

Overview of Transverse Spin and TMDs (Theory)

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What is spin?

Spin = angular momentum of a particle when it is at rest

□ For an elementary particle:

Spin is a fundamental and intrinsic property of the particle

Classically, a particle's angular momentum

is a consequence of its motion: $\vec{r} \times \vec{p}$

➡ Spin is a pure quantum effect

□ For a composite particle – like a hadron or a nucleus:

Spin is a consequence of internal dynamics of the bound state

For example, the nucleon-nucleon interaction and shell structure determines the observed nuclear spin states

Why we are interested in proton spin?

□ Fundamental questions:

What is the internal structure of proton (or a hadron in general)?

If QCD is the right theory of strong interaction, how quarks and gluons and their interaction make up the spin $\frac{1}{2}$?

If QCD is not a complete theory of the strong interaction, even though it has been very successful in the asymptotic region, what is the correction to the theory in nonperturbative regime?

Lattice calculation (nonperturbative) is very important

□ Spin as a tool to explore QCD quantum effect:

Cross section is a probability – classically measured

Spin asymmetry – the difference of two cross sections
involving two spin states

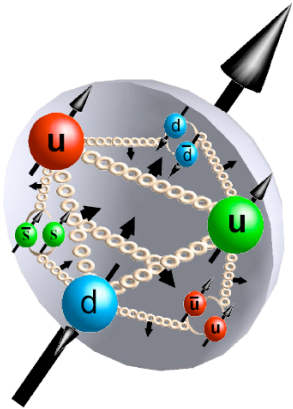
could be a pure quantum effect!

Proton's spin structure

□ Facts – or what our believe:

Proton is made of quarks and gluons, and has spin $\frac{1}{2}$, ...

□ How quarks and gluons make up the spin $\frac{1}{2}$?



Theoretically, $S = \langle P, S_z = 1/2 | \hat{J}^z(\psi, A^\mu) | P, S_z = 1/2 \rangle = \frac{1}{2}$

Controversy in QCD:

How to split the total angular momentum into separate quark and gluon components?

Jaffe-Manohar, Ji, Chen-Lu-Sun-Wang-Goldman, Wakamatsu, ...

Experimentally, we do not see quarks and gluons!

We need – the connection at various momentum scales:

$$\sigma(p_i, s_i) \Leftrightarrow \langle p_i, s_i | \mathcal{O}(\psi, A^\mu) | p_i, s_i \rangle$$

Hadronic cross section in QCD

- Any number of partons could participate in the collision

$$\sigma(Q, \vec{s}) \propto \left| \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \\ \text{Diagram 3} \\ \dots \end{array} \right|^2$$

The diagrams show a hard scattering process with an incoming parton of momentum k and a hard scale Q . The first diagram shows a tree-level process with a propagator $t \sim 1/Q$. The second diagram shows a process with a gluon exchange $A^\mu(k')$. The third diagram shows a process with a gluon exchange and a gluon emission. The diagrams are summed and squared to give the cross section.

- Large momentum transfer simplifies the picture:

$$\sigma_{AB}(Q, \vec{s}) \approx \sigma_{AB}^{(2)}(Q, \vec{s}) + \frac{Q_s}{Q} \sigma_{AB}^{(3)}(Q, \vec{s}) + \frac{Q_s^2}{Q^2} \sigma_{AB}^{(4)}(Q, \vec{s}) + \dots$$

Single hard scale \rightarrow Leading power \rightarrow Collinear factorization

$$\sigma_{AB}^{(2)}(Q, \vec{s}) = \hat{\sigma}_{ab}(x, x', Q) \otimes f_{a/A}(x, Q, \vec{s}) \otimes [f_{b/B}(x', Q) \otimes \dots]$$

Approximation: $k \approx xp$ for parton momentum entering the hard part

- Predictive power:

Short-distance dynamics, PDFs, and FFs, ...

$$\propto \langle p | \bar{\psi}(0) \gamma^+ \psi(y) | p \rangle, \dots, \text{Tr}[\gamma^+ \langle 0 | \bar{\psi}(0) | H(p) X \rangle \langle H(p) X | \psi(y) | 0 \rangle], \dots$$

Hadron structure beyond PDFs

□ Explore new dynamics – vary the spin orientation:

$$\sigma_{AB}(Q, \vec{s}) \approx \sigma_{AB}^{(2)}(Q, \vec{s}) + \frac{Q_s}{Q} \sigma_{AB}^{(3)}(Q, \vec{s}) + \frac{Q_s^2}{Q^2} \sigma_{AB}^{(4)}(Q, \vec{s}) + \dots$$

Spin asymmetries:

- both beams polarized A_{LL}, A_{TT}, A_{LT}

$$A_{LL} = \frac{[\sigma(+, +) - \sigma(+, -)] - [\sigma(-, +) - \sigma(-, -)]}{[\sigma(+, +) + \sigma(+, -)] + [\sigma(-, +) + \sigma(-, -)]} \quad \text{for } \sigma(s_1, s_2)$$

- one beam polarized A_L, A_N

$$A_L = \frac{[\sigma(+, +) - \sigma(+, -)]}{[\sigma(+, +) + \sigma(+, -)]} \quad \text{for } \sigma(s)$$

□ Observables – the leading power does not contribute:

Single transverse-spin asymmetry:

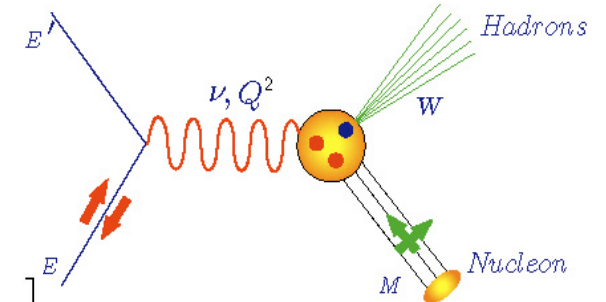
$$A_N = \frac{\sigma(Q, \vec{s}_T) - \sigma(Q, -\vec{s}_T)}{\sigma(Q, \vec{s}_T) + \sigma(Q, -\vec{s}_T)}$$

Inclusive DIS with polarization

□ Inclusive DIS – cleanest measurement:

$$\ell(E, s_e) + h(p, s_p) \rightarrow \ell(E') + X$$

$$x = \frac{Q^2}{2p \cdot q} \quad \nu = E - E' \quad Q^2 = -q^2$$



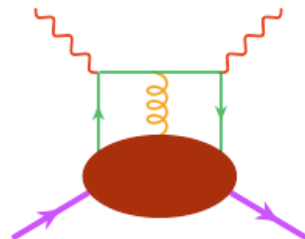
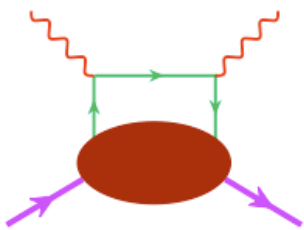
$$\frac{d^2\sigma}{dE'd\Omega}(\downarrow\uparrow - \uparrow\uparrow) = \frac{4\alpha^2}{MQ^2} \frac{E'}{\nu E} \left[(E + E' \cos \theta) g_1(x, Q^2) - \frac{Q^2}{\nu} g_2(x, Q^2) \right]$$

$$\frac{d^2\sigma}{dE'd\Omega}(\downarrow\Rightarrow - \uparrow\Rightarrow) = \frac{4\alpha^2 \sin \theta}{MQ^2} \frac{E'^2}{\nu^2 E} \left[\nu g_1(x, Q^2) + 2E g_2(x, Q^2) \right]$$

