From nucleon structure to nuclear structure and compact astrophysical objects
(KITPC/ITP-CAS, Jun 11 – July 20, 2012)

Lattice study of non-Abelian dual superconductivity for quark confinement

Akihiro Shibata (Computing Research Center, KEK)

Based on works in collaboration with

K.-I. Kondo (Chiba Univ.)

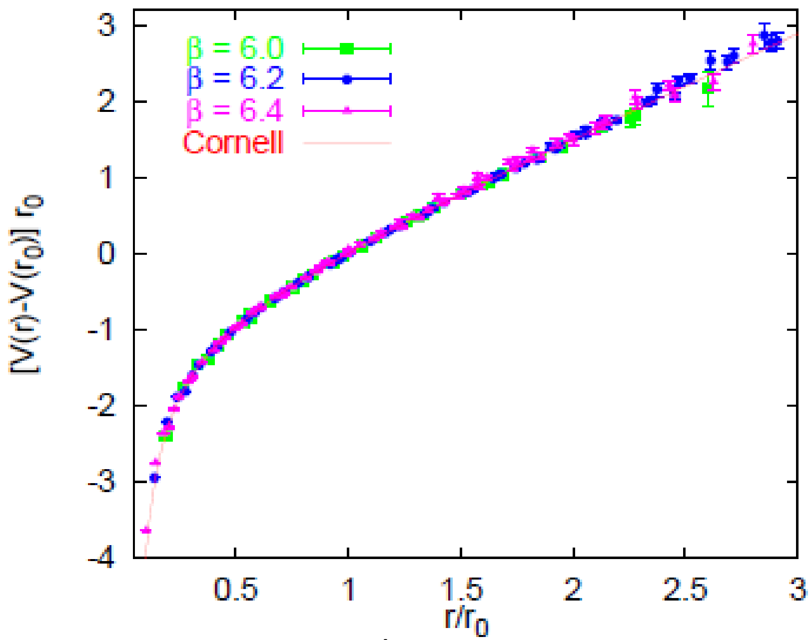
T. Shinohara (Chiba Univ.)

S. Kato (Fukui NCT)

Introduction

- Quark confinement follows from the area law of the Wilson loop average [Wilson,1974]

$$\text{Non-Abelian Wilson loop} \quad \left\langle \mathrm{tr}\left[\mathscr{P} \exp\left\{ ig \oint_C dx^\mu \mathscr{A}_\mu(x) \right\} \right] \right\rangle_{\mathrm{YM}}^{\text{no GF}} \sim e^{-\sigma_{NA}|S|}$$



$$V(r) = -C\frac{g_{\mathrm{YM}}^2(r)}{r} + \sigma r$$

$$F(r) = -\frac{d}{dr}V(r) = -C\frac{g_{\mathrm{YM}}^2(r)}{r^2} - \sigma + \cdots \ (C, \sigma > 0)$$

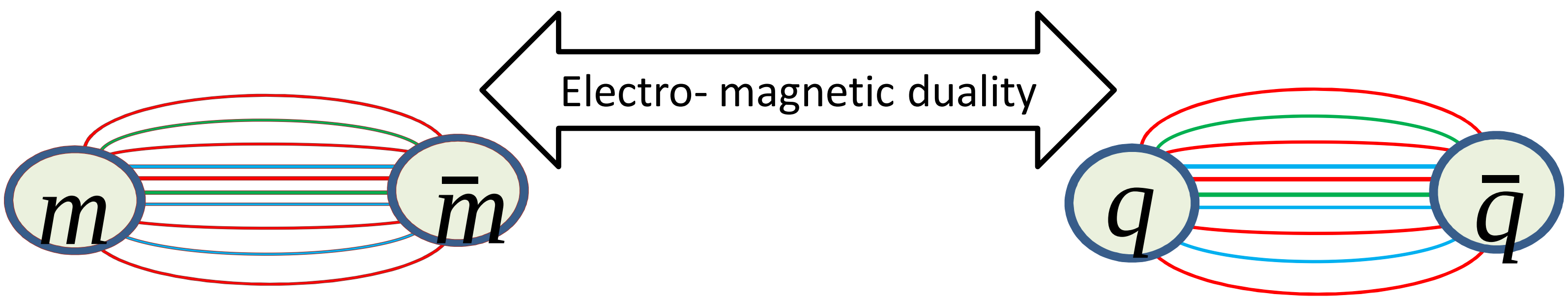
$$V(r) \to \infty \ \text{ for } r \to \infty$$

G.S. Bali, [hep-ph/0001312], Phys. Rept. **343, 1–136 (2001)**

Introduction(cont)

– **Dual superconductivity** is a promising mechanism for the quark confinement. [Y.Nambu (1974). G. ’t Hooft, (1975). S. Mandelstam, (1976) A.M. Polyakov, (1975). Nucl. Phys. B 120, 429(1977).]

Meissner effect Magnetic flux tube Monopole-monople connection Linear potential between monopoles	Dual Meissner effect formation of a hadron string (electric flux tube) Linear potential between quarks



To show the dual superconductivity, we must show the existence of the magnetic monopole and the monopole play the dominant role for the quark confinement.

Introduction (cont.)

SU(2) case

Abelian-projected Wilson loop $\left\langle \exp\left\{ ig \oint_C dx^\mu A_\mu^3(x) \right\} \right\rangle_{\rm YM}^{\rm MAG} \sim e^{-\sigma_{Abel}|S|}$!?

- There exist many numerical simulations that support dual superconductor picture based on Abelian projection such as
  - Abelian dominance [Suzuki & Yotsuyanagi, 1990]
  - Monopole dominance [Stack, Neiman and Wensley,1994][Shiba & Suzuki, 1994]
  - Center vortex dominance [e.g. Greensite (2007)]

Abelian part is obtained by decomposition $U_{x,\mu} = X_{x,\mu} V_{x,\mu}$:

$$V_{x,\mu} := \frac{u_0 + i\sigma_3 u_3}{\sqrt{u_0^2 + u_3^2}} \text{ with } U_{x,\mu} = u_0 \mathbf{1} + i \sum_{k=1}^{3} \sigma_k u_k$$

and $X_{x,\mu}$ is given by the remainder $X_{x,\mu} := U_{x,\mu} V_{x,\mu}^{-1}$.

Introduction (cont)

Problems: These result are obtained
only for gauge fixing of YM field by the special gauges such as the
maximal Abelian (MA) gauge and the Laplacian gauge
and the gauge fixing also breaks color symmetry.

- ***How can we establish "Abelian" dominance and magnetic monopole dominance in the gauge independent way (gauge-invariant way)?***

[Phys.Lett.B632:326-332,2006],[Phys.Lett.B645:67-74,2007][Phys.Lett.B653:101-108,2007]

For SU(2) case, we have proposed the decomposition of gauge link, $U=XV$, which can extract the relevant mode V for quark confinement such that

- The compact representation of Cho- Duan-Ge-Faddeev-Niemi-Shabanov (CDGFNS) decomposition on a lattice.
- V and X transform under the SU(2) gauge transformation.
- V corresponds to the conventional "Abelian" part, which reproduces the "Abelian" dominance for the Wilson loop

Introduction (cont.) : result for SU(2) case

- quark-antiquark potential from Wilson loop operator
- *gauge-independent "Abelian" Dominance*

The decomposed V field reproduced the potential of original YM field.

$$\sigma_{full} \sim \sigma_V \quad (93 \pm 16\%)$$

- *gauge-independent monopole dominance*

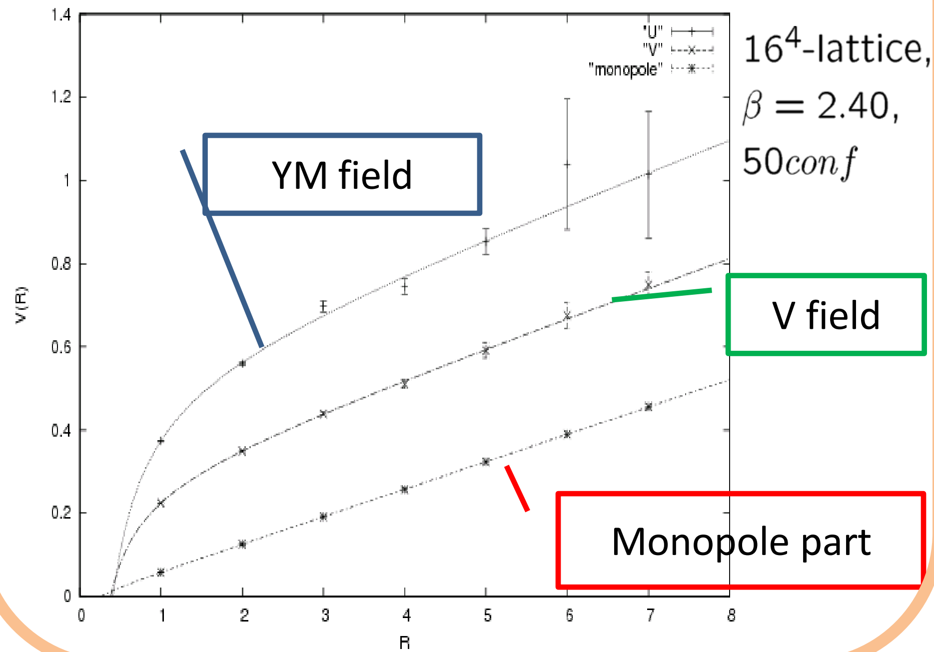
The string tension is reproduced by only magnetic monopole part.

$$\sigma_V \sim \sigma_{monopole} \quad (94 \pm 9\%)$$

$$\sigma_{full} \sim \sigma_{monopole} \quad (88 \pm 13\%)$$

arXiv:0911.0755 [hep-lat]

$$V(R) = c + \frac{\alpha}{R} + \sigma R$$



16^4-lattice, $\beta = 2.40$, $50 conf$

contents

- Introduction
- A new lattice formulation of the SU(N) Yang-Mills theory
  - Brief view of SU(2) case
  - SU(3) minimal option
- Numerical analysis
  - Algorithm of numerical analysis
  - Restricted U(2)-dominance and Non-Abelian monopole dominance
  - Measurement of color flux and dual Meissner effect
  - Propagators (correlation)
- Summary and discussion

A new lattice formulation of Yang-Mills theory
CDGFNS decomposition / non-linear change of variables

Kondo,Murakami,Shinohara (05))

Continuum theory: Cho-Duan-Ge-Faddeev-Niemi-Shabanov (CDGFNS) decomposition: $\mathbf{A}_\mu(x) = \mathbf{V}_\mu(x) + \mathbf{X}_\mu(x)$

By introducing color field $\mathbf{n}(x) \in SU(2)/U(1)$, decomosed field satisfy following eq

(i) $D_\mu[\mathbf{V}_\mu]\mathbf{n}(x) := \partial_\mu \mathbf{n}(x) + g\mathbf{V}_\mu(x) \times \mathbf{n}(x) = 0$

(ii) $\mathbf{n}(x) \cdot \mathbf{X}_\mu(x) = 0$

The decomposition is given by using $\mathbf{A}_\mu(x)$

$$\mathbf{V}_\mu(x) = c_\mu(x)\mathbf{n}(x) + g^{-1}\partial_\mu \mathbf{n}(x) \times \mathbf{n}(x) \quad \text{with} \quad c_\mu(x) = \mathbf{n}(x) \cdot \mathbf{A}_\mu(x),$$

$$\mathbf{X}_\mu(x) = g^{-1}\mathbf{n}(x) \times D_\mu[\mathbf{A}]\mathbf{n}(x)$$

Because of introducing the color field **n**(x), this theory has extended gauge symmetry **SU(2)×[SU(2)/U(1)]**. To obtain the equipollent theory with the original YM theory, we need the reduction condition (enlarged gauge fixing).

$$\chi := D_\mu[\mathbf{V}]\mathbf{X}_\mu(x) = 0 \quad \text{or} \quad \mathbf{n}(x) \times D_\mu[\mathbf{A}]D_\mu[\mathbf{A}]\mathbf{n}(x) = 0$$

CDGFNS decomposition on a lattice : SU(2) case

On a Lattice:

The decomposition of gauge field in the continuum theory

➔ **decomposition of link variable on a lattice**

$$U_{x,\mu} = X_{x,\mu}V_{x,\mu}$$

$$U_{x,\mu} = \exp(-ig\epsilon \mathbf{A}_\mu(x+\epsilon\hat{\mu}/2)),$$

$$V_{x,\mu} = \exp(-ig\epsilon \mathbf{V}_\mu(x+\epsilon\hat{\mu}/2)), \qquad X_{x,\mu} = \exp(-ig\epsilon \mathbf{X}_\mu(x+\epsilon\hat{\mu}/2))$$

are transformed under the gauge transformation Ω as

$$U_{x,\mu} \to U'_{x,\mu} = \Omega_x U_{x,\mu} \Omega^\dagger_{x+\mu}$$

$$V_{x,\mu} \to V'_{x,\mu} = \Omega_x V_{x,\mu} \Omega^\dagger_{x+\mu} \qquad X_{x,\mu} \to X'_{x,\mu} = \boxed{\Omega_x X_{x,\mu} \Omega^\dagger_x}$$

The lattice version of defining equation

$$\text{(i) } D^\epsilon_\mu[V]\mathbf{n}_x := \frac{1}{\epsilon}(V_{x,\mu}\mathbf{n}_{x+\mu} - \mathbf{n}_x V_{x,\mu}) = 0$$

$$\text{(ii) } \mathrm{tr}(X_{x,\mu}\mathbf{n}_x) = 0$$

The decomposition of link variables: SU(2)

$$W_C[U] := \mathrm{Tr}\left[P \prod_{\langle x, x+\mu \rangle \in C} U_{x,\mu} \right] / \mathrm{Tr}(\mathbf{1})$$

