone's theorem
  has a pointwise expression in QCD;

  Namely, in the chiral limit the wave-function for the two-body bound-state Goldstone mode is intimately connected with, and almost completely specified by, the fully-dressed one-body propagator of its characteristic constituent

  - The one-body momentum is equated with the relative momentum of the two-body system



# Deep inelastic scattering

- Quark discovery experiment at SLAC
  (1966-1978, Nobel Prize in 1990)
- Completely different to elastic scattering
  - *Blow the target to pieces instead of keeping only those events where it remains intact.*
- Cross-section is interpreted as a measurement of the momentum-fraction probability distribution for quarks and gluons within the target hadron: $q(x)$, $g(x)$ ← *Probability that a quark/gluon within the target will carry a fraction x of the bound-state's light-front momentum*

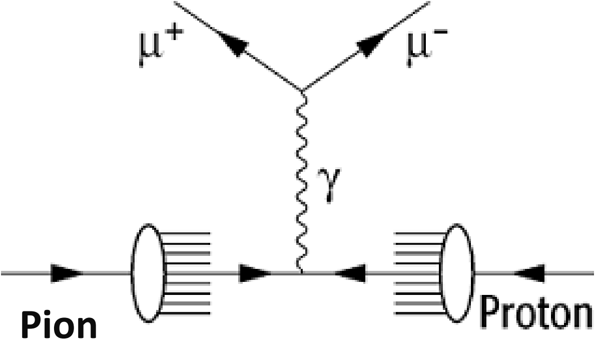
*Distribution Functions of the Nucleon and Pion in the Valence Region*, Roy J. Holt and Craig D. Roberts, arXiv:1002.4666 [nucl-th], Rev. Mod. Phys. **82** (2010) pp. 2991-3044

# Pion distribution function

- The pion plays a key role in nucleon and nuclear structure. It has been used ; e.g., to explain
  - the long-range nucleon-nucleon interaction, forming a basic part of the “Standard Model” of nuclear physics
  - and also the flavor asymmetry observed in the quark sea in the nucleon.
- However, compared to that of other hadrons, the pion mass is anomalously small.
  - This owes to dynamical chiral-symmetry breaking
  - Any veracious description of the pion must properly account for its dual role as a quark-antiquark bound state and the Nambu-Goldstone boson associated with DCSB.
- It is this dichotomy and its consequences that make an experimental and theoretical elucidation of pion properties so essential to understanding the strong interaction.

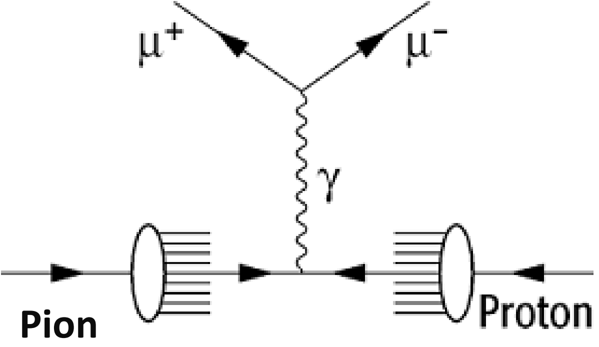
# Kaon distribution function

- The valence structure of the kaon is comprised of a light *up or down* quark (or antiquark) and a *strange* antiquark (or quark).
- If our understanding of meson structure is correct, then the large difference between the current mass of the *s* quark and that of the *u* and *d* quarks should give rise to some interesting effects in the kaon structure function.
  - For example, owing to its larger mass, the *s* quark should carry more of the charged kaon's momentum than the *u quark.*
  - Hence the $u_v$ quark distribution in the kaon should be weighted to lower values in *x* than that in the pion.
  - If such a shift exists, then what sets the scale … current-quark mass or something else?



# Models of the Pion's valence-quark distributions

- *$(1-x)^{\beta}$ with $\beta=0$ (i.e., a* constant – any fraction is equally probable! )
  - AdS/QCD models using light-front holography
  - Nambu–Jona-Lasinio models, when a translationally invariant regularization is used
- *$(1-x)^{\beta}$ with $\beta=1$*
  - *Nambu*–Jona-Lasinio NJL models with a hard cutoff
  - Duality arguments produced by some theorists
- *$(1-x)^{\beta}$ with $0<\beta<2$*
  - Relativistic constituent-quark models, with power-law depending on the form of model wave function
- *$(1-x)^{\beta}$ with $1<\beta<2$*
  - Instanton-based models, all of which have incorrect large-$k^2$ behaviour



# Models of the Pion's valence-quark distributions

- *(1−x)*$^{\beta}$ *with* $\beta=0$ *(i.e., a* constant – any fraction is equally probable! )
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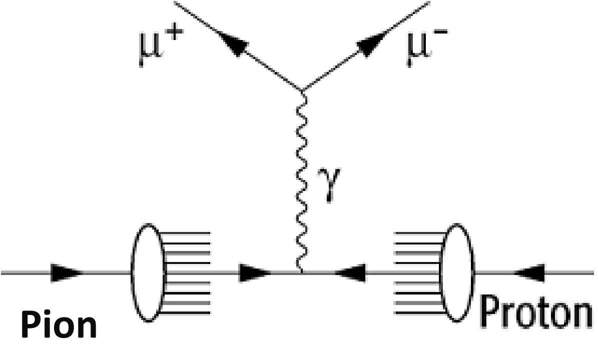
*Unsatisfactory. Impossible to suggest that there's even qualitative agreement!*



# DSE prediciton of the Pion's valence-quark distributions

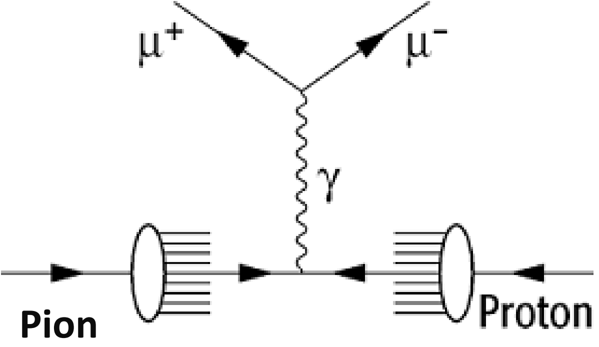
- Consider a theory in which quarks scatter via a vector-boson exchange interaction whose $k^2 >> m_G^2$ behaviour is $(1/k^2)^\beta$,
- Then at a resolving scale $Q_0$

$$u_\pi(x;Q_0) \sim (1-x)^{2\beta}$$

namely, the large-x behaviour of the quark distribution function is a direct measure of the momentum-dependence of the underlying interaction.

- In QCD, β=1 and hence

$$^{QCD}u_\pi(x;Q_0) \sim (1-x)^2$$



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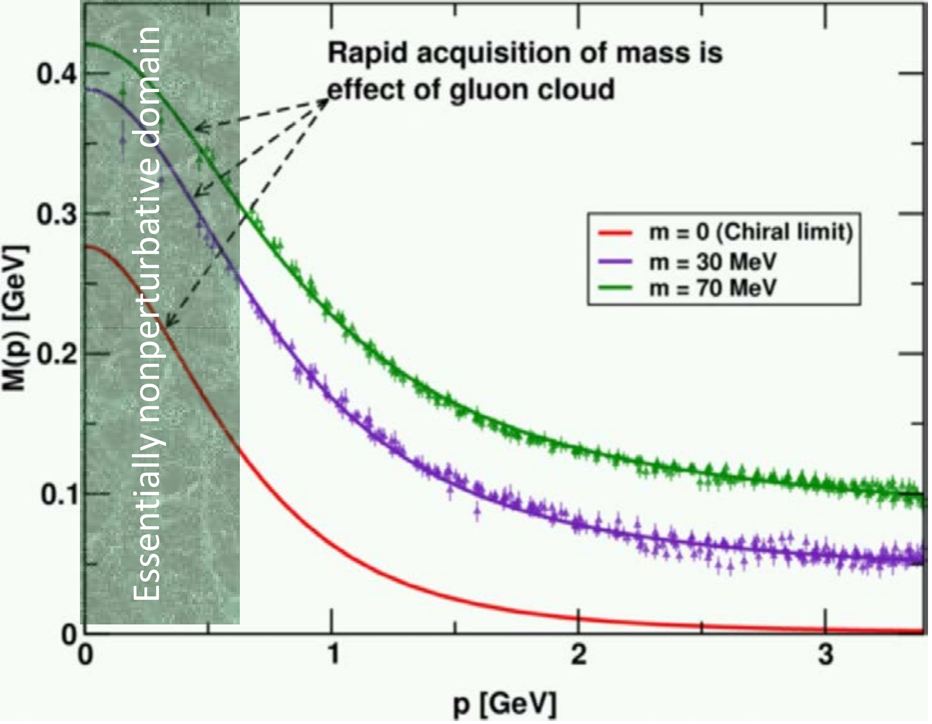
$$^{QCD}u_\pi(x;Q_0) \sim (1-x)^2$$

*Completely unambigous! Direct connection between experiment and theory, empowering both as tools of discovery.*

