Di-hadron Productions

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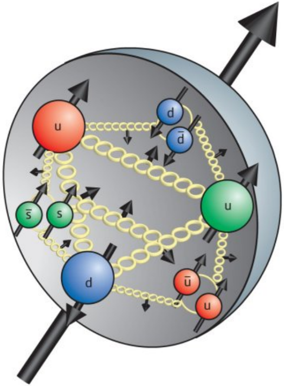
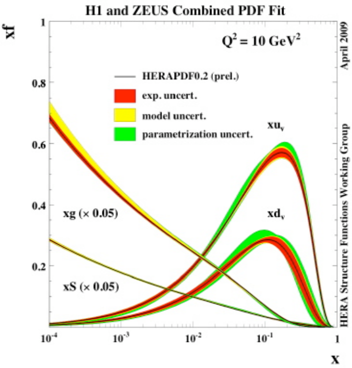
Outline

Deep into low-x region of Protons

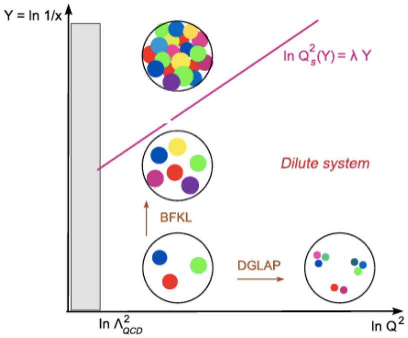




- Partons in the low-x region is dominated by gluons.
- Gluon splitting functions have $1/x$ singularities.
- Resummation of the $\alpha_s \ln \frac{1}{x}$.
- The dynamics becomes non-linear at high gluon density.

Phase diagram in QCD

Consider the evolution inside a hadron:



- Low Q^2 and low x region $\Rightarrow$ saturation region.
- Use BFKL equation and BK equation instead of DGLAP equation.
- BK equation is the non-linear small-x evolution equation which describes the saturation physics.

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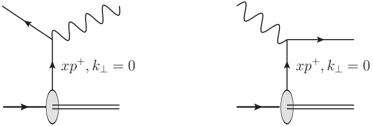
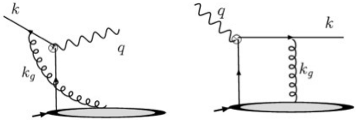
Collinear Factorization vs $k_\perp$ Factorization

Collinear Factorization



$k_\perp$ Factorization(Spin physics and saturation physics)



- The incoming partons carry no $k_\perp$ in the Collinear Factorization.
- In general, there is intrinsic $k_\perp$. It can be negligible for partons in protons, but should be taken into account for the case of nucleus target with large number of nucleons ($A \to \infty$).
- $k_\perp$ Factorization: High energy evolution with $k_\perp$ fixed.
- Initial and final state interactions yield different gauge links. (Process dependent)
- In collinear factorization, gauge links all disappear in the light cone gauge, and PDFs are universal.

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k_t dependent parton distributions

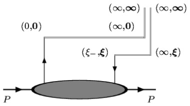
The unintegrated quark distribution

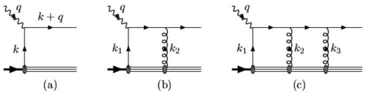
$$f_q(x, k_\perp) = \int \frac{d\xi^- d^2\xi_\perp}{4\pi(2\pi)^2} e^{ixP^+\xi^- + i\xi_\perp \cdot k_\perp} \langle P \left| \bar{\psi}(0)\mathcal{L}^\dagger(0)\gamma^+\mathcal{L}(\xi^-, \xi_\perp)\psi(\xi_\perp, \xi^-) \right| P\rangle$$

as compared to the integrated quark distribution

$$f_q(x) = \int \frac{d\xi^-}{4\pi} e^{ixP^+\xi^-} \langle P \left| \bar{\psi}(0)\gamma^+\mathcal{L}(\xi^-)\psi(0, \xi^-) \right| P\rangle$$

- The dependence of $\xi_\perp$ in the definition.
- Gauge invariant definition.
- Light-cone gauge together with proper boundary condition ⇒ parton density interpretation.
- The gauge links come from the resummation of multiple gluon interactions.
- Gauge links may vary among different processes.





