Nuclear Dependence of Azimuthal Asymmetry in Semi-Inclusive Deep Inelastic Scattering

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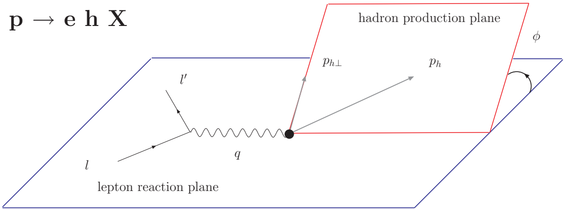
Outline

Azimuthal Asymmetry In SIDIS

Semi-Inclusive Deep Inelastic Scattering

$$\mathbf{e\ p \rightarrow e\ h\ X}$$



Kinematic variables: $x_B = \frac{Q^2}{2P\cdot q}$, $y = \frac{p\cdot q}{p\cdot l}$, $z_h = \frac{p\cdot p_h}{p\cdot q}$

Under the approximation of one photon exchange, the cross section in unpolarized SIDIS can be generally decomposed into:

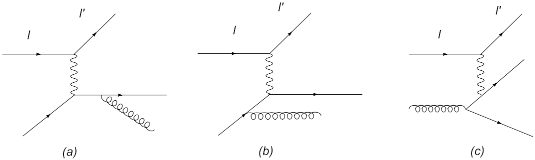
$$\frac{d\sigma}{d\phi} = A + B\cos\phi + C\cos 2\phi$$

The azimuthal angle moments are defined as:

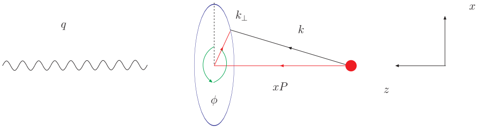
$$\langle\cos\phi\rangle \equiv \frac{\int d\sigma \cos\phi}{\int d\sigma} = \frac{B}{2A}\ , \qquad \langle\cos 2\phi\rangle \equiv \frac{\int d\sigma \cos 2\phi}{\int d\sigma} = \frac{C}{2A}$$

Where the azimuthal asymmetry results from ?

Perturbative QCD effect from $\gamma^* q \to qg\,,\gamma^*\bar{q} \to \bar{q}g\,,\gamma^* g \to q\bar{q}$.
It should be dominant at high $p_{h\perp}$. [H.Georgi and H.D.Politzer PRL 40,3 (1978)]



Nonperturbative effects from intrinsic transverse momentum.
It should be dominant at low $p_{h\perp}$. [R.N.Cahn PLB 78,269 (1978)]



Naive parton distribution function with transverse mometum $k = (xP, 0, k_\perp \cos\phi, k_\perp \sin\phi)$

Interesting nuclear effects

When we replace the proton with the nucleus in DIS, there will be very interesting nuclear effects such as:

- EMC effects
- Nuclear shadowing
- Transverse momentum broadening
- Energy loss from multiple scattering

How about the azimulthal asymmetry ?

What to focus on in this talk

- We will restrict ourselves only on the azimuthal asymmetry $\langle \cos\phi \rangle$ at low $p_{h\perp}$.
- In such kinetic region, we can work in the framework of high twist factorization with TMD distributions.
- In such framework, the cross section can be expressed as a series of products of collinear hard parts and TMD parton distributions or correlations.

Azimulthal asymmetry and TMD distributions

For simplicity, let us first neglect the fragmentation effects and only consider the azimuthal asymmetry of the scattered parton. The azimuthal asymmetry $\langle \cos\phi \rangle$ in the region $k_\perp << Q$ arises at least from the twist-3 contribution:

$$\frac{d\sigma}{dx_B\, dy d^2 k_\perp} = \frac{2\pi\alpha_{em}^2 e_q^2}{Q^2 y}\left\{[1+(1-y)^2] f_q^A(x_B,\vec{k}_\perp) - 4(2-y)\sqrt{1-y}\frac{|\vec{k}_\perp|}{Q} x_B f_{q\perp}^{(1)A}(x_B,\vec{k}_\perp)\cos\phi\right\}$$