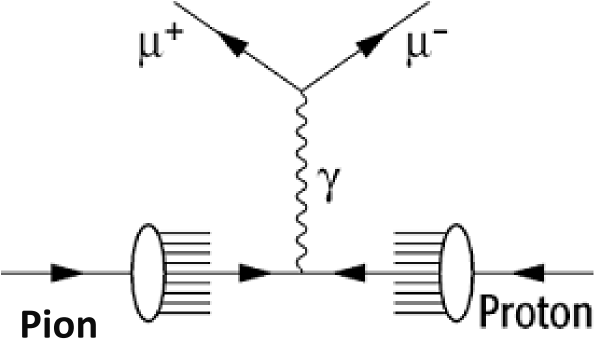


# “Model Scale”

- At what scale $Q_0$ should the prediction be valid?
- Hitherto, PDF analyses within models have used the resolving scale $Q_0$ as a parameter, to be chosen by requiring agreement between the model and low-moments of the PDF that are determined empirically.
- Modern DSE studies have exposed a natural value for the model scale; viz.,

$$Q_0 \approx m_G \approx 0.6 \text{ GeV}$$

which is the location of the inflexion point in the chiral-limit dressed-quark mass function



# Empirical status of the Pion's valence-quark distributions

- Owing to absence of pion targets, the pion's valence-quark distribution functions are measured via the Drell-Yan process:

  $\pi\, p \to \mu^+ \mu^- X$

- Three experiments: CERN (1983 & 1985) and FNAL (1989). No more recent experiments because theory couldn't even explain these!
- Problem

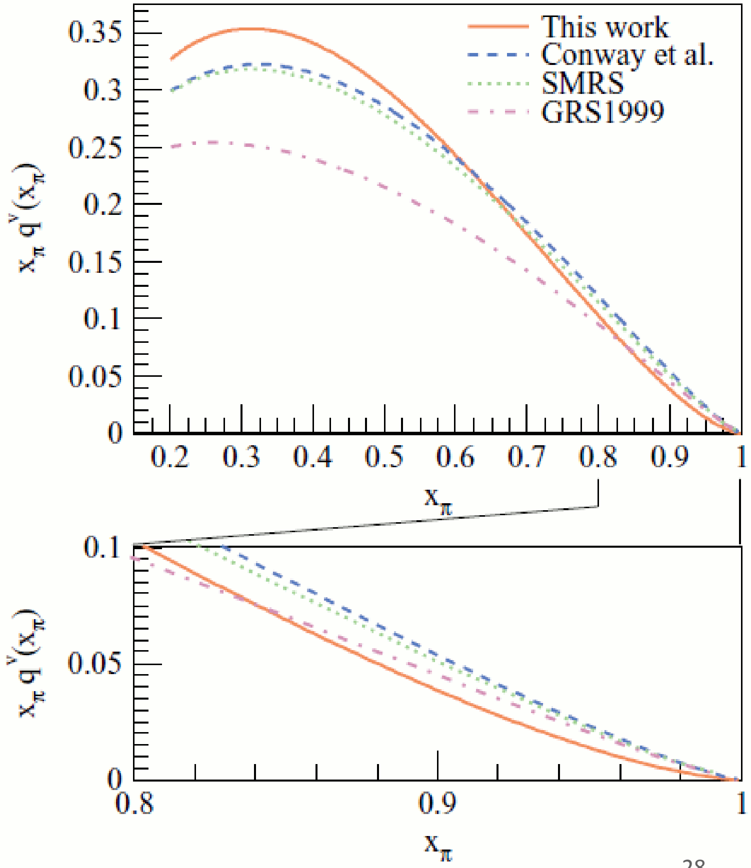
  Conway *et al*. Phys. Rev. D **39**, 92 (1989)

  Wijesooriya *et al*. Phys.Rev. C **72** (2005) 065203

PDF behaviour at large-*x* inconsistent with pQCD; viz,

expt. $(1-x)^{1+\varepsilon}$

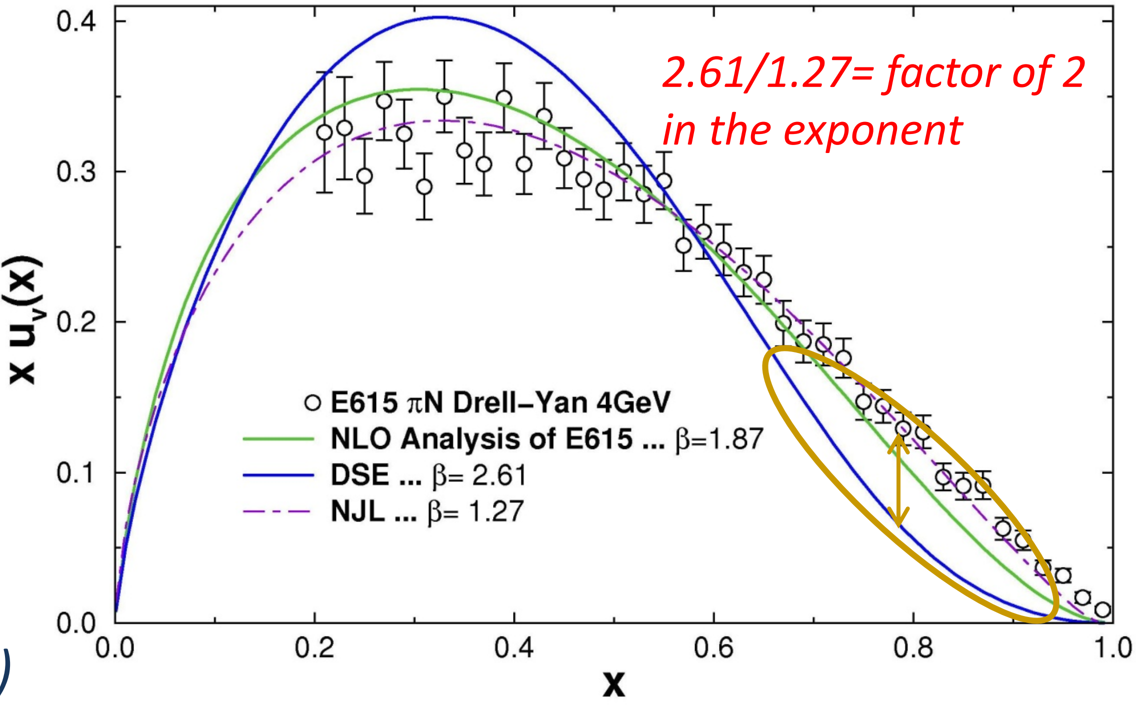
cf. QCD $(1-x)^{2+\gamma}$



Hecht, Roberts, Schmidt
Phys.Rev. C **63** (2001) 025213

# Computation of $q_v^\pi(x)$

- As detailed in preceding transparencies, before first DSE computation, which used the running dressed-quark mass described previously, numerous authors applied versions of the Nambu–Jona-Lasinio model, etc., and were content to vary parameters and $Q_0$ in order to reproduce the data, arguing therefrom that the inferences from pQCD were wrong
- After the first DSE computation, real physicists (i.e., *experimentalists*) again became interested in the process because
  - DSEs agreed with pQCD but disagreed with the data and models
- Disagreement on the "valence domain," which is particularly sensitive to *M(p²)*