□ First hint of quark-gluon correlation:

$$g_2(x, Q^2) = g_2^{WW}(x, Q^2) + \bar{g}_2(x, Q^2) \quad \text{Leading twist term:}$$

$$g_2^{WW}(x, Q^2) = -g_1(x, Q^2) + \int_x^1 g_1(x, Q^2) \frac{dy}{y}$$



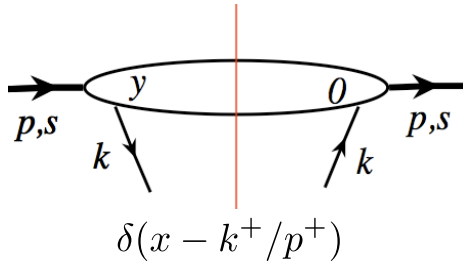
Twist-3 term:

$$\bar{g}_2 \propto \langle p, s_\perp | \bar{\psi} F^{+\perp} \psi | p, s_\perp \rangle$$

Data: SLAC, Jlab, ...

Transversity distributions

□ Spin projection for leading power quark distributions:



$$\Rightarrow \phi(x, p, s)_{ij} = \int \frac{dy^-}{2\pi} e^{ixp^+ y^-} \langle p, s | \bar{\psi}_i(0) \psi_j(y) | p, s \rangle$$

$$\Rightarrow \phi(x, p, s) = \frac{1}{2} \gamma \cdot p \{ q(x) - 2\lambda \Delta q(x) \gamma_5 + h(x) \gamma_5 \gamma \cdot s_T \}$$

wih nucleon helicity $\lambda = \pm 1/2$

□ Transversity distribution $h(x)$ – chiral odd:

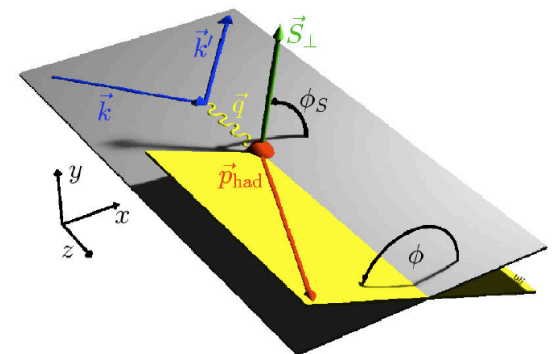
Spin projection for partonic collision: $\frac{1}{2} \gamma \cdot p \gamma \cdot s_T \gamma_5$
(even in gamma matrices)

Need two for a contribution to the cross section or asymmetry:

Drell-Yan: $A_{TT} \propto h(x_1) \otimes h(x_2)$

SIDIS: $A_T^{\sin(\phi+\phi_s)} \propto h(x) \otimes D_{\text{Collins}}(z)$

Soffer's bound: $q(x) + \Delta q(x) \geq 2|h(x)|$



TMD parton distributions

□ Role of parton's transverse momentum:

Collinear approximation/factorization:

Parton's transverse momentum integrated into PDFs: $q(x, \mu^2)$

Transverse momentum dependent (TMD) factorization:

TMD parton distribution functions: $q(x, k_T)$

Relationship: $q(x, \mu^2) = \int d^2 k_T q(x, k_T) + \text{UVCT}(\mu^2)$
Scheme dependence!

□ TMD parton distributions functions:

TMD distributions are natural – no additional UV divergence

TMD distributions, if could be measured, provide much more information on parton structure inside a hadron

Most notable TMD distributions - I

□ Sivers function – transverse polarized hadron:

Sivers function

$$\begin{aligned} f_{q/p,S}(x, \mathbf{k}_\perp) &= f_{q/p}(x, k_\perp) + \frac{1}{2} \Delta^N f_{q/p^\uparrow}(x, k_\perp) \mathbf{S} \cdot (\hat{\mathbf{p}} \times \hat{\mathbf{k}}_\perp) \\ &= f_{q/p}(x, k_\perp) - \frac{k_\perp}{M} f_{1T}^{\perp q}(x, k_\perp) \mathbf{S} \cdot (\hat{\mathbf{p}} \times \hat{\mathbf{k}}_\perp) \end{aligned}$$

□ Boer-Mulder function – transverse polarized quark:

$$\begin{aligned} f_{q,s_q/p}(x, \mathbf{k}_\perp) &= \frac{1}{2} f_{q/p}(x, k_\perp) + \frac{1}{2} \Delta^N f_{q^\uparrow/p}(x, k_\perp) \mathbf{s}_q \cdot (\hat{\mathbf{p}} \times \hat{\mathbf{k}}_\perp) \\ &= \frac{1}{2} f_{q/p}(x, k_\perp) - \frac{1}{2} \frac{k_\perp}{M} h_1^{\perp q}(x, k_\perp) \mathbf{s}_q \cdot (\hat{\mathbf{p}} \times \hat{\mathbf{k}}_\perp) \end{aligned}$$

Boer-Mulder function

Affect angular distribution of Drell-Yan lepton

Most notable TMD distributions - II

□ Collins function – FF of a transversely polarized parton:

$$\begin{aligned} D_{h/q,s_q}(z, \mathbf{p}_\perp) &= D_{h/q}(z, p_\perp) + \frac{1}{2} \Delta^N D_{h/q^\uparrow}(z, p_\perp) \mathbf{s}_q \cdot (\hat{\mathbf{p}}_q \times \hat{\mathbf{p}}_\perp) \\ &= D_{h/q}(z, p_\perp) + \frac{p_\perp}{z M_h} H_1^{\perp q}(z, p_\perp) \mathbf{s}_q \cdot (\hat{\mathbf{p}}_q \times \hat{\mathbf{p}}_\perp) \end{aligned}$$

Collins function

□ Fragmentation function to a polarized hadron:

$$\begin{aligned} D_{\Lambda, S_\Lambda/q}(z, \mathbf{p}_\perp) &= \frac{1}{2} D_{h/q}(z, p_\perp) + \frac{1}{2} \Delta^N D_{\Lambda^\uparrow/q}(z, p_\perp) \mathbf{S}_\Lambda \cdot (\hat{\mathbf{p}}_q \times \hat{\mathbf{p}}_\perp) \\ &= \frac{1}{2} D_{h/q}(z, p_\perp) + \frac{p_\perp}{z M_\Lambda} D_{1T}^{\perp q}(z, p_\perp) \mathbf{S}_\Lambda \cdot (\hat{\mathbf{p}}_q \times \hat{\mathbf{p}}_\perp) \end{aligned}$$

