Gauge invariance, Canonical quantization in the Internal structure of Nucleon

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III. The second proton spin crisis and the quark orbital angular momentum confusion

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I. Introduction

- It is still a quite popular idea that the polarized deep inelastic lepton-nucleon scattering (DIS) measured quark spin invalidates the constituent quark model (CQM).

  I will show that this is not true. After introducing minimum relativistic modification, the DIS measured quark spin can be accomodated in CQM.
- One has either gauge invariant or non-invariant decomposition of the total angular momentum operator of nucleon, but one has no gauge invariance and canonical commutation relation both satisfied decomposition.

  This will cause further confusion in the nucleon spin structure study and might have already caused.

II.The first proton spin crisis and quark spin confusion

quark spin contribution to nucleon spin in naïve non-relativistic quark model

$$\Delta u = \frac{4}{3}, \Delta d = -\frac{1}{3}, \Delta s = 0.$$

$$L_q = 0, \Delta G = 0, L_G = 0.$$

consistent with nucleon magnetic moments.

$$\mu_p / \mu_n = -3/2$$

The DIS measured quark spin contributions are:

$$\Delta\Sigma = \Delta u + \Delta d + \Delta s$$

$$\Delta\Sigma = 0.800 - 0.467 - 0.126 = 0.207 (Q^2 = 1 GeV^2, \Delta G > 0)$$

$$\Delta\Sigma = 0.812 - 0.455 - 0.114 = 0.243 (Q^2 = 1 GeV^2, \Delta G < 0)$$

(E.Leader, A.V.Sidorov and D.B.Stamenov, PRD75,074027(2007); hep-ph/0612360)

$$\Delta\Sigma = 0.813 - 0.458 - 0.114 = 0.242 (Q^2 = 10 GeV^2, \Delta G = -0.084)$$

(D.de Florian, R.Sassot, M.Statmann and W.Vogelsang, PRL101, 072001(2008); 0804.0422[hep-ph])

Contradictions!?

- It seems there are two contradictions between the CQM and measurements:

1. The DIS measured total quark spin contribution to nucleon spin is about 25%, while the naïve quark model is 1;
2. The DIS measured strange quark contribution is nonzero,

   while the naïve quark model is zero.

First confusion

- The DIS measured one is the matrix element of the quark axial vector current operator in a polarized nucleon state,

$$2a_0 S^\mu = \left\langle ps \,|\int d^3x \bar{\psi}\gamma^\mu\gamma^5\psi\,|\, ps\right\rangle$$

Here $a_0 = \Delta u + \Delta d + \Delta s$ which is not the quark spin contributions calculated in CQM. The CQM calculated one is the matrix element of the Pauli spin part only.

The axial vector current operator can be expanded instantaneously as:

$$
\begin{aligned}
\frac{1}{2}\int d^3x\bar{\psi}\vec{\gamma}\gamma^5\psi = &\int d^3x\psi^\dagger\frac{\vec{\Sigma}}{2}\psi \\
&= \sum_{i,\lambda,\lambda'}\int d^3k\chi^\dagger_\lambda\frac{\vec{\sigma}}{2}\chi_{\lambda'}(a^\dagger_{i,\vec{k},\lambda}a_{i,\vec{k},\lambda'} - b^\dagger_{i,\vec{k},\lambda'}b_{i,\vec{k},\lambda}) \\
&-\frac{1}{2}\sum_{i,\lambda,\lambda'}\int d^3k\chi^\dagger_\lambda\frac{\vec{\sigma}\cdot\vec{k}}{k_0(k_0+m_i)}i\vec{\sigma}\times\vec{k}\chi_{\lambda'}(a^\dagger_{i,\vec{k},\lambda}a_{i,\vec{k},\lambda'} - b^\dagger_{i,\vec{k},\lambda'}b_{i,\vec{k},\lambda}) \\
&+\sum_{i,\lambda,\lambda'}\int d^3k\chi^\dagger_\lambda\frac{i\vec{\sigma}\times\vec{k}}{2k_0}\chi_{\lambda'}a^\dagger_{i,\vec{k},\lambda}b^\dagger_{i,-\vec{k},\lambda'} + H.C..
\end{aligned}
$$

- Only the first term of the axial vector current operator, which is the Pauli spin, has been calculated in the non-relativistic quark models.
- The second term, the relativistic correction, has not been included in the non-relativistic quark model calculations. The relativistic quark model does include this correction and it reduces the quark spin contribution about 25%.
- The third term, $q\bar{q}$ creation and annihilation, has not been calculated in models with only valence quark configuration.

An Extended CQM with Sea Quark Components

(D.Qing, X.S.Chen and F.Wang,PRD58,114032(1998))

- To understand the nucleon spin structure quantitatively within CQM and to clarify the quark spin confusion further we developed a CQM with sea quark components,

$$\left|N\right\rangle = c_0 q^3 + \Sigma c_{\alpha\beta}(q^3)_\alpha (q\bar{q})_\beta$$

$$H=\sum_i \left(m_i+\frac{p_i^2}{2m_i}\right)+\sum_{i<j}\left(V_{ij}^c+V_{ij}^G\right)$$

$$+\sum_{i<j}\left(V_{i,i'j'j}+V_{i,i'j'j}^{\dagger}\right),$$

$$V_{ij}^c=-a_c\vec{\lambda}_i\cdot\vec{\lambda}_j r_{ij}^2,$$

$$V_{ij}^G=V_{ij}^{Gs}+V_{ij}^{Ga},$$

$$V_{ij}^{Gs}=\alpha_s\frac{\vec{\lambda}_i\cdot\vec{\lambda}_j}{4}$$

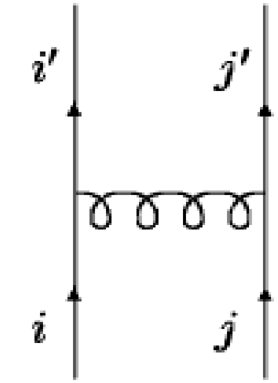
$$\times\left[\frac{1}{r_{ij}}-\frac{\pi}{2}\left(\frac{1}{m_i^2}+\frac{1}{m_j^2}+\frac{4\vec{\sigma}_i\cdot\vec{\sigma}_j}{3m_im_j}\right)\delta(\vec{r}_{ij})+\cdots\right],$$

$$V_{ij}^{Ga}=\pi\alpha_s\left(\frac{\vec{\lambda}_i+\vec{\lambda}_j}{2}\right)^2\left(\frac{1}{3}-\frac{\vec{f}_i\cdot\vec{f}_j}{2}\right)$$