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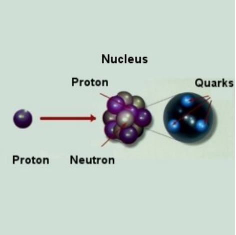
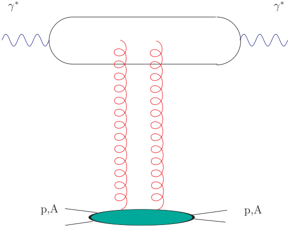
Dilute-Dense factorizations

The effective Dilute-Dense factorization





- Protons and virtual photons are dilute probes of the dense gluons inside target hadrons.
- For pA (dilute-dense system) collisions, there is an effective k_t factorization.

$$\frac{d\sigma^{pA\to qfX}}{d^2P_\perp d^2q_\perp dy_1 dy_2} = x_p q(x_p,\mu^2) x_A f(x_A, q_\perp^2) \frac{1}{\pi}\frac{d\hat{\sigma}}{d\hat{t}}.$$

- For dijet processes in pp, AA collisions, there is no k_t factorization[Collins, Qiu, 08],[Rogers, Mulders; 10].
- At forward rapidity y, $x_p \propto e^y$ is large, while $x_A \propto e^{-y}$ is small.
- Ideal opportunity to search gluon saturation.
- Systematic framework to test saturation physics predictions.

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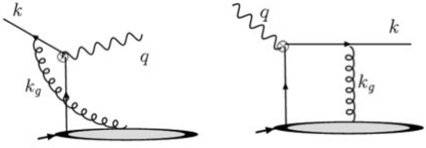
Why is the di-jet production process special?

Initial state interactions and/or final state interactions



- In Drell-Yan process, there are only initial state interactions.

$$\int_{-\infty}^{+\infty} \mathrm{d}k_g^+ \frac{i}{-k_g^+ - i\epsilon} A^+(k_g) = \int_0^{-\infty} \mathrm{d}\zeta^- A^+(\zeta^-)$$

Eikonal approximation $\Longrightarrow$ gauge links.

- In DIS, there are only final state interactions.

$$\int_{-\infty}^{+\infty} \mathrm{d}k_g^+ \frac{i}{-k_g^+ + i\epsilon} A^+(k_g) = \int_0^{+\infty} \mathrm{d}\zeta^- A^+(\zeta^-)$$

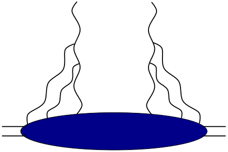
Eikonal approximation $\Longrightarrow$ gauge links.

- However, there are both initial state interactions and final state interactions in the di-jet process for all the active partons.

A Tale of Two Gluon Distributions

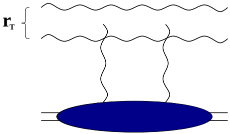
In small-x physics, two gluon distributions are widely used:[Kharzeev, Kovchegov, Tuchin; 03]
I. Weizsäcker Williams gluon distribution ([KM, 98] and MV model):

$$xG^{(1)} = \frac{S_\perp}{\pi^2\alpha_s}\frac{N_c^2-1}{N_c} \Leftarrow$$
$$\times \int \frac{d^2r_\perp}{(2\pi)^2}\frac{e^{-ik_\perp\cdot r_\perp}}{r_\perp^2}\left(1-e^{-\frac{r_\perp^2 Q_{sg}^2}{2}}\right)$$



II. Color Dipole gluon distributions:

$$xG^{(2)} = \frac{S_\perp N_c}{2\pi^2\alpha_s}k_\perp^2 \Leftarrow$$
$$\times \int \frac{d^2r_\perp}{(2\pi)^2}e^{-ik_\perp\cdot r_\perp}e^{-\frac{r_\perp^2 Q_{sq}^2}{4}}$$



Remarks:

- The WW gluon distribution simply counts the number of gluons.
- The Color Dipole gluon distribution often appears in calculations.
- Does this mean that gluon distributions are non-universal? Answer: Yes and No!

