Nucleon Structure from Lattice QCD

- Synopsis of Lattice QCD
- Glue and Quark Momenta
- Renormalization with Sum Rules
- $\langle x \rangle_s$, $\langle x \rangle_{u+d}$ (D.I.), $\langle x^2 \rangle$, $\langle x \rangle_g$
- Quark and glue angular momenta

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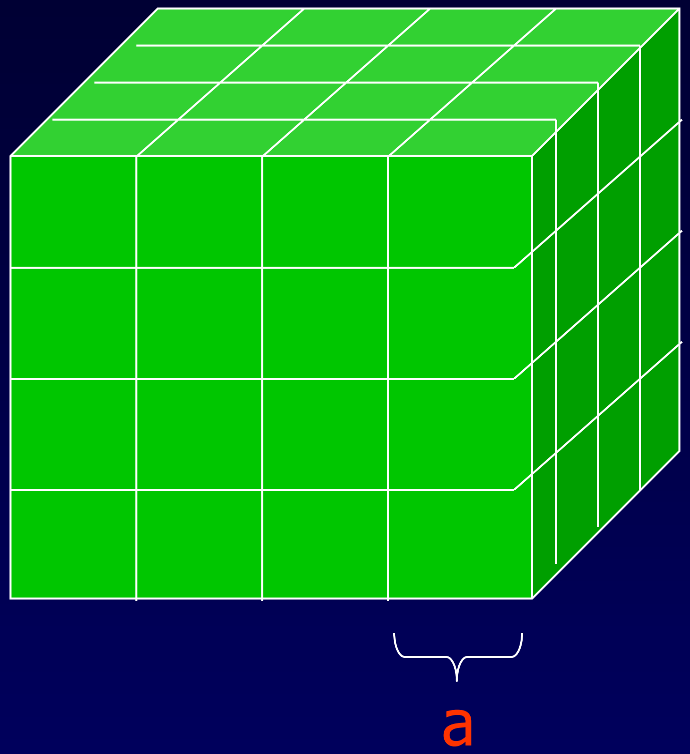
Lattice QCD – Path integral in Euclidean Space

Why Lattice?

- Regularization
  - Lattice spacing a
  - Hard cutoff, $p \leq \pi/a$
  - Scale introduced (dimensional transmutation)
- Renormalization
  - Perturbative
  - Non-perturbative

    Regularization independent Scheme
    Schroedinger functional
    Current algebra relations

- Numerical Simulation
  - Quantum field theory → classical statistical mechanics
  - Monte Carlo simulation (importance sampling)



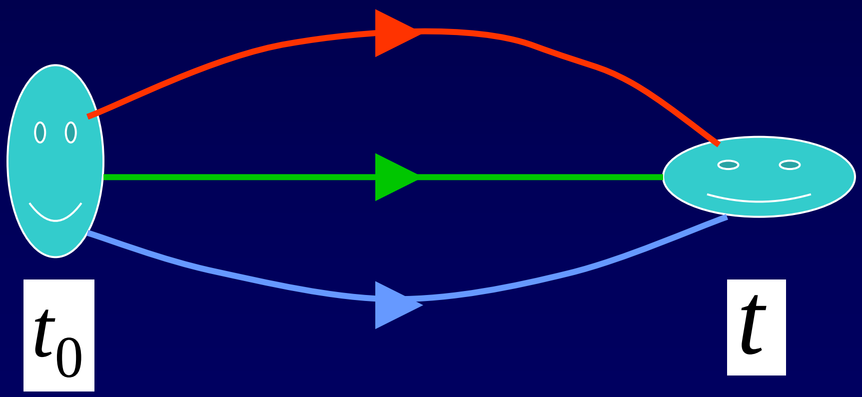
$$e^{-S_G} \det M \geq 0$$

Hadron Mass and Decay Constant

The two-point Green's function decays exponentially at large separation of time

$$G_{NN}^{\alpha\alpha}(t,t_0,\vec{p}) \equiv \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \langle T(\chi^\alpha(x)\bar{\chi}^\alpha(x_0))\rangle$$

$$\xrightarrow{t-t_0>>1} \langle 0|\chi^\alpha|N^\alpha(\vec{p})\rangle\langle N^\alpha(\vec{p})|\bar{\chi}^\alpha|0\rangle \frac{e^{-E_p(t-t_0)}}{2E_pV_3} \equiv \frac{E_p+m}{E_p}|\phi|^2 e^{-E_p(t-t_0)}$$



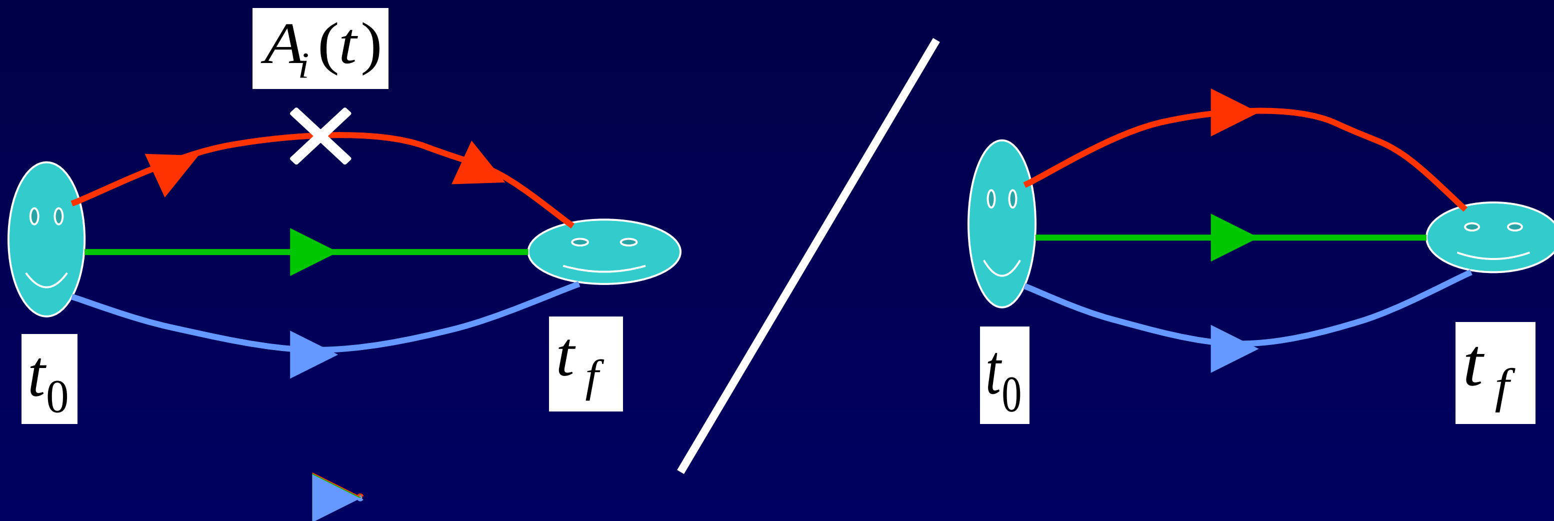
Mass M= $E_{\mathbf{p}}$(p=0), decay constant ~ Φ

Nucleon Form Factor

The three-point Green's function for the iso-vector axial current is proportional to $\langle N(\vec{p}\,') | A_\mu | N(\vec{p}) \rangle$ asymptotically.

$$\Gamma_{\alpha\beta} G_{NN}^{\beta\alpha}(t_f, t, t_0, \vec{q}) \equiv \Gamma_{\alpha\beta} \sum_{\vec{x}, \vec{x}_f} e^{i\vec{q}\cdot\vec{x}} \langle T(\chi^\alpha(x_f) A_i \bar{\chi}^\beta(x_0)) \rangle$$

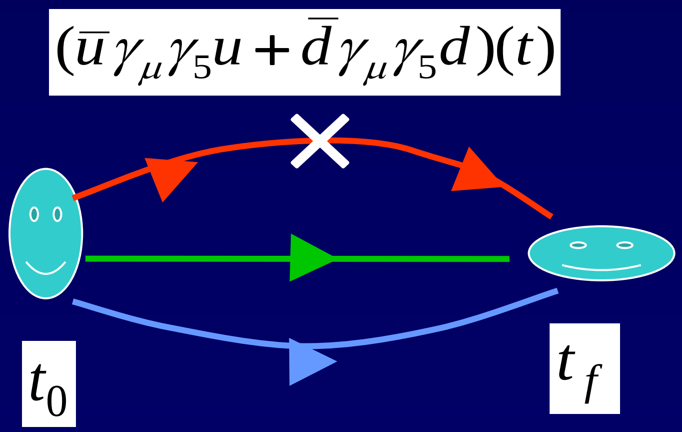
$$\xrightarrow{t_f - t,\, t - t_0 >> 1,} \frac{E_q + m}{E_q} |\phi|^2 \, e^{-m(t_f - t) - E_q(t - t_0)} [g_A(q^2) - h_A(q^2) \frac{q_i^2}{E_q + m}]; \Gamma = \begin{pmatrix} \sigma_i & 0 \\ 0 & 0 \end{pmatrix}$$

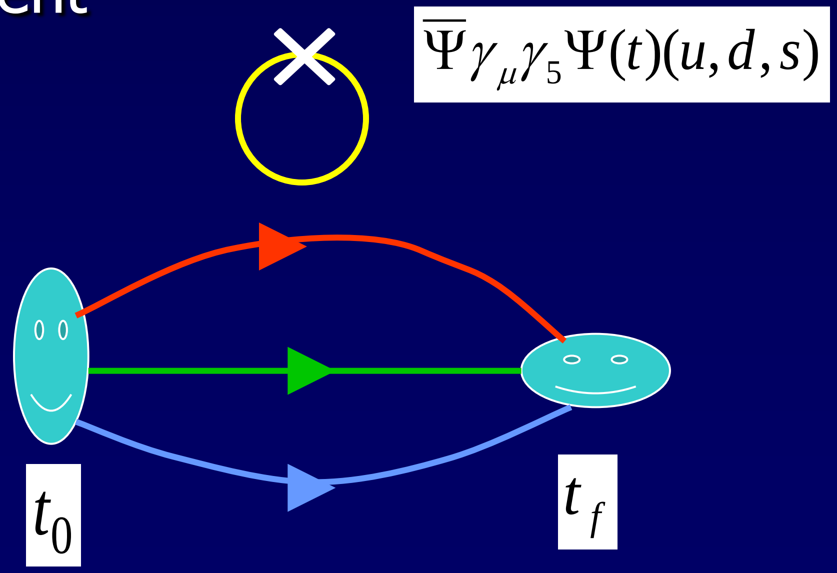


$$\frac{\Gamma_{\alpha\beta} G_{NN}^{\beta\alpha}(t_f, t, t_0, \vec{q})}{G_{NN}^{\alpha\alpha}(t_f, t_0, \vec{p} = 0)} \equiv \xrightarrow{t_f - t,\, t - t_0 >> 1,} \frac{E_q + m}{2E_q} e^{-(E_q - m)(t - t_0)} [g_A(q^2) - h_A(q^2) \frac{q_i^2}{E_q + m}]$$