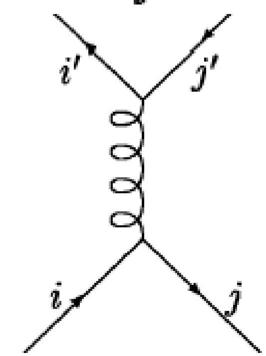
$$\times\left(\frac{\vec{\sigma}_i+\vec{\sigma}_j}{2}\right)^2\frac{2}{3}\frac{1}{(m_i+m_j)^2}\;\delta(\vec{r}_{ij}),$$

$$V_{i,i'j'j}=i\alpha_s\frac{\vec{\lambda}_i\cdot\vec{\lambda}_j}{4}\frac{1}{2r_{ij}}$$

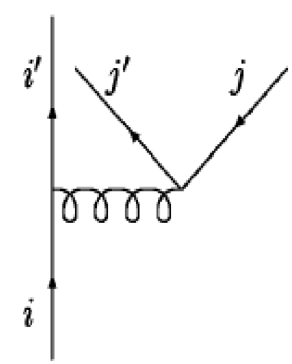
$$\times\left\{\left[\left(\frac{1}{m_i}+\frac{1}{m_j}\right)\vec{\sigma}_j+\frac{i\vec{\sigma}_j\times\vec{\sigma}_i}{m_i}\right]\cdot\frac{\vec{r}_{ij}}{r_{ij}^2}-\frac{2\vec{\sigma}_j\cdot\vec{\nabla}_i}{m_i}\right\}$$



V_{ij}^{Gs}



V_{ij}^{Ga}



$V_{i,i'j'j}$

TABLE I. Proton model wave function.

q^3	$N\eta$	$N\pi$	$\Delta\pi$	$N\eta'$	ΛK	ΣK	$\Sigma^* K$
-0.923	0.044	0.232	-0.252	0.065	0.109	-0.036	-0.106

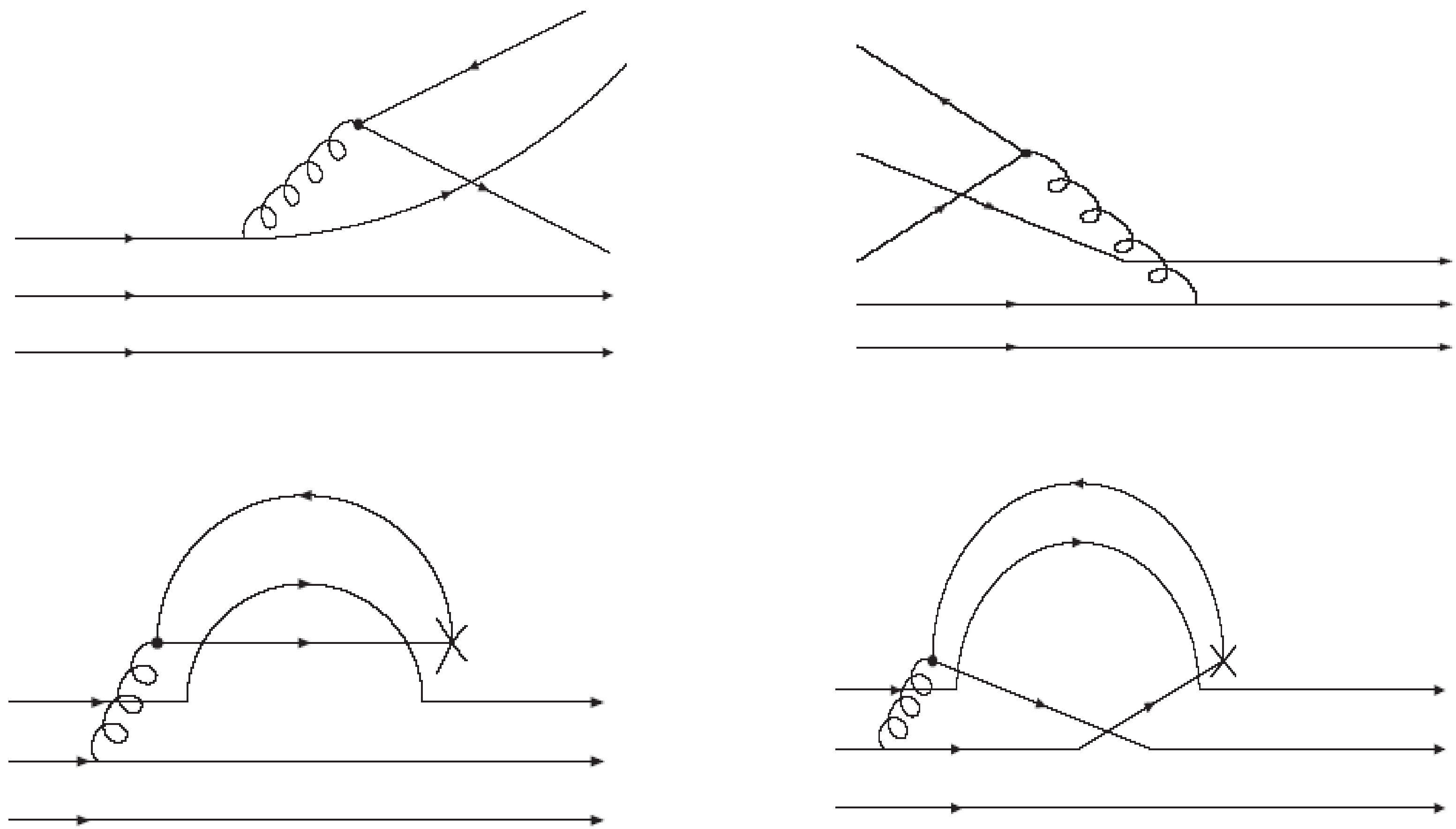
TABLE II. Masses and magnetic moments of the baryon octet and decuplet. $m=330$ (MeV), $m_s=564$ (MeV), $b=0.61$ (fm), $\alpha_s=1.46$, $a_c=48.2$ (MeV fm^{-2}).

