

Scale Dependence of Transversity Distributions and Tensor Charges of the Nucleon

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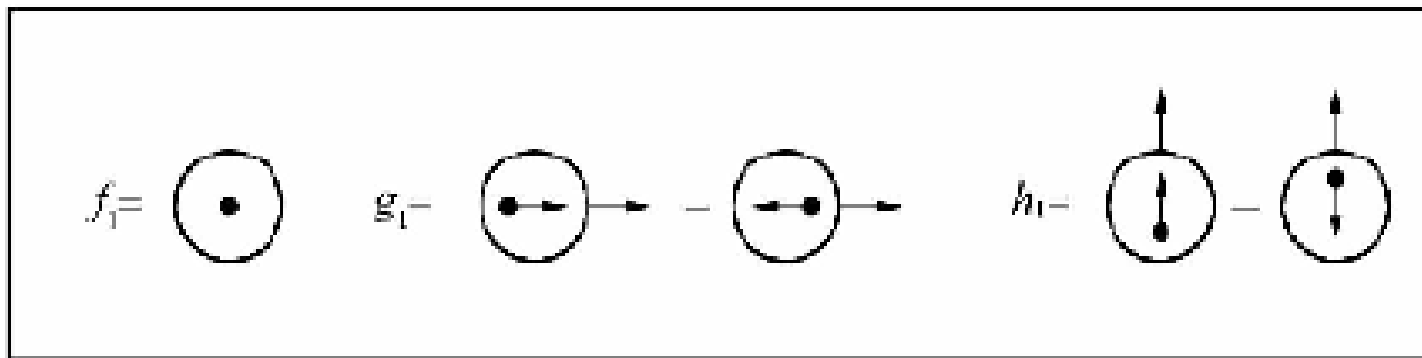
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1. Quark distribution functions and their first moments

夸克分布	扭 度	手征性	螺旋性振幅	测 量
f_1	2	偶	$A_{\frac{1}{2}\frac{1}{2},\frac{1}{2}\frac{1}{2}} + A_{\frac{1}{2}-\frac{1}{2},\frac{1}{2}-\frac{1}{2}}$	自旋平均
g_1	2	偶	$A_{\frac{1}{2}\frac{1}{2},\frac{1}{2}\frac{1}{2}} - A_{\frac{1}{2}-\frac{1}{2},\frac{1}{2}-\frac{1}{2}}$	螺旋性差
h_1	2	奇	$A_{\frac{1}{2}\frac{1}{2},-\frac{1}{2}-\frac{1}{2}}$	螺旋性反转



Longitudinally polarized

Transversely polarized

Quark distribution functions at leading twist

$$g_1(x) = \frac{1}{2} \sum_f e_f^2 [q_f^\uparrow(x) - q_f^\downarrow(x)] = \frac{1}{2} \sum_f e_f^2 \Delta q_f(x)$$

$$F_1(x) = \frac{1}{2} \sum_f e_f^2 [q_f^\uparrow(x) + q_f^\downarrow(x)] = \frac{1}{2} \sum_f e_f^2 q_f(x)$$

$$\Delta q_f(x) = q_f^\uparrow(x) - q_f^\downarrow(x), \quad q_f(x) = q_f^\uparrow(x) + q_f^\downarrow(x).$$

Where $q_f(x)$ ---find a quark parton probability in a nucleon

Quark distribution functions at leading twist

$$h_1(x) = \frac{1}{2} \sum_f e_f^2 [\delta q_f(x) + \delta \bar{q}_f(x)] \quad (4.85)$$

$$\delta q_f(x) = q_f^{\uparrow}(x) - q_f^{\downarrow}(x) \quad (4.86)$$

↑ (↓) 指 f 味夸克的自旋是平行 (反平行) 于核子的横向极化,

The transversity distributions measure the difference of number of quarks(antiquarks) with transverse polarization parallel and antiparallel to the nucleon likewise polarized

Nucleon charges \longleftrightarrow Quark distributions

Baryon charge $B_f \longleftrightarrow$ spin average $g_f^{(s)}$

$$B_f = \int_0^1 dx g_f^{(s)} \quad f=u,d,s.$$

Axial charge $g_A \longleftrightarrow$ quark helicity $\Delta g_f^{(h)}$

$$g_A^{(3)} = \int_0^1 dx (\Delta u(x) - \Delta d(x))$$

$$g_A^{(8)} = \Delta \Sigma = \int_0^1 dx (\Delta u(x) + \Delta d(x) + \Delta s(x))$$

Tensor Charge $g_T \longleftrightarrow$ quark transversity $\delta g_f^{(t)}$

$$g_T^8 = \delta g = \int_0^1 dx (\delta u(x) - \delta d(x))$$

$$\delta \Sigma = \int_0^1 dx (\delta u(x) + \delta d(x) + \delta s(x))$$

Helicity distributions and axial charges

$$\begin{aligned}\int_0^1 dx g_1^p &= \frac{1}{2} \left[\frac{4}{9} \Delta u + \frac{1}{9} \Delta d + \frac{1}{9} \Delta s \right] \\ &= \frac{1}{12} \left[a_3 + \frac{1}{\sqrt{3}} a_8 + \frac{4}{3} a_0 \right]\end{aligned}$$

$$\begin{aligned}\int_0^1 dx g_1^n &= \frac{1}{2} \left[\frac{4}{9} \Delta d + \frac{1}{9} \Delta u + \frac{1}{9} \Delta s \right] \\ &= \frac{1}{12} \left[-a_3 + \frac{1}{\sqrt{3}} a_8 + \frac{4}{3} a_0 \right]\end{aligned}$$

$$a_0 = \Delta u + \Delta d + \Delta s = \Delta \Sigma \quad (\text{Flavor singlet})$$

$$a_3 = \Delta u - \Delta d = F + D = 1.2601$$

$$a_8 = \Delta u + \Delta d - 2\Delta s = 3F - D = 0.588$$

Where flavour singlet axial charge gives the quark spin contribution to nucleon's spin

Transversity distributions and tensor charges

$$h_1(x) = \frac{1}{2} \sum_f e_f^2 [\delta q_f(x) + \delta \bar{q}_f(x)]$$

$$\int_0^1 dx h_1^{\text{P}}(x) = \frac{1}{2} \left[\frac{4}{9} \delta u + \frac{1}{9} \delta d + \frac{1}{9} \delta s \right]$$

$$\int_0^1 dx h_1^{\text{M}}(x) = \frac{1}{2} \left[\frac{4}{9} \delta d + \frac{1}{9} \delta u + \frac{1}{9} \delta s \right]$$

The flavor singlet tensor charge and isovector tensor charge can be defined

2. Axial Charges and Quark Spin Contribution to Nucleon Spin

- Nucleon spin sum rules

$$\frac{1}{2} = \langle P + | J_z | P + \rangle = \frac{1}{2} \Delta \Sigma + \Delta G + L_q + L_g$$