Hadron Structure with Disconnected Insertion Calculation

- Pion-Nucleon Sigma Term, Strangeness Content in N
- Quark Spin and Orbital Angular Momentum in Nucleon
- Sea Quark Contributions in $<x>$, and $<x^2>$
- Strangeness Electric and Magnetic Form Factors
- Muon Anomalous M. M. (g-2) (light-by-light)
- Neutron Electric Dipole Moment





Momenta and Angular Momenta of Quarks and Glue

- One can decompose the energy momentum tensor into quark part and gluon part ***gauge invariantly***

$$T_q^{\mu\nu} = \frac{i}{4}\left[\bar{q}\gamma^\mu \overrightarrow{D}^\nu q - \bar{q}\gamma^\mu \overleftarrow{D}^\nu q + (\mu \leftrightarrow \nu)\right]$$

$$T_g^{\mu\nu} = \frac{1}{4}g^{\mu\nu}F^2 - F^{\mu\alpha}F^\nu{}_\alpha$$

X.Ji (1997)

Orbital part

$$T_q^{\mu\nu} \to \bar{q}\vec{\gamma}\gamma_5 q + \boxed{\bar{q}[\vec{x}\times(-i\vec{D})]q}$$

$$T_g^{\mu\nu} \to \vec{x}\times(\vec{E}\times\vec{B})$$

- Nucleon matrix elements can be decomposed as

$$\langle p,s|T^{\mu\nu}|p',s'\rangle = \bar{u}(p,s)\Big[T_1(q^2)\gamma^\mu\bar{p}^\nu + T_2(q^2)\bar{p}^\mu i\sigma^{nu\alpha}/2m + T_3(q^2)(q^\mu q^\nu - g^{\mu\nu}q^2)/2m + T_4(q^2)g^{\mu\nu}m/2\Big]u(p',s')$$

  - where the angular momentum is

$$J = \frac{1}{2}[T_1(0) + T_2(0)]$$

Methodology

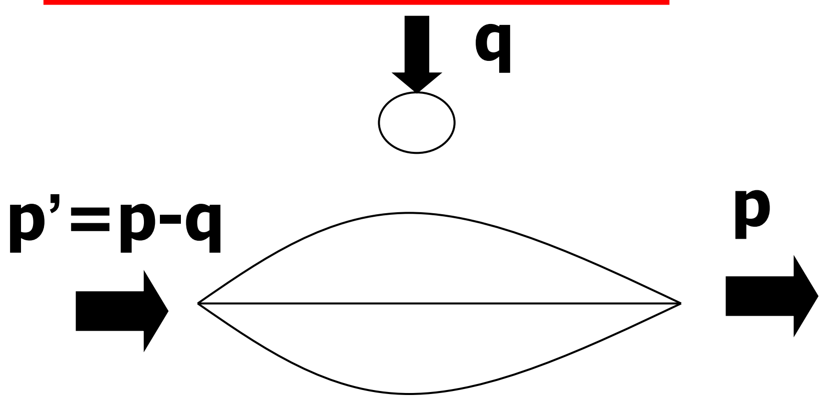
- How to extract T1(q^2) and T2(q^2) ?

$$\Pi^{3pt}(\vec{p}, t_2; \vec{q}, t_1) = \sum_{\vec{x}_2, \vec{x}_1} e^{-i\vec{p}\cdot\vec{x}_2} e^{+i\vec{q}\cdot\vec{x}_1} \langle 0 | \mathrm{T}\left[J_N(\vec{x}_2, t_2) T^{\mu\nu}(\vec{x}_1, t_1) \bar{J}_N(0) \right] 0 \rangle$$

$$\mathrm{Tr}\left[\Gamma_m \Pi^{3pt}_{T_{4i}}(\vec{p} = \vec{0}, t_2; \vec{q}, t_1) \right] = C \cdot e^{-m(t_2 - t_1)} e^{-E t_1} \left[-i\epsilon_{ijm} q_j \left(T_1(-q^2) + T_2(-q^2) \right) \right]$$

N.B. we need one more equation to extract T1 and T2 separately

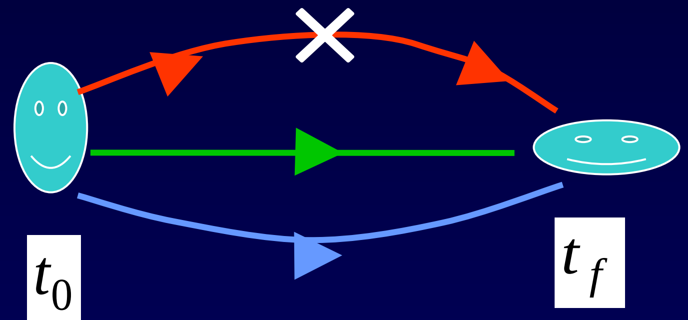
(q^2 dependence could be different)

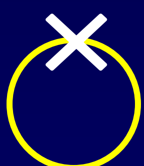


Hadron Structure with Quarks and Glue

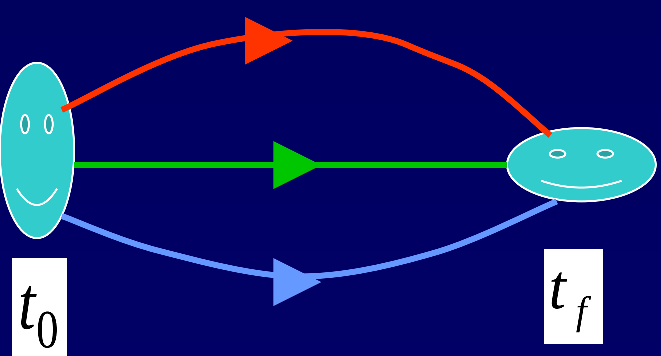
- Quark and Glue Momentum and Angular Momentum in the Nucleon

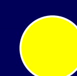
$$(\bar{u}\gamma_\mu D_\nu u + \bar{d}\gamma_\mu D_\nu d)(t)$$



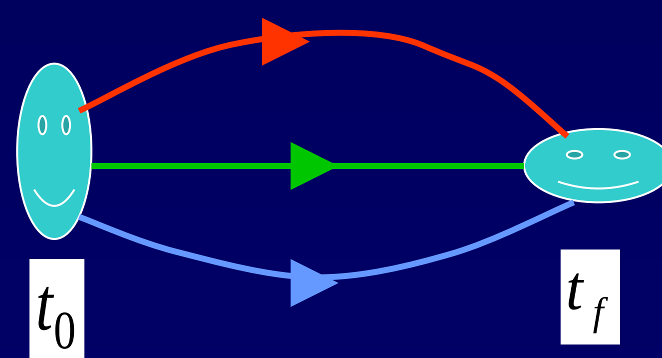


$$\bar{\Psi}\gamma_\mu D_\nu \Psi(t)(u,d,s)$$





$$F_{\mu\alpha}F_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}F^2$$



Renormalization and Quark-Glue Mixing

$$\begin{bmatrix} \langle x \rangle_q^{\overline{MS}}(\mu) \\ \langle x \rangle_g^{\overline{MS}}(\mu) \end{bmatrix} = \begin{bmatrix} C_{qq}(\mu) & C_{qg}(\mu) \\ C_{gq}(\mu) & C_{gg}(\mu) \end{bmatrix} \begin{bmatrix} \langle x \rangle_q^R \\ \langle x \rangle_g^R \end{bmatrix}$$

Momentum and Angular Momentum Sum Rules

$$\langle x \rangle_q^R = Z_q \langle x \rangle_q^L, \quad \langle x \rangle_g^R = Z_g \langle x \rangle_g^L,$$

$$J_q^R = Z_q J_q^L, \quad J_g^R = Z_g J_g^L,$$

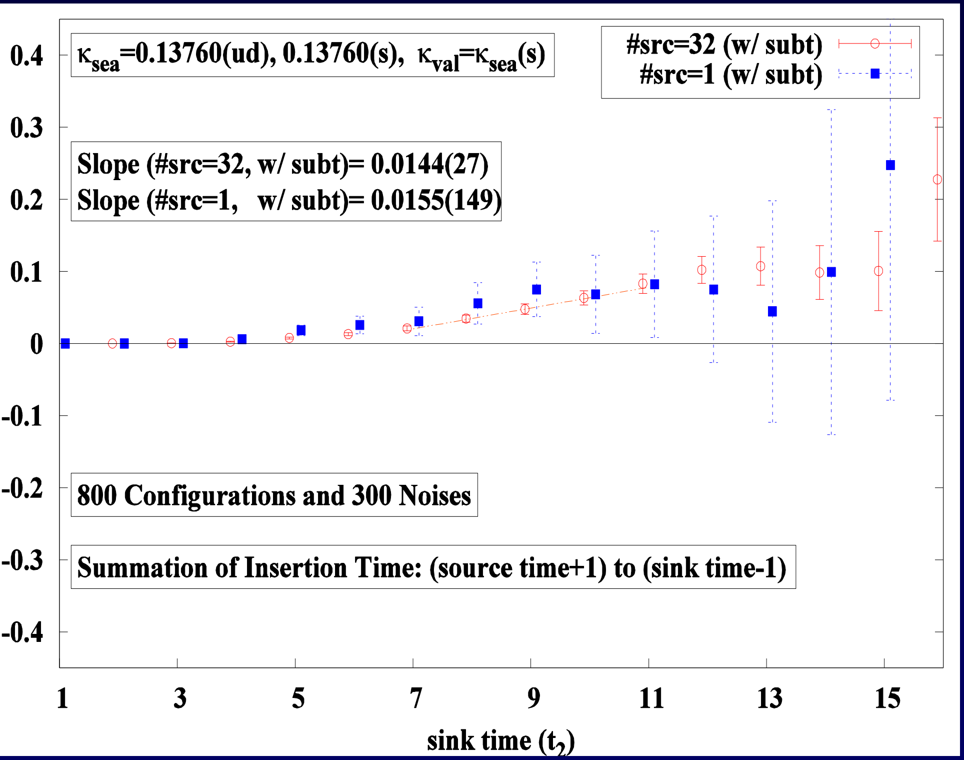
With $Z_q \langle x \rangle_q^L + Z_g \langle x \rangle_g^L = 1,$