		p	n	Λ	Σ^+	Σ^-	Ξ^0	Ξ^-	Δ	Σ^*	Ξ^*	Ω
	M(Mev)	939		1116	1193		1346		1232	1370	1523	1659
Theor.	E1(MeV)	2203		2323	2306		2409		2288	2306	2450	2638
	$\mu(\mu_N)$	2.780	-1.818	-0.522	2.652	-1.072	-1.300	-0.412				
	$\sqrt{\langle r^2\rangle}$(fm)	0.802	0.124									
	M(MeV)	939		1116	1189		1315		1232	1385	1530	1672
Exp.	$\mu(\mu_N)$	2.793	-1.913	-0.613	2.458	-1.160	-1.250	-0.651				
	$\sqrt{\langle r^2\rangle}$(fm)	0.836	0.34									

Model prediction of quark spin contribution to nucleon spin

TABLE III. The spin contents of the proton.

	q^3	$q^3-q^4\bar{q}$	$q^4\bar{q}-q^4\bar{q}$	sum	exp.	lattice [9]	lattice [9,15]
Δu	0.773	−0.125	0.100	0.75	0.80	0.79(11)	0.638(54)
Δd	−0.193	−0.249	−0.041	−0.48	−0.46	−0.42(11)	−0.347(46)
Δs	0	−0.064	−0.002	−0.07	−0.12	−0.12(1)	−0.109(30)



ment of the QAVCO, the axial charge, will undergo such a relativistic reduction due to quark orbital motion.

If one allows sea quark Fock component mixing as shown in Eq. (6) used in our model, then the third term of Eq. (11), the quark-antiquark pair creation and annihilation term, will contribute to the matrix element of QAVCO. Table III shows our model results of the quark spin contents Δq of proton, in fact the matrix element of the QAVCO (axial charge). The experimental value and lattice QCD results are listed for comparison. In Table III, the second column is the q^3 valence quark contribution, where

$$\Delta u = \frac{4}{3}(1-0.32)(-0.923)^2,$$

$$\Delta d = -\frac{1}{3}(1-0.32)(-0.923)^2,$$

$$\Delta s = 0, \tag{12}$$

the first factors $\frac{4}{3}$, $-\frac{1}{3}$, 0 are the well known proton spin contents of the nonrelativistic quark model. $-0.32 = -1/3m^2b^2$ is the relativistic reduction and -0.923 is the amplitude of the q^3 component of our model. The third column is the contribution of the quark-antiquark pair creation (annihilation) term. It is another important reduction of the quark spin contribution and Δs is mainly due to this term. The fourth column lists the contribution of $q^3 q\bar{q}$ Fock components; due to quark antisymmetrization it cannot be separated into the valence and sea quark part. However, the antiquark contribution is very small (the largest one is $\Delta\bar{d} = 0.004$), and has not been listed in Table III. The fifth column lists the sum. Our model quark spin contents Δu, Δd, and Δs are quite close to the experiment ones in Eq. (3) and column 6, even though we have not made any model parameter adjustments aimed at fitting the proton spin content.

As a QCD model, our model results do not have any QCD scale dependence and the fit to $Q^2 = 5$ GeV^2 experimental results is not perfect. However, it seems fair to say that a problem that has lasted for a decade, the ''proton spin crisis,'' can be settled by noting that, first, the quark spin content measured in polarized DIS is the matrix element of the quark axial vector current operator (the axial charge), not just the Pauli spin operator as usually used in the quark spin content calculation in a nonrelativistic quark model. Second, in addition to the relativistic reduction, the quark-antiquark pair creation (annihilation) term gives rise to another important reduction of the quark spin contribution to the proton spin content if minor (15% in our model) sea quark Fock components are mixed in the proton state. Such mixing has been observed both by hadron phenomenology and Fock space expansion theory and certainly does not ruin the success of the pure valence constituent quark model. On the contrary, the pure valence constituent quark model is a good approximation of this extended one especially for those hadron properties which are not sensitive to the sea quark Fock components. For observables such as the nucleon radii, (OZI) rule violation, nucleon spin σ term, and the spin content measured in polarized DIS, these additional sea quark Fock components are crucial. Especially, the quark-antiquark pair creation and annihilation terms contribute significantly via the Z diagram and disconnected diagram.

IV. WHERE DOES THE PROTON GET ITS SPIN?

The quark spin contribution or the matrix element of the quark axial vector current operator (the axial charge) of the proton is suppressed. Then where does the proton get its remaining spin? To answer this question from a nonrelativistic quark model is very simple. If we directly calculate the matrix element of the nonrelativistic spin $\vec{S}_q^{NR}$, the first term of Eq. (11), with our model wave function Eq. (6) and Table I, we will get the nonrelativistic proton spin contents listed in

III.The second proton spin crisis and the quark orbital angular momentum confusion

