Tensor and Flavor-singlet Axial Charges and Their Scale Dependencies

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1. Introduction:

The quark-gluon structure of nucleon is described by a set of parton distribution functions.

- The helicity and transversity distributions and their first moments---the axial and tensor charges provide important information on the nucleon spin structure.

- Since the tensor current is not conserved, the tensor charge defined by the matrix element of tensor current is strong scale dependent especially at low-energy region.
- The axial current is conserved in the DIS region where the quark mass can be neglected, thus the flavor singlet (scalar) axial charge has weak scale dependence due to the couping with gluon at NLO perturbation QCD evolution.
- However the scale dependence of the flavor singlet (scalar) axial charge in low-energy region may be strong due to PCAC.
- How to study the scale dependencies of these charges in low energy region ?

All Leading Twist TMDs

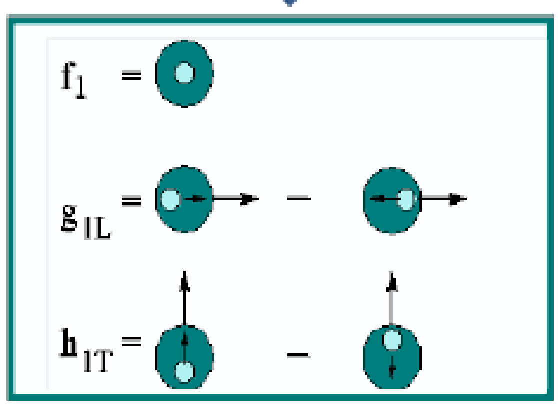
→ Nucleon
→ Quark Spin

		Quark polarization		
		Un-Polarized	Longitudinally Polarized	Transversely Polarized
Nucleon Polarization	U	f_1 =		$h_1^\perp$ = − Boer-Mulder
	L		g_1 = − Helicity	$h_{1L}^\perp$ = −
	T	$f_{1T}^\perp$ = − Sivers	$g_{1T}^\perp$ = −	h_{1T} = − Transversity $h_{1T}^\perp$ = − Pretzelosity

-D description in momentum space?

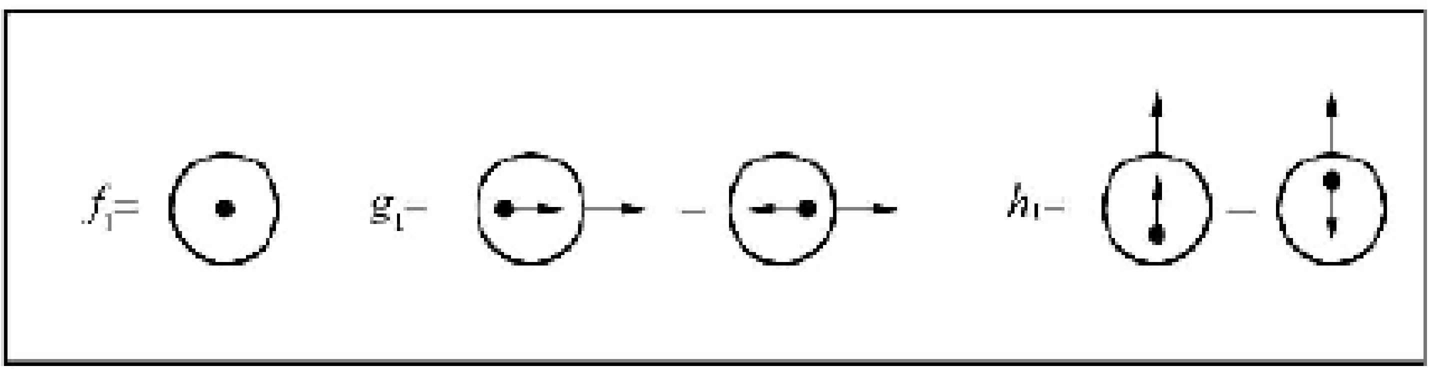
Transverse Momentum-dependent parton distributions (TMDs)

At leading twist 8 total, only 3 TMDs non vanishing upon integrating over transverse momentum of the quark



Quark distribution functions at leading twist

夸克分布	扭度	手征性	螺旋性振幅	测量
f_1	2	偶	$A_{\frac{1}{2}\frac{1}{2},\frac{1}{2}\frac{1}{2}} + A_{\frac{1}{2}-\frac{1}{2},\frac{1}{2}-\frac{1}{2}}$	自旋平均
g_1	2	偶	$A_{\frac{1}{2}\frac{1}{2},\frac{1}{2}\frac{1}{2}} - A_{\frac{1}{2}-\frac{1}{2},\frac{1}{2}-\frac{1}{2}}$	螺旋性差
h_1	2	奇	$A_{\frac{1}{2}\frac{1}{2},-\frac{1}{2}-\frac{1}{2}}$	螺旋性反转



Longitudinally polarized **Transversely polarized**

Quark distribution functions at leading twist.1

$$g_1(x) = \frac{1}{2}\sum_f e_f^2 [q_f^{\uparrow}(x) - q_f^{\downarrow}(x)] = \frac{1}{2}\sum_f e_f^2 \Delta q_f(x)$$

$$F_1(x) = \frac{1}{2}\sum_f e_f^2 [q_f^{\uparrow}(x) + q_f^{\downarrow}(x)] = \frac{1}{2}\sum_f e_f^2 q_f(x)$$

$$\Delta q_f(x) = q_f^{\uparrow}(x) - q_f^{\downarrow}(x), \quad q_f(x) = q_f^{\uparrow}(x) + q_f^{\downarrow}(x).$$

Where q_f(x) ---find a quark parton probability in a nucleon

Quark distribution functions
at leading twist.2

$$h_1(x) = \frac{1}{2}\sum e_f^2[\delta q_f(x) + \delta\bar{q}_f(x)] \qquad (4.85)$$

$$\delta q_f(x) = q_f^{\uparrow}(x) - q_f^{\downarrow}(x) \qquad (4.86)$$

↑(↓)指 f 味夸克的自旋是平行(反平行)于核子的横向极化,

The transversity distributions measure the difference of number of quarks(antiquarks) with transverse polarization parallel and antiparallel to the nucleon likewise polarized

Nucleon charges $\longleftrightarrow$ Quark distributions

Baryon charge B_q $\longleftrightarrow$ spin average $q_f(x)$

$$B_f = \int_0^1 dx\, q_f(x) \qquad f = u, d, s.$$

Axial charge g_A $\longleftrightarrow$ quark helicity $\Delta q_f(x)$

$$g_A^{(3)} = \int_0^1 dx\, (\Delta u(x) - \Delta d(x))$$

$$g_A^{(0)} = \Delta\Sigma = \int_0^1 dx\, (\Delta u(x) + \Delta d(x) + \Delta s(x))$$

Tensor charge g_T $\longleftrightarrow$ quark transversity $\delta q_f(x)$

$$g_T^q = \delta q = \int_0^1 dx\, (\delta q(x) - \delta \bar{q}(x))$$

$$\delta\Sigma = \int_0^1 dx\, (\delta u(x) + \delta d(x) + \delta s(x))$$

Helicity distributions and their first moments---axial charges

$$\int_0^1 \mathrm{d}x g_1^p = \frac{1}{2}\left[\frac{4}{9}\Delta u + \frac{1}{9}\Delta d + \frac{1}{9}\Delta s\right]$$

$$= \frac{1}{12}\left[a_3 + \frac{1}{\sqrt{3}}a_8 + \frac{4}{3}a_0\right]$$

$$\int_0^1 \mathrm{d}x g_1^n = \frac{1}{2}\left[\frac{4}{9}\Delta d + \frac{1}{9}\Delta u + \frac{1}{9}\Delta s\right]$$

$$= \frac{1}{12}\left[-a_3 + \frac{1}{\sqrt{3}}a_8 + \frac{4}{3}a_0\right]$$

$$a_0 = \Delta u + \Delta d + \Delta s = \Delta\Sigma$$ **(Flavor singlet)**

$$a_3 = \Delta u - \Delta d = F + D$$ **=1.2601**

$$a_8 = \Delta u + \Delta d - 2\Delta s = 3F - D$$ =0.588

*** Where flavor singlet axial charge in $\overline{\text{MS}}$ frame gives the quark spin contribution to nucleon's spin**

