From nucleon structure to nuclear structure and compact astrophysical objects: part 2 (QCD and hadron physics)

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Light-by-light scattering sum rules

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CRC1044

JG|U

Outline

- Introduction and motivation: $(g-2)_\mu$
- $\gamma\gamma$ sum rules: general derivation
- $\gamma\gamma$ sum rules in perturbative quantum field theory
- $\gamma\gamma$ sum rules for meson production
- Conclusions

V. Pascalutsa, M. Vdh : Phys. Rev. Lett. 105, 201603 (2010)
V. Pascalutsa, V. Pauk, M. Vdh : Phys. Rev. D 85, 116001 (2012)
V. Pauk, V. Pascalutsa, M. Vdh : publication (perturbative QFT) in preparation

magnetic moment of muon: $(g-2)_\mu$

- magnetic moment $\vec{m} = \mu_B \, g \, \vec{S}$

  μ_B: Bohr magneton
  g: gyromagnetic factor ~ 2

- anomalous part:

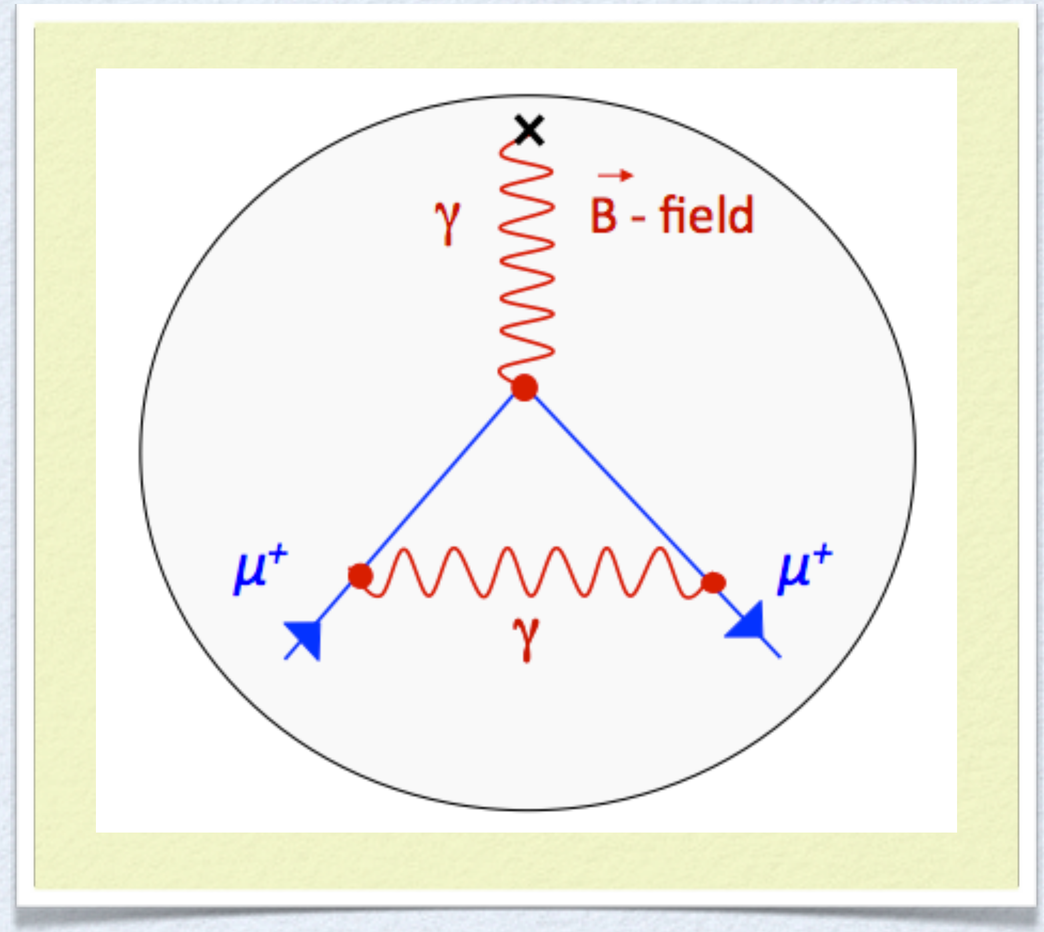
  $a_\mu = (g-2)_\mu/2 = \alpha_{em}/2\pi + \ldots = 0.00116\ldots$

- $a_\mu^{exp} - a_\mu^{th} = (28.7 \pm 8.0) \times 10^{-10}$

  3.6σ deviation from SM value !

- recent $O(\alpha_{em}^5)$ QED calculation

  $a_\mu^{exp} - a_\mu^{th} = (24.9 \pm 8.7) \times 10^{-10}$ (2.9σ)



sensitive test of the standard model (SM) and of physics beyond SM

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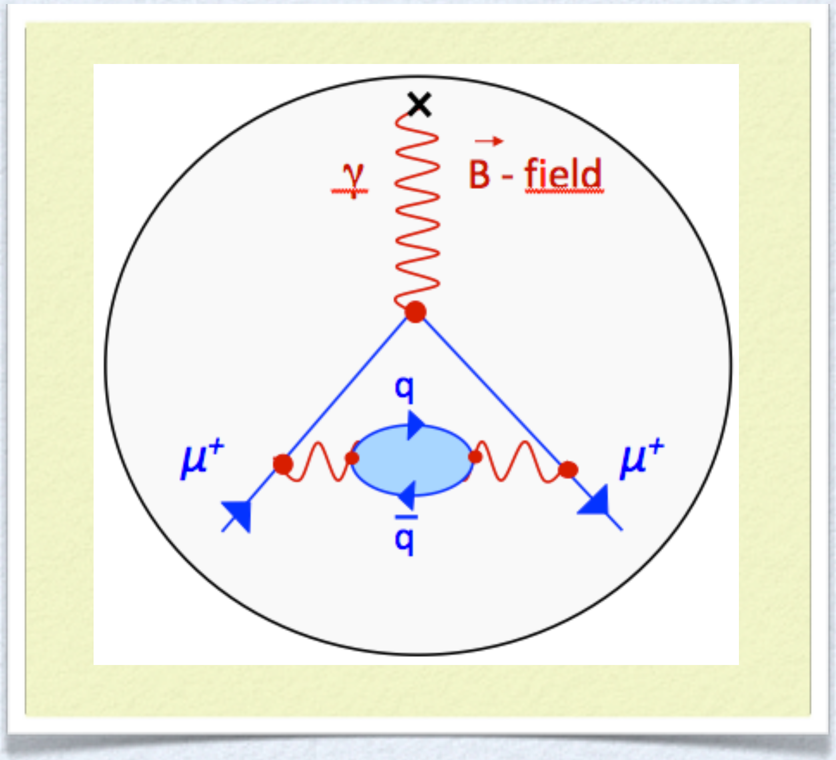
need experimental and theoretical program in hadron physics to reduce uncertainty in a_μ^{strong}

strong contributions to $(g-2)_\mu$

contributions from strong interactions NOT calculable within perturbative QCD

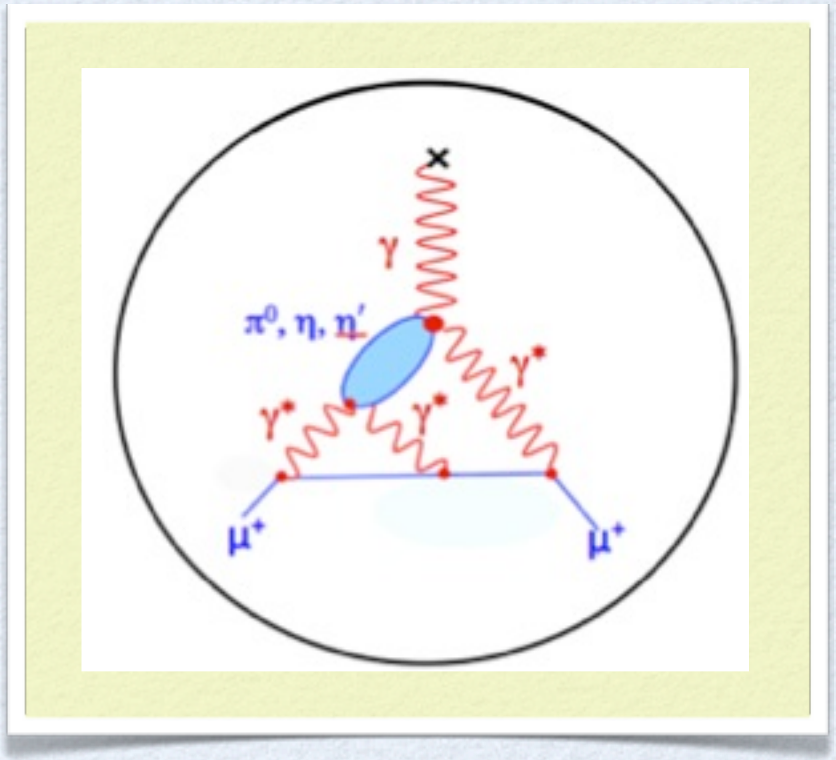
hadronic vacuum polarization



$a_\mu^{had, VP} = (692.3 \pm 4.2) \times 10^{-10}$

hadronic vacuum polarization determined by cross section measurements of e^+e^- -> hadrons

hadronic light-by-light scattering



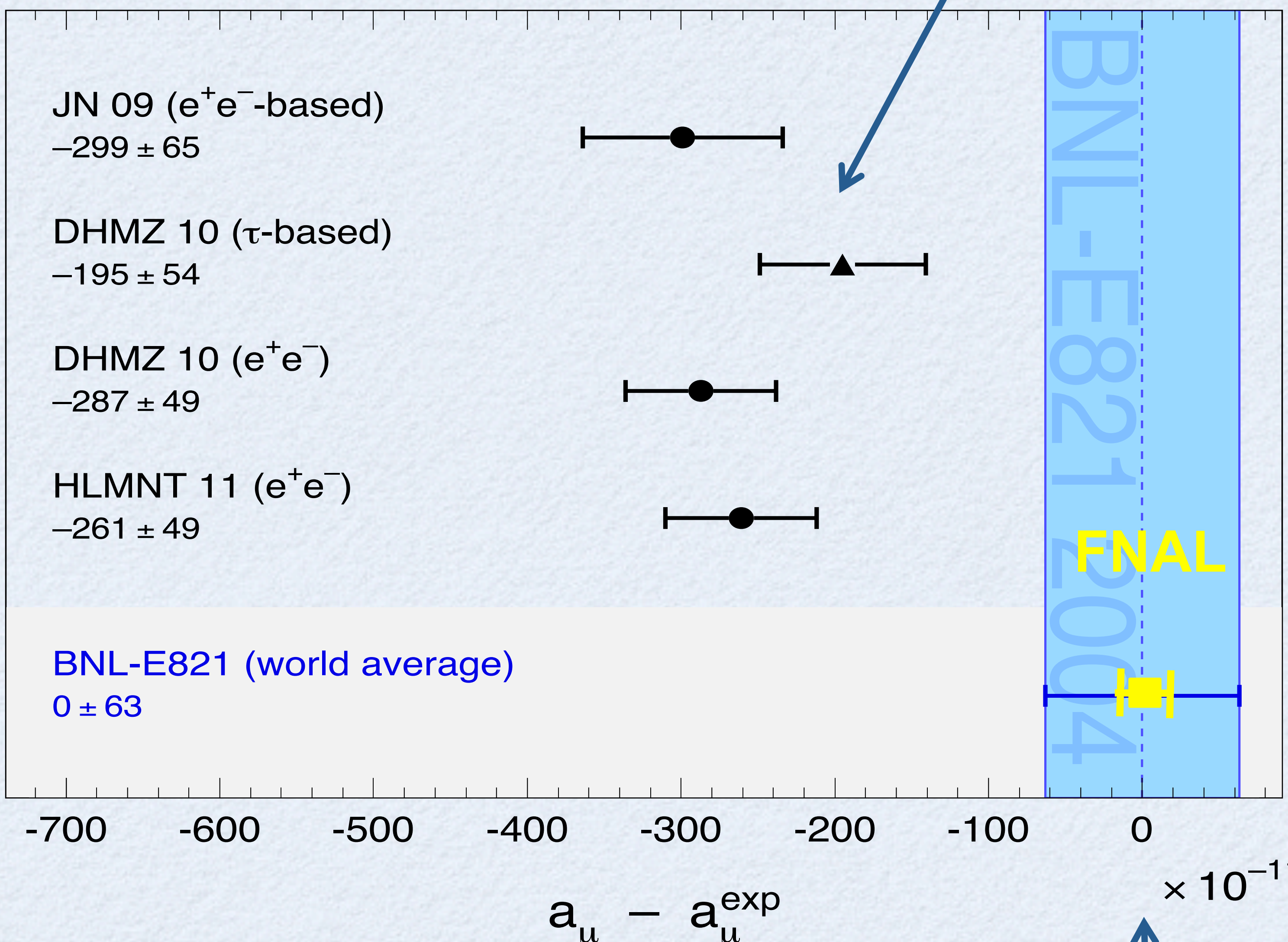
$a_\mu^{had, LbL} = (11.6 \pm 4.0) \times 10^{-10}$

Jegerlehner, Nyffeler (2009)

measurements of meson transition form factors required as input to reduce uncertainty

$(g-2)_\mu$: theory vs experiment

Standard Model predictions $(g-2)_\mu$



E821 measurement of $(g-2)_\mu$

$a_\mu^{exp} = (11\ 659\ 208.9 \pm 6.3) \cdot 10^{-10}$

$\Delta a_\mu = a_\mu^{exp} - a_\mu^{SM} =$

$(28.7 \pm 8.0) \cdot 10^{-10}$ (3.6 σ)

Error(s) or New Physics ?

→ Clarify situation !

New FNAL $(g-2)_\mu$ measurement (2015):

Factor 4 improvement in experimental error

→ Improve a_μ^{had} !

