Single Transverse-Spin Asymmetries and Twist-3 Factorization

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- Partonic states and SSA
- SSA in Drell-Yan Processes
- Evolutions of Twist-3 operators
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Introduction

Single transverse-Spin Asymmetries(SSA) are asymmetries in the case where one initial hadron or one produced hadron is transversely polarized.

Taking Drell-Yan processes as an example:

$$h_A(P_A, s) + h_B(P_B) \to \gamma^*(q) + X \to \ell^- + \ell^+ + X,$$

$$A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}, \quad s^\mu = (0, 0, \vec{s}_\perp)$$

The initial hadron is transversely polarized.

From general principles, SSA can only be generated
if there exist scattering absorptive parts in scattering amplitudes (T-odd effect)
AND helicity-flip interactions.

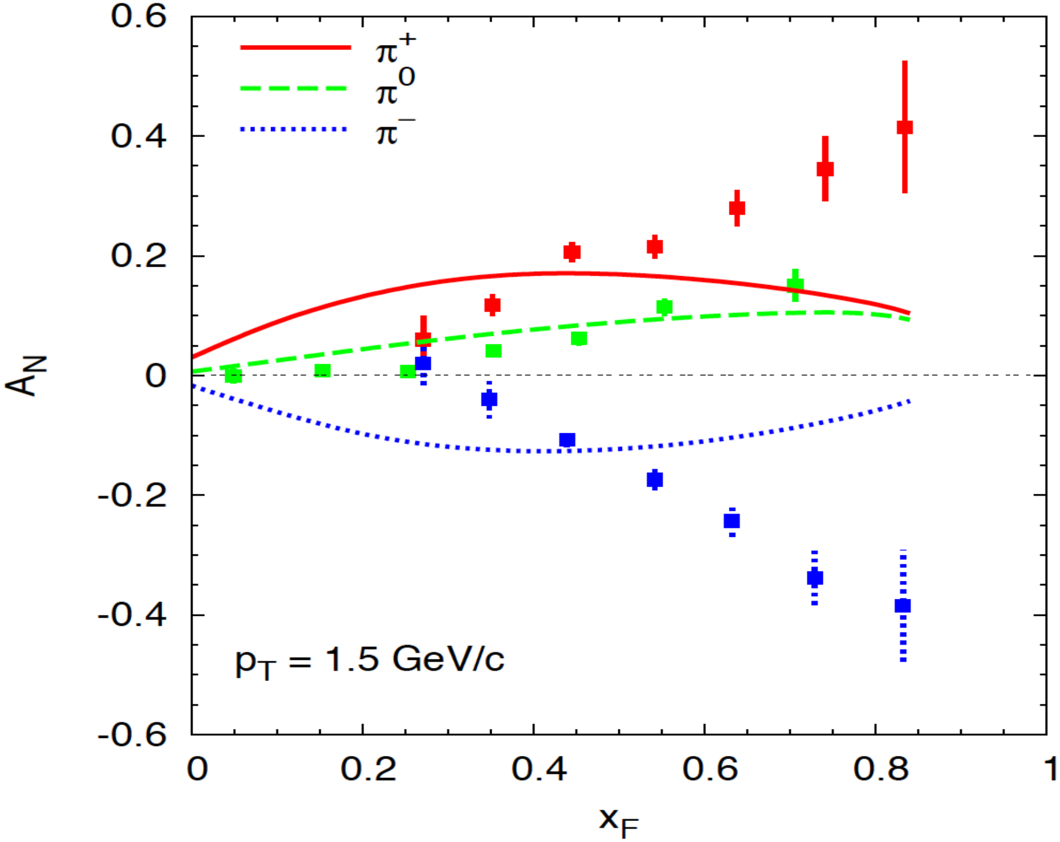
Two known facts:

from perturbation theory: Absorptive parts only exists beyond tree-level for two particle scattering

from QCD: Helicity of quarks are conserved. (massless quarks)

One may expect: SSA =0 or very small (???)

Experiments:



$$p^{\uparrow}p \rightarrow h + X$$

E581, E704, (1991)

Rather large asymmetries……

Various asymmetries have been observed in experiment.

Theoretically, SSA can be predicted with concepts of QCD factorizations.

Two factorizations: Collinear factorization

TMD factorization for certain kinematical regions.

Collinear factorization: Efremov and Teryaev, Qiu and Sterman.