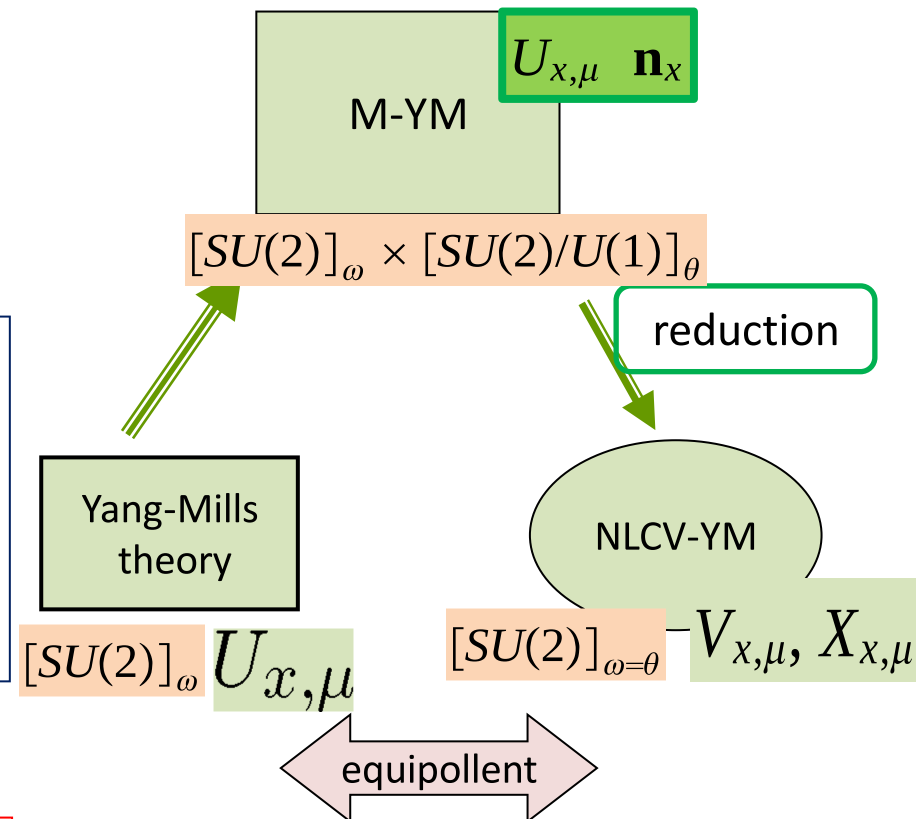
$$U_{x,\mu} = X_{x,\mu} V_{x,\mu}$$

$$U_{x,\mu} \to U'_{x,\mu} = \Omega_x U_{x,\mu} \Omega^{\dagger}_{x+\mu}$$

$$V_{x,\mu} \to V'_{x,\mu} = \Omega_x V_{x,\mu} \Omega^{\dagger}_{x+\mu}$$

$$X_{x,\mu} \to X'_{x,\mu} = \Omega_x X_{x,\mu} \Omega^{\dagger}_{x}$$

$$W_C[V] := \mathrm{Tr}\left[P \prod_{\langle x, x+\mu \rangle \in C} V_{x,\mu} \right] / \mathrm{Tr}(\mathbf{1})$$



$$W_C[U] = \mathrm{const.} W_C[V] \quad !!$$

Decomposition of SU(N) gauge links

- The decomposition as the extension of the SU(2) case.
- For SU(N) YM gauge link, there are several possible options of decomposition corresponding to its stability groups:
- SU(3) Yang-Mills link variables: Two options

minimal option : $U(2) \cong SU(2) \times U(1) \subset SU(3)$

✓ Minimal case is derived for the Wilson loop, which gives the static potential of the quark and anti-quark for the fundamental representation.

maximal option : $U(1) \times U(1) \subset SU(3)$

- Maximal case is gauge invariant version of Abelian projection in the maximal Abelian (MA) gauge. (the maximal torus group) :: *PoS*(*LATTICE 2007*)331

The decomposition of SU(3) link variable: the minimal option

$$W_C[U] := \mathrm{Tr}\left[P \prod_{\langle x, x+\mu \rangle \in C} U_{x,\mu} \right] / \mathrm{Tr}(\mathbf{1})$$

$$U_{x,\mu} = X_{x,\mu} V_{x,\mu}$$

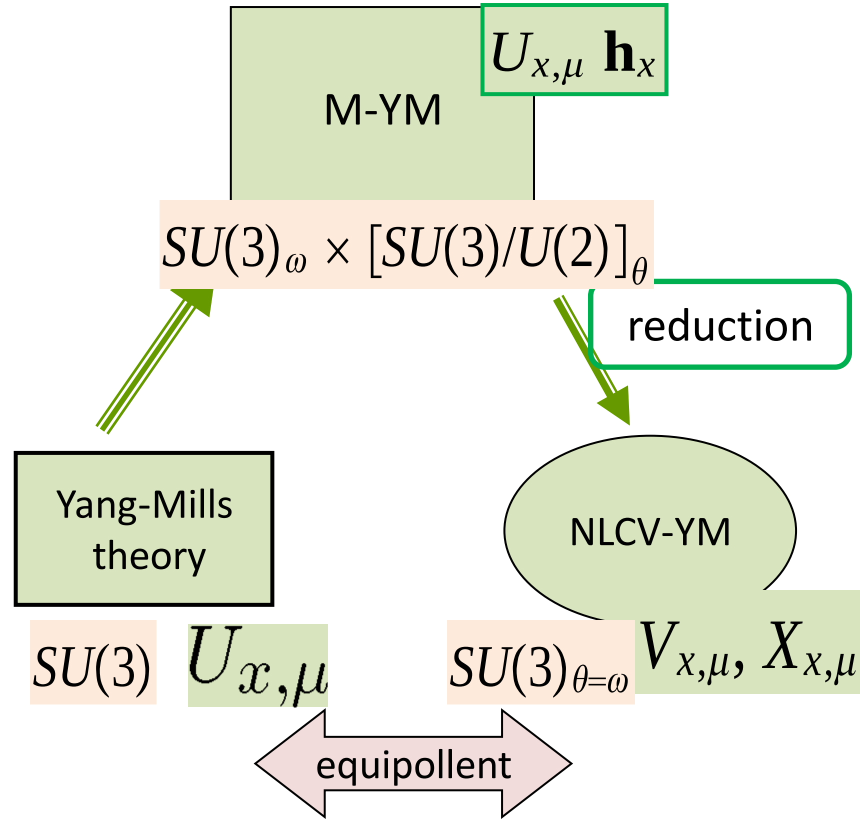
$$U_{x,\mu} \to U'_{x,\mu} = \Omega_x U_{x,\mu} \Omega^\dagger_{x+\mu}$$

$$V_{x,\mu} \to V'_{x,\mu} = \Omega_x V_{x,\mu} \Omega^\dagger_{x+\mu}$$

$$X_{x,\mu} \to X'_{x,\mu} = \Omega_x X_{x,\mu} \Omega^\dagger_x$$

$$\Omega_x \in G = SU(N)$$

$$W_C[V] := \mathrm{Tr}\left[P \prod_{\langle x, x+\mu \rangle \in C} V_{x,\mu} \right] / \mathrm{Tr}(\mathbf{1})$$



$$W_C[U] = \mathrm{const.} W_C[V] \quad !!$$

Defining equation

- The decomposition is obtained as the extension of the CFNS decomposition of SU(2) case.
- Decomposed ***V*** variables can be a dominant role for the quark confinement, i.e., the Wilson loop operator by original YM theory can be reproduced by the new variables.

Introducing a color field $\mathbf{h}_x = \xi(\lambda^8/2)\xi^\dagger \in SU(3)/U(2)$ with $\xi \in SU(3)$, a set of the definining equastion of decomposition $U_{x,\mu} = X_{x,\mu}V_{x,\mu}$ is given by

$$D^\epsilon_\mu[V]\mathbf{h}_x = \frac{1}{\epsilon}(V_{x,\mu}\mathbf{h}_{x+\mu} - \mathbf{h}_x V_{x,\mu}) = 0,$$

$$g_x = e^{-2\pi q_x/N}\exp(-ia^{(0)}_x\mathbf{h}_x - i\sum_{i=1}^{3} a^{(l)}_x\mathbf{u}^{(l)}_x) = 1$$

which correspod to the continume version of the decomposition $\mathcal{A}_\mu(x) = \mathcal{V}_\mu(x) + \mathcal{X}_\mu(x)$:

$$D_\mu[\mathcal{V}]\mathbf{h}(x) = 0, \qquad \mathrm{tr}(\mathbf{h}(x)\mathcal{X}_\mu(x)) = 0.$$

The defining equation and implication to the Wilson loop for the fundamental representation

K.-I. Kondo, Phys.Rev.D77:085029,2008

K.-I. Kondo, A. Shibata arXiv:0801.4203 [hep-th]

By inserting the complete set of the coherent state $|\xi_x, \Lambda\rangle$ at every site on the Wislon loop C, $1 = \int |\xi_x, \Lambda\rangle d\mu(\xi_x) \langle \Lambda, \xi_x|$ we obtain

$$W_C[U] = \mathrm{tr}\left(\prod_{<x>\in C} U_{x,\mu} \right) = \prod_{<x,x+\mu>\in C} \int d\mu(\xi_x) \langle \Lambda, \xi_x | U_{x,\mu} | \xi_{x+\mu}, \Lambda\rangle$$

$$= \prod_{<x,x+\mu>\in C} \int d\mu(\xi_x) \langle \Lambda, |(\xi_x^\dagger X_{x,\mu} \xi_x)(\xi_x^\dagger V_{x,\mu} \xi_{x+\mu})|, \Lambda\rangle$$

where we have used $\xi_x \xi_x^\dagger = 1$.

For the stability group of $\tilde{H}$, the 1st defining equation

$$\xi V_{x,\mu} \xi^\dagger \in \tilde{H} \iff [\xi_x^\dagger V_{x,\mu} \xi_{x+\mu}, \tilde{H}] \iff \mathbf{h}_x V_{x,\mu} - V_{x,\mu} \mathbf{h}_{x+\mu} = 0$$

implies that $|\Lambda\rangle$ is eigenstate of $\xi_x^\dagger V_{x,\mu} \xi_{x+\mu}$:

$$(\xi_x^\dagger V_{x,\mu} \xi_{x+\mu}) |\Lambda\rangle = |\Lambda\rangle e^{i\phi}, \quad e^{i\phi} := \langle \Lambda | \xi_x^\dagger V_{x,\mu} \xi_{x+\mu} | \Lambda \rangle = \langle \Lambda, \xi_x | V_{x,\mu} | \xi_{x+\mu}, \Lambda \rangle.$$

Then we have

$$W_C[U] = \int d\mu(\xi_x) \rho[X;\xi] \prod_{<x,x+\mu>\in C} \langle \Lambda, \xi_x | V_{x,\mu} | \xi_{x+\mu}, \Lambda\rangle$$

$$\rho[X;\xi] := \prod_{<x>\in C} \langle \Lambda, \xi_x | X_{x,\mu} | \xi_{x+\mu}, \Lambda\rangle$$

The defining equation and the Wilson loop for the fundamental representation (2)

By using the expansion of $X_{x,\mu}$: the 2nd defining equaiton, $\mathrm{tr}(\mathcal{X}_\mu(x)\mathbf{h}(x)) = 0$, derives

$$\langle \Lambda, \xi_x | X_{x,\mu} | \xi_{x+\mu}, \Lambda \rangle = \mathrm{tr}(X_{x,\mu})/\mathrm{tr}(\mathbf{1}) + 2\mathrm{tr}(X_{x,\mu}\mathbf{h}_x)$$
$$= 1 + 2ig\epsilon tr(\mathcal{X}_\mu(x)\mathbf{h}(x)) + O(\epsilon^2).$$

Then we have $\rho[X;\xi] = 1 + O(\epsilon^2)$.

Therefore, we obtain

$$W_c[U] = \int d\mu(\xi_x) \prod_{<x,x+\mu>\in C} \langle \Lambda, \xi_x | V_{x,\mu} | \xi_{x+\mu}, \Lambda \rangle = W_C[V]$$

By using the non−Abalian Stokes theorem, Wilson loop along the path C is written to area integral on $\Sigma : C = \partial\Sigma$;

$$W_C[\mathcal{A}] := \mathrm{tr}\left[P\exp\left(-ig \oint_C dx^\mu \mathcal{A}_\mu(x) \right) \right] / \mathrm{tr}(\mathbf{1}) = \int d\mu_\Sigma(\xi) \exp\left(\int_{S:\, C=\partial\Sigma} dS^{\mu\nu} F_{\mu\nu}[\mathcal{V}] \right),$$

(no path ordering), and the decomposed $V_{x,\mu}$ corresponds to the Lie algebra value of $\mathcal{V}_{x,\mu}$ and the field strength on a lattice is given by plaquet of $V_{x,\mu}$

The decomposition of the gauge link

The solution of the defining equation is given by

Phys.Lett.B691:91-98,2010 ; arXiv:0911.5294（hep-lat）

$$L_{x,\mu} = \frac{N^2-2N+2}{N}\mathbf{1} + (N-2)\sqrt{\frac{2(N-1)}{N}}(\mathbf{h}_x + U_{x,\mu}\mathbf{h}_{x+\mu}U_{x,\mu}^{-1}) + 4(N-1)\mathbf{h}_x U_{x,\mu}\mathbf{h}_{x+\mu}U_{x,\mu}^{-1},$$

$$L_{x,\mu} = \sqrt{L_{x,\mu}L_{x,\mu}^{\dagger}}\,\hat{L}_{x,\mu} \Leftrightarrow \hat{L}_{x,\mu} = (\sqrt{L_{x,\mu}L_{x,\mu}^{\dagger}})^{-1}L_{x,\mu}.$$

$$X_{x,\mu} = \hat{L}_{x,\mu}^{\dagger}(\det(\hat{L}_{x,\mu}))^{1/N}g_x^{-1}$$

$$V_{x,\mu} = X_{x,\mu}^{\dagger}U_{x,\mu} = g_x\hat{L}_{x,\mu}U_{x,\mu}(\det(\hat{L}_{x,\mu}))^{-1/N}$$

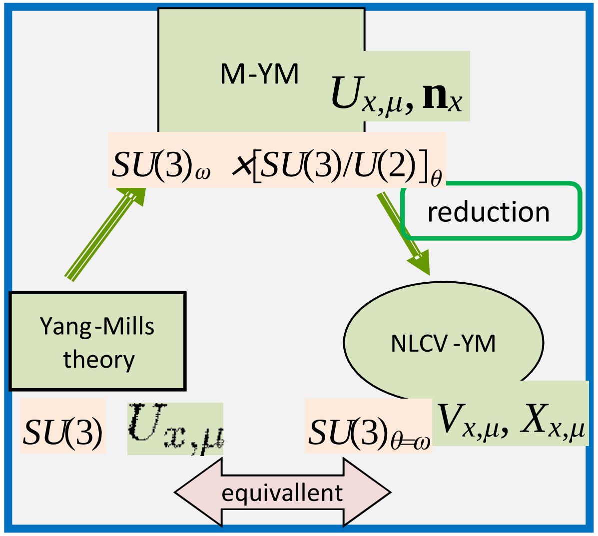
In the (naive) continuum limit , we have the continuum version of change of variables:

$$\mathbf{V}_\mu(x) = \mathbf{A}_\mu(x) - \frac{2(N-1)}{N}[\mathbf{h}(x),[\mathbf{h}(x),\mathbf{A}_\mu(x)]] - ig^{-1}\frac{2(N-1)}{N}[\partial_\mu\mathbf{h}(x),\mathbf{h}(x)],$$

$$\mathbf{X}_\mu(x) = \frac{2(N-1)}{N}[\mathbf{h}(x),[\mathbf{h}(x),\mathbf{A}_\mu(x)]] + ig^{-1}\frac{2(N-1)}{N}[\partial_\mu\mathbf{h}(x),\mathbf{h}(x)].$$

Reduction Condition

- The decomposition is uniquely determined for a given set of link variables $U_{x,\mu}$ and color fields h_x.
- The reduction condition is introduced such that the theory in terms of new variables is equipollent to the original Yang-Mills theory $SU(3)_\omega \times [SU(3)/U(2)]_\theta \rightarrow SU(3)_{\omega=\theta}$



- This is invariant under the gauge transformation θ=ω
- the extended gauge symmetry is reduced to the same symmetry with Original YM theory:
- We chose a reduction condition as same type with SU(2) case

Determining $\mathbf{h}_x$ to minimize the reduction function for given $U_{x,\mu}$

$$F_{\text{red}}[\mathbf{h}_x, U_{x,\mu}] = \sum_{x,\mu} \text{tr}\left\{ (D_\mu^\epsilon[U_{x,\mu}]\mathbf{h}_x)^\dagger (D_\mu^\epsilon[U_{x,\mu}]\mathbf{h}_x) \right\}$$

Non-Abelian magnetic monopole

From the non-Abelian Stokes theorem and the Hodge decomposition, the magnetic monopole is derived without using the Abelian projection

$$W_C[\mathcal{A}] = \int d\mu_\Sigma(\xi) \exp\left(\int_{S:\, C=\partial\Sigma} dS^{\mu\nu} F_{\mu\nu}[\mathcal{V}] \right)$$

$$= \int d\mu_\Sigma(\xi) \exp\left[ig\sqrt{\frac{N-1}{N}} (k, \Xi_\Sigma) + ig\sqrt{\frac{N-1}{N}} (j, N_\Sigma) \right]$$

$$k := \delta^* F = {}^* dF, \qquad \Xi_\Sigma := \delta^* \Theta_\Sigma \Delta^{-1}$$

$$j := \delta F, \qquad N_\Sigma := \delta \Theta_\Sigma \Delta^{-1}$$

$$\Delta := d\delta + \delta d$$

$$\Theta_\Sigma^{\mu\nu} := \int_\Sigma d^2 S^{\mu\nu}(x(\sigma)) \delta^D(x - x(\sigma))$$

k and j are gauge invariant and conserved current $\delta k = 0 = \delta j$.