$(g-2)_\mu$: hadronic vacuum polarization

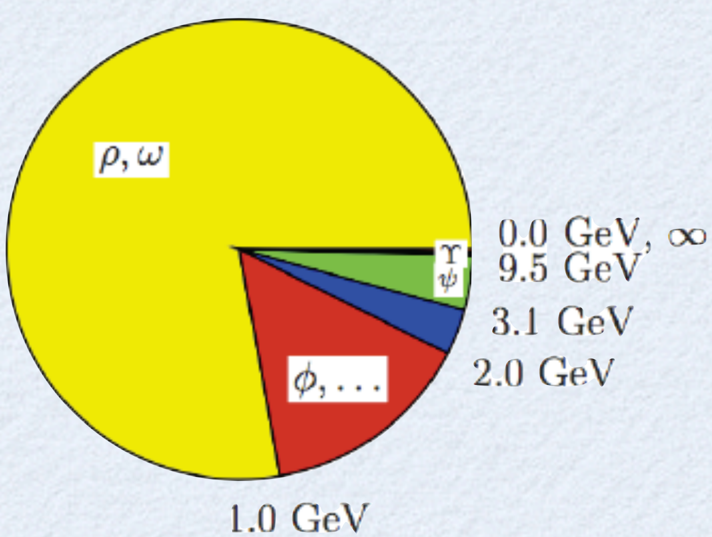
Optical theorem and analyticity allow to relate HVP contribution to $(g-2)_\mu$ with $\sigma_{had} = \sigma(e^+e^- \rightarrow hadrons)$

$$a_\mu^{had,VP} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^{\infty} ds\, K(s)\, \sigma_{had}$$

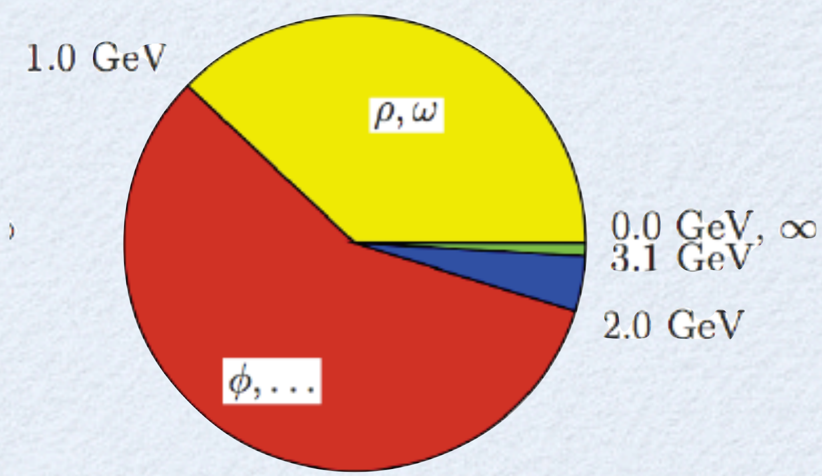
Kernel function → $K(s)$

Hadronic cross section → σ_{had}

a_μ^{had}

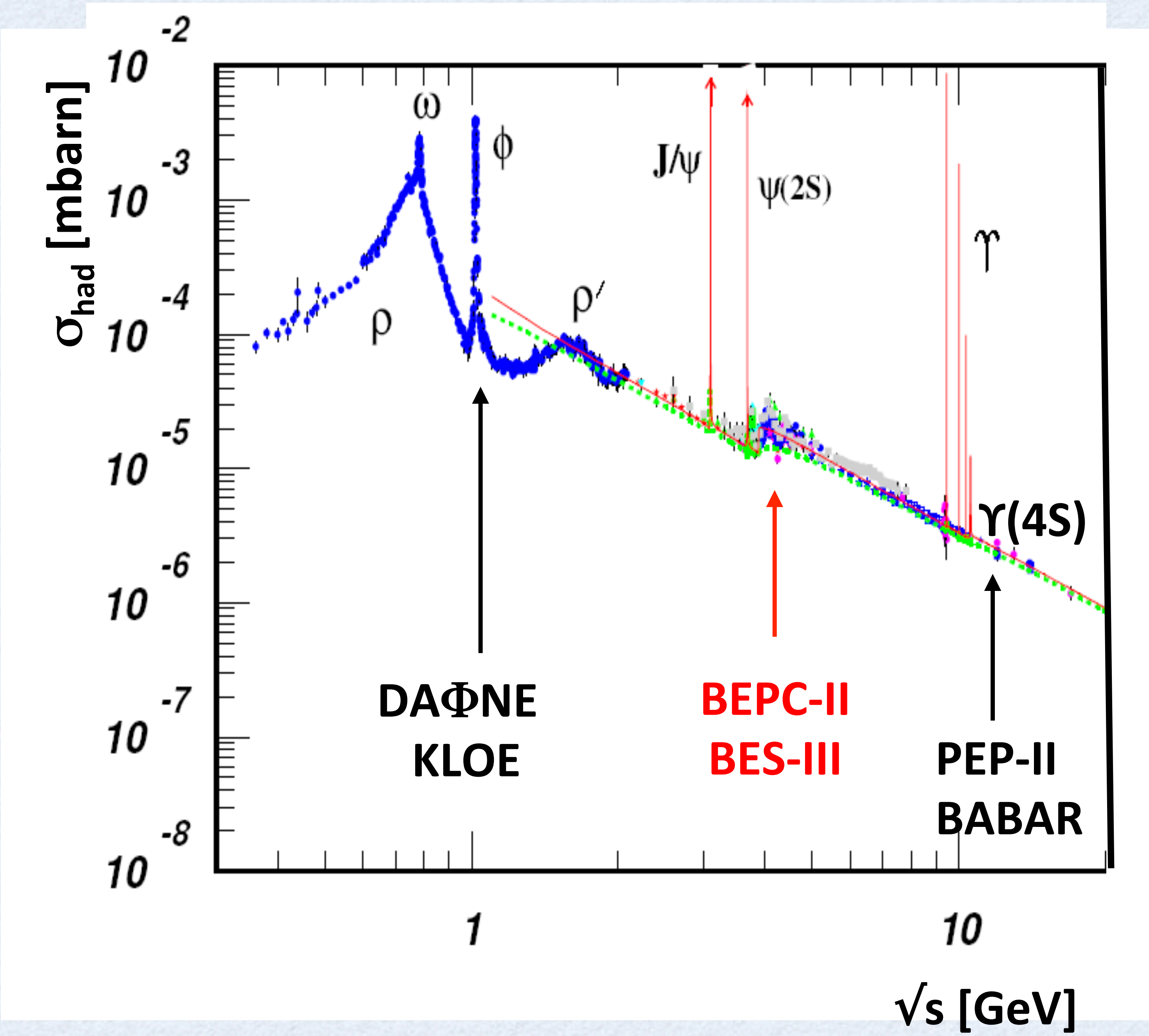


δa_μ^{had}



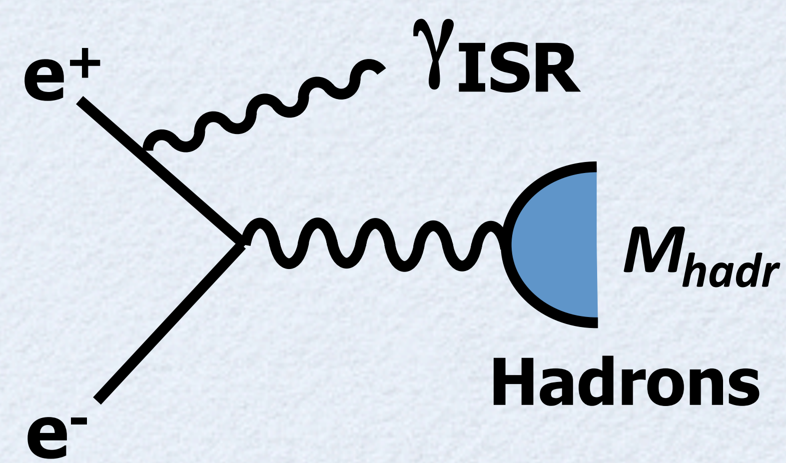
σ_{had} : energy range up to 3 GeV essential!

measure σ_{had} via ISR at BES-III



Approach for measuring hadronic cross section at modern particle factories with fixed c.m.s. energy √s:

Initial State Radiation (ISR)



ISR method allows access to mass range M_{hadr}< 3 GeV at BES-III

→ Continue ISR success story in CRC1044

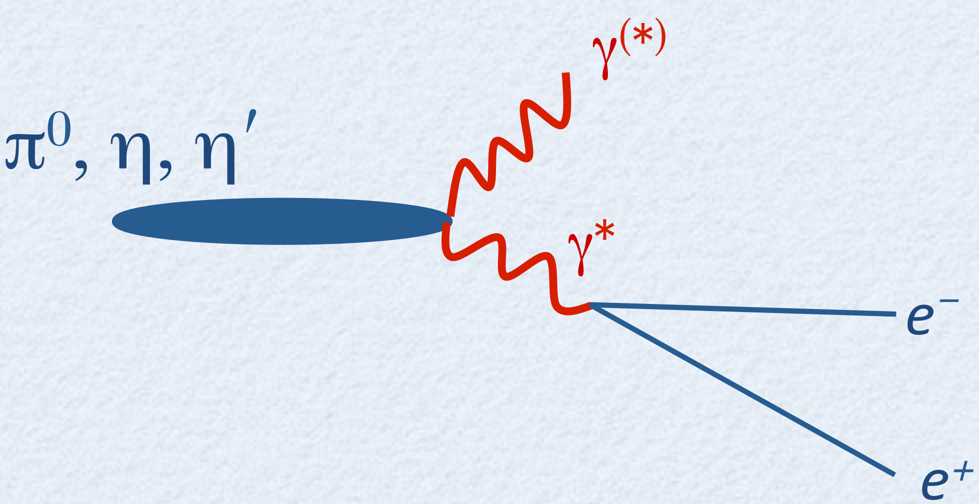
hadronic light-by-light scattering

- strong dependence of hadronic LbL estimate on hadronic models !

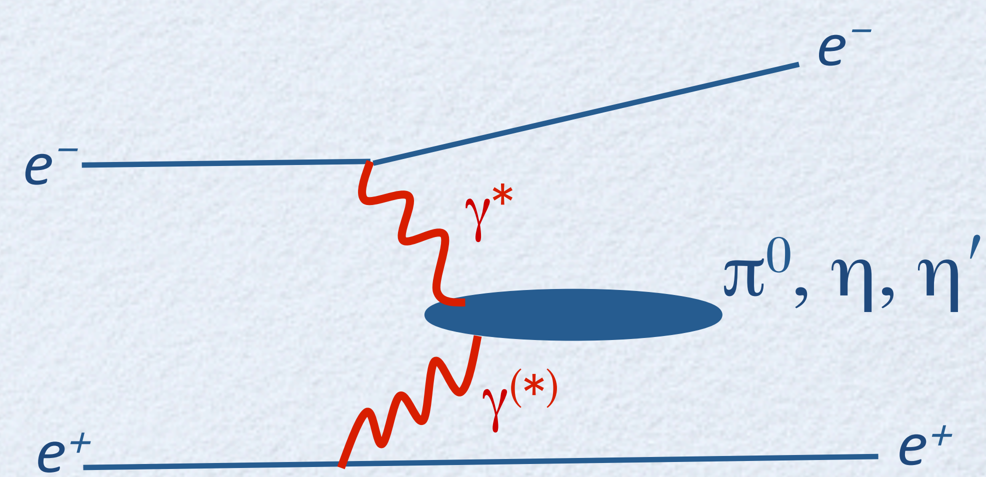
  $a_\mu^{had,\,LbL} = (11.6 \pm 4.0) \times 10^{-10}$ Jegerlehner, Nyffeler (2009)

- measurement of meson transition form factors
  validate hadronic models / most relevant range at low Q^2

timelike
(A2 @ MAMI)



spacelike
(γγ program @ BES-III)



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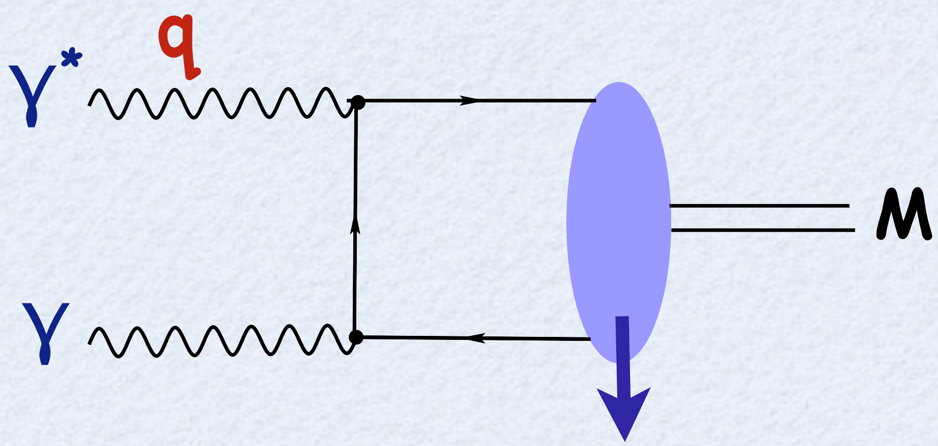
$\gamma\ \gamma$ physics: quark structure of mesons

spacelike

$q^2 = -Q^2 < 0$

timelike

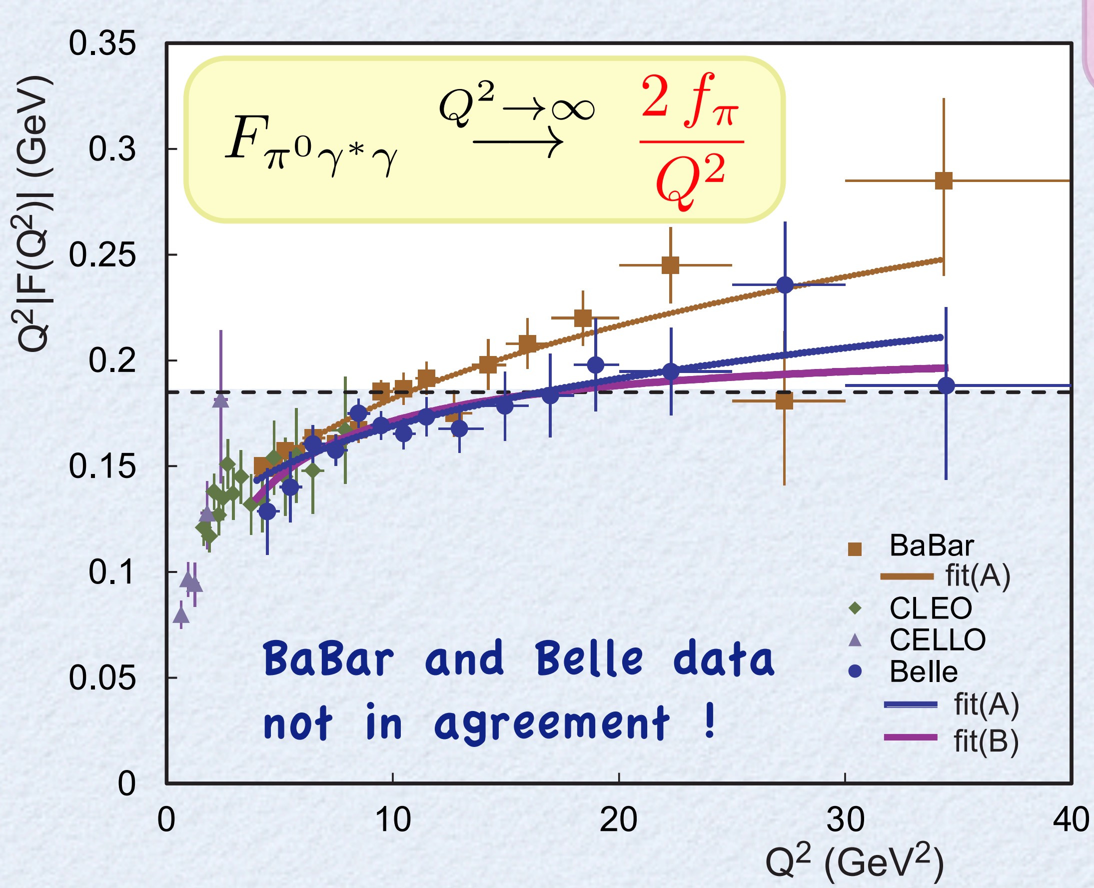
$q^2 > 0$

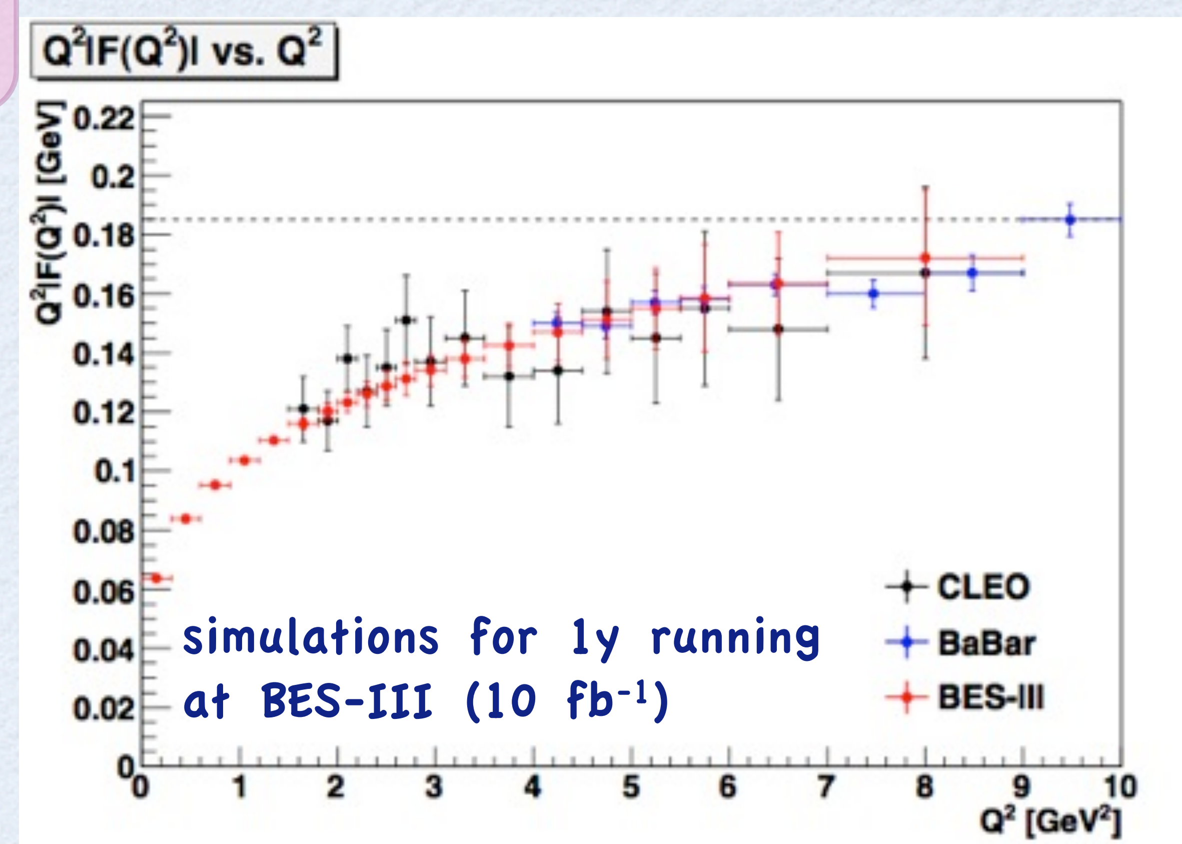


meson distribution amplitude

- paradigm for a whole program of hard processes processes planned at JLab12, Compass, EIC/ENC
- transition region to perturbative QCD remains to be understood