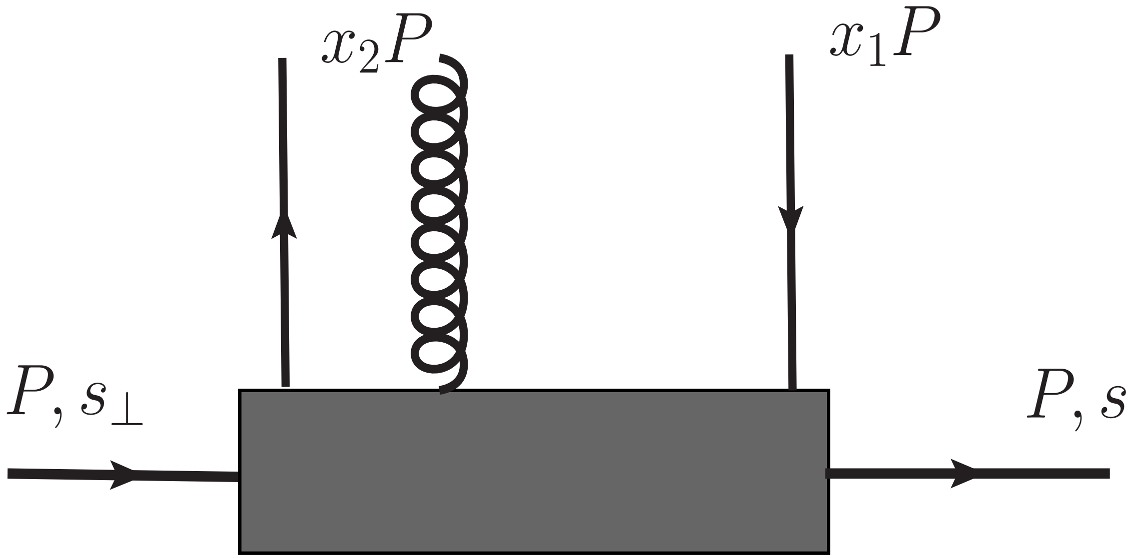
The nonperturbative effects are factorized with twist-3 matrix elements. E.g., quark-gluon correlators: (ETQS matrix elements)

$$T_F(x_1,x_2,\mu) = \quad -\tilde{s}_\mu g_s \int \frac{dy_1 dy_2}{4\pi} e^{-iP^+(y_2(x_2-x_1)+y_1x_1)} \langle P,s|\bar{\psi}(y_1n)\gamma^+ G^{+\mu}(y_2n)\psi(0)|P,s\rangle,$$

$$T_{\Delta,F}(x_1,x_2,\mu) = \quad is_\mu g_s \int \frac{dy_1 dy_2}{4\pi} e^{-iP^+(y_2(x_2-x_1)+y_1x_1)} \langle P,s|\bar{\psi}(y_1n)\gamma^+\gamma_5 G^{+\mu}(y_2n)\psi(0)|P,s\rangle$$

There are 4 twist-3 operators only with gluon fields.