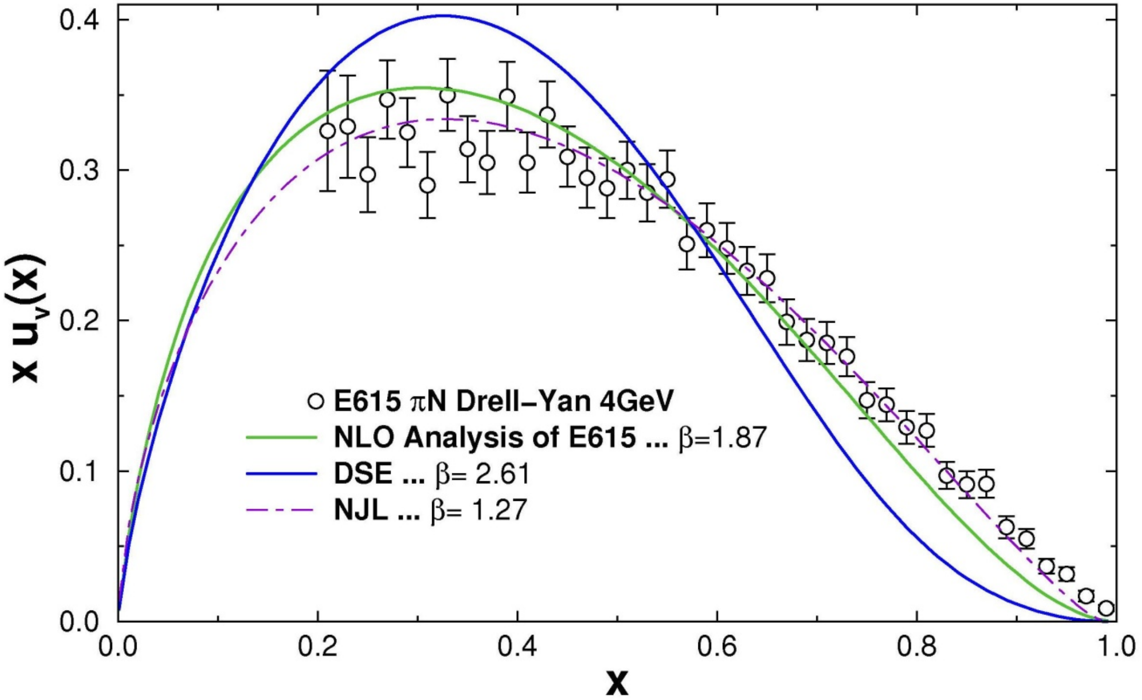


*2.61/1.27= factor of 2 in the exponent*

Hecht, Roberts, Schmidt
Phys.Rev. C **63** (2001) 025213

# Reanalysis of $q_v^\pi(x)$

- After first DSE computation, the "Conway *et al.*" data were reanalysed, this time at next-to-leading-order (Wijesooriya *et al.* Phys.Rev. C **72** (2005) 065203)
- The new analysis produced a much larger exponent than initially obtained; viz., β=*1.87*, but now it disagreed equally with model results and the DSE prediction
  - ✓ NB. Within pQCD, one can readily understand why adding a higher-order correction leads to a suppression of $q_v^\pi(x)$ at large-*x*.
- New experiments were proposed … for accelerators that do not yet exist but the situation remained otherwise unchanged
- Until the publication of *Distribution Functions of the Nucleon and Pion in the Valence Region*, Roy J. Holt and Craig D. Roberts, arXiv:1002.4666 [nucl-th], Rev. Mod. Phys. **82** (2010) pp. 2991-3044



*Distribution Functions of the Nucleon and Pion in the Valence Region*, Roy J. Holt and Craig D. Roberts, arXiv:1002.4666 [nucl-th], Rev. Mod. Phys. **82** (2010) pp. 2991-3044

# Reanalysis of $q_v^\pi(x)$

PRL **105,** 252003 (2010)     PHYSICAL REVIEW LETTERS     week ending 17 DECEMBER 2010

**Soft-Gluon Resummation and the Valence Parton Distribution Function of the Pion**

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(Received 15 September 2010; published 16 December 2010)

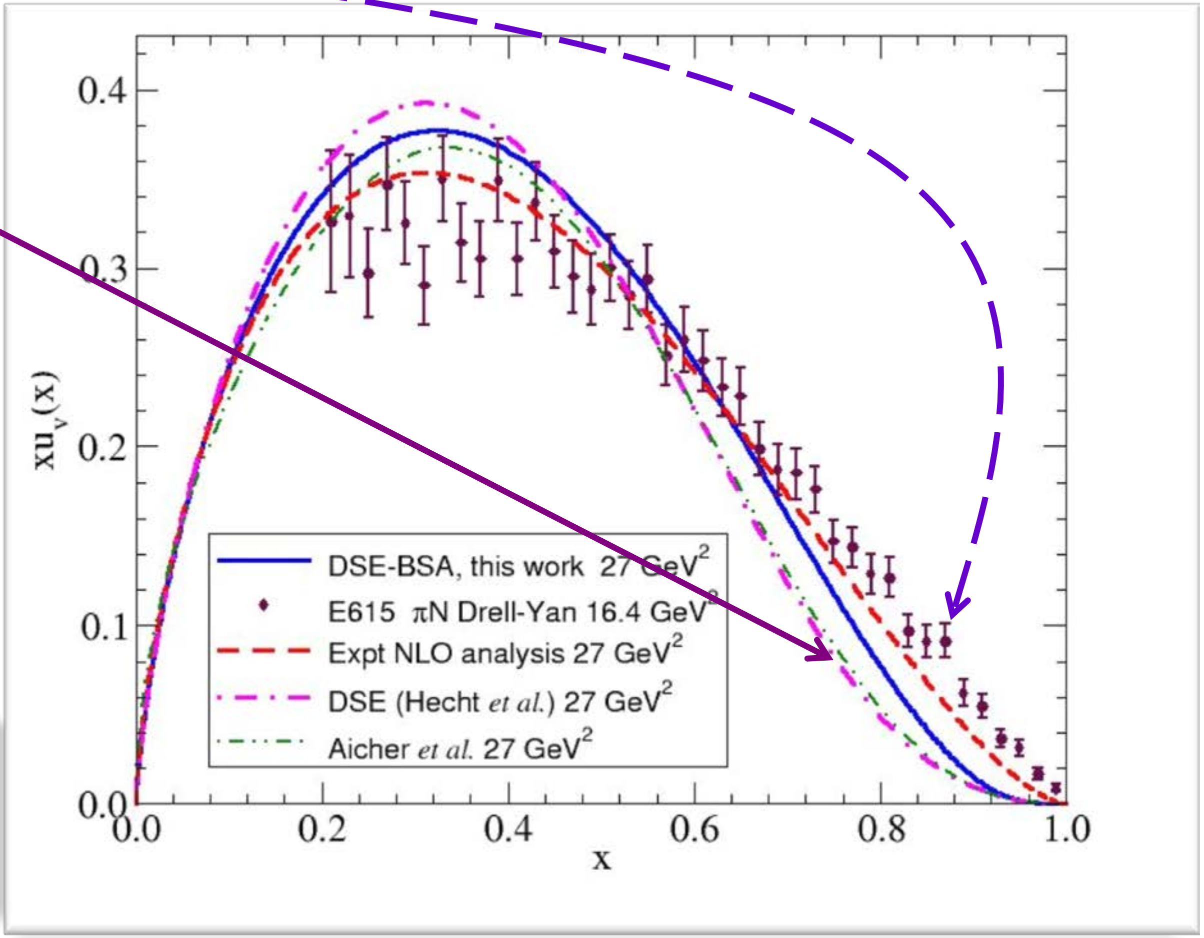
- This article emphasised and explained the importance of the persistent discrepancy between the DSE result and experiment as a challenge to QCD
- It prompted another reanalysis of the data, which accounted for a long-overlooked effect: viz., “soft-gluon resummation,”
  - *Compared to previous analyses, we include next-to-leading-logarithmic threshold resummation effects in the calculation of the Drell-Yan cross section. As a result of these, we find a considerably softer valence distribution at high momentum fractions x than obtained in previous next-to-leading-order analyses, in line with expectations based on perturbative-QCD counting rules or Dyson-Schwinger equations.*

*Aicher, Schäfer, Vogelsang, “Soft-Gluon Resummation and the Valence Parton Distribution Function of the Pion,” Phys. Rev. Lett.* ***105*** *(2010) 252003*

*Trang, Bashir, Roberts & Tandy,* "Pion and kaon valence-quark parton distribution functions," arXiv:1102.2448 [nucl-th], Phys. Rev. C **83**, 062201(R) (2011) [5 pages]

# Current status of $q_v^\pi(x)$

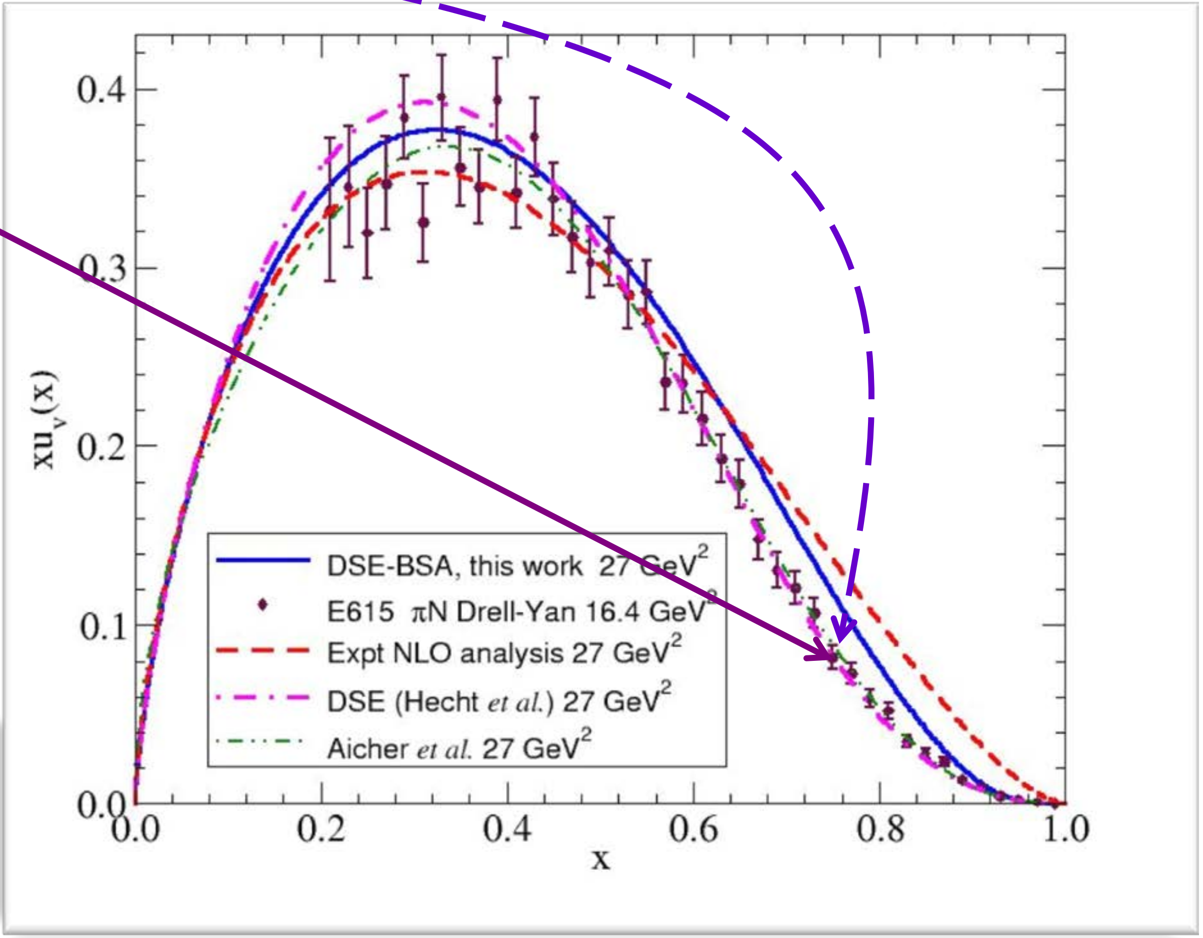
- Data as reported byE615
- DSE prediction (2001)