Transversity distributions and their first moments--- tensor charges

$$h_1(x) = \frac{1}{2}\sum_f e_f^2 \left[\delta q_f(x) + \delta\bar{q}_f(x)\right]$$

$$\int_0^1 \mathrm{d}x h_1^{\mathrm{p}}(x) = \frac{1}{2}\left[\frac{4}{9}\delta u + \frac{1}{9}\delta d + \frac{1}{9}\delta s\right]$$

$$\int_0^1 \mathrm{d}x h_1^{\mathrm{n}}(x) = \frac{1}{2}\left[\frac{4}{9}\delta d + \frac{1}{9}\delta u + \frac{1}{9}\delta s\right]$$

The flavor singlet tensor charge and isovector tensor charge can be defined

2. Tensor Charges and their scale dependencies

$$\int_0^1 \mathrm{d}x \left[\delta q_f(x) - \delta \bar{q}_f(x) \right] = \delta q_f$$

$$\langle PS \mid \bar{q}\sigma^{\mu\nu} q \mid PS \rangle = \delta q \bar{U}(P,S)\sigma^{\mu\nu} U(P,S)$$

Theory estimates of tensor charges —model calculation results

途径和模型	理论值 δu	δd
QCD Sum Rules：三点函数途径[81]	1.0 ± 0.5	0.0 ± 0.5
QCD Sum Rules：二点函数途径[82]	1.29 ± 0.25	0.02 ± 0.02
Bag Model[81]	1.17	−0.29
相对论组分夸克模型[85]	1.17	−0.29
流夸克对张量荷的贡献[85,86]	0.89	−0.22
Melosh 变换途径[87]	1.17	−0.29
Chiral Quark-Soliton Model[88]	1.07	−0.38
Lattice QCD[84]	$\delta u = 0.839(60)$ $\delta u_{\text{con}} = 0.893(22)$	$\delta d = -0.231(55)$ $\delta d_{\text{con}} = -0.180(10)$
Fllavor-Spin Symmetry Estimate[89]	$(0.58—1.01) \pm 0.20$	$-(0.11—0.20) \pm 0.20$

"Complete" Measurements

BNL - Star/Phoenix/Phobos

$\pi^+\pi^-$ Interference Fragmentation: $A_T\left(p_\perp + p \to jet(\pi^+,\pi^-) + X\right) \Leftrightarrow \underline{\delta q} \cdot \delta\hat{q}_I$

Collins Effect : $A_T(p_\perp + p \to jet(h) + X) \Leftrightarrow \underline{\delta q} \cdot C$

Drell Yan: $A_{TT}(p_\perp p_\perp \to ll) \Leftrightarrow \underline{\delta q} \cdot \delta\overline{q}$

Inclusive hadron: $A_N(p_\perp p \to h) \Leftrightarrow \underline{\delta q} \cdot C$ + 2 other terms

DESY-Hermes & CERN-Compass (BNL eRHIC, DESY Tesla-N)

Collins Effect : $A_T(lp_\perp \to l + \pi + X) \Leftrightarrow \underline{\delta q} \cdot C$

$\pi^+\pi^-$ Interference Fragmentation: $A_T\left(lp_\perp \to jet(\pi^+,\pi^-) + X\right) \Leftrightarrow \underline{\delta q} \cdot \delta\hat{q}_I$

e+e- collider

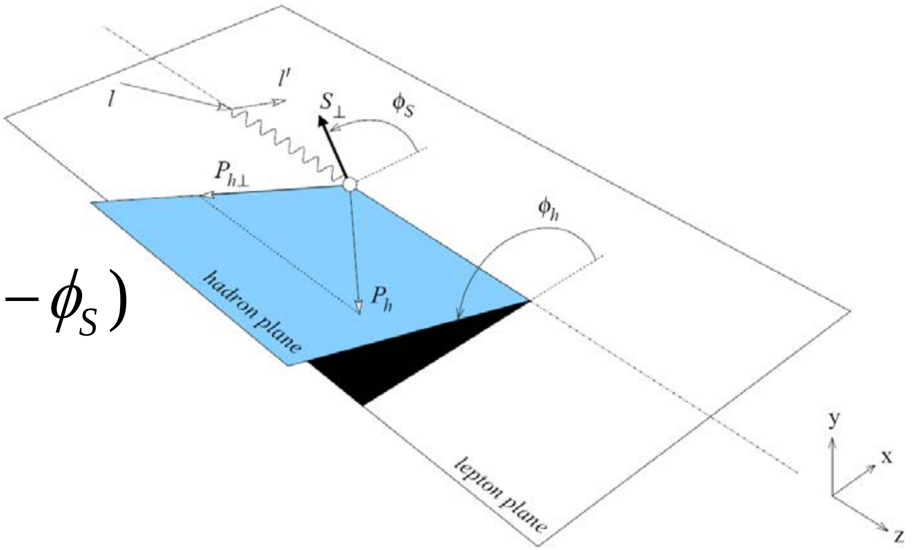
$e^+e^- \to dijet \quad C \cdot C\ ,\ \delta\hat{q}_I \cdot \delta\hat{q}_I\ \ \&\ C \cdot \delta\hat{q}_I$

Separation of Collins, Sivers and pretzelocity effects through angular dependence in SIDIS

$$A_{UT}(\varphi_h^l, \varphi_S^l) = \frac{1}{P}\frac{N^{\uparrow} - N^{\downarrow}}{N^{\uparrow} + N^{\downarrow}}$$

$$= A_{UT}^{Collins}\sin(\phi_h + \phi_S) + A_{UT}^{Sivers}\sin(\phi_h - \phi_S)$$

$$+ A_{UT}^{Pretzelosity}\sin(3\phi_h - \phi_S)$$



$$A_{UT}^{Collins} \propto \langle \sin(\phi_h + \phi_S) \rangle_{UT} \propto h_1 \otimes H_1^{\perp}$$

$$A_{UT}^{Sivers} \propto \langle \sin(\phi_h - \phi_S) \rangle_{UT} \propto f_{1T}^{\perp} \otimes D_1$$

$$A_{UT}^{Pretzelosity} \propto \langle \sin(3\phi_h - \phi_S) \rangle_{UT} \propto h_{1T}^{\perp} \otimes H_1^{\perp}$$

Measurements of transversity distributions and tensor charges

- Jlab measured the single target spin asymmetry(SSA) in SIDIS

$$A_{UT} \equiv \frac{1}{|S_T|}\frac{\mathrm{d}\sigma_{UT}}{\mathrm{d}\sigma_{UU}} = \frac{1}{|S_T|}\frac{\mathrm{d}\sigma(\phi_h,\phi_S) - \mathrm{d}\sigma(\phi_h,\phi_S+\pi)}{\mathrm{d}\sigma(\phi_h,\phi_S) + \mathrm{d}\sigma(\phi_h,\phi_S+\pi)}$$

$$= A_{UT}^{\mathrm{Collins}}\sin(\phi_h+\phi_S) + A_{UT}^{\mathrm{Sivers}}\sin(\phi_h-\phi_S)$$