这里

$$\Delta \Sigma = \langle P + | \hat{S}_{3q} | P + \rangle = \langle P + | \int d^3 x \bar{\psi} \gamma^3 \gamma_5 \psi | P + \rangle$$

$$\Delta G = \langle P + | \hat{S}_{3g} | P + \rangle = \langle P + | \int d^3 x (E^1 A^2 - E^2 A^1) | P + \rangle$$

$$L_q = \langle P + | \hat{L}_{3q} | P + \rangle = \langle P + | \int d^3 x i \bar{\psi} \gamma^0 (x^2 \partial^1 - x^1 \partial^2) \psi | P + \rangle$$

$$L_g = \langle P + | \hat{L}_{3g} | P + \rangle = \langle P + | \int d^3 x E^i (x^1 \partial^2 - x^2 \partial^1) A^i | P + \rangle$$

All components are scale-dependent

* Nucleon spin sum rules

--satisfying angular momentum commutation relation

规范场分解为两部分： $\mathbf{A} = \mathbf{A}_{\text{phys}} + \mathbf{A}_{\text{pure}}$ ，相应的 QCD 角动量：

$$\begin{aligned} J_{\text{QCD}} = & \int d^3x \psi^\dagger \frac{1}{2} \boldsymbol{\Sigma} \psi + \int d^3x \psi^\dagger \mathbf{x} \times \frac{1}{i} \mathbf{D}_{\text{pure}} \psi \\ & + \int d^3x \mathbf{E}^a \times \mathbf{A}_{\text{phys}}^a + \int d^3x \mathbf{E}^{ai} \mathbf{x} \times \nabla \mathbf{A}_{\text{phys}}^{ai} \quad (4) \end{aligned}$$

Perturbative Evolutions of QCD angular momentum operators

$$\frac{d}{dt} \begin{pmatrix} \Delta\Sigma \\ \Delta g \end{pmatrix} = \frac{\alpha_s(t)}{2\pi} \begin{pmatrix} \Delta P_{qq}^s & 2n_f \Delta P_{qg} \\ \Delta P_{gq} & \Delta P_{gg} \end{pmatrix} \otimes \begin{pmatrix} \Delta\Sigma \\ \Delta g \end{pmatrix}$$

$t = \ln(Q^2 / \Lambda_{\text{QCD}}^2)$, ΔP_{qq}^s 中的上标 s 指味单态, 劈裂函数 P (NLO) 为

$$\Delta P(x, \alpha_s) = \Delta P^{(0)}(x) + \frac{\alpha_s}{2\pi} \Delta P^{(1)}(x) + \dots$$

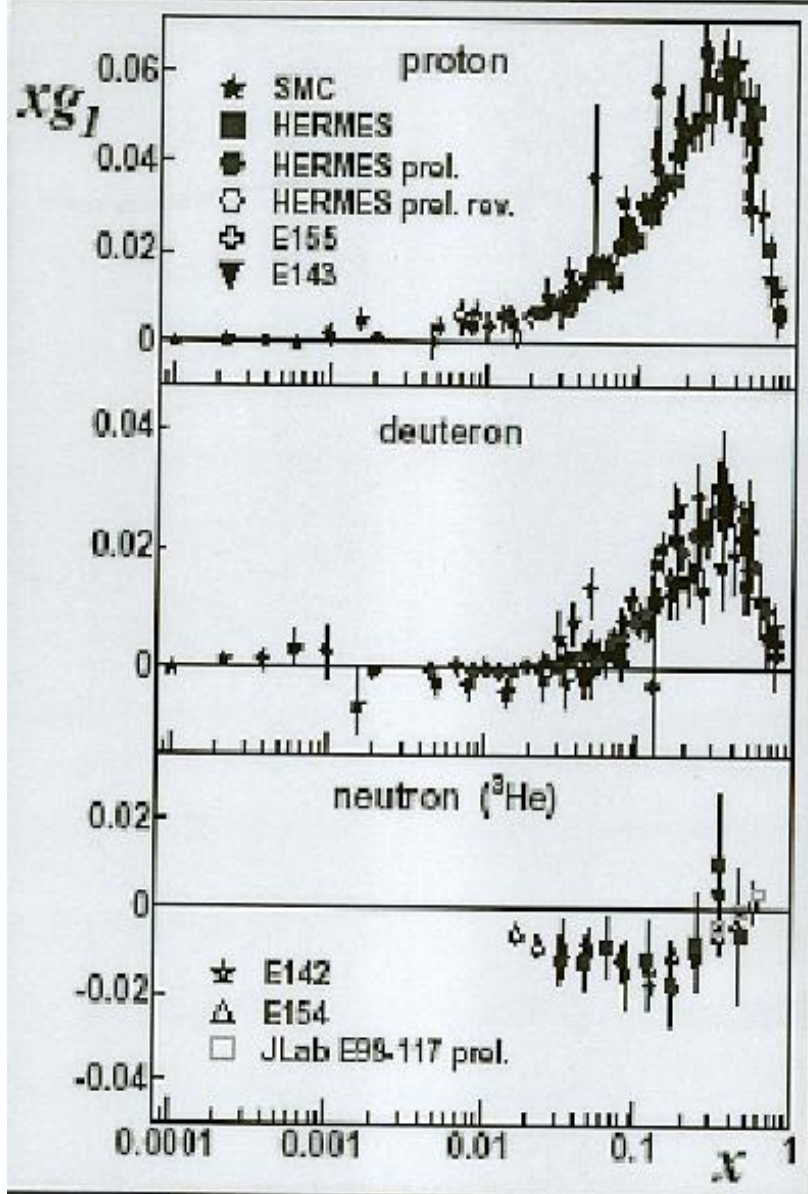
Quark spin contribution to nucleon spin

$$\begin{aligned}\Gamma_1^{\text{p(n)}}(Q^2) &= \int_0^1 g_1^{\text{p(n)}}(x, Q^2) dx \\ &= \left(\pm \frac{1}{12} a_3 + \frac{1}{36} a_8 \right) C_{NS} + \frac{1}{9} a_0 C_S + \delta\Gamma_{\text{1HT}}\end{aligned}$$

$$\begin{aligned}\Gamma_1^{\text{d}}(Q^2) &= \int_0^1 g_1^{\text{d}}(x, Q^2) dx \\ &= \left(1 - \frac{3}{2} \omega_D \right) \left(\frac{1}{36} a_8 C_{NS} + \frac{1}{9} a_0 C_S \right) + \delta\Gamma_{\text{1HT}}\end{aligned}$$

$$a_i \cdot S^\mu = \langle PS | \bar{q} \frac{\lambda_i}{2} \gamma^\mu \gamma_5 | PS \rangle, \quad i = 0, 3, 8$$

