Nucleon Momentum and Spin Decomposition

—Old Crisis and New Resolution—

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Motivation and Plan

A. Canonical versus Kinematic Momentum

- Can we have a gauge independent decomposition of momentum and spin of a composite particle to those of the constituents?

- Can we define the quark momentum independent of gluon?

- What is the measurable momentum of quarks, canonical or kinematic, in nucleon? And how do we measure it?

Theoretical and Experimental Challenges

B. QCD Issue: What are the constituents of nucleons?

- Quark model tells that nucleons have no valence gluons, only valence quarks. What are the valence gluons, and how can we define them?

- The chromo-electric flux generates pair-annhilation of gluons, which suggests that the valence gluons are not the constituents of nucleons.

- If so, what gluons are actually in nucleons, and how can we confirm this? Nucleon momentum and spin decpmposition can answer this.

A Most Important Problem in Low Energy QCD

- **Contents**

  1. Momentum Decomposition in QED
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Momentum Decomposition in QED

A) Origin of Problem

- Using Noether's theorem we obtain the conserved total momentum from QED Lagrangian

$$P_\mu = P_\mu^e + P_\mu^\gamma = i \int \psi^\dagger \partial_\mu \psi d^3 x + \int [(\partial_\mu A_\alpha) F^{\alpha 0} + \frac{1}{4} \delta_\mu^0 F_{\alpha\beta}^2] d^3 x.$$

- Adding a surface term $\int \partial_\alpha (A_\mu F^{\alpha 0}) d^3 x$ we can change it to the gauge invariant expression

$$P_\mu = \bar{P}_\mu^e + \bar{P}_\mu^\gamma = i \int \psi^\dagger D_\mu \psi d^3 x + \int (F_{\mu\alpha} F^{\alpha 0} + \frac{1}{4} \delta_\mu^0 F_{\alpha\beta}^2) d^3 x.$$

- The first provides a decomposition of the constituents, but is not gauge invariant. The second provides a gauge invariant decomposition, but not of the constituents. So neither can provide the desired decomposition.

- Questions:

  1. How can we make a gauge invariant momentum and spin decompositions?

  2. What is the measurable decomposition?

B) Canonical versus Kinematic Momentum

- The electron has two momentums, the canonical p_μ and the kinematic Π_μ,

$$p_\mu = i\partial_\mu, \qquad \Pi_\mu = iD_\mu = p_\mu - eA_\mu,$$

but only the kinematic momentum is (assumed to be) observable because the canonical momentum is not gauge invariant.

- Classically the conserved momentum of charged particles in electromagnetic field is the sum of the photon momentum (the Poynting vector) and the kinematic momentum of the particle.

- Quantum mechanically, however, the kinematic momentum does not satisfy the canonical commutation relation

$$[\Pi_\mu, \ \Pi_\nu] \neq 0.$$

so that it can not be the physical (observable) momentum.

- Question: Can we define the gauge invariant canonical momentum which satisfies the canonical commutation relation?

C) Vacuum Decomposition of QED

- Let $U = \exp i\theta$ be an arbitrary element of U(1), and decompose the gauge potential to the vacuum (pure gauge) part Ω_μ and the physical (transverse) part B_μ,

$$A_\mu = \Omega_\mu + B_\mu.$$
$$\Omega_\mu = -i\, U^{-1}\partial_\mu U = \partial_\mu\theta, \quad \partial_\mu B_\mu = 0.$$

- Under the gauge transformation we have

$$\delta\Omega_\mu = \partial_\mu\alpha, \quad \delta B_\mu = 0.$$

So the decomposition is gauge independent and Lorentz covariant. In particular, the physical part becomes a Lorentz covariant four-vector.

D) Gauge Invariant Canonical Momentum

- Define the gauge covariant canonical momentum operator

$$\bar{p}_\mu = -i\bar{D}_\mu = p_\mu - e\Omega_\mu, \qquad [\bar{p}_\mu, \ \bar{p}_\nu] = 0.$$

- With this we have

$$P_\mu = i \int \psi^\dagger \bar{D}_\mu \psi d^3x + \int [(\partial_\mu B_\alpha) F^{\alpha 0} + \frac{1}{4}\delta^0_\mu F^2_{\alpha\beta}]d^3x.$$

  adding a surface term to P_μ. Now, each term is gauge invariant and at the same time involves only one constituent.