$\gamma^*\ \gamma \rightarrow \pi^0$



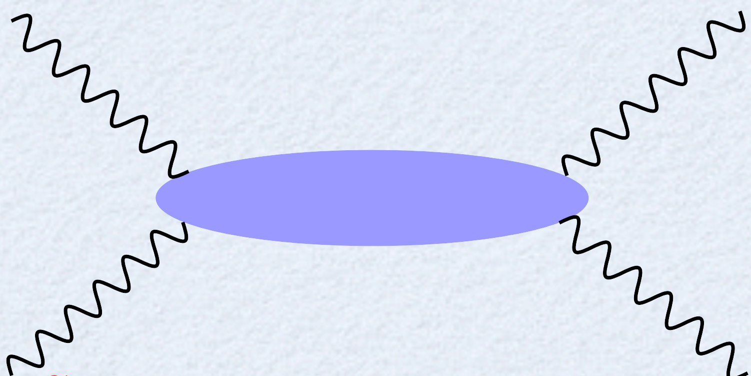


B. Kloss (2011)

$\gamma^* \gamma^*$ sum rules:

general derivation

$\gamma^* \gamma^* \rightarrow \gamma^* \gamma^*$ forward scattering



$$\gamma^*(\lambda_1, q_1) + \gamma^*(\lambda_2, q_2) \rightarrow \gamma^*(\lambda_1', q_1) + \gamma^*(\lambda_2', q_2)$$

kinematical invariants:

$$s = (q_1 + q_2)^2, \qquad u = (q_1 - q_2)^2$$

$$\nu \equiv \frac{s-u}{4}, \quad Q_1^2 \equiv -q_1^2, \quad Q_2^2 \equiv -q_2^2$$

helicity amplitudes: $M_{\lambda_1'\lambda_2',\lambda_1\lambda_2}(\nu, Q_1^2, Q_2^2)$ $\qquad \lambda = 0, \pm 1$

discrete symmetries:

$$P: \quad M_{\lambda_1'\lambda_2',\lambda_1\lambda_2} = M_{-\lambda_1'-\lambda_2',-\lambda_1-\lambda_2}$$

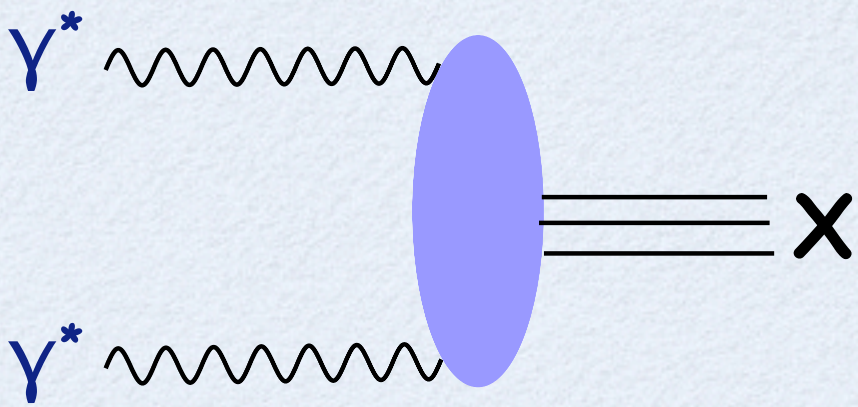
$$T: \quad M_{\lambda_1'\lambda_2',\lambda_1\lambda_2} = M_{\lambda_1\lambda_2,\lambda_1'\lambda_2'}$$

→ **8 independent amplitudes:**

$M_{++,++},\ M_{+-,+-},\ M_{++,--},$ **T**

$M_{00,00},\ M_{+0,+0},\ M_{0+,0+},\ M_{++,00},\ M_{0+,-0}$ **T and L**

$\gamma^* \gamma^* \to \gamma^* \gamma^*$ forward scattering



Unitarity (optical theorem): link to $\gamma^* \gamma^* \to X$ cross sections

$$W_{\lambda_1' \lambda_2', \lambda_1 \lambda_2} \equiv \mathrm{Im}\, M_{\lambda_1' \lambda_2', \lambda_1 \lambda_2}$$

$$W_{\lambda_1' \lambda_2', \lambda_1 \lambda_2} = \frac{1}{2} \int d\Gamma_{\mathrm{X}} (2\pi)^4 \delta^4(q_1 + q_2 - p_{\mathrm{X}})\, \mathcal{M}_{\lambda_1 \lambda_2}(q_1, q_2; p_{\mathrm{X}})\, \mathcal{M}^*_{\lambda_1' \lambda_2'}(q_1, q_2; p_{\mathrm{X}})$$

can be expressed in terms of $\gamma^* \gamma^* \to X$ cross sections, which can be measured

$$W_{++,++} + W_{+-,+-} \equiv 2\sqrt{X}\ (\sigma_0 + \sigma_2) = 2\sqrt{X}\ (\sigma_\parallel + \sigma_\perp) \equiv 4\sqrt{X}\, \sigma_{TT},$$

$$W_{++,++} - W_{+-,+-} \equiv 2\sqrt{X}\ (\sigma_0 - \sigma_2) \equiv 4\sqrt{X}\, \tau^a_{TT},$$

$$W_{++,--} \equiv 2\sqrt{X}\ (\sigma_\parallel - \sigma_\perp) \equiv 2\sqrt{X}\, \tau_{TT},$$

$$W_{00,00} \equiv 2\sqrt{X}\, \sigma_{LL},$$

$$W_{+0,+0} \equiv 2\sqrt{X}\, \sigma_{TL},$$

$$W_{0+,0+} \equiv 2\sqrt{X}\, \sigma_{LT},$$

$$W_{++,00} + W_{0+,-0} \equiv 4\sqrt{X}\, \tau_{TL},$$

$$W_{++,00} - W_{0+,-0} \equiv 4\sqrt{X}\, \tau^a_{TL}.$$

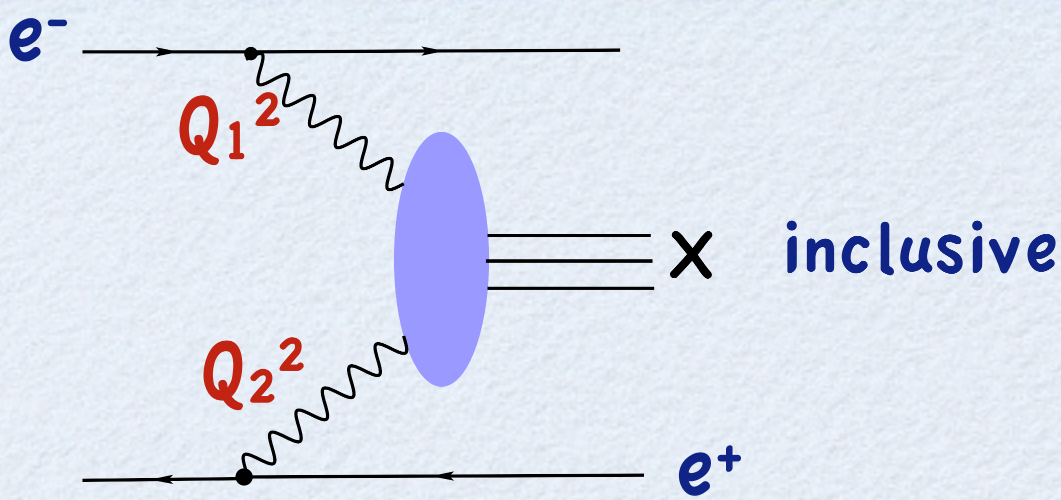
virtual photon flux:

$$X \equiv \nu^2 - Q_1^2 Q_2^2$$

$ee \longrightarrow eeX$ cross sections

$$e(p_1) + e(p_2) \to e(p_1') + e(p_2') + X$$

$$s = (p_1 + p_2)^2$$



virtual photons

$$q_1 \equiv p_1 - p_1', \qquad q_2 \equiv p_2 - p_2'$$

$$Q_1^2 \equiv -q_1^2, \qquad Q_2^2 \equiv -q_2^2$$

$$d\sigma = \frac{\alpha^2}{16\pi^4\, Q_1^2\, Q_2^2}\, \frac{2\sqrt{X}}{s(1-4m^2/s)} \cdot \frac{d^3\vec{p}_1'}{E_1'} \cdot \frac{d^3\vec{p}_2'}{E_2'}$$
$$\times \{4\,\rho_1^{++}\,\rho_2^{++}\,\sigma_{TT} + \rho_1^{00}\,\rho_2^{00}\,\sigma_{LL} + 2\,\rho_1^{++}\,\rho_2^{00}\,\sigma_{TL} + 2\,\rho_1^{00}\,\rho_2^{++}\,\sigma_{LT}$$
$$+2\,(\rho_1^{++}-1)\,(\rho_2^{++}-1)\,\left(\cos 2\tilde{\phi}\right)\,\tau_{TT} + 8\left[\frac{(\rho_1^{00}+1)\,(\rho_2^{00}+1)}{(\rho_1^{++}-1)\,(\rho_2^{++}-1)}\right]^{1/2}\left(\cos\tilde{\phi}\right)\,\tau_{TL}$$
$$+h_1 h_2\, 4\left[(\rho_1^{00}+1)\,(\rho_2^{00}+1)\right]^{1/2}\tau_{TT}^a + h_1 h_2\, 8\left[(\rho_1^{++}-1)\,(\rho_2^{++}-1)\right]^{1/2}\left(\cos\tilde{\phi}\right)\,\tau_{TL}^a\}$$

lepton beam polarization

$\rho_1^{++}, \rho_2^{++}, \rho_1^{00}, \rho_2^{00}$: **kinematical coefficients**

$\tilde{\phi}$: **angle between both lepton planes in $\gamma^*\,\gamma^*$ c.m. frame**

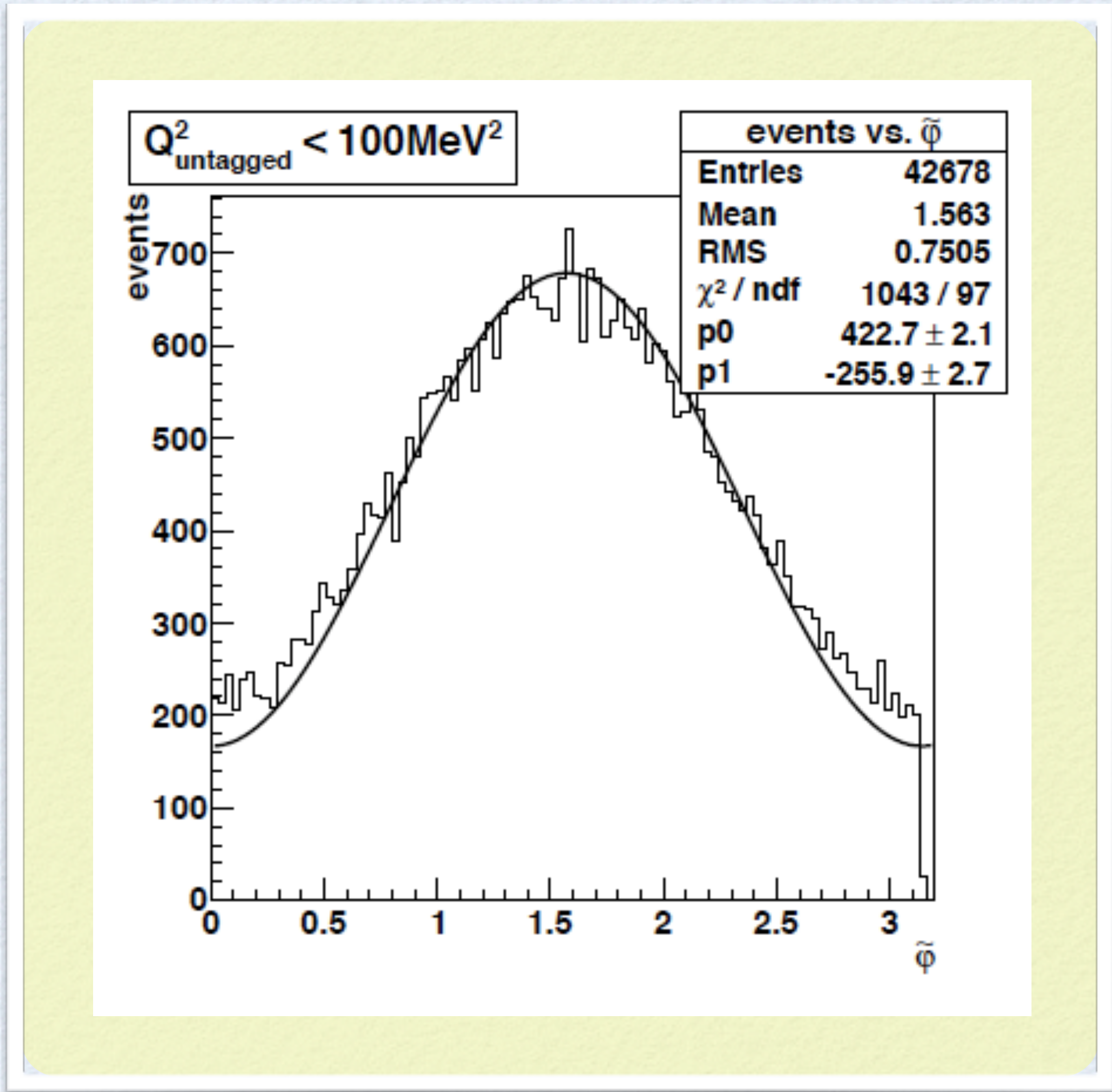
$ee \longrightarrow eeX$ cross sections

extraction of cos(2φ) moment

$$\tau_{TT} = \sigma_{\parallel} - \sigma_{\perp}$$

possible at BES-III

- has never been extracted at an e^+e^- collider
- through SR: measures forward LbL scattering coefficient $c_1 - c_2$



simulations for 1y running at BES-III (10 fb^{-1})

B. Kloss (2011)

$\gamma^* \gamma^* \to \gamma^* \gamma^*$ forward scattering

Causality,
photon crossing symmetry ($\nu \to -\nu$)

➡ dispersion relations

$$f_{even}(\nu) = \frac{2}{\pi} \int_{\nu_0}^{\infty} d\nu' \frac{\nu'}{\nu'^2 - \nu^2 - i0^+} \mathrm{Im}\, f_{even}(\nu')$$

$$f_{odd}(\nu) = \frac{2\nu}{\pi} \int_{\nu_0}^{\infty} d\nu' \frac{1}{\nu'^2 - \nu^2 - i0^+} \mathrm{Im}\, f_{odd}(\nu')$$