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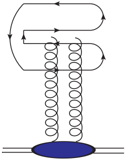
A Tale of Two Gluon Distributions

[F. Dominguez, C. Marquet, BX and F. Yuan, 11]

I. Weizsäcker Williams gluon distribution

$$xG^{(1)} = \frac{S_\perp}{\pi^2\alpha_s}\frac{N_c^2-1}{N_c} \times \int \frac{d^2r_\perp}{(2\pi)^2}\frac{e^{-ik_\perp\cdot r_\perp}}{r_\perp^2}\left(1-e^{-\frac{r_\perp^2 Q_{sg}^2}{2}}\right) \Leftarrow$$



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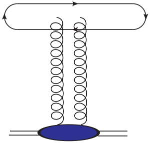
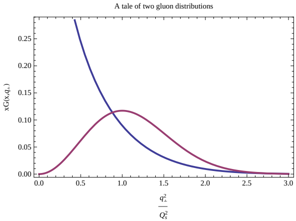




A Tale of Two Gluon Distributions

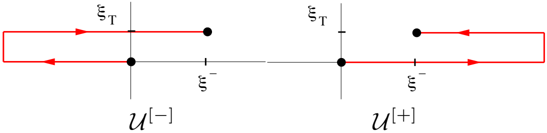
In terms of operators (known from TMD factorization), we find these two gluon distributions can be defined as follows: [F. Dominguez, C. Marquet, BX and F. Yuan, 11]

I. Weizsäcker Williams gluon distribution:

$$xG^{(1)} = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \mathrm{Tr}\langle P|F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[+]\dagger}F^{+i}(0)\mathcal{U}^{[+]}|P\rangle.$$

II. Color Dipole gluon distributions:

$$xG^{(2)} = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \mathrm{Tr}\langle P|F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[-]\dagger}F^{+i}(0)\mathcal{U}^{[+]}|P\rangle.$$



Remarks:

- The WW gluon distribution is the conventional gluon distributions. In light-cone gauge, it is the gluon density. (Only final state interactions.)
- The dipole gluon distribution has no such interpretation. (Initial and final state interactions.)
- Both definitions are gauge invariant.
- Same after integrating over $q_\perp$.

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A Tale of Two Gluon Distributions

In terms of operators, we find these two gluon distributions can be defined as follows: [F. Dominguez, C. Marquet, BX and F. Yuan, 11]

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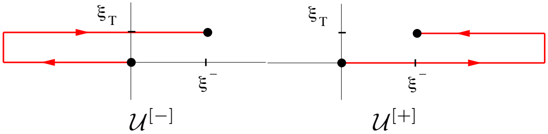

Questions:

- Can we distinguish these two gluon distributions? Yes, We Can.
- How to measure $xG^{(1)}$ directly? DIS dijet.
- How to measure $xG^{(2)}$ directly? Direct γ+Jet in pA collisions.
  For single-inclusive particle production in pA up to all order.
- What happens in gluon+jet production in pA collisions? It's complicated!

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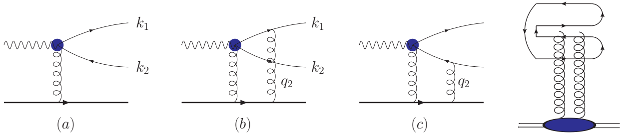
DIS dijet

[F. Dominguez, C. Marquet, BX and F. Yuan, 11]



$$\frac{d\sigma^{\gamma_T^* A\to q\bar{q}+X}}{d\mathcal{P}.\mathcal{S}.} \propto N_c\alpha_{em}e_q^2\int\frac{d^2x}{(2\pi)^2}\frac{d^2x'}{(2\pi)^2}\frac{d^2b}{(2\pi)^2}\frac{d^2b'}{(2\pi)^2}e^{-ik_{1\perp}\cdot(x-x')}$$

$$\times e^{-ik_{2\perp}\cdot(b-b')}\sum\psi_T^*(x-b)\psi_T(x'-b')$$

$$\underbrace{\left[1+S_{x_g}^{(4)}(x,b;b',x')-S_{x_g}^{(2)}(x,b)-S_{x_g}^{(2)}(b',x')\right]}_{-u_iu'_j\frac{1}{N_c}\langle\mathrm{Tr}\left[\partial^i U(v)\right]U^\dagger(v')\left[\partial^j U(v')\right]U^\dagger(v)\rangle_{x_g}\Rightarrow\text{Operator Def}},$$

- Eikonal approximation ⇒ Wilson Line approach [Kovner, Wiedemann, 01].
- In the dijet correlation limit, where $u = x - b \ll v = zx + (1-z)b$
- $S_{x_g}^{(4)}(x,b;b',x') = \frac{1}{N_c}\left\langle\mathrm{Tr}U(x)U^\dagger(x')U(b')U^\dagger(b)\right\rangle_{x_g} \neq S_{x_g}^{(2)}(x,b)S_{x_g}^{(2)}(b',x')$
- Quadrupoles are generically different objects and only appear in dijet processes.

