Recent progress from China Lattice QCD

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Outline

- Current status of collaboration
- Recent progress
  - D^*D_1 scattering and Z(4430)
  - J/psi radiative decays
  - Other works…
- Summary

Basic status of the collaboration

- People
  - Y. Chen (IHEP, CAS)
  - C. Liu (Peking University)
  - Y.B. Liu (Nankai University)
  - J.P. Ma (ITP, CAS)
  - J.B. Zhang (Zhejiang University)
  - Postdocs+students
- Funding
  - Funded by NSFC, etc.
- Computing facilities: Shanghai, Tianjin, Beijing…

D^*D_1 scattering and Z(4430)

Signal for Z(4430)

S.K. Choi et al., PRL 100,142001 (2008)

- **Resonance structure Z(4430)**
  - **Close to threshold** $D^{*+}\overline{D}_1^0$
  - $Q=+1,\ J^P=0^-,1^-,2^-$
  - M=4433 MeV
  - Γ=45 MeV

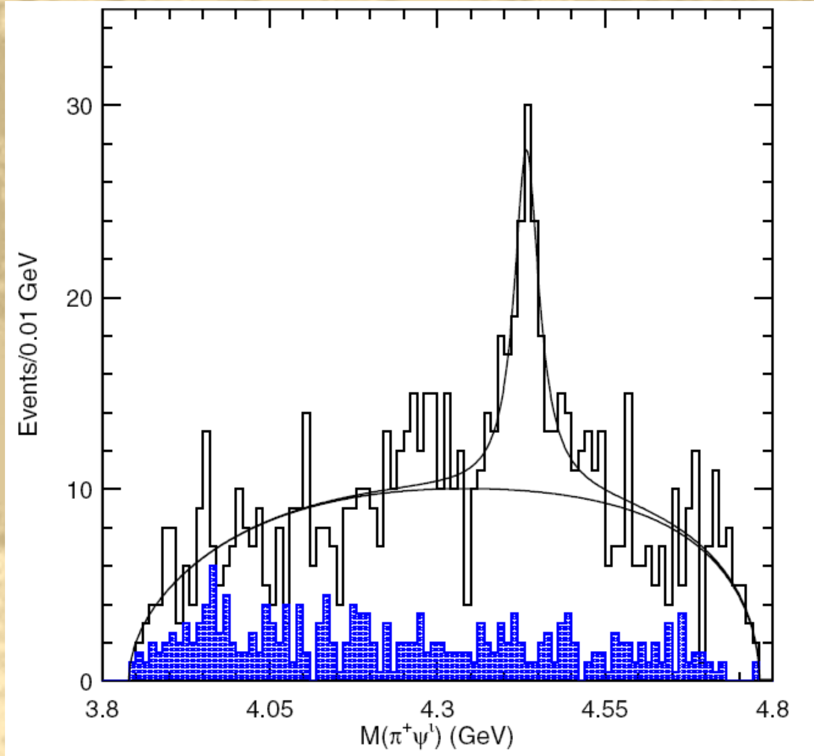


FIG. 2 (color online). The $M(\pi^+\psi')$ distribution for events in the $M_{bc} - \Delta E$ signal region and with the K^* veto applied. The shaded histogram show the scaled results from the ΔE sideband. The solid curves show the results of the fit described in the text.

Theoretical investigations

- **Phenomenology**
  - Shallow bound state (S.L. Zhu et al, PRD77,034003)
  - Tetra-quark resonance above threshold (X.-H Liu et al, PRD77, 094005)
  - Threshold enhancement (J.L. Rosner, PRD76,114002)
- **Need to study scattering near threshold**
  - Scattering length a_0 and effective range r_0
  - Lattice QCD study (quenched)

Utilizing Lüscher's formalism

- Consider two hadrons in a finite box
- interaction ： shift of two-particle energy
- interaction ： scattering phase shifts
- Lüscher's formula:

$$\delta(E_{1\bullet 2}) \Leftrightarrow E_{1\bullet 2}(L)$$

Basic formulae

- A box of size ***L***, periodic in all three spatial directions:

$$\vec{k} = (2\pi / L)\vec{n} \;,\;\; \vec{n} \in Z^3$$

- Two interacting hadrons

$$E_{1\bullet 2}(\bar{k}) = \sqrt{m_1^2 + \bar{k}^2} + \sqrt{m_2^2 + \bar{k}^2} \;,\;\; q^2 = \bar{k}^2 L^2 / (2\pi)^2$$

$$\tan\delta(q) = \frac{\pi^{3/2} q}{Z_{00}(1; q^2)}$$

Taking advantage of asymmetric box

- Use asymmetric volumes: $L \times (\eta_2 L) \times (\eta_3 L)$
- Take: $\eta_2 = 1,\ \eta_3 > 1$
- The symmetry for the box is D_4

$$\tan\delta(q) = \frac{\pi^{3/2} q \eta_2 \eta_3}{Z_{00}(1; q^2; \eta_2, \eta_3)}$$

Single hadron operators

- Single particle operators

D^{*+}

$\overline{D_1}^0$

$$Q_i(x) = [\bar{d}\gamma^i c](x), \qquad P_i(x) = [\bar{c}\gamma^i\gamma^5 u](x),$$

- Angular momentum decomposition

$$\mathbf{0} = A_1, \qquad \mathbf{1} = E \oplus A_2, \qquad \mathbf{2} = A_1 \oplus B_1 \oplus B_2 \oplus E.$$

Double hadron operators

TABLE I. The two-particle operators defined in Eq. (12) and their corresponding angular momentum quantum number J in the continuum.

J^P	Two-particle operators
$\mathbf{0}^-$	$O^{(A_1)(1)}(t)$
$\mathbf{1}^-$	$O^{(A_2)}(t)$, $O_1^{(E)(1)}(t)$, $O_2^{(E)(1)}(t)$
$\mathbf{2}^-$	$O^{(A_1)(2)}(t)$, $O^{(B1)}(t)$, $O^{(B_2)}(t)$, $O_1^{(E)(2)}(t)$, $O_2^{(E)(2)}(t)$

$$O^{(A_1)(1)}(t) = \sum_{R\in G}[Q_1(t+1, -R\circ \mathbf{k})P_1(t, R\circ \mathbf{k}) + Q_2(t+1, -R\circ \mathbf{k})P_2(t, R\circ \mathbf{k}) + Q_3(t+1, -R\circ \mathbf{k})P_3(t, R\circ \mathbf{k})],$$

Correlation matrix

- Only the correlation matrix in A_1 channel shows signal

$$C_{mn}^{(A_1)(1)}(t) = \langle O_m^{(A_1)(1)\dagger}(t) O_n^{(A_1)(1)}(0) \rangle,$$

- We have computed 5 lowest non-zero momentum modes together with the zero momentum mode.