➡ high-energy behavior: may require some subtractions

➡ Low-energy expansion: for real parts at $\nu = 0$

- $\mathcal{L}^{(8)} = c_1 (F_{\mu\nu} F^{\mu\nu})^2 + c_2 (F_{\mu\nu} \tilde{F}^{\mu\nu})^2$ Euler, Heisenberg (1936)

  c_1 and c_2 describe low-energy (at order ν^2) LbL scattering

- at next order: 6 new constants enter Pauk, Pascalutsa, Vdh (2012)

sum rules for $\gamma^* \gamma^*$ scattering

- 3 super-convergence relations

helicity difference sum rule

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)} [\sigma_0 - \sigma_2]_{Q_2^2=0}$$

for $Q^2 = 0$
cfr. GDH SR
Gerasimov, Moulin (1975)
Brodsky, Schmidt (1995)

sum rules involving longitudinal photons

$$0 = \int_{s_0}^{\infty} ds \left[\frac{\tau_{TL}}{Q_1 Q_2}\right]_{Q_2^2=0}$$

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)^2} \left[\sigma_{\parallel} + \sigma_{LT} + \frac{(s+Q_1^2)}{Q_1 Q_2} \tau_{TL}^a\right]_{Q_2^2=0}$$

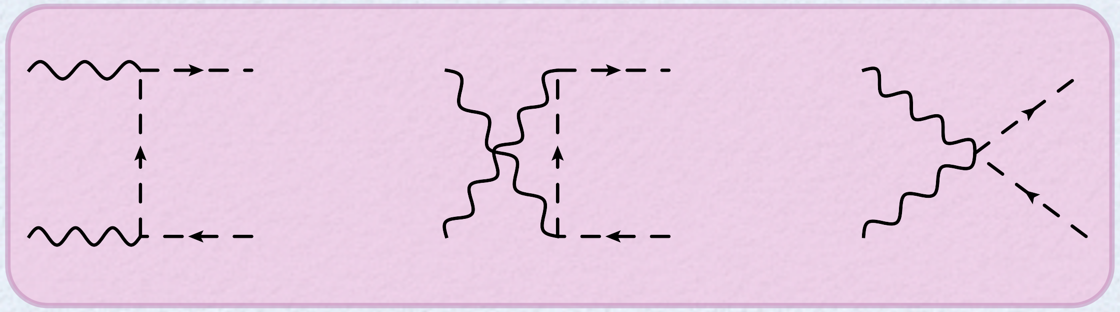
- SRs involving LbL low-energy coefficients

$$c_1 \pm c_2 = \frac{1}{8\pi} \int_{s_0}^{\infty} ds \frac{[\sigma_{\parallel} \pm \sigma_{\perp}](s)}{s^2}$$

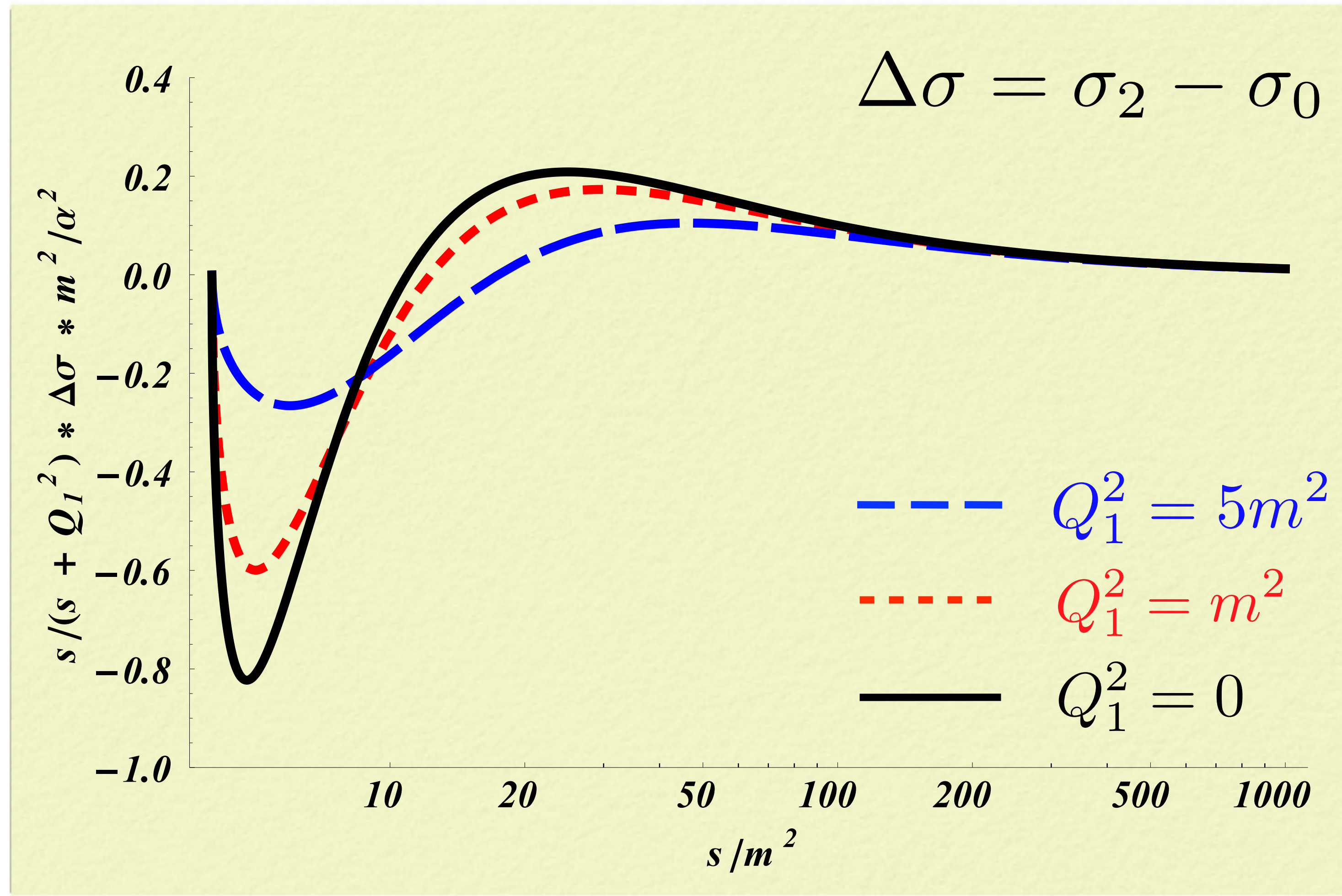
V. Pauk, V. Pascalutsa, M. Vdh (2010, 2012)

$\gamma^* \gamma^*$ sum rules in perturbative quantum field theory

sum rules in scalar QED: tree level



$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)} \left[\sigma_0 - \sigma_2\right]_{Q_2^2=0}$$



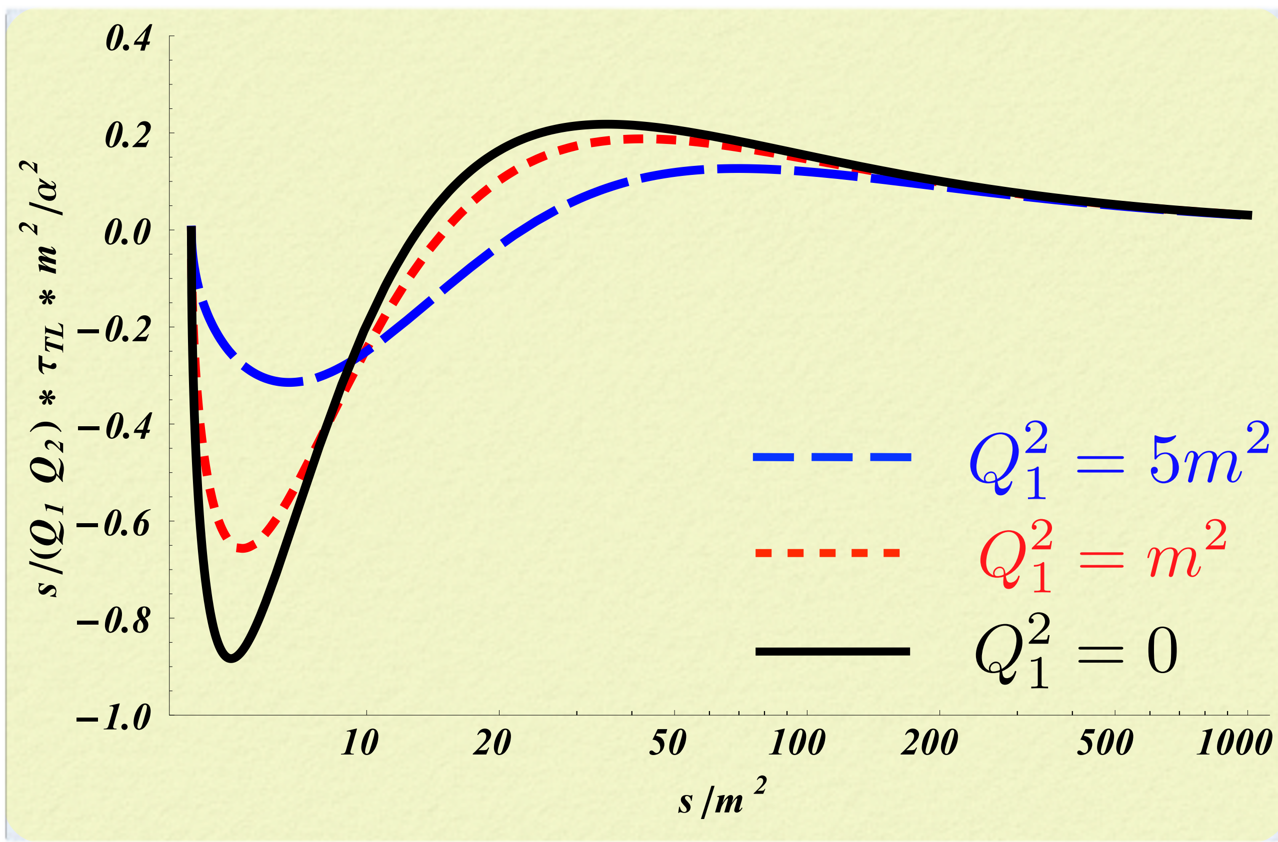
$$\left[\sigma_0 - \sigma_2\right]_{Q_2^2=0} = \alpha^2 4\pi \frac{s}{(s+Q_1^2)^2} \left\{ -\sqrt{1-\frac{4m^2}{s}}\left(1-\frac{Q_1^2}{s}\right) + \frac{8m^2}{s} \ln\left(\frac{\sqrt{s}}{2m}\left[1+\sqrt{1-\frac{4m^2}{s}}\right]\right)\right\}$$

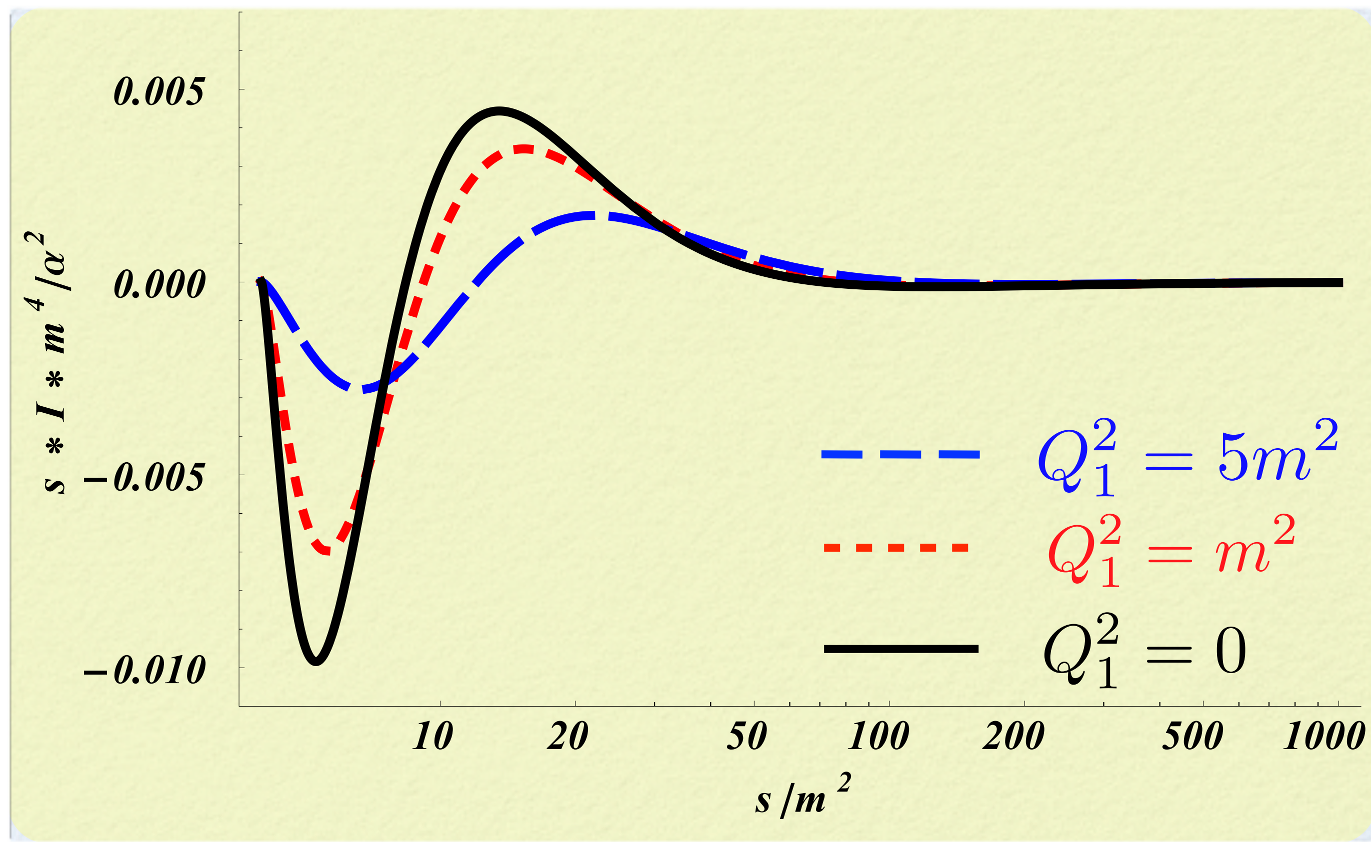
V. Pascalutsa, M. Vdh (2010)

sum rules in scalar QED: tree level

$$0 = \int_{s_0}^{\infty} ds \left[\frac{\tau_{TL}}{Q_1 Q_2} \right]_{Q_2^2=0}$$

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)^2} \left[\sigma_{\parallel} + \sigma_{LT} + \frac{(s+Q_1^2)}{Q_1 Q_2} \tau_{TL}^a \right]_{Q_2^2=0}$$