$A_{UT}^{\mathrm{Collins}}$由(4.96)式给出，$A_{UT}^{\mathrm{Sivers}}$的形式为

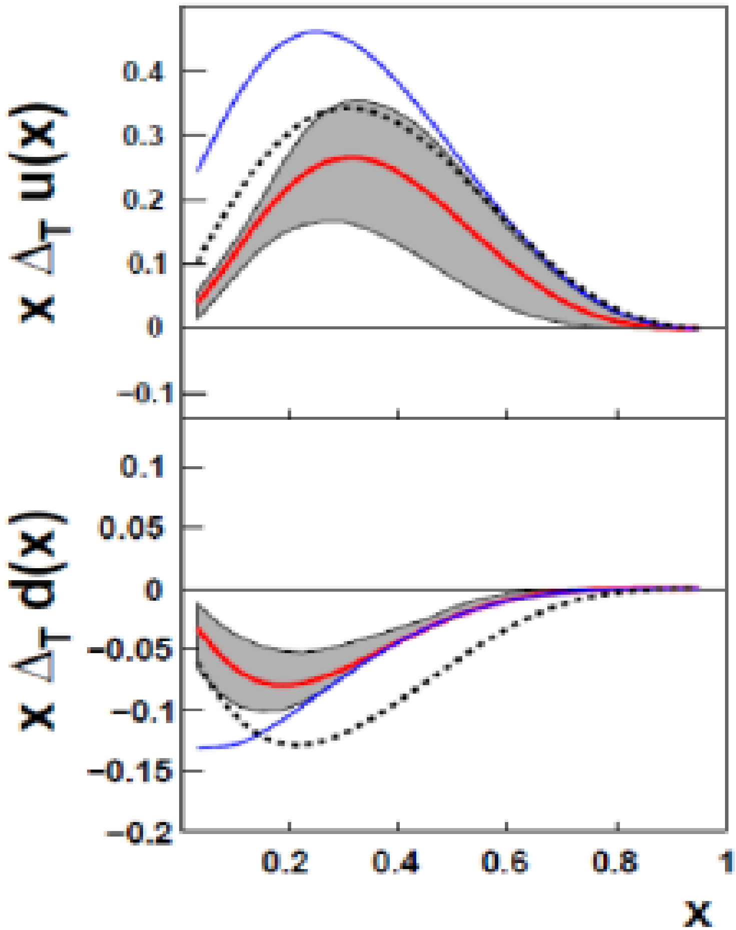
$$A_{UT}^{\mathrm{Sivers}} = \frac{\sum_q e_q^2 f_{1T}^{\perp(1)}(x)\cdot D_1^q(z)}{\sum_q e_q^2\, q(x)\cdot D_1^q(z)}$$

$f_{1T}^{\perp(1)q}(x)$称为 Sivers 函数。

A Global Analysis of Experimental Data

Transversity Distributions

A global fit to the HERMES p, COMPASS d and BELLE e+e- data by the Torino group, Anselmino et al., arXiv:0812.4366



Solid red line : transversity distribution, analysis at Q^2=2.4 (GeV/c)^2

Solid blue line: Soffer bound $|h_{1T}| <= (f_1+g_{1L})/2$
GRV98LO + GRSV98LO

Dashed line: helicity distribution g_{1L}, GRSV98LO

$$\Delta_T = h_{1T}$$

A. Prokudin, Talk 1059

Experimental extraction of tensor charges

- **Extraction of tensor charges was given by Anselmino et al. based on the combined global fit to semi-inclusive DIS(SIDIS) data from HERMES and COMPASS and those in (BELLE)**

$$e^+e^- \longrightarrow h_1 h_2 X$$

The results are(central values)

$$\delta u = 0.59, \quad \delta d = -0.20, \quad \delta\Sigma_q = 0.39, \quad (Q^2 = 0.8 Gev^2)$$

(From Phys.Rev.D75,054032(2007; arXiv:0809.3743)

Scale dependence of tensor Charge:

- QCD perturbative evolutions

$$\frac{\mathrm{d}}{\mathrm{d}t}\delta q(x,t) = \frac{\alpha_s(t)}{2\pi}\int_x^1 \frac{\mathrm{d}z}{z} P_h(z)\delta q(x/z,t)$$

Note that gluons do not enter the evolution equation for transversity distributions due to the chiral-odd property.

The evolution equation of tensor charge at leading order

$$\delta q(Q^2) = \left[\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)}\right]^{-4/27} \delta q(Q_0^2)$$

* Perturbutive evolutions of tensor charges to next leading order(NLO)

The next leading order Q^2 evolution of tensor charge is given by

$$\delta q(Q^2) = \delta q(\mu^2)[\frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)}]^{4/27}[\frac{\beta_0+\beta_1\alpha_s(Q^2)/4\pi}{\beta_0+\beta_1\alpha_s(\mu^2)/4\pi}]^{\gamma^{(1)}/\beta_1-\gamma^{(0)}/\beta_0},$$

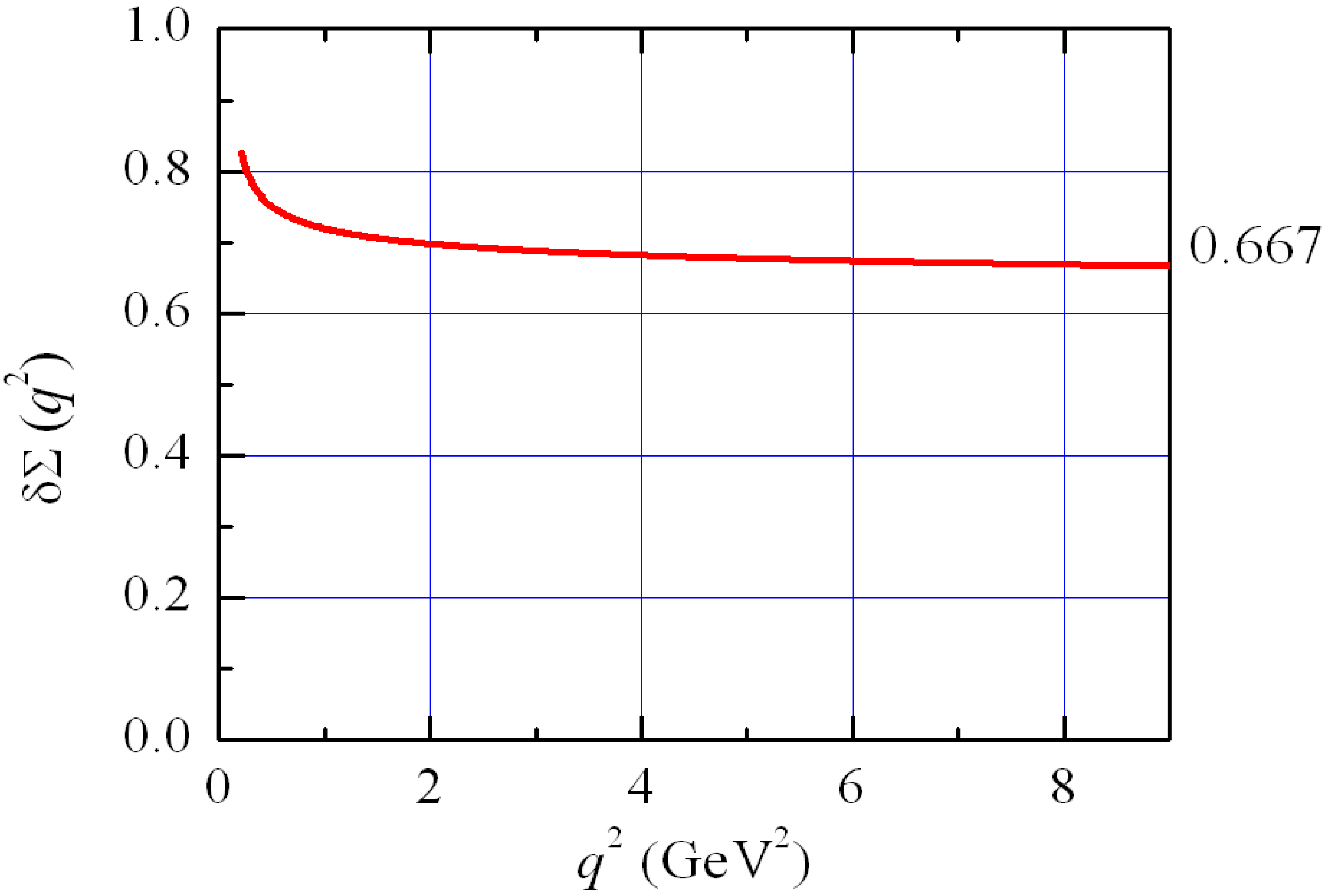
$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0 ln(Q^2/\Lambda^2)}[1-\frac{\beta_1}{\beta_0^2}\frac{lnln(Q^2/\Lambda^2)}{ln(Q^2/\Lambda^2)}],$$

where

$$\beta_0 = 11 - 2n_f/3 = 9, \quad \beta_1 = 102 - 38n_f/3 = 64,$$

and $\Lambda = 0.248GeV$ for $n_f = 3$ case.

