Proper Identification of the Gluon Spin

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Outline:

- Various versions of the gluon spin
- Critical comparison
- Implication for the nucleon spin structure
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Gluon spin in the nucleon ★

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Abstract

We study the gluon spin contribution (Γ) to the nucleon's spin as measured in hard QCD processes. Γ can be related to the forward matrix element of a local gluon operator in $A^+ = 0$ gauge. In quark models the nucleon contains ambient color electric and magnetic fields. The latter are thought to be responsible for spin splittings among the light baryons. These fields give rise to a significant *negative* contribution to Γ at the quark model renormalization scale, μ_0^2. In a generic non-relativistic quark model $\Gamma_{\rm NQM} = -\frac{8}{9}(\alpha_{\rm NQM}/m_q)\langle\frac{1}{r}\rangle$, in the bag model $\Gamma_{\rm bag} = -0.1\alpha_{\rm bag}$. These correspond to $\Gamma_{\rm NQM} \approx -0.7$ and $\Gamma_{\rm bag} \approx -0.4$ at $\alpha_{\rm QCD} \approx 1.0$.

Various versions of the gluon spin: Jaffe's work

$$\Gamma(Q^2) = \int_0^1 dx\, \Delta g(x, Q^2)$$

$$= \int_0^1 dx (g_\uparrow(x, Q^2) - g_\downarrow(x, Q^2))$$

$$= \frac{1}{2M} \langle \hat{e}_3 | 2\,\mathrm{Tr}\{(\boldsymbol{E} \times \boldsymbol{A})^3 + \boldsymbol{A}_\perp \cdot \boldsymbol{B}_\perp\}\Big|_{Q^2} |\hat{e}_3\rangle$$

$$M_\Gamma^{\mu\nu\lambda} \equiv 2\,\mathrm{Tr}\{F^{\mu\nu} A^\lambda + F^{\lambda\mu} A^\nu\}. \qquad (10)$$

Then comparison with Eq. (7) shows that

$$\Gamma(Q^2) = \frac{1}{2S^+} \langle P, \hat{e}_3 | M_\Gamma^{+12} \Big|_{Q^2} | P, \hat{e}_3 \rangle, \qquad (11)$$

in $A^+ = 0$ gauge. This identification makes physical sense since the parton model distribution should measure helicity along the $\hat{e}_3$-axis (hence $\nu = 1$, $\lambda = 2$) in an infinite momentum frame, which corresponds to $\mu = +$ in the laboratory. The restriction to $A^+ = 0$ gauge is natural in the parton model. Needless to say, this restriction does not render Γ gauge dependent:

$$\Delta g/A(x) = \frac{i}{4\pi x P^+} \int d\xi^- e^{-ix\xi^- P^+} \langle A, P | G_a^{+\alpha}(0, \xi^-, 0_\perp) \mathcal{W}^a{}_b \tilde{G}_\alpha^{+b}(0, 0, 0_\perp) - G_a^{+\alpha}(0, 0, 0_\perp) \mathcal{W}^{\dagger a}{}_b \tilde{G}_\alpha^{+b}(0, \xi^-, 0_\perp) | A, P \rangle .$$

Various versions of the gluon spin -II

The standard form: $S^i = \Pi_a \left(\Sigma^i\right)_{ab} \Phi_b, \quad \vec{P} = \Pi_a (-i\vec{\nabla})\Phi_a$

For a vector field: $S^i = \Pi_j \left(\Sigma^i\right)_{jk} A_k$

$$= (iE^j)(-i\varepsilon_{ijk})A_k$$

$$= (\vec{E}\times\vec{A})^i$$

Orbital angular momentum: $\vec{L} = \vec{r}\times\vec{P} = \vec{r}\times(E^i\vec{\nabla}A^i)$

Problem for the gauge field: A_i are not independent variables, and not gauge invariant!

The "inclusive" approach

$$\vec{J}'_g = \int d^3x\, \vec{r} \times (\vec{E} \times \vec{B})$$

$$= \int d^3x\, \vec{E} \times \vec{A} + \int d^3x \vec{r} \times (E^i \vec{\nabla} A^i)$$

$$+ \int d^3x \vec{r} \times \psi^+ g \vec{A} \psi$$

Not zero in the presence of quarks

$$1)\quad \boxed{\vec{J}_{\text{total}}} = \int d^3x \psi^+ \frac{1}{2} \vec{\Sigma} \psi + \int d^3x \vec{x} \times \psi^+ \frac{1}{i} \vec{\nabla} \psi + \int d^3x \vec{E} \times \vec{A} + \int d^3x E^i \vec{x} \times \vec{\nabla} A^i$$

$$\equiv \vec{\mathrm{S}}_{\mathrm{q}} + \vec{\mathrm{L}}_{\mathrm{q}} + \vec{\mathrm{S}}_{\mathrm{g}} + \vec{\mathrm{L}}_{\mathrm{g}}$$

$$2)\quad \boxed{\vec{J}_{\text{total}}} = \int d^3x \psi^+ \frac{1}{2} \vec{\Sigma} \psi + \int d^3x \vec{x} \times \psi^+ \frac{1}{i} \vec{D} \psi + \int d^3x \vec{x} \times \left(\vec{E} \times \vec{B} \right)$$

$$\equiv \vec{\mathrm{S}}_{\mathrm{q}} + \vec{\mathrm{L}}'_{\mathrm{q}} + \vec{\mathrm{J}}'_{\mathrm{g}}$$