Unpolarized parton fragments into a polarized hadron - Λ

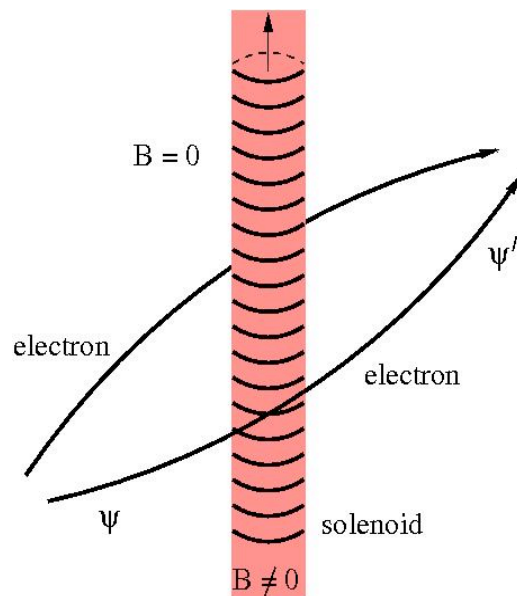
QCD factorization

□ Connect hadrons to partons:

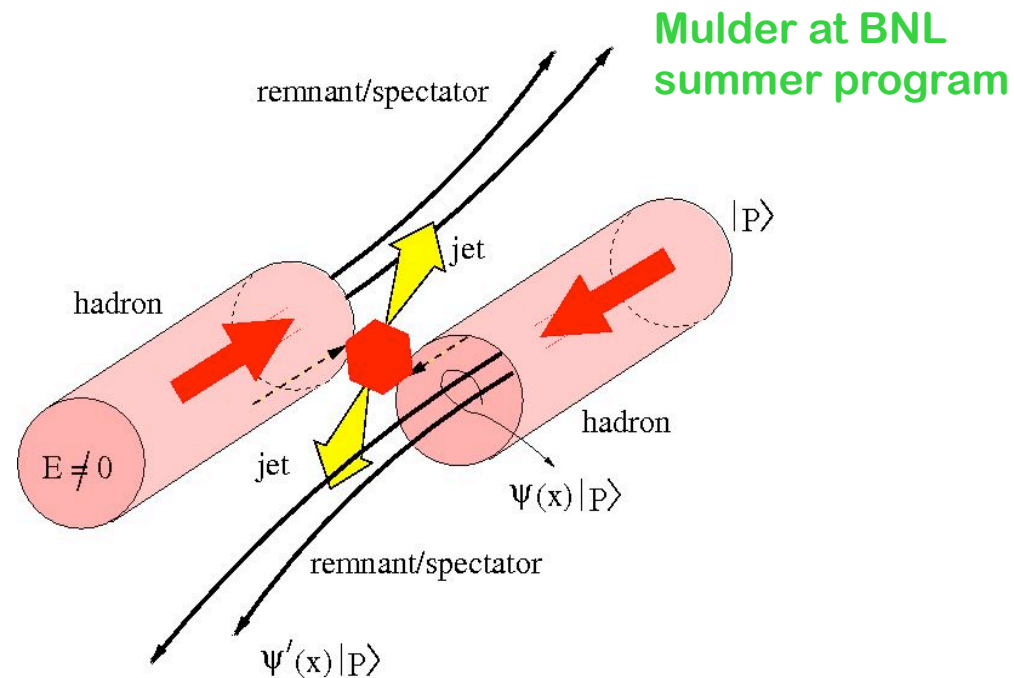
$$\sigma_{\text{hadron}}(Q, \Lambda_{\text{QCD}}) = \sum_{\text{parton}} \phi_{\text{hadron} \rightarrow \text{parton}}(\Lambda_{\text{QCD}}) \otimes \hat{\sigma}_{\text{parton}}(Q) \{ \otimes D_{\text{parton} \rightarrow \text{hadron}}(\Lambda_{\text{QCD}}) \} + \mathcal{O}(\Lambda_{\text{QCD}}/Q)$$

Very non-trivial – due to the phase of gauge theory,
long range soft gluon integrations, ...

□ Phase in gauge theory:



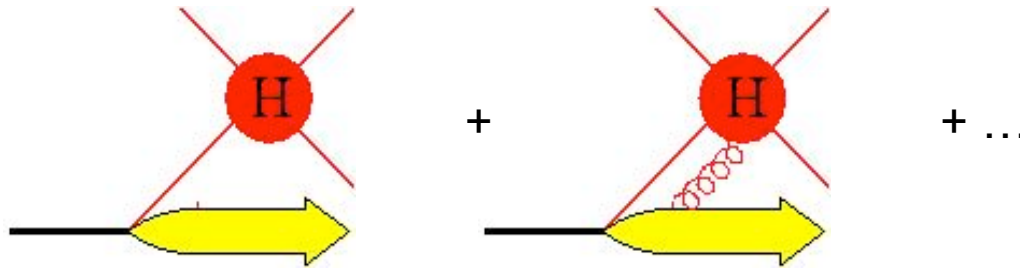
$$\psi' = e^{ie \int ds \cdot A} \psi$$



$$\psi_i(x)|P\rangle = e^{-ig \int_x^{x'} ds_\mu A^\mu} \psi_i(x')|P\rangle$$

Gauge invariance and Universality of PDFs

□ Gauge links:



Summation of leading power gluon field contribution produces the gauge link:

$$U_{[0,\xi]}^{[C]} = P \exp \left(-ig \int_0^\xi ds^\mu A_\mu \right)$$

Gauge invariant PDFs:

$$\Phi_{ij}^{[C]}(p; P) = \int \frac{d^4\xi}{(2\pi)^4} e^{ip \cdot \xi} \langle P | \bar{\psi}_j(0) U_{[0,\xi]}^{[C]} \psi_i(\xi) | P \rangle$$

□ Universality of PDFs:

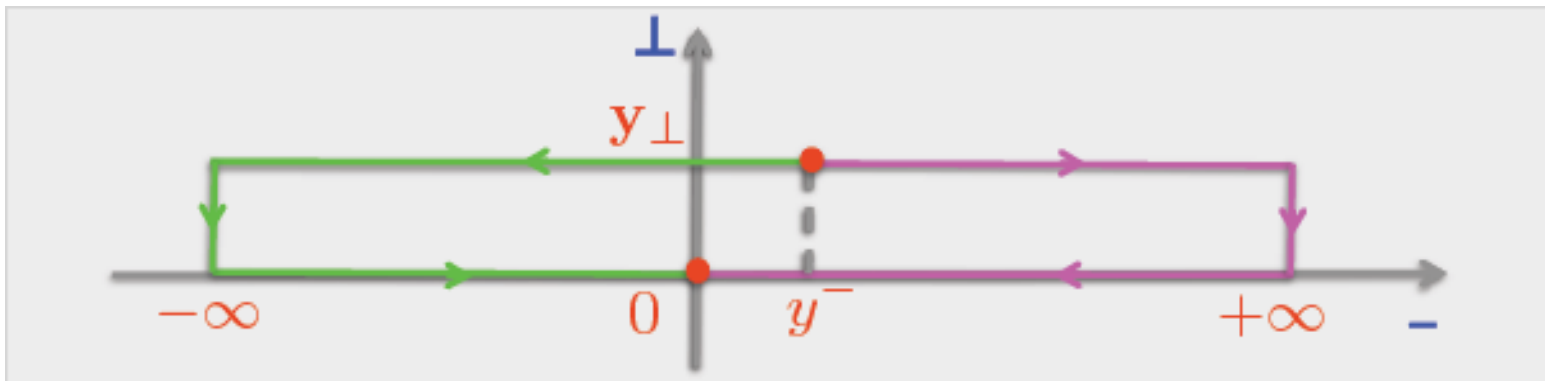
Gauge link should be process independent!