K.-I. Kondo PRD77 085929(2008)

Note that the Wilson loop operator knows the non-Abelian magnetic monopole ***k*** .

Non-Abelian Magnetic monopole on a lattice

The magnetic monopole currents are calculated from decomposed variable $V_{x,\mu}$ as

$$V_{x,\mu}V_{x+\mu,\nu}V^{\dagger}_{x+\nu,\mu}V^{\dagger}_{x,\nu} = \exp(-ig\mathcal{F}[\mathbf{V}_{\mu}(x)]_{\mu\nu}) = \exp(-ig\Theta^{8}_{\mu\nu}\mathbf{h}_{x'}),$$

$$\Theta^{8}_{\mu\nu} = -\arg\mathrm{Tr}\left[\left(\frac{1}{3}\mathbf{1} - \frac{2}{\sqrt{3}}\mathbf{h}_{x}\right)V_{x,\mu}V_{x+\mu,\nu}V^{\dagger}_{x+\nu,\mu}V^{\dagger}_{x,\nu}\right],$$

$$k_{x,\mu} := \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\partial_{\nu}\Theta^{8}_{\alpha\beta}.$$

Integer valued monopole charge is defined by $n_{x,\mu} = k_{x,\mu}/(2\pi)$.

The magnetic monopole is derived as Hodge decomposition of field strength *F[V], so* the magnetic monopole current, *k,* is defined *in the gauge invariant way.*

The ***V*** field is an element of U(2) stability sub-group in SU(3) gauge group, it is a non-Abelian magnetic monopole.

contents

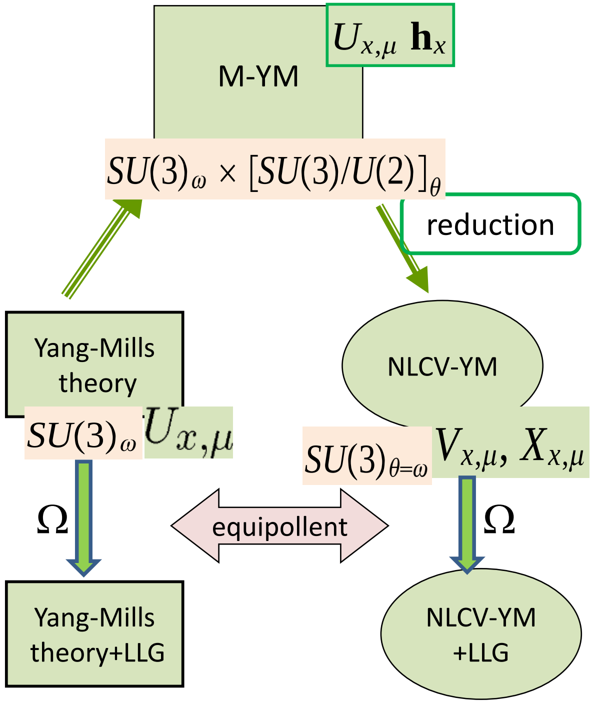
- Introduction
- A new lattice formulation of the SU(N) Yang-Mills theory
  - Brief view of SU(2) case
  - SU(3) minimal option
- Numerical analysis
  - Algorithm of numerical analysis
  - Restricted U(2)-dominance and Non-Abelian monopole dominance
  - Measurement of color flux and dual Meissner effect
  - Propagators (correlation)
- Summary and outlook

Numerical Analysis: Algorithm

- *The decomposition is uniquely determined for a given set of link variables $U_{x,m}$ and color fields h_x.*
- *The reduction function is invariant under the gauge trans formation* θ=ω

Algorithms:

1. The configurations of YM field are generated for the standard Wilson action by using the standard algorithms.
2. The configurations of color field are obtained by solving the reduction condition.
3. New variables are obtained by using the decomposition formula.
4. Measurement by ensemble <O(V,X)>



- The new variables ***V, X*** transform under the same the same gauge transformation: physical quantity is gauge invariant.

Static potential

- Wilson loop by the decomposed variable V
- Dose Wilson of V loop reproduces the original one?

$$W_C[U] = \mathrm{const.} W_C[V] \quad !!$$

- To get the static potential

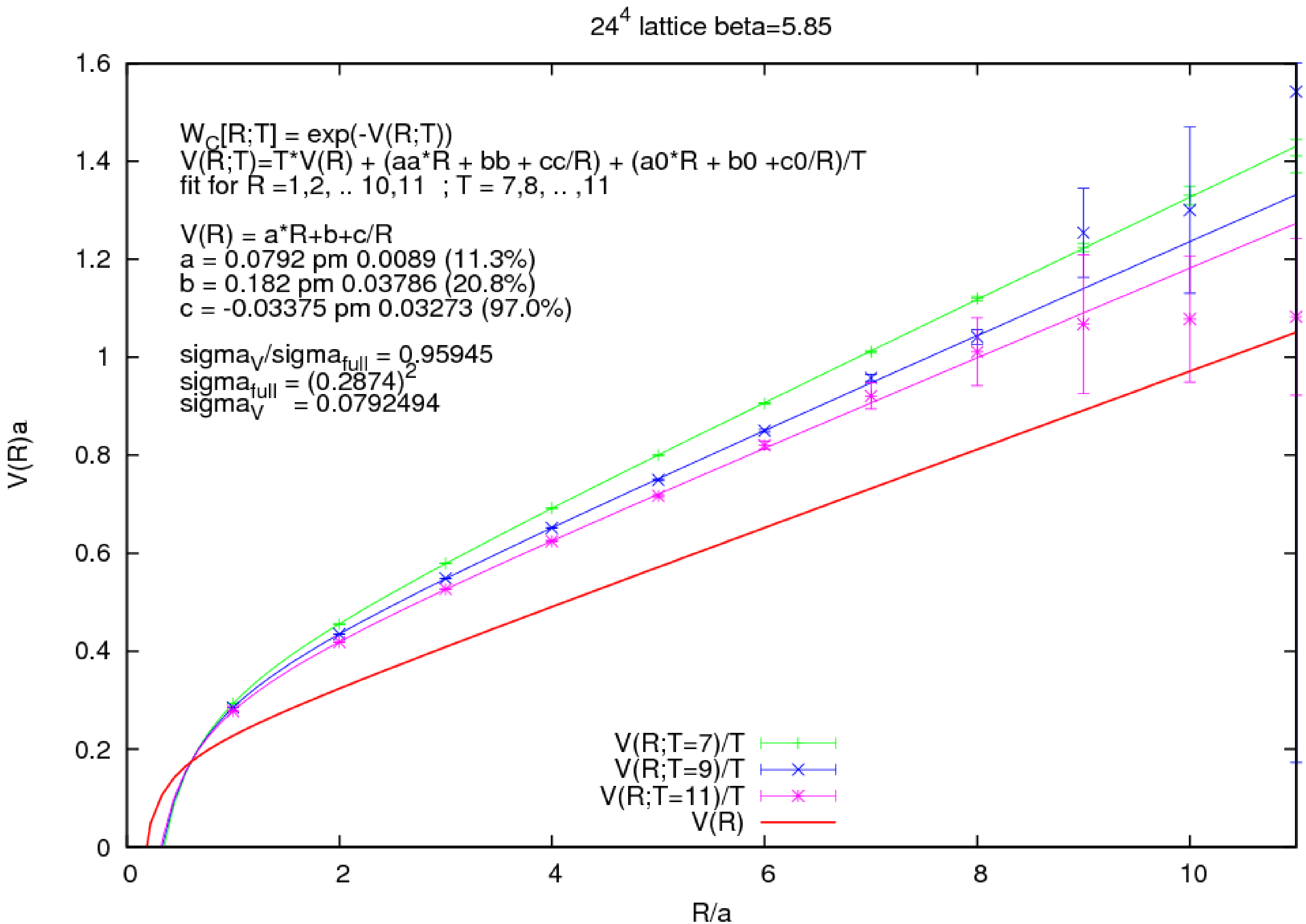
$$V(R) = -\lim_{T\to\infty} \frac{1}{T} \log\langle W_{(R,T)}[V]\rangle$$

- We fit the Wilson loop *$W_C[V]$* by the function *$V(R,T)$*

$$\langle W_{(R,T)}[V]\rangle = \exp(-V(R,T))$$

$$V(R,T) := T \times V(R) + (a'R + b' + c'/R) + (a''R + b'' + C''/R)/T$$

$$V(R) = \sigma R + b + c/R$$



24^4 lattice beta=5.85
W_C[R;T] = exp(-V(R;T))
V(R;T)=T*V(R) + (aa*R + bb + cc/R) + (a0*R + b0 +c0/R)/T
fit for R =1,2, .. 10,11 ; T = 7,8, .. ,11
V(R) = a*R+b+c/R
a = 0.0792 pm 0.0089 (11.3%)
b = 0.182 pm 0.03786 (20.8%)
c = -0.03375 pm 0.03273 (97.0%)
sigma_V/sigma_full = 0.95945
sigma_full = (0.2874)^2
sigma_V = 0.0792494
V(R)a
R/a
V(R;T=7)/T
V(R;T=9)/T
V(R;T=11)/T
V(R)

Wilson loop operator and magnetic monopole on a lattice

- Non-Abrelian Stokes' theorm *e.g. K.-I. Kondo PRD77 085929(2008)*

$$W_C[\mathbf{A}] = \mathrm{tr}\left[P \exp ig \oint_C dx^\mu A_\mu(x) \right] / \mathrm{tr}(\mathbf{1}) = \int [d\mu(\xi)]_\Sigma \exp\left\{ \int_{S:C=\partial S} dS^{\mu\nu} \mathcal{F}_{\mu\nu}[V] \right\}$$

$$= \int [d\mu(\xi)]_\Sigma \exp\left\{ ig\sqrt{\frac{N-1}{2N}}(k, \Xi_\Sigma) + ig\sqrt{\frac{N-1}{2N}}(j, N_\Sigma) \right\}$$

$$\Xi_\Sigma := *d\Theta_\Sigma \Delta^{-1} = \delta * \Theta_\Sigma \Delta^{-1}, N_\Sigma := \delta\Theta_\Sigma\Delta^{-1}$$

D−dimensional Laplacian $\Delta = d\delta + \delta d$

Θ_Σ : the vorticity tensor with support on the surface Σ_C sppaned by Willson loop C

$$\Theta^{\mu\nu}_\Sigma(x) = \int_\Sigma dS^{\mu\nu}(X(\sigma))\delta^D(x - X(\sigma)))$$

lattice version

$$\langle W_C[V] \rangle \simeq \langle W_C[Mono] \rangle = \left\langle \exp\left\{ i \sum k_{x,\mu} \Xi_{x,\mu} \right\} \right\rangle$$

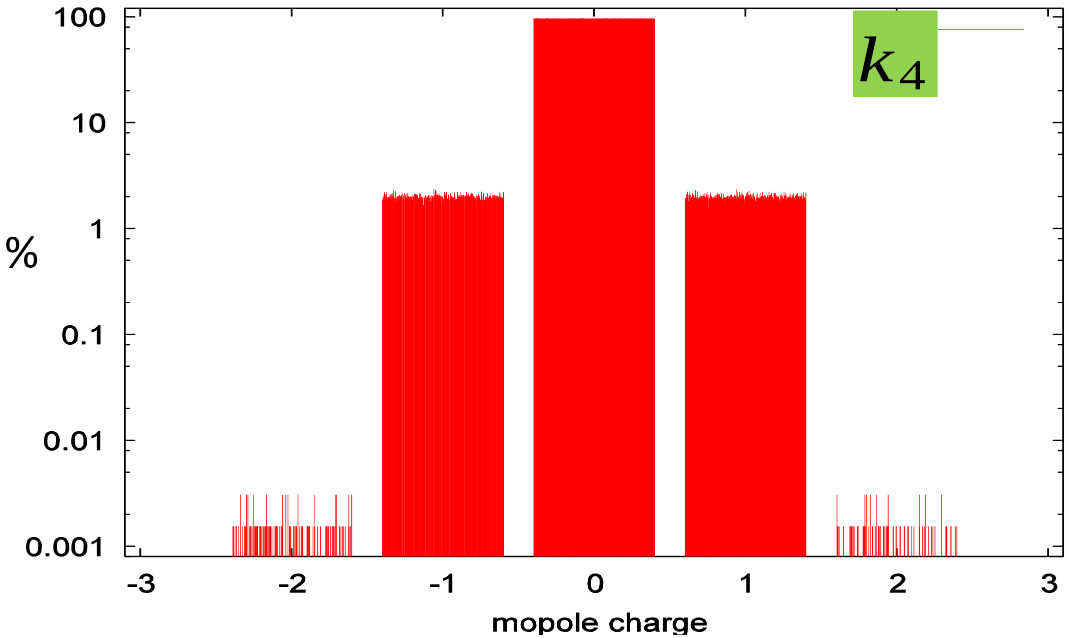
$$\Xi_{x,\mu} = \sum_{\sigma(y)\in\Sigma} \epsilon^{\mu\nu\alpha\beta} \partial_\nu \Delta^{-1}(x-y)\sigma^{\alpha\beta}(y)$$

$$\Theta^8_{x,\mu\nu} \equiv -arg Tr[(\frac{1}{3}\mathbf{1} - \frac{2}{\sqrt{3}}\mathbf{h}_x) V_{x,\mu} V_{x+\hat{\mu},\nu} V^\dagger_{x+\hat{\nu},\mu} V^\dagger_{x,\nu}]$$

$$k_{x,\mu} = -\frac{1}{4\pi}\epsilon_{\mu\nu\rho\sigma}\partial_\nu \Theta^8_{x,\rho\sigma}$$

Distribution of the magnetic currents (monipoles)

$$k_{x,\mu} = -\frac{1}{4\pi}\epsilon_{\mu\nu\rho\sigma}\partial_\nu\Theta^8_{x,\rho\sigma} \qquad \Theta^8_{x,\mu\nu} \equiv -argTr[(\frac{1}{3}\mathbf{1} - \frac{2}{\sqrt{3}}\mathbf{h}_x)V_{x,\mu}V_{x+\hat{\mu},\nu}V^\dagger_{x+\hat{\nu},\mu}V^\dagger_{x,\nu}]$$

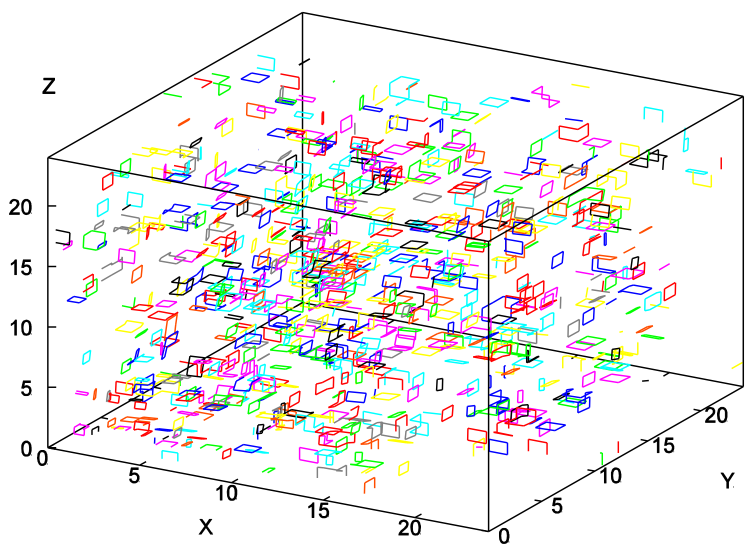


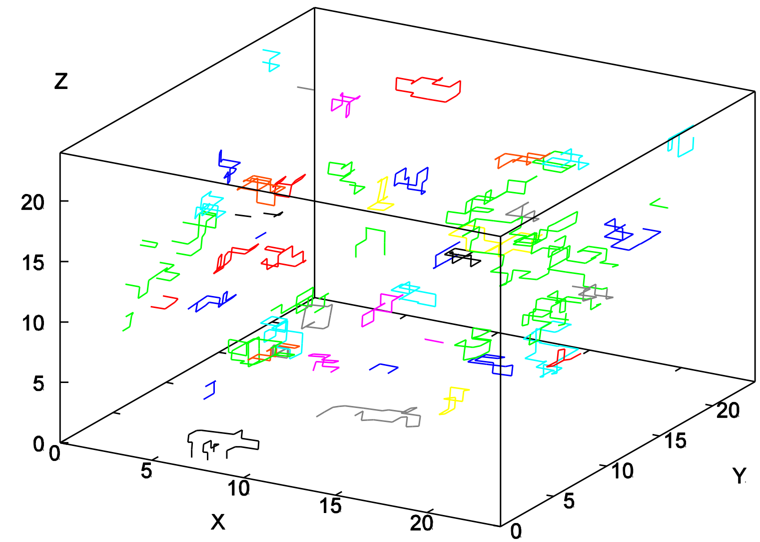
•The distribution of the monopole charges for 16^4 lattice β=5.7
400 configurations. The distribution of each configuration is shown by thin bar chart.