World data on $g_1(x, Q^2)$



Virtual Photon Asymmetries:

$$A_1 = \frac{\frac{\sigma_1^- - \sigma_1^+}{2}}{\frac{\sigma_1^- + \sigma_1^+}{2}} \sim \frac{g_1}{F_1}$$

$$\begin{aligned} F_1(x) &= \frac{1}{2} \sum_i e_i^2 (q_i^+(x) + q_i^-(x)) \\ &= \frac{1}{2} \sum_i e_i^2 q_i(x) \end{aligned}$$

$2xF_1$: momentum distribution

$$\begin{aligned} g_1(x) &= \frac{1}{2} \sum_i e_i^2 (q_i^+(x) - q_i^-(x)) \\ &= \frac{1}{2} \sum_i e_i^2 \Delta q_i(x) \end{aligned}$$

g_1 : spin distribution of quarks

Data given at measured $\langle Q^2 \rangle$: 0.02 - 58 GeV²

Quark spin contribution to nucleon spin

实 验	靶	$\langle Q^2 \rangle$	x -范围	$\Gamma_{\text{靶}}$
EMC(87)	p	10.7	0.1–0.7	$0.126 \pm 0.010 \pm 0.015$
SMC(93)	d	4.6	0.006–0.7	$0.023 \pm 0.020 \pm 0.015$
SMC(94)	p	10.0	0.003–0.7	$0.136 \pm 0.011 \pm 0.011$
SMC(95)	d	10.0	0.003–0.7	$0.034 \pm 0.009 \pm 0.006$
E142(93)	n	2.0	0.03–0.6	$-0.022 \pm 0.007 \pm 0.006$
E143(95)	p	3.0	0.03–0.8	$0.127 \pm 0.004 \pm 0.010$
E143(95)	d	3.0	0.03–0.8	$0.042 \pm 0.003 \pm 0.004$
E154(95)	n	5	0.014–0.7	$-0.036 \pm 0.004 \pm 0.005$
SMC(98)	p	5	0.003–0.8	$0.130 \pm 0.003 \pm 0.005$
SMC(98)	d	5	0.003–0.8	$0.036 \pm 0.004 \pm 0.003$
SMC(98)	n	5	0.003–0.8	$-0.054 \pm 0.007 \pm 0.005$
HERMES(06)	p	5	0.021–0.9	$0.121 \pm 0.003 \pm 0.007 \pm \dots$
HERMES(06)	d	5	0.021–0.9	$0.044 \pm 0.001 \pm 0.002 \pm \dots$
HERMES(06)	n	5	0.21–0.9	$-0.027 \pm 0.004 \pm 0.008 \pm \dots$

Quark spin contribution to nucleon spin

最近, HERMES 实验得到的结果为(至 $O(\alpha_s^2)$ 即 *NNLO*)^[78]:

$$\Delta u = 0.842 \pm 0.004 \pm 0.008 \pm \dots$$

$$\Delta d = -0.427 \pm 0.004 \pm 0.008 \pm \dots$$

$$\Delta s = -0.085 \pm 0.013 \pm 0.008 \pm \dots$$

$$\Delta \Sigma = 0.330 \pm 0.011(\text{theo}) \pm 0.025(\text{exp}) \pm 0.028(\text{evol})$$

如果计算至(*N*) *NNLO* 阶(ΔC_{NS} 分析至 $O(\alpha_s^3)$, ΔC_S 至 $O(\alpha_s^2)$), 夸克自旋对核子自旋的贡献为^[78]

$$\Delta \Sigma = 0.333 \pm 0.011(\text{theo}) \pm 0.025(\text{exp}) \pm 0.028(\text{evol})$$

The measurement results are given at $Q^2 = 5 \text{ GeV}^2$

3. Transversity Distributions and Tensor Charges

The first moment of transversity distribution gives the tensor charge

$$\int_0^1 dx [\delta q_f(x) - \delta \bar{q}_f(x)] = \delta q_f$$

$$\langle PS | \bar{q} \sigma^{\mu\nu} q | PS \rangle = \delta q \bar{U}(P, S) \sigma^{\mu\nu} U(P, S)$$

Theory estimates of tensor charges

—model calculation results

途径和模型	理论值 δu	δd
QCD Sum Rules: 三点函数途径 ^[81]	1.0 ± 0.5	0.0 ± 0.5
QCD Sum Rules: 二点函数途径 ^[82]	1.29 ± 0.25	0.02 ± 0.02
Bag Model ^[81]	1.17	-0.29
相对论组分夸克模型 ^[85]	1.17	-0.29
流夸克对张量荷的贡献 ^[85-86]	0.89	-0.22
Melosh 变换途径 ^[87]	1.17	-0.29
Chiral Quark-Soliton Model ^[88]	1.07	-0.38
Lattice QCD ^[84]	$\delta u = 0.839(60)$ $\delta u_{\text{con}} = 0.893(22)$	$\delta d = -0.231(55)$ $\delta d_{\text{con}} = -0.180(10)$
Flavor-Spin Symmetry Estimate ^[89]	$(0.58-1.01) \pm 0.20$	$-(0.11-0.20) \pm 0.20$

Theoretical Estimates of Tensor Charges

- QCD sum rules

$$\left. \begin{aligned} \delta u &= - \frac{4(2\pi)^2 \langle \bar{q}q \rangle}{m_N^3} \left(1 - \frac{9m_0^2}{16m_N^2} \right) \\ \delta d &= \frac{\langle g_c^2 G^2 \rangle}{36m_N^4} \end{aligned} \right\}$$

,则上式给出 $\delta u = 1.29, \delta d = 0.02,$

- Lattice QCD

δu	0.839(60)
δu_{em}	0.893(22)
δd	-0.231(55)
δd_{em}	-0.180(10)

Tensor Charges in Effective Theory

$$\delta M_q = \frac{4}{3} \left(\frac{2}{3} + \left\langle \frac{m}{3E} \right\rangle \right), \quad \delta d_q = -\frac{1}{3} \left(\frac{2}{3} + \left\langle \frac{m}{3E} \right\rangle \right)$$

$$\delta M_q = 1.17, \quad \delta d_q = -0.29$$

价流夸克对轴荷和张量荷的贡献

$$\left. \begin{aligned} \Delta u_q^{(v)} &= \frac{4}{9}, & \Delta d_q^{(v)} &= -\frac{1}{9} \\ \delta u_q^{(v)} &= \frac{8}{9}, & \delta d_q^{(v)} &= -\frac{2}{9} \end{aligned} \right\}$$

Scale dependence of tensor Charge:

- QCD perturbative evolutions

$$\frac{d}{dt} \delta q(x, t) = \frac{\alpha_s(t)}{2\pi} \int_x^1 \frac{dz}{z} P_h(z) \delta q(x/z, t)$$

Note that gluons do not enter the evolution equation for transversity distributions due to the chiral-odd property.