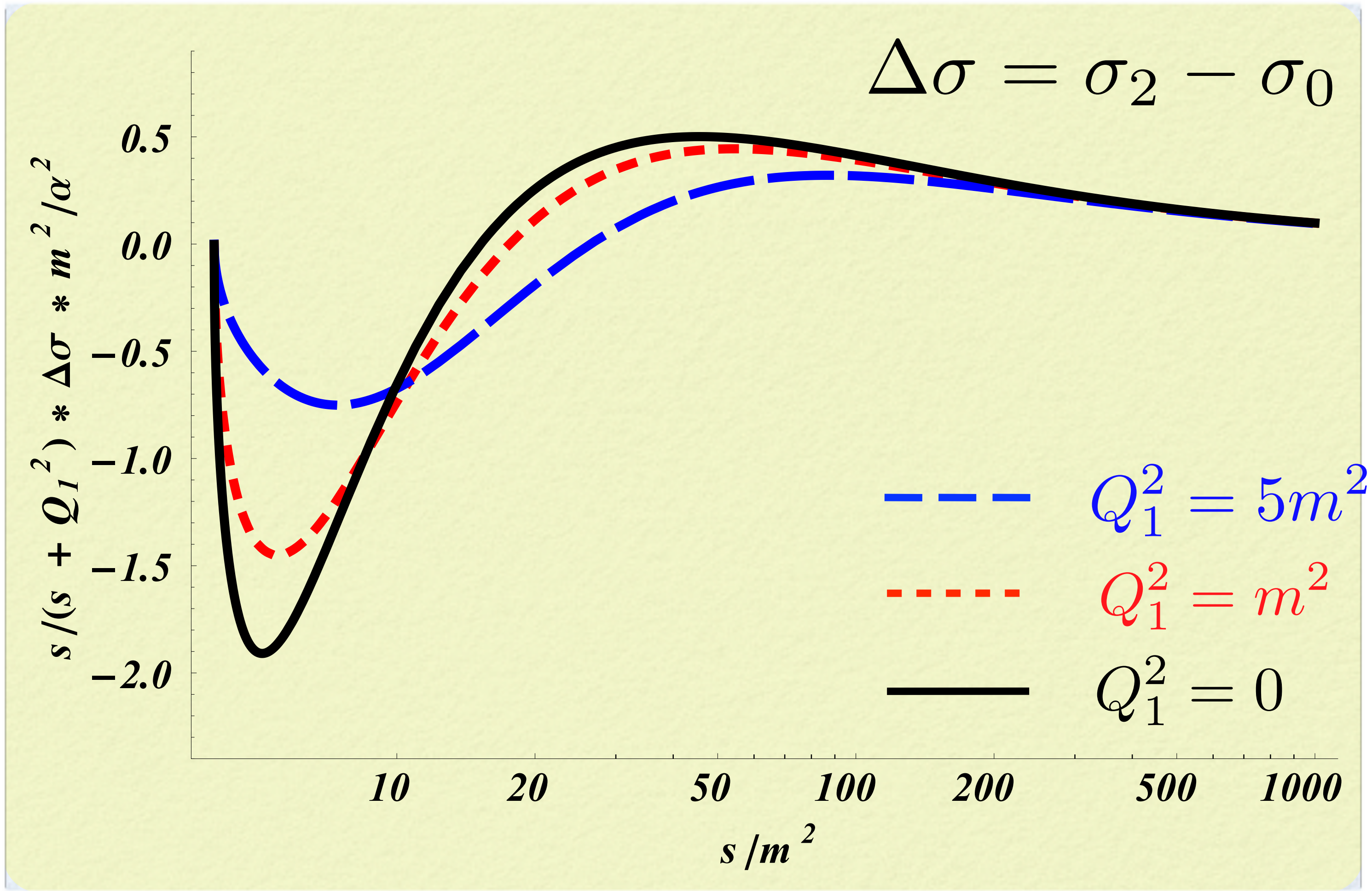




V. Pauk, V. Pascalutsa, M. Vdh (2012)

sum rules in spinor QED: tree level

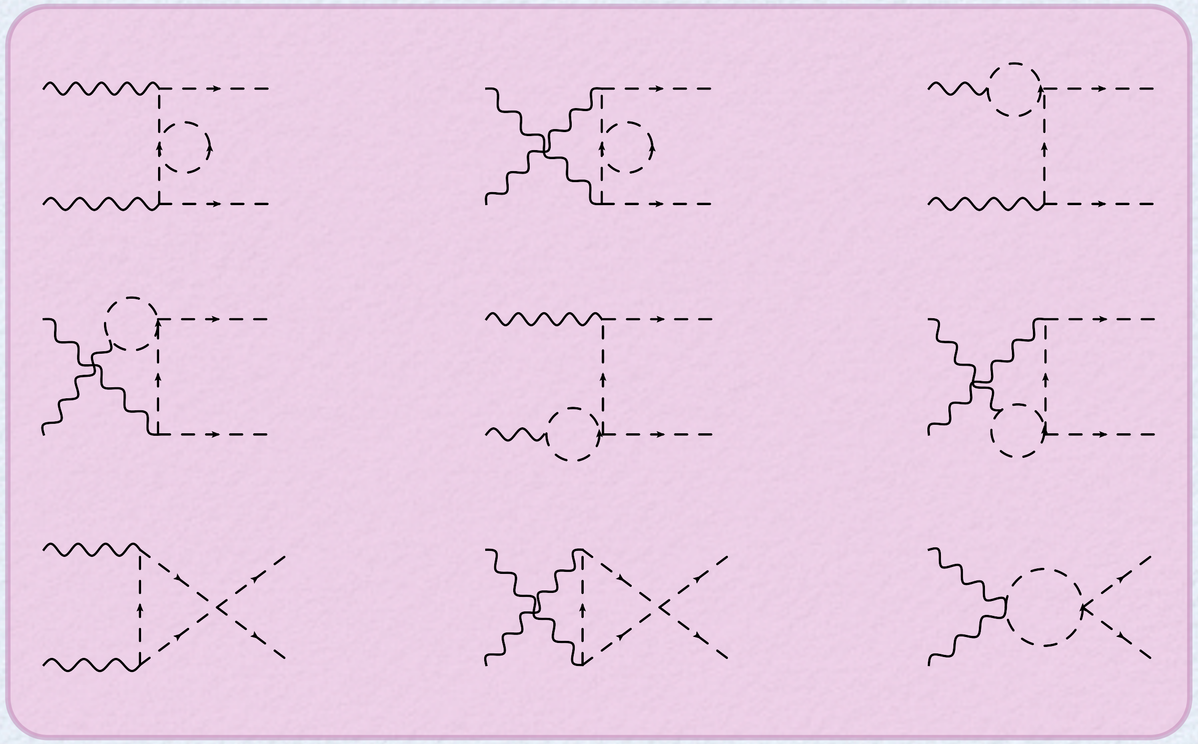
$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)} \left[\sigma_0 - \sigma_2\right]_{Q_2^2=0}$$

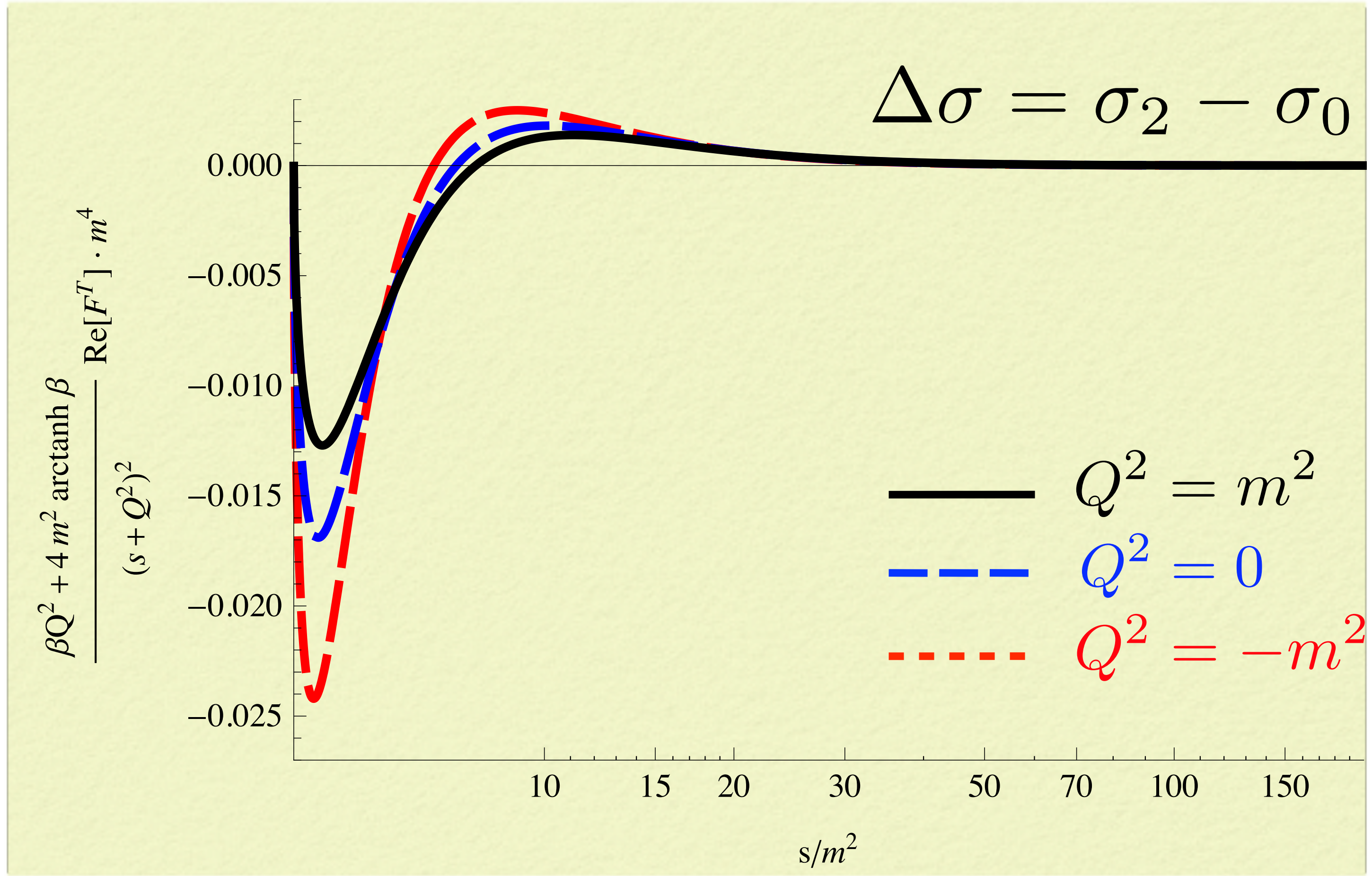


$$\left[\sigma_0 - \sigma_2\right]_{Q_2^2=0} = \alpha^2 8\pi \frac{s}{(s+Q_1^2)^2} \left\{ \sqrt{1-\frac{4m^2}{s}} \left(3 - \frac{Q_1^2}{s}\right) - 2\left(1-\frac{Q_1^2}{s}\right) \ln\left(\frac{\sqrt{s}}{2m}\left[1+\sqrt{1-\frac{4m^2}{s}}\right]\right) \right\}$$

sum rules in ϕ^4 theory : loop level

$$\mathcal{L}_I = -\frac{\lambda}{4}(\phi^\dagger\phi)^2$$ **interaction**





helicity difference sum rule at order λ

$$0 = \int_{4m^2}^{\infty} ds \frac{1}{(s+Q^2)^2} \underbrace{\left(\beta Q^2 + 4m^2 \operatorname{arctanh}\beta\right)}_{\text{tree level}} \underbrace{\operatorname{Re} F^T(\nu, Q^2, 0)}_{\text{1-loop}}$$

$$F^T(s,Q^2,0) = \frac{1}{(2\pi)^2}\frac{1}{(s+Q^2)^3}\left(s + Q^2 + 4m^2 \operatorname{arctanh}^2\frac{1}{\beta} + 2Q^2 \operatorname{arctanh}\frac{1}{\beta} - m^2 \ln^2\frac{\gamma+1}{\gamma-1} - Q^2\gamma\ln\frac{\gamma+1}{\gamma-1}\right)$$

$$\beta = \sqrt{1 - \frac{4m^2}{s}}$$

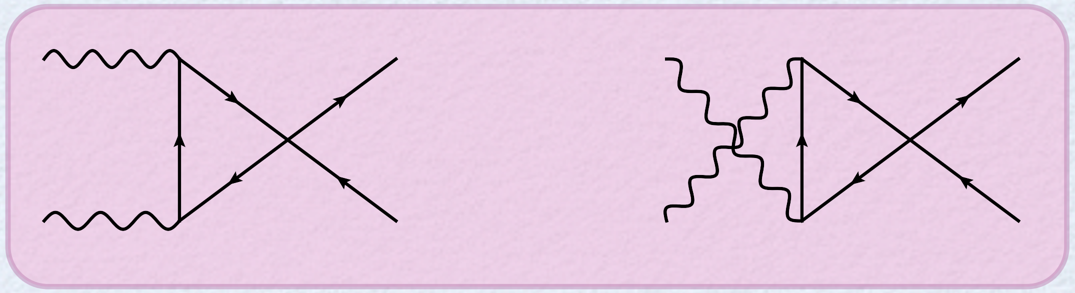
$$\gamma = \sqrt{1 + \frac{4m^2}{Q^2}}$$

sum rules in perturbative QFT : summary

sum rules for $\gamma^* \gamma^*$ forward scattering:

- verified at tree level for scalar QED, spinor QED / QCD
- verfied at one-loop level in Φ^4 theory
- verified at one-loop level in spinor QED with NJL-type interaction (scalar and pseudo-scalar 4-fermion interaction)

$$\mathcal{L}_I = \frac{G_S}{2}(\bar{\psi}\psi)^2 + \frac{G_P}{2}(\bar{\psi}i\gamma_5\psi)^2$$

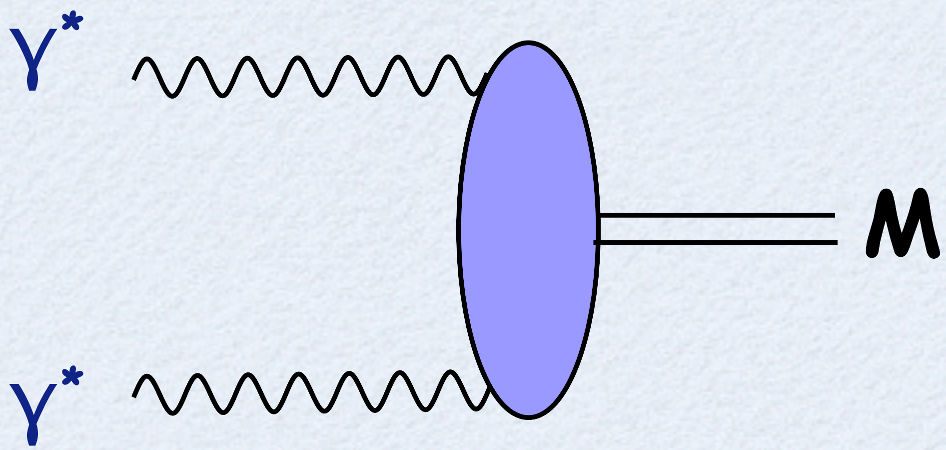


- resummation proof to all orders given within Φ^4 theory

V. Pauk et al.

$\gamma^* \gamma^*$ sum rules for meson production

meson production in $\gamma\gamma$ collisions



with J^{PC}

- two-photon state: produced meson M has $C = +1$
- for production by 2 real photons $\gamma\,\gamma \rightarrow M$:

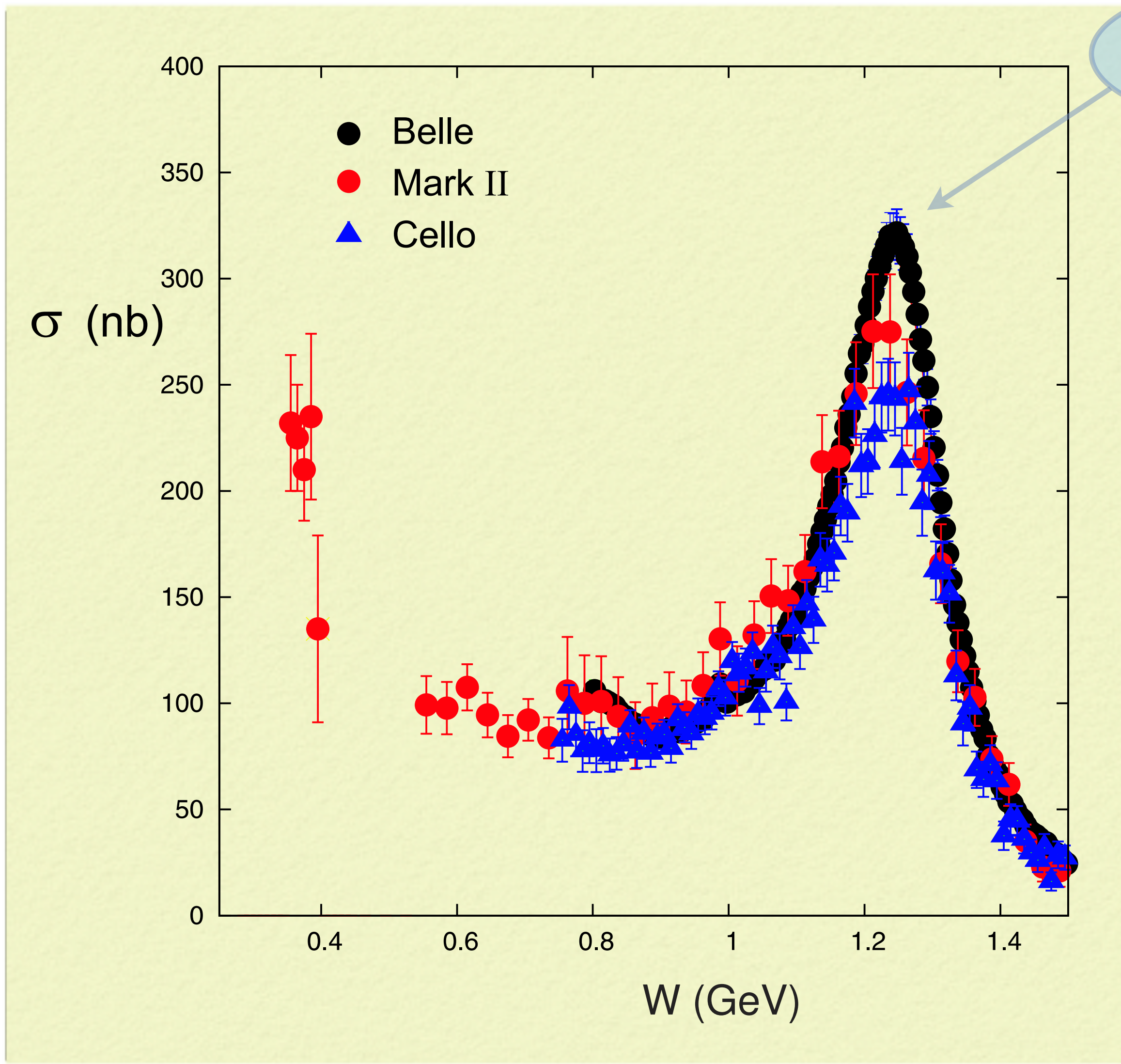
  $J = 1$ is forbidden (Landau-Yang theorem)

  predominantly $J = 0$: 0^{-+} (pseudo-scalar) and 0^{++} (scalar)

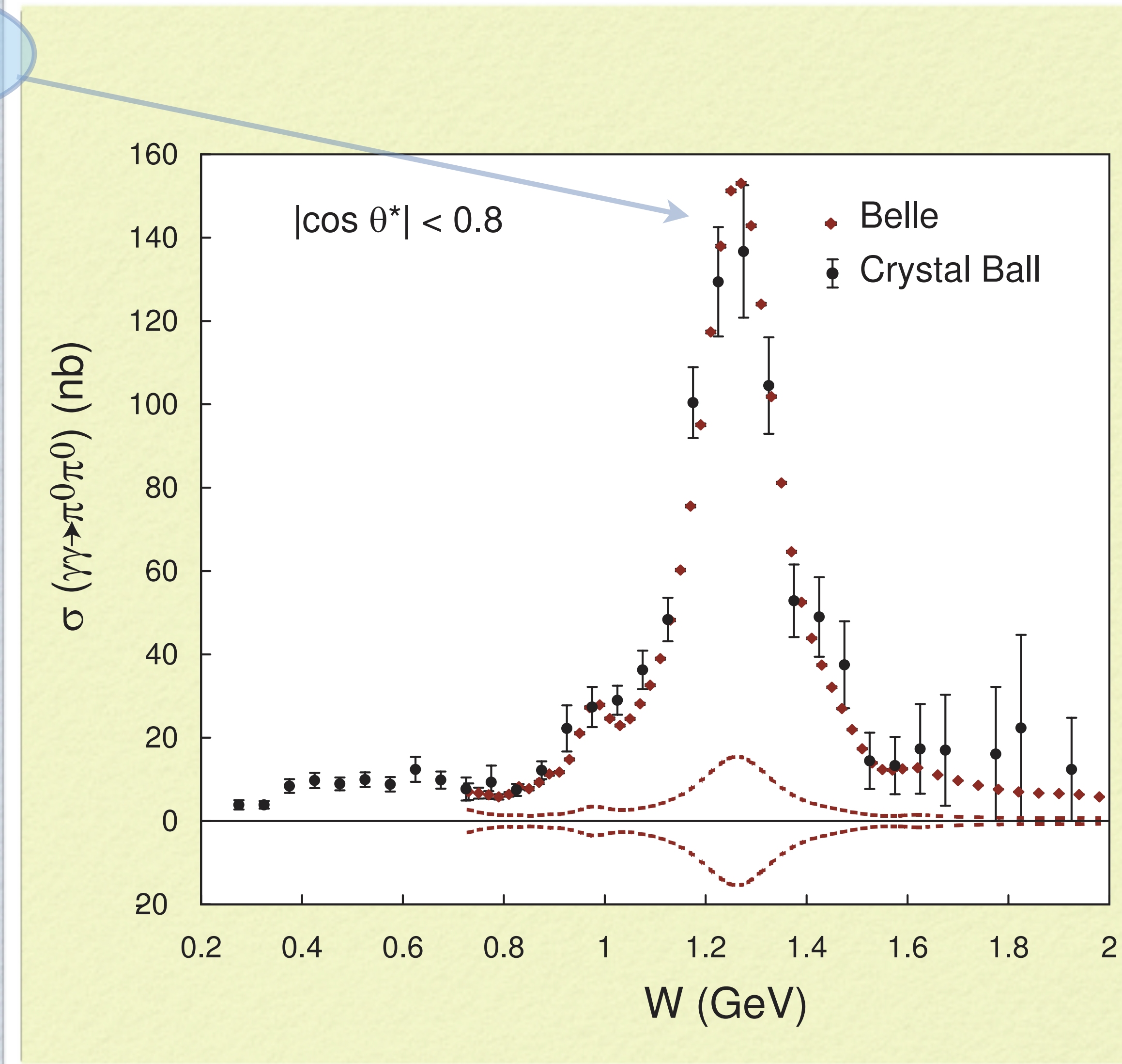
  or $J = 2$: 2^{++} (tensor)

meson production in γγ collisions: I = 0,2

$\gamma\,\gamma \rightarrow \pi^+\,\pi^-$ $\gamma\,\gamma \rightarrow \pi^0\,\pi^0$



$f_2(1270)$



amplitude analysis: $f_2(1270)$ produced predominantly in helicity-2 state

M. Pennington et al. (2008)

γγ → ππ production accesses I = 0 and I = 2 states

meson production in γγ collisions: I = 0

sum rules evaluated in I = 0 state

$$0 = \int_{s_0}^{\infty} ds \, \frac{[\sigma_2 - \sigma_0](s)}{s}$$

$$c_1 \pm c_2 = \frac{1}{8\pi} \int_{s_0}^{\infty} ds \, \frac{[\sigma_\parallel \pm \sigma_\perp](s)}{s^2}$$

		m_M [MeV]	$\Gamma_{\gamma\gamma}$ [keV]	$\int \frac{ds}{s}\,(\sigma_2 - \sigma_0)$ [nb]	c_1 [$10^{-4}\mathrm{GeV}^{-4}$]	c_2 [$10^{-4}\mathrm{GeV}^{-4}$]
0^{-+}	η	547.853 ± 0.024	0.510 ± 0.026	-191 ± 10	0	0.65 ± 0.03
	η'	957.78 ± 0.06	4.29 ± 0.14	-300 ± 10	0	0.33 ± 0.01
0^{++}	$f_0(980)$	980 ± 10	0.29 ± 0.07	-19 ± 5	0.020 ± 0.005	0
	$f_0'(1370)$	$1200 - 1500$	3.8 ± 1.5	-91 ± 36	0.049 ± 0.019	0
2^{++}	$f_2(1270)$	1275.1 ± 1.2	3.03 ± 0.35	449 ± 52	0.141 ± 0.016	0.141 ± 0.016
	$f_2'(1525)$	1525 ± 5	0.081 ± 0.009	7 ± 1	0.002 ± 0.000	0.002 ± 0.000
	$f_2(1565)$	1562 ± 13	0.70 ± 0.14	56 ± 11	0.012 ± 0.002	0.012 ± 0.002
	Sum			-89 ± 66	0.22 ± 0.03	1.14 ± 0.04

helicity difference SR: η, η' contributions entirely compensated by $f_2(1270)$, $f_2(1565)$

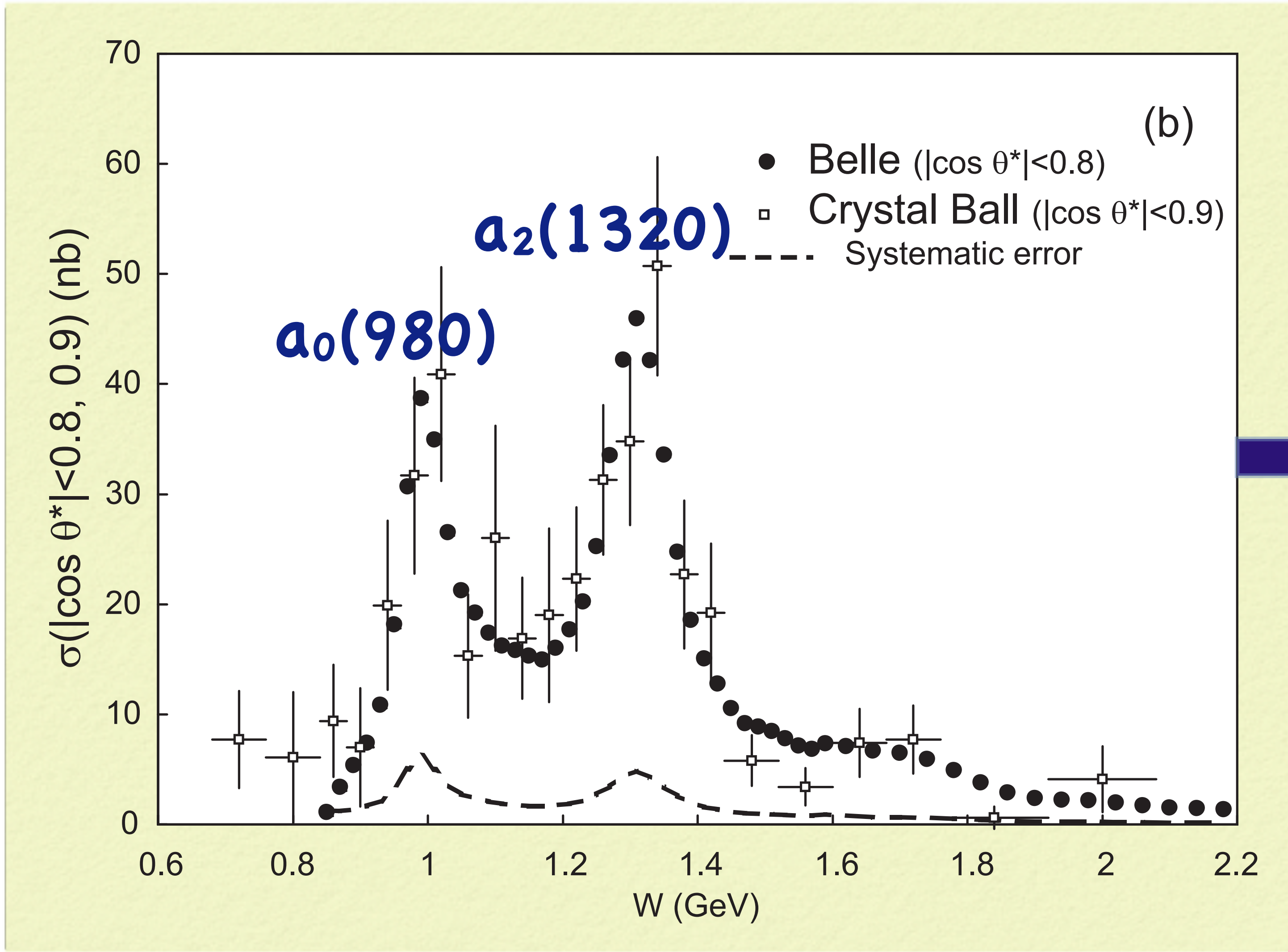
Note: $f_0(600)$ contribution requires to go beyond narrow resonance estimate

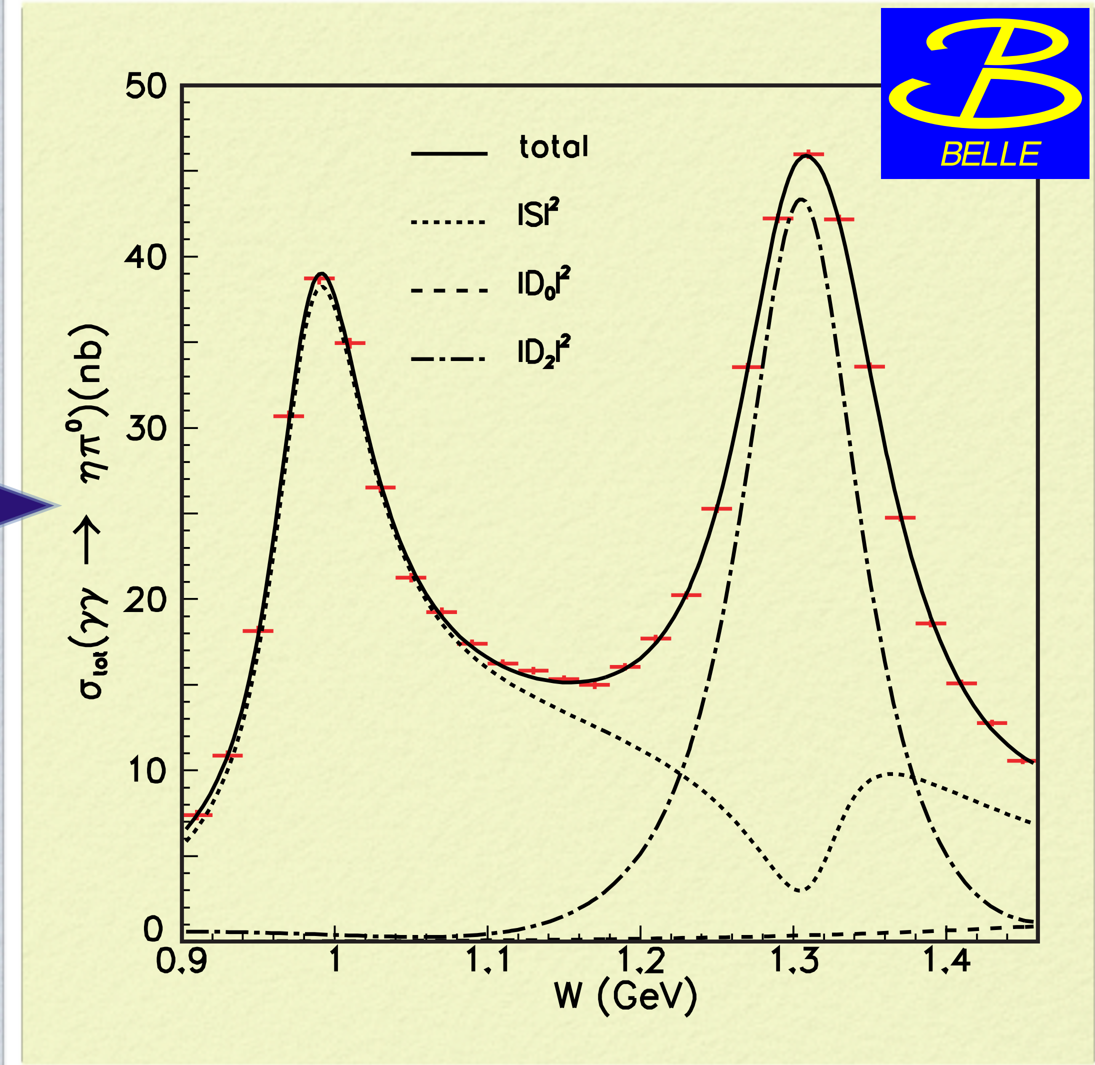
meson production in γγ collisions: I = 1