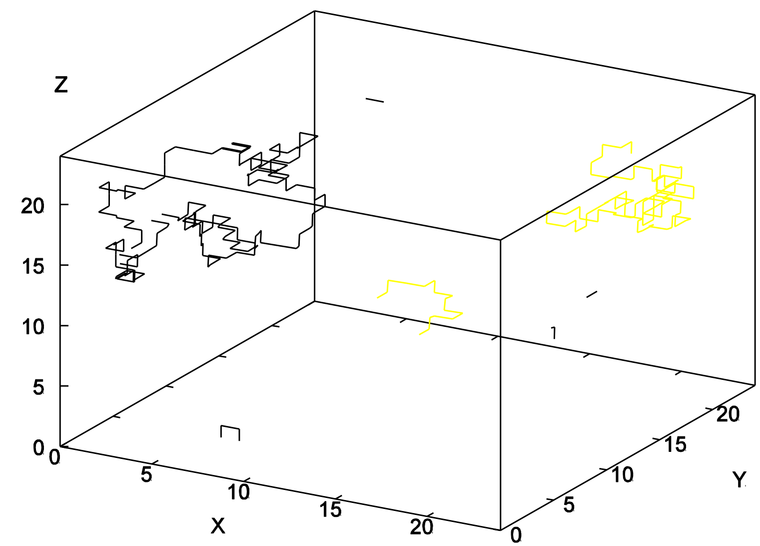
Non-Abelian magnetic monopole loops: 24^4 laiitce $\beta=6.0$

Projected view (x,y,z,t) ➔(x,y,z)

(left lower) loop length 1-10
(right upper) loop length 10 -- 100
(right lower) loop length 100 -- 1000





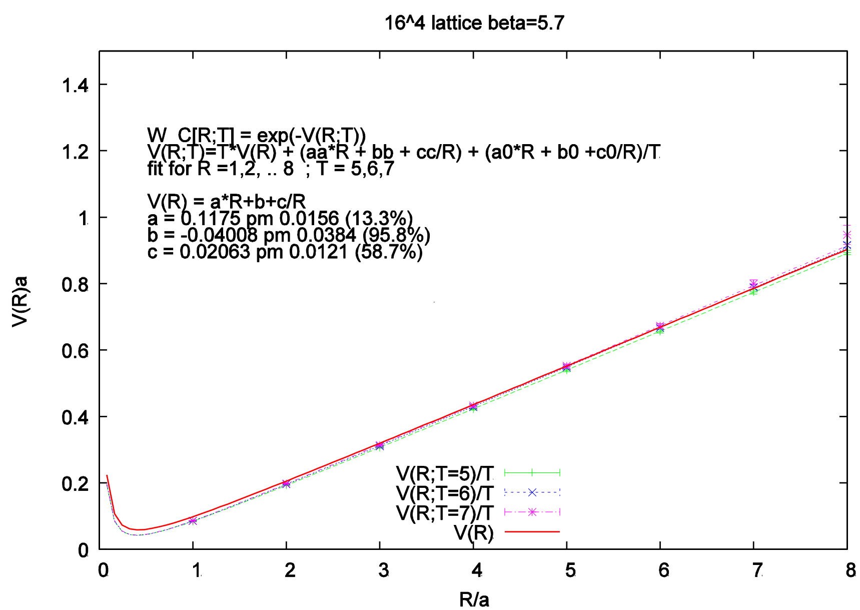


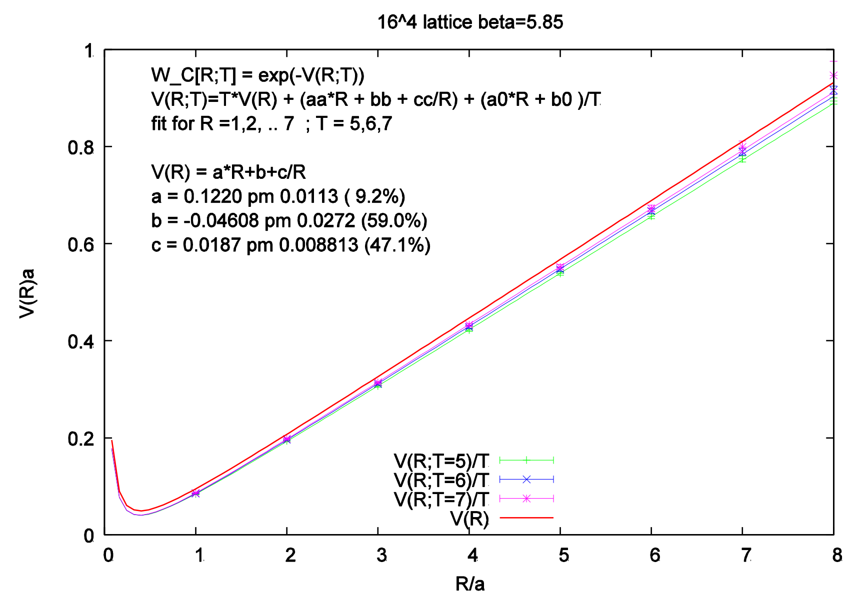
Static potential by non-Abelaian magnetic monopole

$$\langle W_C[V] \rangle \simeq \langle W_C[Mono] \rangle = \left\langle \exp\left\{ i \sum_{x,\mu} k_{x,\mu} \Xi_{x,\mu} \right\} \right\rangle$$

$$V(R,T) := T \times V(R) + (a'R + b' + c'/R)$$

$$V(R) = \sigma R + b + c/R$$





SU(3) YM theory: minimal option

- *gauge independent "Abelian" dominance*

$$\frac{\sigma_V}{\sigma_U} = 0.92$$
$$\frac{\sigma_V}{\sigma_{U^*}} = 0.78 - 0.82$$

- *Gauge independent non-Abelian monople dominance*

-

$$\frac{\sigma_M}{\sigma_U} = 0.85$$
$$\frac{\sigma_M}{\sigma_{U^*}} = 0.72 - 0.76$$

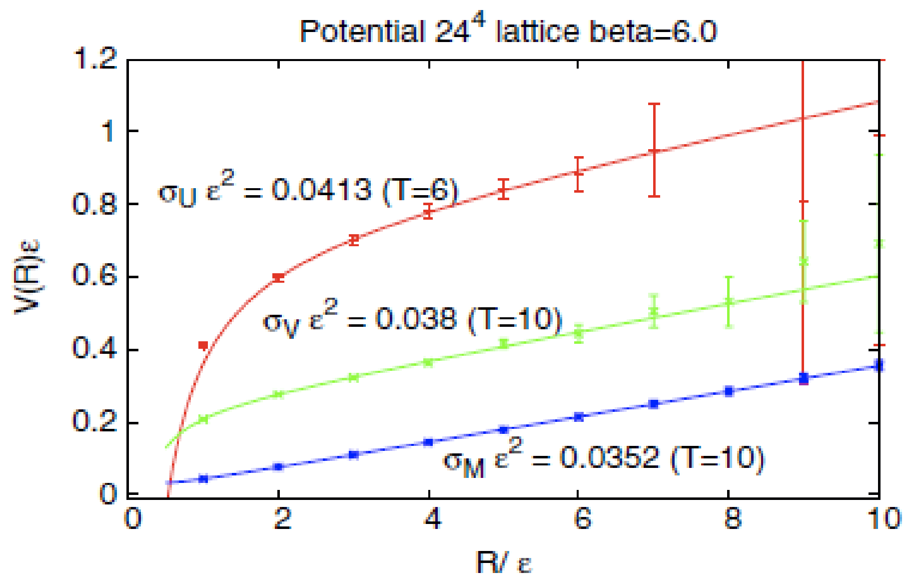


FIG. 1 (color online). $SU(3)$ quark-antiquark potentials as functions of the quark-antiquark distance R: (from tob to bottom) (i) full potential $V_f(R)$ (red curve), (ii) restricted part $V_r(R)$ (green curve), and (iii) ma;gnetic-monopole part $V_m(R)$ (blue curve), measured at $\beta = 6.0$ on 24^4 using 500 configurations where ϵ is the lattice spacing.

PRD 83, 114016 (2011)

We focus on the dual Meissner effect in SU(3) Yang-Mills theory. By measuring the distribution of chromo-electric field strength created by a static quark-antiquark pair, we discuss whether or not the non-Abelian dual superconductivity claimed by us is indeed a mechanism of quark confinement in SU(3) Yang-Mills theory.

MEASUREMENT OF COLOR FLUX

Color Flux measurement of SU(3)-YM field

Many works on measurement of color flux by using Wilson line/loop operator of the original YM field,;

- Mario Salvatore Cardaci, Paolo Cea, Leonardo Cosmai, Rossella Falcone and Alessandro Papa, Phys.Rev.D83:014502,2011 (also lattice2011)
- N. Cardoso, M. Cardoso, P. Bicudo, arXiv:1107.1355 [hep-lat] (also lattice2011)
- Ahmed S. Bakry, Derek B. Leinweber, Anthony G. Williams, e-Print: arXiv:1107.0150 [hep-lat]
- Pedro Bicudo,Marco Cardoso, Nuno Cardoso, PoS LATTICE2010:268, 2010.
- Paolo Cea, Leonardo Cosmai, Phys.Rev.D52:5152-5164,1995
- …..

We directly measure the color flux of **restricted non-Abelian variable which play a dominant role in quark confinement**

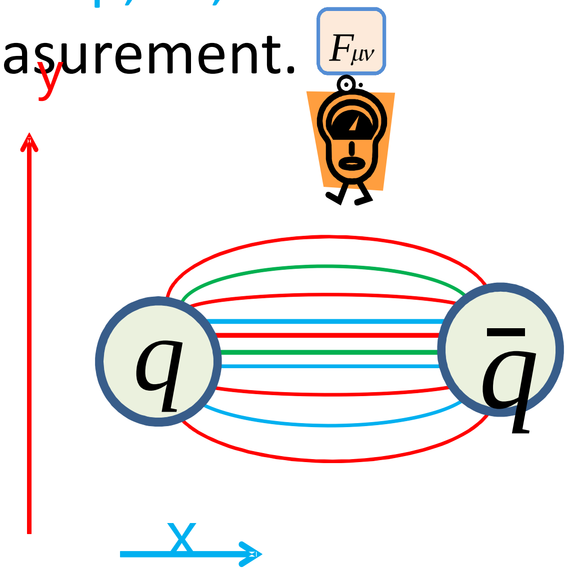
Measurements of Color Flux

■ Basic idea

- Color flux between quark and antiquark is obtained by measuring field strength.
- In order to measure it **in gauge invariant way**, the sources (pair of quark and antiquark) can be presented by Wilson loop (line) operator.
- Thus, correlation function between Wilson loop, *W*, and plaquette, *Up* can be an operator of flux measurement.

Measurement is done by using two type of operator

- The original YM field ***U***
- The restricted U(2) field ***V***



$$\rho_W = \frac{\langle \mathrm{tr}(WLU_pL^\dagger)\rangle}{\langle \mathrm{tr}(W)\rangle} - \frac{1}{N}\frac{\langle \mathrm{tr}(W)\mathrm{tr}(U_p)\rangle}{\langle \mathrm{tr}(W)\rangle}$$

Proposed by Adriano Di Giacomo et.al.
[Phys.Lett.B236:199,1990]
[Nucl.Phys.B347:441-460,1990]

This operator is sensitive to the field strength rather than square of the field strength,

$$\rho'_W = \frac{\langle \mathrm{tr}(W)\mathrm{tr}(U_p)\rangle}{\langle \mathrm{tr}(W)\rangle} - \langle \mathrm{tr}(U_p)\rangle$$

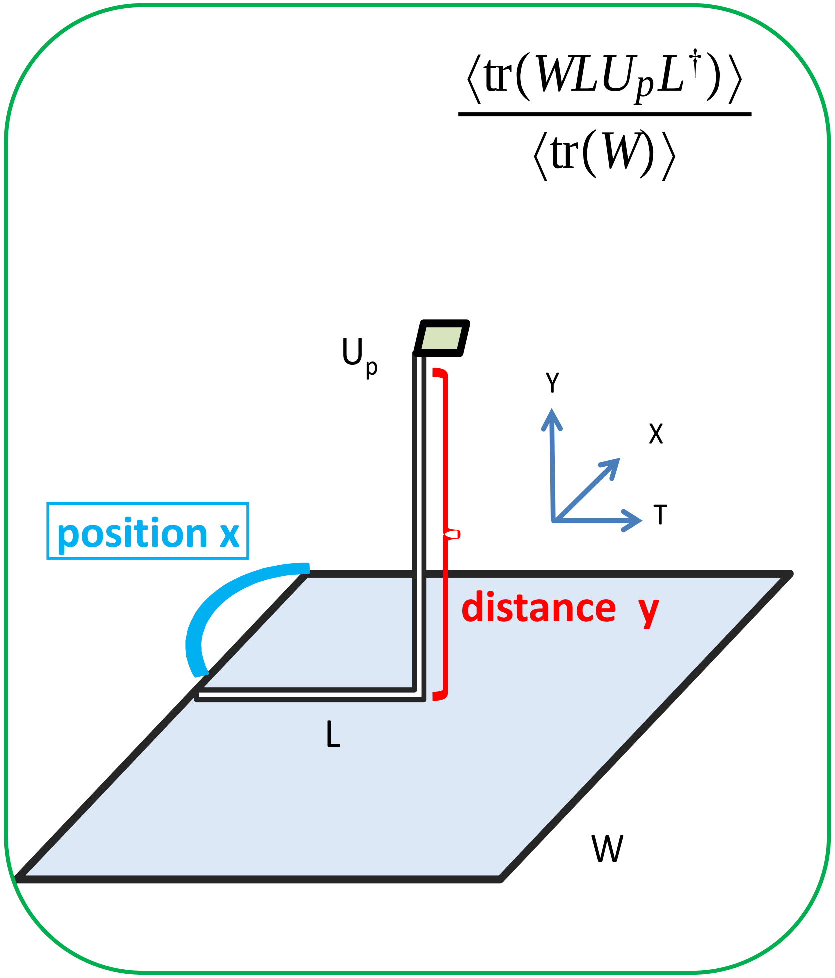
since in the (naive) continuum limit, we have

$$\rho'_W \overset{\epsilon\to 0}{\simeq} \frac{g^2\epsilon^2}{2}\left[\langle \mathcal{F}^2_{\mu\nu}\rangle_{\bar{q}q} - \langle \mathcal{F}^2_{\mu\nu}\rangle_0\right]$$