- Which momentum decomposition do we measure in experiments?

Spin Decomposition in QED

- The canonical angular momentum in QED obtained from Noether's theorem is given by

$$J_{\mu\nu}^{(qed)} = S_{\mu\nu}^e + L_{\mu\nu}^e + S_{\mu\nu}^\gamma + L_{\mu\nu}^\gamma$$
$$= \int \psi^\dagger \frac{\Sigma_{\mu\nu}}{2} \psi d^3x - i \int \psi^\dagger x_{[\mu} \partial_{\nu]} \psi d^3x$$
$$- \int A_{[\mu} F_{\nu]0} d^3x - \int F_{0\alpha} x_{[\mu} \partial_{\nu]} A_\alpha d^3x.$$

Notice that three terms except for the first are not gauge invariant.

- We can change it to the popular decomposition made of the kinematic momentum

$$J_{\mu\nu}^{(qed)} = S_{\mu\nu}^e + L_{\mu\nu}^e + J_{\mu\nu}^\gamma$$
$$= \int \psi^\dagger \frac{\Sigma_{\mu\nu}}{2} \psi d^3x - i \int \psi^\dagger x_{[\mu} D_{\nu]} \psi d^3x$$
$$- \int F_{0\alpha} x_{[\mu} F_{\nu]\alpha} d^3x.$$

- But this provides only a partial decomposition of spin. Moreover, the second term does not satisfy the angular momentum commutation relation.

- Using the gauge invariant canonical momentum, however, we can modify it to (adding a surface term)

$$J_{\mu\nu}^{(qed)} = \int \psi^\dagger \frac{\Sigma_{\mu\nu}}{2} \psi d^3x - i \int \psi^\dagger x_{[\mu} \bar{D}_{\nu]} \psi d^3x$$
$$- \int B_{[\mu} F_{\nu]0} d^3x - \int F_{0\alpha} x_{[\mu} \partial_{\nu]} B_\alpha d^3x,$$

where each term becomes gauge invariant. This perfectly accounts for the experiments which measure each term separately.

- How can we generalize this to QCD?

- **Motivation**

  1. To understand the non-Abelian dynamics we need to know the anatomy of the non-Abelian gauge theory.

  2. To demonstrate the Abelian dominance in QCD we must know how to separate the colored (non-Ableian) part from the neutral (Abelian) part.

A) Abelian Decomposition

- 'tHooft proposal: Let $(\hat{n}_1, \hat{n}_2, \hat{n}_3)$ be an orthonormal basis and choose $\hat{n} = \hat{n}_3$ to be the Abelian direction. Let

$$\vec{A}_\mu = A_\mu^1 \hat{n}_1 + A_\mu^2 \hat{n}_2 + A_\mu^3 \hat{n}_3 = \hat{\mathcal{A}}_\mu + \vec{\mathcal{X}}_\mu,$$

$$\hat{\mathcal{A}}_\mu = A_\mu \hat{n}, \quad \vec{\mathcal{X}}_\mu = A_\mu^1 \hat{n}_1 + A_\mu^2 \hat{n}_2 \quad (A_\mu = A_\mu^3).$$

Then in the maximal Abelian gauge (MAG)

$$\hat{\mathcal{D}}_\mu \vec{\mathcal{X}}_\mu = 0 \quad (\hat{\mathcal{D}}_\mu = \partial_\mu + g\hat{\mathcal{A}}_\mu \times)$$

$\hat{\mathcal{A}}_\mu$ becomes the Abelian part.

- This decomposition has become very popular in lattice QCD, but has problems.

1. The decomposition $\vec{A}_\mu = \hat{\mathcal{A}}_\mu + \vec{\mathcal{X}}_\mu$ is ad hoc (i.e., gauge dependent).

2. The MAG is a constraint on $\vec{\mathcal{X}}_\mu$, not on $\hat{\mathcal{A}}_\mu$. So logically it is independent of $\hat{\mathcal{A}}_\mu$.