Perturbative evolution (to NLO)of flavor singlet tensor charge



* How to describe scale dependencies of tensor charges in low-energy region ?

- Lattice QCD.

  We use:

- Effective quark model of tensor charges
- Cooperating with dynamically quark mass generating to account nonperturbative QCD effects

* Effective theory analyses

QCD at low-energy is equivalent to an effective quark theory

用泛函积分技术可以在形式上完成对胶子场的积分，导致一有效 QCD 拉氏量 $\mathscr{L}_{\mathrm{eff}}^{\mathrm{QCD}}$。利用相平行的手续，我们可以导出有效拉氏量 $\mathscr{L}_{\mathrm{eff}}^{\mathrm{QCD}}$ 的角动量密度 $M_{\mathrm{eff}}^{\mu\nu\lambda}$，由此可定义有效 QCD 理论中的角动量算符[328]

$$\left.\begin{aligned}
J_{\mathrm{eff}} &= J_{\mathrm{quark}} + J_{\mathrm{q\bar{q}}} \\
J_{\mathrm{q\bar{q}}} &= \int \mathrm{d}^3 x \, \frac{\mathrm{i}}{4}\left[x^j \, \bar{\psi}(\sigma^{0k}\hat{M} - \hat{M}\sigma^{0k})\psi - (j \leftrightarrow k)\right] \\
\hat{M} &= \sum_{n=2}\int \mathrm{d}^4 x_2 \cdots \mathrm{d}^4 x_n \, \frac{(\mathrm{i}g)^n}{n!} G^{\alpha_1\cdots\alpha_n}_{\mu_1\cdots\mu_n}(x_1,\cdots,x_n) \\
&\quad \times \left(\frac{\lambda_{\alpha_1}}{2}\gamma^{\mu_1}\right)\bar{\psi}(x_2)\frac{\lambda_{\alpha_2}}{2}\gamma^{\mu_2}\psi(x_2)\cdots\bar{\psi}(x_n)\frac{\lambda_{\alpha_n}}{2}\gamma^{\mu_n}\psi(x_n)
\end{aligned}\right\} \quad (10.115)$$

Transversity distributions and tensor charges are valence quark dominated since

- Tensor current operator is odd under charge conjugation $\longrightarrow$ the quark-antiquark sea does not contribute

- Gluons do not contribute under QCD evolution of trensversity distributions

Axial charges and tensor charges in the effective theory

$$\langle PS \mid \bar{q}\gamma^{\mu}\gamma^{5} q \mid_{\mu_0^2} \mid PS \rangle = 2\Delta q(\mu_0^2) S^{\mu}$$

$$\langle PS \mid \bar{q}\mathrm{i}\sigma^{\mu\nu}\gamma^{5} q \mid_{\mu_0^2} \mid PS \rangle = 2\delta q(\mu_0^2)(S^{\mu}P^{\nu} - S^{\nu}P^{\mu})$$

where

$$\left.\begin{aligned} \Delta q &= \Delta q_{\mathrm{q}} - \Delta \bar{q} \\ \delta q &= \delta q_{\mathrm{q}} - \delta \bar{q} \end{aligned}\right\}$$

Axial and tensor charges
in the effective quark picture

$$\left.\begin{aligned} \Delta q_{\rm q} &= \langle M_A \rangle \Delta q_{\rm NR} \\ \delta q_{\rm q} &= \langle M_T \rangle \delta q_{\rm NR} \end{aligned}\right\}$$

$$M_A = \frac{1}{3} + \frac{2m}{3E}, \quad M_T = \frac{2}{3} + \frac{m}{3E}$$

Where m is dynamical quark mass

*Nonperturbative evolution:

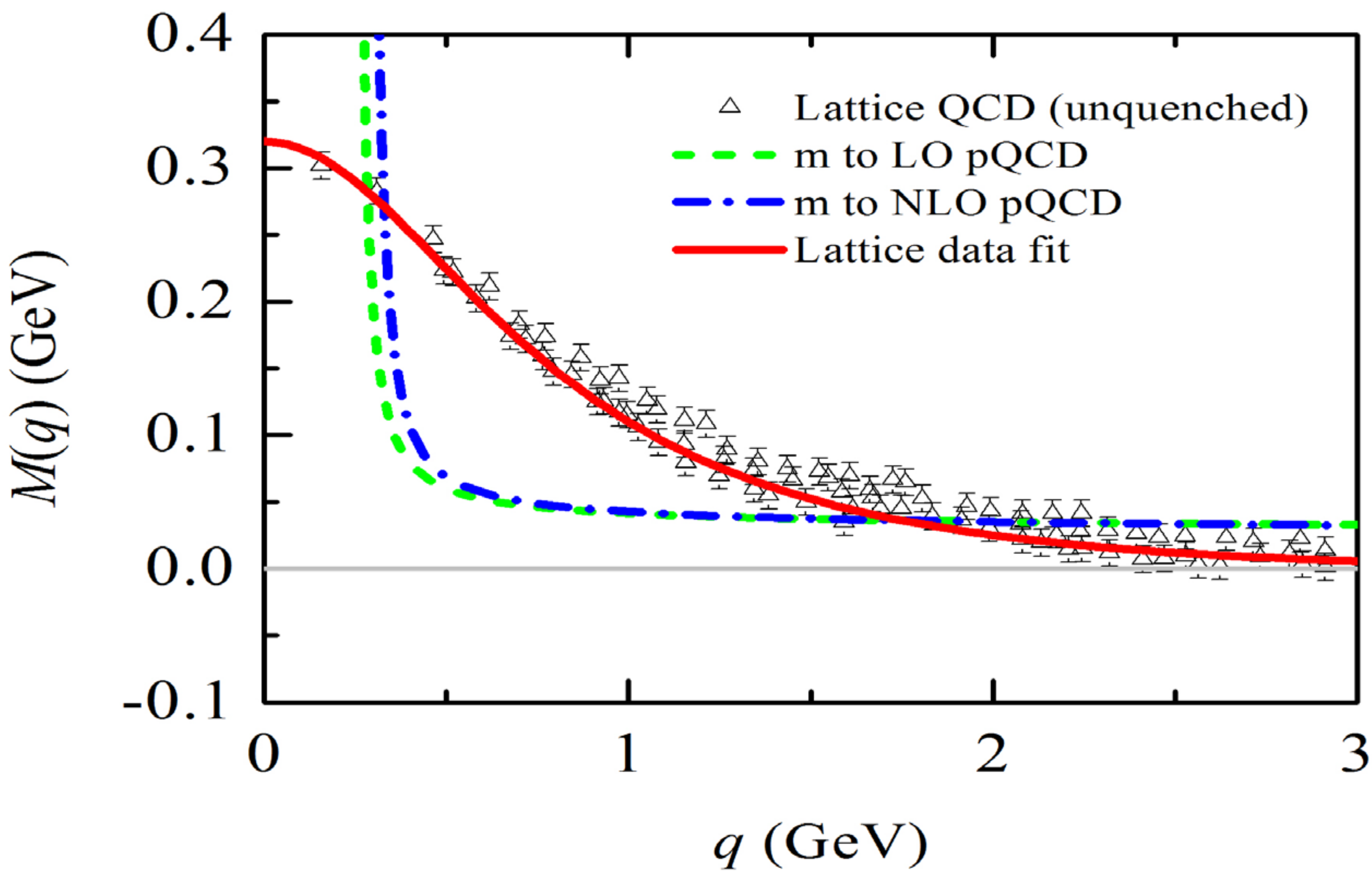
Effective theory analyses of tensor charges

$$\delta u_q = \frac{4}{3}\left(\frac{2}{3} + \left\langle \frac{m}{3E} \right\rangle\right), \quad \delta d_q = -\frac{1}{3}\left(\frac{2}{3} + \left\langle \frac{m}{3E} \right\rangle\right)$$