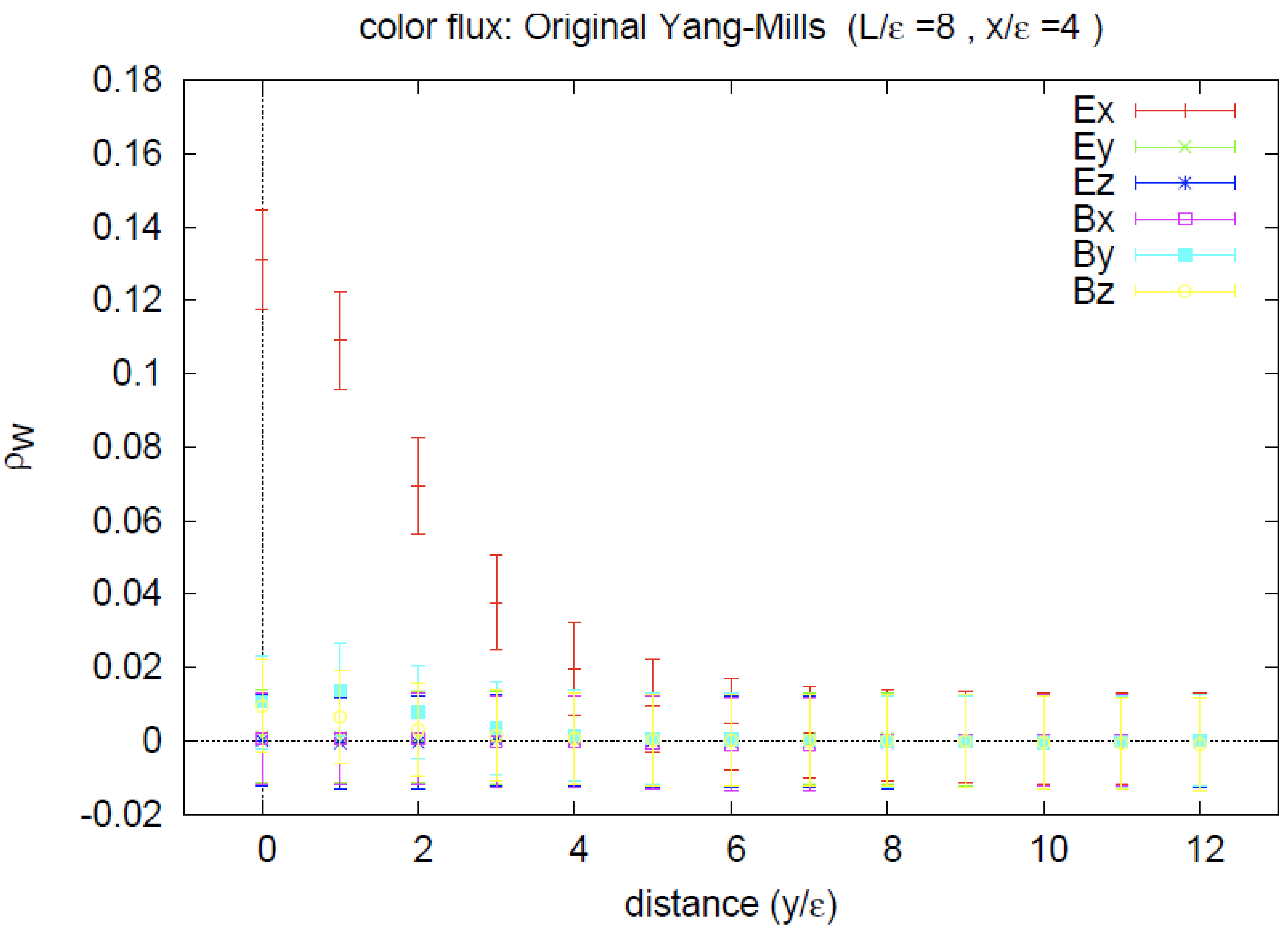
$$\rho_W \overset{\epsilon\to 0}{\simeq} g\epsilon\frac{\langle \mathrm{tr}(\mathcal{F}_{\mu\nu}L^\dagger WL)\rangle}{\langle \mathrm{tr}(L^\dagger WL)\rangle} = g\epsilon\langle\langle \mathcal{F}_{\mu\nu}\rangle\rangle_{q\bar{q}}$$

The field strength by quark and anti quark can be defined as

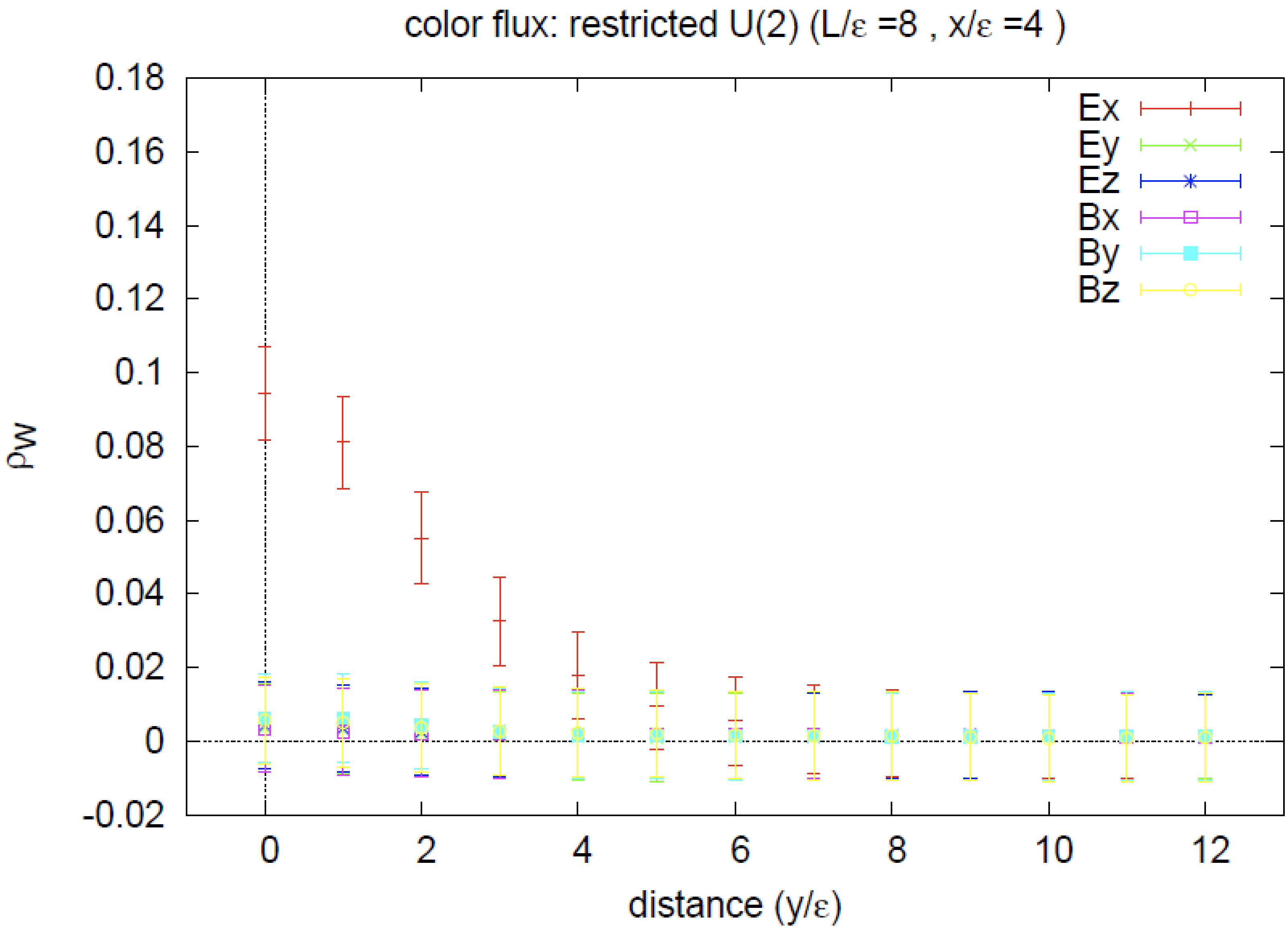
$$F_{\mu\nu}(x) = \sqrt{\frac{\beta}{2N}}\,\rho_W(x)$$



Color Flux by Original YM Field

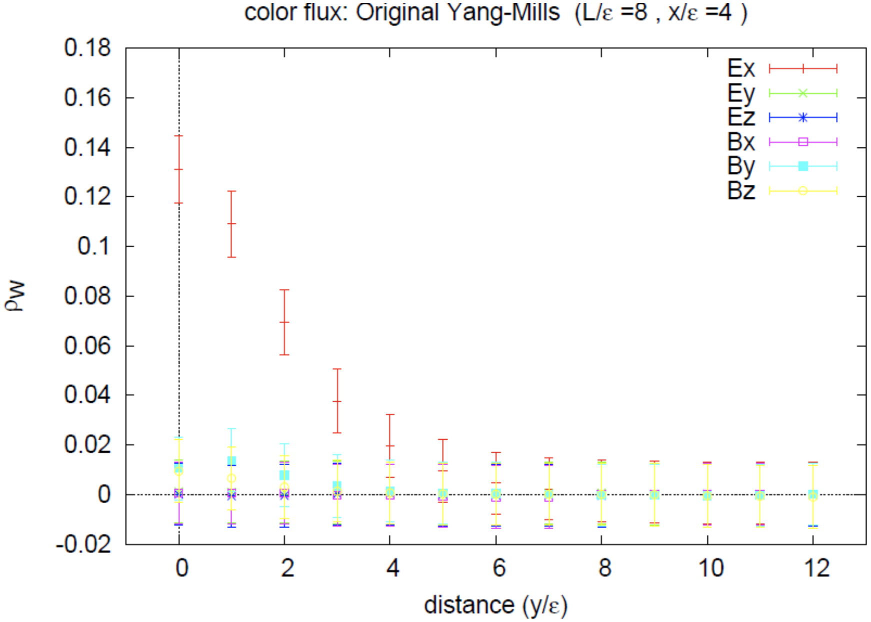


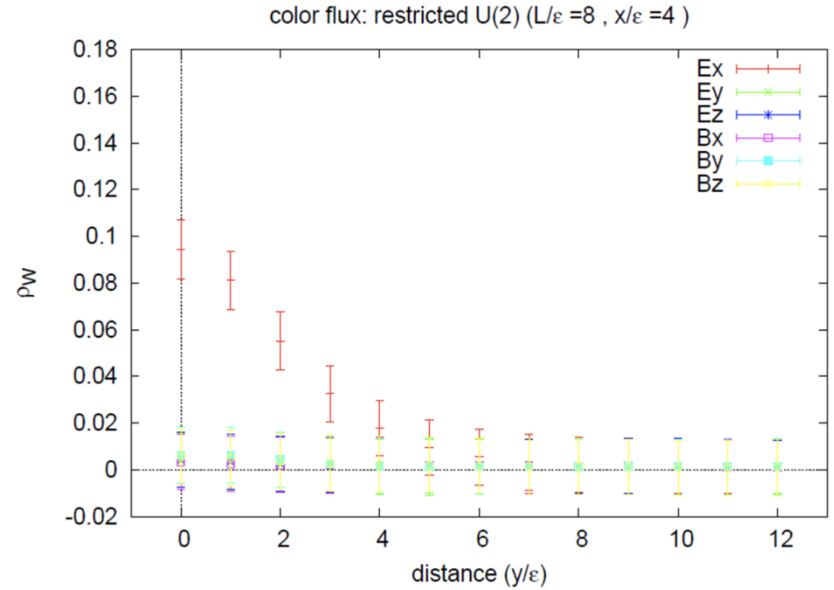
Color flux by new variables (our new formulation)



Original YM filed

Restricted U(2) field

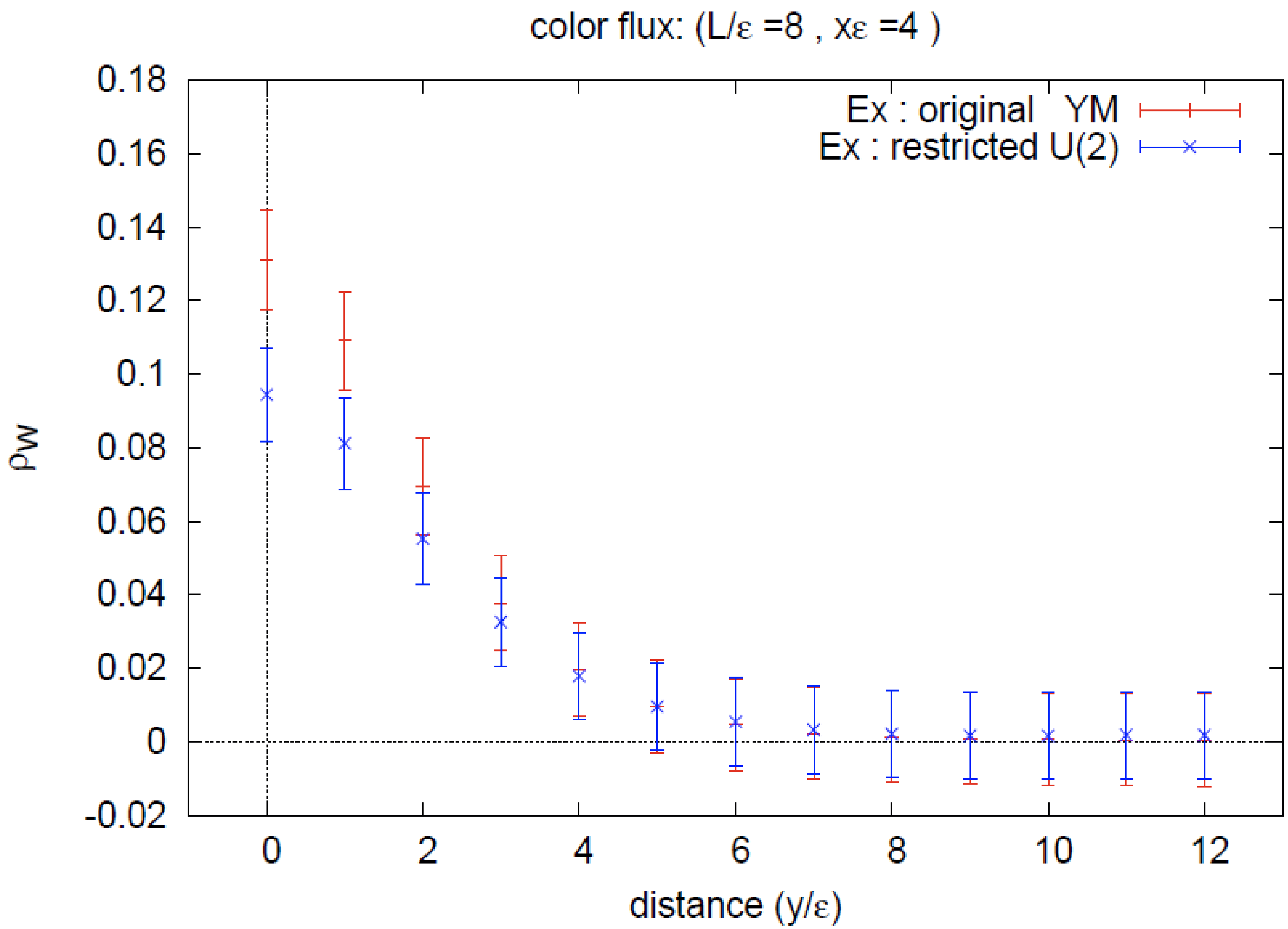




Measurement of color flux by Wilson line operator of the original YM field ($\boldsymbol{U}_\mu$) and by the operator of Restricted U(2) field ($\boldsymbol{V}_\mu$) .