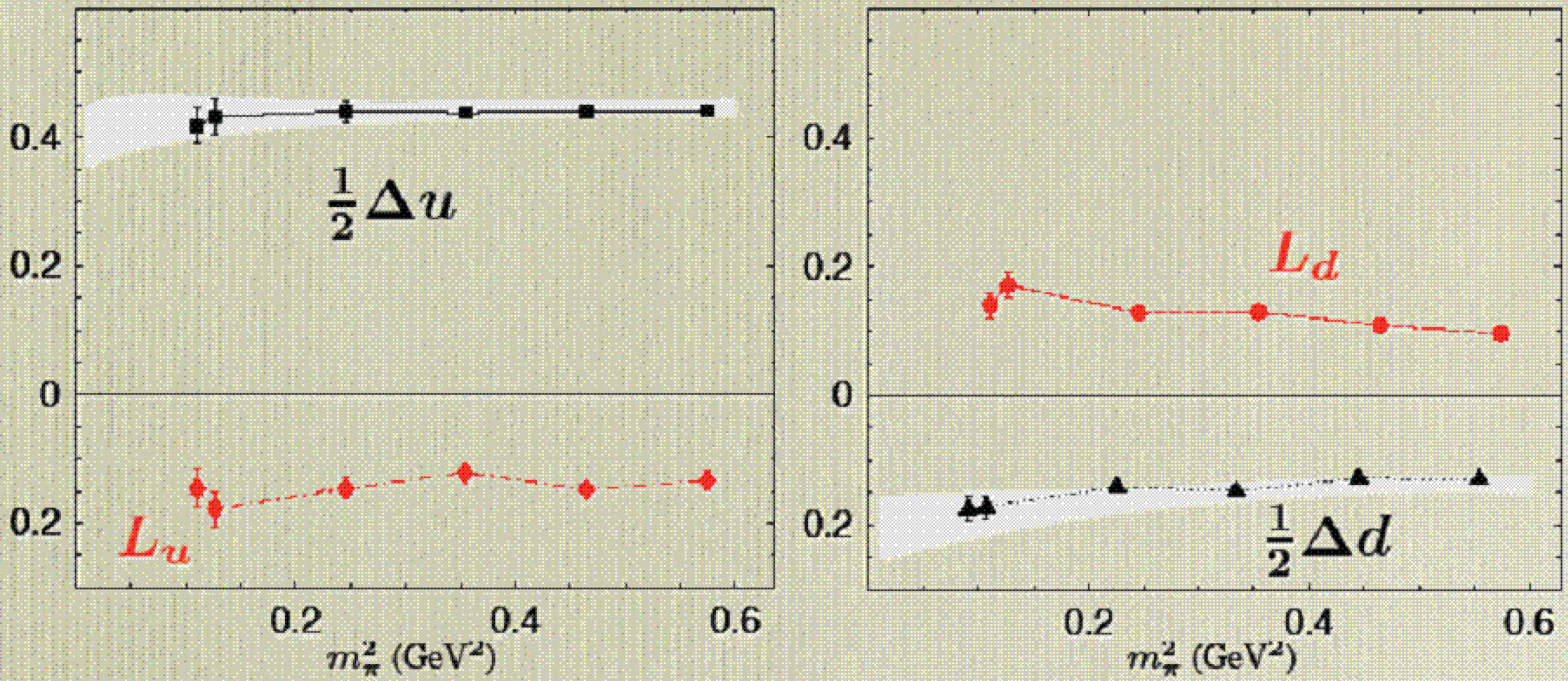
R.L.Jaffe gave a talk at 2008 1th International Symposium on Science at J-PARK, raised the second proton spin crisis, mainly

$$\Delta\Sigma \;\square\; 0$$

$$L_q \;\square\; 0$$

$$\Delta G \;\square\; 0$$

Hägler et al also calculate L_u and L_d separately.



The real enigma is that quark models generically predict that orbital and spin angular momentum should be parallel. So

(Ask me offline)

$$\Delta u > 0 \quad \Rightarrow \quad L_u > 0$$
$$\Delta d < 0 \quad \Leftrightarrow \quad L_d < 0$$

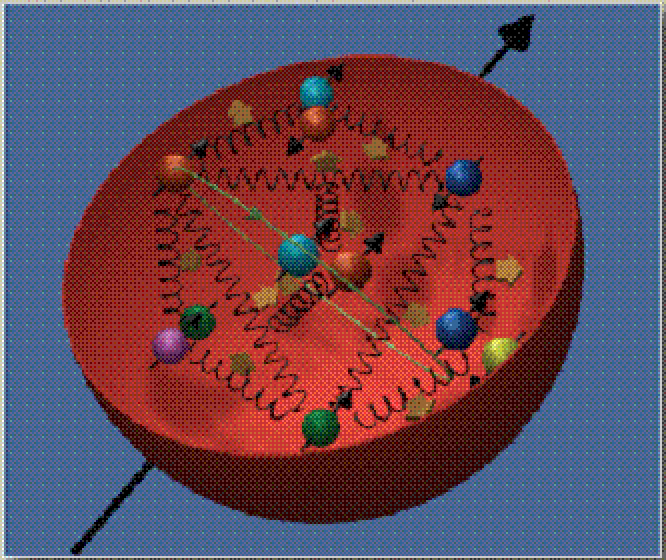
Note that $L_u - L_d$ is an isovector and not effected by disconnected diagrams

This is the opposite of what is seen!

Everything's coming out zero!

$$\begin{aligned}\Delta u + \Delta d + \Delta s &\approx 0 \\ \frac{1}{2}\Delta u + L_u \equiv J_u &\approx 0 \\ \frac{1}{2}\Delta d + L_d \equiv J_d &\approx 0 \\ L_u + L_d &\approx 0 \text{ mod disconnected diagrams} \\ \Delta G &\approx \text{small}\end{aligned}$$

So, what remains of the quark model?
And, where is the proton's spin?



with unpolarized beam and a transversely polarized proton target [233]. The kinematic dependences of two asymmetry amplitudes of particular interest are shown in figure 42 in comparison with calculations based on two different types of GPD models. The data is clearly able to discriminate among these GPD models. The model variants that agree best with the data, are used in the next section to derive constraints from the data on the total angular momentum of u quarks. Sensitivity exists, as can be seen from the top panel of the figure.

4.8. Quark total angular momenta

In the theoretical description of the DVCS process, experimental observables like cross section differences or asymmetries are related to various combinations of the Compton Form Factors $\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}}, \widetilde{\mathcal{E}}$, which can be calculated from the respective GPDs $H^f, \widetilde{H}^f, E^f, \widetilde{E}^f$ (see section 4.5). Parameterizations of the spin-flip GPD E^f embody explicitly or implicitly the quark total angular momenta J_u and J_d [125]. Hence model-

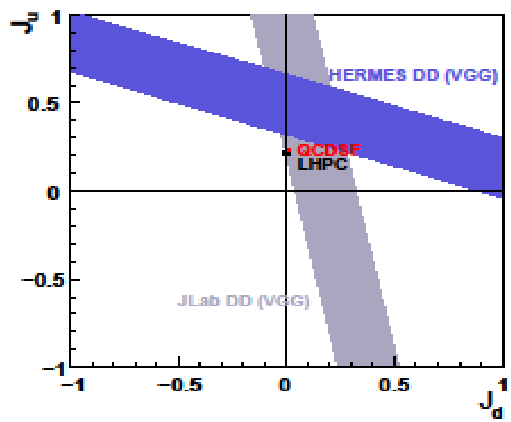


Figure 43. Model-dependent constraints on u-quark total angular momentum J_u vs d-quark total angular momentum J_d, obtained by comparing DVCS experimental results and theoretical calculations using the double-distribution GPD model of [125, 198]. The constraints based on the HERMES data [233] for the azimuthal amplitudes $A_{\mathrm{UT,I}}^{\sin(\phi-\phi_S)\cos\phi}$ and $A_{\mathrm{UT,I}}^{\sin(\phi-\phi_S)}$ are labelled HERMES DD (VGG). Those based on the JLAB neutron cross section data [238] are labelled JLAB DD (VGG). Also shown as small (overlapping) rectangles are results from lattice gauge theory by the QCDSF [239] and LHPC [165] collaborations, as well as a result for only the valence-quark contribution (DFJK) based on zero-skewness GPDs extracted from nuclear form factor data [193, 240]. The sizes of the small rectangles represent the statistical uncertainties of the lattice results and the parameter range for which a good DFJK fit to the nucleon form factor data was achieved. Theoretical uncertainties are unavailable. The figure is taken from [233].