Process dependence of TMDs

□ The form of gauge link is a result of factorization:

$$f_{q/h^\uparrow}(x, \mathbf{k}_\perp, \vec{S}) = \int \frac{dy^- d^2 y_\perp}{(2\pi)^3} e^{ixp^+ y^- - i\mathbf{k}_\perp \cdot \mathbf{y}_\perp} \langle p, \vec{S} | \bar{\psi}(0^-, \mathbf{0}_\perp) \boxed{\text{Gauge link}} \frac{\gamma^+}{2} \psi(y^-, \mathbf{y}_\perp) | p, \vec{S} \rangle$$

- **SIDIS:** $\Phi_n^\dagger(\{+\infty, 0\}, \mathbf{0}_\perp) \Phi_{n_\perp}^\dagger(+\infty, \{\mathbf{y}_\perp, \mathbf{0}_\perp\}) \Phi_n(\{+\infty, y^-\}, \mathbf{y}_\perp)$
- **DY:** $\Phi_n^\dagger(\{-\infty, 0\}, \mathbf{0}_\perp) \Phi_{n_\perp}^\dagger(-\infty, \{\mathbf{y}_\perp, \mathbf{0}_\perp\}) \Phi_n(\{-\infty, y^-\}, \mathbf{y}_\perp)$



$$f_{q/h^\uparrow}^{\text{SIDIS}}(x, \mathbf{k}_\perp, \vec{S}) \neq f_{q/h^\uparrow}^{\text{DY}}(x, \mathbf{k}_\perp, \vec{S})$$

Collinear factorized PDFs are process independent

Modified universality

□ Parity – Time reversal invariance:

$$f_{q/h^\uparrow}^{\text{SIDIS}}(x, \mathbf{k}_\perp, \vec{S}) = f_{q/h^\uparrow}^{\text{DY}}(x, \mathbf{k}_\perp, -\vec{S})$$

□ Definition of Sivers function:

$$f_{q/h^\uparrow}(x, \mathbf{k}_\perp, \vec{S}) \equiv f_{q/h}(x, k_\perp) + \frac{1}{2} \Delta^N f_{q/h^\uparrow}(x, k_\perp) \vec{S} \cdot \hat{p} \times \hat{\mathbf{k}}_\perp$$

□ Modified universality:

$$\Delta^N f_{q/h^\uparrow}^{\text{SIDIS}}(x, k_\perp) = -\Delta^N f_{q/h^\uparrow}^{\text{DY}}(x, k_\perp)$$

Same applies to TMD gluon distribution

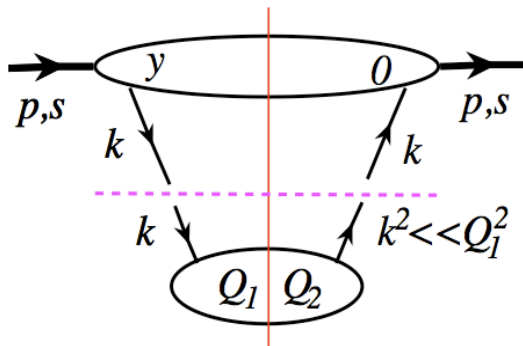
Spin-averaged TMD is process independent

TMD factorization

- More relevant to observables with two very different momentum scales: $Q_1 \gg Q_2 \sim \Lambda_{\text{QCD}}$

$Q_1 \gg \Lambda_{\text{QCD}}$ Makes it possible to have pQCD factorization

$Q_2 \sim \Lambda_{\text{QCD}}$ Sensitive to parton's transverse motion



$k^2 \approx 0$ to hard part

$$k^\mu \approx xp^\mu + \frac{k_T^2}{2x^+} n^\mu + k_T^\mu$$

$$d^4k \Rightarrow \frac{dk^+}{k^+} d^2k_T dk^2$$

Complication: soft gluon interactions between hadrons

- Valid for processes involving only two hadrons:

e^+e^- : $e^+ + e^- \rightarrow h_1(p_1) + h_2(p_2)$ where p_1, p_2 almost back-to-back

SIDIS: $e(l) + h(p) \rightarrow e(l') + h'(p') + X$ with $Q \gg q_T$

Drell-Yan: $h_1(p_1) + h_2(p_2) \rightarrow ll'(q) + X$ with $Q = \sqrt{q^2} \gg q_T$

Key: color flow + locality Collins, Qiu; Yuan, Vogelsang; Rogers, Mulder, ..

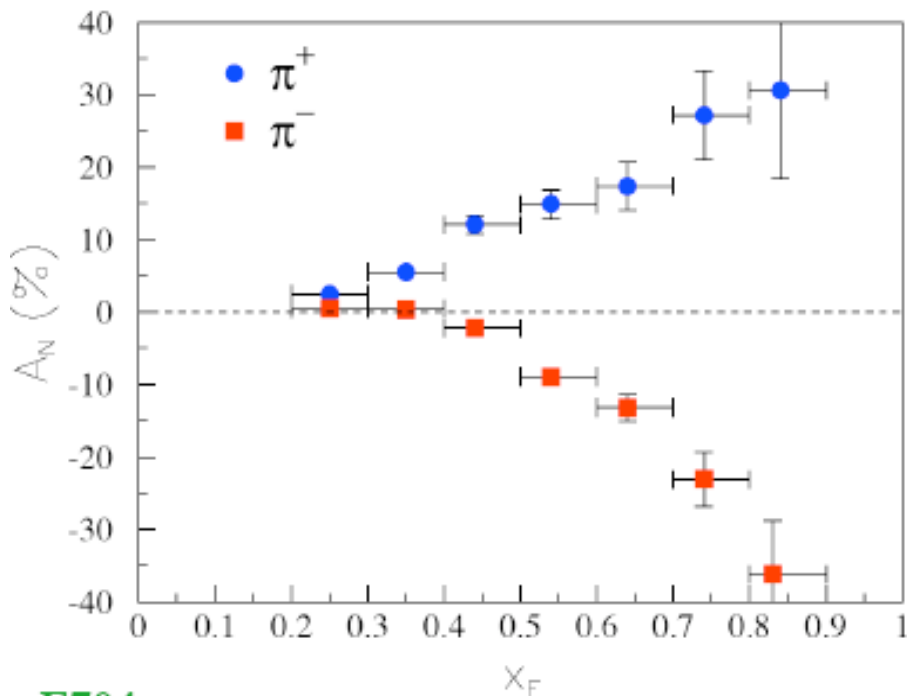
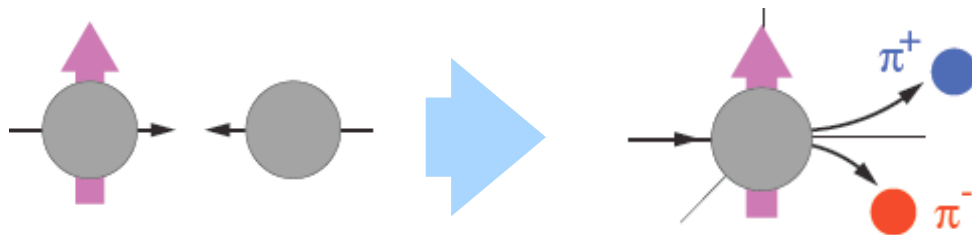
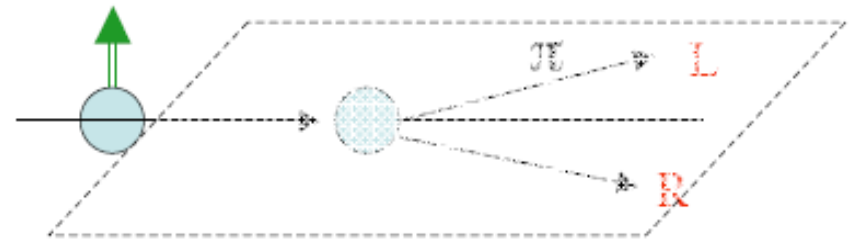
Single transverse spin asymmetry