Simulation setup

- Tadpole improved clover action on anisotropic lattices

TABLE II. Simulation parameters in this study. All lattices have the same aspect ratio $\xi = 5$.

	$\beta = 2.5$	$\beta = 2.8$	$\beta = 3.2$
N_{conf}	700	500	200
u_s^4	0.4236	0.4630	0.50679
ν_c	0.732	0.79	0.89
ν_{ud}	0.9305	0.96	1.0
$a_s(fm)$	0.2037	0.1432	0.0946
Lattice	$8 \times 8 \times 12 \times 40$	$12 \times 12 \times 20 \times 64$	$16 \times 16 \times 24 \times 80$
$\kappa^c_{\max}$	0.0577	0.0598	0.0595
$\kappa^{ud}_{\max}$	0.0613	0.0611	0.0606

Checking single particle states

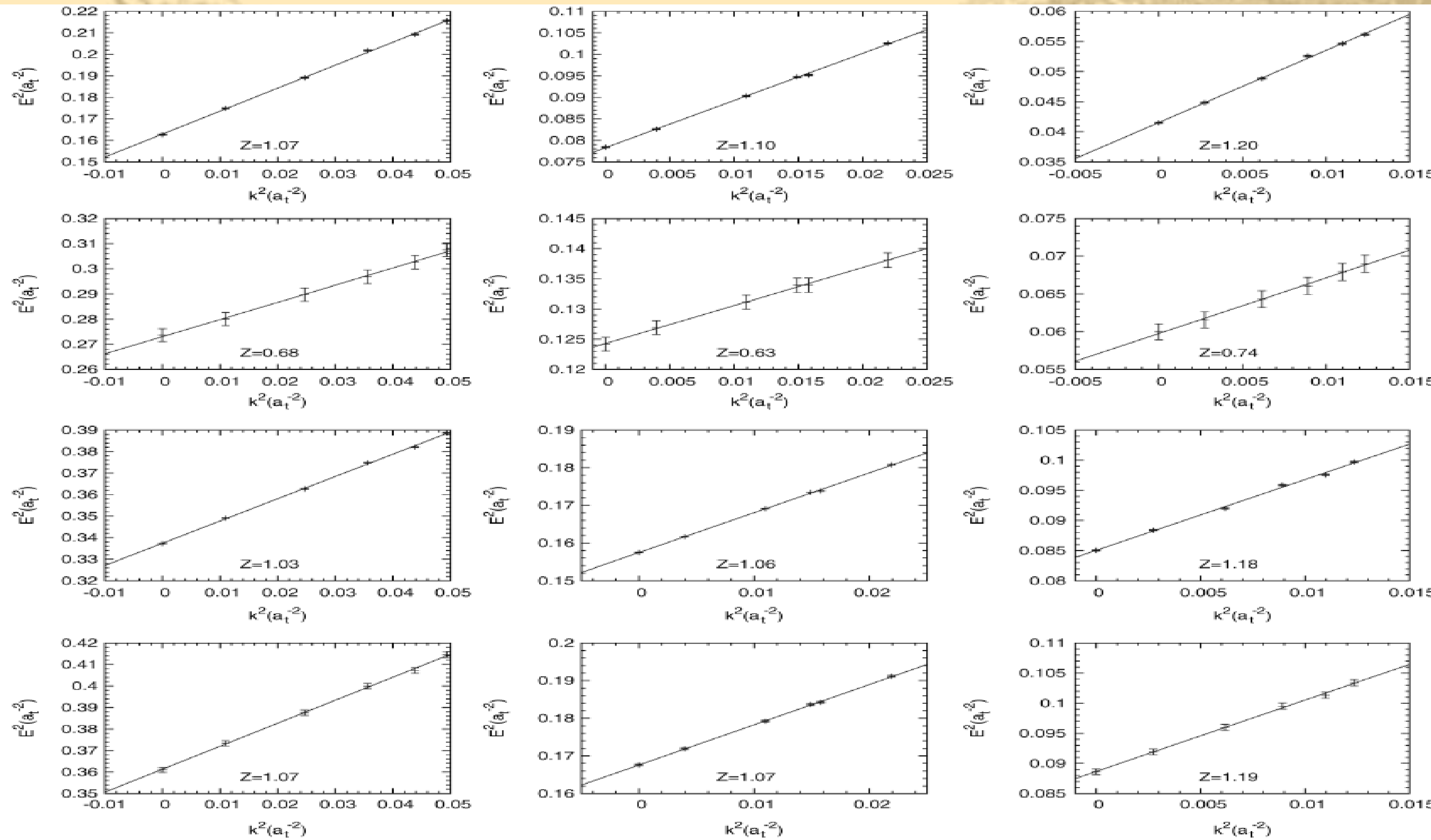