3. It is insensitive to the non-Abelia topology. In particular, it does not tell anything about the monopole.

4. This trivializes QCD to Maxwell-type Abelian gauge theory coupled to colored gluons.

- Gauge independent Abelian decomposition: Consider SU(2) and let $\hat{n}$ be an isotriplet which selects the "Abelian" direction (the color direction). Make the Abelian projection by

$$D_\mu \hat{n} = \partial_\mu \hat{n} + g\vec{A}_\mu \times \hat{n} = 0 \quad (\hat{n}^2 = 1),$$

$$\vec{A}_\mu \to \hat{A}_\mu = A_\mu \hat{n} - \frac{1}{g}\hat{n} \times \partial_\mu \hat{n} = \mathcal{A}_\mu + \vec{C}_\mu,$$

$$\vec{C}_\mu = -\frac{1}{g}\hat{n} \times \partial_\mu \hat{n}, \quad A_\mu = \hat{n} \cdot \vec{A}_\mu.$$

- With this we have the Abelian (Cho-Duan-Ge) decomposition

$$\vec{A}_\mu = A_\mu \hat{n} - \frac{1}{g}\hat{n} \times \partial_\mu \hat{n} + \vec{X}_\mu \quad (\hat{n} \cdot \vec{X}_\mu = 0).$$

- Under the infinitesimal gauge transformation we have

$$\delta \hat{A}_\mu = \frac{1}{g} \hat{D}_\mu \vec{\alpha}, \quad \delta \vec{X}_\mu = -\vec{\alpha} \times \vec{X}_\mu. \quad (\hat{D}_\mu = \partial_\mu + g \hat{A}_\mu \times)$$

1. $\hat{A}_\mu$ is essentially Abelian, but has the full SU(2) gauge degrees of freedom. So it can be identified as the binding (neutral) gluon.

2. $\vec{X}_\mu$ transforms covariantly, and describes the valence (colored) gluons.

- $\hat{A}_\mu$ has a dual structure. In particular it contains the non-Abelian monopole

$$\vec{H}_{\mu\nu} = \partial_\mu \vec{C}_\nu - \partial_\nu \vec{C}_\mu + g\vec{C}_\mu \times \vec{C}_\nu = -\frac{1}{g}\partial_\mu \hat{n} \times \partial_\nu \hat{n}$$

$$= (\partial_\mu C_\nu - \partial_\nu C_\mu)\hat{n}, \quad C_\mu = \frac{1}{g}\,\hat{n}_1 \cdot \partial_\mu \hat{n}_2.$$

- $\hat{F}_{\mu\nu}$ is made of two potentials, "electric" A_μ and "magnetic" C_μ,

$$\hat{F}_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu + g\hat{A}_\mu \times \hat{A}_\nu = (F_{\mu\nu} + H_{\mu\nu})\hat{n},$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad H_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu.$$

Gauge Independent Separation of Monopole

B) Vacuum Decomposition

- Let $\hat{n}_i$ ($i = 1, 2, 3$; $\hat{n}_1 \times \hat{n}_2 = \hat{n}_3 = \hat{n}$) be the orthonormal isotriplets, and impose the maximal isometry,

$$^{\forall_i} D_\mu \hat{n}_i = 0 \Rightarrow^{\forall_i} [D_\mu, D_\nu] \hat{n}_i = \vec{F}_{\mu\nu} \times \hat{n}_i = 0$$
$$\Rightarrow \vec{F}_{\mu\nu} = 0.$$

- Solving this we can construct the most general vacuum potential

$$\hat{\Omega}_\mu = C_\mu{}^k \ \hat{n}_k = -\frac{1}{2g} \epsilon_{ij}{}^k \ (\hat{n}_i \cdot \partial_\mu \hat{n}_j) \ \hat{n}_k.$$

- $\hat{n}$ defines the mapping $\Pi_3(S^3) \simeq \Pi_3(S^2)$ which classifies topologically distinct vacua $|n\rangle$ by the knot quantum number $\Pi_3(S^2)$,

$$n = -\frac{g^3}{96\pi^2} \int \epsilon_{\alpha\beta\gamma}\epsilon_{ijk}C_\alpha{}^i C_\beta{}^j C_\gamma{}^k d^3x. \quad (\alpha, \beta, \gamma = 1, 2, 3)$$