dependent constraints on these two parameters can be derived by fitting them to

Second confusion

- The quark "orbital angular momentum"

$$\vec{L}_q = \int d^3x \vec{x} \times \psi_q^+ (\vec{p} - g\vec{A}) \psi_q$$

calculated in LQCD and measured in DVCS is not the real orbital angular momentum used in quantum mechanics. It does not satisfy the Angular Momentum Algebra,

$$\vec{L} \times \vec{L} = i\vec{L}$$

and the gluon contribution is ENTANGLED in it.

Where does the nucleon get spin?

Real quark orbital angular momentum contribution

- As a QCD system the nucleon spin consists of the following four terms (in Coulomb gauge),

$$\vec{J} = \vec{S}_q + \vec{L}_q + \vec{S}_G + \vec{L}_G$$

$$\vec{S}_q = \frac{1}{2}\int d^3x \psi^\dagger \vec{\Sigma} \psi$$

$$\vec{L}_q = \int d^3x \psi^\dagger \vec{r} \times \frac{\nabla}{i} \psi$$

$$\vec{S}_G = 2\int d^3x T_r\{\vec{E} \times \vec{A}\}$$

$$\vec{L}_G = 2\int d^3x T_r\{E_i \vec{r} \times \nabla A_i\}$$

- The real quark orbital angular momentum operator can be expanded instantaneously as,

$$
\begin{aligned}
\vec{L}_q &= \int d^3x\psi^\dagger \vec{r}\times\frac{\nabla}{i}\psi \\
&= \sum_{i,\lambda}\int d^3k(a^\dagger_{i,\vec{k},\lambda} i\vec{\nabla}_k\times\vec{k}a_{i,\vec{k},\lambda} + b^\dagger_{i,\vec{k},\lambda} i\vec{\nabla}_k\times\vec{k}b_{i,\vec{k},\lambda}) \\
&+\frac{1}{2}\sum_{i,\lambda,\lambda'}\int d^3k\chi^\dagger_\lambda\frac{\vec{\sigma}\cdot\vec{k}}{k_0(k_0+m_i)}i\vec{\sigma}\times\vec{k}\chi_{\lambda'}(a^\dagger_{i,\vec{k},\lambda}a_{i,\vec{k},\lambda'} - b^\dagger_{i,\vec{k},\lambda'}b_{i,\vec{k},\lambda}) \\
&-\sum_{i,\lambda,\lambda'}\int d^3k\chi^\dagger_\lambda\frac{i\vec{\sigma}\times\vec{k}}{2k_0}\chi_{\lambda'}a^\dagger_{i,\vec{k},\lambda}b^\dagger_{i,-\vec{k},\lambda'} + H.C.
\end{aligned}
$$

Quark orbital angular momentum will compensate the quark spin reduction

- The first term is the non-relativistic quark orbital angular momentum operator used in CQM, which does not contribute to nucleon spin in the naïve CQM.
- The second term is again the relativistic correction, which will compensate the relativistic quark spin reduction.
- The third term is again the $q\bar{q}$ creation and annihilation contribution, which will compensate the quark spin reduction due to $q\bar{q}$ creation and annihilation.

- It is most interesting to note that the relativistic correction and the $q\bar{q}$ creation and annihilation terms of the quark spin and the orbital angular momentum operator are exact the same but with opposite sign. Therefore if we add them together we will have

$$\vec{S}_q + \vec{L}_q = \vec{S}_q^{NR} + \vec{L}_q^{NR}$$

where the $\vec{S}_q^{NR} \cdot \vec{L}_q^{NR}$ are the non-relativistic quark spin and orbital angular momentum operator used in quantum mechanics.

$$\vec{S}_q + \vec{L}_q = \vec{S}_q^{NR} + \vec{L}_q^{NR}$$

- The above relation tell us that the quark contribution to nucleon spin can be either attributed to the quark Pauli spin, as did in the last thirty years in CQM, and the non-relativistic quark orbital angular momentum which does not contribute to the nucleon spin in naïve CQM; or
- part of the quark contribution is attributed to the relativistic quark spin as measured in DIS, the other part is attributed to the relativistic quark orbital angular momentum which will provide

the exact compensation of the missing part in the relativistic “quark spin”

- one must use the right combination otherwise will misunderstand the nucleon spin structure.

Prediction

- Based on the LQCD and the extended quark model calculation of quark spin;
- and the analysis of quark spin and orbital angular momentum operators;

The matrix elements of the real relativistic quark orbital angular momentum should be:

$$L_u > L_d > 0$$

under a reasonable assumption that the non-relativistic quark orbital angular momentum contributions are not a too large negative value.

- This can be first checked by the LQCD calculation of the matrix elements of the real quark orbital angular momentum.

IV.A proper decomposition of the momentum and angular momentum of a gauge system

Jaffe-Manohar decomposition:

$$\vec{J} = \vec{S}_q + \vec{L}_q + \vec{S}_G + \vec{L}_G$$

$$\vec{S}_q = \frac{1}{2}\int d^3x \psi^\dagger \vec{\Sigma} \psi$$

$$\vec{L}_q = \int d^3x \psi^\dagger \vec{r} \times \frac{\nabla}{i} \psi$$

$$\vec{S}_G = 2\int d^3x T_r\{\vec{E} \times \vec{A}\}$$

$$\vec{L}_G = 2\int d^3x T_r\{E_i \vec{r} \times \nabla A_i\}$$

R.L.Jaffe and A. Manohar,Nucl.Phys.B337,509(1990).

- Each term in this decomposition satisfies the canonical commutation relation of angular momentum operator, so they are qualified to be called quark spin, orbital angular momentum, gluon spin and orbital angular momentum operators.
- However they are not gauge invariant except the quark spin.