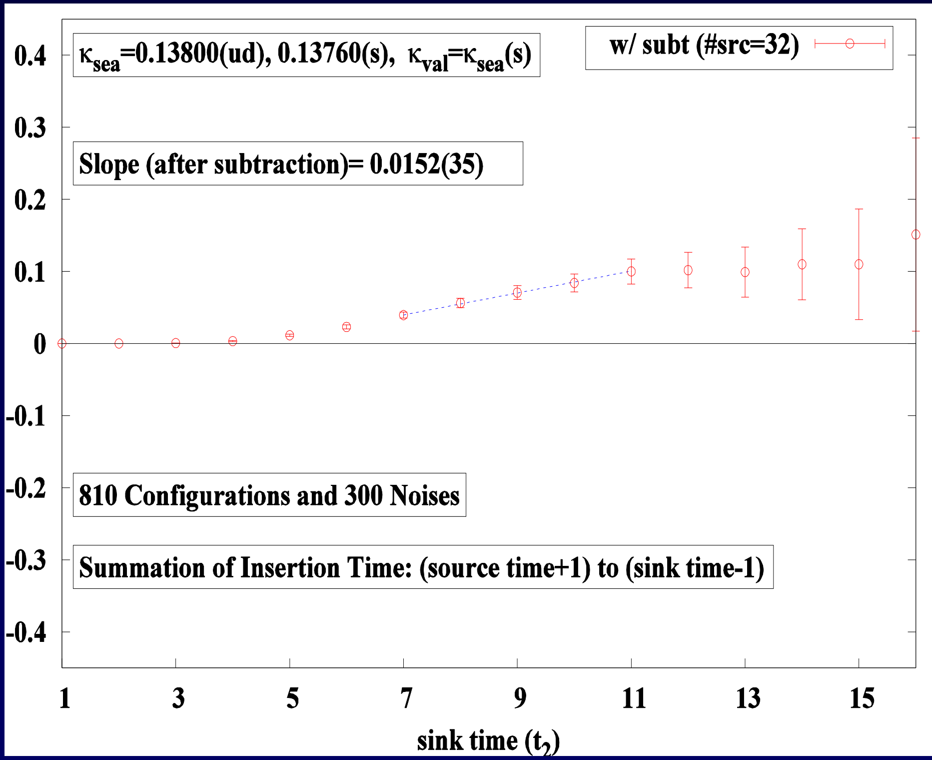
$$Z_q J_q^L + Z_g J_g^L = \frac{1}{2}$$

Strange Parton Moments

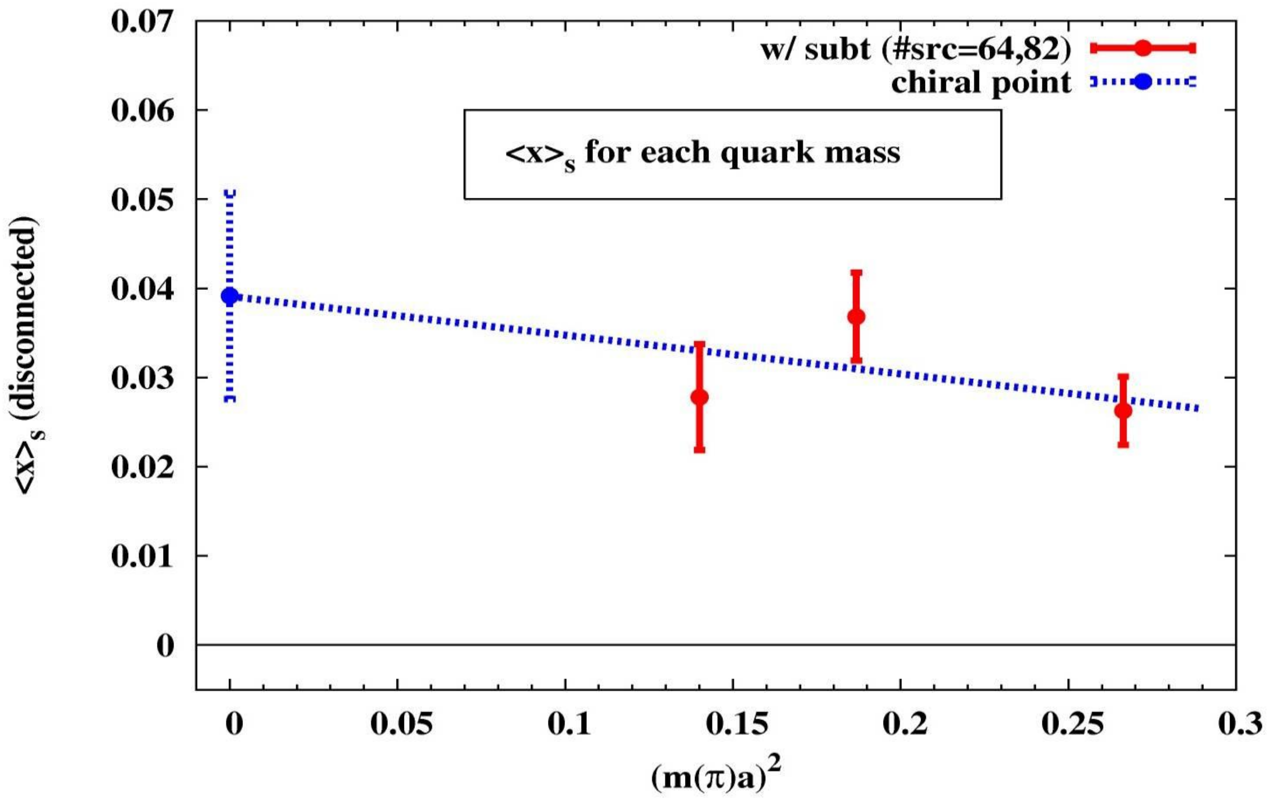
- Strange parton distribution is poorly known – H.L. Lai et al. (CTEQ), JHEP 0704:089 (2007)
- $0.018 < \langle x \rangle_s < 0.040$
- NuTeV measurement of $\sin^2 \theta_W$ is 3 σ above the standard model ⟶ strangeness asymmetry, i.e. $s(x) \neq \bar{s}(x)$?
- CTEQ: the sign of $\int dx\, x[s(x) - \bar{s}(x)]$ is uncertain.
- Lattice can calculate $\langle x^2 \rangle_s = \int dx\, x^2 [s(x) - \bar{s}(x)]$

$\langle x \rangle_s$ Full QCD with 2+1 Flavor Clover Fermions m_π = 800, 700, 600 MeV



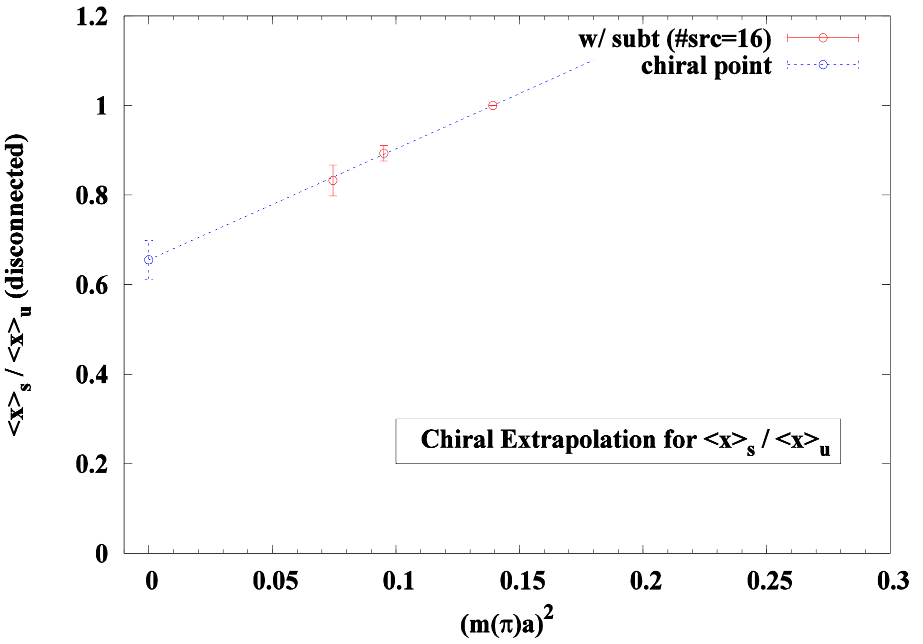


$\langle x \rangle_{s+\bar{s}}$

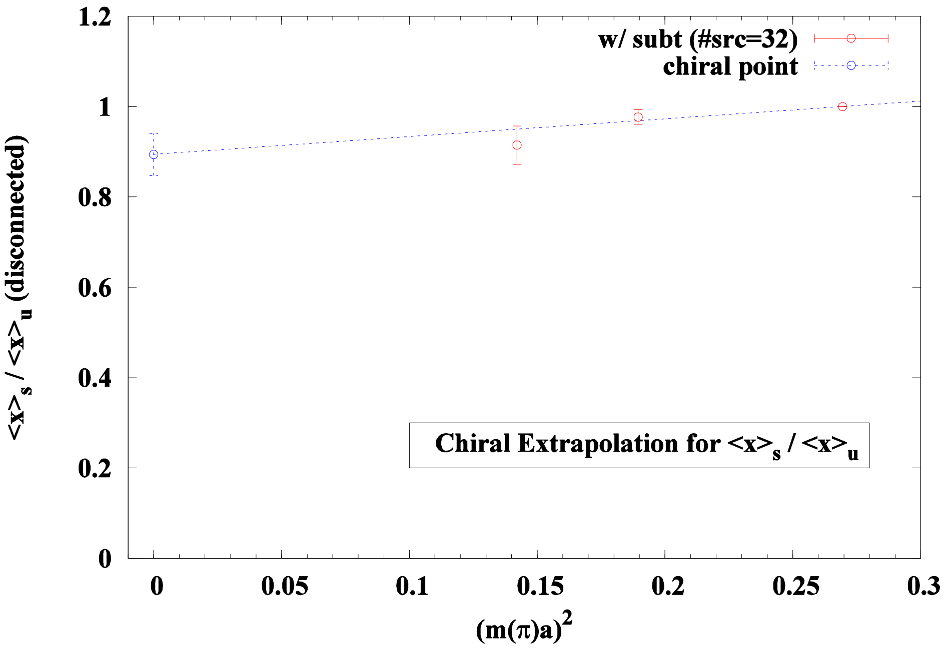


$$\frac{\langle x \rangle_{s+\bar{s}}}{\langle x \rangle_{u+\bar{u}}(\mathrm{DI})}$$