The evolution equation of tensor charge at leading order

$$\delta q(Q^2) = \left[\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \right]^{-4/27} \delta q(Q_0^2)$$

* Perturbative evolutions of tensor charges to next leading order(NLO)

The next leading order Q^2 evolution of tensor charge is given by

$$\delta q(Q^2) = \delta q(\mu^2) \left[\frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)} \right]^{4/27} \left[\frac{\beta_0 + \beta_1 \alpha_s(Q^2)/4\pi}{\beta_0 + \beta_1 \alpha_s(\mu^2)/4\pi} \right]^{\gamma^{(1)}/\beta_1 - \gamma^{(0)}/\beta_0},$$

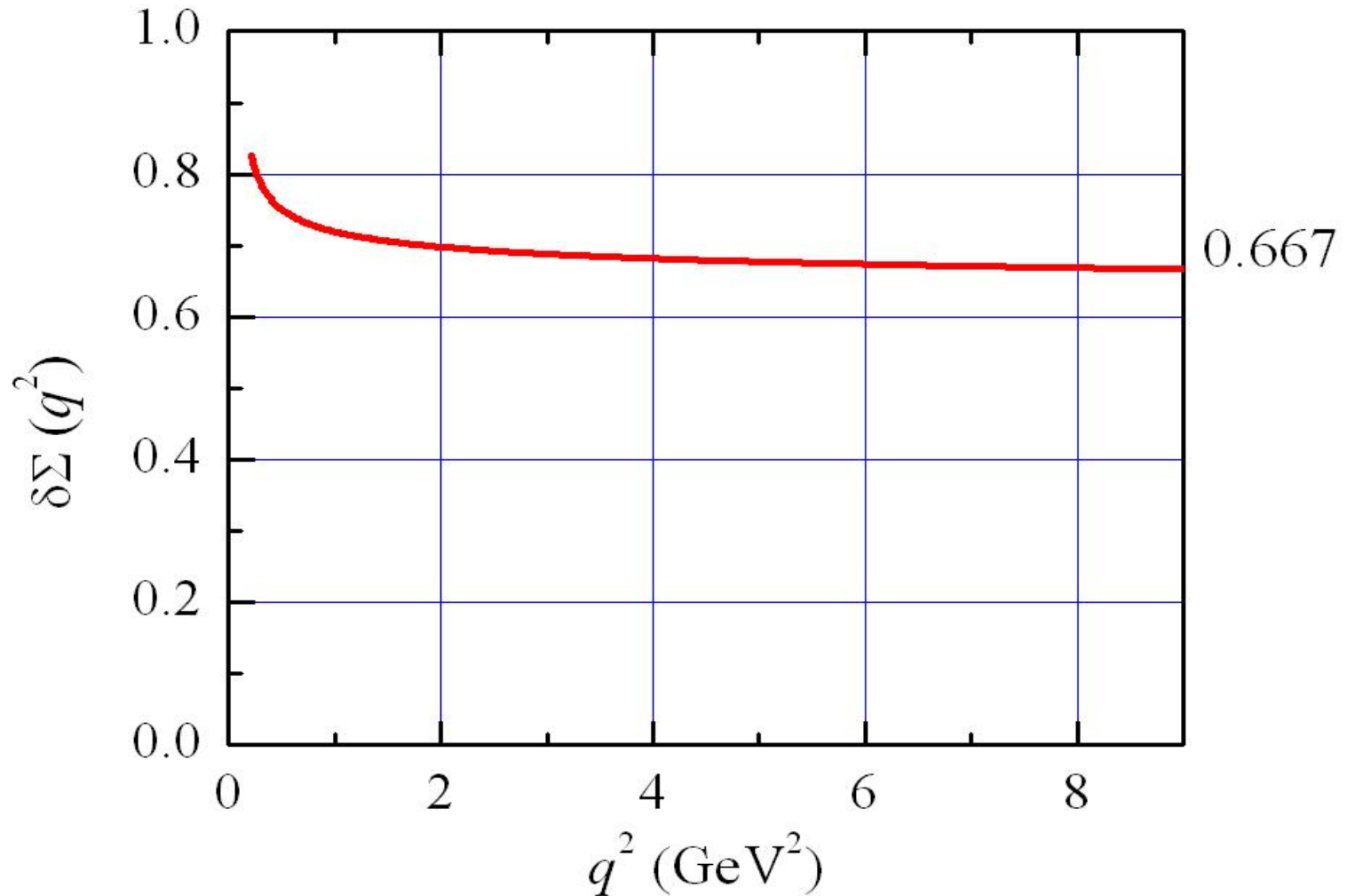
$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda^2)} \left[1 - \frac{\beta_1 \ln \ln(Q^2/\Lambda^2)}{\beta_0^2 \ln(Q^2/\Lambda^2)} \right],$$

where

$$\beta_0 = 11 - 2n_f/3 = 9, \quad \beta_1 = 102 - 38n_f/3 = 64,$$

and $\Lambda = 0.248 \text{ GeV}$ for $n_f = 3$ case.

Perturbative evolution (to NLO) of flavor singlet tensor charge



* Effective theory analyses

QCD at low-energy is equivalent to an effective quark theory

用泛函积分技术可以在形式上完成对胶子场的积分, 导致一有效 QCD 拉氏量 $\mathcal{L}_{\text{eff}}^{\text{QCD}}$. 利用相平行的手续, 我们可以导出有效拉氏量 $\mathcal{L}_{\text{eff}}^{\text{QCD}}$ 的角动量密度 $M_{\text{eff}}^{\mu\nu}$, 由此可定义有效 QCD 理论中的角动量算符^[328]

$$\left. \begin{aligned}
 J_{\text{eff}} &= J_{\text{quark}} + J_{\text{q}\bar{\text{q}}} \\
 J_{\text{q}\bar{\text{q}}} &= \int d^3x \frac{i}{4} [x^j \bar{\psi} (\sigma^{0k} \hat{M} - \hat{M} \sigma^{0k}) \psi - (j \leftrightarrow k)] \\
 \hat{M} &= \sum_{n=2} \int d^4x_2 \cdots d^4x_n \frac{(ig)^n}{n!} G_{\mu_1 \cdots \mu_n}^{a_1 \cdots a_n}(x_1, \cdots, x_n) \\
 &\quad \times \left[\frac{\lambda_{a_1}}{2} \gamma^{\mu_1} \right] \bar{\psi}(x_2) \frac{\lambda_{a_2}}{2} \gamma^{\mu_2} \psi(x_2) \cdots \bar{\psi}(x_n) \frac{\lambda_{a_n}}{2} \gamma^{\mu_n} \psi(x_n)
 \end{aligned} \right\} (10.115)$$

