Quark and gluon momentum and spin operators in nucleon spin problem

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Content:

- Introduction to problem
- Spin decomposition based on gauge invariant variables
- Helicity notion in non-Abelian gauge theory
- Spin decompositions with generalized axial (light-cone) constraints
- Spin decompositions with color Killing vector
- Conclusions

Outlook of nucleon spin decomposition problem

- Canonical total angular momentum decomposition:

$$J^{can}_{\mu\nu} = \int \bar{\psi}\gamma^0 \frac{\Sigma_{\mu\nu}}{2}\psi d^3x - i\int \bar{\psi}\gamma^0 x_{[\mu}\partial_{\nu]}\psi d^3x$$
$$-\int \vec{A}_{[\mu}\cdot\vec{F}_{\nu]0}d^3x - \int \vec{F}_{0\alpha}\cdot x_{[\mu}\partial_{\nu]}\vec{A}_\alpha d^3x =$$
$$S_q + L_q + S_g + L_g. \qquad (1)$$

$$\Gamma(Q^2) = \int_0^1 dx\Delta g(x, Q^2) \to \frac{1}{2S^+} < P, \hat{e}_3|M^{+12}|_{Q^2}|P, \hat{e}_3 >$$
$$M^{\mu\nu\lambda} \equiv 2Tr(F^{\mu\nu}A^\lambda - F^{\mu\lambda}A^\nu) \qquad (2)$$

/ Jaffe, PLB, 1996

- X. Ji / PRL, 1997:

$$\vec{J} = \int d^3x \psi^\dagger [\vec{\gamma}\gamma^5 + (\vec{x} \times i\vec{D})]\psi + (\vec{x} \times (\vec{E} \times \vec{B})) \qquad (3)$$

- Chen et al / PRL 2008, 2009, PLB 2011: Gauge invariant definition of photon and gluon spin density operator does exist!
- X. Ji, E. Leader' criticisms, 2008,2011
- Wakamatsu: further development, two issues of problem: mathematical definitions and measurement problem
- Y.M. Cho et al 2010, 2011: Lorentz invariant spin decomposition
- Y. Hatta, 2011: spin decomposition consistent with canonical one in light-cone gauge.

The main points of the present talk:

- To answer the principal questions on nucleon spin decomposition problem:
- (i) Does gauge invariant spin decomposition exist?
- (ii) Is it unique or not?
- (iii) Physical implications.

Chen et al idea of spin decomposition:QED

- QED case: the basic idea:to separate a pure gauge potential

$$A_i = A_i^{pure} + A_i^{phys} = \frac{\partial_i \partial_j}{\Delta} A_j + (\delta_{ij} - \frac{\partial_i \partial_j}{\Delta}) A_j \qquad (4)$$

$$(A^{pure})' = U(\partial + A^{pure}) U^{-1}, \qquad (A^{phys})' = A^{phys}$$

after adding a surface term $\int d^3x \partial_\alpha (F_{0\alpha} \cdot x_{[\mu} A_{\nu]}^{pure})$ to $J_{\mu\nu}^{can}$ one obtains a gauge invariant decomposition:

$$J_{\mu\nu}^{can} = \int d^3x \Big\{ \bar{\psi} \gamma^0 \frac{\Sigma_{\mu\nu}}{2} \psi - i \bar{\psi} \gamma^0 x_{[\mu} \mathcal{D}_{\nu]} \psi - F_{0[\mu} \cdot A_{\nu]}^{phys} - F_{0\alpha} \cdot x_{[\mu} \mathcal{D}_{\nu]} A_\alpha^{phys} \Big\}, \qquad (5)$$

$$\mathcal{D}_\mu = \partial_\mu + A_\mu^{pure}$$

One can gauge out $A^{pure} = 0$, which is equivalent to imposing the Coulomb gauge $\partial_i A_i = 0$. In this gauge $A^{phys} = A$, so (5) reduces to the canonical decomposition in Coulomb gauge.