Quenched



2+1 Flavor Full QCD



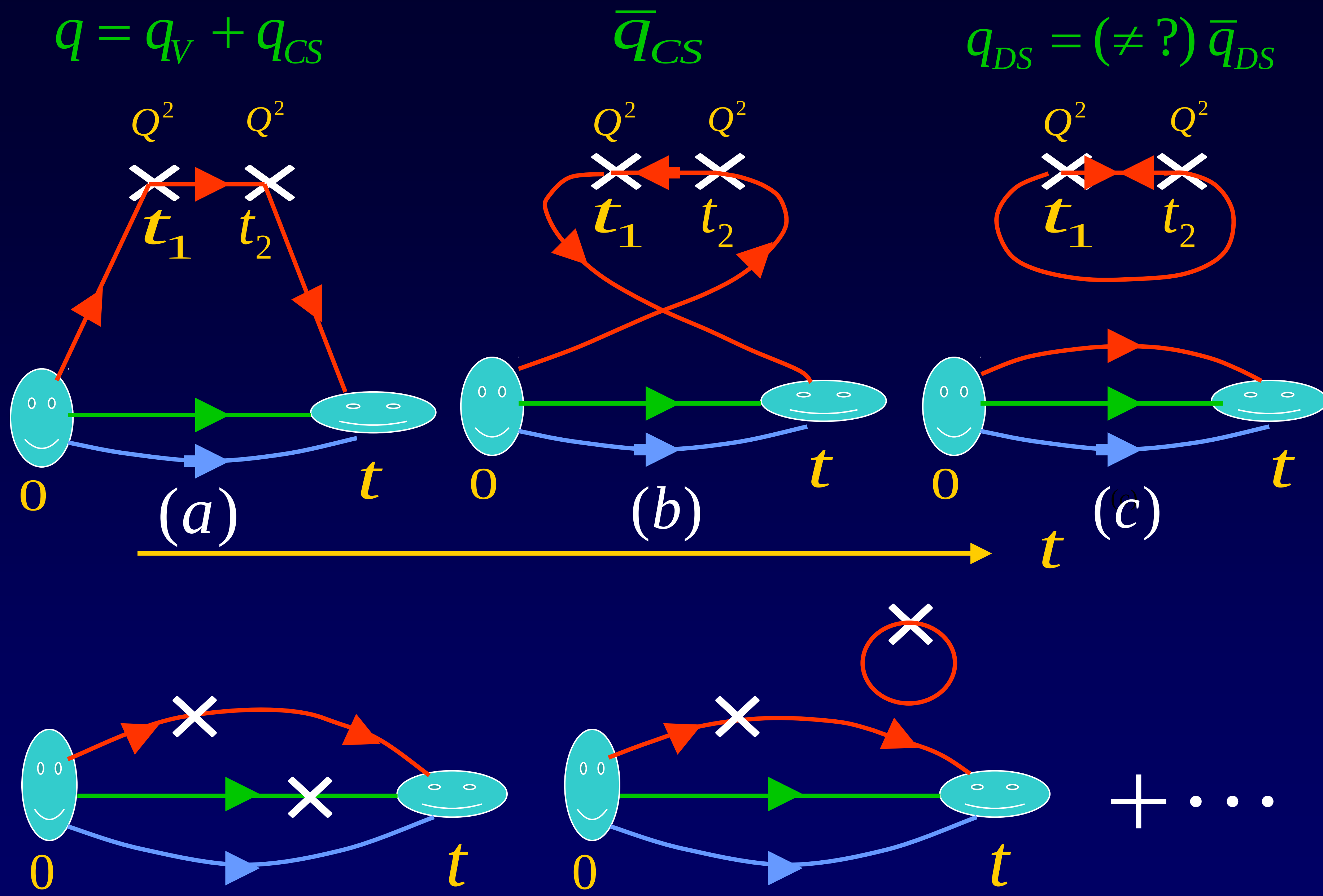
Implication on Fitting of PDF

$$\frac{\langle x \rangle_{s+\bar{s}}}{\langle x \rangle_{u+\bar{u}}(\mathrm{DI})} = \begin{cases} 0.655(43)\,[\text{quenched}] \\ 0.894\,(47)\,[\text{full QCD}] \end{cases}$$

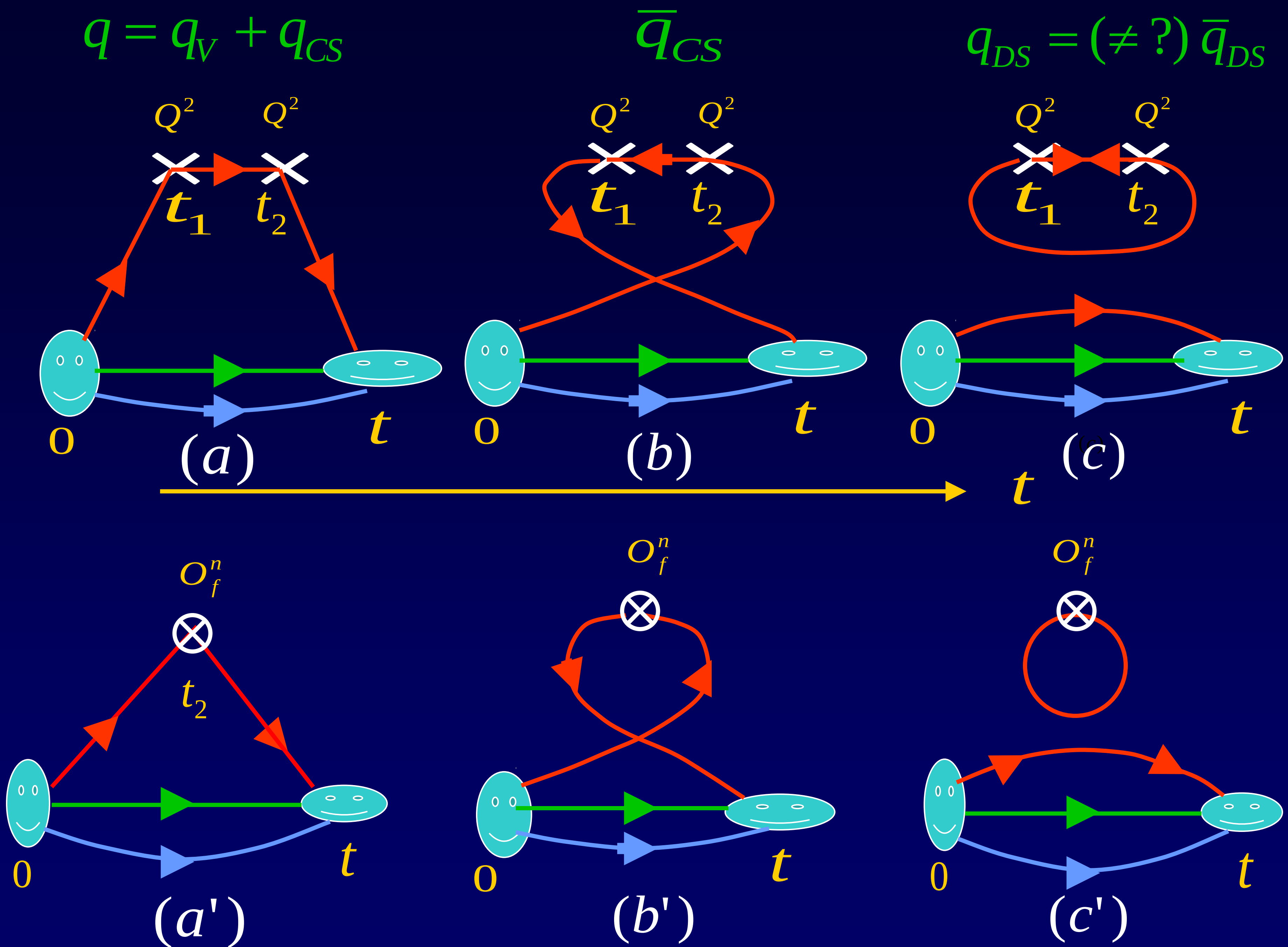
$$\frac{\langle x \rangle_{\bar{s}}}{\langle x \rangle_{(\bar{u}+\bar{d})/2}} \sim 0.4$$

Global PDF Fitting à la CTEQ

Is there a discrepancy?



Cat’s ears diagrams are suppressed by O(1/Q²).



$q = q_V + q_{CS}$
$\overline{q}_{CS}$
$q_{DS} = (\neq ?)\ \overline{q}_{DS}$
Q^2 Q^2
t_1 t_2
0
t
(a)
(b)
(c)
O_f^n
(a')
(b')
(c')

- $W_{\mu\nu}(p,q) = -W_1(q^2,\nu)(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) + W_2(q^2,\nu)(p_\mu - \frac{p\cdot q}{q^2}q_\mu)(p_\nu - \frac{p\cdot q}{q^2}q_\nu)$

- Large momentum frame

$$\nu W_2(q^2,\nu) \xrightarrow[|\vec{p}|>>|\vec{q}|]{} F_2(x,Q^2) = x\sum_i e_i^2(q_i(x,Q^2) + \bar{q}_i(x,Q^2)); \quad x = \frac{Q^2}{2M_N\nu}$$

- Parton degrees of freedom: valence, connected sea and disconnected sea

u	d	s
$u_V(x) + u_{CS}(x)$	$d_V(x) + d_{CS}(x)$	
$\bar{u}_{CS}(x)$	$\bar{d}_{CS}(x)$	
$u_{DS}(x) + \bar{u}_{DS}(x)$	$d_{DS}(x) + \bar{d}_{DS}(x)$	$s_{DS}(x) + \bar{s}_{DS}(x)$

Physical Consequences

- Is it necessary to separate out the CS from the DS?