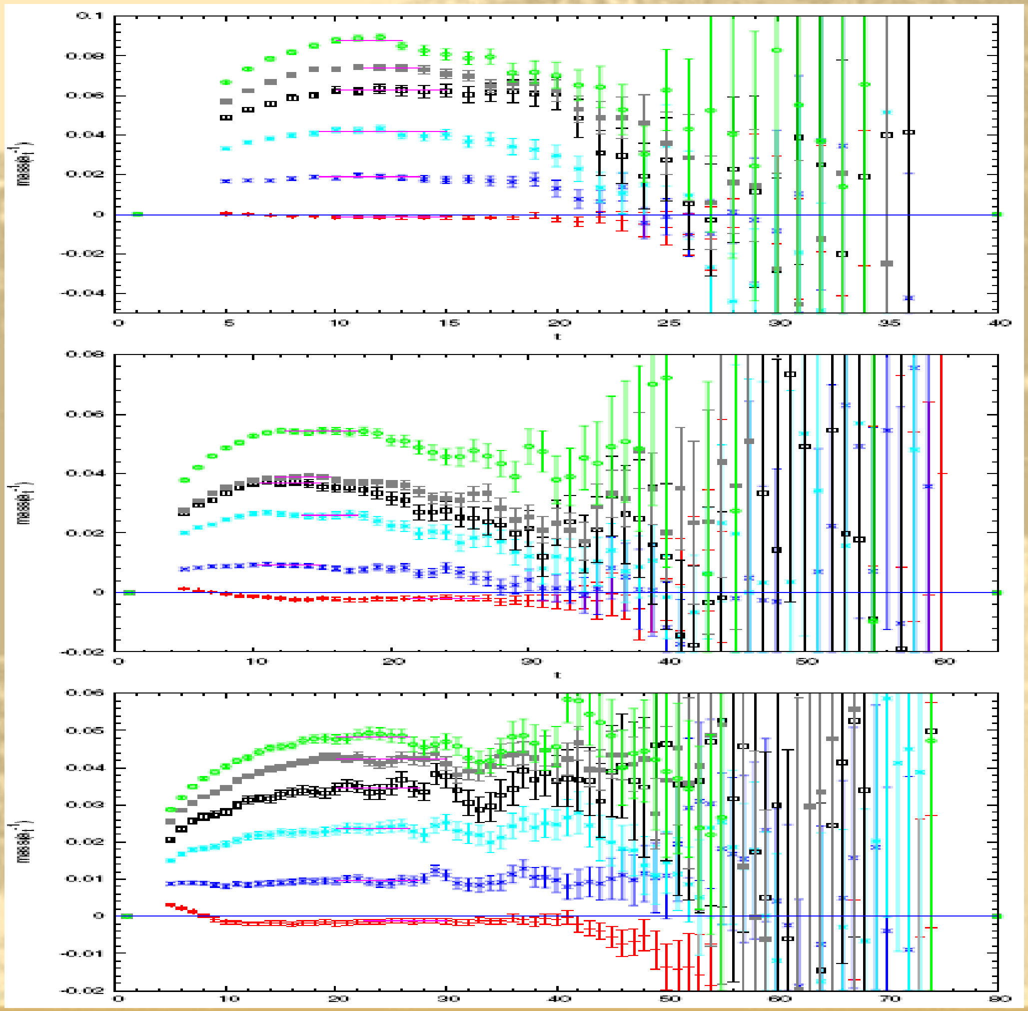
FIG. 5. Dispersion relations for various mesons obtained from single meson energies. From top to bottom: D^*, $\bar{D}_1$, η_c, J/ψ; from left to right: $\beta = 2.5, 2.8, 3.2$.

Extract two-hadron energy

- Appropriate ratios are taken:

$$R(t) = \frac{\lambda_i(t)}{C_{D^*}(t) C_{D_1}(t)}$$

$$\Rightarrow \delta E_i^{eff} = E_{1\bullet 2} - m_{D^*} - m_{D_1}$$



Obtaining a_0 & r_0

⌘ Scattering length a_0 & effective range r_0

$$\frac{k}{\tan\delta(k)} = \frac{1}{a_0} + \frac{1}{2} r_0 k^2 + \cdots,$$

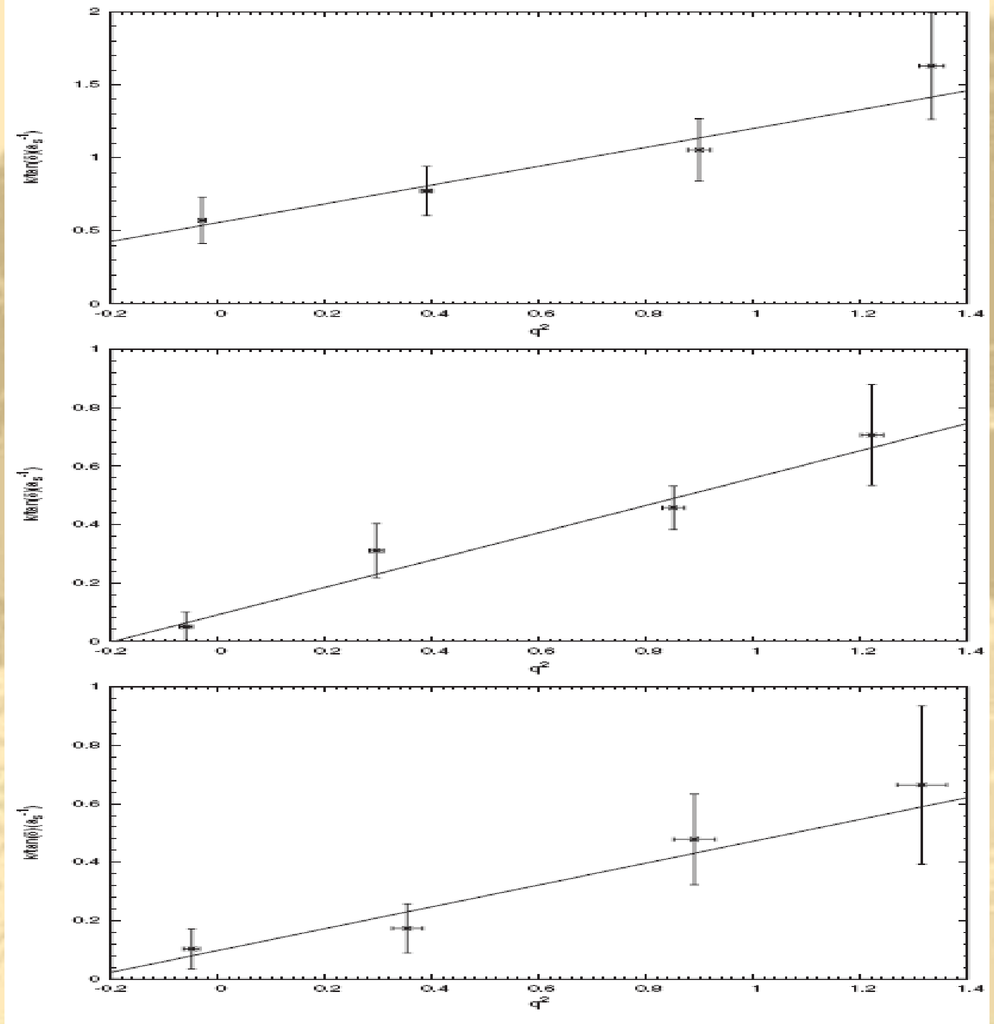
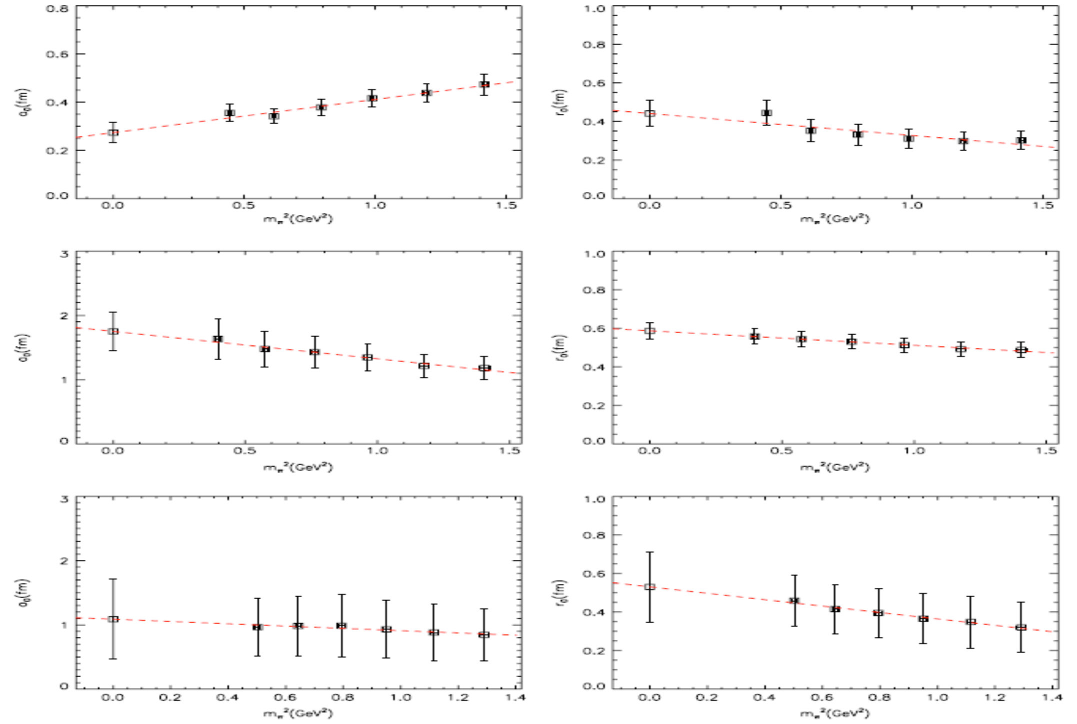