- With $\hat{\Omega}_\mu$ we have the vacuum decomposition

$$\vec{A}_\mu = \hat{A}_\mu + \vec{X}_\mu = \hat{\Omega}_\mu + \vec{Z}_\mu,$$
$$\hat{A}_\mu = \hat{\Omega}_\mu + \vec{B}_\mu, \quad \vec{Z}_\mu = \vec{B}_\mu + \vec{X}_\mu,$$
$$\vec{B}_\mu = B_\mu \hat{n} = (A_\mu + C_\mu)\,\hat{n}.$$

- $\hat{\Omega}_\mu$ (just like $\hat{A}_\mu$) forms its own SU(2) connection space, and $\vec{B}_\mu$ transforms covariantly,
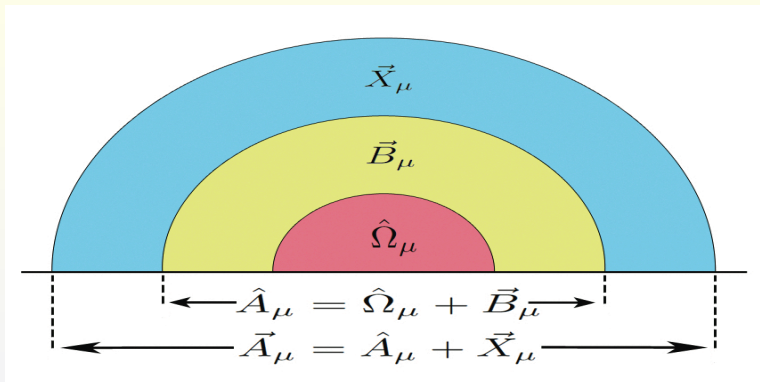
$$\delta \hat{\Omega}_\mu = \frac{1}{g} \bar{D}_\mu \vec{\alpha}, \qquad \delta \vec{B}_\mu = -\vec{\alpha} \times \vec{B}_\mu,$$

$$\bar{D}_\mu = \partial_\mu + g \, \hat{\Omega}_\mu \times .$$

- With the transversality conditions

$$\bar{D}_\mu \vec{B}_\mu = 0, \qquad \bar{D}_\mu \vec{X}_\mu = 0,$$

we can make $\vec{B}_\mu$ and $\vec{X}_\mu$ physical. Notice that the transversality conditions are gauge independent.

Figure: The affine structure of non-Abelian connection space: It has two proper subspaces made of the restricted potentials $\hat{A}_\mu$ and the vacuum potentials $\hat{\Omega}_\mu$ which form their own non-Abelian connection spaces.

- With the vacuum decomposition we have the gauge covariant canonical momentum operator

$$\bar{p}_\mu = -i\bar{D}_\mu, \qquad [\bar{p}_\mu, \ \bar{p}_\nu] = 0.$$

This confirms that in gauge theories the charged particles do have the gauge invariant canonical momentum.

- Obtain the momentum and spin decomposition using the gauge invariant quark canonical momentum.

Gauge Invariant Quark Canonical Momentum

A) Restricted Chromodynamics (RCD)

- Define the restricted chromodynamics (RCD) by

$$\mathcal{L}_{RCD} = -\frac{1}{4}\hat{F}_{\mu\nu}{}^2 = -\frac{1}{4}(F_{\mu\nu} + H_{\mu\nu})^2.$$

  It has the full non-Abelian gauge symmetry, but simpler than QCD.

- RCD describes the core dynamics, the Abelian (dual) sub-dynamics made of two Abelian potentials, of QCD.

A) Extended Chromodynamics (ECD)

- Recover QCD adding the valence gluons to RCD. Find

$$\mathcal{L} = -\frac{1}{4}\vec{F}_{\mu\nu}^2 = -\frac{1}{4}\hat{F}_{\mu\nu}^2$$

$$-\frac{1}{4}(\hat{D}_\mu\vec{X}_\nu - \hat{D}_\nu\vec{X}_\mu)^2 - \frac{g}{2}\hat{F}_{\mu\nu} \cdot (\vec{X}_\mu \times \vec{X}_\nu) - \frac{g^2}{4}(\vec{X}_\mu \times \vec{X}_\nu)^2.$$

So QCD is RCD which has the valence gluons as colored source.

- The restricted part and the valence part can be interpreted as the slow-varying classical field and the fast-varying quantum field.