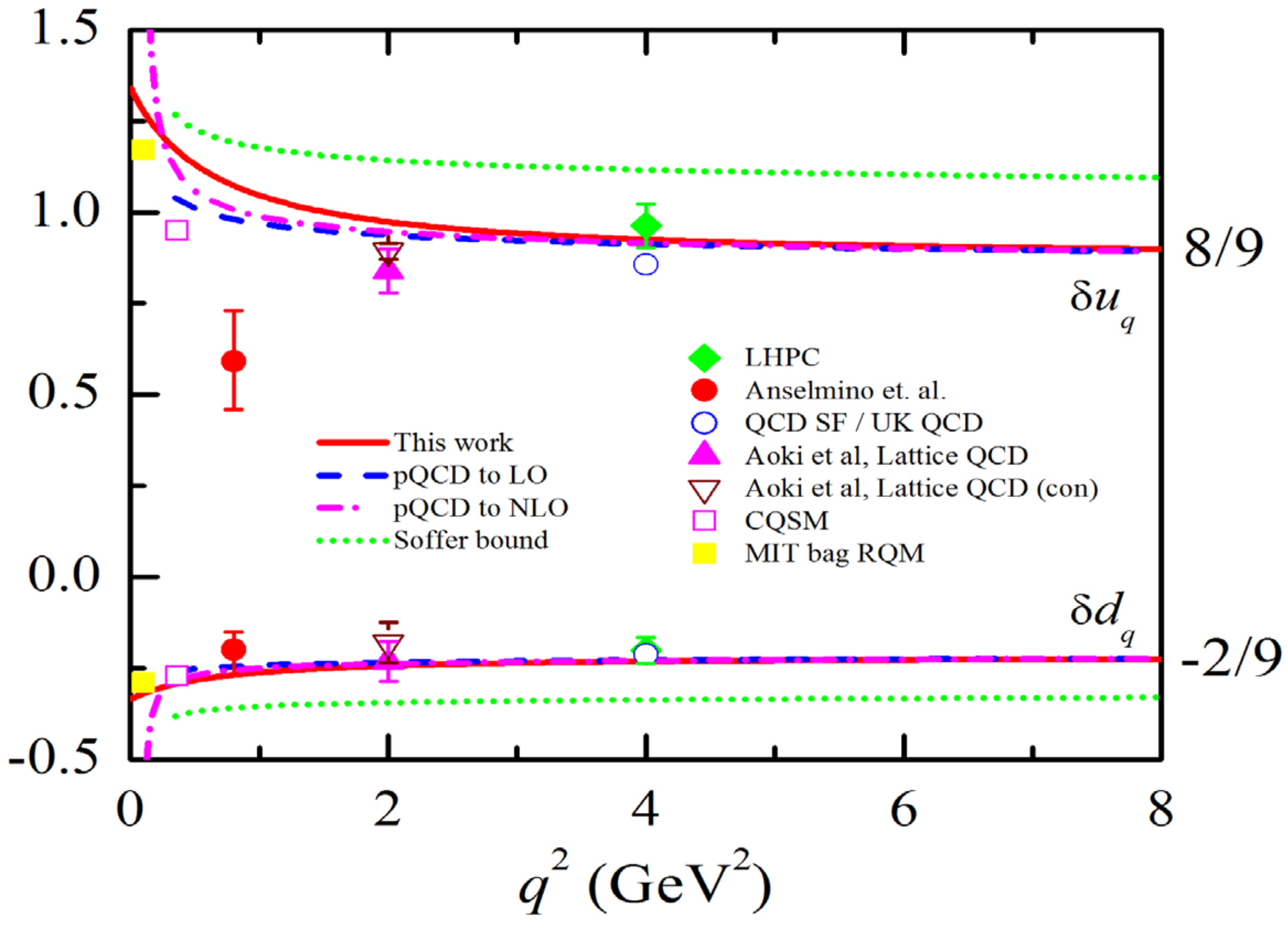
- Here m is dynamical quark mass.
- Generally one may set definite m to calculate the matrix element.
- The quark mass in QCD is a running quantity, different mass corresponds to different scale.
- Flavor singlet tensor charge of current quark contributions (in the asymptotic limit)

$$\delta\Sigma_v = \delta u_q^v + \delta d_q^v = \frac{2}{3} = 0.667$$

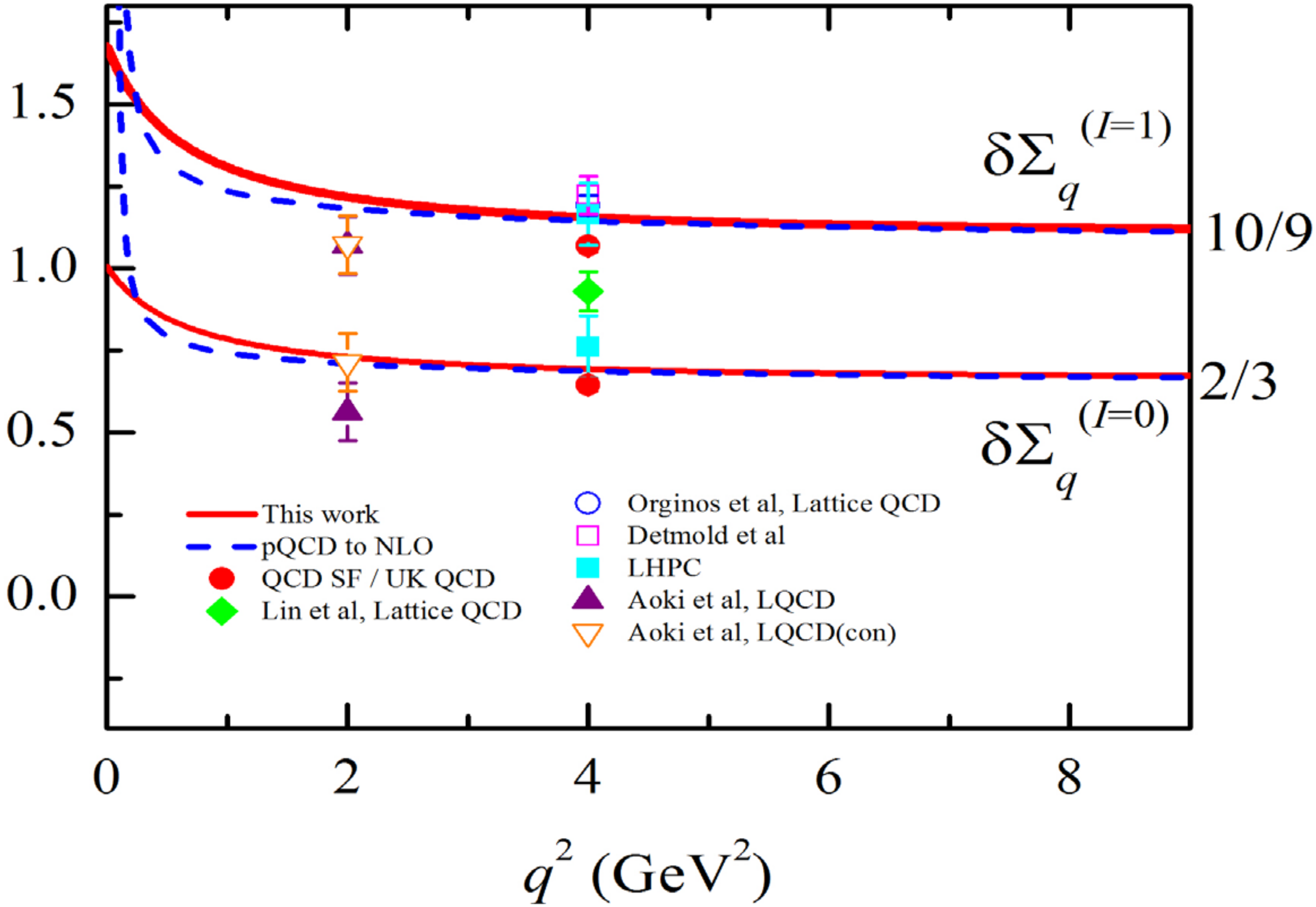
Running quark mass:
Scale evolutions of quark mass