The gauge-invariant “exclusive” approach

$$\vec{J}_{\text{total}} = \int d^3x \vec{E} \times \vec{A}_{\text{phys}} + \int d^3x \vec{x} \times E^i \vec{\mathcal{D}}_{\text{pure}} A^i_{\text{phys}} + \int d^3x \, \psi^\dagger \frac{1}{2} \vec{\Sigma} \psi + \int d^3x \vec{x} \times \psi^\dagger \frac{1}{i} \vec{D}_{\text{pure}} \psi .$$

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1) $$\boxed{\vec{J}_{\text{total}}} = \int d^3x \psi^+ \frac{1}{2} \vec{\Sigma} \psi + \int d^3x \vec{x} \times \psi^+ \frac{1}{i} \vec{\nabla} \psi + \int d^3x \vec{E} \times \vec{A} + \int d^3x E^i \vec{x} \times \vec{\nabla} A^i$$

$$\equiv \vec{S}_q + \vec{L}_q + \vec{S}_g + \vec{L}_g$$

2) $$\boxed{\vec{J}_{\text{total}}} = \int d^3x \psi^+ \frac{1}{2} \vec{\Sigma} \psi + \int d^3x \vec{x} \times \psi^+ \frac{1}{i} \vec{D} \psi + \int d^3x \vec{x} \times \left(\vec{E} \times \vec{B} \right)$$

$$\equiv \vec{S}_q + \vec{L}'_q + \vec{J}'_g$$

Gauge-invariant polarized gluon PDF and gauge-invariant gluon spin

$$P_{\Delta g/A}(\xi) = \int_{-\infty}^{\infty} \frac{dx^-}{2\pi} e^{-i\xi P^+ x^-} \langle F^{+i}(0, x^-, 0_\perp)$$

$$\mathcal{P} \exp\{ig \int_0^{x^-} dy^- A^+_{\text{pure}}(0, y^-, 0_\perp)\} \epsilon_{ij+} A^j_{\text{phys}}(0) \rangle_A$$

Its first moment gives the gauge-invariant local operator:

$$M_g^{+ij} = F^{+i} \varepsilon_{ij+} A^j_{\text{phys}},$$

which is the + component of the gauge-invariant gluon spin

$$\vec{S}_g = \vec{E} \times \vec{A}_{\text{phys}}$$

Critical comparison: angular momentum for a static field

Abelian Field

$$\int d^3x\left[\vec{E}\times\vec{A}_{\text{phys}}+E^i\vec{x}\times\vec{\nabla}A^i_{\text{phys}}\right]$$

$$=\int d^3x\left[-(\vec{\nabla}\phi)\times\vec{A}_{\text{phys}}-(\nabla^i\phi)\vec{x}\times\vec{\nabla}A^i_{\text{phys}}\right]$$

$$=\int d^3x\left[\phi\vec{x}\times\vec{\nabla}(\vec{\nabla}\cdot\vec{A}_{\text{phys}})\right]=0!$$

Static field:

$$0=\partial_t\vec{B}=-\vec{\nabla}\times\vec{E}$$

$$\Rightarrow \vec{E}=-\vec{\nabla}\phi$$

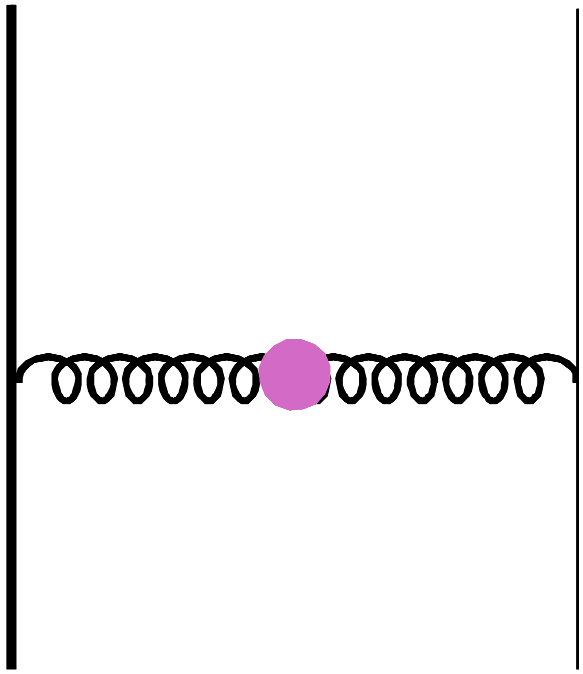
Non-Abelian Field

$$\vec{E}=-\vec{D}_{pure}A^0{}_{phys}-D_t^{pure}\vec{A}_{phys}+ig[\vec{A}_{phys},A^0{}_{phys}]$$