- ECD has two types of gauge invariance, the invariance under the active (background) gauge transformation

$$\delta \hat{A}_\mu = \frac{1}{g} \hat{D}_\mu \vec{\alpha}, \qquad \delta \vec{X}_\mu = -\vec{\alpha} \times \vec{X}_\mu,$$

as well as the passive (quantum) gauge transformation

$$\delta \hat{A}_\mu = \frac{1}{g} (\hat{n} \cdot D_\mu \vec{\alpha}) \hat{n}, \qquad \delta \vec{X}_\mu = -\vec{\alpha} \times \vec{X}_\mu.$$

- To fix the quantum gauge freedom we need a gauge fixing on $\vec{X}_\mu$,

$$\hat{D}_\mu \vec{X}_\mu = 0.$$

Compare this with MAG.

B) Monopole Condensation and Color Confinement

- Abelian Dominance: RCD is responsible for the color confinement, because the valence gluons play no role in confinement mechanism. But RCD is made of two parts, and we must tell which makes the confinement.

- QCD one-loop effective action based on the Abelian decomposition establishes the stable monopole condensation.

- The recent KEK lattice calculation established the color confinement by monopole condensation. This is made possible with the gauge independent separation of the monopole.
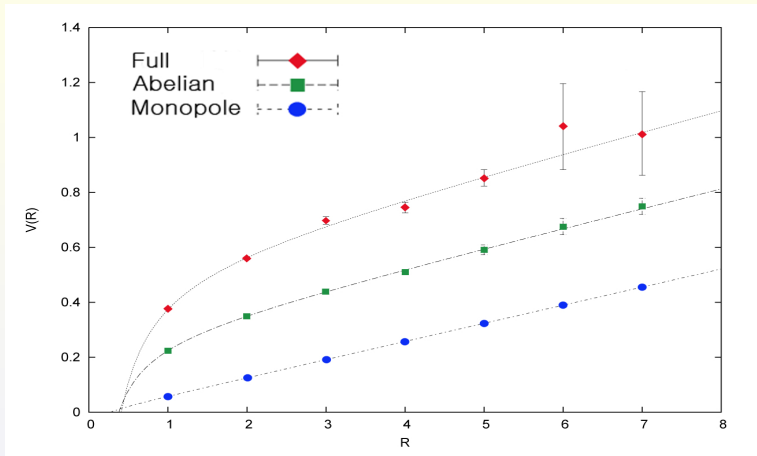
Figure: The KEK lattice calculation of the monopole dominance in the confining force in QCD.

- The old Saviddy-Nielsen-Olesen (SNO) vacuum is not gauge invariant nor parity conserving. Moreover, it does not represent the monopole condensation.

- The C-projection, just like the GSO-projection in string theory, projects out the tachyons in the gluon functional determinant and makes the monopole condensation stable.

- Warning: The Abelian projection based on MAG can not demonstrate the monopole condensation.

Momentum Decomposition in SU(2) QCD

- In QCD we have the conserved momentum obtained by Noether's theorem

$$P_\mu^{(qcd)} = -i \int \psi^\dagger \partial_\mu \psi d^3 x + \int [(\partial_\mu \vec{A}_\alpha) \cdot \vec{F}^{\alpha 0} + \frac{1}{4} \delta_\mu^0 \vec{F}_{\alpha\beta}^2] d^3 x.$$

- Adding a surface term, we obtain a gauge invariant decomposition of total momentum

$$P_\mu^{(qcd)} = -i \int \psi^\dagger \bar{D}_\mu \psi d^3 x + \int [(\bar{D}_\mu \vec{Z}_\alpha) \cdot \vec{F}^{\alpha 0} + \frac{1}{4} \delta_\mu^0 \vec{F}_{\alpha\beta}^2] d^3 x.$$

End of the Story? No!

- According to quark model the nucleons are made of three valence (i.e., colored) quarks, without any valence gluons (qqq singlets, but not qqqg or qqqgg...singlets). This tells that the valence gluons are not in nucleons.

- QCD effective action tells that the chromo-electric flux induces the pair annhilation of the valence gluons. This means that the valence gluons are not the constituents of nucleons.

- So we have to exclude the valence gluons. How?