Gauge invariant decomposition:

$$\vec{J} = \vec{S}_q + \vec{L'}_q + \vec{J'}_G$$

$$\vec{S}_q = \frac{1}{2}\int d^3x \psi^\dagger \vec{\Sigma} \psi$$

$$\vec{L'}_q = \int d^3x \psi^\dagger \vec{r} \times \frac{\vec{D}}{i} \psi$$

$$\vec{J'}_G = 2\int d^3x T_r\{\vec{r} \times (\vec{E} \times \vec{B})\}$$

X.S.Chen and F.Wang, Commun.Theor.Phys. 27,212(1997).

X.Ji, Phys.Rev.Lett.,78,610(1997).

- However each term no longer satisfies the canonical commutation relation of angular momentum operator except the quark spin, in this sense the second and third term is not the real quark orbital and gluon angular momentum operator.
- One can not have gauge invariant gluon spin and orbital angular momentum operator separately, the only gauge invariant one is the total angular momentum of gluon.
- In QED this means there is no photon spin and orbital angular momentum! Contradict to the well established multipole radiation analysis.

Gauge invariance and canonical quantization satisfied decomposition

- Gauge invariance is not sufficient to fix the decomposition of the angular momentum of a gauge system.
- Canonical quantization rule of the angular momentum operator must be respected. It is also an additional condition to fix the decomposition.

X.S.Chen, X.F.Lu, W.M.Sun, F.Wang and T.Goldman Phys.Rev.Lett. 100(2008) 232002.

arXiv:0806.3166; 0807.3083; 0812.4366[hep-ph]; 0909.0798[hep-ph]

$$\vec{J}_{QED} = \vec{S}_e + \vec{L}''_e + \vec{S}''_\gamma + \vec{L}''_\gamma$$

$$\vec{S}_e = \int d^3x \psi^+ \frac{\vec{\Sigma}}{2} \psi$$

$$\vec{L}''_e = \int d^3x \psi^+ \vec{x} \times \frac{\vec{D}_{pure}}{i} \psi$$

$$\vec{S}''_\gamma = \int d^3x \vec{E} \times \vec{A}_{phy}$$

$$\vec{L}''_\gamma = \int d^3x E^i \vec{x} \times \vec{D}_{pure} A^i_{phys}$$

$$\vec{A} = \vec{A}_{phys} + \vec{A}_{pure}, \qquad \nabla \cdot \vec{A}_{phys} = 0, \nabla \times \vec{A}_{phys} = \nabla \times \vec{A}.$$

$$\vec{A}_{phy} = \nabla \times \frac{1}{4\pi} \int d^3x' \frac{\vec{B}(x')}{\left|\vec{x} - \vec{x}'\right|}, \qquad \vec{D}_{pure} = \vec{\nabla} - ie\vec{A}_{pure}.$$

It provides the theoretical basis of the multipole radiation analysis

QCD

$$\vec{J}_{QCD} = \vec{S}_q + \vec{L}_q^{\,''} + \vec{S}_g^{\,''} + \vec{L}_g^{\,''}$$

$$\vec{S}_q = \int d^3x \psi^+ \frac{\vec{\Sigma}}{2} \psi$$

$$\vec{L}_q^{\,''} = \int d^3x \psi^+ \vec{x} \times \frac{\vec{D}_{pure}}{i} \psi$$

$$\vec{S}_g^{\,''} = \int d^3x \vec{E}^a \times \vec{A}^a_{phys}$$

$$\vec{L}_g^{\,''} = \int d^3x E^{ia} \vec{x} \times \vec{D}^{adj}_{pure} A^{ia}_{phys}$$

Non Abelian complication

$$\vec{A} = \vec{A}_{pure} + \vec{A}_{phy} \qquad \vec{A}_{pure} = T^a \vec{A}^a_{pure}$$

$$\vec{D}_{pure} \times \vec{A}_{pure} = \vec{\nabla} \times \vec{A}_{pure} - ig \vec{A}_{pure} \times \vec{A}_{pure} = 0$$

$$\vec{D}_{pure} = \vec{\nabla} - ig \vec{A}_{pure}$$

$$\vec{D}^{adj}_{pure} \cdot \vec{A}_{phys} = \nabla \cdot \vec{A}_{phys} - ig[A^i_{pure,} A^i_{phys}] = 0$$

$$\vec{D}^{adj}_{pure} = \nabla - ig[\vec{A}_{pure},]$$

Proper separation of nucleon momentum and angular momentum

Sq Lq Sg Lg

$$\vec{J}_{\text{total}} = \int d^3x \psi^\dagger \frac{1}{2}\vec{\Sigma}\psi + \int d^3x \vec{x} \times \psi^\dagger \frac{1}{i}\vec{D}_{\text{pure}}\psi + \int d^3x \vec{E} \times \vec{A}_{\text{phys}} + \int d^3x \vec{x} \times E^i \vec{\mathcal{D}}_{\text{pure}} A^i_{\text{phys}}$$

$$\vec{P}_{\text{total}} = \int d^3x \psi^\dagger \frac{1}{i}\vec{D}_{\text{pure}}\psi + \int d^3x E^i \vec{\mathcal{D}}_{\text{pure}} A^i_{\text{phys}}$$

Pq Pg

Standard construction of orbital angular momentum $\vec{L} = \int d^3x \, \vec{x} \times \vec{P}$

- Each term is gauge invariant and so in principle measurable.
- Each term satisfies angular momentum commutation relation and so can be compared to quark model ones.
- In Coulomb gauge it reduces to Jaffe-Manohar decomposition.
- In other gauge, Jaffe-Manohar’s quark, gluon orbital angular momentum and gluon spin are gauge dependent. Ours are gauge invariant.

- It is not a special problem for quark and gluon angular momentum operators;
- But a fundamental problem for a gauge field system. Operators for the individual part of a gauge system need such kind modifications.

X.S.Chen, X.F.Lu, W.M.Sun, F.Wang and T.Goldman, Phys.Rev.Lett. 100,232002(2008), arXiv:0806.3166; 0807.3083; 0812.4336[hep-ph]; 0909.0798[hep-ph]

V. Summary

- There are different quark and gluon momentum and orbital angular momentum operators. Confusions disturbing or even misleading the nucleon spin structure studies.
- Quark spin missing can be understood within the CQM.
- Quite possible, the real relativistic quark orbital angular momentum will compensate the missing quark spin.
- A LQCD calculation of the matrix elements of u,d quark real orbital angular momentum might lighten the nucleon spin structure study.