where the parton distributions are defined as

twist-2 : $$f_q^A = \int \frac{dy^- d^2\vec{y}_\perp}{(2\pi)^3} e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp} \langle A|\bar{\psi}(0)\frac{\gamma^+}{2}\mathcal{L}(0,y)\psi(y)|A\rangle$$

twist-3 : $$f_{q\perp}^A = \int \frac{dy^- d^2\vec{y}_\perp}{(2\pi)^3} e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp} \langle A|\bar{\psi}(0)\frac{\not{k}_\perp}{2k_\perp^2}\mathcal{L}(0,y)\psi(y)|A\rangle$$

It follows that the azimuthal asymmetry is given by the ratio of twist-2 TMD distribution to twist-3 TMD distribution :

$$\langle \cos\phi \rangle_{eA} = -\frac{2(2-y)\sqrt{1-y}}{1+(1-y)^2}\frac{|\vec{k}_\perp|}{Q}\frac{x_B f_{q\perp}^A(x_B,k_\perp)}{f_q^A(x_B,k_\perp)}.$$

Nuclear effects and TMD distributions

The above formula are valid for both A(nucleus) and N(nucleon):

$$\langle \cos\phi\rangle_{eA} = -\frac{2(2-y)\sqrt{1-y}}{1+(1-y)^2}\frac{|\vec{k}_\perp|}{Q}\frac{x_B f^A_{q\perp}(x_B,k_\perp)}{f^A_q(x_B,k_\perp)}.$$

$$\langle \cos\phi\rangle_{eN} = -\frac{2(2-y)\sqrt{1-y}}{1+(1-y)^2}\frac{|\vec{k}_\perp|}{Q}\frac{x_B f^N_{q\perp}(x_B,k_\perp)}{f^N_q(x_B,k_\perp)}.$$

Nuclear effect can be measured by the ratio :

$$\frac{\langle \cos\phi\rangle_{eA}}{\langle \cos\phi\rangle_{eN}} = \frac{f^A_{q\perp}(x_B,k_\perp)/f^A_q(x_B,k_\perp)}{f^N_{q\perp}(x_B,k_\perp)/f^N_q(x_B,k_\perp)}$$

Hence all the nuclear effects come from the differences between

$$f^A_q, f^A_{q\perp} \quad \text{and} \quad f^N_q, f^N_{q\perp}$$

How can we estimate such differences between them ?

Some approximation or simplification

In order to estimate such nuclear effect, we need make some approximation or simplification on the initial distribution of the nucleon in the nucleus:

- We will assume the nucleus is large and weakly bound, in which we can neglect multiple-nucleon correlation. It follows that the nuclear effect can only arise from the final state interaction from multiple parton scattering which have been ecoded into the gauge link.

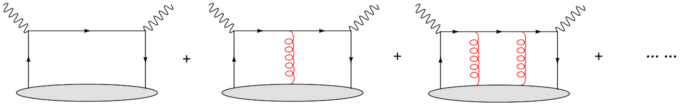
Gauge Link and Final state interaction

Recall the definition of the TMD distributions which are relevant to the azimulthal asymmetry $\cos\phi$:

$$f_q^A = \int \frac{dy^- d^2\vec{y}_\perp}{(2\pi)^3} e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp} \langle A|\bar{\psi}(0)\frac{\gamma^+}{2}\mathcal{L}(0,y)\psi(y)|A\rangle$$

$$f_{q\perp}^A = \int \frac{dy^- d^2\vec{y}_\perp}{(2\pi)^3} e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp} \langle A|\bar{\psi}(0)\frac{\not{k}_\perp}{2k_\perp^2}\mathcal{L}(0,y)\psi(y)|A\rangle$$

where $\mathcal{L}(0,y)$ is the so-called gauge link. It is indispensable to keep the distribution functions gauge invariant. It results from the final interaction between the scattered parton and the remnants of the broken nucleon or nucleus:



Gauge Link and Final state interaction

It should be emphasized that the gauge link has important physical effect. For example, in SIDIS on the tranversely polarized proton, the TMD quark distribution can be generally decomposed into :

$$f_q^P(x,k_\perp)=\int \frac{dy^- d^2\vec{y}_\perp}{(2\pi)^3} e^{ixp^+y^- -i\vec{k}_\perp\cdot\vec{y}_\perp} \langle P,S|\bar{\psi}(0)\frac{\gamma^+}{2}\mathcal{L}(0,y)\psi(y)|P,S\rangle$$

$$=f(x,k_\perp)+\vec{S}\cdot(\hat{\vec{p}}\times\vec{k}_\perp)f_{1T}^\perp(x,k_\perp)/M$$

where $\vec{S}$ is the spin of the target proton and $f_{1T}^\perp$ is Sivers function.

Gauge link $\Rightarrow$ Sivers function $f_{1T}^\perp(x,k_\perp)\neq 0$ $\Rightarrow$ single spin asymmetry

Is it possible such gauge link gives rise to nuclear effect?

Multiple Scattering and Nuclear Effects

In order to extract the gauge invariant nuclear effects, we need perform some manipulation on the TMD distribution function :

$$\int \frac{dy^-}{2\pi}\frac{d^2y_\perp}{(2\pi)^2} e^{-i\vec{k}_\perp\cdot\vec{y}_\perp} \langle A|\bar{\psi}(0)\frac{\gamma_\alpha}{2}\mathcal{L}(0,y)\psi(y)|A\rangle$$

$$\Downarrow$$

$$\int d^2l_\perp \int \frac{dy^-}{2\pi}\frac{d^2y_\perp}{(2\pi)^2} e^{-i\vec{l}_\perp\cdot\vec{y}_\perp} \langle A|\bar{\psi}(0)\frac{\gamma_\alpha}{2} e^{i\vec{\nabla}_{y_\perp}\cdot\vec{\nabla}_{k_\perp}}\mathcal{L}(0,y)\psi(y)|A\rangle\delta^{(2)}(k_\perp)$$

Note : The transverse derivative $i\vec{\nabla}_{y_\perp}$ should act on not only the quark field ψ but also the gauge link $\mathcal{L}(0,y)$:

$i\vec{\nabla}_{y_\perp}\mathcal{L}(0,y)=\mathcal{L}(0,y)\vec{W}_\perp(y^-)$, where $\vec{W}_\perp(y^-)\equiv i\vec{D}_\perp(y^-)-g\int_{y^-}^{\infty} d\xi^-\vec{F}_{+\perp}(\xi^-)$

We will show that the nuclear effect results just from $\vec{W}_\perp(y^-)$.
We can formally finish integrating over both $l_\perp$ and $y_\perp$:

$$\int \frac{dy^-}{2\pi}\langle A|\bar{\psi}(0)\frac{\gamma_\alpha}{2} e^{\vec{W}_\perp(y^-)\cdot\vec{\nabla}_{k_\perp}}\psi(y^-)|A\rangle\delta^{(2)}(\vec{k}_\perp)$$

Multiple Scattering and Nuclear Effects

The nuclear effect arises when $\vec{W}_\perp(y^-)$ acts on different nucleons:

$$\langle A|\bar{\psi}(0)\frac{\gamma^\alpha}{2}\left[\vec{W}_\perp(y^-)\cdot\vec{\nabla}_{k_\perp}\right]^n\psi(y^-)|A\rangle$$
$$\Downarrow$$
$$\langle\langle A|\left[\vec{W}_\perp(y^-)\cdot\vec{\nabla}_{k_\perp}\right]^{2m}|A\rangle\rangle\cdot\langle N|\bar{\psi}(0)\frac{\gamma^\alpha}{2}\left[\vec{W}_\perp(y^-)\cdot\vec{\nabla}_{k_\perp}\right]^{n-2m}\psi(y^-)|N\rangle$$