1) Small x behavior

- $q_V \, , q_{CS} \, , \bar{q}_{CS} \sim_{x \to 0} x^{-\alpha_R} (x^{-1/2})$

  (Reggeon exchange, no pomeron exchange)

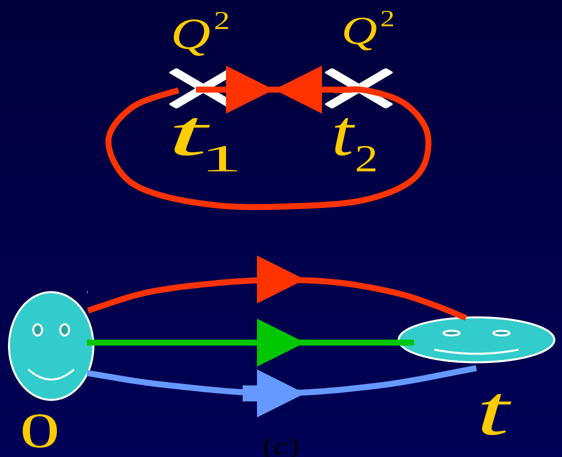
- $q_{DS} \, , \bar{q}_{DS} \sim_{x \to 0} x^{-1}$

  (Pomeron exchange)

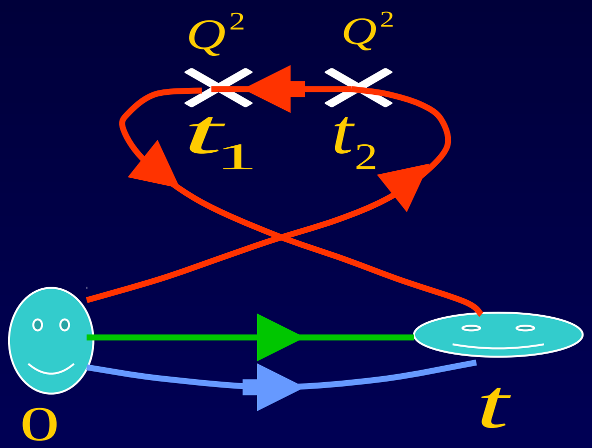
2) Gottfried Sum Rule Violation

$$S_G(0,1;Q^2) = \frac{1}{3} + \frac{2}{3}\int_0^1 dx\,(\bar{u}_P(x) - \bar{d}_P(x)); \quad S_G(0,1;Q^2) = \frac{1}{3}\,(\text{Gottfried Sum Rule})$$

NMC: $S_G(0,1;4\,\text{GeV}^2) = 0.240 \pm 0.016\ (5\sigma \text{ from GSR})$



two flavor traces ($\bar{u}_{DS} = \bar{d}_{DS}$)



one flavor trace ($\bar{u}_{CS} \neq \bar{d}_{CS}$)

K.F. Liu and S.J. Dong, PRL 72, 1790 (1994)

$$Sum = \frac{1}{3} + \frac{2}{3}\int_0^1 dx\,(\bar{u}_{CS}(x) - \bar{d}_{CS}(x)),$$

$$= \frac{1}{3} + \frac{2}{3}\left[n_{\bar{u}_{CS}} - n_{\bar{d}_{CS}}\right](1 + O(\alpha_s))$$

3) Fitting of experimental data

CTEQ4, MRST $\bar{u} - \bar{d} \xrightarrow[x \to 0]{} x^{-1/2}$ O.K.

But $\bar{u} + \bar{d} \propto \bar{s} \xrightarrow[\mathrm{x} \to 0]{} x^{-1}$

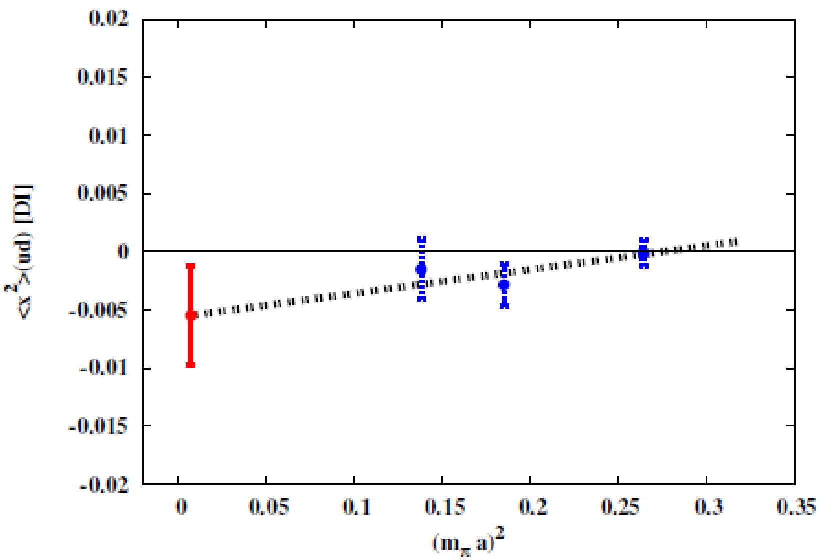
A better fit $\dfrac{\bar{u}(x) + \bar{d}(x)}{2} = f\ \bar{s}(x) + c(x)$

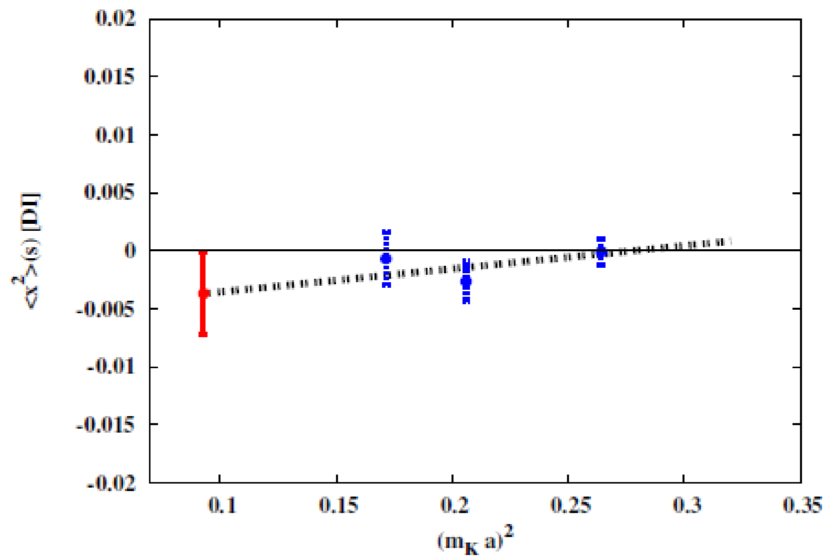
where $c(x) \xrightarrow[x \to 0]{} x^{-1/2}$ like in $\bar{u}(x) - \bar{d}(x)$

Results for <x^2>

Nf=2+1

Preliminary





<x^2>(ud) [DI]

<x^2>(s)

c.f. $\langle x \rangle_{s-\bar{s}} = 0.0038 \rightarrow$ No NuTeV anomaly

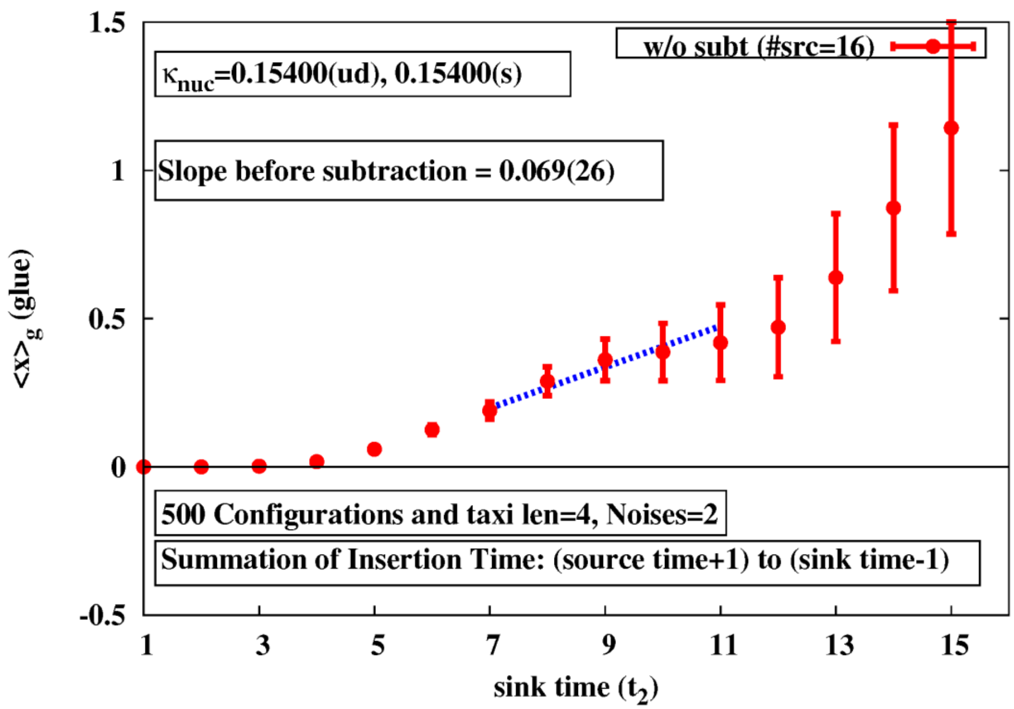
$$Z_{\text{pert}}(\mu, a) \simeq 1.1$$

$$\mu = 2\text{GeV}$$

Glue Momentum in Nucleon

Results for <x>(g) from overlap op

Glue operator from the overlap operator



Linear slope corresponds to signal

Able to obtain a clear signal of glue in the nucleon.

c.f. M.Gockeler et al., Nucl.Phys.Proc.supp..53(1997)324

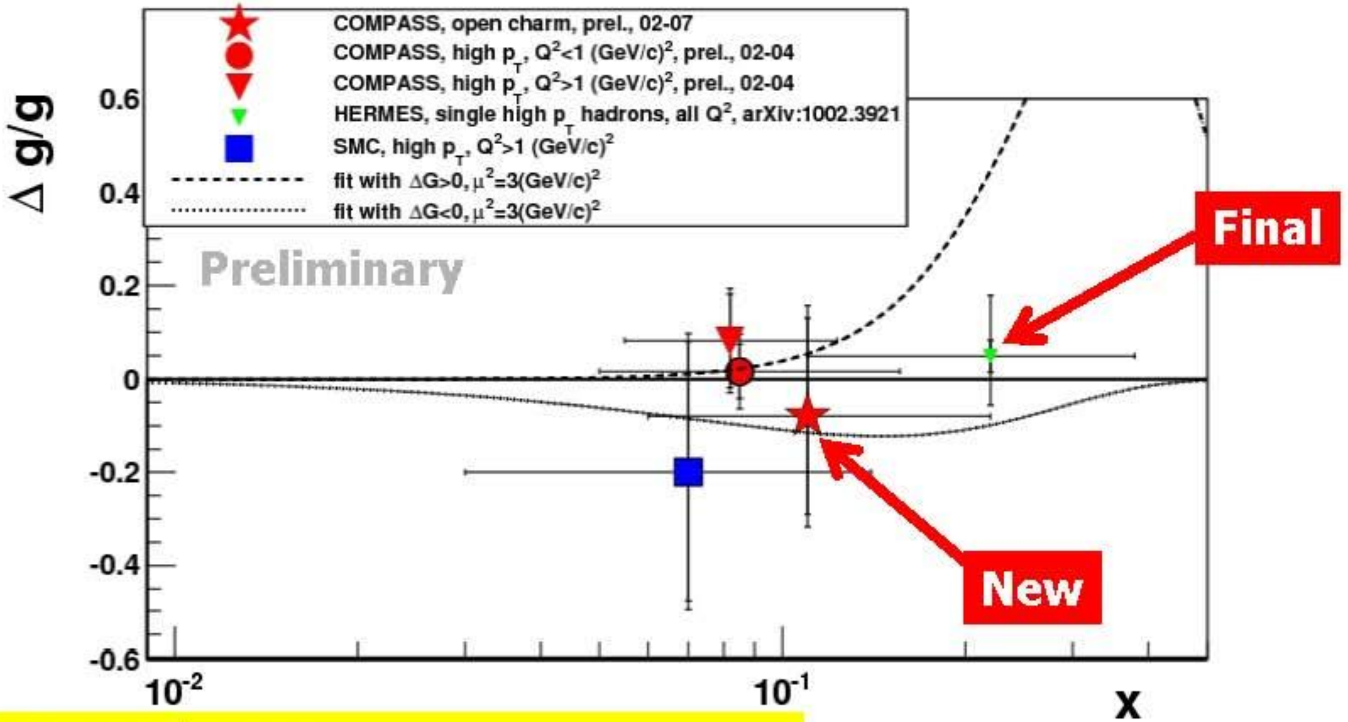
Nucleon Spin

- Quark spin ΔΣ ~ 30% (DIS, Lattice)
- Quark orbital angular momentum? (recent lattice calculation→ ~ 0)
- Glue spin ΔG/G small (COMPASS, STAR) ?
- Glue orbital angular momentum is small (Brodsky and Gardner) ?