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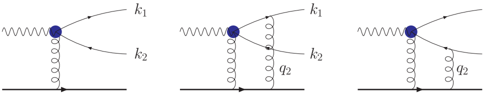
DIS dijet

The dijet production in DIS.



(a) (b) (c)

TMD factorization approach:

$$\frac{d\sigma^{\gamma_T^* A\to q\bar{q}+X}}{d\mathcal{P}.\mathcal{S}.} = \delta(x_{\gamma^*}-1)x_g G^{(1)}(x_g, q_\perp)H_{\gamma_T^* g\to q\bar{q}},$$

Remarks:

- Dijet in DIS is the only physical process which can measure Weizsäcker Williams gluon distributions.
- Golden measurement for the Weizsäcker Williams gluon distributions of nuclei at small-x. The cross section is directly related to the WW gluon distribution.
- EIC and LHeC will provide us perfect machines to study the strong gluon fields in nuclei. Important part in EIC and LHeC physics.

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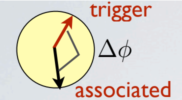
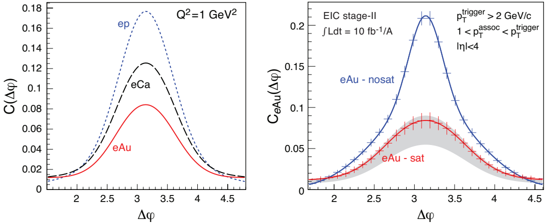
Di-Hadron correlations in DIS

Di-pion correlations at EIC

$$C(\Delta\phi) = \frac{\int_{|p_{1\perp}|,|p_{2\perp}|} \frac{d\sigma^{eA\to h_1 h_2}}{dy_1 dy_2 d^2p_{1\perp} d^2p_{2\perp}}}{\int_{|p_{1\perp}|} \frac{d\sigma^{eA\to h_1}}{dy_1 d^2p_{1\perp}}}$$

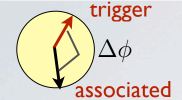




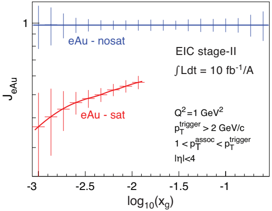
- EIC stage II energy 30×100GeV.
- Using: $Q^2_{sA} = c(b)A^{1/3}Q^2_s(x)$.
- Physical picture: Dense gluonic matter suppresses the away side peak.

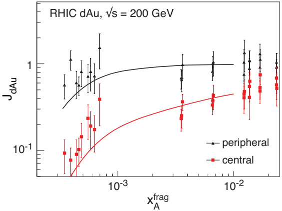
Di-Hadron correlations in DIS

The estimate of di-pion correlations at EIC



$$J_{eA} = \frac{1}{\langle N_{\rm coll}\rangle} \frac{\sigma_{eA}^{\rm pair}/\sigma_{eA}}{\sigma_{ep}^{\rm pair}/\sigma_{ep}}$$





- J is normalized to unity in the dilute regime.
- Physical picture: The cross sections saturates at low-x.

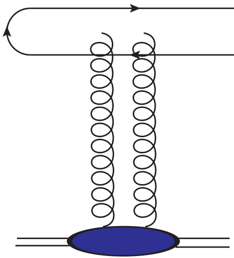
γ+Jet in pA collisions

The direct photon + jet production in pA collisions. (Drell-Yan follows the same factorization.)
TMD factorization approach:

$$\frac{d\sigma^{(pA\to\gamma q+X)}}{d\mathcal{P}.\mathcal{S}.} = \sum_f x_1 q(x_1,\mu^2) x_g G^{(2)}(x_g, q_\perp) H_{qg\to\gamma q}.$$