FIG. 7. The quantity $k/\tan\delta(k)$ versus q^2 in the $A_1^{(1)}$ channel. From top to bottom: $\beta = 2.5$, 2.8, and 3.2.

extrapolations

- After all the extrapolations

$$a_0 = 2.53 \pm 0.47 \text{ fm}, \qquad r_0 = 0.70 \pm 0.10 \text{ fm}.$$



discussions

- Possible bound state?
  - For shallow bound state, $a_0 \to -\infty$
  - But our scattering lengths are all positive
  - On the verge of developing a bound state
  - Using the square well potential model to estimate, the potential well is V_0=70(10) MeV, R=0.7fm.
- Our results is **not** in favor of a shallow bound state

Glueballs in J/psi radiative decays

I. Introduction

- QCD predicts the existence of glueballs
- Quenched LQCD predicts glueball spectrum in the range 1~3GeV
- candidates: f0(1370), f0(1500), f0(1710) etc.
- J/psi radiative decay can be the best hunting ground.
- BESIII is producing 10^{10} J/psi events

J^{PC}	$r_0 M_G$	M_G (MeV)
0^{++}	4.16(11)(4)	1710(50)(80)
2^{++}	5.83(5)(6)	2390(30)(120)
0^{-+}	6.25(6)(6)	2560(35)(120)
1^{+-}	7.27(4)(7)	2980(30)(140)
2^{-+}	7.42(7)(7)	3040(40)(150)
3^{+-}	8.79(3)(9)	3600(40)(170)
3^{++}	8.94(6)(9)	3670(50)(180)
1^{--}	9.34(4)(9)	3830(40)(190)
2^{--}	9.77(4)(10)	4010(45)(200)
3^{--}	10.25(4))(10)	4200(45)(200)
2^{+-}	10.32(7)(10)	4230(50)(200)
0^{+-}	11.66(7)(12)	4780(60)(230)

Y. Chen et al,
Phys. Rev. D 73, 014516 (2006)

Hadronic matrix element

- The computation of J/psi radiative decay into glueballs breaks up into perturbative part (QED part) and non-perturbative part
- The non-perturbative part requires the computation of matrix element

$$\left\langle G \,|\, j_{\mu}^{(e.m.)}(x) \,|\, J/\Psi \right\rangle$$

Glueball state

EM current

J/Ψ state

Three-point function

$$\Gamma^{(3)}(\vec{p}_f,\vec{q};t_f,t) = -\sum_{\vec{x},\vec{y}} e^{-i\vec{p}_f\cdot\vec{x}} e^{+i\vec{q}\cdot\vec{y}} \langle O_S(\vec{x},t_f) j^\mu(\vec{y},t) O_V^\dagger(0,0)\rangle$$

$$(t_f \geq t \geq 0)$$

Scalar glueball operator

Electromagnetic current operator

Charmonium operator

After the intermediate state insertion, the three-point function can be written as

$$\Gamma^{(3),\mu j}(\vec{p}_f,\vec{q};t_f,t) = \sum_{f,i,r} \frac{e^{-E_f(t_f-t)}e^{-E_i t}}{2E_f(\vec{p}_f)2E_i(\vec{p}_i)} \times \langle 0|O_S(0)|f(\vec{p}_f)\rangle\langle f(\vec{p}_f)|j^\mu(0)|i(\vec{p}_i,r)\rangle\langle i(\vec{p}_i,r)|O_V^{(j)\dagger}|0\rangle$$

$$\langle 0|O_V^\mu|n(\vec{p},r)\rangle \equiv Z_n \epsilon^\mu(\vec{p},r)$$

$$\sum_r \epsilon^\mu(\vec{p},r)\epsilon^{\nu *}(\vec{p},r) = -g^{\mu\nu} + \frac{p^\mu p^\nu}{m_n^2}$$

$$\vec{p}_i = \vec{p}_f - \vec{q}.$$

Two-point function of vector meson

$$\begin{aligned} C_2^{ij}(\vec{p},t) &= \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}}\langle 0|O_V^{(i)}(\vec{x},t)O_V^{(j),\dagger}((\vec{0}),0)|0\rangle \\ &= \sum_{n,r} \frac{1}{2E_n(\vec{p})}\langle 0|O_V^{(i)}(0)|n(\vec{p},r)\rangle\langle n(\vec{p},r)|O_V^{(j),\dagger}(0)|0\rangle e^{-E_n t} \\ &= \sum_n \frac{Z_n Z_n^*}{2E_n(\vec{p})}\left(\delta_{ij} + \frac{p^j p^j}{m_n^2}\right) e^{-E_n t}. \end{aligned}$$