Chen et al idea of spin decomposition: QCD case

$$\vec{A}_\mu = \vec{A}^{pure}_\mu(\vec{A}) + \vec{A}^{phys}_\mu(\vec{A}). \tag{6}$$

vector notations stand for color vectors, $\vec{A}_\mu = A^a_\mu$
Self-consistent construction by solving the equations:

$$\vec{\mathcal{D}}^{pure}_i \vec{A}^{phys}_i = 0,$$
$$\vec{F}^{pure}_{\mu\nu} = 0. \tag{7}$$

$$J^{can}_{\mu\nu} = \int d^3x \Big\{ \bar{\psi}\gamma^0 \frac{\Sigma_{\mu\nu}}{2}\psi - i\bar{\psi}\gamma^0 x_{[\mu}\vec{\mathcal{D}}_{\nu]}\psi - \vec{F}_{0[\mu} \cdot \vec{A}^{phys}_{\nu]} - \vec{F}_{0\alpha} \cdot x_{[\mu}\vec{\mathcal{D}}_{\nu]}\vec{A}^{phys}_\alpha \Big\}, \tag{8}$$

One can choose a gauge $\vec{A}^{pure} = 0$, i.e. $\partial_i \vec{A}_i = 0$:

$$\vec{A}^{phys}_i = (\delta_{ij} - \frac{\partial_i \partial_j}{\Delta})\vec{A}_j + O(A^{n>1}) \rightarrow \vec{A}_i \tag{9}$$

On Lorentz invariant spin decomposition

The defining equation for the physical field is given by the constraint of Lorenz gauge type Y.M. Cho a.o.,2010

$$\mathcal{D}^\mu \vec{A}_\mu^{phys} = 0. \quad (10)$$

In the Maxwell theory where the formal solution is given by

$$A_\mu^{phys}(\vec{x}, t) = \int d^3\vec{x}' \frac{\partial^\nu F_{\nu\mu}(x', t - |\vec{x} - \vec{x}'|/c)}{|\vec{x} - \vec{x}'|} \quad (11)$$

so, on-shell the A_μ^{phys} can not be determined. This is a well-known consequence of the incompleteness of the Lorenz gauge. Another Lorentz invariant constraint for A_μ^{phys} is given by Fock-Schwinger gauge type condition $x^\mu A_\mu^{phys} = 0$ which has a solution W.M. Sun, F. Wang, 2011

$$A_\mu = \int_0^1 d\alpha \alpha x^\nu F_{\nu\mu}(\alpha x). \quad (12)$$

Unfortunately, it lacks the invariance under translations.

Spin decomposition with using gauge invariant variables

The idea: / Pervushin, TMF, 1981: Using the Eq. of motion $\vec{D}_i^2 A_0^a = (\vec{D}_i \partial_0 A_i)^a + j_0^a$ one can postulate the Eqn. for pure gauge matrix $\hat{v}$:

$$\partial_0 \hat{v}(A) = \hat{v}(A)\Big(\frac{1}{\vec{D}^2(A)} \vec{D}_j(A) \partial_0 \hat{A}_j\Big), \tag{13}$$

$$\hat{v}(A^g) = \hat{v} g^{-1}, \tag{14}$$

in this section "^" stands for Lie algebra valued functions. The solution is the time exponent:

$$\hat{v}(A) = T \exp\Big\{ \int^t dt \frac{1}{\vec{D}^2(A)} \vec{D}_j(A) \partial_0 \hat{A}_j \Big\}. \tag{15}$$

This allows to define the gauge invariant variable $\hat{A}_i^{\mathrm{I}}$

$$\hat{A}_i^{\mathrm{I}}(A) = \hat{v}(A)(\partial_i + \hat{A}_i)\hat{v}^{-1}(A), \qquad \vec{D}_i(A^{\mathrm{I}}) \partial_0 \hat{A}_i^{\mathrm{I}} = 0 \tag{16}$$

Split into pure gauge and physical gluons

$$\begin{aligned}\hat{A}_i &= \hat{v}^{-1}(A)\partial_i \hat{v}(A) + \hat{v}^{-1}(A)\hat{A}^{\mathrm{I}}_i(A)\hat{v}(A) \\ &\equiv \hat{A}^{pure}_i + \hat{A}^{phys}_i, \end{aligned} \tag{17}$$

In the gauge $\vec{A}^{pure} = 0$ one has a generalized Coulomb gauge condition for $\vec{A}^{phys} \equiv \vec{A}$: $\vec{D}_i \partial_0 \hat{A}_i = 0$.

Generalization:

$$\partial_0 \tilde{v} = \tilde{v}\Big[\frac{1}{\vec{D}^2(A)}(\vec{D}_j(A)\partial_0 \hat{A}_j + \hat{f}[A^{\mathrm{I}}, v, T])\Big]. \tag{18}$$

This implies a constraint on physical field

$$\vec{D}_i(\tilde{A}^{phys})\partial_0 \tilde{A}^{phys}_i + \hat{f}[A^{\mathrm{I}}, v, T] = 0 \tag{19}$$

In a special case $\hat{f} = \hat{j}_0$ one has a decomposition which on-shell has a constraint like a temporal gauge $\vec{A}^{phys}_0 = 0$.