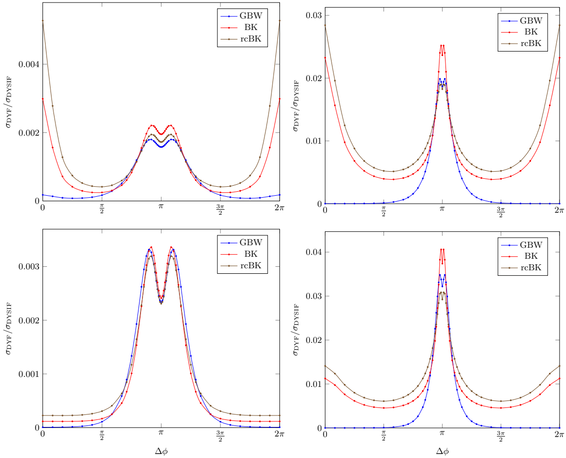
Remarks:

- Independent CGC calculation gives the identical result in the correlation limit.
- Direct measurement of the Color Dipole gluon distribution.



DY correlations in *pA* collisions

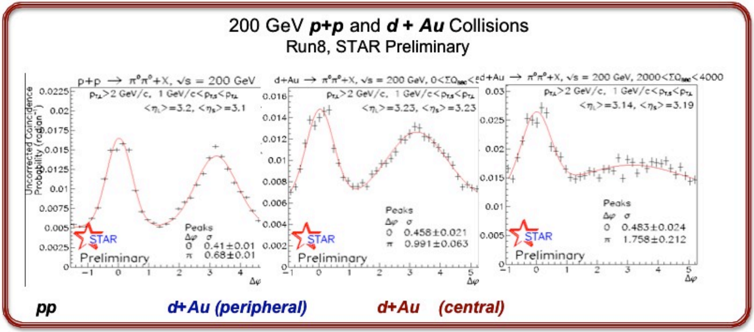
[Stasto, BX, Zaslavsky, 12]



$M = 0.5, 4\text{GeV}, Y = 2.5$ at RHIC dAu. $M = 4, 8\text{GeV}, Y = 4$ at LHC pPb.

- Partonic cross section vanishes at $\pi \Rightarrow$ Dip at π.
- Prompt photon calculation [J. Jalilian-Marian, A. Rezaeian, 12]

STAR measurement on di-hadron correlation in *dA* collisions



- There is no sign of suppression in the $p + p$ and $d + Au$ peripheral data.
- The suppression and broadening of the away side jet in $d + Au$ central collisions is due to the multiple interactions between partons and dense nuclear matter (CGC).
- Probably the best evidence for saturation.

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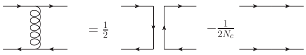
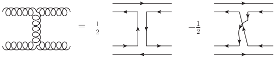
Dijet processes in the large N_c limit

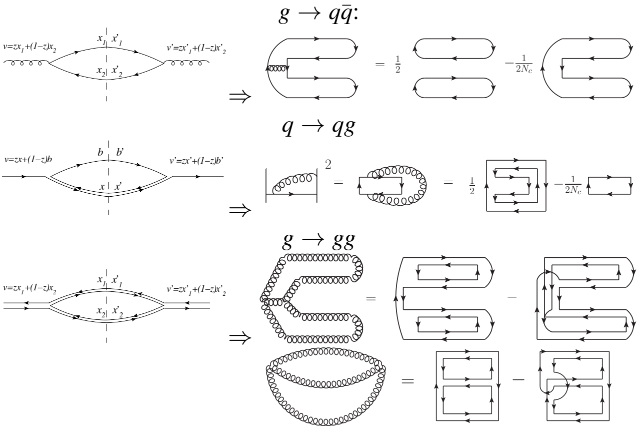
The Fierz identity:



and



Graphical representation of dijet processes



The Octupole and the Sextupole are suppressed.