$\gamma\,\gamma \to \pi^0\,\eta$

amplitude analysis

S. Uehara et al. (2009)





a₂(1320) produced predominantly in helicity-2 state

meson production in γγ collisions: I = 1

sum rules evaluated in I = 1 state

$$0 = \int_{s_0}^{\infty} ds \, \frac{[\sigma_2 - \sigma_0](s)}{s}$$

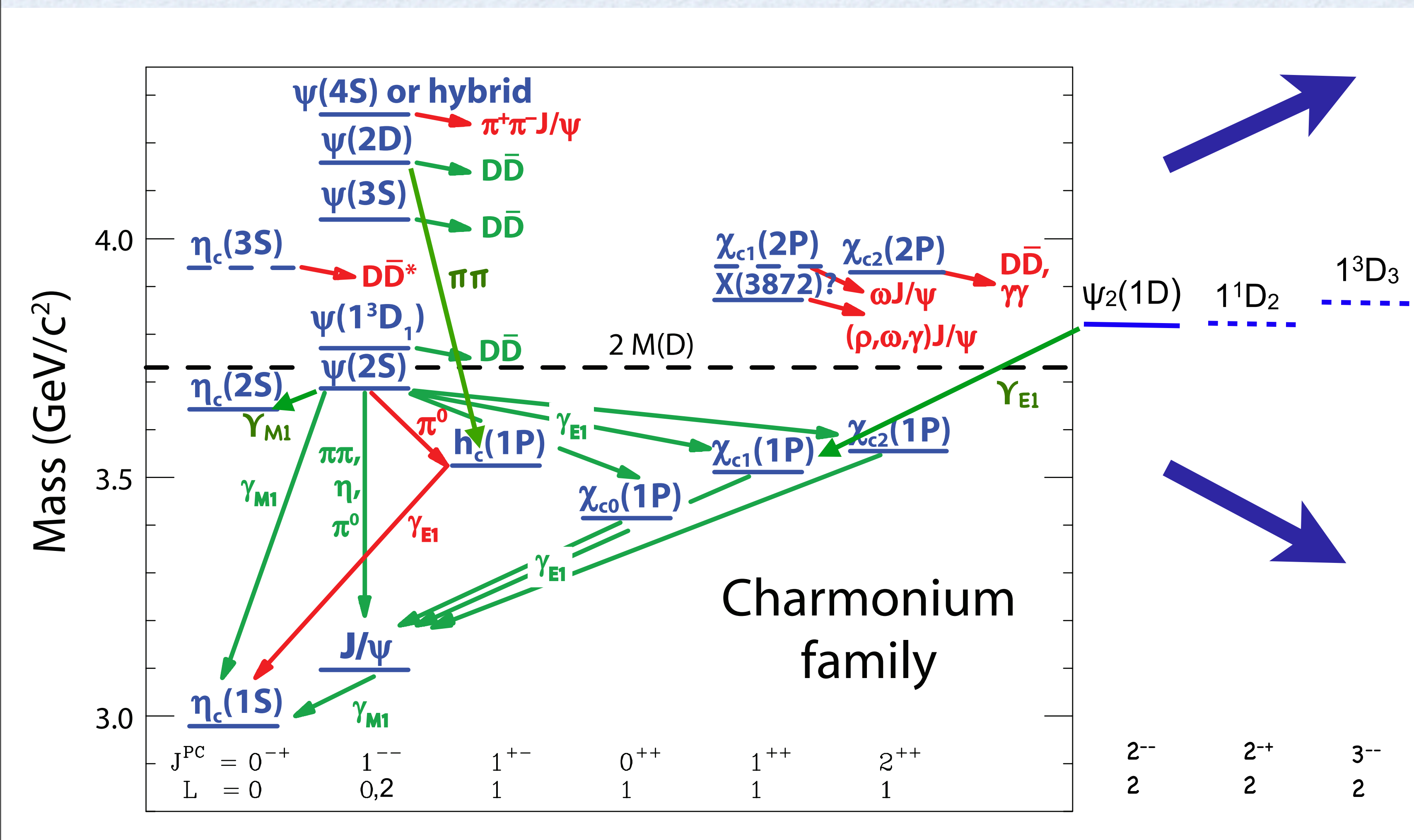
$$c_1 \pm c_2 = \frac{1}{8\pi} \int_{s_0}^{\infty} ds \, \frac{[\sigma_\parallel \pm \sigma_\perp](s)}{s^2}$$

		m_M [MeV]	$\Gamma_{\gamma\gamma}$ [keV]	$\int \frac{ds}{s} (\sigma_2 - \sigma_0)$ [nb]	c_1 $[10^{-4}\ \mathrm{GeV}^{-4}]$	c_2 $[10^{-4}\ \mathrm{GeV}^{-4}]$
0^{-+}	π^0	134.9766 ± 0.0006	$(7.8 \pm 0.5) \times 10^{-3}$	-195 ± 13	0	10.94 ± 0.70
0^{++}	$a_0(980)$	980 ± 20	0.3 ± 0.1	-20 ± 8	0.021 ± 0.007	0
2^{++}	$a_2(1320)$	1318.3 ± 0.6	1.00 ± 0.06	134 ± 8	0.039 ± 0.002	0.039 ± 0.002
	$a_2(1700)$	1732 ± 16	0.30 ± 0.05	18 ± 3	0.003 ± 0.001	0.003 ± 0.001
	Sum			-63 ± 17	0.06 ± 0.01	10.98 ± 0.70

- helicity difference SR: $a_2(1320)$ compensates π^0 contribution to 70%
- dominant contribution to LbL scattering coefficient c_2 comes from π^0

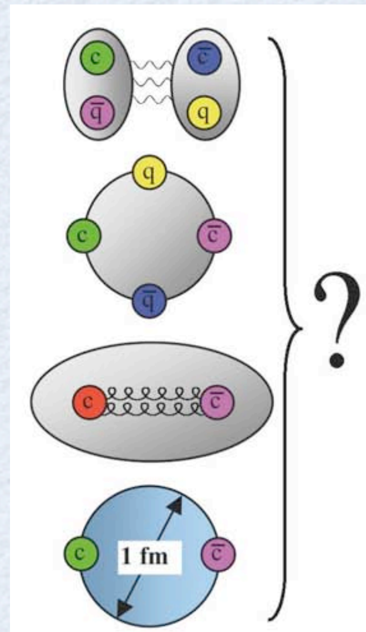
charmonium states

charmonium spectrum



above $D\bar{D}$ threshold:

- plethora of new states
- nature ?
  molecules
  tetra-quarks,
  hybrids, ...
  BABAR, BELLE,
  BES-III, PANDA



narrow states:

- well understood $c\bar{c}$ states
- only 2 remain to be observed

S. Godfrey, H. Mahlke, J.L. Rosner,
E. Eichten (2008)

$c\bar{c}$ meson production in γγ collisions

sum rules evaluated
for $c\bar{c}$ states

$$0 = \int_{s_0}^{\infty} ds \, \frac{[\sigma_2 - \sigma_0](s)}{s}$$

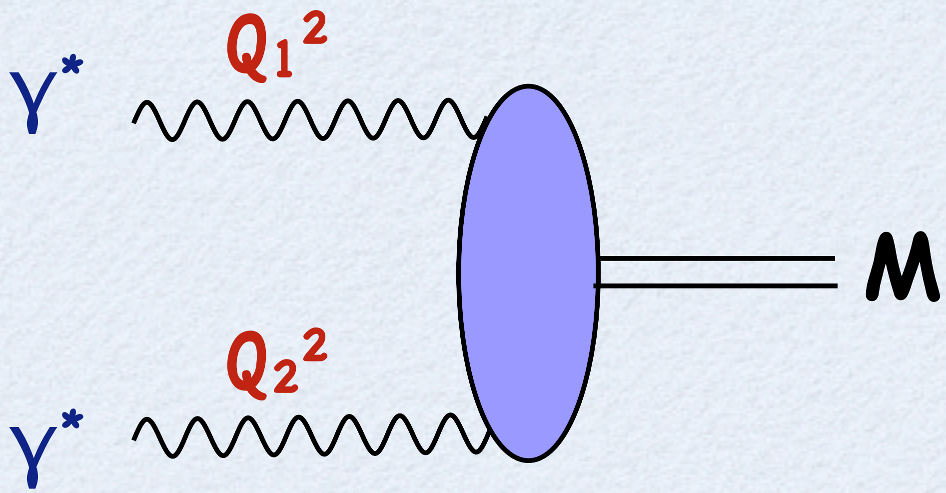
		m_M [MeV]	$\Gamma_{\gamma\gamma}$ [keV]	$\int \frac{ds}{s}\,(\sigma_2 - \sigma_0)$ [nb]	c_1 [10^{-7}GeV^{-4}]	c_2 [10^{-7}GeV^{-4}]
0^{-+}	$\eta_c(1S)$	2980.3 ± 1.2	6.7 ± 0.9	-15.6 ± 2.1	0	1.79 ± 0.24
0^{++}	$\chi_{c0}(1P)$	3414.75 ± 0.31	2.32 ± 0.13	-3.6 ± 0.2	0.31 ± 0.02	0
2^{++}	$\chi_{c2}(1P)$	3556.2 ± 0.09	0.50 ± 0.06	3.4 ± 0.4	0.14 ± 0.02	0.14 ± 0.02
	Sum resonances			-15.8 ± 2.1	0.49 ± 0.03	1.97 ± 0.24
	duality estimate continuum ($\sqrt{s} \geq 2m_D$)			15.1		
	resonances + continuum			-0.7 ± 2.1		

duality estimate for continuum contribution, above $D\bar{D}$ threshold

$$\int_{s_D}^{\infty} ds \, \frac{1}{s} \left[\sigma_2 - \sigma_0\right](\gamma\gamma \to X) \approx \int_{s_D}^{\infty} ds \, \frac{1}{s} \left[\sigma_2 - \sigma_0\right](\gamma\gamma \to c\bar{c})$$

interplay between hidden charm mesons ($c\bar{c}$ states)
and production of charmed mesons

meson production in $\gamma^*\gamma^*$ collisions



with J^{PC}

- one photon virtual Q_1^2
  second photon: real or quasi-real $Q_2^2 \approx 0$

- axial-vector mesons 1^{++} are also allowed if one of the photons is virtual
  $\gamma^*\gamma^* \rightarrow$ $f_1(1285)$ / $f_1(1420)$ measured L3 Coll.

equivalent 2γ decay width

$$\tilde{\Gamma}_{\gamma\gamma}(\mathcal{A}) \equiv \lim_{Q_1^2 \to 0} \frac{m_A^2}{Q_1^2} \frac{1}{2} \Gamma\left(\mathcal{A} \to \gamma_L^* \gamma_T\right)$$

- information on meson transition FFs

meson production in $\gamma^*\gamma^*$ collisions

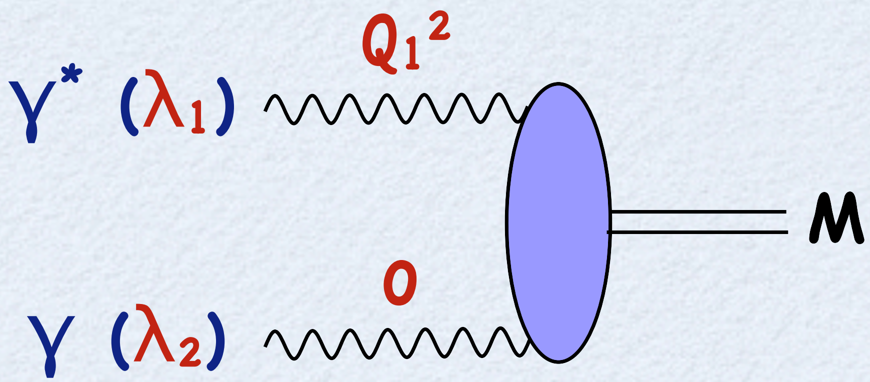
both photons real
or quasi-real

$$0 = \int_{s_0}^{\infty} ds \left[\frac{\sigma_\parallel}{s^2} + \frac{1}{s} \frac{\tau^a_{TL}}{Q_1 Q_2} \right]_{Q_1^2=Q_2^2=0}$$

		m_M [MeV]	$\Gamma_{\gamma\gamma}$ [keV]	$\int \frac{ds}{s^2} \sigma_\parallel(s)$ [nb / GeV^2]	$\int ds \left[\frac{1}{s}\frac{\tau^a_{TL}}{Q_1 Q_2}\right]_{Q_i^2=0}$ [nb / GeV^2]	$\int ds \left[\frac{1}{s^2}\sigma_\parallel + \frac{1}{s}\frac{\tau^a_{TL}}{Q_1 Q_2}\right]_{Q_i^2=0}$ [nb / GeV^2]
1^{++}	$f_1(1285)$	1281.8 ± 0.6	3.5 ± 0.8	0	-93 ± 21	-93 ± 21
	$f_1(1420)$	1426.4 ± 0.9	3.2 ± 0.9	0	-50 ± 14	-50 ± 14
0^{++}	$f_0(980)$	980 ± 10	0.29 ± 0.07	20 ± 5	0	20 ± 5
	$f_0'(1370)$	$1200 - 1500$	3.8 ± 1.5	48 ± 19	0	48 ± 19
2^{++}	$f_2(1270)$	1275.1 ± 1.2	3.03 ± 0.35	138 ± 16	$\gtrsim 0$	138 ± 16
	$f_2'(1525)$	1525 ± 5	0.081 ± 0.009	1.5 ± 0.2	$\gtrsim 0$	1.5 ± 0.2
	$f_2(1565)$	1562 ± 13	0.70 ± 0.14	12 ± 2	$\gtrsim 0$	12 ± 2
	Sum					76 ± 36

SR involving L photons: $f_1(1285)$, $f_1(1420)$ contributions compensated by $f_2(1270)$

meson production in $\gamma^*\gamma^*$ collisions



for 2^{++} : dominant FF for

helicity $\Lambda = \lambda_1 - \lambda_2 = 2$

tensor FFs $T^{(2)}(Q_1^2, 0)$ totally unknown

- use helicity difference SR: assume $a_2(1320)$ compensated by π^0

$$\frac{T^{(2)}_{a_2}(Q_1^2,0)}{T^{(2)}_{a_2}(0,0)} \simeq \frac{1}{(1+Q_1^2/\Lambda_\pi^2)}$$

using PS FF:

$$\frac{F_{\mathcal{P}\gamma^*\gamma^*}(Q_1^2,0)}{F_{\mathcal{P}\gamma^*\gamma^*}(0,0)} = \frac{1}{1+Q_1^2/\Lambda_P^2}$$

- use helicity difference SR: assume $f_2(1270)$ compensated by η, η'

$$\frac{T^{(2)}_{f_2}(Q_1^2,0)}{T^{(2)}_{f_2}(0,0)} \simeq \left[\frac{c_\eta}{c_\eta + c_{\eta'}} \frac{1}{(1+Q_1^2/\Lambda_\eta^2)^2} + \frac{c_{\eta'}}{c_\eta + c_{\eta'}} \frac{1}{\left(1+Q_1^2/\Lambda_{\eta'}^2\right)^2} \right]^{1/2}$$