Axial charges and tensor charges in the effective theory

$$\langle PS | \bar{q} \gamma^\mu \gamma^5 q |_{\mu_0^2} | PS \rangle = 2\Delta q(\mu_0^2) S^\mu$$

$$\langle PS | \bar{q} i\sigma^{\mu\nu} \gamma^5 q |_{\mu_0^2} | PS \rangle = 2\delta q(\mu_0^2) (S^\mu P^\nu - S^\nu P^\mu)$$

where

$$\left. \begin{aligned} \Delta q &= \Delta q_q - \Delta \bar{q} \\ \delta q &= \delta q_q - \delta \bar{q} \end{aligned} \right\}$$

Axial charges and tensor charges in the effective theory

$$\left. \begin{aligned} \Delta q_q &= \langle M_A \rangle \Delta q_{\text{NR}} \\ \delta q_q &= \langle M_T \rangle \delta q_{\text{NR}} \end{aligned} \right\}$$

$$M_A = \frac{1}{3} + \frac{2m}{3E}, \quad M_T = \frac{2}{3} + \frac{m}{3E}$$

Where **m** is dynamical mass

Dynamically generated quark mass

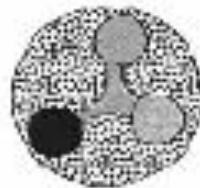
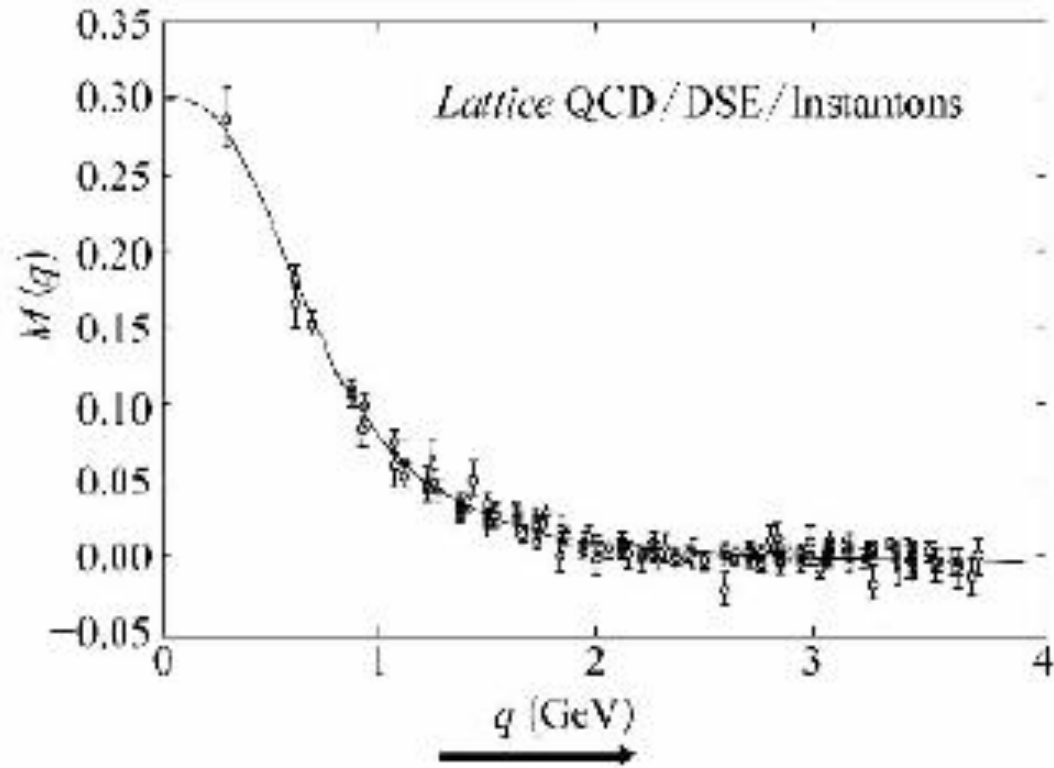
QCD at low-energy scale is spontaneously chiral symmetry breaking, where quarks get dynamical mass

$$\left. \begin{aligned}
 M_{\text{eff}}(p^2) &= \frac{m_f + g_s^2 |\langle \bar{q}q \rangle| (3 + \xi)/9p^2 + g_s^2 \langle GG \rangle m_f p^2 / [12(p^2 - m_f^2)^3]}{1 + g_s^2 |\langle \bar{q}q \rangle| \xi m / 9p^4 + g_s^2 \langle GG \rangle m_f^2 / [12(p^2 - m_f^2)^3]} \\
 N_\psi(p^2) &= \left\{ 1 + g_s^2 (\xi m |\langle \bar{q}q \rangle| / 9p^2 + \langle GG \rangle m_f^2 / [12(p^2 - m_f^2)^3]) \right\}^{-1}
 \end{aligned} \right\} \quad (10.55)$$

这里 $\langle \bar{q}q \rangle$ 和 $\langle GG \rangle$ 分别为夸克和胶子凝聚,参数化 QCD 非微扰效应, ξ 为规范参数, m_f 为 QCD 拉氏量中的流夸克质量, M_{eff} 为有效夸克质量,当 $p^2 \rightarrow \infty$ 时 $M_{\text{eff}} \rightarrow m_f$,这正如所期望的,由上式可得到规范不变的“在壳”质量关系

$$M = \tilde{m}_f + \frac{g_s^2 |\langle \bar{q}q \rangle|}{3M^2} \quad (10.56)$$

Dynamically generated quark mass



Axial charges of the proton and current quark spin contribution to proton spin

$$\Delta u_q = \frac{4}{3} \left(\frac{1}{3} + \left\langle \frac{2m}{3E} \right\rangle \right), \quad \Delta d_q = -\frac{1}{3} \left(\frac{1}{3} + \left\langle \frac{2m}{3E} \right\rangle \right)$$