*Trang, Bashir, Roberts & Tandy,* "Pion and kaon valence-quark parton distribution functions," arXiv:1102.2448 [nucl-th], Phys. Rev. C **83**, 062201(R) (2011) [5 pages]

# Current status of $q_v^\pi(x)$

- Data after inclusion of soft-gluon resummation
- DSE prediction and modern representation of the data are *indistinguishable* on the valence-quark domain
- Emphasises the value of using a single internally-consistent, well-constrained framework to correlate and unify the description of hadron observables



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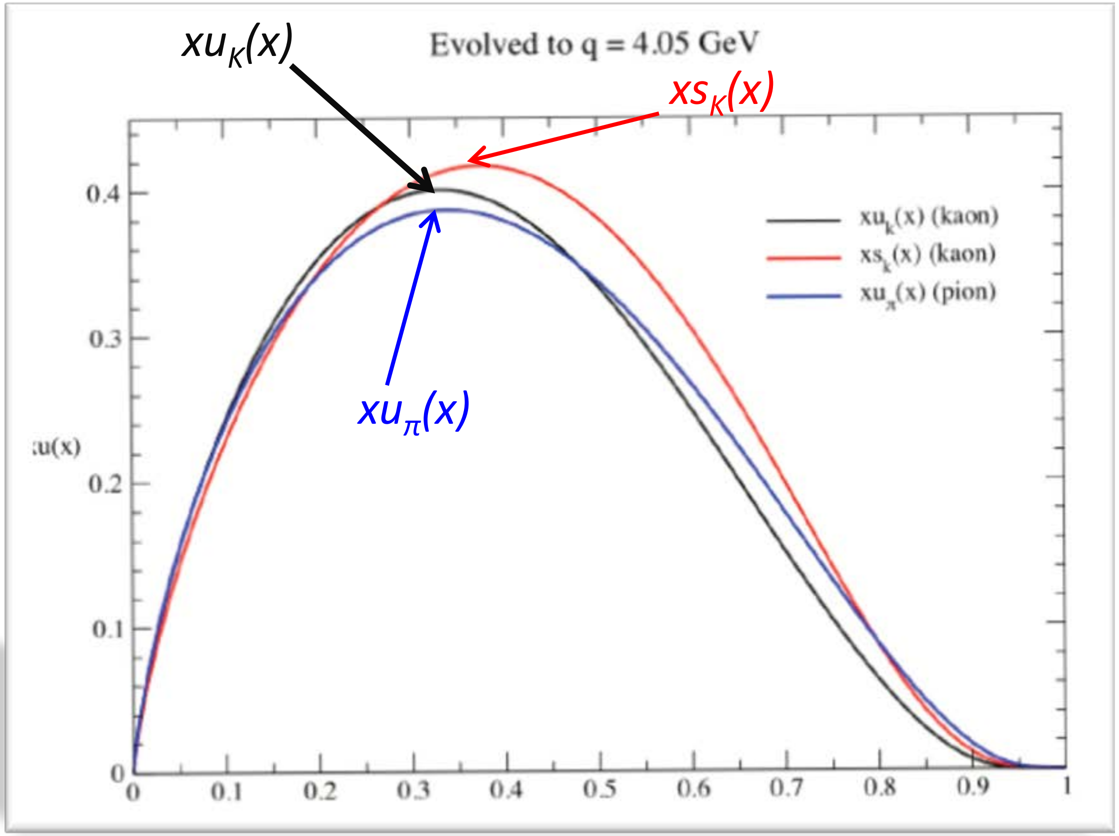
# $q_v^{\pi}(x)$ & $q_v^{K}(x)$

- $m_s \approx 24\, m_u$ & $M_s \approx 1.25\, M_u$ Expect the *s*-quark to carry more of the kaon’s momentum than the *u*-quark, so that $xs_K(x)$ peaks at larger value of $x$ than $xu_K(x)$
- Expectation confirmed in computations, with *s*-quark distribution peaking at 15% larger value of $x$
- *Even though deep inelastic scattering is a high-$Q^2$ process, constituent-like mass-scale explains the shift*



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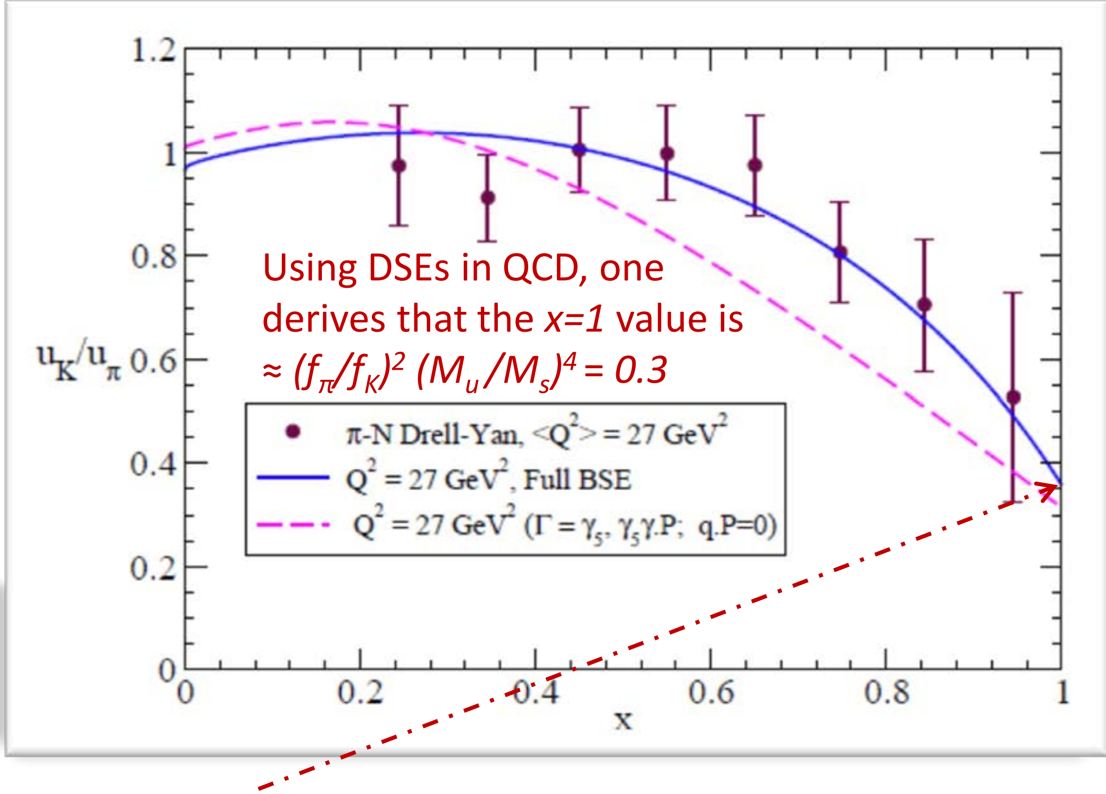
# $u_K(x)/u_\pi(x)$

- Drell-Yan experiments at CERN (1980 & 1983) provide the only extant measurement of this ratio
- DSE result in complete accord with the measurement
- New Drell-Yan experiments are capable of validating this comparison
- It should be done so that complete understanding can be claimed

*Value of ratio at x=0 will approach "1" under evolution to higher resolving scales. This is a feature of perturbative dynamics*