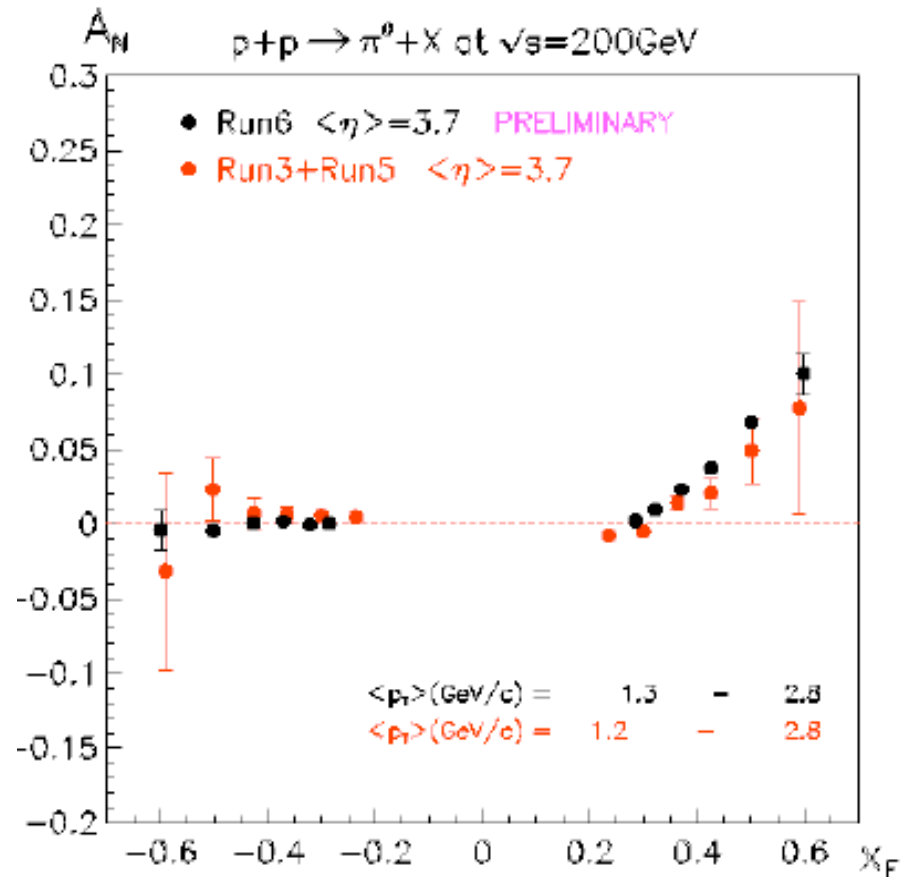
$$p \uparrow + p \rightarrow \pi(l)X$$

Hadronic

$$A_N = \frac{1}{P_{\text{beam}}} \frac{N_{\text{left}}^{\pi} - N_{\text{right}}^{\pi}}{N_{\text{left}}^{\pi} + N_{\text{right}}^{\pi}}$$



E704



STAR (BRAHMS, too)

Necessary condition to have A_N

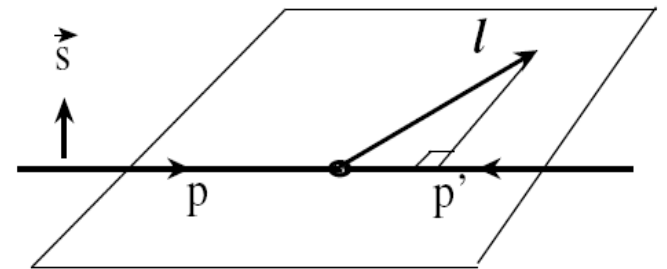
□ Fundamental symmetry and vanishing asymmetry:

- ❖ $A_L=0$ (longitudinal) for Parity conserved interactions
- ❖ $A_N=0$ (transverse) for inclusive DIS – Time-reversal invariance
– proposed to test T-invariance by Christ and Lee (1966)

Even though the cross section is finite!

□ SSA corresponds to a T-odd triple product

$$A_N \propto i\vec{s}_p \cdot (\vec{p} \times \vec{\ell}) \Rightarrow i\varepsilon^{\mu\nu\alpha\beta} p_\mu s_\nu \ell_\alpha p'_\beta$$



Novanishing A_N requires a phase, a spin flip, and enough vectors to fix a scattering plan

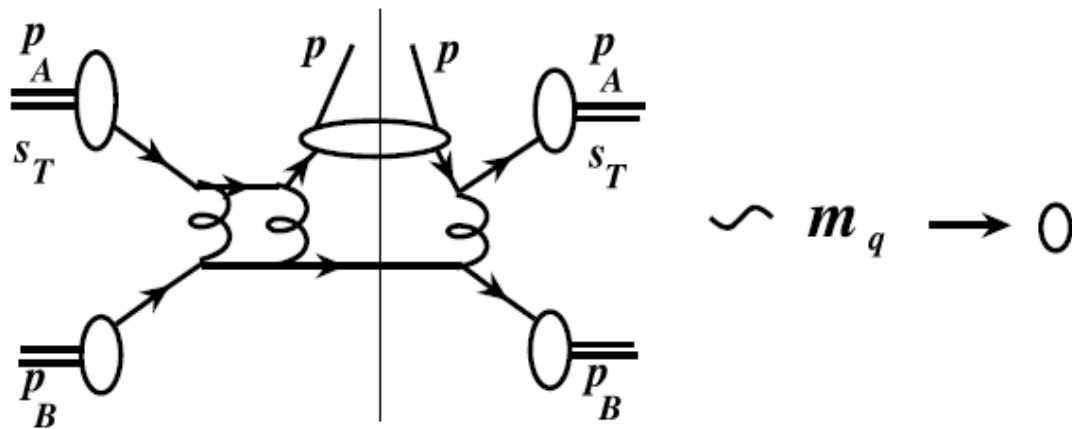
SSA in parton model

□ The spin flip at leading twist – transversity:

$$\delta q(x) = \begin{array}{c} \uparrow \\ \circlearrowleft \\ \uparrow \\ \bullet \\ \downarrow \end{array} - \begin{array}{c} \uparrow \\ \circlearrowright \\ \uparrow \\ \bullet \\ \downarrow \end{array} \propto \langle P, \vec{S}_\perp | \bar{\psi}_q \left[\gamma^+ \gamma \cdot \vec{S}_\perp \right] \psi_q | P, \vec{S}_\perp \rangle$$

Chiral-odd helicity-flip density

- ❖ the operator for δq has even γ 's \rightarrow quark mass term
- ❖ the phase requires an imaginary part \rightarrow loop diagram