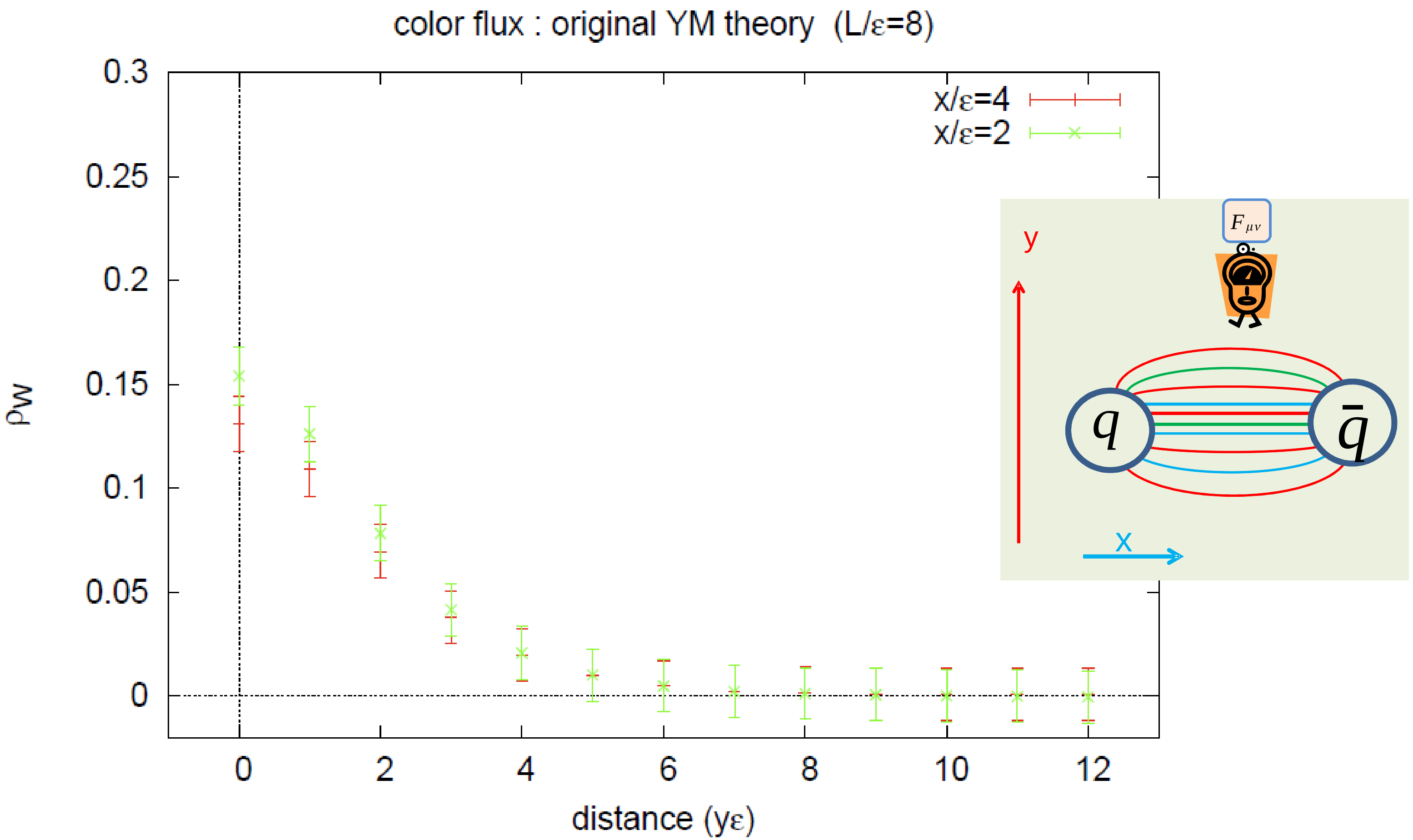
- Only Ex component of the chromo-electro field is detected and damps quickly as getting off from center.
- Restricted U(2) field almost reproduce the color flux of the original YM field.

Restricted U(2) dominance in color flux



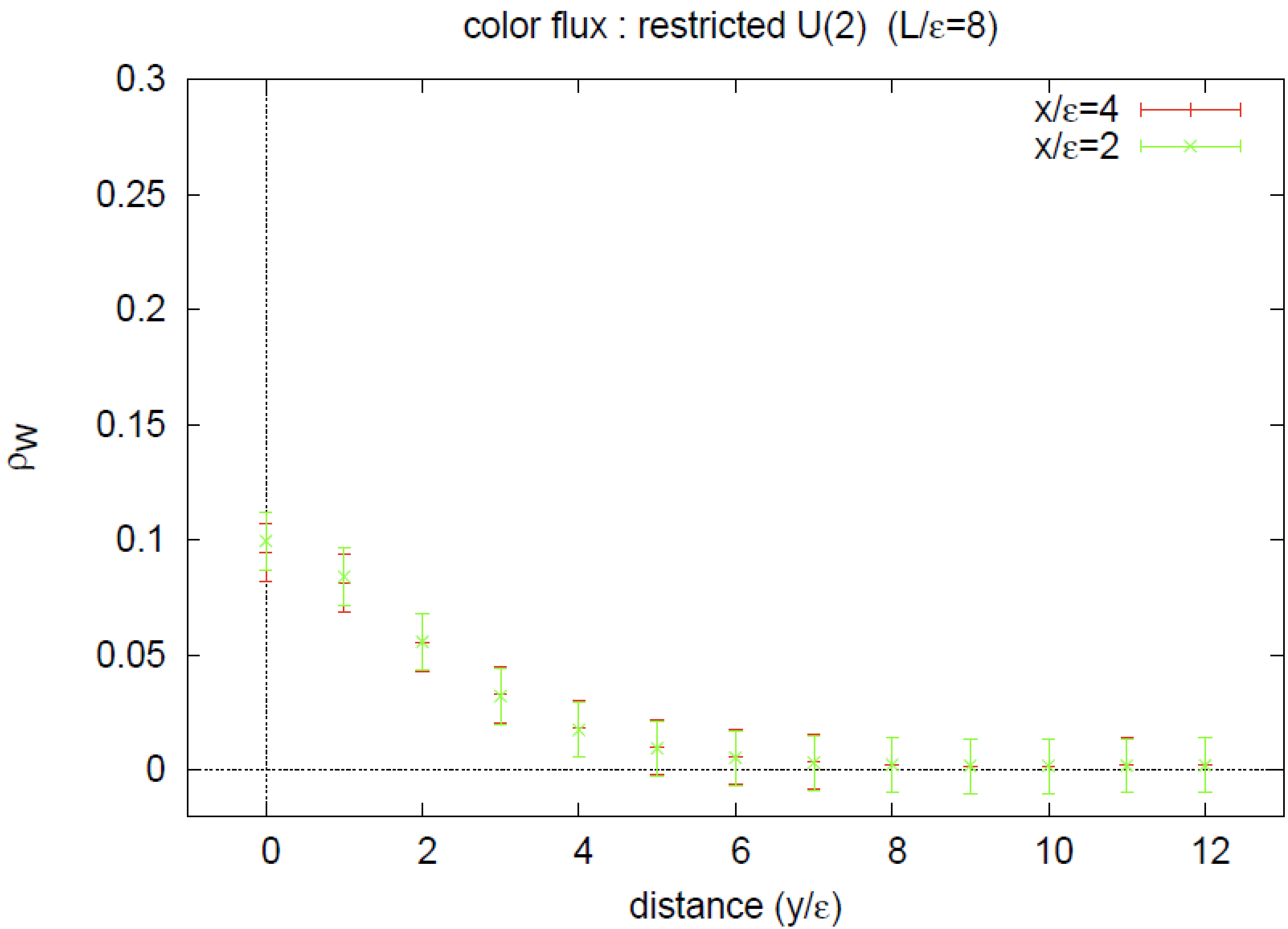
- Comparison of the electric field Ex shows good agreement between color flux generated by the original YM fields and one by restricted U(2) field

Color flux tube of original YM fields



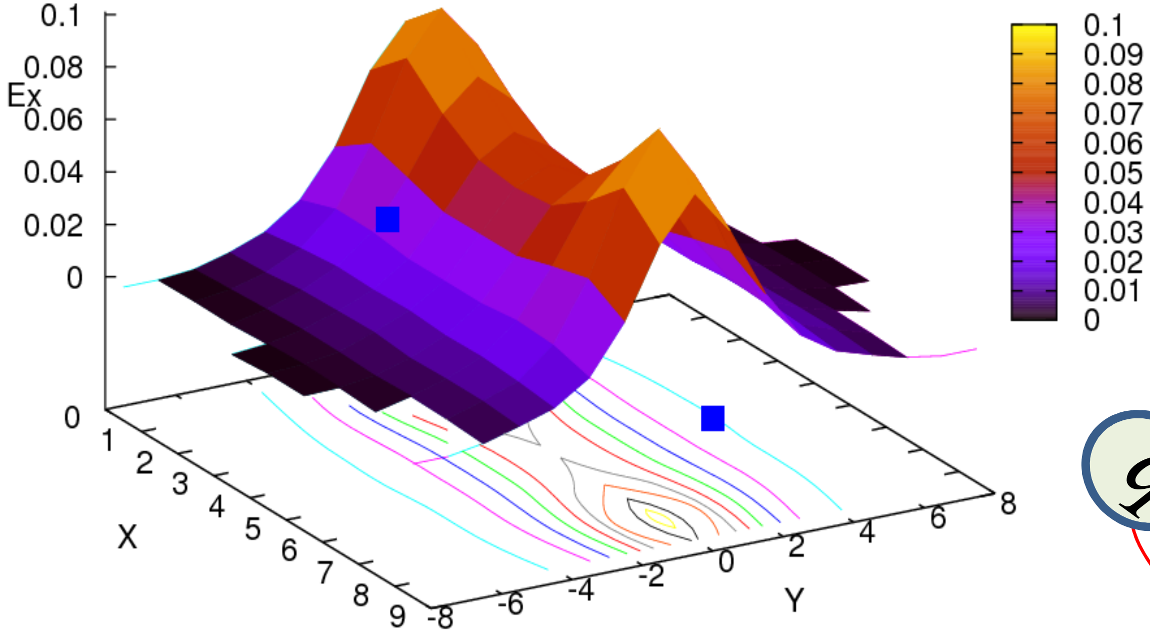
- Check of color flux tube by changing the position of the plaquette *Up*.

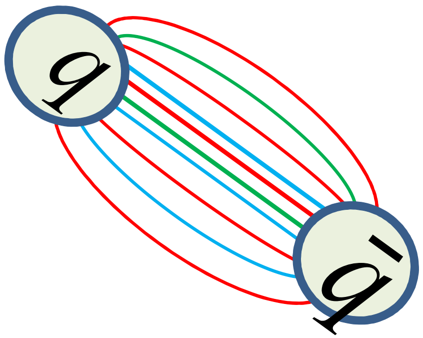
Color flux tube in restricted U(2) field.



Color Flux Tube

- Original YM filed

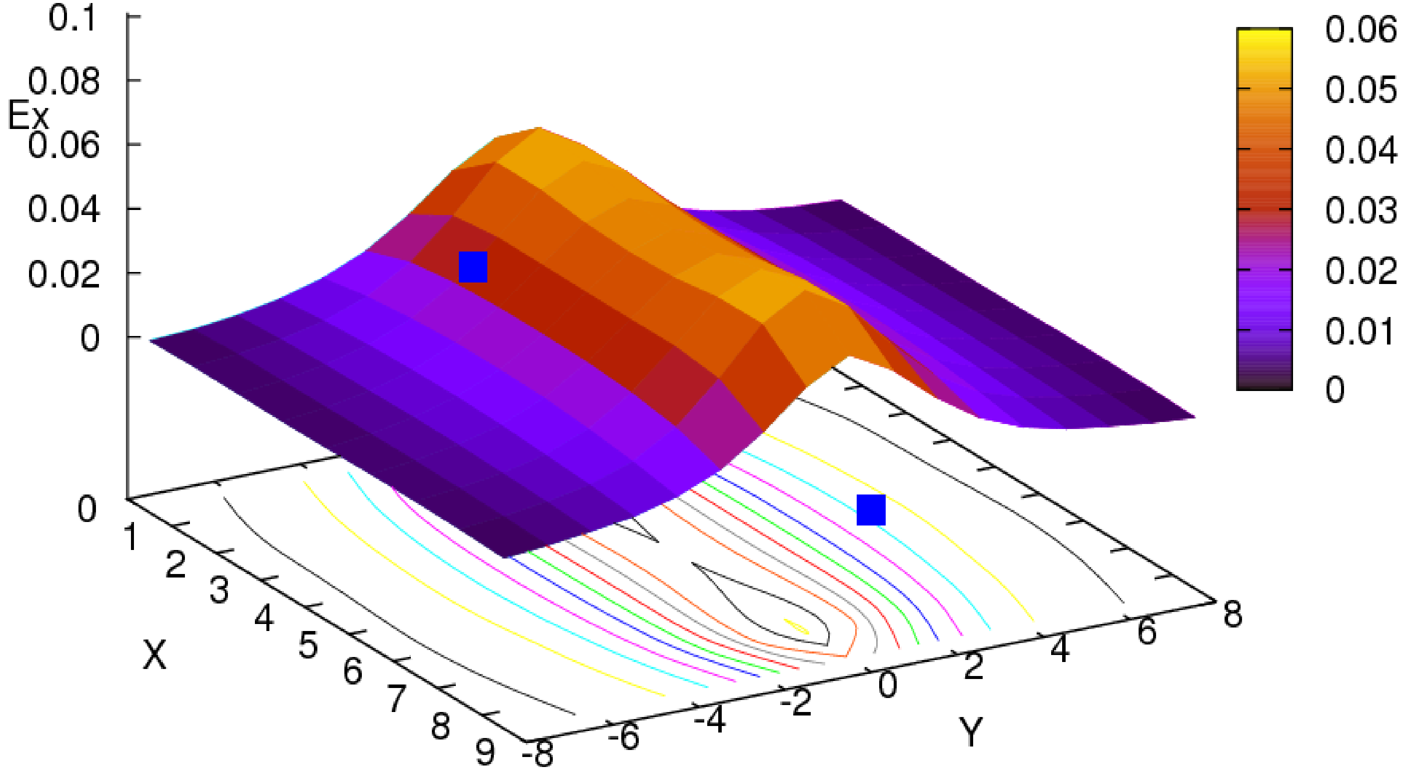




Measurement of color flux in X-Y plain.