Using DSEs in QCD, one derives that the *x=1* value is $\approx (f_\pi/f_K)^2\,(M_u/M_s)^4 = 0.3$

*Value of ratio at x=1 is a fixed point of the evolution equations*
*Hence, it's a very strong test of nonperturbative dynamics*

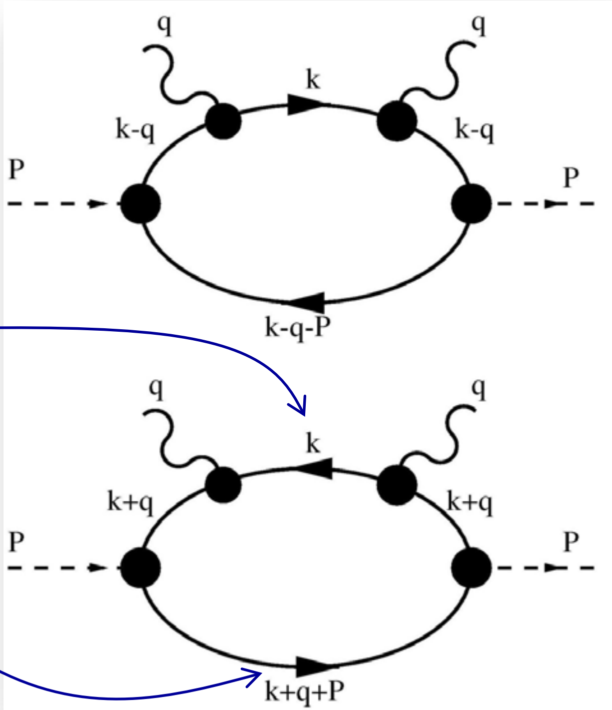
*Khitrin, Roberts & Tandy,* in progress.

# Reconstructing PDF from moments

- Suppose one cannot readily compute the PDF integral,
  - perhaps because one has employed a Euclidean metric, such as is typical of *nonperturbative* studies with QCD connection
- Preceding computations employed a *dirty trick* to proceed from Euclidean space to the light-front; viz.,
  - Spectator pole approximation:

    $S_{dressed}(p) \to 1/(i\,\gamma{\cdot}p + M)$

    for internal lines
- Can one otherwise determine the PDF, without resorting to artifices?



*Khitrin, Roberts & Tandy,* in progress.

# Reconstructing PDF from moments

- Rainbow-ladder truncation – general expression for PDF moments:

π Bethe-Salpeter amplitude

Dressed-quark propagator

$$(n\cdot P)^{m+1}\langle x^m\rangle = \frac{3}{2i}\int\frac{d^4k}{(2\pi)^4}(n\cdot k)^m\,\mathrm{tr}\Big[\bar{\Gamma}_\pi(k-P/2)S(k)$$

Dressed-quark-photon vertex

$$\times\, n_\mu\Gamma_\mu(k,k)S(k)\Gamma_\pi(k-P/2)S(k-p)\Big]$$

$n^2=0$, $n.P= -m_\pi$

- Consider vector-vector interaction with exchange $(1/k^2)^n$, $n=0$ then

$$\langle x^m\rangle = 1/(m+1)$$

- To which distribution does this correspond?

  Solve $\int_0^1 dx\, x^m\, u_\pi(x) = 1/(m+1)$ for $u_\pi(x)$

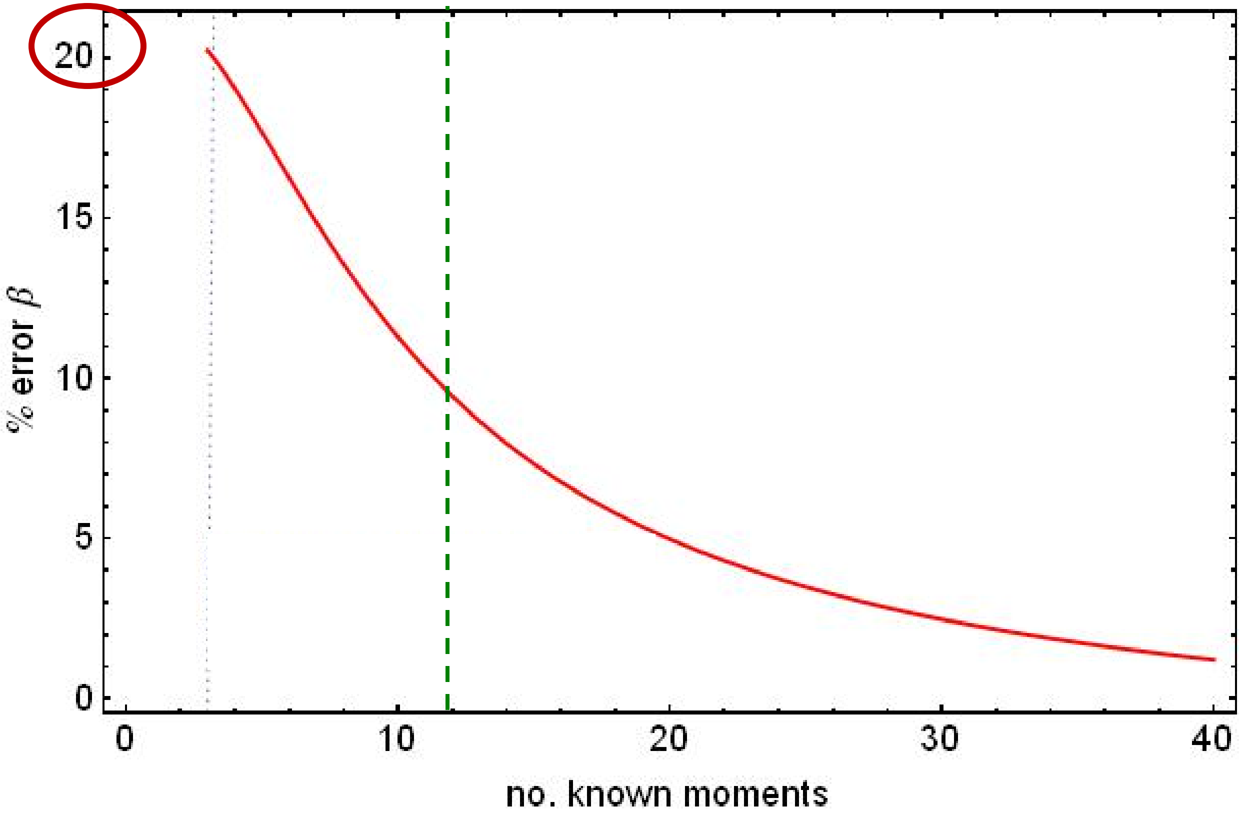
  Answer $u_\pi(x)=1$, which can be verified by direct substitution

- Many numerical techniques available for more interesting interactions

*Khitrin, Roberts & Tandy,* in progress.

# Reconstructing the Distribution Function

- Suppose one has "N" nontrivial moments of the quark distribution function & assume $u_\pi(x) \sim x^\alpha (1-x)^\beta$
- Then, how accurately can one obtain the large-*x* exponent, β?
  - Available moments from lattice-QCD … not better than 20%
  - 12 moments needed for 10% accuracy
- Lower bound … For a more complicated functional form, one needs more moments.