→ Dark Spin ?

Summary Gluon Polarization

Presently all Analysis in LO only



COMPASS Open Charm:

$\Delta G/G$=-0.08+- 0.21(stat) +- 0.11(sys.)
(Systematic error still under investigations)

(Value supersedes previous publication)

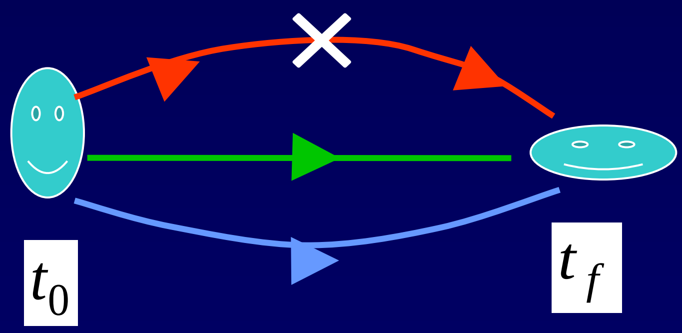
C.Franco

See Talk 1193 by F. Kunne

Flavor-singlet g_A

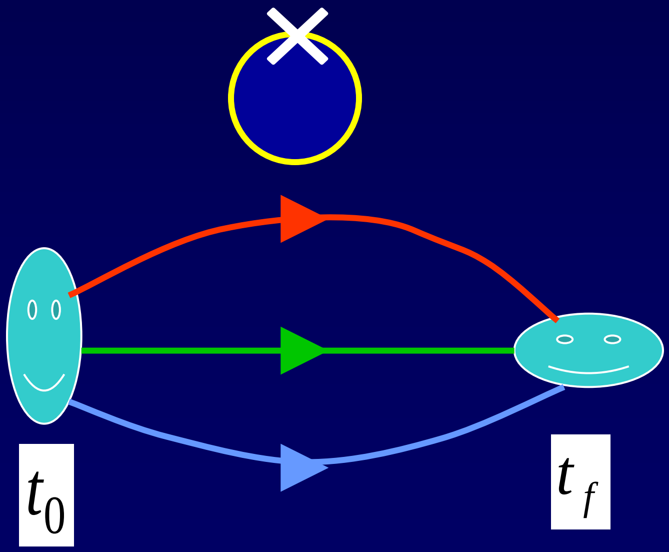
- Quark spin puzzle (dubbed `proton spin crisis')
  - $$g_A^0 = \Delta u + \Delta d + \Delta s = \begin{cases} 1 & \text{NRQM} \\ 0.75 & \text{RQM} \end{cases}$$
  - Experimentally (EMC, SMC, … $\Delta\Sigma = g_A^0 \sim 0.2 - 0.3$

$$(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d)(t)$$

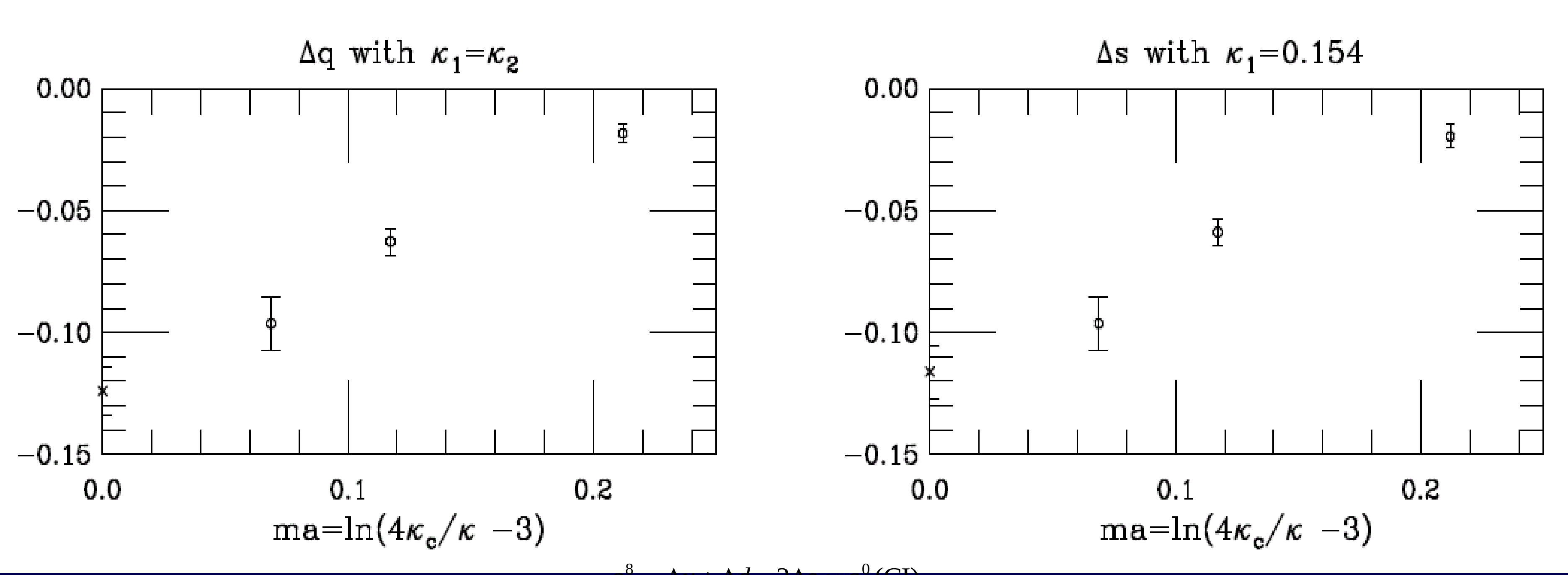


$$g^0_{A,con} = (\Delta u + \Delta d)_{con}$$

$$\overline{\Psi}\gamma_\mu\gamma_5\Psi(t)(u,d,s)$$



$$g^0_{A,dis} = (\Delta u + \Delta d + \Delta s)_{dis}$$



$g_A^8 = \Delta u + \Delta d - 2\Delta s \approx g_A^0(\mathrm{CI})$

S.J. Dong, J.-F. Lagae, and KFL, PRL 75, 2096 (1995)

- DI sea contribution independent of quark mass

$$\Delta u = \Delta d \cong \Delta s$$

- This suggests U(1) anomaly at work.
- $g_A^8 = \Delta u + \Delta d - 2\Delta s \approx g_A^0(\mathrm{CI})$

Lattice resolution: U(1) anomaly

$$g_A^0 = (\Delta u + \Delta d)_{con} + (\Delta u + \Delta d + \Delta s)_{dis} = 0.62(9) + 3(-0.12(1)) = 0.25(12)$$

	Lattice	Expt. (SMC)	NRQM	RQM
$g_A^0 = \Delta u + \Delta d + \Delta s$	0.25(12)	0.22(10)	1	0.75
$g_A^3 = \Delta u - \Delta d$	1.20(10)	1.2573(28)	5/3	1.25
$g_A^8 = \Delta u + \Delta d - 2\Delta s$	0.61(13)	0.579(25)	1	0.75
Δu	0.79(11)	0.80(6)	1.33	1
Δd	-.42(11)	-0.46(6)	-0.33	-0.25
Δs	-.12(1)	-0.12(4)	0	0
F_A	0.45(6)	0.459(8)	0.67	0.5
D_A	0.75(11)	0.798(8)	1	0.75
F_A / D_A	0.60(2)	0.575(16)	0.67	0.67

$$F_A = (\Delta u - \Delta s)/2; \quad D_A = (\Delta u - 2\Delta d + \Delta s)/2$$

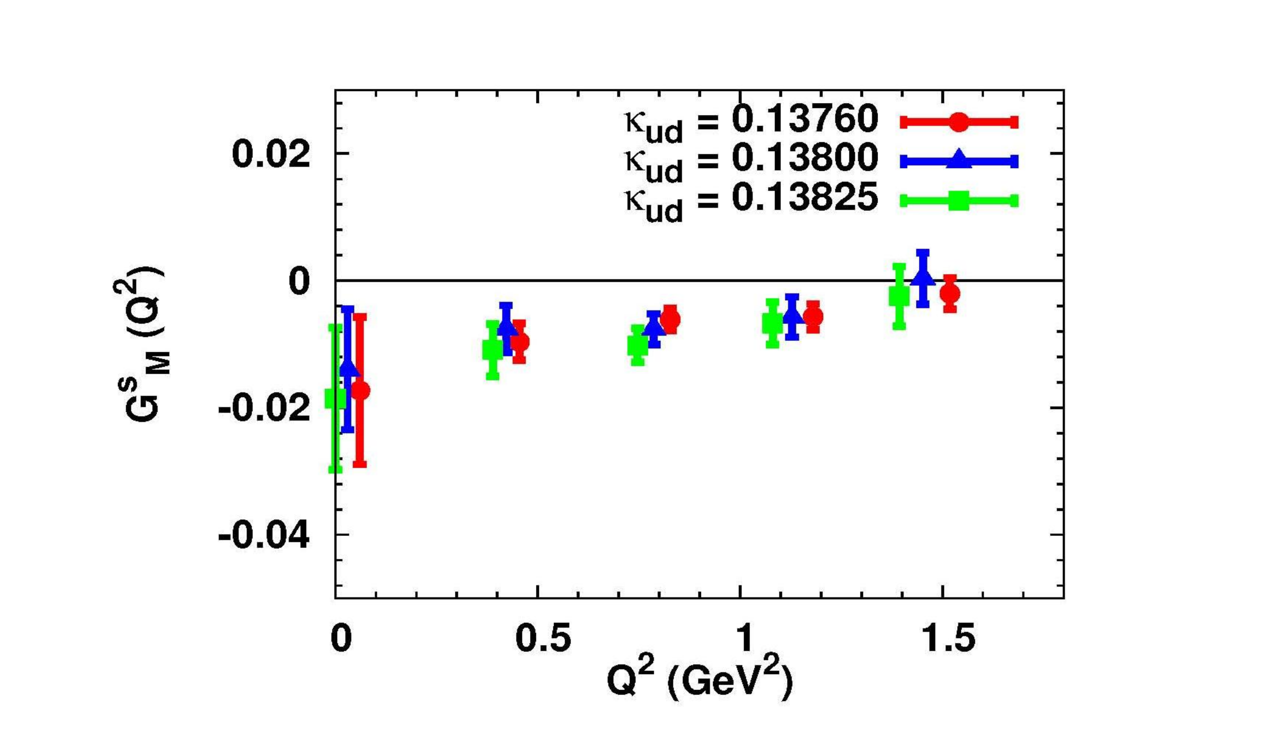
Summary

- Momentum fraction of quarks (both valence and sea) and gluons have been calculated for quenched and 2+1 flavor dynamical fermions.
- Glue mometum fraction is ~ 50%.
- g_A ~ 0.25 in agreement with expt.
- Glue angular momentum (gauge invariant) is very small.
- Quark orbital angular momentum is small for the valence, but large for the sea quarks.