Notion of helicity within the framework of a little group $E(2) \subset SO(1,3)$: Wigner, Ann.Math.'1939. Gauge invariant helicity in Maxwell theory:S. Weinberg,PRD'1964; D. Han, Y.S. Kim and D. Son,PRD'1985. Gluon momentum is directed along $z-$ axis, $p_\mu = (\omega, 0, 0, \omega)$. Generators of the little group $E(2)$:

$$J_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad N_1 = K_1 - J_2, \quad N_2 = K_2 + J_1. \quad (20)$$

Polarization vectors $\epsilon_\pm = (0, 1, \pm i, 0)$ are eigenstates of J_3. This implies helicity conditions:

$$\vec{A}_0^{phys} = 0, \qquad \vec{A}_3^{phys} = 0. \quad (21)$$

To provide both helicity conditions in a consistent manner with equations of motion has been a principal obstacle toward generalization of the helicity notion to the case of non-Abelian gauge theory. Since one has the condition $\vec{A}_0^{phys} = 0$ on mass-shell by construction, it is possible to provide the second helicity condition $\vec{A}_3^{phys} = 0$ by choosing a gauge of either Coulomb or axial or light-cone type. There should exist a class of gauge equivalent spin decompositions which satisfy on mass-shell the same helicity conditions for the physical field. One such possible decomposition has been proposed by Hatta, PRD'2011

Let us define the physical gauge potential $\vec{A}_\mu^{phys}$ by a generalized axial gauge type constraint

$$n^\mu \vec{A}_\mu^{phys} = 0, \tag{22}$$

where the vector n^μ specifies the axial or light-cone gauge condition. Solution for the physical gauge field $\vec{A}_\mu^{phys}$ in terms of the general field strength is very simple:

$$\vec{A}_\mu^{phys} = -\int_0^\infty d\lambda n^\nu \vec{F}_{\nu\mu}(x + \lambda n). \tag{23}$$

A pure gauge field $\vec{A}_\mu^{pure}$ is defined by

$$\vec{A}_\mu^{pure} = \vec{A}_\mu - \vec{A}_\mu^{phys}. \tag{24}$$

Choosing a proper n_μ one can define $\vec{A}_\mu^{phys}$ by axial, $\vec{A}_3^{phys} = 0$, or light-cone type, $\vec{A}_+^{phys} = 0$, constraint. The both helicity gauge conditions can be easily reached by imposing the temporal gauge fixing condition $\vec{A}_0^{phys} = 0$. The advantage of the decomposition with the light-cone type constraint, $n^2 = 0$, is that the corresponding non-local operator $\vec{A}_\mu^{phys}(A)$ reduces to the canonical spin density operator in a special gauge $\vec{A}_\mu^{pure} = 0$, i.e., explicitly in the light-cone gauge.
This allows to make straightforward one-to-one correspondence of the gauge invariant spin density operator to the gluon helicity Δg measured in experiment. The gluon spin operator corresponding to the canonical gluon spin density is

$$S_\mu^{gluon} = \epsilon_{\mu\nu\rho\sigma} \vec{F}_{\nu\rho} \cdot \vec{A}_\sigma^{phys}. \quad (25)$$

In light-cone gauge one has:

$$S_\mu^{gluon} = Tr \int_0^\infty d\lambda n^\nu F_{\nu\xi}(\lambda n + z) P \exp \Big(ig \cdot \int_0^\lambda du n^\nu A_\nu(un + z) \Big) \tilde{F}_{\xi\mu}(z) + n_\mu(\mathcal{O}(A^3)), \qquad (26)$$

where $\tilde{F}^a_{\xi\mu}$ is the dual field strength. On the other hand, one has the following expression for the gluon helicity at light-cone $x^2 = 0$ /Manohar, PRL'1990, Jaffe, PLB'1996

$$(sx)\Delta g =< N| \int_0^\infty d\lambda x^\mu F_{\xi\mu}(\lambda x) \cdot P \exp \Big(ig \int_0^\lambda du x^\nu A_\nu(ux) \Big) x^\nu \tilde{F}_{\nu\xi}(0)|N >, \qquad (27)$$

where $s_\mu = \bar{u}(p,s)\gamma_\mu\gamma_5 u(p,s)$ is the four-vector of nucleon spin.With (26) one results in the known relationship between Δg and the nucleon expectation value of the transverse part of S_μ

$$< N|x^\mu S_\mu^{gluon}|N >= -(sx)\Delta g. \qquad (28)$$

$$\vec{A}_\mu = A_\mu \hat{n} + \vec{C}_\mu + \vec{X}_\mu \equiv \hat{A}_\mu + \vec{X}_\mu,$$
$$\vec{C}_\mu = -\frac{1}{g}\hat{n} \times \partial_\mu \hat{n}, \qquad \vec{X}_\mu \cdot \hat{n} = 0, \tag{29}$$