With 40 nontrivial moments, obtain β=2.03 from *1/k*$^2$ input

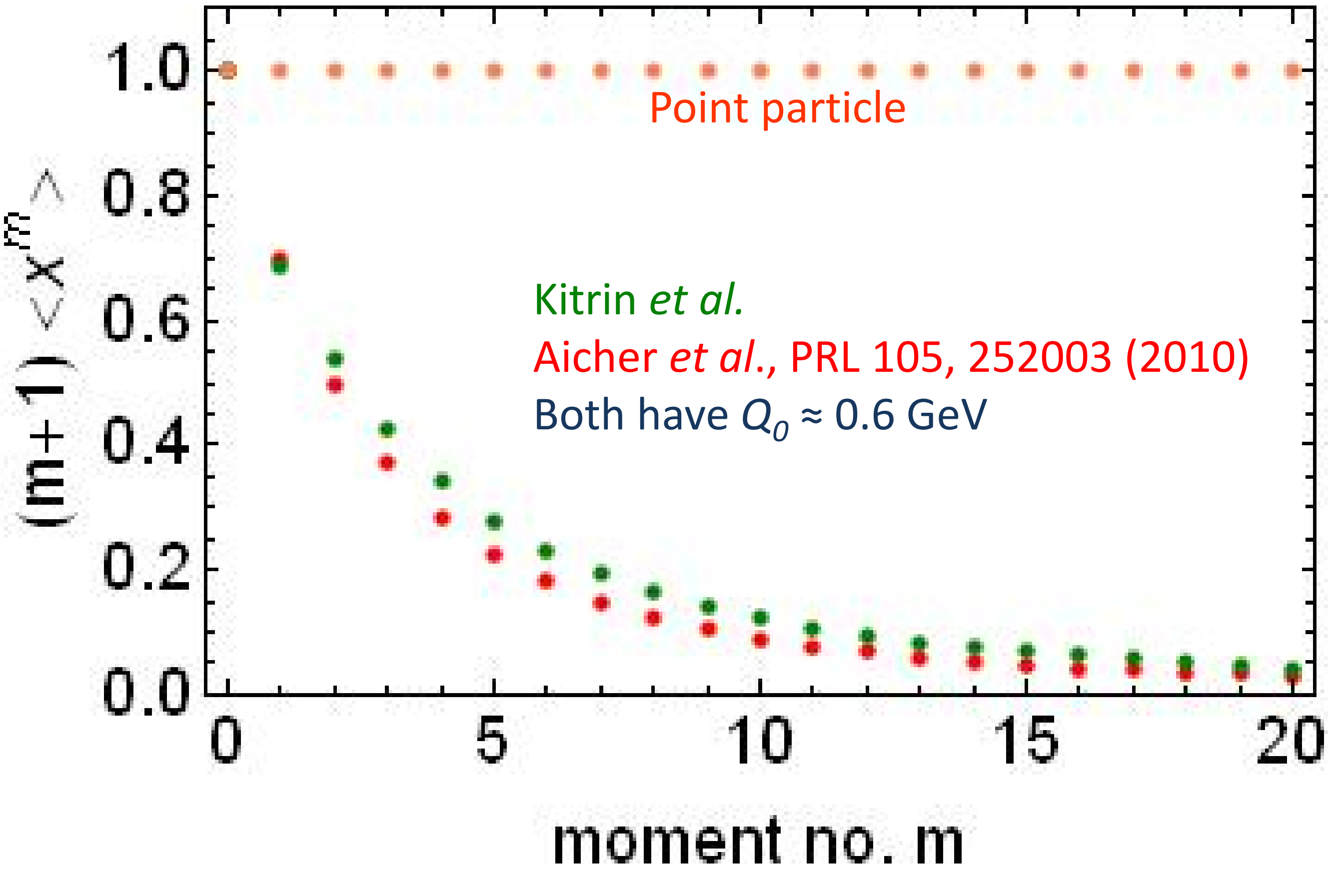
*Khitrin, Roberts & Tandy,* in progress;
Si-xue Qin, Lei Chang, Yu-xin Liu, Craig Roberts and David Wilson,
arXiv:1108.0603 [nucl-th], Phys. Rev. C **84** 042202(R) (2011)

# Euclidean Space Moments of the Distribution Function

- Best rainbow-ladder interaction available for QCD:

$$|\pi_{\text{bound-state}}>$$

- Refined to reflect inclusion of sea-quarks via pion cloud: $Z_D = 0.87$
- Origin in comparison with ChPT; viz., dressed-quark core produces 80% of ≈ $r_\pi^2$ and chiral-logs produce ≈ 20%



Kitrin *et al.*

Aicher *et al.*, PRL 105, 252003 (2010)

Both have $Q_0 \approx 0.6$ GeV

*Expression exact in QCD*
*– no corrections*

# Pion's valence-quark Distribution Amplitude

- Moments method is also ideal for $\varphi_\pi(x)$:

$$\varphi_\pi(x) = Z_2 \mathrm{tr}_{CD} \int \frac{d^4k}{(2\pi)^4} \delta(n\cdot k - x n\cdot P)\, \gamma_5 \gamma\cdot n\, \underbrace{S(k)\Gamma_\pi(k;P)S(k-P)}$$

entails

$$(n\cdot P)^{m+1} \int_0^1 dx\, x^m\, \varphi_\pi(x) = Z_2 \mathrm{tr}_{CD} \int \frac{d^4k}{(2\pi)^4} (n\cdot k)^m \gamma_5 \gamma\cdot n\, \chi_\pi(k;P)$$

Pion's Bethe-Salpeter wave function

- Contact interaction

  *$(1/k^2)^\nu$ , $\nu=0$*

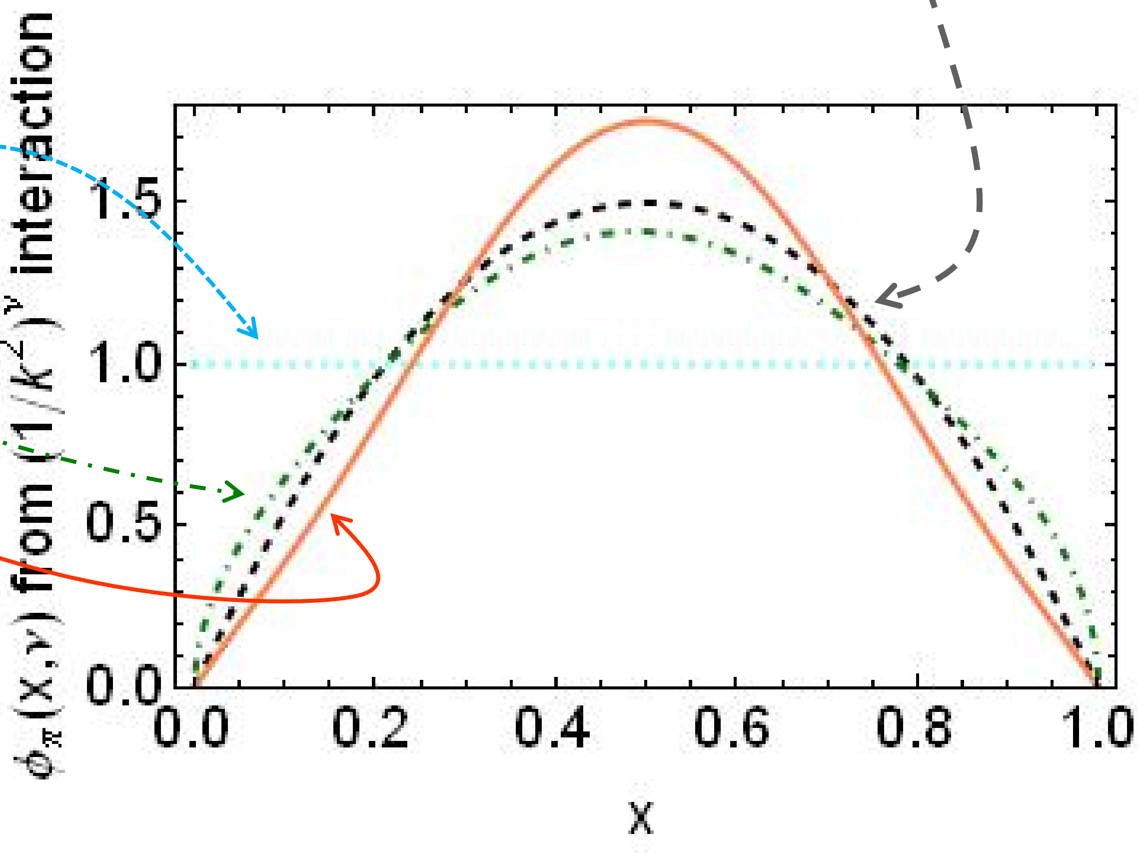
  Straightforward exercise to show

  $\int_0^1 dx\, x^m\, \varphi_\pi(x) = f_\pi\, 1/(1+m)$ , hence $\varphi_\pi(x) = f_\pi\, \Theta(x)\Theta(1-x)$

Work now underway with sophisticated rainbow-ladder interaction: *Chang, Cloët, Roberts, Schmidt & Tandy*

# Pion's valence-quark Distribution Amplitude

- Using simple parametrisations of solutions to the gap and Bethe-Salpeter equations, rapid and semiquantitatively reliable estimates can be made for $\varphi_\pi(x)$
  - $(1/k^2)^{\nu=0}$
  - $(1/k^2)^{\nu=½}$
  - $(1/k^2)^{\nu=1}$
- Again, unambiguous and direct mapping between behaviour of interaction and behaviour of distribution amplitude



Leading pQCD $\varphi_\pi(x)=6\,x\,(1-x)$

*Chang, Cloët, Roberts, Schmidt & Tandy,* in progress;
Si-xue Qin, Lei Chang, Yu-xin Liu, Craig Roberts and David Wilson,
arXiv:1108.0603 [nucl-th], Phys. Rev. C **84** 042202(R) (2011)