Future

- Dynamical domain-wall fermion gauge (RBC + UKQCD configurations, lowest pion mass ~ 180 MeV on 4.5 fm box)
  + overlap fermion for the valence.
- The next set of lattices will have physical pion mass and 7 fm box.

Strangeness Magnetic Form Factors with 3 Quark Masses (m_π = 0.6, 0.7, 0.8 GeV); T. Doi et al. (χQCD) arXiV:0903.3232 PRD (2009)

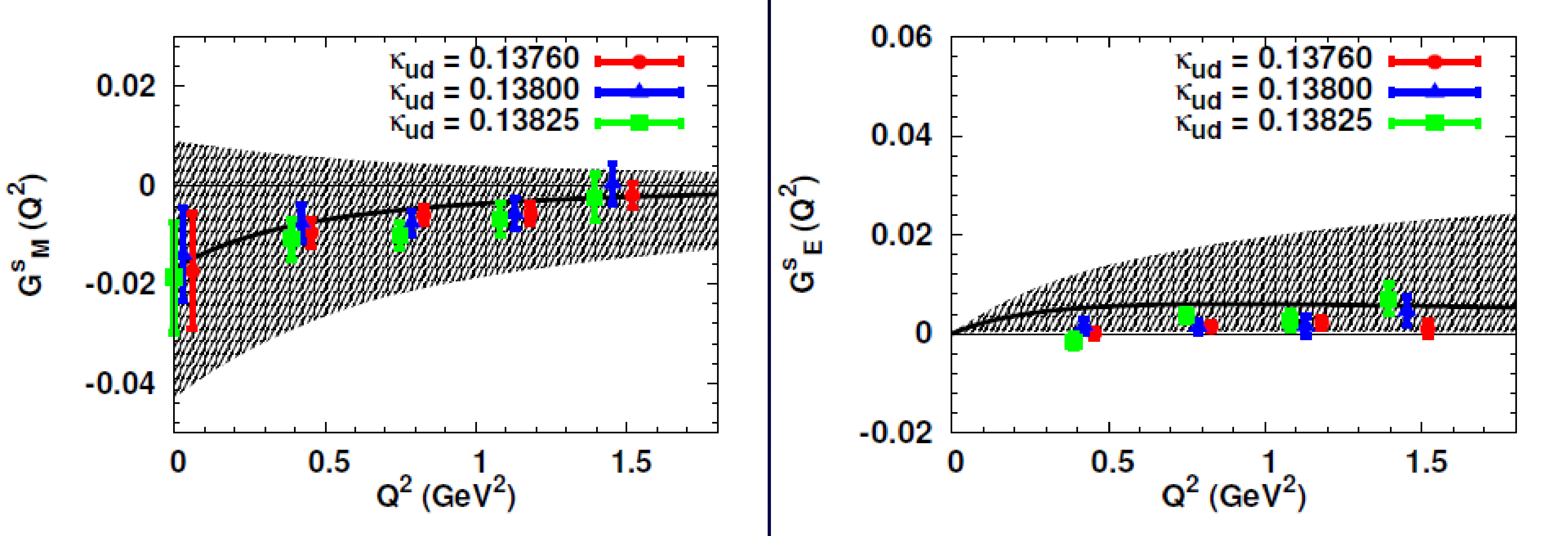


$G_M^S(Q^2 = 0.1\ \text{GeV}^2) = 0.29(21)\ \mu_N$ Liu, McKeown, Ramsey-Musolf (2007);

$G_M^S(Q^2 = 0.22\ \text{GeV}^2) = -\ 0.14(11)(11)\ \mu_N$ S. Baunack et al., (2009)

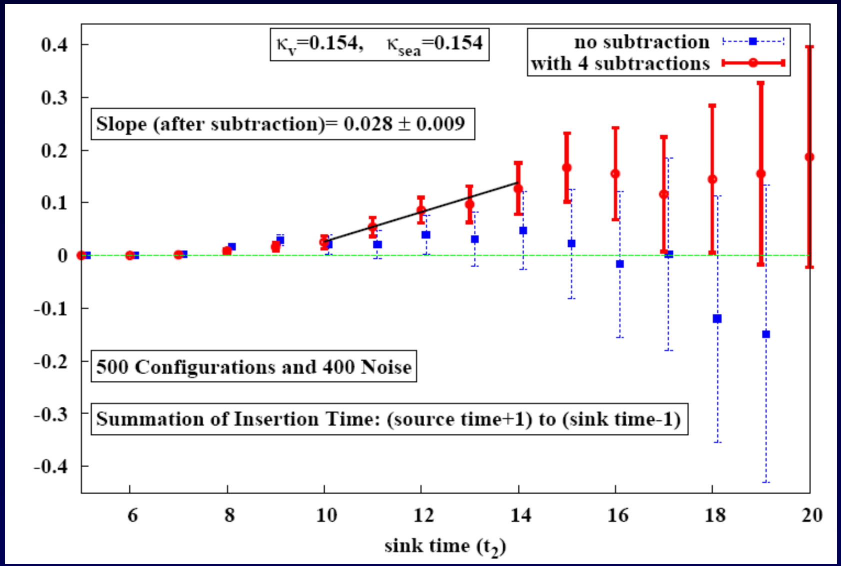
$$G_M^S(Q^2 = 0) = -\ 0.017(25)(07)\ \mu_N\ \ \chi\text{QCD (T. Doi et al.)}$$

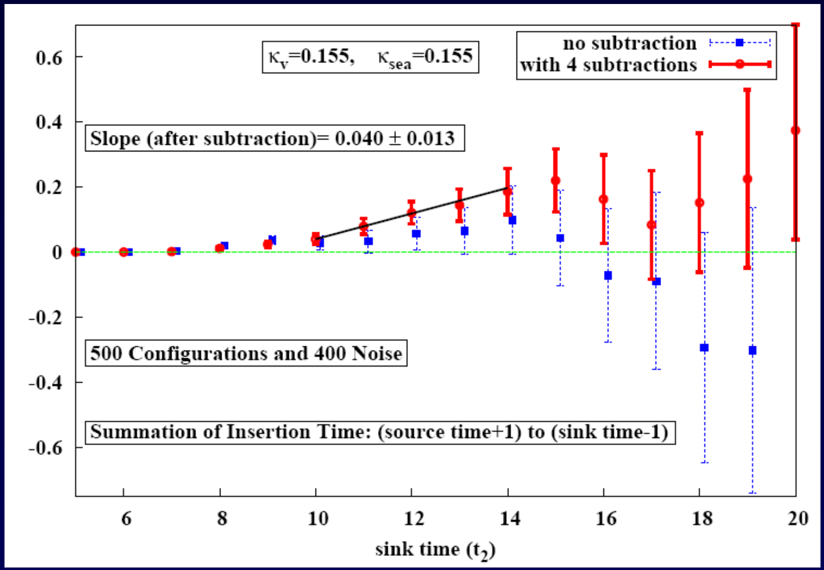
$G_M^S(Q^2 = 0) = -0.046(19)\mu_N$ D. Leinweber at al. (2005)

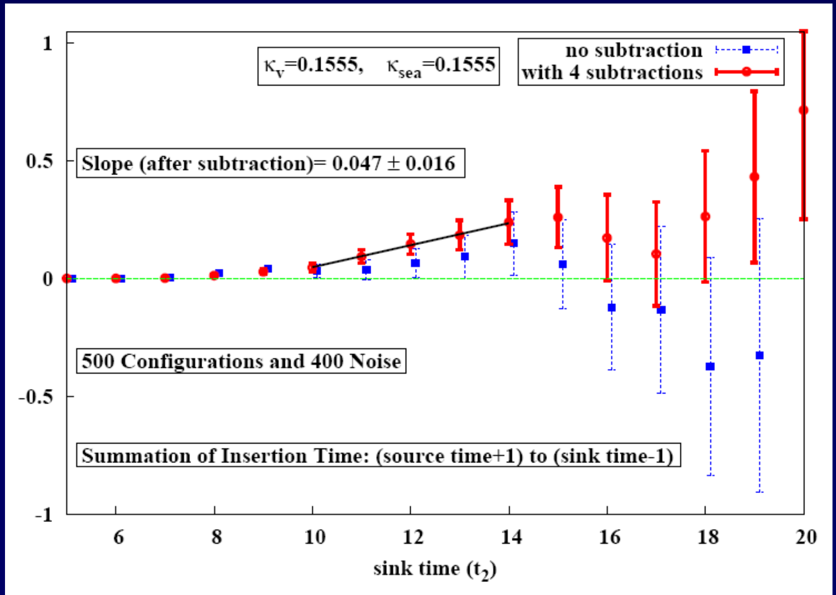


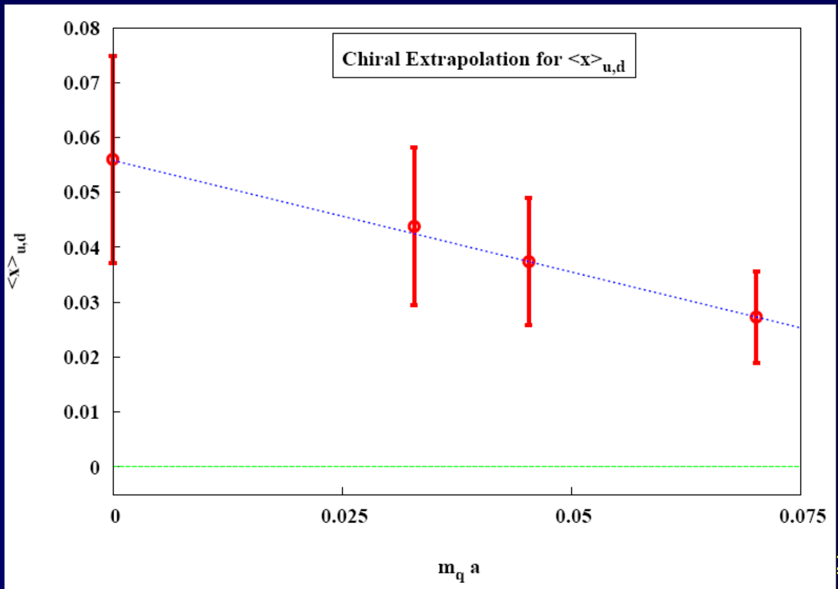
- The error band is due to chiral extrapolation. As a consequence, $G_M^S(Q^2=0) = -0.017(25)(07)\ \mu_N$ is only ~ 1 σ. Need larger lattices and smaller quark masses to be closer to the physical u/d mass.
- The error is about a factor of 10 smaller than previous direct lat calc and several times smaller than current experimental results.
- $G_M^S = -0.015(23),\quad G_E^S = +0.0022(19)\ \text{ at } Q^2 = 0.1\ \text{GeV}^2$