- Start from the gauge invariant decomposition of total momentum

$$P_\mu^{(qcd)} = i \int \psi^\dagger \bar{D}_\mu \psi \, d^3x + \int [(\bar{D}_\mu \vec{Z}_\alpha) \cdot \vec{F}^{\alpha 0} + \frac{1}{4} \delta_\mu^0 \vec{F}_{\alpha\beta}^2] d^3x.$$

- Replace $\vec{Z}_\mu$ and $\vec{F}_{\mu\nu}$ by $\vec{B}_\mu$ and $\hat{F}_{\mu\nu}$, and find

$$P_\mu^{(rcd)} = i \int \psi^\dagger \bar{D}_\mu \psi \, d^3x + \int [(\bar{D}_\mu \vec{B}_\alpha) \cdot \hat{F}^{\alpha 0} + \frac{1}{4} \delta_\mu^0 \hat{F}_{\alpha\beta}^2] d^3x.$$

Notice that this is physically very similar to the QED expression.

- RCD has the conserved momentum given by Noether's theorem,

$$P_\mu^{(rcd)} = i \int \psi^\dagger \partial_\mu \psi d^3x + \int [(\partial_\mu \hat{A}_\alpha) \cdot \hat{F}^{\alpha 0} + \frac{1}{4}\delta_\mu^0 \hat{F}_{\alpha\beta}^2] d^3x.$$

From this we can rederive the above result, adding a surface term. This is because RCD has only the binding gluons.

- This shows that all we need for the nucleon momentum decomposition is RCD.

- Find the canonical (Norther) angular momentum decomposition

$$J_{\mu\nu}^{(qcd)} = S_{\mu\nu}^q + L_{\mu\nu}^q + S_{\mu\nu}^g + L_{\mu\nu}^g = \int \psi^\dagger \frac{\Sigma_{\mu\nu}}{2} \psi d^3x$$

$$-i\int \psi^\dagger x_{[\mu}\partial_{\nu]}\psi d^3x - \int \vec{A}_{[\mu} \cdot \vec{F}_{\nu]0}d^3x - \int \vec{F}_{0\alpha} \cdot x_{[\mu}\partial_{\nu]}\vec{A}_\alpha d^3x.$$

- Adding a surface term we obtain

$$J_{\mu\nu}^{(qcd)} = \int \psi^\dagger \frac{\Sigma_{\mu\nu}}{2} \psi d^3x - i\int \psi^\dagger x_{[\mu}\bar{D}_{\nu]}\psi d^3x$$

$$-\int \vec{Z}_{[\mu} \cdot \vec{F}_{\nu]0}d^3x - \int \vec{F}_{0\alpha} \cdot x_{[\mu}\bar{D}_{\nu]}\vec{Z}_\alpha d^3x.$$

But again this decomposition contains the valence gluons.

- Replace $\vec{A}_\mu$ and $\vec{F}_{\mu\nu}$ by $\hat{A}_\mu$ and $\hat{F}_{\mu\nu}$, and find

$$J_{\mu\nu}^{(qcd)} = \int \psi^\dagger \frac{\Sigma_{\mu\nu}}{2} \psi d^3x - i \int \psi^\dagger x_{[\mu} \bar{D}_{\nu]} \psi d^3x$$

$$- \int \vec{B}_{[\mu} \cdot \hat{F}_{\nu]0} d^3x - \int \hat{F}_{0\alpha} \cdot x_{[\mu} \bar{D}_{\nu]} \vec{B}_\alpha d^3x.$$

This is a straightforward generalization of QED result.

- This is precisely the angular momentum decomposition we obtain from RCD Lagrangian.

SU(3) QCD

A) Abelian Decomposition and RCD

- In reality we have SU(3) QCD, but the generic feature remains the same. The Abelian projection is described by two isometries,

$$D_\mu \hat{n} = 0, \quad D_\mu \hat{n}' = 0, \quad (\hat{n}^2 = \hat{n}'^2 = 1)$$

where $\hat{n}$ and $\hat{n}' = \hat{n} * \hat{n}$ are λ_3-like and λ_8-like octet unit vectors.