Scale dependencies of tensor charges for u- and d-quarks



Scale dependencies of isovector- and isoscalar tensor charges



3. Flavor singlet axial charge and its scale dependence

$$a_i \cdot S^\mu = \langle PS \mid \bar{q} \frac{\lambda_i}{2} \gamma^\mu \gamma_5 \mid PS \rangle, \; i = 0,3,8$$

$$a_0 = \Delta u + \Delta d + \Delta s = \Delta\Sigma$$

$$a_3 = \Delta u - \Delta d = F + D$$

$$a_8 = \Delta u + \Delta d - 2\Delta s = 3F - D$$

- In $\overline{\mathrm{MS}}$ frame the flavor singlet axial charge gives quark spin contribution to nucleon spin

*Axial charges and quark spin contribution to nucleon spin

- Nucleon spin sum rules

$$\frac{1}{2} = \langle P + | J_z | P + \rangle = \frac{1}{2}\Delta\Sigma + \Delta G + L_{\mathrm{q}} + L_{\mathrm{g}}$$

这里

$$\Delta\Sigma = \langle P + | \hat{S}_{3\mathrm{q}} | P + \rangle = \left\langle P + \left| \int \mathrm{d}^3 x\, \bar{\psi} \gamma^3 \gamma_5 \psi \right| P + \right\rangle$$

$$\Delta G = \langle P + | \hat{S}_{3\mathrm{g}} | P + \rangle = \left\langle P + \left| \int \mathrm{d}^3 x \left(E^1 A^2 - E^2 A^1 \right) \right| P + \right\rangle$$

$$L_{\mathrm{q}} = \langle P + | \hat{L}_{3\mathrm{q}} | P + \rangle = \left\langle P + \left| \int \mathrm{d}^3 x\, \mathrm{i}\, \bar{\psi} \gamma^0 \left(x^2 \partial^1 - x^1 \partial^2 \right) \psi \right| P + \right\rangle$$

$$L_{\mathrm{g}} = \langle P + | \hat{L}_{3\mathrm{g}} | P + \rangle = \left\langle P + \left| \int \mathrm{d}^3 x\, E^i \left(x^1 \partial^2 - x^2 \partial^1 \right) A^i \right| P + \right\rangle$$

All components are scale-dependent

* Nucleon spin sum rules

--satisfying angular momentum commutation relation

规范场分解为两部分：$\boldsymbol{A} = \boldsymbol{A}_{\text{phys}} + \boldsymbol{A}_{\text{pure}}$，相应的 QCD 角动量：

$$\boldsymbol{J}_{\text{QCD}} = \underline{\int \mathrm{d}^3 x \psi^{+} \frac{1}{2} \boldsymbol{\Sigma} \psi} + \int \mathrm{d}^3 x \psi^{+} \boldsymbol{x} \times \frac{1}{\mathrm{i}} \boldsymbol{D}_{\text{pure}} \psi + \int \mathrm{d}^3 x \boldsymbol{E}^{a} \times \boldsymbol{A}^{a}_{\text{phys}} + \int \mathrm{d}^3 x E^{ai} \boldsymbol{x} \times \boldsymbol{\nabla} A^{ai}_{\text{phys}} \qquad (4.$$

Quark spin contribution to nucleon spin

$$\Gamma_1^{p(n)}(Q^2) = \int_0^1 g_1^{p(n)}(x, Q^2)\mathrm{d}x$$

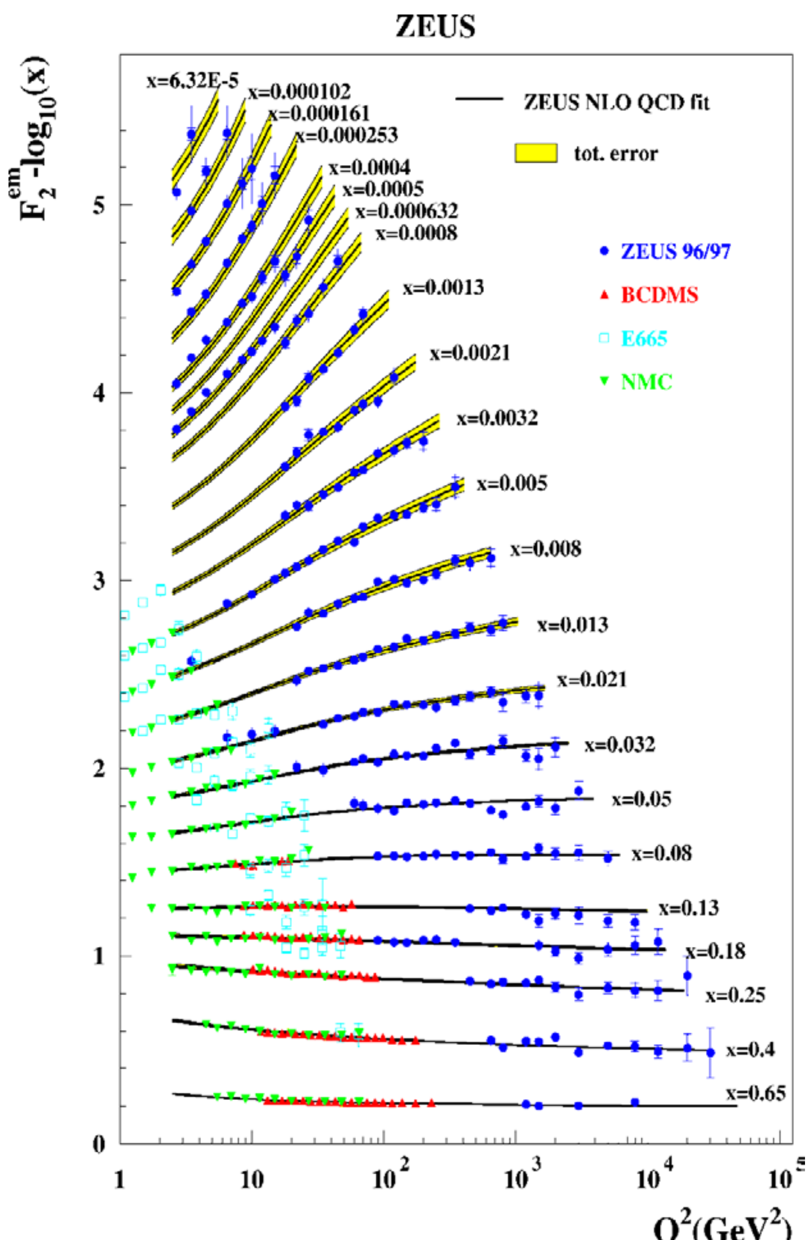
$$= \left(\pm\frac{1}{12}a_3 + \frac{1}{36}a_8\right)C_{NS} + \frac{1}{9}a_0 C_S + \delta\Gamma_{1HT}$$