- Field strength of Ex field is plotted for the original YM field (upper) and the restricted U(2) field (lower).
- quark-antiquark source is given by 9x11 Wilson loop in X-T plain. Thus, the quark and antiquark (marked by blue solid box) are located at (0,0) and (9,0) in the X-Y plain.

U(2) restricted field (V-field)



In what follows , we discuss the propagators (correlation functions), and the Yang-Mills field is fixed to Landau gauge.

CORRELATION FUNCTIONS OF DECOMPOSED VARIABLES

global SU(3) (color) symmetry

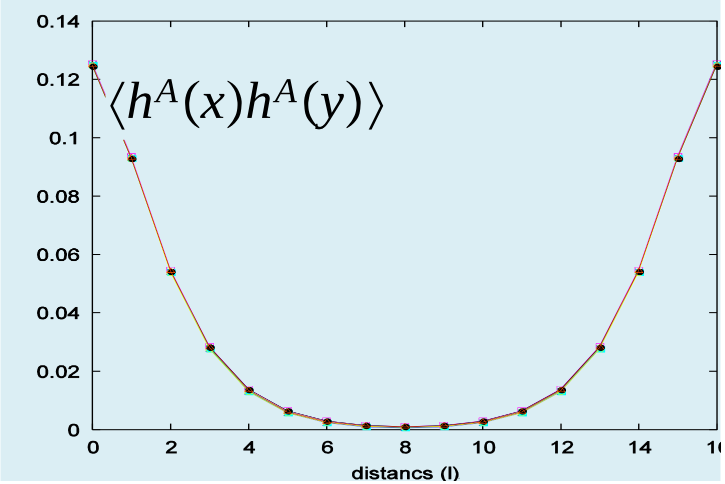
- **YM filed in the Landau gauge has global SU(3) symmetry.**

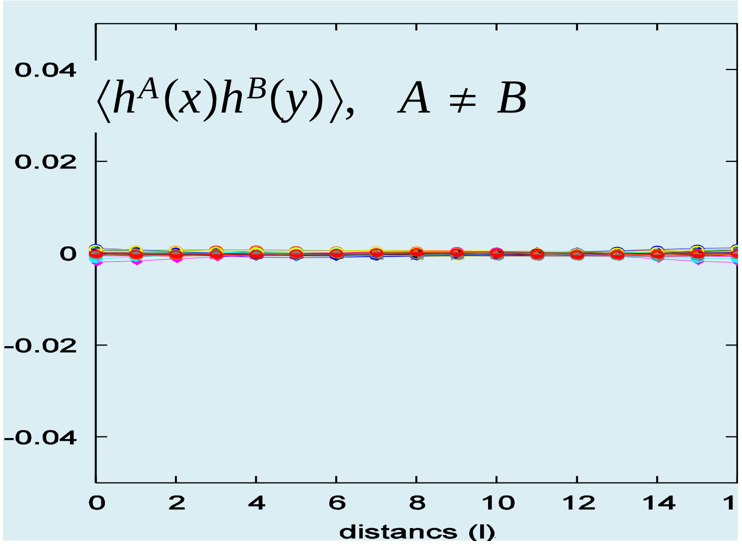
- **VEV of color field**

$$\langle h^A(x)\rangle = 0 \pm 0.002$$

$$\langle h_x^A h_y^B\rangle = \delta^{AB} D(x-y)$$

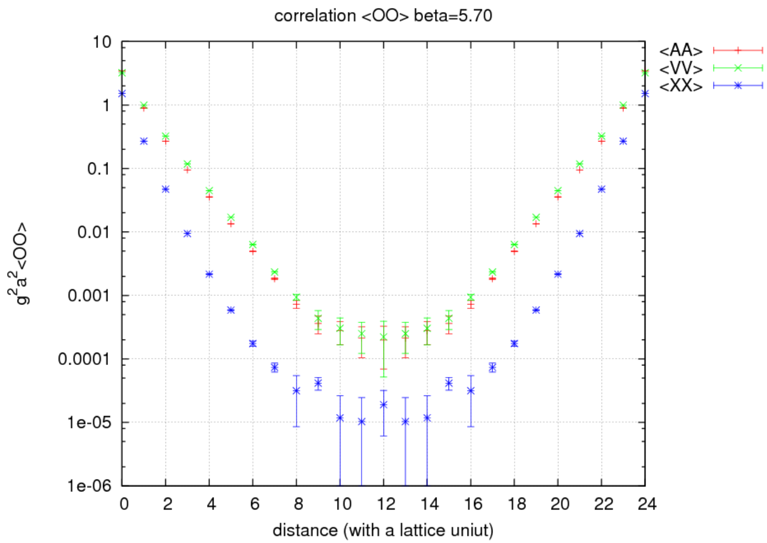
- **Two point correlation function** of color vector fields. (right figures)

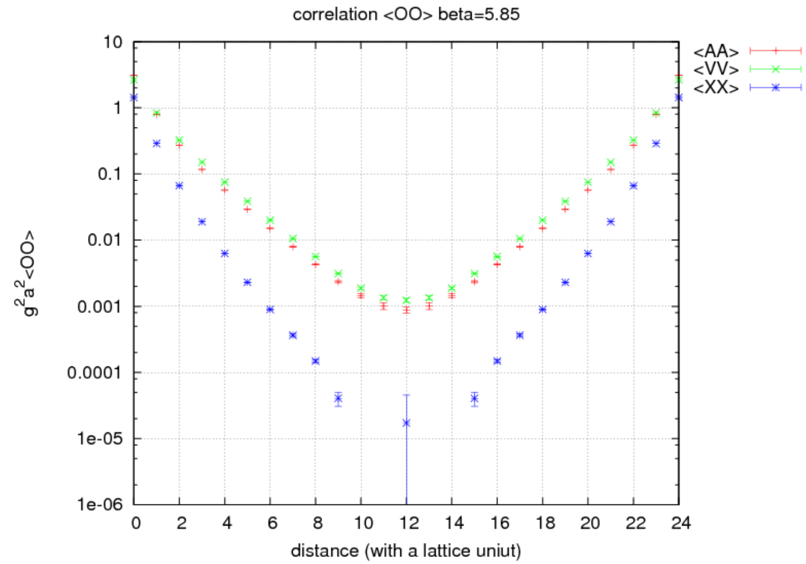


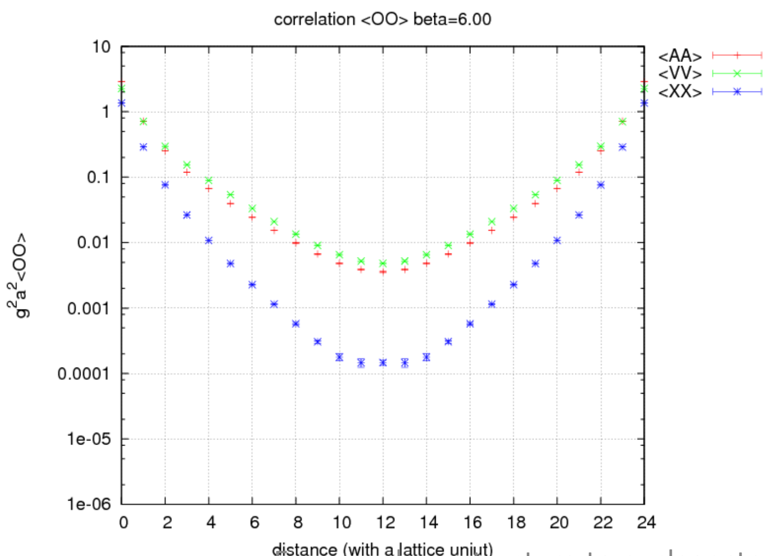


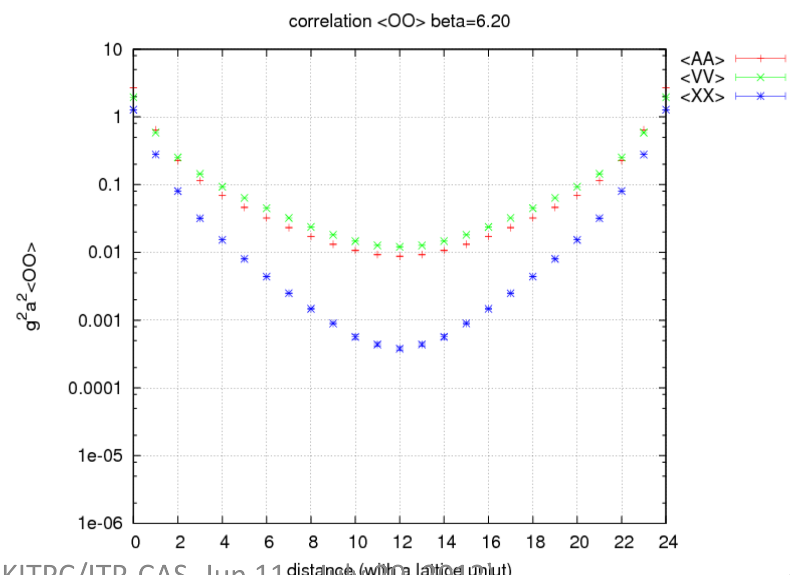
Color symmetry is preserved.

Correlation functions





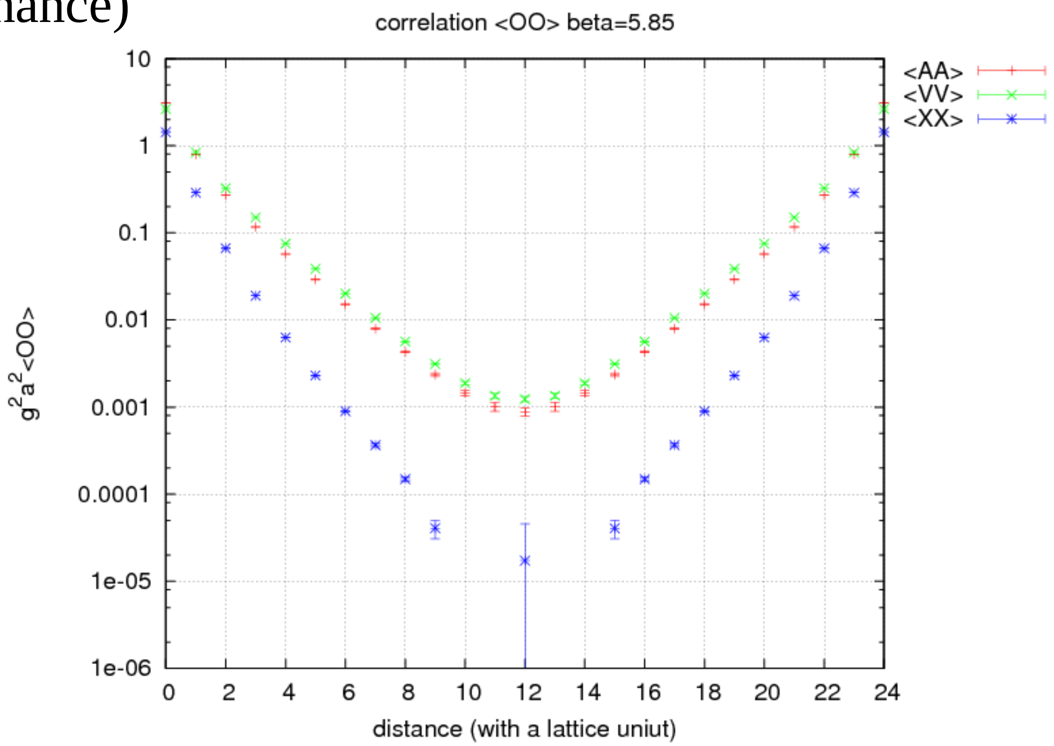




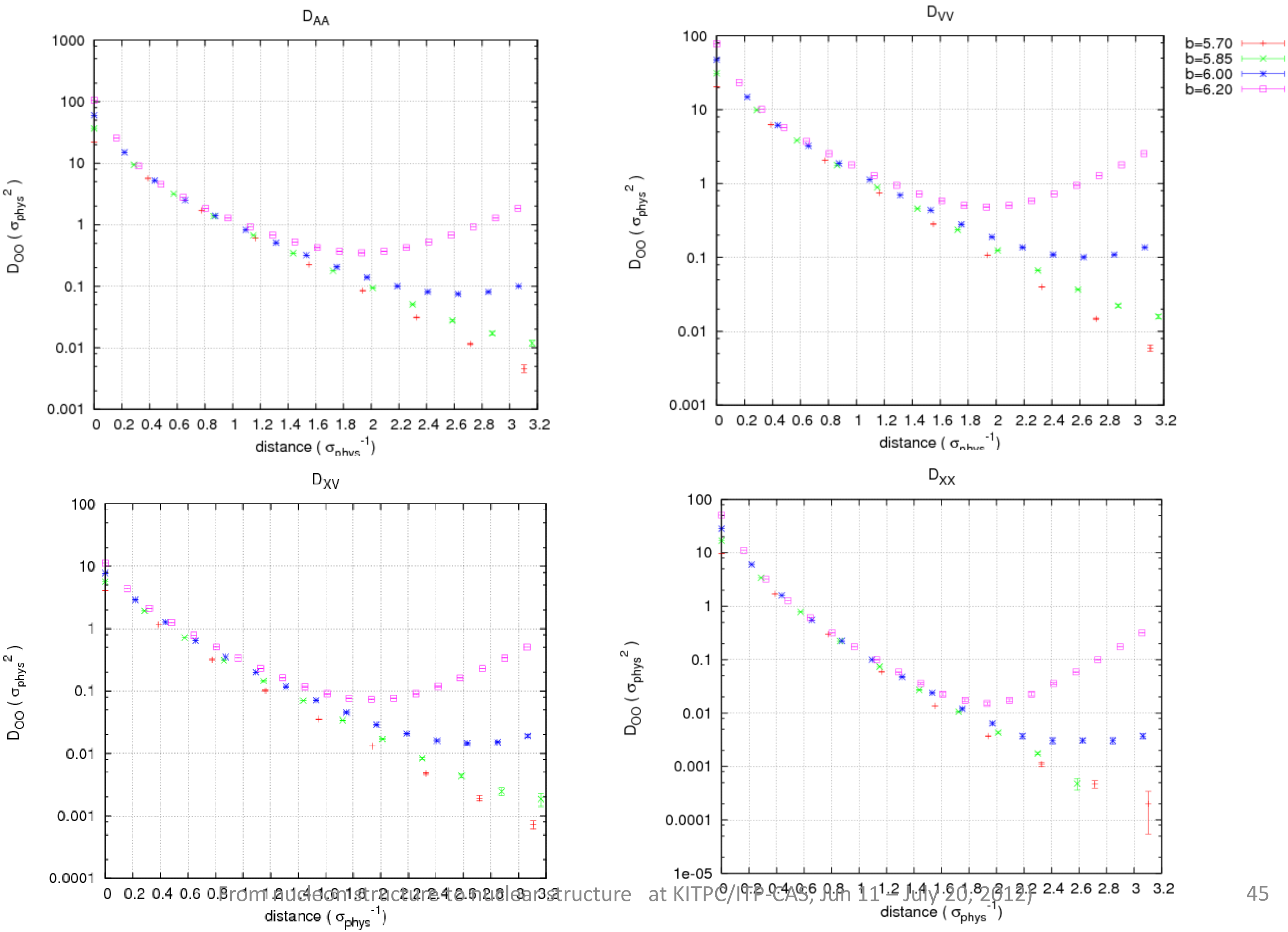
Infrared U(2) dominance

- The correlation function for the original YM filed in the Landau gauge and new variables, V, X.
- <VV> is almost the same as <AA> .
- <XX> is damping quickly .