- With this we have the following Abelian decomposition,

$$\vec{A}_\mu = \hat{A}_\mu + \vec{X}_\mu, \quad \hat{A}_\mu = A_\mu \hat{n} + A'_\mu \hat{n}' - \frac{1}{g}(\hat{n} \times \partial_\mu \hat{n} + \hat{n}' \times \partial_\mu \hat{n}')$$

$$A_\mu = \hat{n} \cdot \vec{A}_\mu, \quad A'_\mu = \hat{n}' \cdot \vec{A}_\mu, \quad \hat{n} \cdot \vec{X}_\mu = \hat{n}' \cdot \vec{X}_\mu = 0.$$

- From the vacuum isometry $D_\mu \hat{n}_a = 0, \ (a = 1, 2, ..., 8)$ we have

$$\hat{\Omega}_\mu = \frac{1}{3}(\hat{\Omega}_\mu^{(i)} + \hat{\Omega}_\mu^{(u)} + \hat{\Omega}_\mu^{(v)}),$$

where $\hat{\Omega}_\mu^{(i)}, \hat{\Omega}_\mu^{(u)}, \hat{\Omega}_\mu^{(v)}$ are the i-spin, u-spin, v-spin SU(2) vacuum.

- Moreover, we have

$$\hat{A}_\mu = \hat{\Omega}_\mu + \vec{B}_\mu, \qquad \vec{B}_\mu = (A_\mu + C_\mu)\hat{n} + (A'_\mu + C'_\mu)\hat{n}',$$

$$\vec{A}_\mu = \hat{A}_\mu + \vec{X}_\mu = \hat{\Omega}_\mu + \vec{Z}_\mu, \qquad \vec{Z}_\mu = \vec{B}_\mu + \vec{X}_\mu.$$

$$\delta\hat{\Omega}_\mu = \frac{1}{g}\bar{D}_\mu\vec{\alpha}, \quad \delta\hat{A}_\mu = \frac{1}{g}\hat{D}_\mu\vec{\alpha}, \quad \delta\vec{X}_\mu = -\alpha \times \vec{X}_\mu.$$

- Find the Abelianized QCD Lagrangian

$$\mathcal{L} = \sum_p \Big\{ -\frac{1}{6}(\mathcal{G}^p_{\mu\nu})^2 - \frac{1}{2}|D_\mu W^p_\nu - D_\nu W^p_\mu|^2$$

$$+ ig\, \mathcal{G}^p_{\mu\nu} W^{p*}_\mu W^p_\nu - \frac{g^2}{2}\big[(W^{p*}_\mu W^p_\mu)^2 - (W^{p*}_\mu)^2 (W^p_\nu)^2\big] \Big\},$$

$$\mathcal{G}^p_{\mu\nu} = \partial_\mu \mathcal{B}^p_\nu - \partial_\nu \mathcal{B}^p_\mu, \quad D_\mu W^p_\nu = (\partial_\mu + ig\mathcal{B}^p_\mu)W^p_\nu, \quad (p = 1, 2, 3)$$

$$\mathcal{B}^1_\mu = B_\mu, \quad \mathcal{B}^2_\mu = -\frac{1}{2}B_\mu + \frac{\sqrt{3}}{2}B'_\mu, \quad \mathcal{B}^3_\mu = -\frac{1}{2}B_\mu - \frac{\sqrt{3}}{2}B'_\mu,$$

$$B_\mu = A_\mu + C_\mu, \quad B'_\mu = A'_\mu + C'_\mu,$$

$$W^1_\mu = \frac{X^1_\mu + iX^2_\mu}{\sqrt{2}}, \quad W^2_\mu = \frac{X^6_\mu + iX^7_\mu}{\sqrt{2}}, \quad W^3_\mu = \frac{X^4_\mu - iX^5_\mu}{\sqrt{2}}.$$

- Notice that $\mathcal{B}_\mu^p$ $(p = 1, 2, 3)$ are precisely the i-spin, u-spin, and v-spin dual potentials which couple to three colored valence gluons W_μ^p.

- This shows that all fundamental features remain exactly the same in SU(3) QCD, except that here we have two binding gluons and six valence gluons.

- With this we obtain RCD made of two binding gluons (with $\vec{X}_\mu = 0$) which has the full SU(3) gauge symmetry. Moreover, we have the Abelian dominance.