$$\int d^3x\,(-\vec{D}_{pure}A^0_{phys}\times\vec{A}_{phys})+\int d^3x\vec{r}\times[(-D^i_{pure}A^0_{phys})\vec{D}_{pure}A^i_{phys}]$$

$$=\int d^3x\,(-\vec{\nabla}A^0\times\vec{A})+\int d^3x\vec{r}\times[(-\vec{\nabla}A^0)\vec{\nabla}A^i]|_{\vec{\nabla}\cdot\vec{A}=0}=0$$

Critical comparison: One-Gluon Exchange



$$\vec{J}'_g = \int d^3x\, \vec{r} \times (\vec{E} \times \vec{B})$$

$$> 0$$

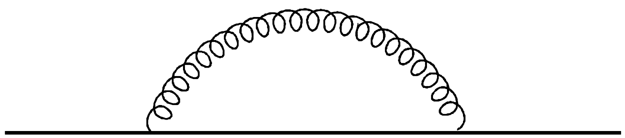
$$\vec{J}_g = \int d^3x\, \vec{E} \times \vec{A}_{phys} + \int d^3x \vec{r} \times (E^i \vec{\nabla} A^i{}_{phys})$$

$$= 0$$

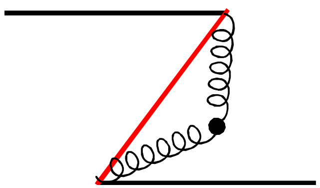
$$\int d^3x \left[-\dot{\vec{A}}_{\rm phys} \times \vec{A}_{\rm phys} - \dot{A}^i_{\rm phys} \vec{x} \times \vec{\nabla} A^i_{\rm phys} \right] \equiv \vec{S}_\gamma + \vec{L}_\gamma$$

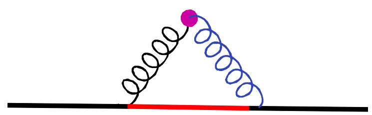
Both Zero!

Critical comparison: 1-Loop contribution



$$\Delta g \equiv \langle p\sigma | \int d^3x (\vec{E}\times\vec{A})^3 | p\sigma\rangle_{A^+=0} = \sigma \cdot 2\frac{\alpha_s}{\pi}\ln\frac{Q^2}{m^2}$$





$$\begin{aligned} S_g &\equiv \langle p\sigma | \int d^3x (\vec{E}\times\vec{A}_\perp)^3 | p\sigma\rangle \\ &= \langle p\sigma | \int d^3x (\vec{E}\times\vec{A})^3 | p\sigma\rangle_{\vec{\nabla}\cdot\vec{A}=0} = \frac{5}{9}\Delta g, \end{aligned}$$

$$\alpha_s(Q^2) = \frac{g^2(Q^2)}{4\pi} = \frac{12\pi}{(33-2n_f)\ln(Q^2/\Lambda^2)}.$$