II. Numerical details

1. Lattice and parameters

Anisotropic lattice: $L^3 \times T = 8^3 \times 96$ $\xi = a_s / a_t = 5$

Strong coupling: $\beta = 2.4$ $a_s = 0.222(2)\,fm$

2. Actions

$$S_{IA} = \beta\{\frac{5}{3}\frac{\Omega_{sp}}{\xi u_s^4} + \frac{4}{3}\frac{\xi\Omega_{tp}}{u_t^2 u_s^2} - \frac{1}{12}\frac{\Omega_{sr}}{\xi u_s^6} - \frac{1}{12}\frac{\xi\Omega_{str}}{u_s^4 u_t^2}\}$$

$$\mathcal{A}_{xy} = \delta_{xy}[1/(2\kappa_{max}) + \rho_t \sum_{i=1}^{3} \sigma_{0i}\mathcal{F}_{0i} + \rho_s(\sigma_{12}\mathcal{F}_{12} + \sigma_{23}\mathcal{F}_{23} + \sigma_{31}\mathcal{F}_{31})]$$
$$- \sum_{\mu} \eta_\mu[(1-\gamma_\mu)U_\mu(x)\delta_{x+\mu,y} + (1+\gamma_\mu)U_\mu^+(x-\mu)\delta_{x-\mu,y}]$$

$$\eta_i = \nu/(2u_s), \eta_0 = \xi/2, \sigma = 1/(2\kappa) - 1/(2\kappa_{max}),$$

$$\rho_t = c_{SW}(1+\xi)/(4u_s^2), \rho_s = c_{SW}/(2u_s^4).$$

3. Configurations and quark propagators

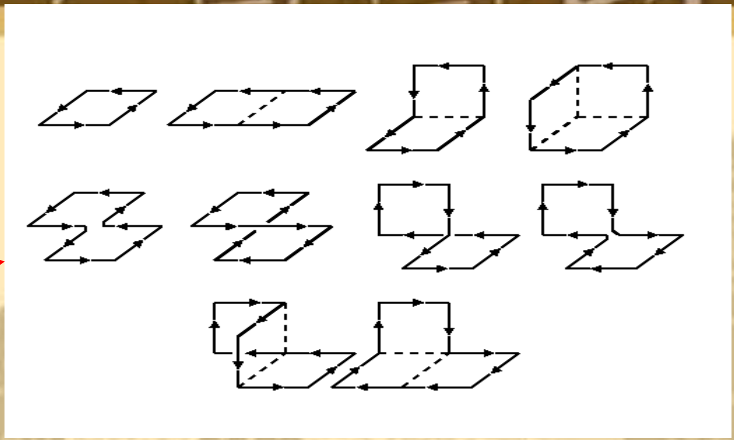
In order to get fair signals of the three point functions, a large enough statistics is required.

- 5000 gauge configurations, separated by 100 HB sweeps
- Charm quark mass is set by the physical mass of J/psi
- On each configuration, 96 charm quark propagators are calculated with point sources on all the 96 time slices. The periodic boundary conditions are used both for the spatial and temporal directions.

$$\Gamma^{(3)\mu i}(\vec{p}_f,\vec{q};t_f,t)=\frac{1}{T}\sum_{\tau=0}^{T-1}\sum_{\vec{y}}e^{+i\vec{q}\cdot\vec{y}}\left\langle O_G(\vec{p}_f,t_f+\tau)\,j^{\mu}(\vec{y},t+\tau)O^{i,+}_{J/\psi}(\tau)\right\rangle$$

4. The glueball operators

Building prototypes
(various Wilson loops)



Smearing: Single link scheme (APE) and double link scheme (fuzzying)

The essence of the VM is to find a set of combinational coefficients

$$\{v_\alpha, \alpha = 1,2,\ldots 24\}$$

such that the operator

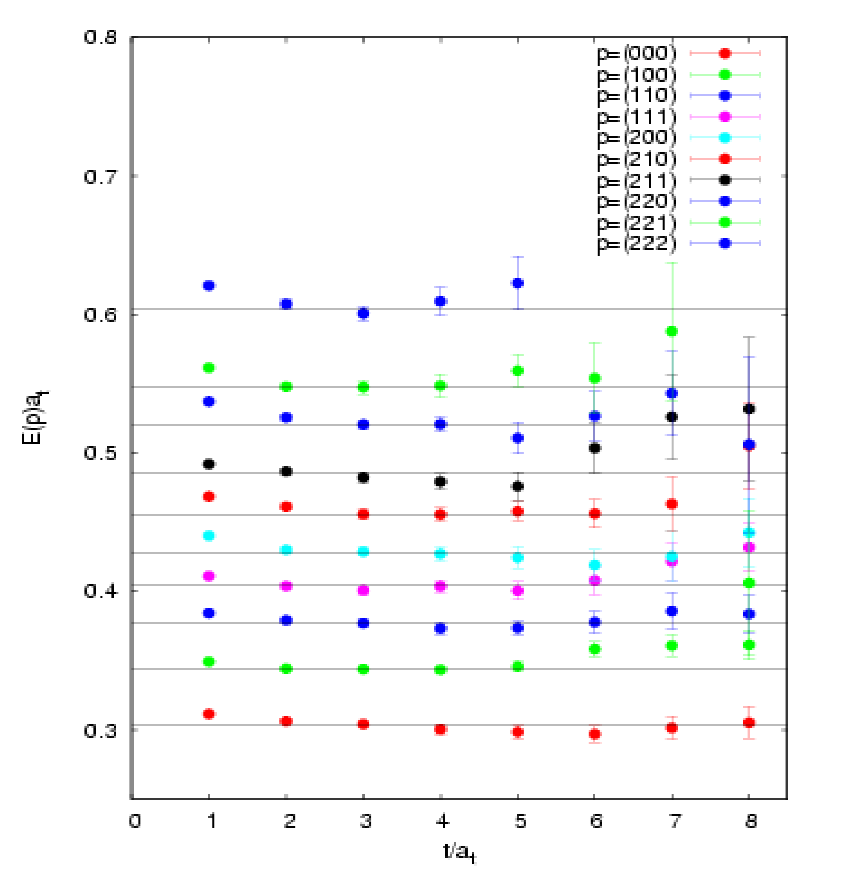
$$\Phi = \sum_\alpha v_\alpha \phi_\alpha$$