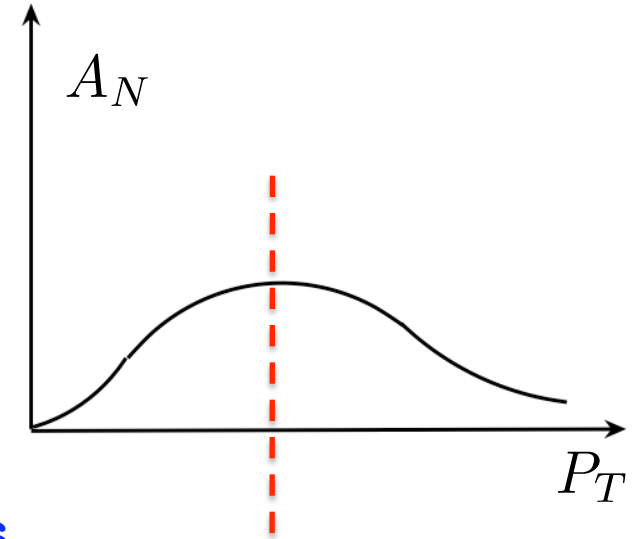
\rightarrow SSA vanishes in the parton model
connects to parton's transverse motion

Generic features of A_N

□ **SIDIS as an example ($Q \gg 1/\text{fm}$):**

$$A_N \longrightarrow 0 \text{ as } p_T \rightarrow 0$$

$$A_N \longrightarrow 0 \text{ as } p_T \rightarrow \text{“}\infty\text{”} (\gg Q)$$

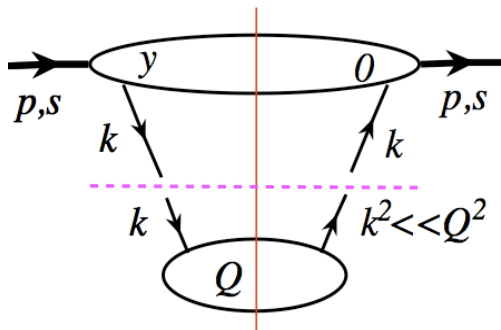


□ **Low p_T region:** $1/\text{fm} \ll p_T \ll Q$

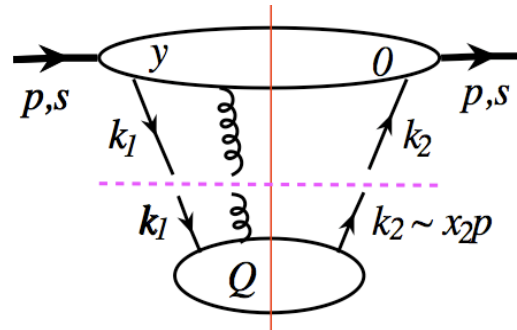
TMD factorization – TMD parton distributions

– direct information on parton’s transverse momentum distributions

□ **High p_T region:** $p_T \gtrsim Q$ – **Collinear factorization:**



+



$$\Rightarrow \langle p, s | \bar{\psi} D^\perp \psi | p, s \rangle$$

$$\langle p, s | \bar{\psi} F^{+\perp} \psi | p, s \rangle$$

Net effect of total “ k_T ”

□ **Transition region:**

If both factorizations are valid, should predict the same A_N !

A_N in collinear factorization

□ A_N – twist-3 effect:

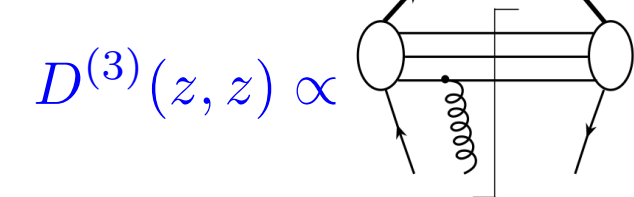
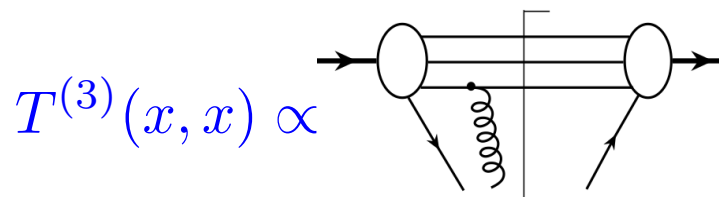
$$\sigma(Q, \vec{s}) \propto \left| \begin{array}{c} \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \dots \end{array} \right|^2$$

Diagram 1: A vertex with incoming lines p, \vec{s} and outgoing lines k, t . A gluon line connects the vertex to another vertex. $t \sim 1/Q$.

Diagram 2: Similar to Diagram 1, but with a different gluon line configuration.

Diagram 3: Similar to Diagram 1, but with a different gluon line configuration.

$$\Delta(s_T) \propto T^{(3)}(x, x) \otimes \hat{\sigma}_T \otimes D_f(z) + \delta q_f(x) \otimes \hat{\sigma}_D \otimes D^{(3)}(z, z)$$



Qiu, Sterman, 1991

Kang, Yuan, Zhou, 2010

□ Spin flip:

– Interference of single parton and a two-parton composite state

□ The phase:

– Interference of Real and Imaginary part of scattering amplitude

– gluon pole: $\propto T^{(3)}(x, x)$

– fermion pole contribution: $\propto T^{(3)}(x, 0)$ or $T^{(3)}(0, x)$ Kang, Qiu, Zhang, 2010

Expected to be smaller!

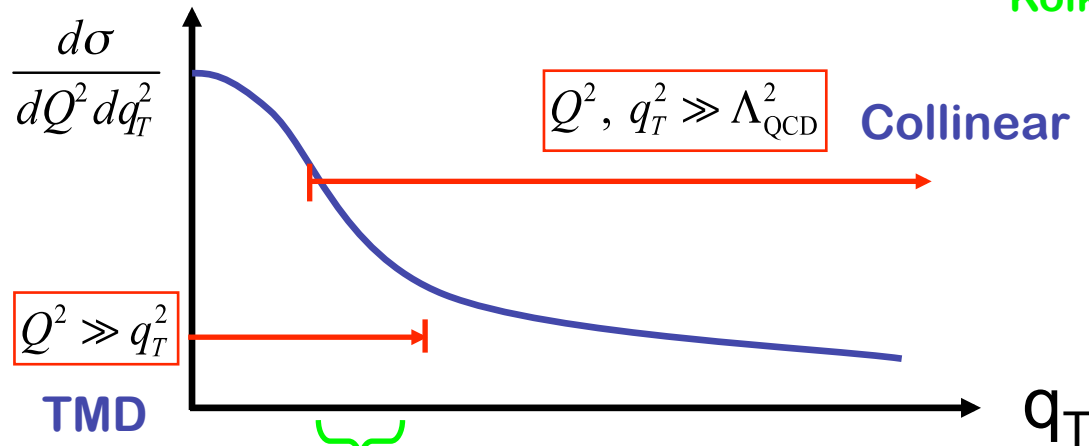
The consistency check

□ **IF both factorizations are proved to be valid,**

✧ both formalisms should yield the same result in overlap region

✧ Case studies – Drell-Yan/SIDIS

Ji, Qiu, Vogelsang, and Yuan
Koike, Vogelsang, and Yuan



In this overlap region, both formalisms indeed give the same result

□ **IF one factorization formalism is valid,**

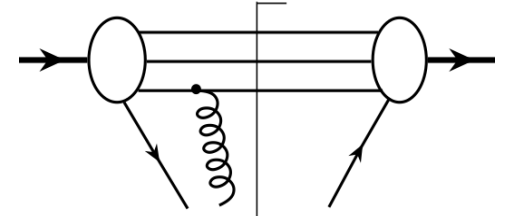
Qiu, Vogelsang, and Yuan

✧ Its asymptotic form in the overlap region is a necessary condition for the other formalism to match