- For a gauge system, the momentum and angular momentum operators of the individual part (quark~gluon, electron~photon), the existed ones are either gauge invariant or satisfy the canonical commutation relation only but not both.
- We suggest a decomposition which satisfies both the gauge invariance and canonical commutation relations. It might be useful and modify our picture of nucleon internal structure.

Quantum Mechanics

- The fundamental operators in QM

$$\vec{r}, \qquad \vec{p} = -i\nabla,$$

$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times \frac{\nabla}{i},$$

$$H = \frac{(\vec{p} - e\vec{A})^2}{2m} + e\varphi$$

For a charged particle moving in em field, the canonical momentum is,

$$\vec{p} = m\dot{\vec{r}} + q\vec{A}$$

- It is gauge dependent, so classically it is Not measurable.
- In QM, we quantize it as $\vec{p} = \frac{\vec{\nabla}}{i}$, no matter what gauge is.
- It appears to be gauge invariant, but in fact Not!

Under a gauge transformation

$$\psi \rightarrow \psi' = e^{-iq\omega(x)}\psi,$$

$$\vec{A} \rightarrow \vec{A'} = \vec{A} - \vec{\nabla}\omega, \qquad \varphi \rightarrow \varphi' = \varphi + \partial_t \omega,$$

The matrix elements transformed as

$$\langle \psi | p | \psi \rangle \rightarrow \langle \psi | p | \psi \rangle - \langle \psi | q\nabla\omega | \psi \rangle,$$

$$\langle \psi | L | \psi \rangle \rightarrow \langle \psi | L | \psi \rangle - \langle \psi | q\vec{r} \times \nabla\omega | \psi \rangle,$$

$$\langle \psi | H | \psi \rangle \rightarrow \langle \psi | H | \psi \rangle + \langle \psi | q\partial_t\omega | \psi \rangle,$$

New momentum operator

- Old generalized momentum operator for a charged particle moving in em field,

$$\vec{p} = m\dot{\vec{r}} + q\vec{A}_{\perp} + q\vec{A}_{\parallel} = \frac{\vec{\nabla}}{i}$$

$$\vec{\nabla} \cdot \vec{A}_{\perp} = 0, \qquad \vec{\nabla} \times \vec{A}_{\parallel} = 0, \qquad \vec{\nabla} \times \vec{A}_{\perp} = \vec{\nabla} \times \vec{A}.$$

It satisfies the canonical momentum commutation relation, but its matrix elements are not gauge invariant.

- New momentum operator we proposed,

$$\vec{p}_{pure} = \vec{p} - q\vec{A}_{\parallel} = m\dot{\vec{r}} + q\vec{A}_{\perp}.$$

It is both gauge invariant and canonical commutation relation satisfied.

We call

$$\frac{\vec{D}_{pure}}{i} = \vec{p} - q\vec{A}_{//} = \frac{1}{i}\vec{\nabla} - q\vec{A}_{//}$$

physical momentum.

It is neither the canonical momentum

$$\vec{p} = m\vec{\dot{r}} + q\vec{A} = \frac{1}{i}\vec{\nabla}$$

nor the mechanical momentum

$$\vec{p} - q\vec{A} = m\vec{\dot{r}} = \frac{1}{i}\vec{D}$$

Gauge transformation

$$\psi' = e^{-iq\omega(x)}\psi, \qquad A'^{\mu} = A^{\mu} + \partial^{\mu}\omega(x),$$

only affects the longitudinal part of the vector potential

$$A'_{//} = A_{//} - \nabla\omega(x),$$

and time component

$$\varphi' = \varphi + \partial_t \omega(x),$$

it does not affect the transverse part,

$$A'_{\perp} = A_{\perp},$$

so $A_{\perp}$ is physical and which is completely determined by the em field tensor $\vec{E}, and, \vec{B}$.

$\vec{A}_{//}$ is unphysical, it is caused by gauge transformation.

Separation of the gauge potential

$$A^{\mu} = A^{\mu}_{pure} + A^{\mu}_{phys}$$

$$F^{\mu\nu}_{pure} = \partial^{\mu} A^{\nu}_{pure} - \partial^{\nu} A^{\mu}_{pure} = 0, \qquad \vec{\nabla} \times \vec{A}_{pure} = 0.$$

$$\vec{\nabla} \cdot \vec{A}_{phys} = 0, \qquad \vec{\nabla} \times \vec{A}_{phys} = \vec{\nabla} \times \vec{A}.$$

$$\vec{A}_{phys}(x) = \vec{\nabla} \times \int d^{3}x' \, \frac{\vec{B}(x')}{4\pi \left| \vec{x} - \vec{x}' \right|}$$

$$\partial_{i} A^{0}_{phys} = \partial_{i} A^{0} + \partial_{t} (A^{i} - A^{i}_{phys}),$$

$$A^{0}_{phys}(x) = \int_{-\infty}^{x} dx^{i} (\partial_{t} (\vec{\nabla}' \times \frac{\vec{B}}{\nabla'^{2}})^{i} - E^{i}).$$

Gauge transformation

Under a gauge transformation,

$$A'^{\mu} = A^{\mu} + \partial^{\mu}\omega(x),$$

$$\mathbf{A}'^{\mu}_{phys} = \mathbf{A}^{\mu}_{phys},$$

$$A'^{\mu}_{pure} = A^{\mu}_{pure} + \partial^{\mu}\omega(x).$$

Non Abeliean case

$$A^{\mu} = A^{\mu}_{phys} + A^{\mu}_{pure}, \qquad A^{\mu} = A^{\mu a} T^{a}.$$

$$F^{\mu\nu}_{pure} = \partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu} + ig[A^{\mu}_{pure}, A^{\nu}_{pure}] = 0,$$

$$\vec{D}_{pure} \times \vec{A}_{pure} = \vec{\nabla} \times \vec{A}_{pure} - ig\vec{A}_{pure} \times \vec{A}_{pure} = 0$$

$$\vec{D}^{adj}_{pure} \cdot \vec{A}_{phys} = \vec{\nabla} \cdot \vec{A}_{phys} - ig[A^{i}_{pure}, A^{i}_{phys}] = 0$$

$$\vec{D}_{pure} = \vec{\nabla} - ig\vec{A}_{pure}, \qquad \vec{D}^{adj}_{pure} = \vec{\nabla} - ig[\vec{A}_{pure},]$$

$$[A^{i}_{pure}, A^{i}_{phys}] = \Sigma i c_{abc} A^{ib}_{pure} A^{ic}_{phys} T^{a}$$