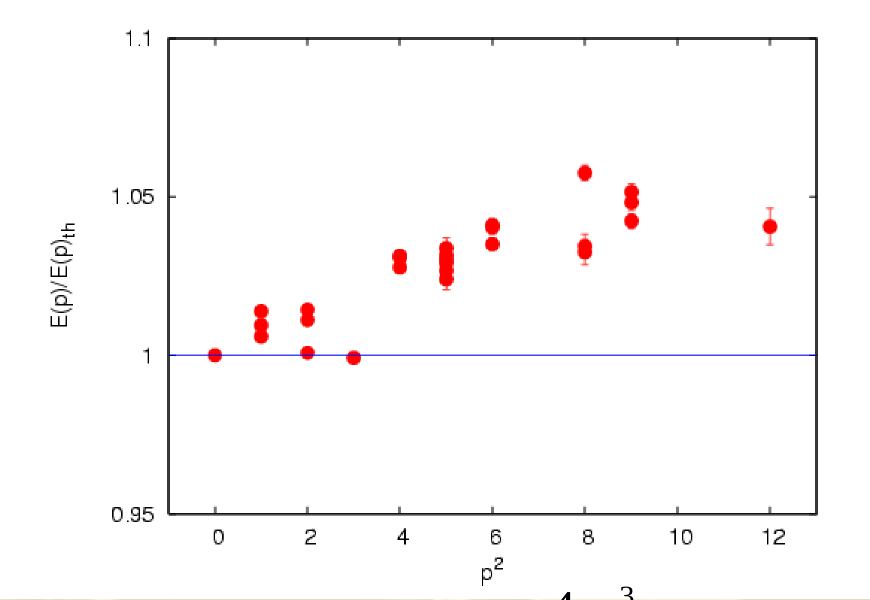
couples mostly to a specific state.

$$\tilde{C}(t_D)\mathbf{v}^{(R)} = e^{-t_D \tilde{m}(t_D)}\tilde{C}(0)\mathbf{v}^{(R)}$$

$$\tilde{C}_{\alpha\beta}(t) = \sum_\tau \langle 0|\phi_\alpha(t+\tau)\phi_\beta(\tau)|0\rangle$$

$$\tilde{m}(t_D) = -\frac{1}{t_D}\ln\frac{\sum_{\alpha\beta} v_\alpha v_\beta \tilde{C}_{\alpha\beta}(t_D)}{\sum_{\alpha\beta} v_\alpha v_\beta \tilde{C}_{\alpha\beta}(0)}$$





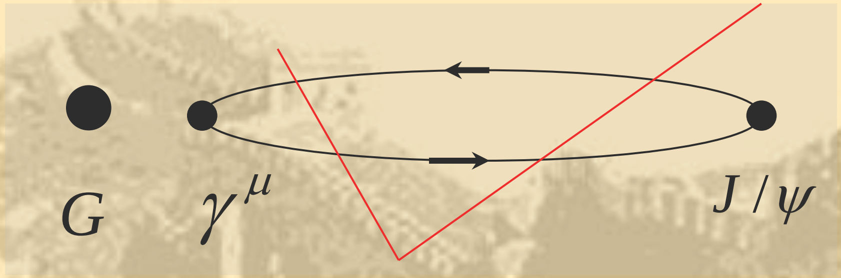
$$a_t^2 E_{th}^2(\vec{p}) = a_t^2 m^2 + \frac{4}{\xi^2}\sum_{i=1}^{3}\sin^2\frac{p_i}{2},$$

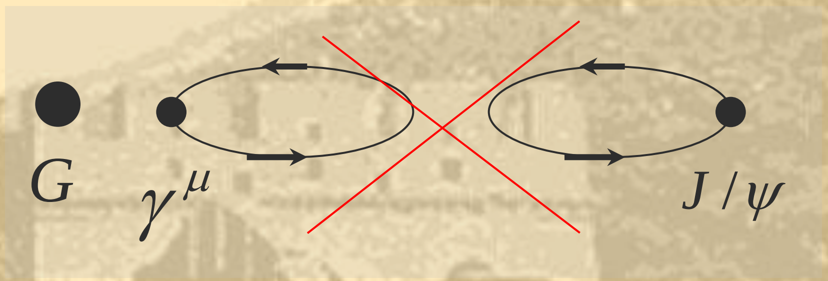
$$\vec{p} = \frac{2\pi}{L}(n_1, n_2, n_3)$$

The energies of 27 momentum modes of scalar glueballs are calculated. Plotted are the plateaus using the optimized operators

The horizontal line is the theoretical prediction with $\xi = 5$.
It is seen that the deviations are less than 5%

5. Three point functions





$$\Gamma^{(3),\mu j}(\vec{p}_f,\vec{q};t_f,t) = \sum_{f,i,r} \frac{e^{-E_f(t_f-t)}e^{-E_i t}}{2E_f(\vec{p}_f)2E_i(\vec{p}_i)} \times \langle 0|O_S(0)|f(\vec{p}_f)\rangle\langle f(\vec{p}_f)|j^\mu(0)|i(\vec{p}_i,r)\rangle\langle i(\vec{p}_i,r)|O_V^{(j)\dagger}|0\rangle$$

$$\vec{p}_i = \vec{p}_f - \vec{q}.$$

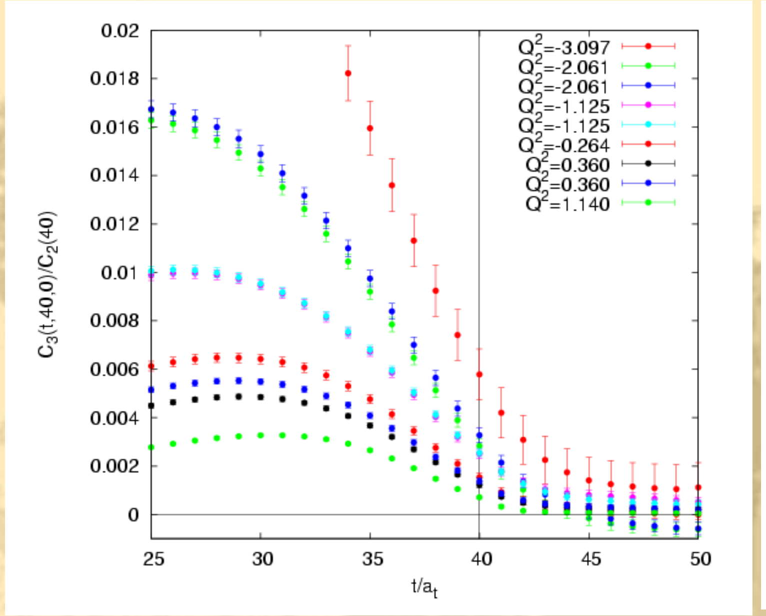
Temporarily, we only analyze the following cases:

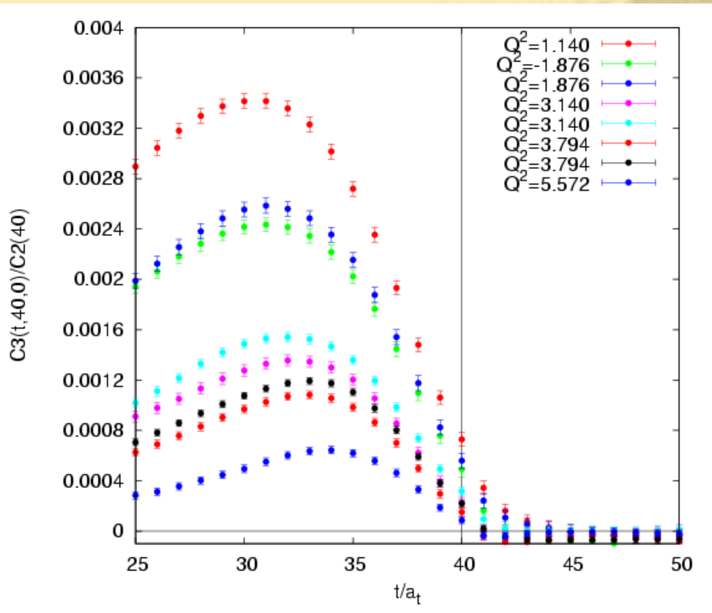
$$\vec{p}_i = (0,0,0) \quad \Longleftrightarrow \quad \vec{p} = \vec{q} = (000),(100),\cdots,(222)$$

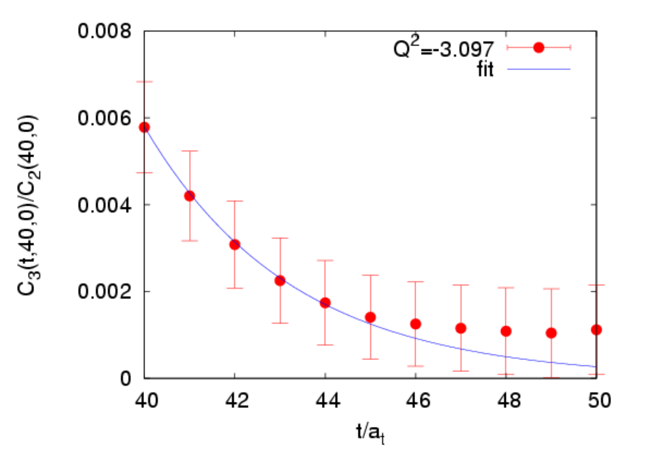
$$\mu = i = 1,2,3$$

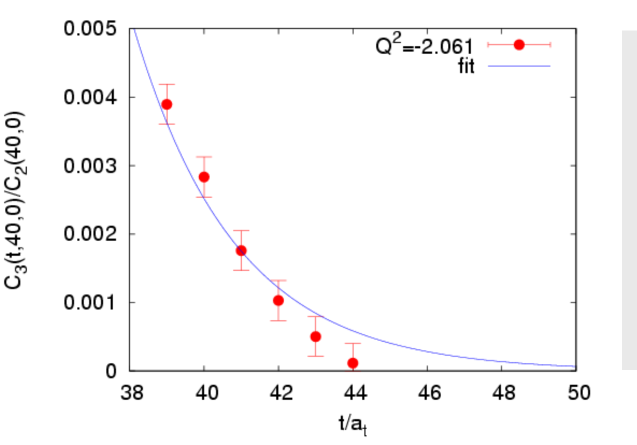
$$Q^2 = -(p_f - p_i)^2 = \vec{p}_f^2 - (M_i - E_f)^2$$