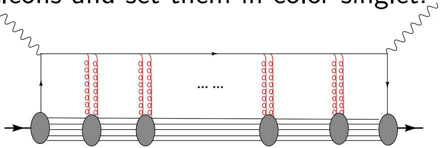
In order to get the maximal nuclear effect, we will only need keep $-g\int_{y^-}^{\infty}d\xi^-\vec{F}_{+\perp}(\xi^-)$ in the definition of $\vec{W}_\perp(y^-)$

$$\langle\langle A|\left[\vec{W}_\perp(y^-)\cdot\vec{\nabla}_{k_\perp}\right]^{2m}|A\rangle\rangle\Rightarrow\langle\langle\left[g^2\int_{y^-}^{\infty}d\xi_1^-\int_{y^-}^{\infty}d\xi_2^-\vec{F}_{+\perp}(\xi_1^-)\vec{F}_+^\perp(\xi_2^-)\nabla^2_{k_\perp}\right]^m\rangle\rangle_A$$

Besides, we should make each pair of gauge field strength tensor be attached to different nucleons and set them in color singlet.

$|\xi_1^- - \xi_2^-|\sim r_N$

$(\xi_1^- + \xi_2^-)/2\sim R_A\gg r_N$



r_N: the radius of the nucleon. R_A: the radius of the nucleus.

[M. Luo,J. W. Qiu,G. Stermann PRD50,1951,(1994)] [Z. T. Liang,X. N. Wang, J. Zhou PRD77,125010,(2008)]

Multiple Scattering and Nuclear Effects

Under the assumption that the nucleus is large and weakly bound and the distribution of the nucleon in nucleus is homogenous. We can finally obtain two valuable relations between the TMD distribution of the nucleus and the nucleon such as:

$$f_q^A(x,k_\perp) \approx \frac{A}{\pi \Delta_{2F}} \int d^2\ell_\perp e^{-(\vec{k}_\perp - \vec{\ell}_\perp)^2/\Delta_{2F}} f_q^N(x,\ell_\perp).$$

$$f_{q\perp}^A(x,k_\perp) \approx \frac{A}{\pi \Delta_{2F}} \left(1 + \frac{\Delta_{2F}}{2\vec{k}_\perp^2} \vec{k}_\perp \cdot \vec{\partial}_{k_\perp}\right) \int d^2\ell_\perp e^{-(\vec{k}_\perp - \vec{\ell}_\perp)^2/\Delta_{2F}} f_{q\perp}^N(x,\ell_\perp)$$

where $\Delta_{2F} = \frac{2\pi^2 \alpha_s}{N_c} \int d\xi_N^- \rho_N^A(\xi_N^-) [x f_g^N(x)]_{x=0}$

It represents the total average-squared transverse momentum $k_\perp$ broadening when a parton propagates in the nucleus.

$\rho_N^A(\xi_N)$: the spatial nucleon number density inside the nucleus.
$x f_g^N(x)$: the gluon distribution function in a nucleon.

$$x f_g^N(x) = -\int \frac{d\xi^-}{2\pi p^+} e^{ixp^+\xi^-} \langle N | F_{+\sigma}(0) F_+{}^{\sigma}(\xi^-) | N \rangle.$$

A simple Gaussian distribution as an example

To illustrate the nuclear dependence of the azimuthal asymmetry qualitatively, we consider an ansatz of Gaussian distribution in $k_\perp$ for both twist-2 and twist-3 quark TMD distribution in the nucleon:

$$f_q^N(x,k_\perp)=\frac{1}{\pi\alpha}f_q^N(x)e^{-k_\perp^2/\alpha}, \quad f_{q\perp}^N(x,k_\perp)=\frac{1}{\pi\beta}f_{q\perp}^N(x)e^{-k_\perp^2/\beta}$$

It follows that the TMD distribution of the nucleus is given by:

$$f_q^A(x,k_\perp)\approx\frac{A}{\pi(\alpha+\Delta_{2F})}f_q^N(x)e^{-k_\perp^2/(\alpha+\Delta_{2F})}, \quad f_{q\perp}^A(x,k_\perp)\approx\frac{A\beta}{\pi(\beta+\Delta_{2F})^2}f_{q\perp}^N(x)e^{-k_\perp^2/(\beta+\Delta_{2F})}$$