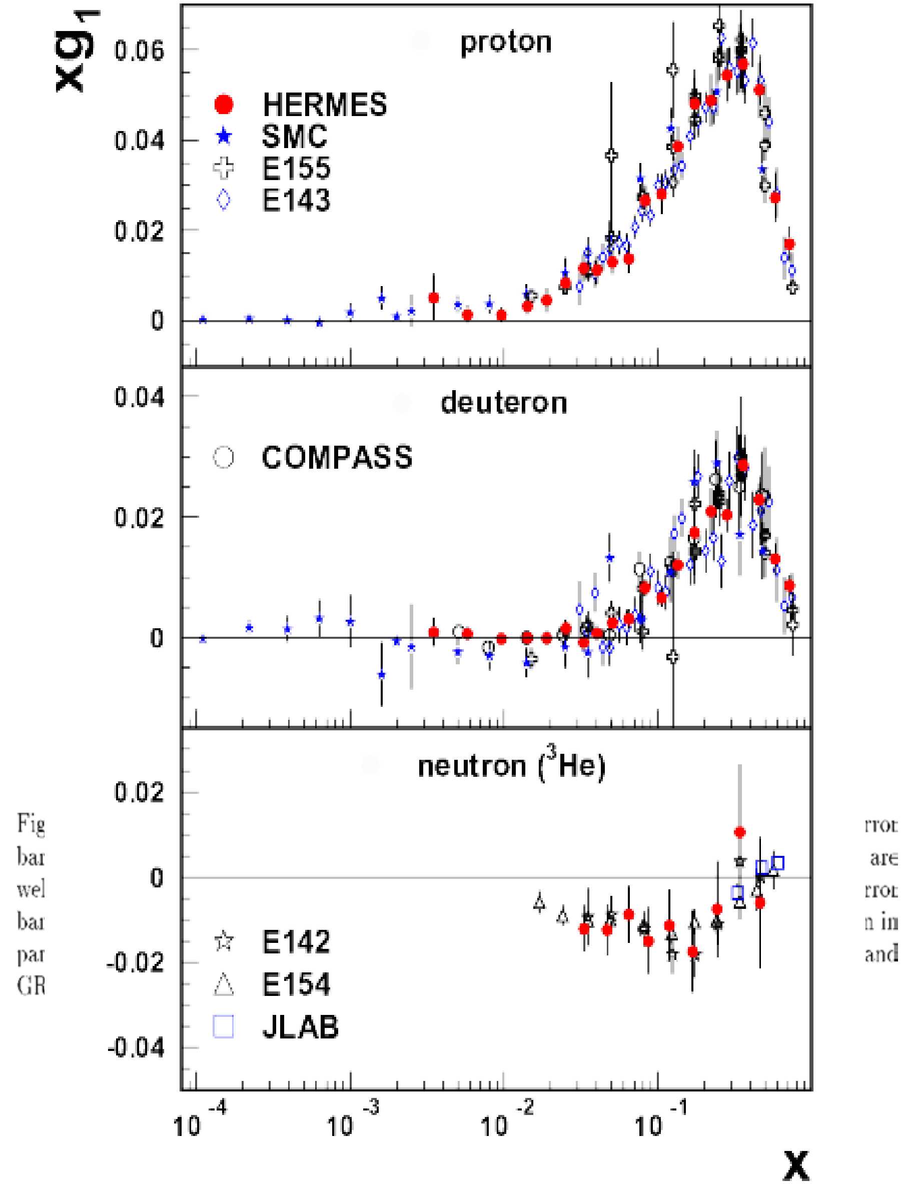
$$\Gamma_1^{d}(Q^2) = \int_0^1 g_1^{d}(x, Q^2)\mathrm{d}x$$

$$= \left(1 - \frac{3}{2}\omega_D\right)\left(\frac{1}{36}a_8 C_{NS} + \frac{1}{9}a_0 C_S\right) + \delta\Gamma_{1HT}$$

$$a_i \cdot S^\mu = \langle PS \mid \bar{q}\frac{\lambda_i}{2}\gamma^\mu\gamma_5 \mid PS\rangle,\ i = 0,3,8$$

Unpolarized and Polarized Structure functions





Quark spin contribution to nucleon spin

最近,HERMES 实验得到的结果为(至 $O(\alpha_s^2)$即 $NNLO$)[78]:

$$\Delta u = 0.842 \pm 0.004 \pm 0.008 \pm \cdots$$

$$\Delta d = -0.427 \pm 0.004 \pm 0.008 \pm \cdots$$

$$\Delta s = -0.085 \pm 0.013 \pm 0.008 \pm \cdots$$

$$\Delta\Sigma = 0.330 \pm 0.011(\text{theo}) \pm 0.025(\text{exp}) \pm 0.028(\text{evol})$$

如果计算至(N)NNLO 阶(ΔC_{NS}分析至 $O(\alpha_s^3)$,ΔC_S 至 $O(\alpha_s^2)$),夸克自旋对核子自旋的贡献为[78]

$$\Delta\Sigma = 0.333 \pm 0.011(\text{theo}) \pm 0.025(\text{exp}) \pm 0.028(\text{evol})$$

The measurement results are given at Q^2 =5 Gev^2

Perturbative Evolutions of QCD angular momentum operators

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\Delta\Sigma \\ \Delta g\end{pmatrix} = \frac{\alpha_s(t)}{2\pi}\begin{pmatrix}\Delta P^s_{\mathrm{qq}} & 2n_f\Delta P_{\mathrm{qg}} \\ \Delta P_{\mathrm{gq}} & \Delta P_{\mathrm{gg}}\end{pmatrix}\otimes\begin{pmatrix}\Delta\Sigma \\ \Delta g\end{pmatrix}$$

$t=\ln(Q^2/\Lambda^2_{\mathrm{QCD}})$，$\Delta P^s_{\mathrm{qq}}$中的上标 s 指味单态.劈裂函数
(NLO)为

$$\Delta P(x,\alpha_s) = \Delta P^{(0)}(x) + \frac{\alpha_s}{2\pi}\Delta P^{(1)}(x) + \cdots$$

* Scale dependence of flavor singlet axial charge due to PCAC

---the axial-vector current operator is not conserved if m does not equal to 0.

Consider PCAC:

$$\partial_x^\mu \bar{\psi}(x)\gamma^\mu\gamma_5\psi(x) = 2im\bar{\psi}(x)\gamma_5\psi(x).$$

Then we obtain

$$a_0(Q^2, m_q) = a_0(Q^2, m=0) + \frac{2m_q}{Q_3}(\langle PS|\bar{u}\gamma^5 u|PS\rangle + \langle PS|\bar{d}\gamma^5 d|PS\rangle + \langle PS|\bar{s}\gamma^5 s|PS\rangle)$$

*Flavor singlet (scalar) axial charge in effective quark model

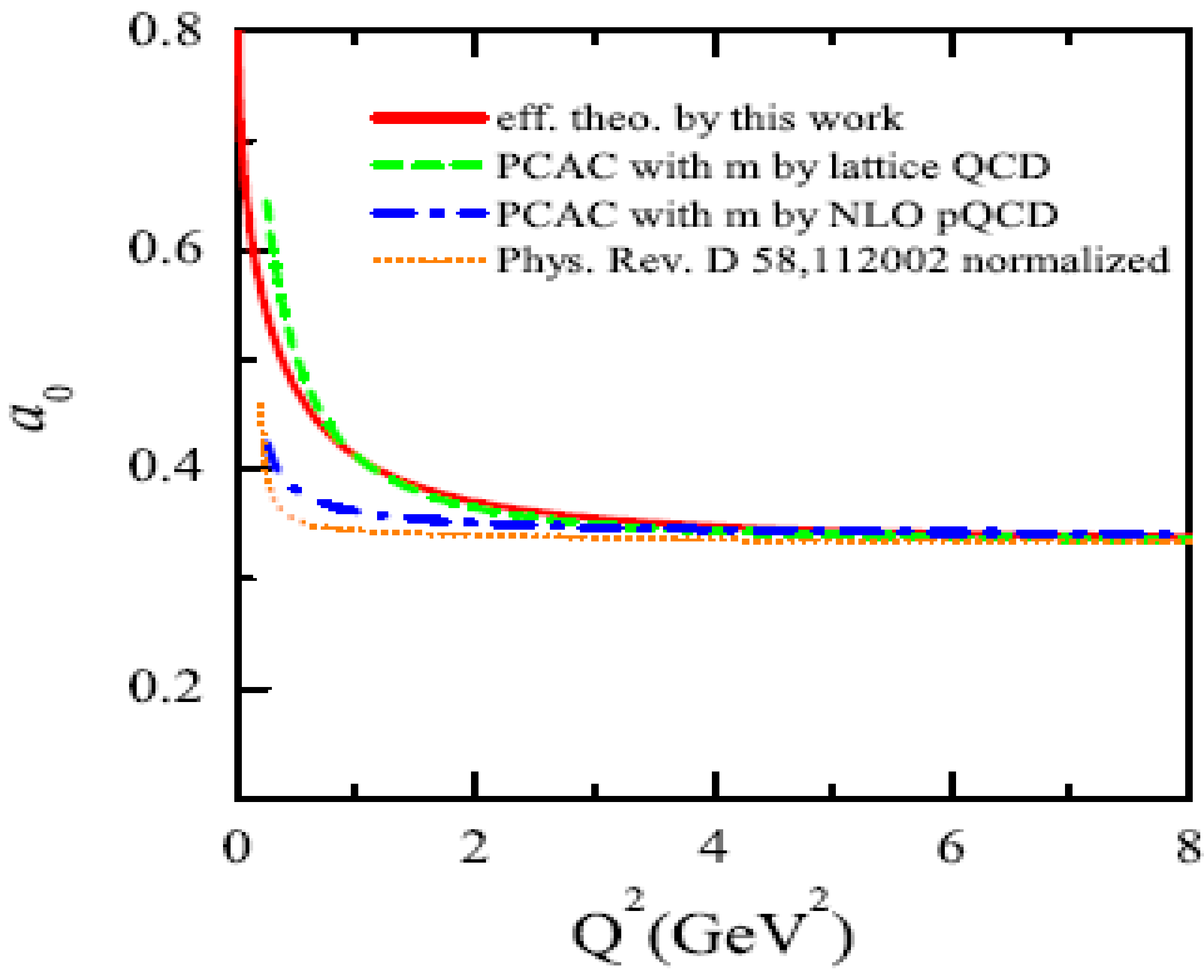
$$\Delta u_{\mathrm{q}} = \frac{4}{3}\left(\frac{1}{3} + \left\langle\frac{2m}{3E}\right\rangle\right), \quad \Delta d_{\mathrm{q}} = -\frac{1}{3}\left(\frac{1}{3} + \left\langle\frac{2m}{3E}\right\rangle\right)$$