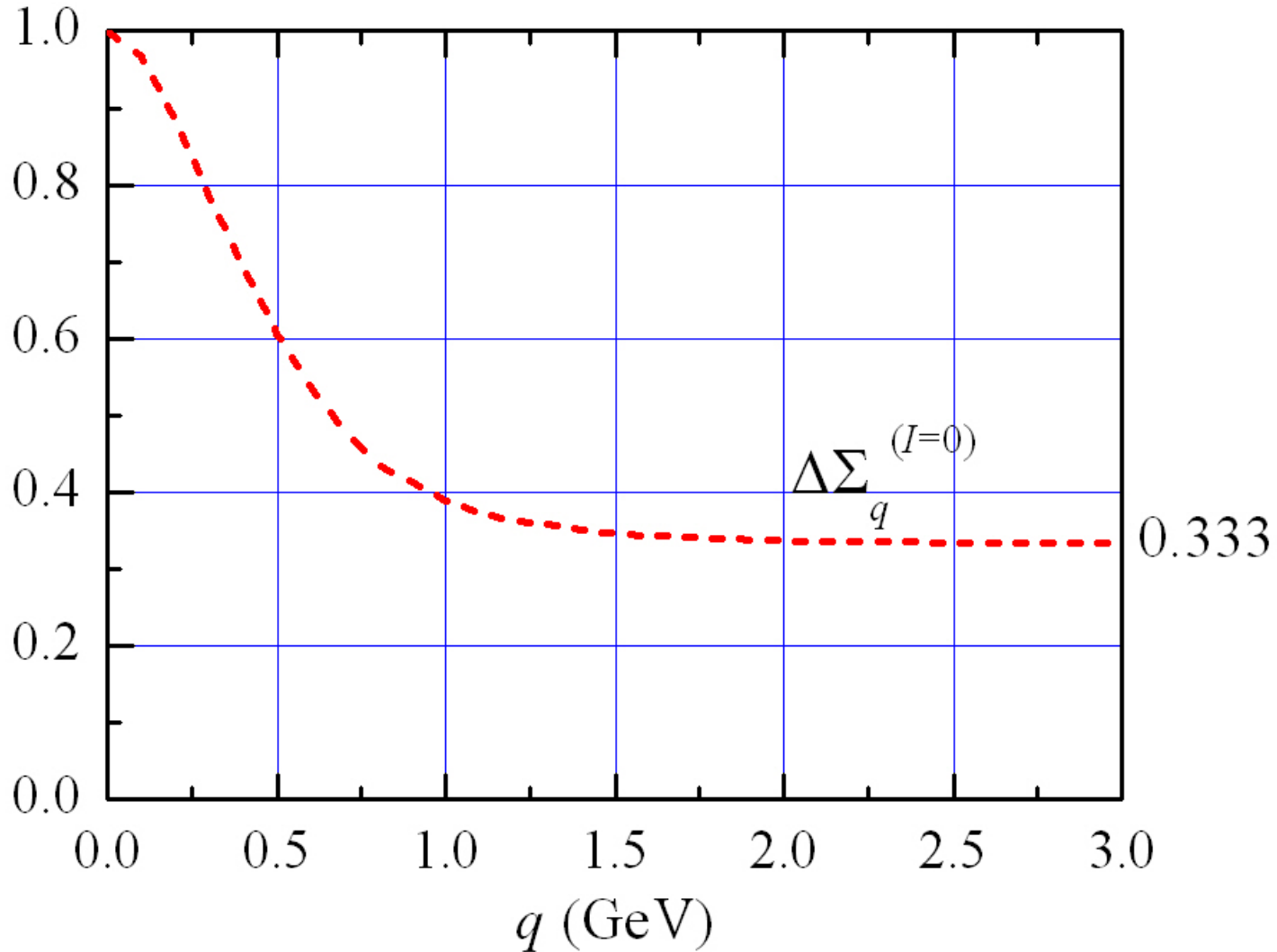
The current quark spin contribution is

$$\Delta \Sigma_v = \Delta u_q^v + \Delta d_q^v = \frac{1}{3} = 0.333'$$

Which is exactly agreement with HERMES's result.

Quark spin contribution to nucleon spin

--- its scale evolution



***Nonperturbative calculations:**
Effective theory analyses of tensor charges

$$\delta u_q = \frac{4}{3} \left(\frac{2}{3} + \left\langle \frac{m}{3E} \right\rangle \right), \quad \delta d_q = -\frac{1}{3} \left(\frac{2}{3} + \left\langle \frac{m}{3E} \right\rangle \right)$$

- Flavor singlet tensor charge of current quark contributions (in the asymptotic limit)

$$\delta \Sigma_v = \delta u_q^v + \delta d_q^v = \frac{2}{3} = 0.667$$

Flavor singlet axial charge

$$\Delta\Sigma_q = \frac{1}{3} + \frac{2}{3} \frac{m}{E},$$

Flavor singlet tensor charge

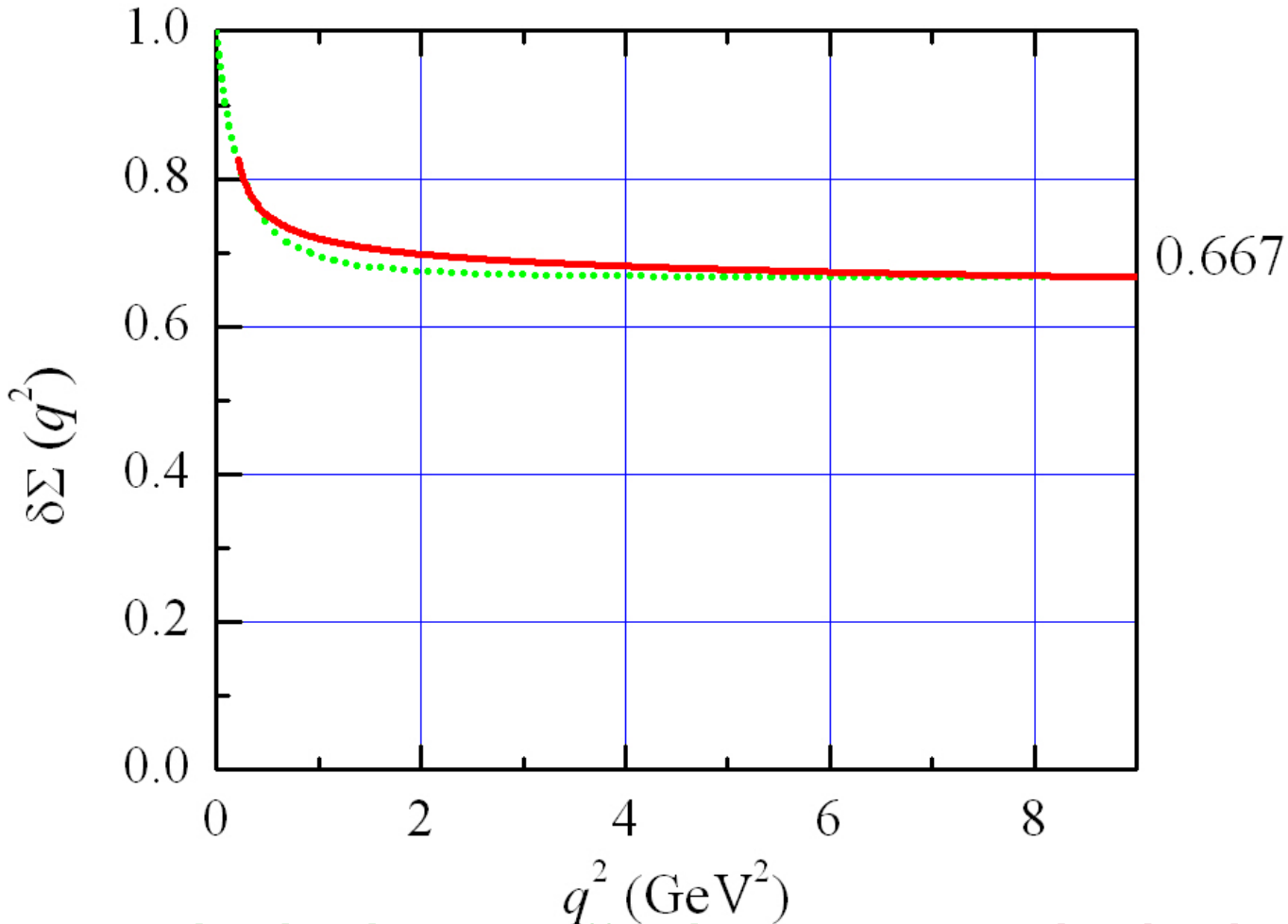
$$\delta\Sigma_q = \frac{2}{3} + \frac{1}{3} \frac{m}{E},$$