/Cho, PRL, Duan and Ge Sci. Sinica, 1979

where A_μ is a binding gluon, $\vec{X}_\mu$ is the valence gluon, and $\hat{n}$ is $SU(2)$ color vector. The restricted potential $\hat{A}_\mu$ transforms as a gauge connection, and $\vec{X}_\mu$ transforms covariantly.

$$\vec{F}_{\mu\nu} = (F_{\mu\nu} + H_{\mu\nu})\hat{n} + \vec{F}_{\mu\nu}(\vec{X}),$$
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$
$$H_{\mu\nu} = \partial_\mu \tilde{C}_\nu(\hat{n}) - \partial_\nu \tilde{C}_\mu(\hat{n}), \tag{30}$$

where the valence gluon field strength is

$$\vec{F}_{\mu\nu}(\vec{X}) = \hat{D}_\mu \vec{X}_\nu - \hat{D}_\nu \vec{X}_\mu + g\vec{X}_\mu \times \vec{X}_\nu. \tag{31}$$

$\hat{D}_\mu$ contains only the restricted potential $\hat{A}_\mu$.

Three types of Lorentz and gauge invariant nucleon spin decompositions

I: $\vec{A}_\mu = \vec{A}_\mu^{pure} + \vec{A}_\mu^{phys}, \qquad \vec{\mathcal{D}}_\mu^{pure} \vec{A}_\mu^{phys} = 0$
like Chen et al. decomposition which reduces to canonical one in the Lorentz gauge $\partial_\mu \vec{A}_\mu = 0$.
II: $\vec{A}_\mu = \hat{A}_\mu + \vec{X}_\mu, \qquad$ like X. Ji, Wakamatsu' approach

$$J_{\mu\nu}^{cov} = \int d^3x \Big\{ \bar{\psi}\gamma^0 \frac{\Sigma_{\mu\nu}}{2}\psi - i\bar{\psi}\gamma^0 x_{[\mu}\hat{D}_{\nu]}\psi - \vec{F}_{0[\mu} \cdot \vec{X}_{\nu]} - \vec{F}_{0\alpha} \cdot x_{[\mu}(\hat{D}_{\nu]}\vec{X}_\alpha - \hat{F}_{\nu]\alpha}(\hat{A})) \Big\}. \qquad (32)$$

III: $\vec{A}_\mu = \vec{A}_\mu^{pure} + \hat{A}_\mu, \qquad$ restricted QCD.

$$J_{\mu\nu}^{res} = \int d^3x \Big\{ \bar{\psi}\gamma^0 \frac{\Sigma_{\mu\nu}}{2}\psi - i\bar{\psi}\gamma^0 x_{[\mu}\partial_{\nu]}\psi - \vec{F}_{0[\mu} \cdot \hat{A}_{\nu]} - \vec{F}_{0\alpha} \cdot x_{[\mu}\partial_{\nu]}\hat{A}_\alpha \Big\}. \qquad (33)$$

/ Cho, Ge, Pak, Zhang, 2011

Implications of Restricted QCD (RCD)

(i) nucleons as lowest energy hadrons are made of quarks and binding gluons in a simple quark model
(ii) Abelian dominance in nucleons, lattice calc. /by Kondo, Shinohara et al, 2007
Estimate of gluon contribution to total nucleon momentum in asymptotic limit using kinematic quark momentum /Georgi, Politzer, Gross, Wilczek, 1974

$$P_\mu^g = \frac{2n_g}{2n_g + 3n_f} P_\mu^{tot} \simeq 1/2, \qquad n_g = 8, n_f = 5. \tag{34}$$

Estimate with canonical momentum decomposition by / Chen et al, PRL 2009:

$$P_\mu^g = \frac{n_g}{n_g + 6n_f} P_\mu^{tot} \simeq 1/5. \tag{35}$$

It is interesting to notice, that this result can be reproduced exactly from (34) within RCD by replacement $n_g \to n_g/4$.
/thanks to O. Teryaev

Restricted QCD (RCD)

Estimate within restricted QCD: /Cho et al, 2011:

$$P_\mu^g = \frac{n_g}{n_g + 6n_f} P_\mu^{tot} \simeq 6\%, \quad n_g = 2. \tag{36}$$