The current quark (m=0) spin contribution to nucleon spin is

$$\Delta\Sigma_v = \Delta u_{\mathrm{q}}^v + \Delta d_{\mathrm{q}}^v = \frac{1}{3} = 0.333$$

Which is well agreement with the results given by HERMES & COMPASS groups.

Scale evolution of quark spin contribution to nucleon spin



4.Comparing scale dependence of tensor charge with that of axial charge

Flavor singlet axial charge

$$\Delta\Sigma_q = \frac{1}{3} + \frac{2}{3}\frac{m}{E},$$

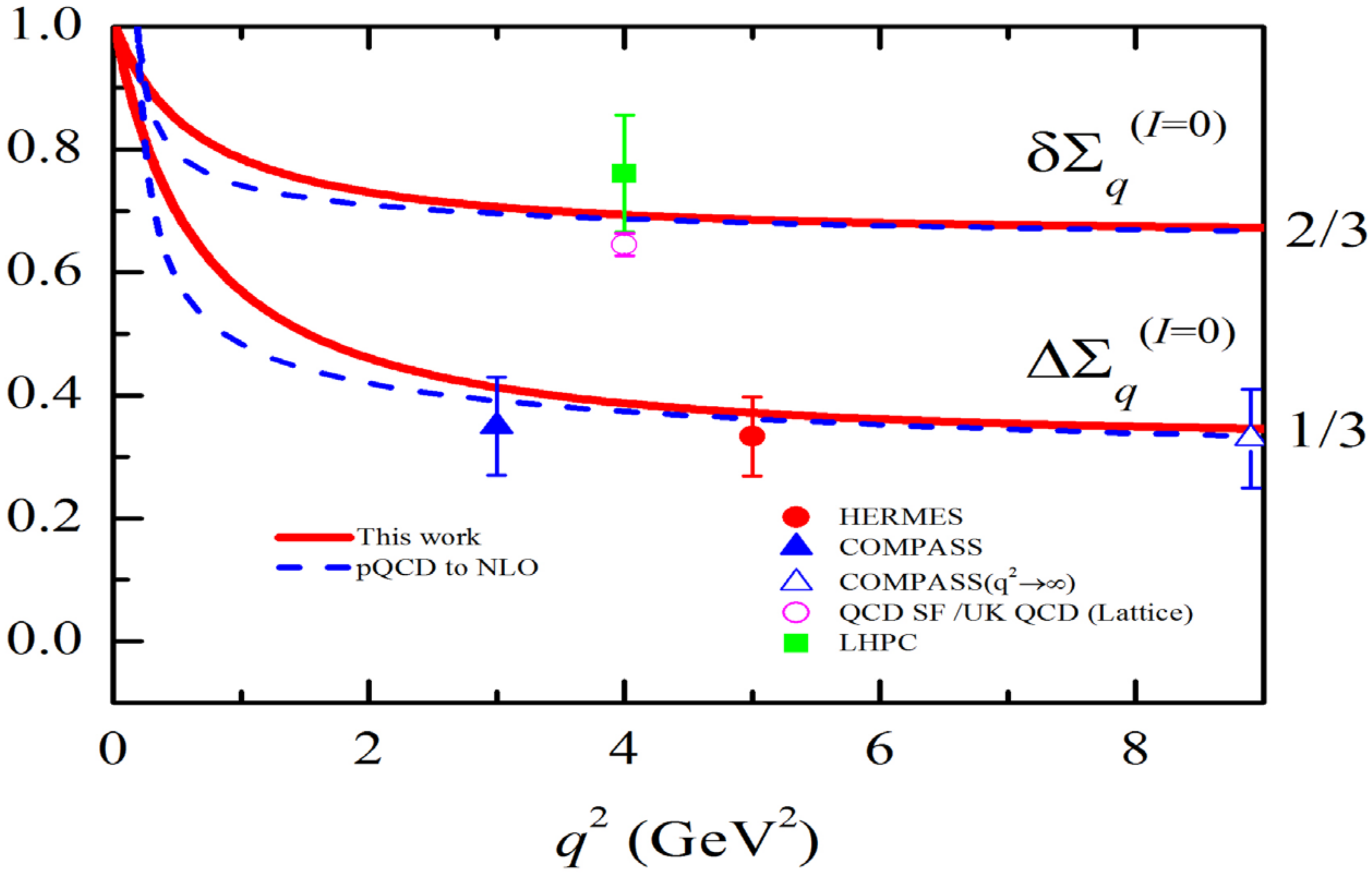
Flavor singlet tensor charge

$$\delta\Sigma_q = \frac{2}{3} + \frac{1}{3}\frac{m}{E},$$

Isovector tensor charge

$$\delta\Sigma_q^{(I=1)} = \frac{10}{9} + \frac{5}{9}\frac{m}{E},$$

Scale dependencies of scalar tensor and axial charges



5. Summary(1)

- Tensor charges are most important quantities that characterize transversities
- Tensor charges are strong scale dependent, especially in the low-energy region.
- Using the quark model in cooperation with dynamical quark mass generating can reasonably describe the scale dependencies from perturbation to nonperturbation region, while the results are well consistent with the QCD perturbation evolution ones in the large momentum region.

Effective model predicts

- In the asymptotic limit, present effective quark model predicts that tensor charges are

For proton

$$\delta u = \frac{8}{9}, \quad \delta d = -\frac{2}{9}, \quad \delta \Sigma_q = \frac{2}{3} = 0.667,$$

For neutron

$$\delta d = \frac{8}{9}, \quad \delta u = -\frac{2}{9}, \quad \delta \Sigma_q = \frac{2}{3} = 0.667.$$

5. Summary(2)

- Flavor singlet (scalar) axial charge is one of most important quantities that characterize helicity, which in the $\overline{\mathrm{MS}}$ scheme gives quark spin contribution to nucleon spin
- Flavor singlet (scalar) axial charge in DIS region has weak scale dependence due to the coupling with gluon at NLO perturbation QCD evolution. However, the flavor singlet (scalar) axial charge in the low-energy region is strong scale dependent due to

5. Summary(3)

- PCAC and dynamical chiral symmetry breaking, which can be reasonably described by using the quark model in cooperation with dynamical quark mass generating.
- This effective model also predicts that the quark spin contributes to 1/3 of nucleon spin in large momentum limit, which is well consistent with HERMES & COMPASS results.
- Comparing the scale evolutions of the flavor singlet (scalar) axial and tensor charges shows that relativistic effects lead to the difference between them: Relativistic effect on scalar axial-charge is larger then one on scalar tensor charge.

Thank you