✧ But, it is not sufficient to prove the other factorization formalism

Twist-3 distributions relevant to SSA

□ Two-sets Twist-3 correlation functions:



$$\tilde{T}_{q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ixP^+ y_1^-} e^{ix_2 P^+ y_2^-} \langle P, s_T | \bar{\psi}_q(0) \frac{\gamma^+}{2} [\epsilon^{s_T \sigma n \bar{n}} F_\sigma^+(y_2^-)] \psi_q(y_1^-) | P, s_T \rangle$$

$$\tilde{T}_{G,F}^{(f,d)} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ixP^+ y_1^-} e^{ix_2 P^+ y_2^-} \frac{1}{P^+} \langle P, s_T | F^{+\rho}(0) [\epsilon^{s_T \sigma n \bar{n}} F_\sigma^+(y_2^-)] F^{+\lambda}(y_1^-) | P, s_T \rangle (-g_{\rho\lambda})$$

$$\tilde{T}_{\Delta q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ixP^+ y_1^-} e^{ix_2 P^+ y_2^-} \langle P, s_T | \bar{\psi}_q(0) \frac{\gamma^+ \gamma^5}{2} [i s_T^\sigma F_\sigma^+(y_2^-)] \psi_q(y_1^-) | P, s_T \rangle$$

$$\tilde{T}_{\Delta G,F}^{(f,d)} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ixP^+ y_1^-} e^{ix_2 P^+ y_2^-} \frac{1}{P^+} \langle P, s_T | F^{+\rho}(0) [i s_T^\sigma F_\sigma^+(y_2^-)] F^{+\lambda}(y_1^-) | P, s_T \rangle (i\epsilon_{\perp\rho\lambda})$$

No probability interpretation!

Kang, Qiu, 2009

□ Twist-2 distributions:

▪ Unpolarized PDFs:

$$q(x) \propto \langle P | \bar{\psi}_q(0) \frac{\gamma^+}{2} \psi_q(y) | P \rangle$$

$$G(x) \propto \langle P | F^{+\mu}(0) F^{+\nu}(y) | P \rangle (-g_{\mu\nu})$$

▪ Polarized PDFs:

$$\Delta q(x) \propto \langle P, S_{\parallel} | \bar{\psi}_q(0) \frac{\gamma^+ \gamma^5}{2} \psi_q(y) | P, S_{\parallel} \rangle$$

$$\Delta G(x) \propto \langle P, S_{\parallel} | F^{+\mu}(0) F^{+\nu}(y) | P, S_{\parallel} \rangle (i\epsilon_{\perp\mu\nu})$$

Evolution equations and evolution kernels

□ Evolution is a prediction of QCD:

Like twist-2 PDFs, both collinear and UV divergence are logarithmic, and share the same slope

Kang, Qiu, 2009

➔ Evolution equation for factorization scale dependence
= renormalization group equation for UV renormalization

Bruan et al, 2009

□ Evolution kernels are process independent:

▪ Calculate directly from the variation of process independent twist-3 distributions

Kang, Qiu, 2009
Yuan, Zhou, 2009

▪ Extract from the scale dependence of the NLO hard part of any physical process

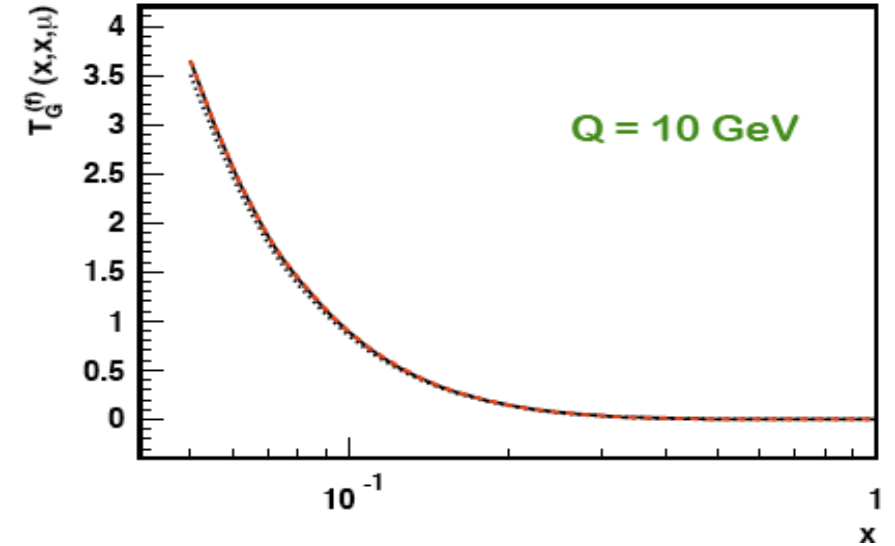
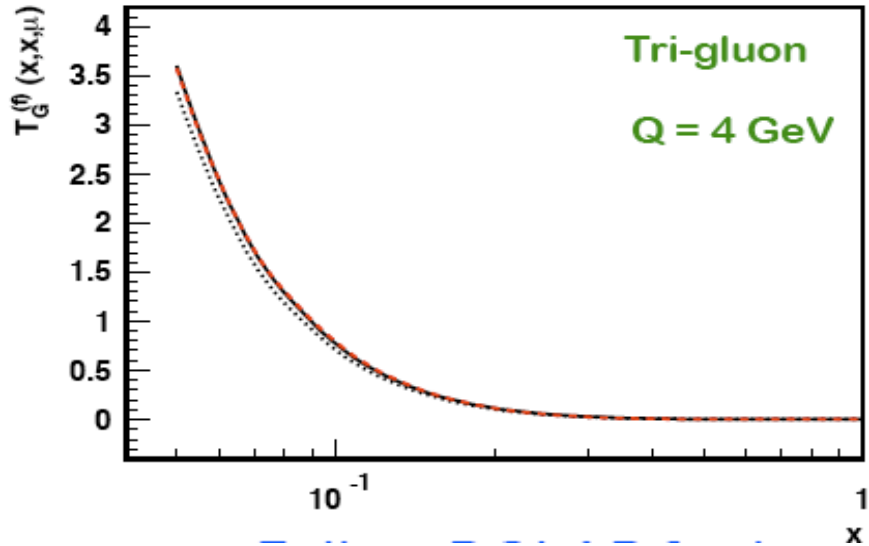
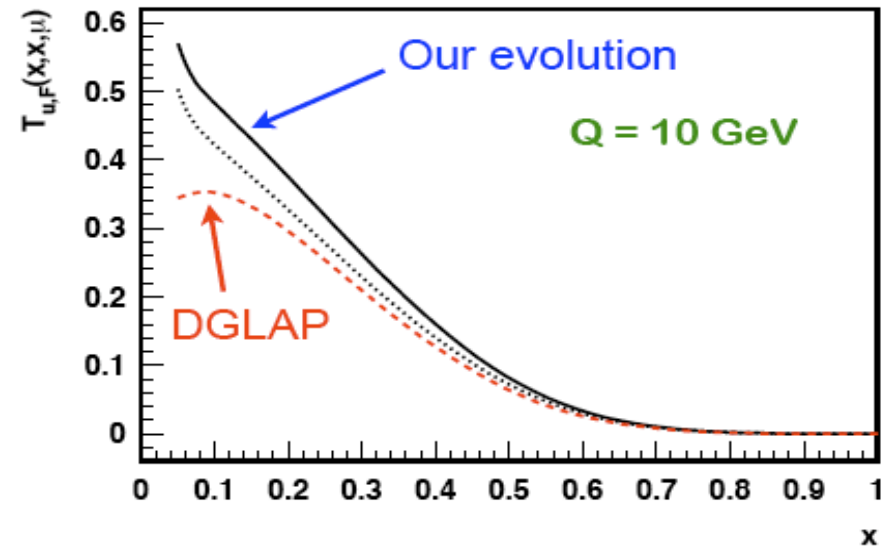
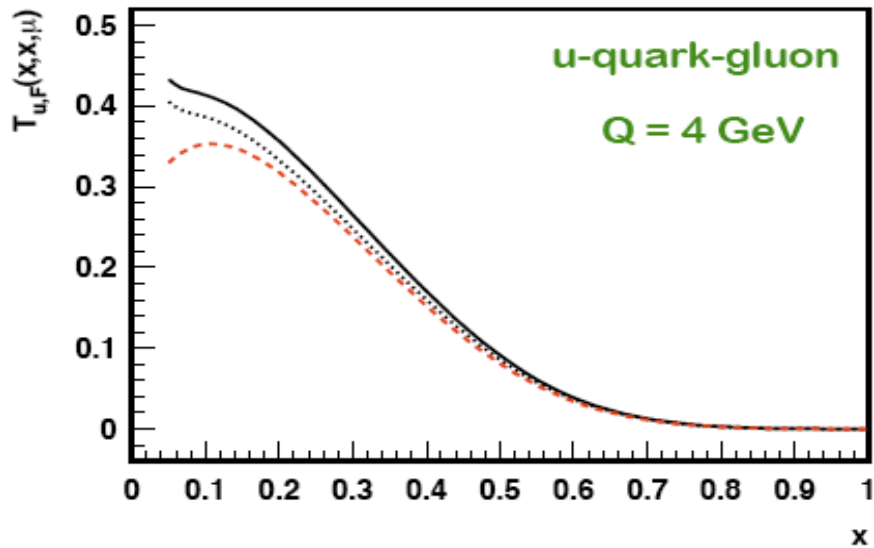
Vogelsang, Yuan, 2009