Graphical representation



$$x_1, x_2$$

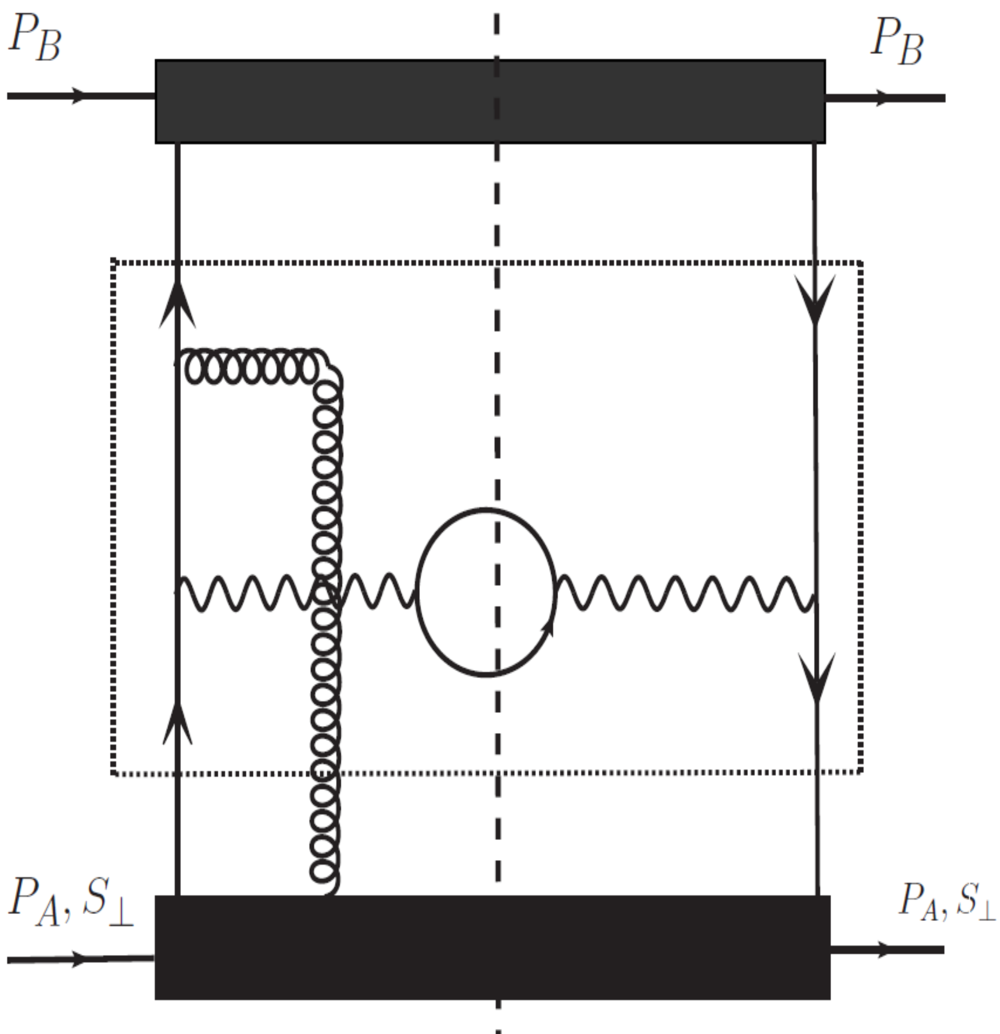
Momentum fraction

Nonperturbative properties of the polarized hadron.

SSA => Information of quark-gluon correlations inside the hadron

So far, only one method exists to derive factorizations for SSA or to predict SSA in terms of ETQS matrix elements.

The diagrammatic approach at hadron level:



$$\Gamma_B(q\ \bar{q})$$

Quark density matrix of B

$$H$$

Hard scattering

$$\Gamma_A(q\ G\ \bar{q})$$

Quark-Gluon density matrix of A

Expanding momenta of incoming partons collinearly, one derives the factorized form. Schematically (E.g., Drell-Yan):

$$
\begin{aligned}
d\sigma(s_\perp) \sim & f_a \otimes \mathcal{H}_h \otimes T_F\,[x_1, x_2] && \longrightarrow \text{Hard-pole contribution} \\
& + f_a \otimes \mathcal{H}_{gs} \otimes T_F\,[x, x] && \longrightarrow \text{Soft-gluon-pole contribution} \\
& + f_a \otimes \mathcal{H}_{fs} \otimes T_F\,[0, x] && \longrightarrow \text{Soft-fermion-pole contribution} \\
& + \cdots
\end{aligned}
$$

f_a The standard parton distribution of the unpolarized hadron

$\mathcal{H}_h, \mathcal{H}_{gs}, \mathcal{H}_{fs}$ The perturbative coefficient functions

Q: Is there another way to derive the factorization ?

A: Yes !

Purpose: independent check, understanding soft-gluon-pole contributions

....

An important fact: QCD factorization, if it is proven, is a general property of QCD. It holds for all states, not only for specific hadrons. It also holds for parton states.

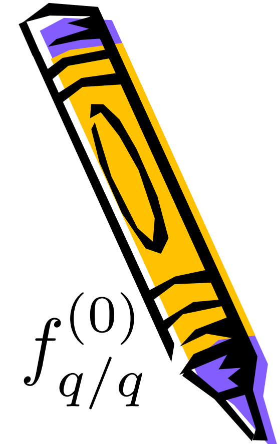
E.g., DIS with H as the initial hadron, the structure function is factorized as:

$$F_2 = \mathcal{H} \otimes f_{q/H} + \cdots \; ,$$

The factorization holds for any hadron, especially if we replace the hadron with partons, H -> q,

The perturbative coefficient function is the same.

With a quark as the initial state, one can calculate the structure function of the quark, and PDF with the same quark state.



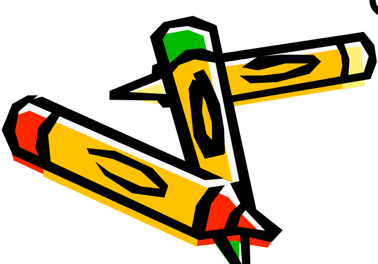
At tree -level: $$F_2^{(0)} = \mathcal{H}^{(0)} \otimes f_{q/q}^{(0)},$$

At one-loop level: $$F_2^{(1)} = \mathcal{H}^{(0)} \otimes f_{q/q}^{(1)} + \mathcal{H}^{(1)} \otimes f_{q/q}^{(0)}$$

The collinear divergence in F_2 is the same as that in the first term, so that H does not contain it. This is the sense of factorization.