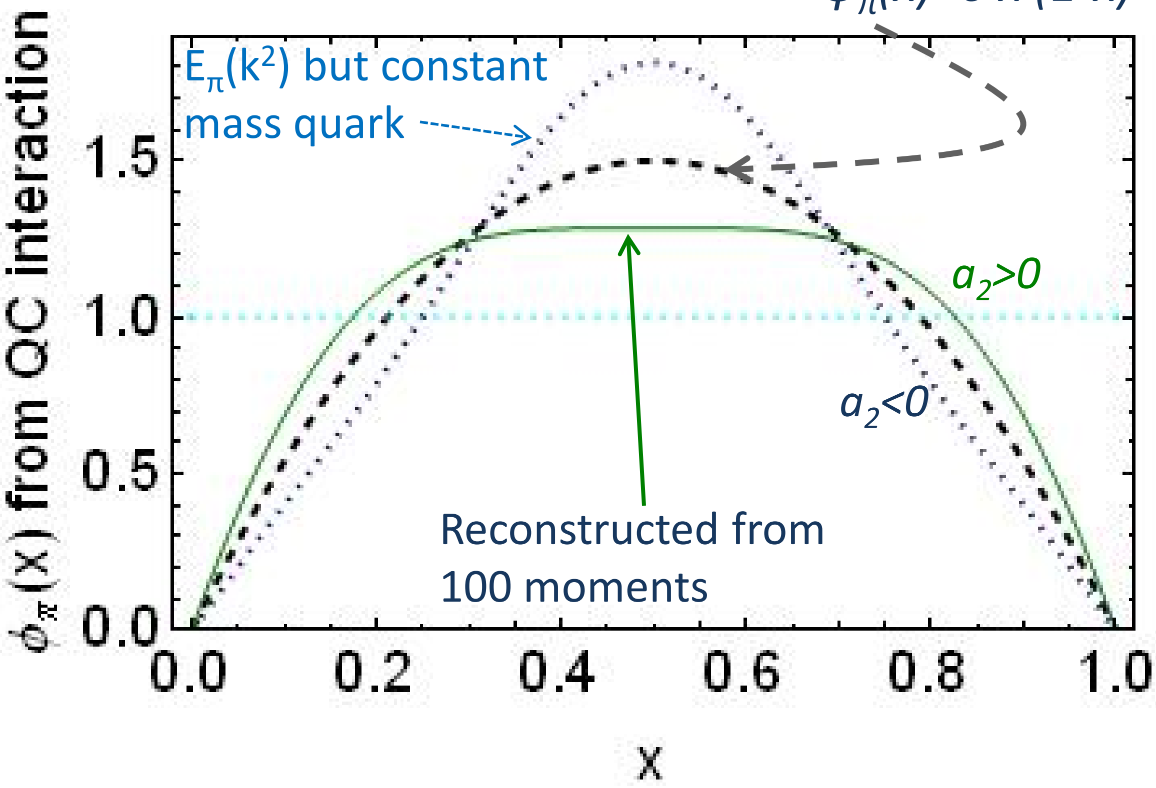
# Pion's valence-quark Distribution Amplitude

- Preliminary results: rainbow-ladder QCD analyses of renormalisation-group-improved *(1/k²)*$^{\nu=1}$ interaction – humped disfavoured but modest flattening

- Such behaviour is only obtained with

  (1) Running mass in dressed-quark propagators

  (2) Pointwise expression of Goldstone's theorem



Leading pQCD
*$\varphi_\pi(x)=6\,x\,(1-x)$*

$E_\pi(k^2)$ but constant mass quark

*$a_2>0$*

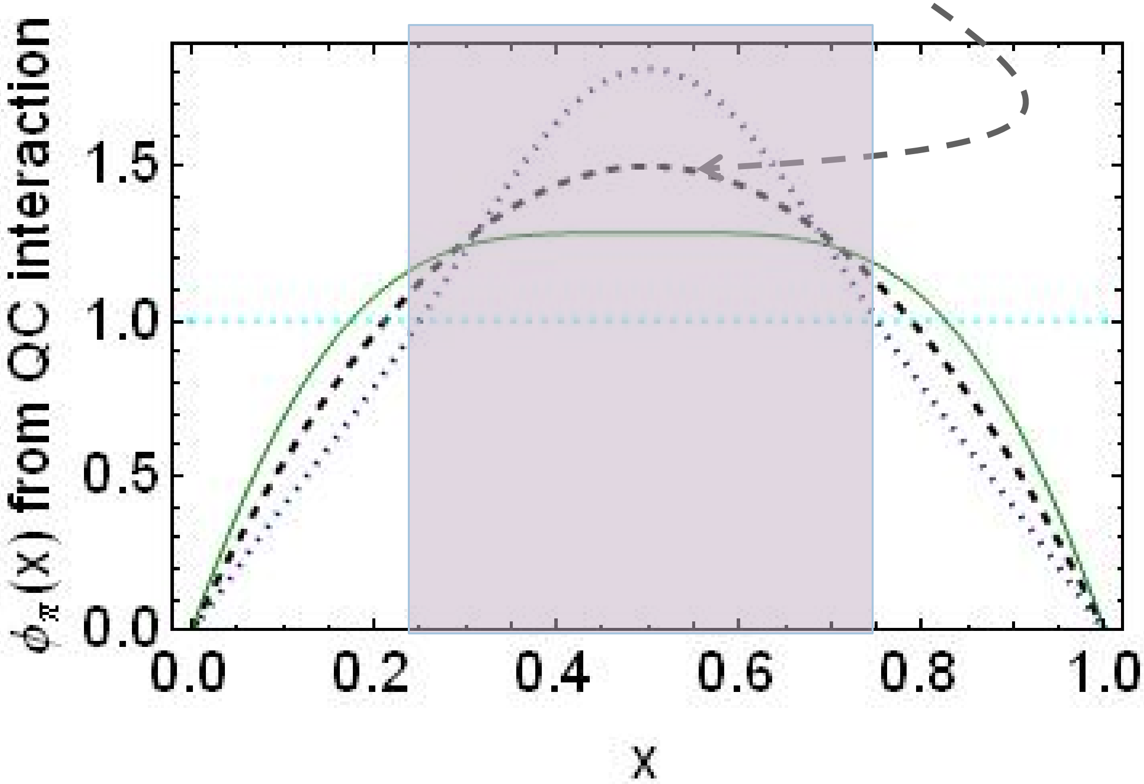
*$a_2<0$*

Reconstructed from 100 moments

# Pion's valence-quark Distribution Amplitude

- $x \approx 0$ & $x \approx 1$ correspond to maximum relative momentum within bound-state
  - *expose pQCD physics*
- $x \approx ½$ corresponds to minimum possible relative momentum
  - *behaviour of distribution around midpoint is strongly influence by DCSB*

Leading pQCD $\varphi_\pi(x)=6\,x\,(1-x)$

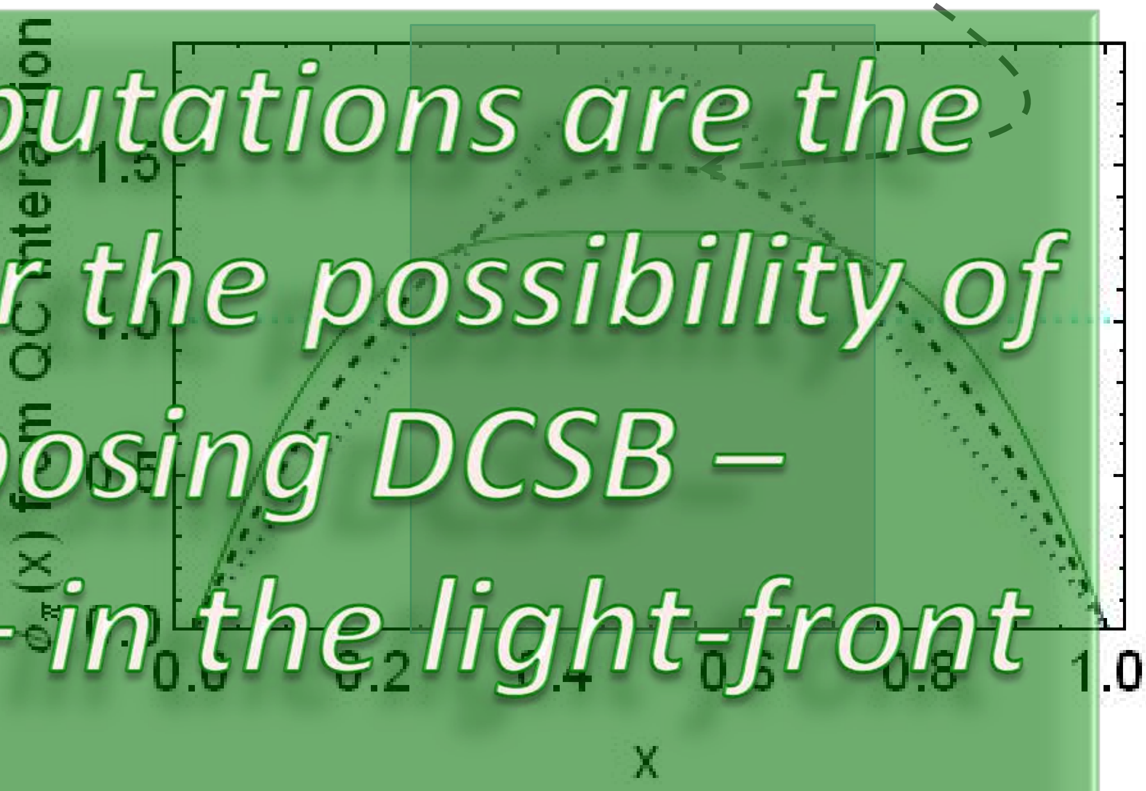


- Preliminary results, rainbow-ladder QCD analyses of $(1/k^2)^{\nu=1}$ interaction humped disfavoured but modest flattening