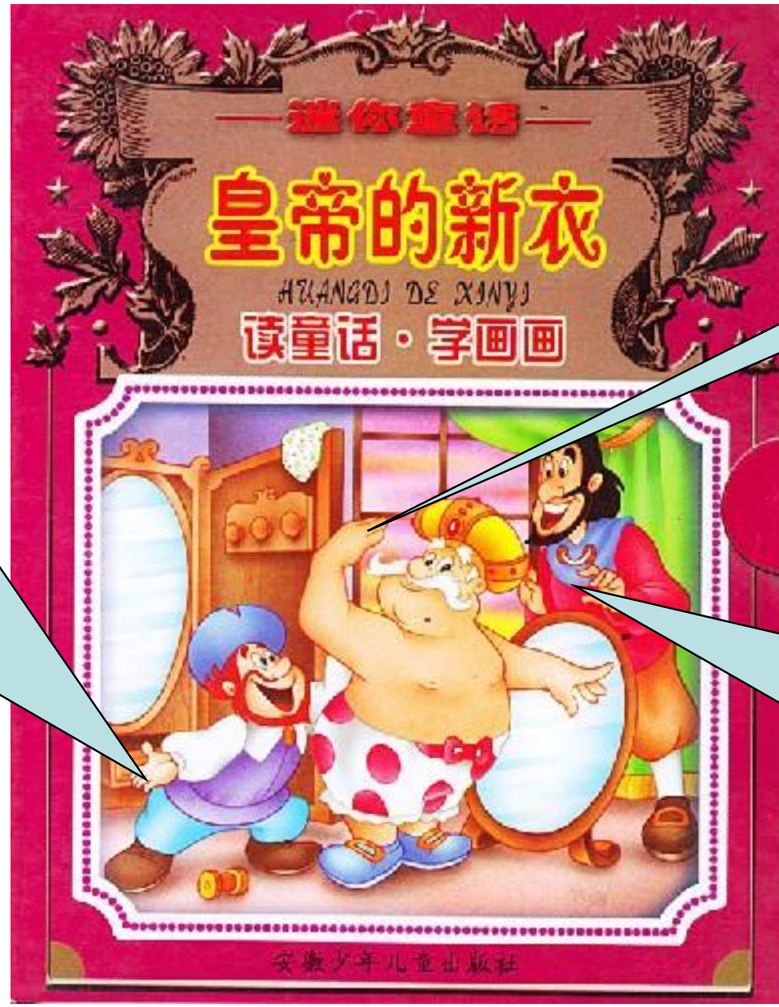
$$\begin{aligned} \vec{S}_g^{\rm st} &= \int d^3x(-\vec{\nabla}\phi)\times\vec{A}_\perp = \int d^3x\phi\vec{\nabla}\times\vec{A}_\perp \\ \vec{L}_g^{\rm st} &= \int d^3x(-\nabla^i\phi)\vec{x}\times\vec{\nabla}A_\perp^i \\ &= \int d^3x\phi(\nabla^i\vec{x})\times\vec{\nabla}A_\perp^i + \int d^3x\vec{x}\times\vec{\nabla}(\nabla^i A_\perp^i) \\ &= -\int d^3x\phi\vec{\nabla}\times\vec{A}_\perp = -\vec{S}_g^{\rm st} \qquad (12) \end{aligned}$$

$$S_g^{\rm dy} = \frac{1}{5}S_g = \frac{1}{9}\Delta g = \sigma\cdot\frac{2}{9}\frac{\alpha_s}{\pi}\ln\frac{Q^2}{m^2}, \qquad (13)$$

which is largely negligible. E.g., the specific renormalization by choosing the ultraviolet cutoff Q^2 to be the same as the scale for $\alpha_s(Q^2)$ gives $S_g^{\rm dy} \simeq 0.1\sigma$.

GDP by selling the Vacuum

This clothes is made of QCD vacuum. Intelligent people see a splendid structure. Stupid people see nothing! Selling for one million dollars!



Wonderful! I buy it.

Great! We have produced GDP of one million dollars!

Gluon contribution to the nucleon spin: Cheater’s money

Further discussion: Photon spin

Dipole rad.

l=1

m=1

$$\vec{B} \propto \frac{e^{ikr}}{ikr} \vec{L} Y_{11}$$

$$\vec{E} = \frac{i}{k} \vec{\nabla} \times \vec{B} = i\omega \vec{A}$$

(rad. gauge)

E Flux

$$\vec{E} \times \vec{B} \Rightarrow dP/d\Omega \propto \left(1 + \cos^2\theta\right)$$

J Flux

$$\vec{E} \times \vec{A} + E_i \vec{x} \times \vec{\nabla} A_i \Rightarrow dJ_z/d\Omega \propto \left(1 + \cos^2\theta\right)$$

$$\vec{x} \times \left(\vec{E} \times \vec{B}\right) \Rightarrow dJ_z/d\Omega \propto 2\sin^2\theta$$

Further discussion: graviton polarization

$$\vec{\nabla} \cdot \vec{A} = 0$$

$$\partial^i h^\rho{}_i - \frac{1}{2}\partial^\rho h^i{}_i = 0.$$

$$\vec{\nabla}^2 A^0 \; = \; -j^0,$$

$$\vec{\nabla}^2 h_{0\mu} \; = \; -S_{0\mu},$$

$$\Box \vec{A}_\perp = -\vec{j}_\perp,$$

$$\Box \hat{h}_{ij} = -\hat{S}_{ij}.$$

$$\vec{j}_\perp = \vec{j} - \vec{\nabla}\frac{1}{\vec{\nabla}^2}(\vec{\nabla}\cdot\vec{j})$$

$$\hat{S}_{ij} = S_{ij} - \frac{1}{\vec{\nabla}^2}(\partial_i\partial_k S^k{}_j + \partial_j\partial_k S^k{}_i - \partial_i\partial_j S^k{}_k).$$

“True Radiation Gauge for Gravity”

Chen & Zhu, arXiv: 1006.3927