▪ UV renormalization of the twist-3 operators

Braun et al, 2009

▪ All approaches are equivalent and should give the same kernel

Scale dependence of twist-3 correlations



- Follow DGLAP at large x
- Large deviation at low x (stronger correlation)

Interpretation of twist-3 distributions?

□ Quark-gluon correlation as an example:

$$T_F(x, x) = \int \frac{dy_1^-}{4\pi} e^{ixP^+ y_1^-} \times \langle P, \vec{s}_T | \bar{\psi}_a(0) \gamma^+ \left[\int dy_2^- \epsilon^{s_T \sigma n \bar{n}} F_\sigma^+(y_2^-) \right] \psi_a(y_1^-) | P, \vec{s}_T \rangle$$

□ Normal twist-2 quark distribution:

$$q(x) = \int \frac{dy_1^-}{4\pi} e^{ixP^+ y_1^-} \langle P, \vec{s}_T | \bar{\psi}_a(0) \gamma^+ \psi_a(y_1^-) | P, \vec{s}_T \rangle$$

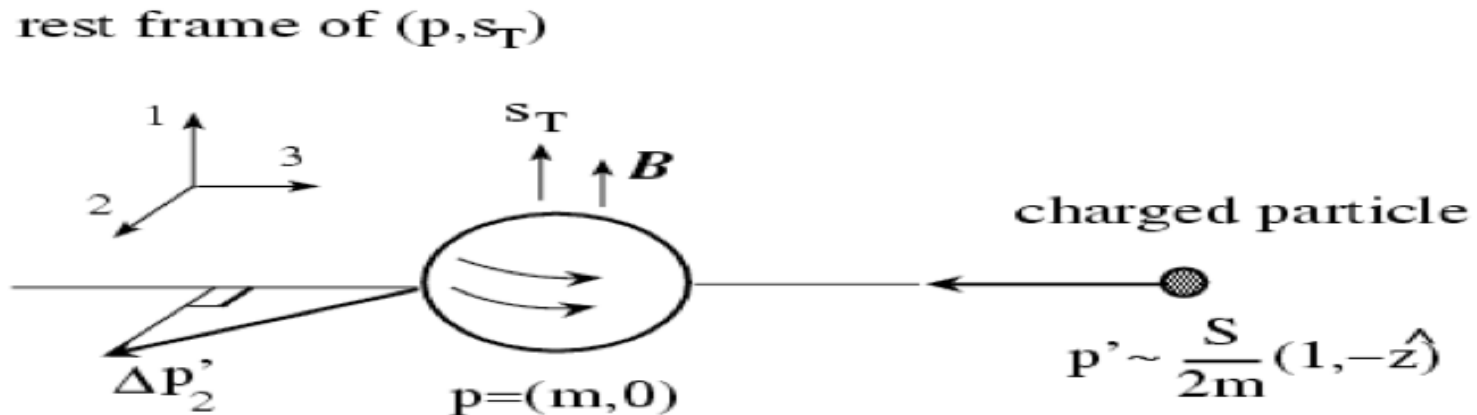
□ Difference – the operator in Red:

$$\left[\int dy_2^- \epsilon^{s_T \sigma n \bar{n}} F_\sigma^+(y_2^-) \right]$$

How can we interpret the “expectation value” of this operator?

What the twist-3 distribution can tell us?

- The operator in Red – a classical Abelian case:



- Change of transverse momentum:

$$\frac{d}{dt} p'_2 = e(\vec{v}' \times \vec{B})_2 = -ev_3 B_1 = ev_3 F_{23}$$

- In the c.m. frame:

$$(m, \vec{0}) \rightarrow \bar{n} = (1, 0, 0_T), \quad (1, -\hat{z}) \rightarrow n = (0, 1, 0_T)$$

$$\implies \frac{d}{dt} p'_2 = e \epsilon^{s_T \sigma n \bar{n}} F_\sigma^+$$

- The total change:

$$\Delta p'_2 = e \int dy^- \epsilon^{s_T \sigma n \bar{n}} F_\sigma^+(y^-)$$

Net quark transverse momentum imbalance caused by color Lorentz force inside a transversely polarized proton

Summery and outlook

- ❑ QCD has been very successful in interpreting high energy data from collisions with hadron(s)
- ❑ Leading power PDFs are more sensitive to the short-distance quantum fluctuation, not hadron structure
- ❑ TMD distributions are more sensitive to hadron structure
- ❑ Transverse spin program opens up many opportunities to explore the parton's transverse motion, parton's 3D structure, and to test QCD in a completely new domain
- ❑ Future EIC is a much needed QCD machine!

Thank you!