Important: The collinear divergence at one-loop in F_2 is "determined" by the tree-level H.....

Can we do the same for SSA??
Yes or No......??



If we replace the hadron A with a transversely polarized quark, one can not have a nonzero SSA, because the helicity conservation of QCD.

One needs to consider multi-parton states for the replacement.

The talk presents a study of QCD factorizations for SSA by using partonic states.

Partonic states and SSA

Transverse spin corresponds to the non-diagonal part of spin density matrix in helicity space.

Define a spin $\frac{1}{2}$ state as:

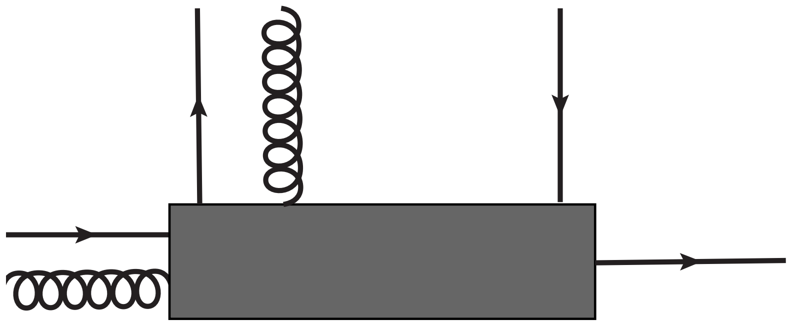
$$|n[\lambda]\rangle = |q(p,\lambda)\rangle + c_1|q(p_1,\lambda_q)g(p_2,\lambda_g)[\lambda=\lambda_q+\lambda_g]\rangle + \cdots\ ,$$

$$p_1 = x_0 p, \quad p_2 = (1-x_0)p$$

Using this state to replace the transversely polarized hadron A, one will get nonzero non-diagonal part of spin density matrix because of the interference between the single quark- and the quark-gluon state. i.e.,

$$T_F \sim \langle q(p,+)|\mathcal{O}|q(p_1,+)g(p_2,-)\rangle + \cdots\ ,$$

Graphical representation:



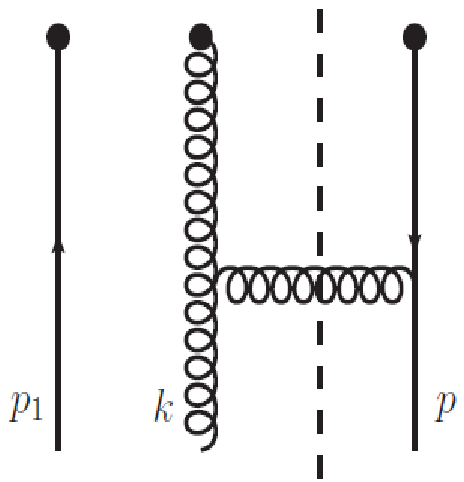
Helicity is flipped by $\frac{1}{2}$.

At tree-level:

$$T_F(x_1,x_2) = c_1 g_s \pi \sqrt{\frac{x_2}{2}}(x_2 - x_1)\left[\delta(1-x_1)\delta(x_2-x_0) - \delta(1-x_2)\delta(x_1-x_0)\right]$$

It is nonzero. It is zero for $x_1 = x_2$

At one loop $T_F(x,x)$ becomes nonzero



Dimensional regularization, U.V. poles are subtracted.

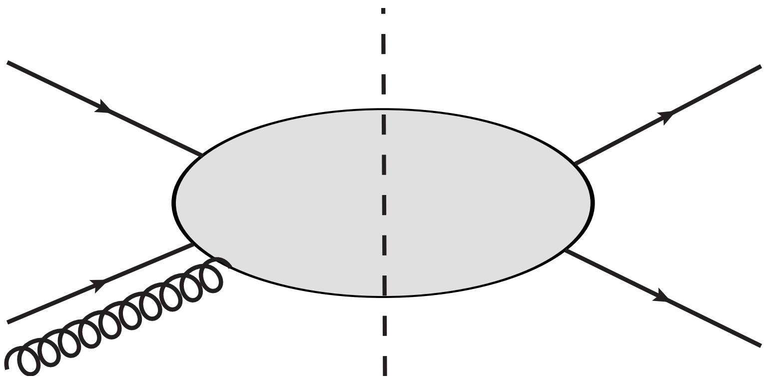
A collinear divergence:

$$T_F(x,x) = -c_1 \frac{g_s \alpha_s}{4} N_c(N_c^2-1) x_0 \sqrt{2x_0}\, \delta(x_0 - x) \left(-\frac{2}{\epsilon_c} + \gamma - \ln \frac{\mu^2}{4\pi\mu_c^2} \right).$$

One can also calculate the function in general cases.