Isovector tensor charge

$$\delta\Sigma_q^{(I=1)} = \frac{10}{9} + \frac{5}{9} \frac{m}{E},$$

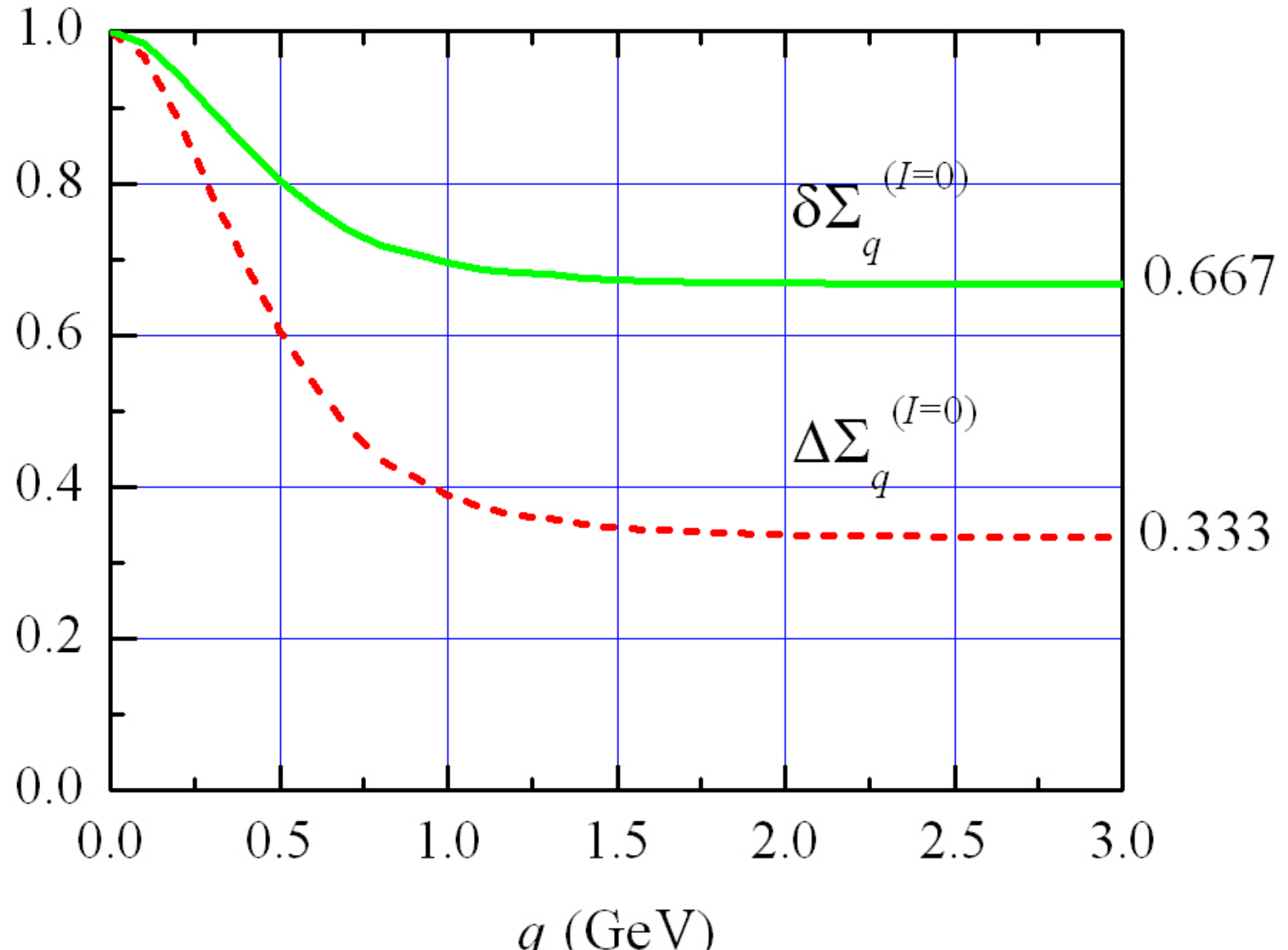
Where **m** is **dynamic mass** of quark in the chiral limit

Scale evolutions of tensor charge



Green line is given by effective theory, red line is given by AP equation of perturbative QCD.

Flavor singlet tensor charge (green line) and axial charge (red line) with scale change



Conclusion(1)

- Tensor charges are strongly scale dependent quantities.
- In the asymptotic limit, tensor charges are

For proton

$$\delta u = \frac{8}{9}, \quad \delta d = -\frac{2}{9}, \quad \delta \Sigma_q = \frac{2}{3} = 0.667,$$

For neutron

$$\delta d = \frac{8}{9}, \quad \delta u = -\frac{2}{9}, \quad \delta \Sigma_q = \frac{2}{3} = 0.667.$$

Conclusion(2)

- Quark spin contribution to the nucleon spin is strongly scale dependent
- Difference between tensor charge and axial charge shows relativistic effects of quark motion in a nucleon