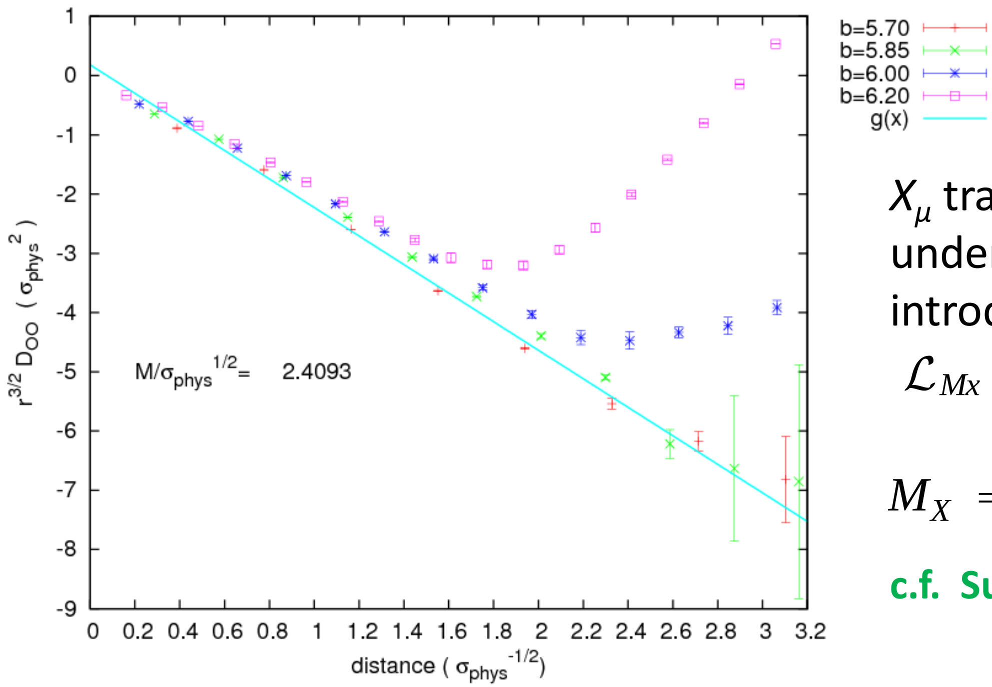
- ➔IR V dominance (U(2)-dominance)



Rescaled correlation function by lattice spacing



Mass generation of the gauge boson



X_μ transforms adjointly $X_{x,\mu} \rightarrow X'_{x,\mu} = \Omega_x X_{x,\mu} \Omega_x^\dagger$ under the gauge transformation, we can introduce the mass term

$$\mathcal{L}_{Mx} = -\frac{1}{2} M_x^2 X_\mu(x) X_\mu(x)$$

$$M_X = 2.409 \sqrt{\sigma_{phys}} = 1.1 GeV$$

c.f. Suganuma et.al in MAG and Abelian projection

The gauge boson propagator $D_{\mu\nu}^{XX}(x-y)$ is related to the Fourier transform of the massive propagator

$$D_{\mu\nu}^{XX}(x-y) = \langle X_\mu(x) X_\nu(y) \rangle = \int \frac{d^4k}{(2\pi)^4} e^{ik(x-y)} D_{\mu\nu}^{XX}(k)$$

The scalar type of propagator as function r should behave for large M_x as

$$D^{XX}(r) = \langle X_\mu(x) X_\mu(y) \rangle = \int \frac{d^4k}{(2\pi)^4} e^{ik(x-y)} \frac{3}{k^2 + M_X^2} \simeq \frac{3\sqrt{M}}{2(2\pi)^{3/2}} \frac{e^{-M_x r}}{r^{3/2}}$$

Summary

- We have presentation a new lattice formulation of Yang-Mills theory, that gives the gauge-link decomposition in the gauge independent way for SU(N) Yang-Mills fields, $U_{x,\mu}=X_{x,\mu}V_{x,\mu}$, such that the decomposed V variable play the dominant role for the quark confinement.
- We have defined non-Abelian magnetic monopole in gauge independent (invariant) way.
- As for the **the fundamental representation of fermion,** we have shown that Wilson loop is represented by V field of minimal option as the result of **non-Abelian stokes theorem**. Note the maximal option (the conventional Abeilan projection in MAG) corresponds to the Wilson loop for the higher representation.

Summary(2)

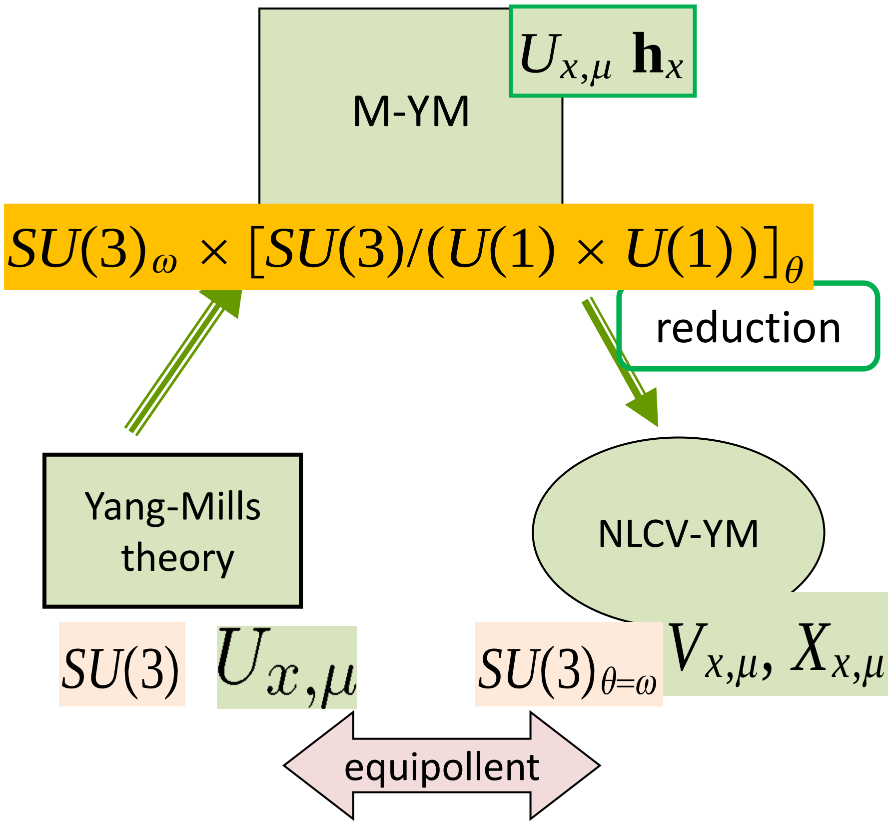
- We have studied the dual Meissner effect in SU(3) Yang-Mills theory by measuring the distribution of chromo-electric field strength created by a static quark-antiquark pair.
  - We have found chromo-electric flux tube both in **the original YM field** and in the **restricted U(2) field**
  - These results confirm the **non-Abelian dual superconductivity** due to non-Abelian magnetic monopoles we have proposed.
- We have performed the numerical simulation in the minimal option of the SU(3) lattice Yang-Mills theory and shown:
  - V-dominance (say, U(2)-dominance) in the string tension (85-95%)
  - Non-Abelian magnetic monopole dominance in string tension (75%)
  - color symmetry preservation, infrared V-dominance (U(2)-dominance) of correlation function of decomposed field in LLG.
  - Mass generation for X-field M_x=1.1GeV in LLG.

Outlook

- Magnetic monopole condensation and phase transition in finite temperature
- Direct measurement of the induced magnetic monopole current.
- Propagators in the momentum space in the deep IR region
  - To examine that whether the propagator in the momentum space is the Gribov-Stingl type or not.

- Study of the maximal case.
  - Confinement of the fermion with the higher representation
  - To study of the gluon confinement.

The decomposition of SU(3) link variable: maximal option



$$U_{x,\mu} = X_{x,\mu} V_{x,\mu}$$

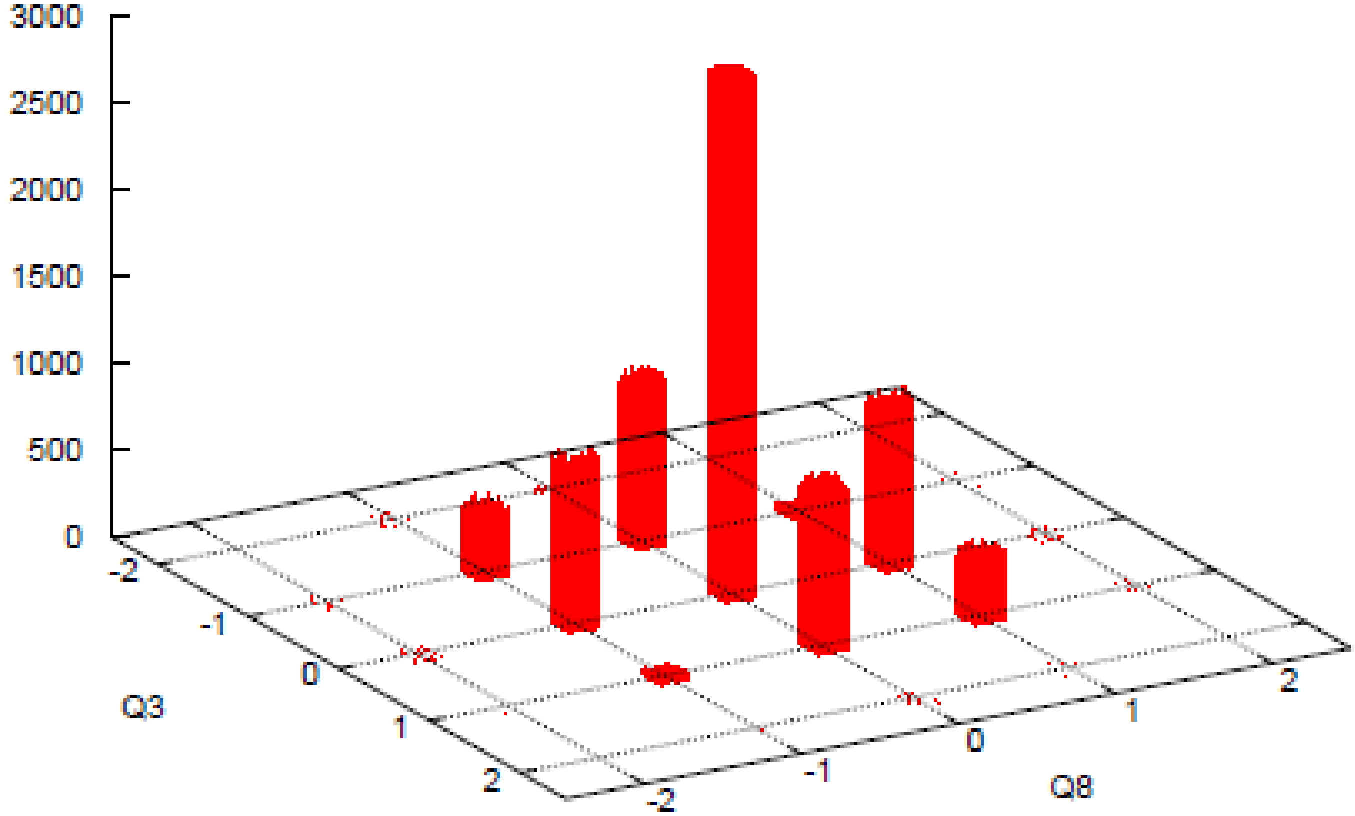
$$U_{x,\mu} \to U'_{x,\mu} = \Omega_x U_{x,\mu} \Omega^\dagger_{x+\mu}$$

$$V_{x,\mu} \to V'_{x,\mu} = \Omega_x V_{x,\mu} \Omega^\dagger_{x+\mu}$$

$$X_{x,\mu} \to X'_{x,\mu} = \Omega_x X_{x,\mu} \Omega^\dagger_{x}$$

$$\Omega_x \in G = SU(N)$$

Distribution of monopole currents (maximal case)

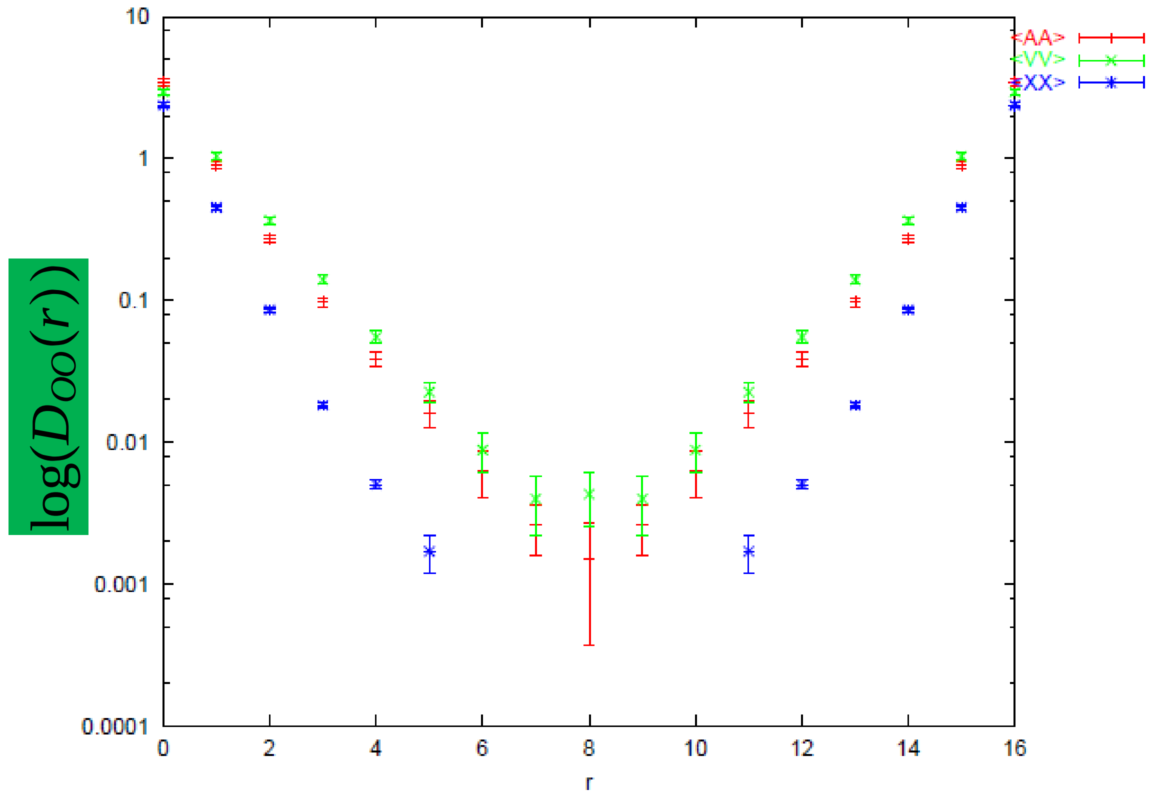


#configurations = 120
distributions are blocked on lattice site (quantized charge)

From nucleon structure to nuclear structure at KITPC/ITP-CAS, Jun 11 – July 20, 2012)

Correlation function for new variables (Propagators)

$$D_{OO}(r) = \langle O(x) O(y) \rangle \quad \text{O=A,V,X}$$



From nucleon structure to nuclear structure at KITPC/ITP-CAS, Jun 11 – July 20, 2012)

THANK YOU FOR ATTENTION