One can use the same multi-parton state to calculate differential cross-sections to find or to establish factorizations.

E.g., hadron-hadron collision:



Standard way to calculate differential cross sections of parton states.

SSA in Drell-Yan process

$$h_A(P_A, s) + h_B(P_B) \to \gamma^*(q) + X \to \ell^- + \ell^+ + X,$$

Consider the differential cross-section:

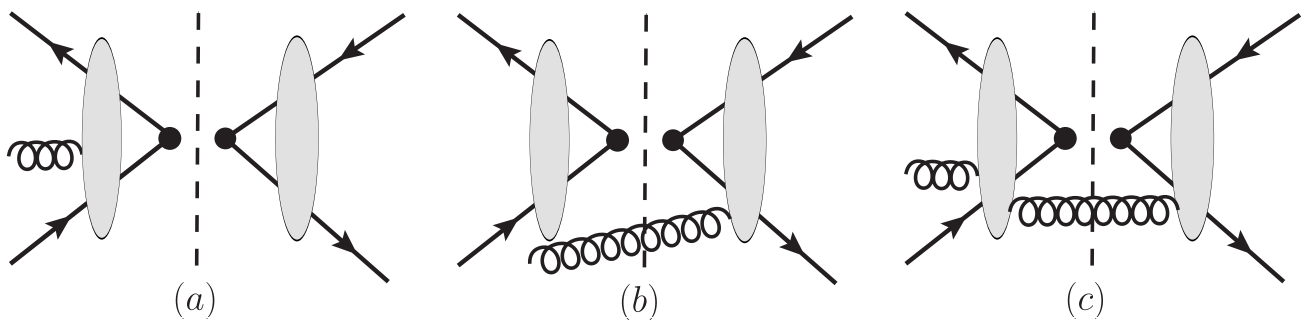
$$\frac{d\sigma(s_\perp)}{dQ^2 d\Omega} = \frac{\alpha^2}{SQ^4} \int d^4q \delta(q^2 - Q^2) \left[k_1^\mu k_2^\nu + k_1^\nu k_2^\mu - k_1 \cdot k_2 g^{\mu\nu}\right] W_{\mu\nu}.$$

Ω The solid angle of the lepton in the rest-frame of the lepton pair.
We take here the Collins-Soper frame.

We replace the hadron A with the multi-parton state
the hadron B with an anti-quark,

and calculate the spin-dependent part.

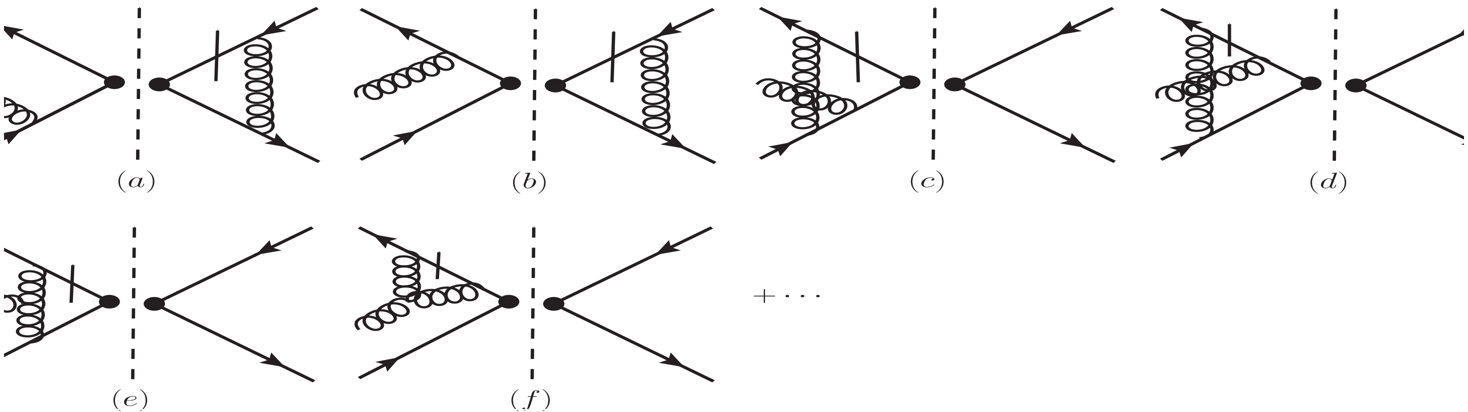
At leading(nonzero order) there are 3 classes of diagrams contributing to the hadronic tensor:



Class (b): No contributions at any order.