Nuclear modification factor:

$$\frac{\langle\cos\phi\rangle_{eA}}{\langle\cos\phi\rangle_{eN}}=\frac{\beta^2(\alpha+\Delta_{2F})}{\alpha(\beta+\Delta_{2F})^2}\exp\left\{\frac{(\alpha-\beta)\Delta_{2F}(\alpha+\beta+\Delta_{2F})}{\alpha\beta(\alpha+\Delta_{2F})(\beta+\Delta_{2F})}\vec{k}_\perp^2\right\}.$$

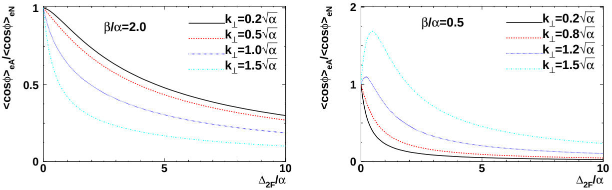
Set $\alpha=\beta$

$$\frac{\langle\cos\varphi\rangle_{eA}}{\langle\cos\varphi\rangle_{eN}} = \frac{\alpha}{\alpha+\Delta_{2F}} < 1$$

The azymuthal asymmetry is suppressed.

The suppression increases with Δ_{2F} and is independent of $k_\perp$.

Nuclear effects dependence on Δ_{2F} and $k_\perp$



In the case $\beta > \alpha$,

- The azimuthal asymmetry is suppressed and the suppression increases with both $k_\perp$ and Δ_{2F}.

In the case $\beta < \alpha$,

- The azimuthal asymmetry is suppressed at small and medium $k_\perp$. The suppression decreases with $k_\perp$ but increases with Δ_{2F}.
- The azimuthal asymmetry could be enhanced for large $k_\perp$, however in such kinematic region it is expected that PQCD process starts to play a good role.

Smearing effects from the fragmentation

In order to conclude the nuclear effects of azimuthal asymmetry, we must estimate the fragmentation effects. Similarly, take another Gaussian distribution for the TMD fragmentation function:

$$D_F^{q\to h}(z,\vec{k}_{F\perp})=D_F^{q\to h}(z)\frac{1}{\pi\alpha_F}e^{-\vec{k}^2_{F\perp}/\alpha_F}.$$

For simplicity, just set $\alpha=\beta$ for the $f_q^N(x,k_\perp)$ and $f_{q\perp}^N(x,k_\perp)$

$$\frac{\langle\cos\phi_h\rangle_{eA}}{\langle\cos\phi_h\rangle_{eN}}=\frac{\alpha z^2+\alpha_F}{(\alpha+\Delta_{2F})z^2+\alpha_F}\geqslant\frac{\alpha}{\alpha+\Delta_{2F}}$$

Averaged azimuthal asymmetry after integrating over $|\vec{p}_{h\perp}|$:

$$\frac{\langle\langle\cos\phi_h\rangle\rangle_{eA}}{\langle\langle\cos\phi_h\rangle\rangle_{eN}}=\sqrt{\frac{\alpha z^2+\alpha_F}{(\alpha+\Delta_{2F})z^2+\alpha_F}}\geqslant\sqrt{\frac{\alpha}{\alpha+\Delta_{2F}}}$$

The nuclear suppression of the azimuthal asymmetry is clearly smeared due to the fragmentation process of the quark

Summary

- The azimuthal asymmetry is suppressed by multiple parton scattering for most cases of the TMD quark distributions.
- The nuclear suppression increases with the average squared transverse-momentum broadening for most cases.
- Nuclear effect of the azimuthal asymmetry is a very sensitive probe of the twist-2 and twist-3 TMD distribution functions.
- The nuclear suppression effects would be smeared due to the parton's fragmentation process.

Thanks for your attention !