Incompleteness of the definition for $\hat{n}$

$$\vec{A}_\mu = A_\mu \hat{n} + \vec{C}_\mu + \vec{X}_\mu \equiv \hat{A}_\mu + \vec{X}_\mu,$$
$$\vec{C}_\mu = -\frac{1}{g}\hat{n}\times\partial_\mu\hat{n}, \qquad \vec{X}_\mu\cdot\hat{n} = 0, \tag{37}$$

What is the color vector $\hat{n}(x)$ in Abelian projection? If $\hat{n}$ is an independent field then one has two types of gauge symmetry

$$(I): \quad \hat{n}' = U\hat{n}, \quad \hat{A}'_\mu = U(\partial_\mu + \hat{A}_\mu)U^{-1}, \quad \vec{X}'_\mu = U\vec{X}_\mu U^{-1},$$

$$(II): \quad \hat{n}' = \hat{n} \quad \hat{A}'_\mu = \hat{A}_\mu, \quad \vec{X}'_\mu = U(\partial_\mu + \vec{A}_\mu)U^{-1}.$$

With given expressions for pure gauge field $\vec{A}_\mu^{pure}$ or $\hat{v}(A)$ we construct $\hat{n}(A)$ in terms of general $\vec{A}_\mu$:

$$\hat{v}(A) = e^{i\omega\hat{n}^i\vec{\tau}^i}. \tag{38}$$

In the pure gauge limit $\hat{v} \to I, \omega \to 0$ the $\hat{n}$ remains undetermined.

Decomposition of gluon into binding and valence parts

Define the vacuum gauge potential /Cho, PLB 2007

$$\vec{A}_\mu^{pure} = -\tilde{C}_\mu \hat{n}(A) + \vec{C}_\mu. \tag{39}$$

Our decomposition with the constraint $\vec{D}_i(\vec{A}^{phys})\partial_0 \vec{A}_i^{phys} + \hat{j}_0 = 0$ which on-shell implies $\vec{A}_0^{phys} = 0$ and can be written as $\vec{D}_i(\vec{A}^{phys})\vec{E}_i^{phys} = 0$. Notice, we do not fix the gauge! The $\vec{A}^{phys}$ is a highly non-linear operator function of $\vec{A}$.
We decompose the gauge potential:

$$\begin{aligned} \vec{A}_\mu &= -\tilde{C}_\mu \hat{n} + \vec{C}_\mu + \vec{A}_\mu^{phys}, \\ \vec{A}_\mu^{phys} &\equiv \vec{A}_\mu - \vec{A}_\mu^{pure} = A_\mu \hat{n}(A) + \vec{X}_\mu. \end{aligned} \tag{40}$$

Split of gluon into binding and valence constituents

The gluon angular momentum is factorized with using $\vec{A}_0^{phys} = 0$:

$$J^{gluon}_{\mu\nu} = \int d^3x \Big\{ - F_{0[\mu} A_{\nu]} - \vec{F}_{0[\mu}(\vec{X}) \cdot \vec{X}_{\nu]} + F_{0\alpha} x_{[\mu}\partial_{\nu]} A_\alpha + \vec{F}_{0\alpha}(\vec{X}) x_{[\mu}\partial_{\nu]} \vec{X}_\alpha \Big\}, \qquad (41)$$

where all operators are physical (the superscript "phys" is omitted). Notice, with type (II) decomposition $\vec{A}_\mu = \hat{A}_\mu + \vec{X}_\mu$ the terms with binding gluons in (41) disappear, so if contribution of valence gluons is small, then the gluon spin contribution to nucleon spin will be zero, this can explain small value of $\Delta G \simeq 0$.

Conclusions

- 1. There is a wide number of Lorentz non-invariant and gauge invariant nucleon spin decompositions. In general they lead to gauge non-equivalent gluon spin operators. For most of such decomposition schemes the definition of the spin operator is frame dependent.
- 2. Poincare group and conformal invariance selects a unique Lorentz gauge type constraint for the physical gluon field and respective spin decomposition. However, the decomposition is not well defined on-shell and its physical meaning remains unclear.

Conclusions

- 3. We have shown that there is a class of gauge equivalent spin decompositions, leading to gauge invariant gluon spin operators consistent with the helicity notion and, so that, such definitions of spin operators are frame independent. The corresponding definitions for the spin operator are gauge equivalent and lead to the same matrix elements. In practical calculations the canonical spin density in light-cone gauge is most suitable.
- 4. Using Cho-Duan-Ge Abelian projection leads to a more wide class of gauge invariant and Lorentz invariant spin decompositions, especially within restricted QCD. In this case an explicit parametrization (definition) of color Killing vector field $\hat{n}$ is needed.

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