Gluon+quark jets correlation

Including all the $qg \to qg$, $gg \to gg$ and $gg \to q\bar{q}$ channels, a lengthy calculation gives

$$\frac{d\sigma^{(pA\to \text{Dijet}+X)}}{d\mathcal{P}.\mathcal{S}.} = \sum_q x_1 q(x_1,\mu^2)\frac{\alpha_s^2}{\hat{s}^2}\left[\mathcal{F}_{qg}^{(1)}H_{qg}^{(1)} + \mathcal{F}_{qg}^{(2)}H_{qg}^{(2)}\right]$$
$$+x_1 g(x_1,\mu^2)\frac{\alpha_s^2}{\hat{s}^2}\left[\mathcal{F}_{gg}^{(1)}\left(H_{gg\to q\bar{q}}^{(1)} + \frac{1}{2}H_{gg\to gg}^{(1)}\right)\right.$$
$$\left.+\mathcal{F}_{gg}^{(2)}\left(H_{gg\to q\bar{q}}^{(2)} + \frac{1}{2}H_{gg\to gg}^{(2)}\right) + \mathcal{F}_{gg}^{(3)}\frac{1}{2}H_{gg\to gg}^{(3)}\right],$$

with the various gluon distributions defined as

$$\mathcal{F}_{qg}^{(1)} = xG^{(2)}(x,q_\perp), \quad \mathcal{F}_{qg}^{(2)} = \int xG^{(1)} \otimes F,$$
$$\mathcal{F}_{gg}^{(1)} = \int xG^{(2)} \otimes F, \quad \mathcal{F}_{gg}^{(2)} = -\int \frac{q_{1\perp}\cdot q_{2\perp}}{q_{1\perp}^2} xG^{(2)} \otimes F,$$
$$\mathcal{F}_{gg}^{(3)} = \int xG^{(1)}(q_1) \otimes F \otimes F,$$

where $F = \int \frac{d^2 r_\perp}{(2\pi)^2} e^{-iq_\perp\cdot r_\perp} \frac{1}{N_c}\left\langle \text{Tr}U(r_\perp)U^\dagger(0)\right\rangle_{x_g}$.

Remarks:

- All the above gluon distributions can be written as combinations and convolutions of two fundamental gluon distributions.
- This describes the dihadron correlation data measured at RHIC in forward dAu collisions.

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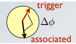
Comparing to STAR and PHENIX data

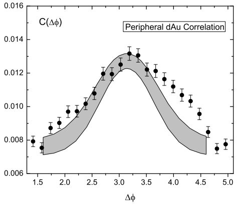
Physics predicted by C. Marquet. Further calculated in[A. Stasto, BX, F. Yuan, 11]

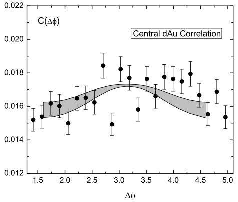


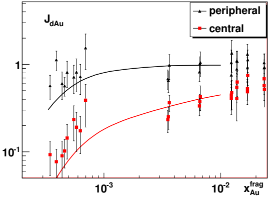
For away side peak in both peripheral and central *dAu* collisions

$$C(\Delta\phi) = \frac{\int_{|p_{1\perp}|,|p_{2\perp}|} \frac{d\sigma^{pA\to h_1 h_2}}{dy_1 dy_2 d^2p_{1\perp} d^2p_{2\perp}}}{\int_{|p_{1\perp}|} \frac{d\sigma^{pA\to h_1}}{dy_1 d^2p_{1\perp}}}$$

$$J_{dA} = \frac{1}{\langle N_{\text{coll}}\rangle} \frac{\sigma_{dA}^{\text{pair}}/\sigma_{dA}}{\sigma_{pp}^{\text{pair}}/\sigma_{pp}}$$







- Using: $Q_{sA}^2 = c(b)A^{1/3}Q_s^2(x)$.
- Physical picture: Dense gluonic matter suppresses the away side peak.

Conclusion

Conclusion:

- DIS dijet provides direct information of the WW gluon distributions. Perfect for testing saturation physics calculation, and ideal measurement for EIC and LHeC.
- Modified Universality for Gluon Distributions:

	Inclusive	Single Inc	DIS dijet	γ +jet	g+jet
$xG^{(1)}$	$\times$	$\times$	$\surd$	$\times$	$\surd$
$xG^{(2)}$, F	$\surd$	$\surd$	$\times$	$\surd$	$\surd$

$\times \Rightarrow$ Do Not Appear. $\surd \Rightarrow$ Apppear.

- Two fundamental gluon distributions. Other gluon distributions are just different combinations and convolutions of these two.
- The small-x evolution of the WW gluon distribution, a different equation from Balitsky-Kovchegov equation;[Dominguez, Mueller, Munier, Xiao, 11]
- Dihadron correlation calculation agrees with the RHIC STAR and PHENIX data.