The plots for the ratios $\Gamma^{(3)}(t,40,0)/C_2(40)$

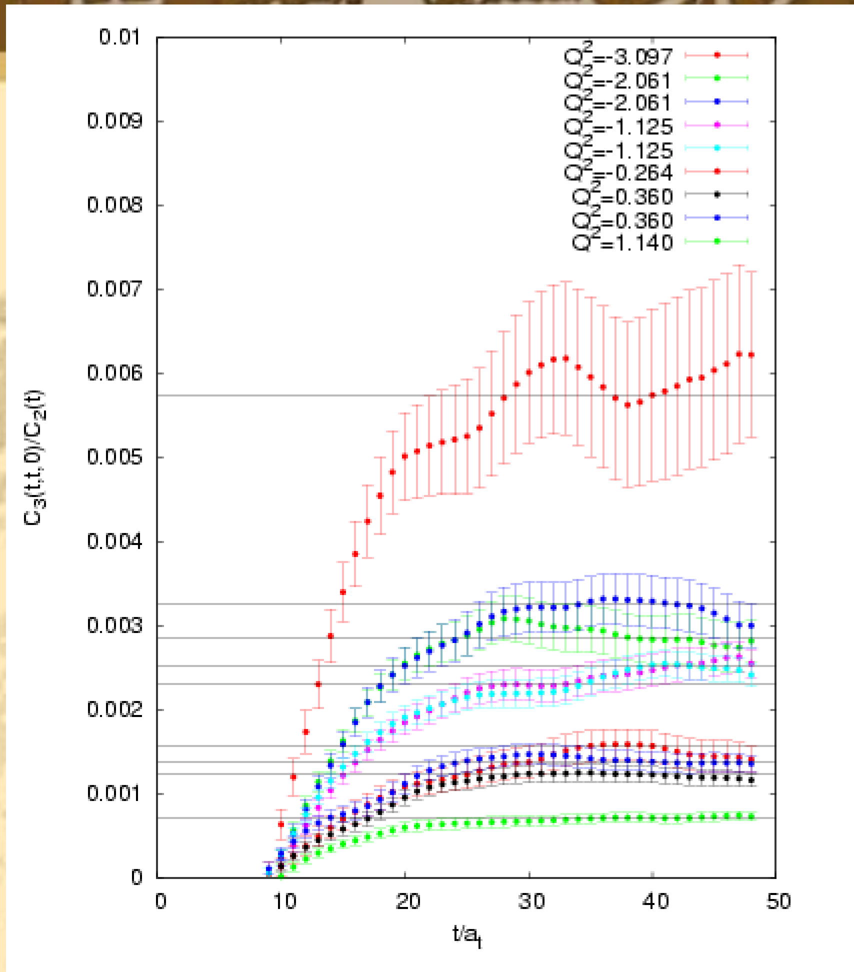


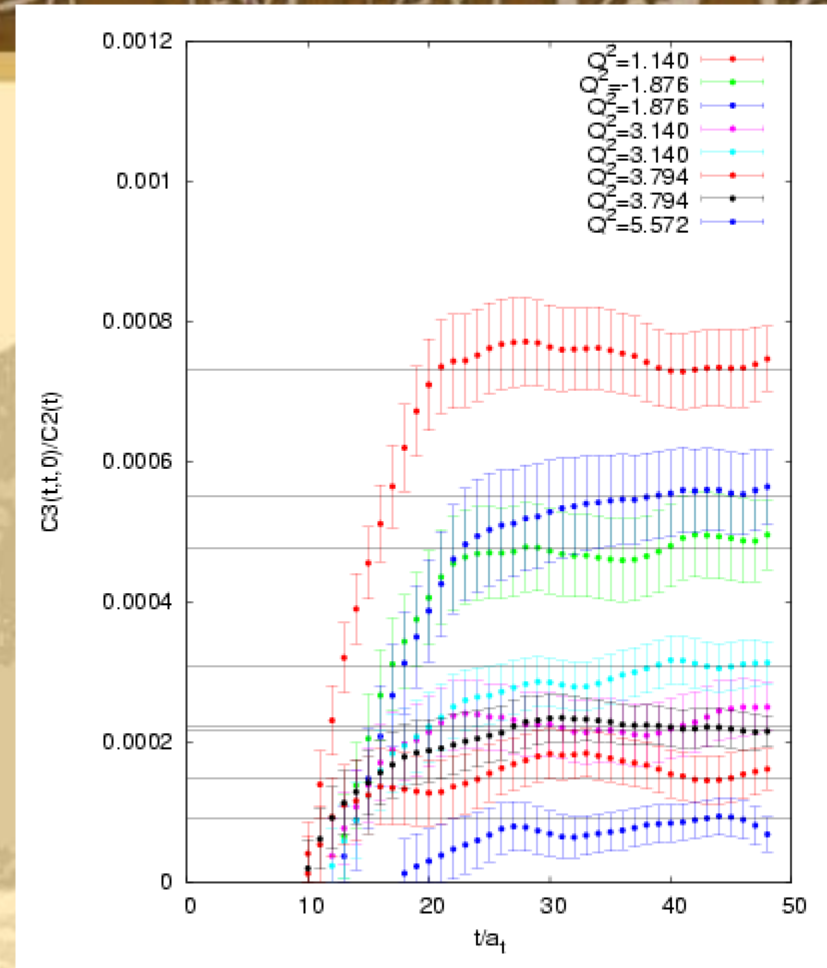






Exponential fit by taking the glueball energies as known parameters





$$\frac{\Gamma^{(3),ii}(\vec{p}_f;t,t)}{C_2^{ii}(\vec{p}_i=0;t)} \equiv \alpha^{ii}(M_f,\vec{p}_f,M_i)E_1(Q^2,t)+\beta^{ii}(M_f,\vec{p}_f,M_i)C_1'(Q^2,t)$$

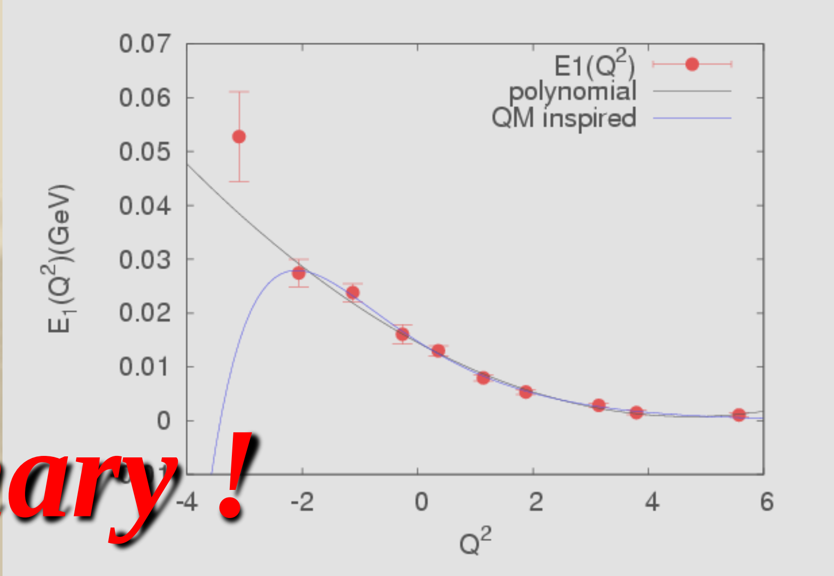
These are known functions

6. The form factor and the decay width

Polynomial fit:

$$E_1(Q^2) = E_1(0) + aQ^2 + bQ^4$$

$$E_1(0) = 0.0145(13) GeV$$



Preliminary !

The branch ratio is

$$\Gamma(J/\psi \to \gamma G_{0^+}) = \frac{4}{27}\alpha \frac{|p|}{M_{J/\psi}^2} \left|E_1(0)\right|^2 = 0.030(5) keV$$

$$\frac{\Gamma}{\Gamma_{tot}} = 0.030(5)/93.2 = 3.2(5)\times 10^{-4}$$

Summary

Summary

- Quenched lattice QCD study on $J^P = 0^-$ channel scattering yields scattering length a_0 and effective range r_0. Two meson interaction is attractive, but unlikely to form a bound state.
  - Unquenched study desired
- Preliminary quenched study on J/psi radiative decay to scalar glueball is calculated with the result: Γ=0.030(5)keV.
  - More to be checked (curent renormalization, extrapolations, other channels etc.)