We first consider the divergent contributions to the differential cross-section, come back later to the finite contributions.

Class (a) contributions are proportional to $\delta^2(\vec{q}_\perp)$



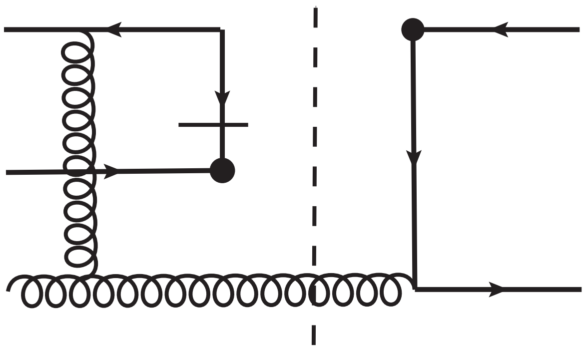
The sum is free of any soft-divergences (Glauber –divergence)

Only finite contributions.

Class (c) : there is a soft divergence in small q_t region.

We scale $q_\perp \sim \lambda,\ \ \lambda \to 0,$

Only one diagram gives the divergence if we integrated over q_t :



$$\begin{aligned}\tilde{W}^{\mu\nu} = \ & -\frac{g_s\alpha_s}{4\pi}(N_c^2-1)\sqrt{2x_0}\,\delta(1-y)\delta(x-x_0)\left[\frac{1}{(q_\perp^2)^2}\left(x_0\tilde{s}\cdot q_\perp g_\perp^{\mu\nu} - \frac{\tilde{s}\cdot q_\perp}{\bar{p}\cdot p}\bar{p}^{\{\mu}q_\perp^{\nu\}}\right)\right. \\ & \left. + \frac{1}{2p\cdot\bar{p}q_\perp^2}\left(x_0 p^{\{\mu}\tilde{s}^{\nu\}} - \bar{p}^{\{\mu}\tilde{s}^{\nu\}}\right)\right] + \mathcal{O}(\lambda^{-1}).\end{aligned}$$

The divergent part of the differential cross section:

$$\frac{d\sigma(\vec{s}_\perp)}{dQ^2 d\Omega} = \left[\frac{g_s \alpha_s}{4}(N_c^2-1)x_0\sqrt{2x_0}\left(-\frac{2}{\epsilon_c}\right)\right](-1+2)\frac{\delta(x_0 s - Q^2)}{128\pi^2 Q^3}\sin(2\theta)\sin\phi + \cdots$$

With the results of T_F and pdf of parton states one finds the factorized form for the asymmetry in the lepton angular distribution:

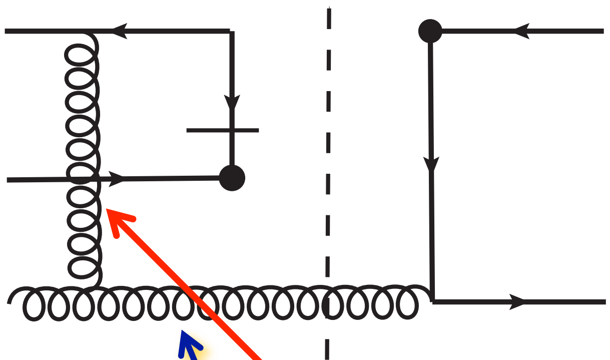
$$A_N = -\frac{\sin(2\theta)\sin\phi}{2Q(1+\cos^2\theta)}\frac{\int dx dy T_F(x,x)\bar{q}(y)\delta(xyS-Q^2)}{\int dx dy q(x)\bar{q}(y)\delta(xyS-Q^2)}.$$

The perturbative coefficient function here is at order of 1.

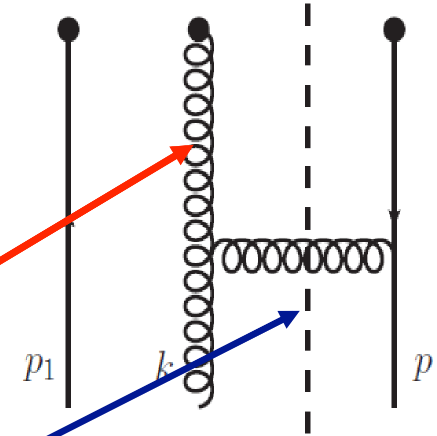
There is a discrepancy about the asymmetry in literature.....