Gauge transformation

$$\psi' = U\psi,$$

$$A'_\mu = UA_\mu U^+ - \frac{i}{g} U\partial_\mu U^+,$$

$$A'^\mu_{phys} = UA^\mu_{phys} U^+,$$

$$A'^\mu_{pure} = UA^\mu_{pure} U^+ - \frac{i}{g} U\partial^\mu U^+.$$

Hamiltonian of hydrogen atom

Coulomb gauge:

$$\vec{A}^{c}_{//} = 0, \qquad \vec{A}^{c}_{\perp} \neq 0, \qquad A^{c}_{0} = \varphi^{c} \neq 0.$$

Hamiltonian of a non-relativistic charged particle

$$H_c = \frac{(\vec{p} - q\vec{A}^{c}_{\perp})^2}{2m} + q\varphi^{c}.$$

Gauge transformed one

$$\vec{A}_{//} = \vec{A}^{c}_{//} + \vec{\nabla}\omega(x) = \vec{\nabla}\omega(x), \vec{A}_{\perp} = \vec{A}^{c}_{\perp}, \varphi = \varphi^{c} - \partial_t \omega(x)$$

$$H = \frac{(\vec{p} - q\vec{A})^2}{2m} + q\varphi = \frac{(\vec{p} - q\vec{\nabla}\omega - q\vec{A}^{c}_{\perp})^2}{2m} + q\varphi^{c} - q\partial_t \omega.$$

Follow the same recipe, we introduce a new Hamiltonian,

$$H_{phy} = H + q\partial_t \omega(x) = \frac{(\vec{p} - q\vec{A}_{//} - q\vec{A}_{\perp}^{c})^2}{2m} + q\varphi^c$$

$$\omega = \nabla^{-2}\nabla \cdot \vec{A}$$

which is gauge invariant, i.e.,

$$\left\langle \psi \,|\, H_{phy} \,|\, \psi \right\rangle = \left\langle \psi^c \,|\, H_c \,|\, \psi^c \right\rangle$$

This means the hydrogen energy calculated in Coulomb gauge is gauge invariant and physical.

A check

- We derived the Dirac equation and the Hamiltonian of electron in the presence of a massive proton from a em Lagrangian with electron and proton and found that indeed the time translation operator and the Hamiltonian are different, exactly as we obtained phenomenologically before.

W.M. Sun, X.S. Chen, X.F. Lu and F. Wang, arXiv:1002.3421[hep-ph]

QED

- Different approach will obtain different energy-momentum tensor and four momentum, they are not unique:

Noether theorem

$$\vec{P} = \int d^3x \{\psi^+ \frac{\nabla}{i}\psi + E^i \nabla A^i\}$$

Gravitational theory (weinberg)

$$\vec{P} = \int d^3x \{\psi^+ \frac{\vec{D}}{i}\psi + \vec{E} \times \vec{B}\}$$

- It appears to be perfect and its QCD version has been used in parton distribution analysis of nucleon, but do not satisfy the momentum algebra.
- Usually one supposes these two expressions are equivalent, because the sum of the integral is the same.

We are experienced in quantum mechanics, so we introduce

$$\vec{P} = \int d^3x \{\psi^+ \frac{\vec{D}_{pure}}{i} \psi + E^i \nabla A_\perp^i\}$$

$$\vec{A} = \vec{A}_{//} + \vec{A}_\perp$$

$$\vec{D}_{pure} = \nabla - ieA_{//}$$

They are both gauge invariant and momentum algebra satisfied. They return to the canonical expressions in Coulomb gauge.

The renowned Poynting vector is not the proper momentum of em field

$$\vec{J}_\gamma = \int d^3x \vec{r} \times (\vec{E} \times \vec{B}) = \int d^3x \vec{E} \times \vec{A}_\perp + \int d^3x \vec{r} \times E^i \nabla A^i{}_\perp$$

It includes photon spin and

orbital angular momentum

QCD

There are three different momentum operators as in QED,

$$\vec{p} = \int d^3x \psi^+ \frac{\vec{\nabla}}{i} \psi + \int d^3x E^{ia} \vec{\nabla} A^{ia}$$

$$\vec{p} = \int d^3x \psi^+ \frac{\vec{D}}{i} \psi + \int d^3x \vec{E}^a \times \vec{B}^a$$

$$\vec{p} = \int d^3x \psi^+ \frac{\vec{D}_{pure}}{i} \psi + \int d^3x E^{ia} \vec{D}^{adj}_{pure} A^{ia}_{phys}$$

$$\vec{D}_{pure} = \vec{\nabla} - ig\vec{A}_{pure}, \qquad \vec{D}^{adj}_{pure} = \vec{\nabla} - ig[\vec{A}_{pure},]$$

Angular momentum operators

The decomposition of angular momentum operators have been discussed before, and will not be repeated here.

VI. Summary

- There is no proton spin crisis but quark spin and orbital angular momentum confusion.
- The DIS measured quark spin can be accomodated in CQM.
- One can either attribute the quark contribution to nucleon spin to the quark Pauli spin and the non-relativistic quark orbital angular momentum or to the relativistic quark spin and orbital angular momentum. The following relation is an operator relation,

$$\vec{S}_q + \vec{L}_q = \vec{S}_q^{NR} + \vec{L}_q^{NR}$$

- The real relativistic quark orbital angular momentum will compensate the quark spin missing.

- The gauge potential can be separated into physical and pure gauge parts. The physical part is gauge invariant and measurable.
- The renowned Poynting vector is not the right momentum operator of em field.
- The photon spin and orbital angular momentum can be separated.
- The quark (electron) and gluon (photon) space-time translation and rotation generators are not observable.
- The gauge invariant and canonical quantization rule satisfied momentum, spin and orbital angular momentum can be obtained. They are observable.
- The unphysical pure gauge part has been gauged away in Coulomb gauge. The operators appear there are physical, including the hydrogen atomic Hamiltonian and multipole radiation.

We suggest to use the physical momentum, angular momentum, etc.

in hadron physics as have been used

in atomic, nuclear physics so long a time.

Quite possible, it will modify our picture of nucleon internal structure.

Thanks