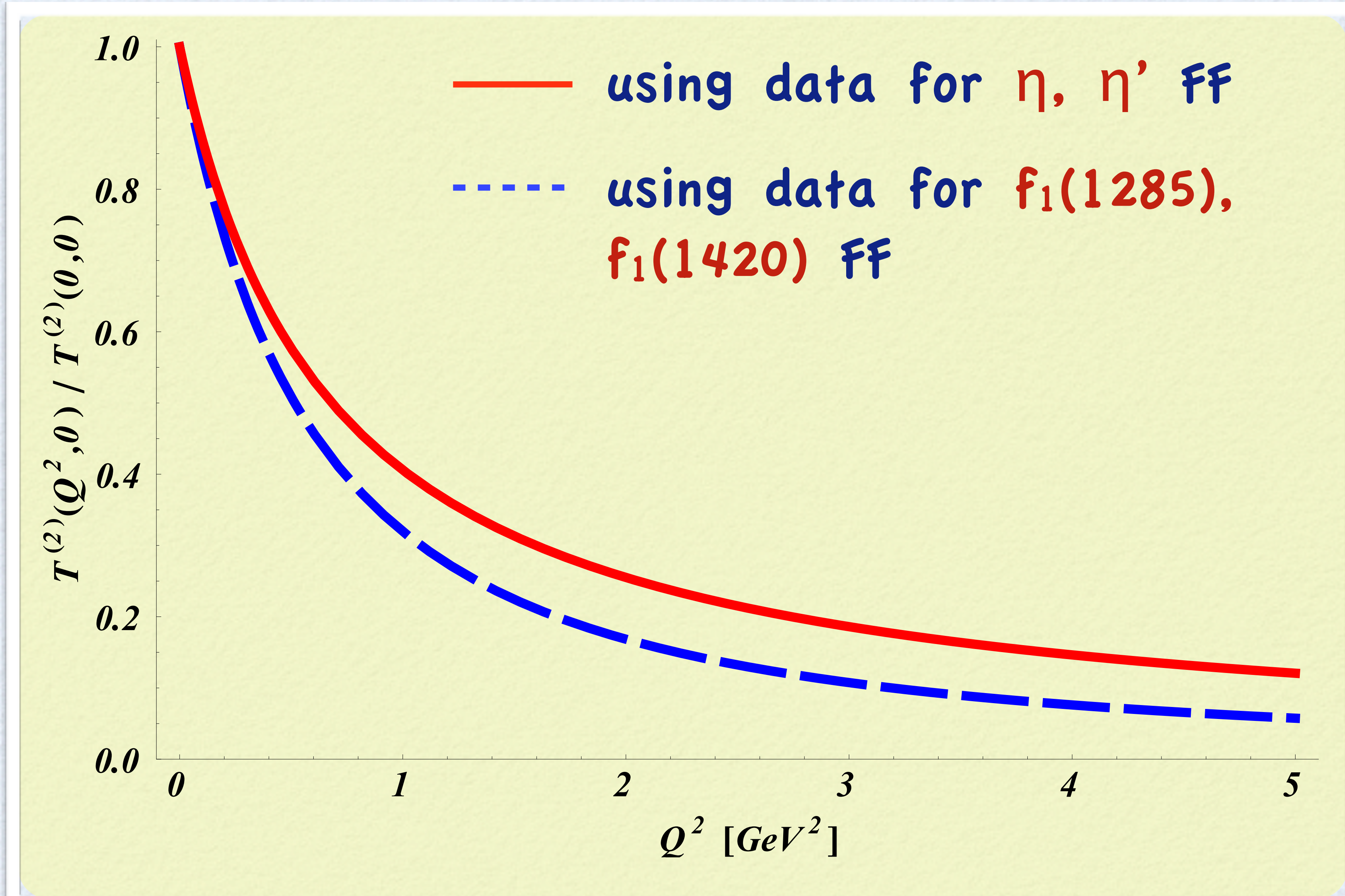
$$c_P \equiv \Gamma_{\gamma\gamma}/m_P^3$$

- use SR involving L photons: $f_2(1270)$ compensated by $f_1(1285)$, $f_1(1420)$

$$\frac{T^{(2)}_{f_2}(Q_1^2,0)}{T^{(2)}_{f_2}(0,0)} \simeq \left(1+\frac{Q_1^2}{m_{f_2}^2}\right)^{1/2} \left[\frac{d_{f_1}}{d_{f_1}+d_{f_1'}} \frac{1}{\left(1+Q_1^2/\Lambda_{f_1}^2\right)^4} + \frac{d_{f_1'}}{d_{f_1}+d_{f_1'}} \frac{1}{\left(1+Q_1^2/\Lambda_{f_1'}^2\right)^4} \right]^{1/2}$$

$$d_A \equiv 3\Gamma_{\gamma\gamma}/m_A^5$$

meson production in $\gamma^*\gamma^*$ collisions



sum rules allow to make a prediction for the (yet unmeasured) helicity-2 FF of $f_2(1270)$

Conclusions

- 3 new super-convergence relations for $\gamma^*\gamma$ scattering,
  sum rules for LbL scattering coefficients
- sum rules verified in perturbative quantum field theory
- $\gamma^*\gamma$ sum rules for meson production:

  compensations between different mesons verified

  π^0 and $a_2(1320)$, η, η' and $f_2(1270)$, $f_2(1565)$,

  $f_2(1270)$ and $f_1(1285)$, $f_1(1420)$, hidden and open charm mesons
- for virtual photons: predictions for meson FFs, sum rule tests,

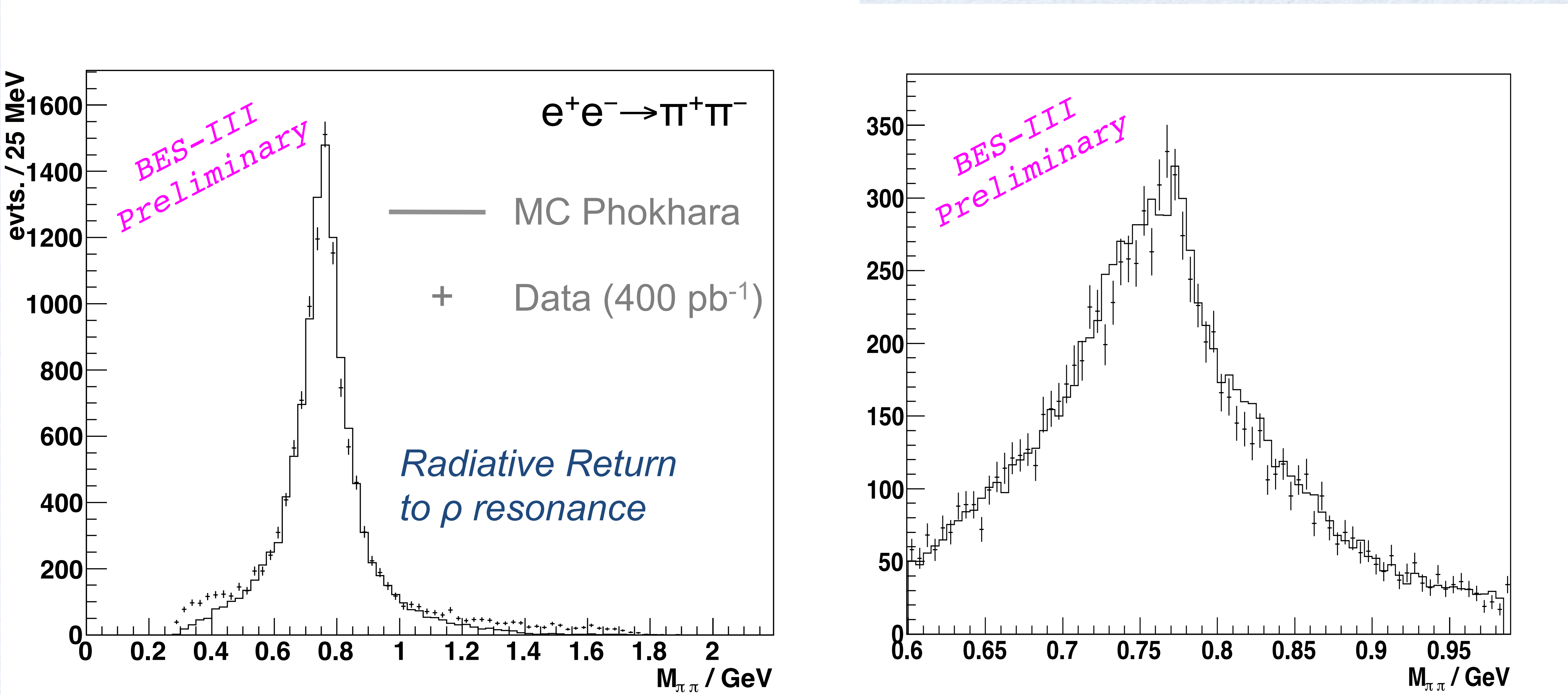
  constraints on models for LbL scattering to $(g-2)_\mu$

Backup

results from data: $e^+e^- \to \pi^+\pi^-$

Czyz, Kühn, Rodrigo, … (since 1999)

- ***Feasibility studies using Monte Carlo generator PHOKHARA***
  → *BES-III data sample of 10 fb^{-1} provides similar statistics as BABAR (454 fb^{-1})*
- ***Analysis on most relevant channel $e^+e^- \to \pi^+\pi^-$ started***



Zimmermann (2011)

$\gamma^*\gamma \to \eta/\eta'$: η-η' mixing

η-η' mixing probes strange quark content of light pseudo-scalar mesons and gluon dynamics of QCD

η/η' transition form factors encode information on different mixing scenarios

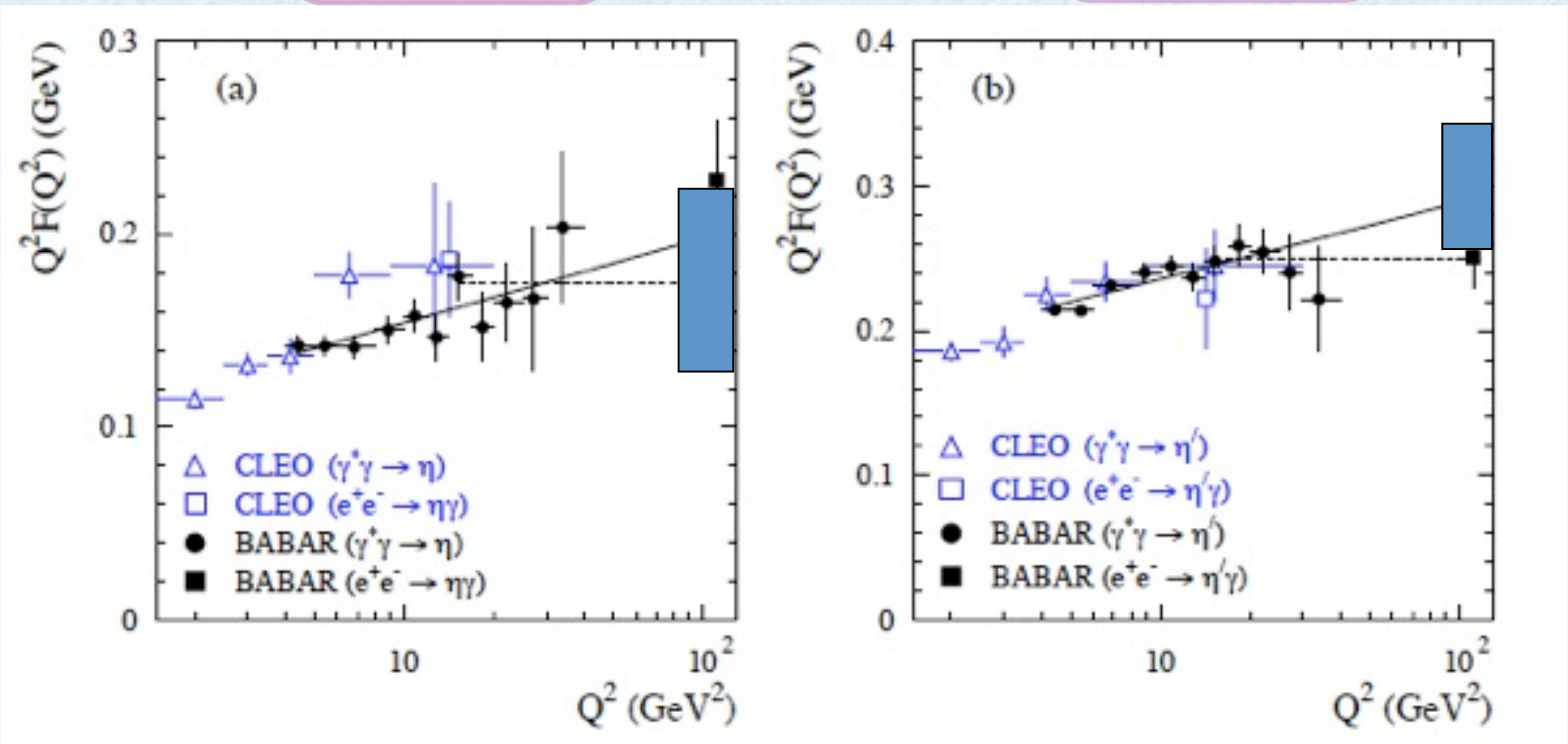
Spacelike ($q^2 < 0$): e^+e^- colliders

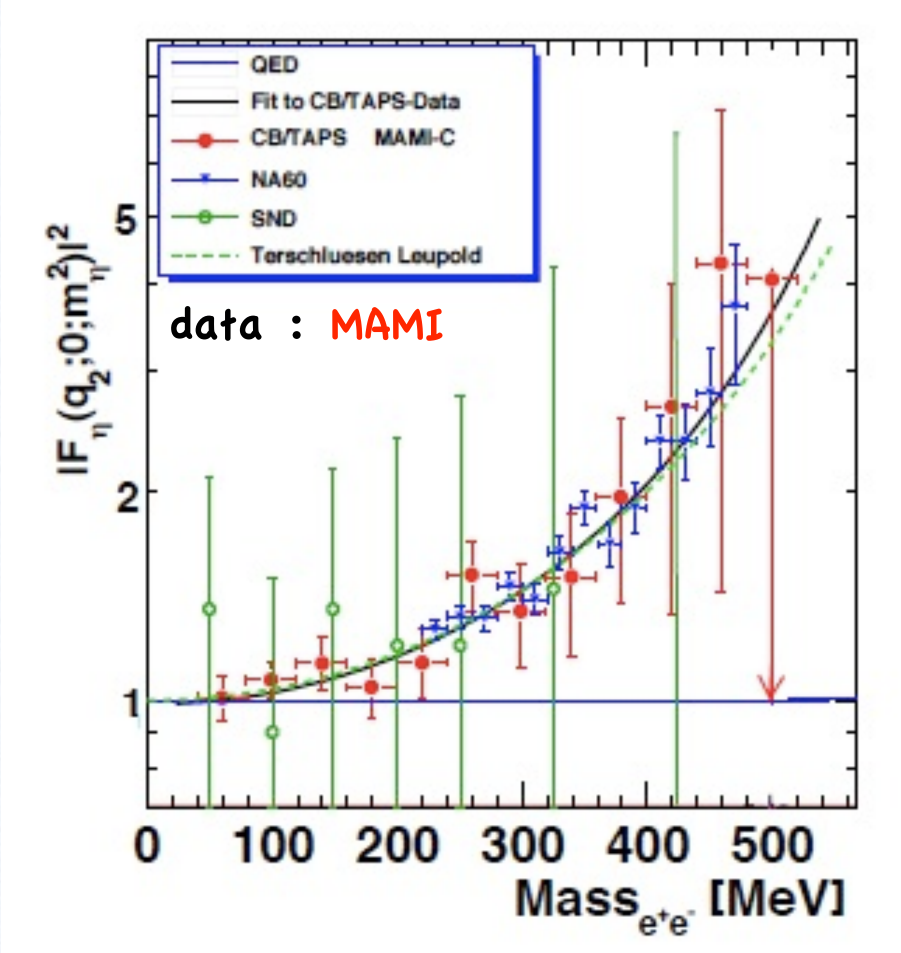
Timelike ($q^2 > 0$): meson decays

$\gamma^*\gamma \to \eta$

$\gamma^*\gamma \to \eta'$

$\eta \to \gamma e^+ e^-$





bands: uncertainties from different mixing scenarios