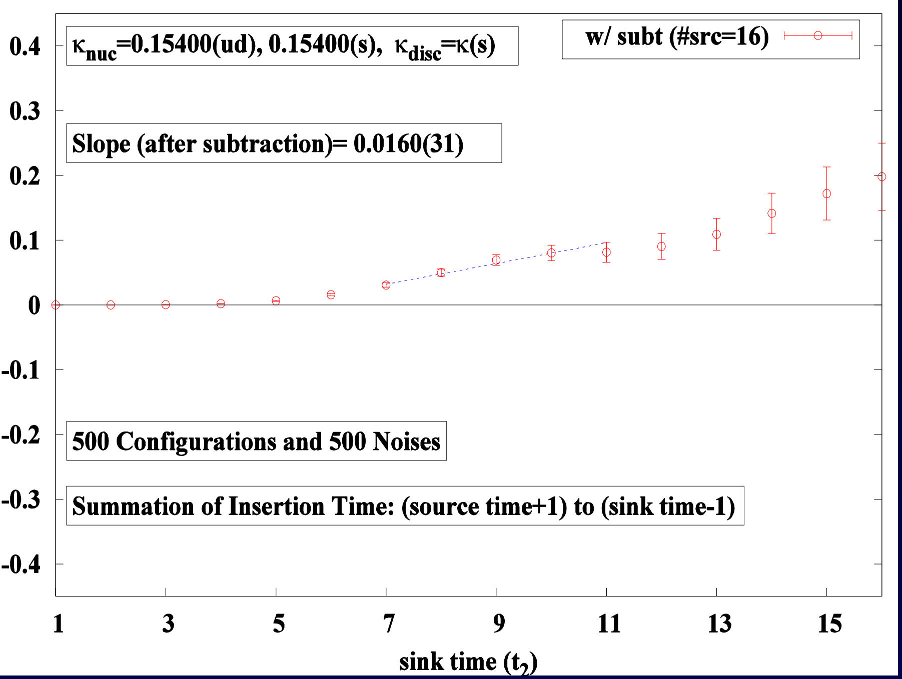
$\langle x \rangle_{u,d}$



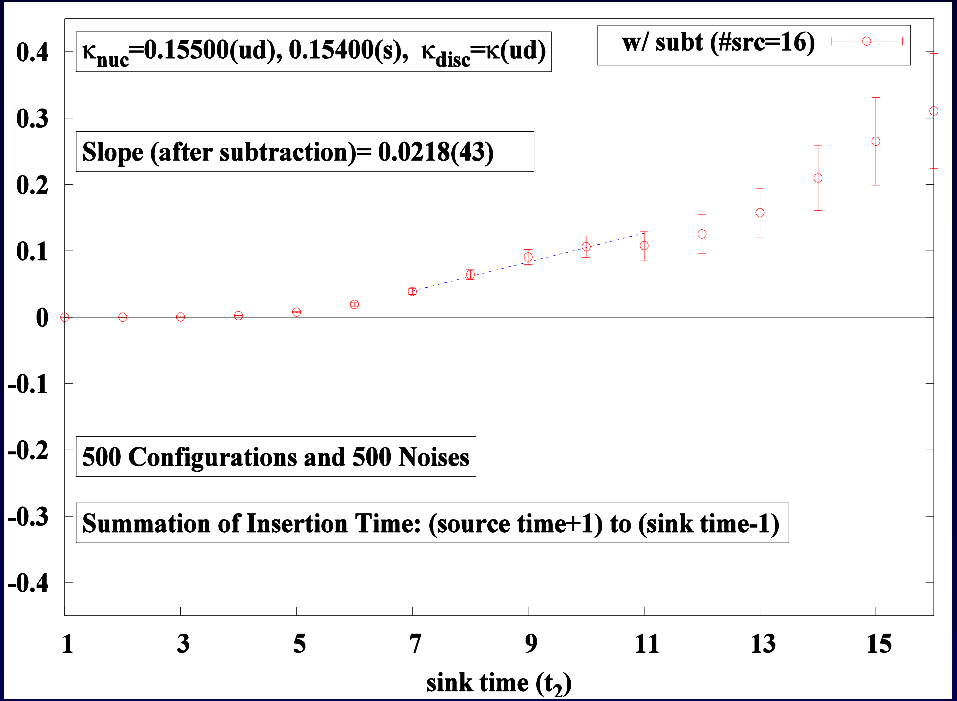




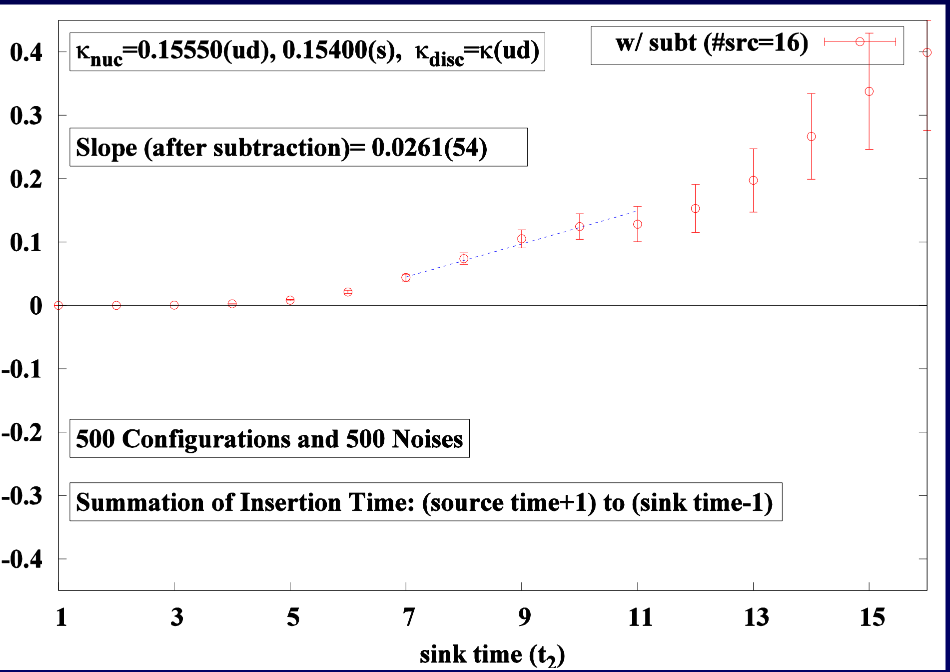




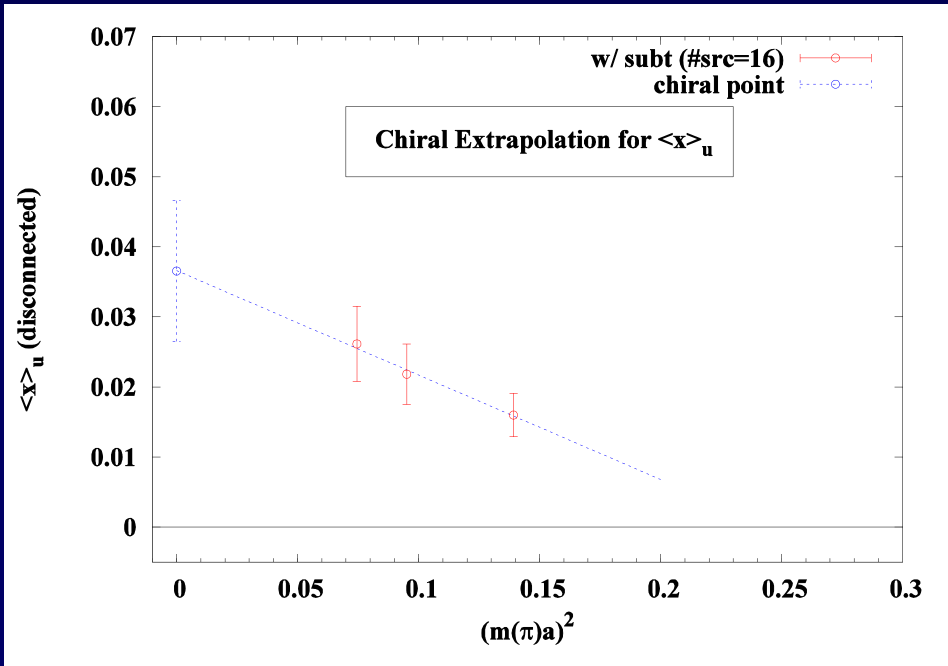
κnuc=0.15400(ud), 0.15400(s), κdisc=κ(s)
w/ subt (#src=16)
Slope (after subtraction)= 0.0160(31)
500 Configurations and 500 Noises
Summation of Insertion Time: (source time+1) to (sink time-1)
sink time (t2)



κnuc=0.15500(ud), 0.15400(s), κdisc=κ(ud)
w/ subt (#src=16)
Slope (after subtraction)= 0.0218(43)
500 Configurations and 500 Noises
Summation of Insertion Time: (source time+1) to (sink time-1)
sink time (t2)



κnuc=0.15550(ud), 0.15400(s), κdisc=κ(ud)
w/ subt (#src=16)
Slope (after subtraction)= 0.0261(54)
500 Configurations and 500 Noises
Summation of Insertion Time: (source time+1) to (sink time-1)
sink time (t2)



w/ subt (#src=16)
chiral point
Chiral Extrapolation for <x>u
<x>u (disconnected)
(m(π)a)2