B) Gluon Momentum in Nucleons

- Assume that only the kinematic momentum of quarks (iD_μ) are measurable. In this case the mixing matrix of P_μ^q and P_μ^g in the asymptotic limit is given by

$$P_\mu^g = \frac{2n_g}{2n_g + 3n_f} P_\mu^{tot}.$$

  So with $n_g = 8$ (and $n_f = 5$), about 52% of total momentum is carried by the gluons in nucleons.

- But in RCD the kinematic momentum is given by $i\hat{D}_\mu$. So with $n_g = 2$, only about 21% of total momentum is carried by gluons. This is in sharp contrast with the standard prediction.

- Now, suppose the gauge invariant canonical momentum of quarks $(i\bar{D}_\mu)$ are measurable. In this case the mixing matrix of P_μ^q and P_μ^g changes, and we may have

$$P_\mu^g = \frac{n_g}{n_g + 6n_f} P_\mu^{tot}.$$

If this is true, about 21% of nucleon momentum is carried by gluons with $n_g = 8$. But with $n_g = 2$, only about 6% of nucleon momentum is carried by gluons. Notice that the quark canonical momentum becomes less than the kinematic momentum.

- Independent of the details, the lesson is that RCD predicts much smaller fraction of gluon momentum, and for obvious reason: RCD has fewer gluons than QCD. And this prediction is endorsed by recent experiments.

C) Angular Momentum Decomposition

- Just like SU(2) QCD we have

$$J_{\mu\nu}^{(qcd)} = \int \psi^\dagger \frac{\Sigma_{\mu\nu}}{2} \psi d^3x - i \int \psi^\dagger x_{[\mu} \bar{D}_{\nu]} \psi d^3x$$
$$- \int \vec{B}_{[\mu} \cdot \hat{F}_{\nu]0} d^3x - \int \hat{F}_{0\alpha} \cdot x_{[\mu} \bar{D}_{\nu]} \vec{B}_\alpha d^3x,$$

where $\vec{B}_\mu$ is the transverse binding gluons.

- Again, independent of the details, the fraction of the angular momentum carried by gluons must be much smaller than the popular prediction.

New resolution to old crisis?

Discussion

- Notice that

  1. Our decomposition is gauge invariant. In particular, $\hat{n}$ represents gauge degrees of freedom, so that it can be left arbitrary. Moreover, it become important only in the infra-red limit.

  2. Our decomposition applies to any Lorentz frame. So we can always choose a preferred Lorentz frame to make it simpler.

- In practical calculations, however, the canonical (Noether) decomposition is equally good. This is because we always fix the gauge to calculate momentum and spin of quarks and gluons.

A. Theoretical Issues

- Which momentum is measurable?

  There appears no reason why the canonical momentum is not measurable.

- Which gluons are in nucleons? Which dynamics, QCD or RCD, describes nucleons?

  The quark model (hadron spectroscopy) strongly implies that only the binding gluons are in nucleons. Moreover, RCD is all we need to explain nucleons.

B. Experimental Questions

- Which momentum do we actually measure in nucleons?

  Hint: In QED we have the momentum and angular momentum decomposition endorsed by experiments. But with the gauge independent Abelianization, RCD (and QCD) is not much different from QED, especially in the asymptotic limit.

- Is there any way to measure both canonical and kinematic momentums of quarks?

- Which gluons are in nucleons?

- References

1. Y.M. Cho, PRD 21 (1980); PRL 46 (1981); PRD 23 (1981); Y.S. Duan and M.L. Ge, S. Sinica 11 (1979).
2. Y.M. Cho, PLB 644 (2007).
3. X. Ji, PRL 78 (1997); PRL 104 (2010).
4. X.S. Chen, X.F. Lu, W.M. Sun, F. Wang, T. Goldman, PRL 100 (2007); 103 (2009).
5. M. Wakamatsu, PRD 81 (2010); E. Leader, hep-th/1101.5956; Y. Hatta, hep-ph/1101.5989.
6. K. Kondo et al. PLB 632 (2006); 645 (2007); 669 (2008).
7. S. van Enk and G. Nienhuis, PRA 76 (2007); M. Andersen et al. PRL 97 (2006); A. Alexandrescu et al. PRL 96 (2006).
8. Y.M. Cho, M.L. Ge, P. Zhang, nucl-th/1010.1080; 1102.1130.
9. Y.M. Cho hep-th/1206.6936.