Details in: J.P. Ma and G.P. Zhang, e-Print: arXiv:1203.6415

The argument for the factorization:
When integrating q_t :



$= H \otimes$



$T_F(x, x)$

The soft gluon, in fact it is a Glauber gluon.

The collinear gluon

The finite contributions will be factorized with tree-level T_F, gives the contributions to the perturbative coefficient function at α_s

There is a order mixing!

Another asymmetry:

$$\frac{d\sigma}{dQ^2 d^2q_\perp dq^+ dq^-} = \frac{4\pi\alpha_{em}^2 Q_q^2}{3SQ^2}\delta(q^2 - Q^2)\left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right)W^{\mu\nu}, \quad S = 2P_A^+ P_B^-.$$

With those multi-parton states we find the factorized form:

$$\frac{d\sigma(s_\perp)}{dQ^2 dq_\perp^2 dq^+ dq^-} \sim f_a \otimes \mathcal{H}_h \otimes T_F\ [x_1, x_2]$$
$$+ f_a \otimes \mathcal{H}_{gs} \otimes T_F\ [x, x]$$
$$+ f_a \otimes \mathcal{H}_{fs} \otimes T_F\ [0, x]$$
$$+ \cdots$$

All perturbative functions start at order α_s

Details in:

J.P. Ma, H.Z. Sang & S.J. Zhu, e-Print: arXiv:1111.3717
J.P. Ma, H.Z. Sang, e-print: arXiv: 1102.2679.

Evolutions of Twist-3 Operators

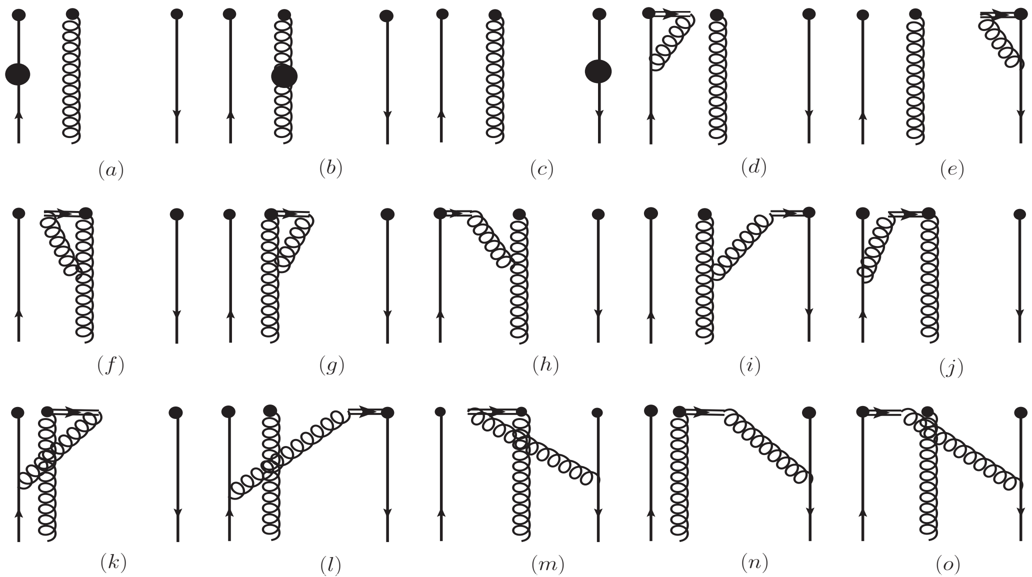
The defined twist-3 operators have scale-dependence, e.g., the non-singlet part of the quark-gluon correlators has:

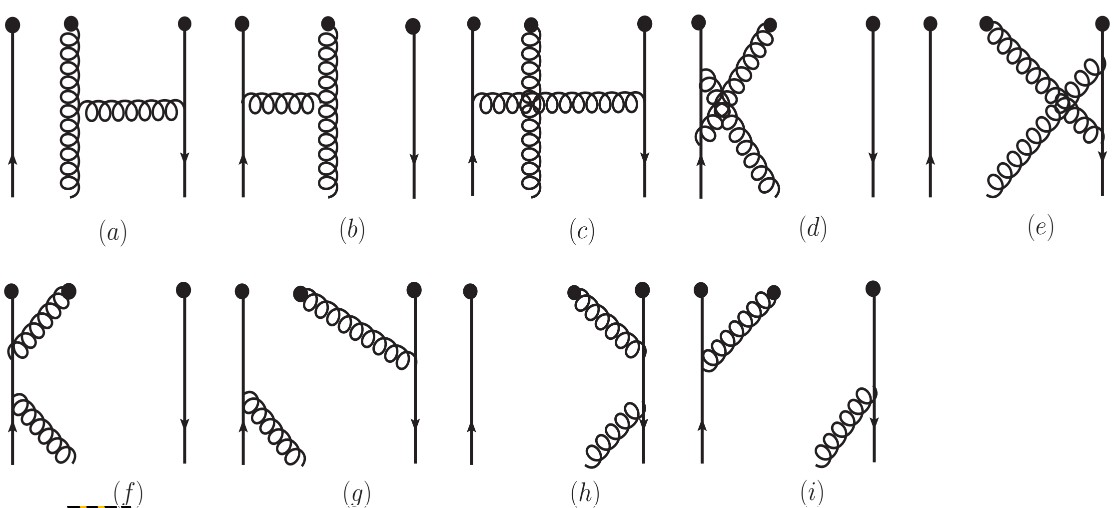
$$\frac{\partial\ T_{\pm}(x_1, x_2, \mu)}{\partial \ln \mu} = \frac{\alpha_s}{\pi} \int d\xi_1 d\xi_2 \mathcal{F}_{\pm}(x_1, x_2, \xi_1, \xi_2) T_{\pm}(\xi_1, \xi_2, \mu).$$

$$T_{\pm}(x_1, x_2) = T_F(x_1, x_2) \pm T_{\Delta, F}(x_1, x_2).$$

Again, the dependence is determined by QCD and is not related to any specific hadron. One can use the multi-parton states to calculate the twist-3 matrix elements and derived the evolution.

One-loop diagrams for the twist-3 matrix elements:





24 diagrams in
Feynman gauge
+
diagrams with
pure gluon-states

The general results are too long to give here. But some special cases are interesting to give here.

The soft-gluon case:

$$
\begin{aligned}
\frac{\partial T_F(x,x,\mu)}{\partial \ln \mu} = \quad & \frac{\alpha_s}{\pi}\left\{ \int_x^1 \frac{dz}{z}\left[P_{qq}(z) T_F(\xi,\xi) + \frac{N_c}{2}\frac{(1+z)T_F(x,\xi)-(1+z^2)T_F(\xi,\xi)}{1-z} + T_{\Delta,F}(x,\xi) \right.\right. \\
& \left. + \frac{1}{2N_c}\Big((1-2z)T_F(x,x-\xi) + T_{\Delta,F}(x,x-\xi) \Big) \right] - N_c T_F(x,x) \\
& \left. - \frac{1}{2}\int_x^1 \frac{dz}{z}\frac{(1-z)^2+z^2}{\xi}\Big(T_G^{(f)}(\xi,\xi) + T_G^{(d)}(\xi,\xi) \Big) \right\},
\end{aligned}
$$

$$z = x/\xi$$

Twist-3 gluonic matrix elements

There were discrepancies about terms in the second line in literature.

Soft-Fermion cases:

$$
\begin{aligned}
\frac{\partial T_+(0,x,\mu)}{\partial \ln \mu} = \frac{\alpha_s}{\pi}\Bigg\{ \int_x^1 \frac{dz}{z}\Bigg[-\frac{1}{2N_c}\frac{T_+(\xi - x,\xi)}{(1-z)_+} + \frac{N_c}{2}\frac{1+z^3}{(1-z)_+}T_+(0,\xi)\Bigg] \\
+ \frac{3(N_c^2-1)}{4N_c}T_+(0,x) + \int_x^1 \frac{dz}{z}\Bigg[\frac{N_c}{2}\frac{z^2}{(1-z)_+}T_+(x-\xi,x) + \frac{1}{2N_c}(1-z)^2 T_+(0,-\xi)\Bigg] \\
- \frac{1}{2x}T_{+G}(0,x) - \frac{1}{2}\int_x^1 \frac{dz}{z\xi}T_{+G}(\xi,\xi-x)\Bigg\},
\end{aligned}
$$

Details in e-Print: arXiv:1205.0611 [hep-ph] by J.P. Ma and Q. Wang

Summary

Soft gluons for soft-gluon-pole contributions are Glauber gluons.

There is a non-trivial order mixing in the collinear factorization of SSA.

The “best” way to access the soft-gluon-pole contributions is to measure the asymmetry in the lepton angular distribution in Drell-Yan.

Questions:

SIDIS? (a simple prediction with the soft-gluon-pole contribution)

Higher-order corrections ?

A proof for the factorization?

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