# Pion's valence-quark Distribution Amplitude

- $x \approx 0$ & $x \approx 1$ correspond to maximum relative momentum within bound-state
  - *expose pQCD physics*
- $x \approx \frac{1}{2}$ corresponds to minimum possible relative momentum
  - *behaviour of distribution around midpoint is strongly influence by DCSB*

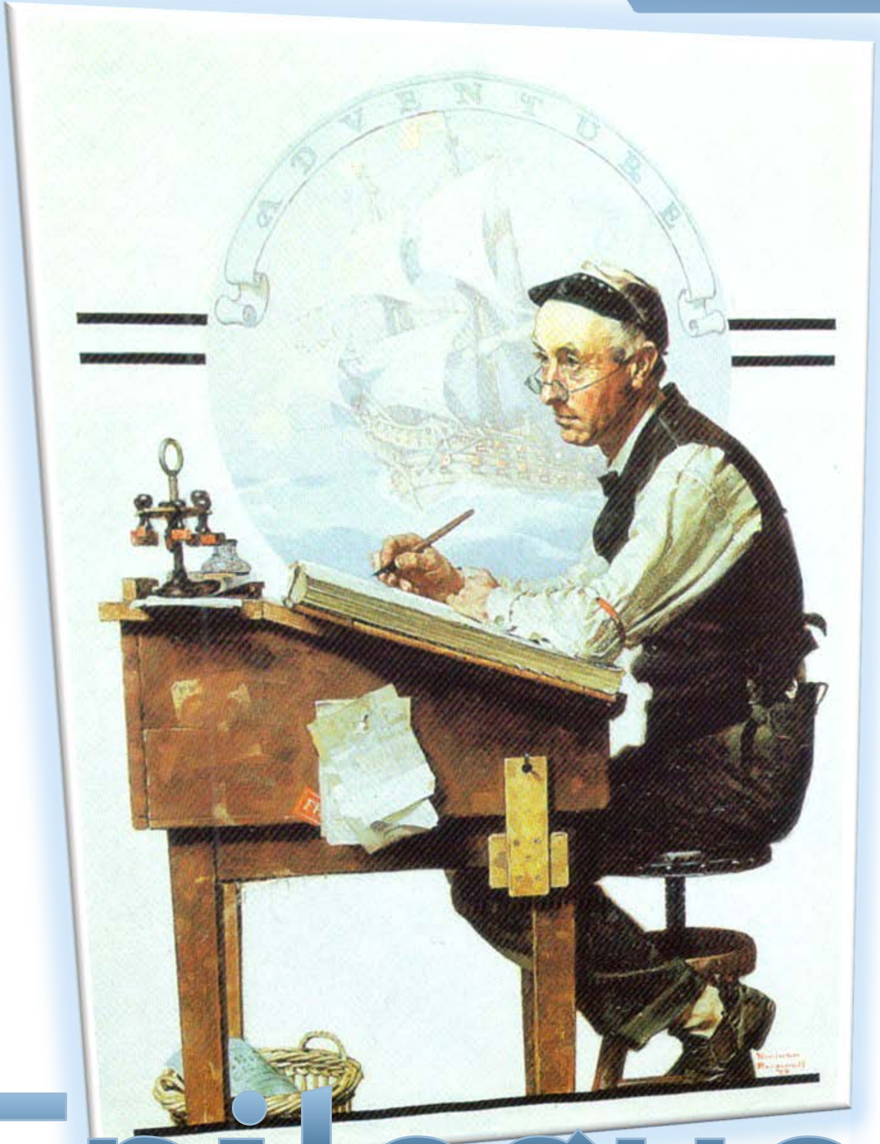
Leading pQCD $\varphi_\pi(x)=6\,x\,(1-x)$



*These computations are the first to offer the possibility of directly exposing DCSB – pointwise – in the light-front frame.*

- Preliminary results, rainbow-ladder QCD analyses of $(1/k^2)^{\nu=1}$ interaction

humped disfavoured but modest flattening



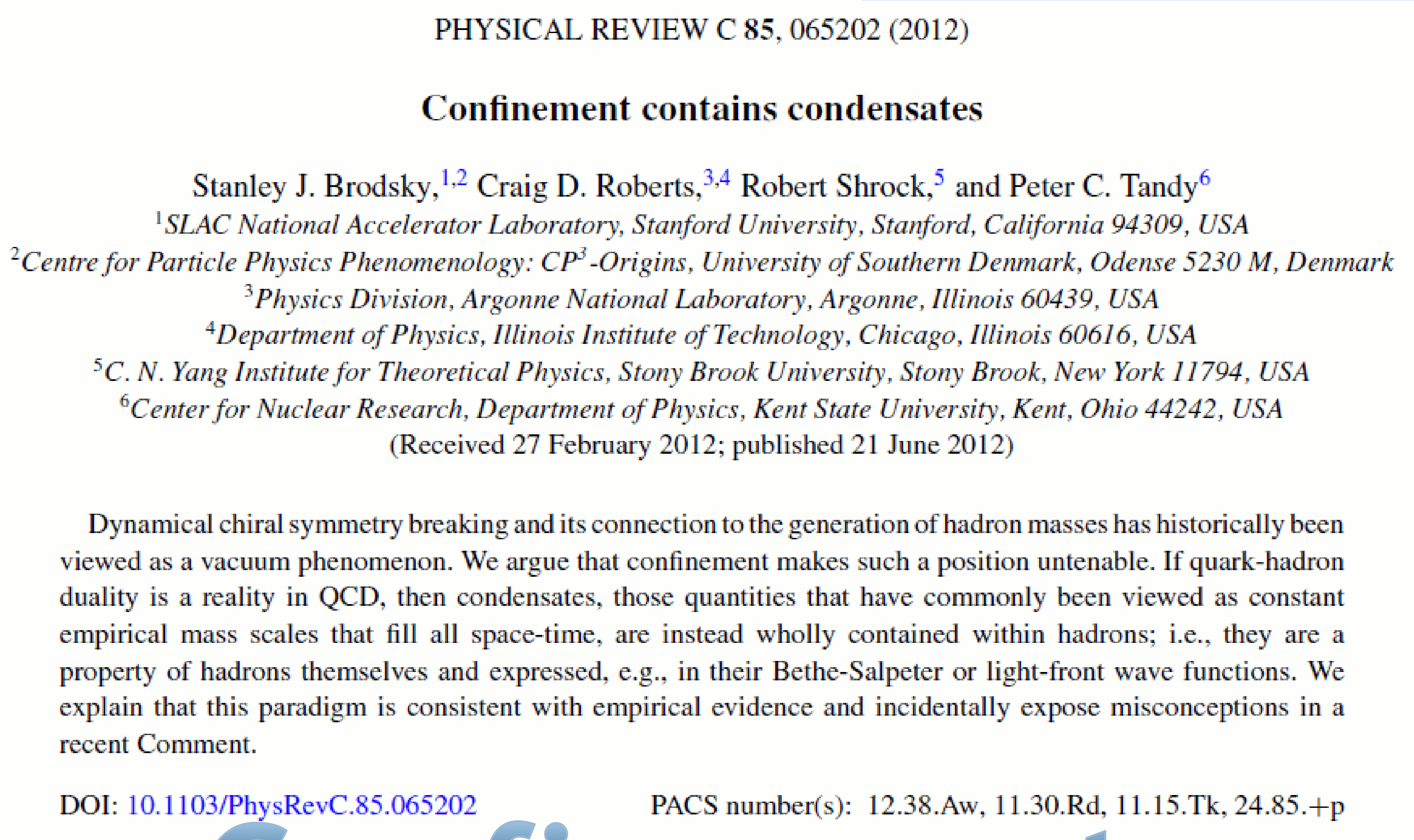
# Epilogue

*QCD is the most interesting part of the standard model - Nature's only example of an essentially nonperturbative fundamental theory,*

# Epilogue

- Confinement with light-quarks is not connected in any known way with a linear potential; not with a potential of any kind.
- Confinement with light-quarks is associated with a dramatic change in the infrared structure of the parton propagators.
- Dynamical chiral symmetry breaking, the origin of 98% of visible matter in universe, is manifested fundamentally in an equivalence between the one- and two-body problem in QCD
- Working together to chart the behaviour of the running masses in QCD, experiment and theory can potentially answer the questions of *confinement* and *dynamical chiral symmetry breaking*; a task that currently each alone find hopeless.

# This is not the end



PHYSICAL REVIEW C **85**, 065202 (2012)

## Confinement contains condensates

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Dynamical chiral symmetry breaking and its connection to the generation of hadron masses has historically been viewed as a vacuum phenomenon. We argue that confinement makes such a position untenable. If quark-hadron duality is a reality in QCD, then condensates, those quantities that have commonly been viewed as constant empirical mass scales that fill all space-time, are instead wholly contained within hadrons; i.e., they are a property of hadrons themselves and expressed, e.g., in their Bethe-Salpeter or light-front wave functions. We explain that this paradigm is consistent with empirical evidence and incidentally expose misconceptions in a recent Comment.

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# Confinement contains condensates