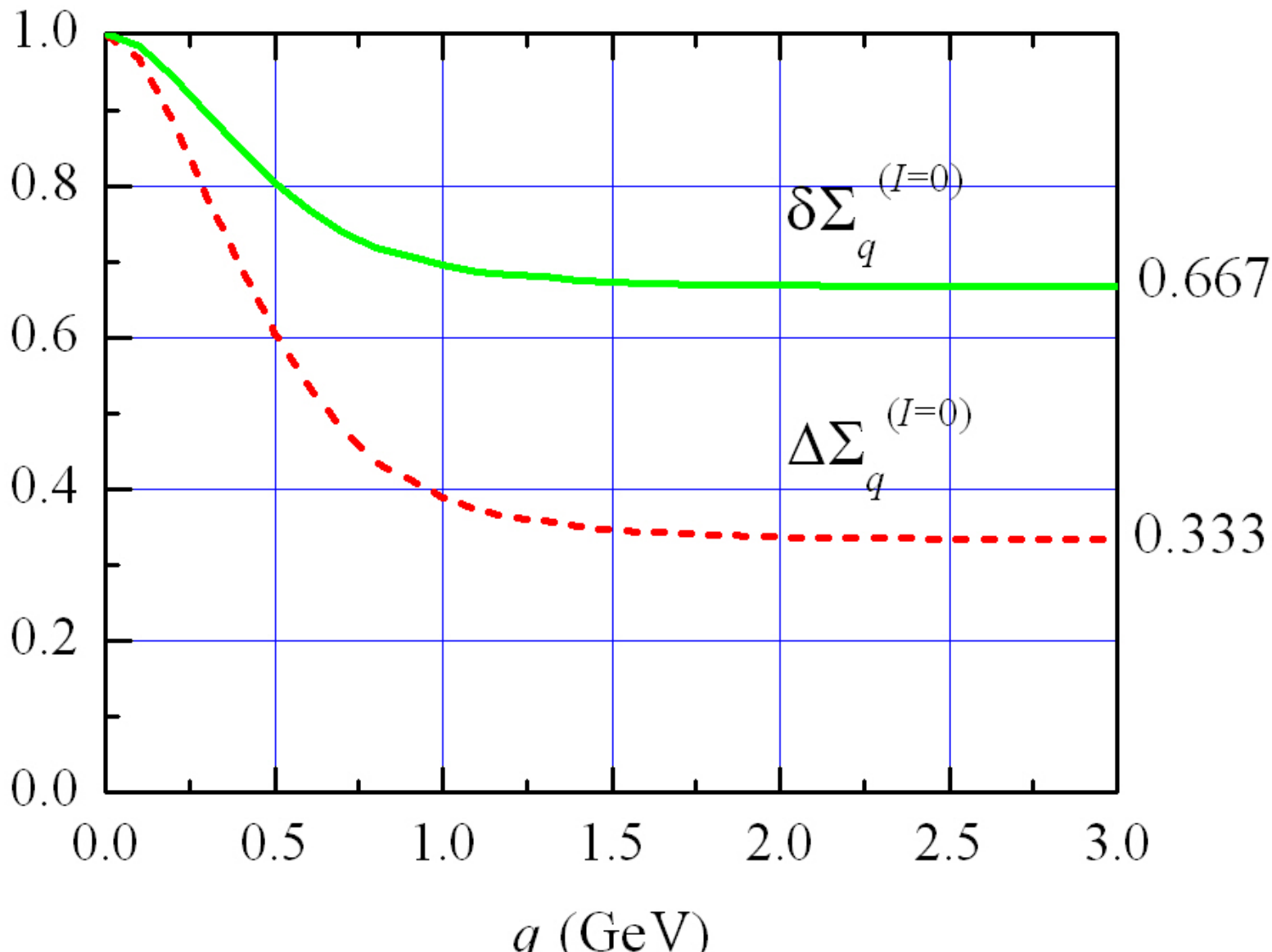
$$\delta\Sigma_q - \Delta\Sigma_q = 0.333, \quad q \geq 5\text{Gev.} \quad \text{In asymptotic limit}$$

$$\delta\Sigma_q - \Delta\Sigma_q = 0.31, \quad q = 1\text{Gev.}$$

$$\delta\Sigma_q - \Delta\Sigma_q = 0.20, \quad q = 0.5\text{Gev.}$$

$$\delta\Sigma_q - \Delta\Sigma_q = 0, \quad q = 0\text{Gev.} \quad \text{Non-relativistic limit}$$

Scale dependence of flavor singlet tensor charge (green line) and axial charge (red line)



Measurements of transversity distributions and tensor charges

- **HERMES, COMPASS and BELLE Collaborations**
based on the combined global analysis of measured **azimuthal asymmetries in semi-inclusive DIS(SIDIS)** and **those in processes**

$$e^+ e^- \longrightarrow h_1 h_2 X$$

The results are(central values)

$$\delta u = 0.59, \quad \delta d = -0.20, \quad \delta \Sigma_q = 0.39, \quad (Q^2 = 0.8 \text{ GeV}^2)$$

(From Phys.Rev.D75,054032(2007; arXiv:0809.3743)

Measurements of transversity distributions and tensor charges

- **Jlab** measured the **single target spin asymmetry(SSA)** in **SIDIS**

$$\begin{aligned} A_{UT} &\equiv \frac{1}{|S_T|} \frac{d\sigma_{UT}}{d\sigma_{UU}} = \frac{1}{|S_T|} \frac{d\sigma(\phi_h, \phi_s) - d\sigma(\phi_h, \phi_s + \pi)}{d\sigma(\phi_h, \phi_s) + d\sigma(\phi_h, \phi_s + \pi)} \\ &= A_{UT}^{\text{Collins}} \sin(\phi_h + \phi_s) + A_{UT}^{\text{Sivers}} \sin(\phi_h - \phi_s) \end{aligned}$$

A_{UT}^{Collins} 由(4.96)式给出, A_{UT}^{Sivers} 的形式为

$$A_{UT}^{\text{Sivers}} = \frac{\sum_q e_q^2 f_{1T}^{\perp(1)q}(x) \cdot D_1^q(z)}{\sum_q e_q^2 q(x) \cdot D_1^q(z)}$$

$f_{1T}^{\perp(1)q}(x)$ 称为 **Sivers 函数**.

“Complete” Measurements

BNL - Star/Phoenix/Phobos

$$\pi^+ \pi^- \text{ Interference Fragmentation: } A_T(p_\perp + p \rightarrow \text{jet}(\pi^+, \pi^-) + X) \Leftrightarrow \underline{\delta q} \cdot \hat{\delta q}_I$$

$$\text{Collins Effect: } A_T(p_\perp + p \rightarrow \text{jet}(h) + X) \Leftrightarrow \underline{\delta q} \cdot C$$

$$\text{Drell Yan: } A_{TT}(p_\perp p_\perp \rightarrow ll) \Leftrightarrow \underline{\delta q} \cdot \delta \bar{q}$$

$$\text{Inclusive hadron: } A_N(p_\perp p \rightarrow h) \Leftrightarrow \underline{\delta q} \cdot C + 2 \text{ other terms}$$

DESY-Hermes & CERN-Compass (BNL eRHIC, DESY Tesla-N)

$$\text{Collins Effect: } A_T(lp_\perp \rightarrow l + \pi + X) \Leftrightarrow \underline{\delta q} \cdot C$$

$$\pi^+ \pi^- \text{ Interference Fragmentation: } A_T(lp_\perp \rightarrow \text{jet}(\pi^+, \pi^-) + X) \Leftrightarrow \underline{\delta q} \cdot \hat{\delta q}_I$$

e+e- collider

$$e^+ e^- \rightarrow \text{dijet} \quad C \cdot C, \delta \hat{q}_I \cdot \delta \hat{q}_I \text{ \& } C \cdot \delta \hat{q}_I$$