Low Mass Meson Spectrum in Covariant Bethe-Salpeter Equation

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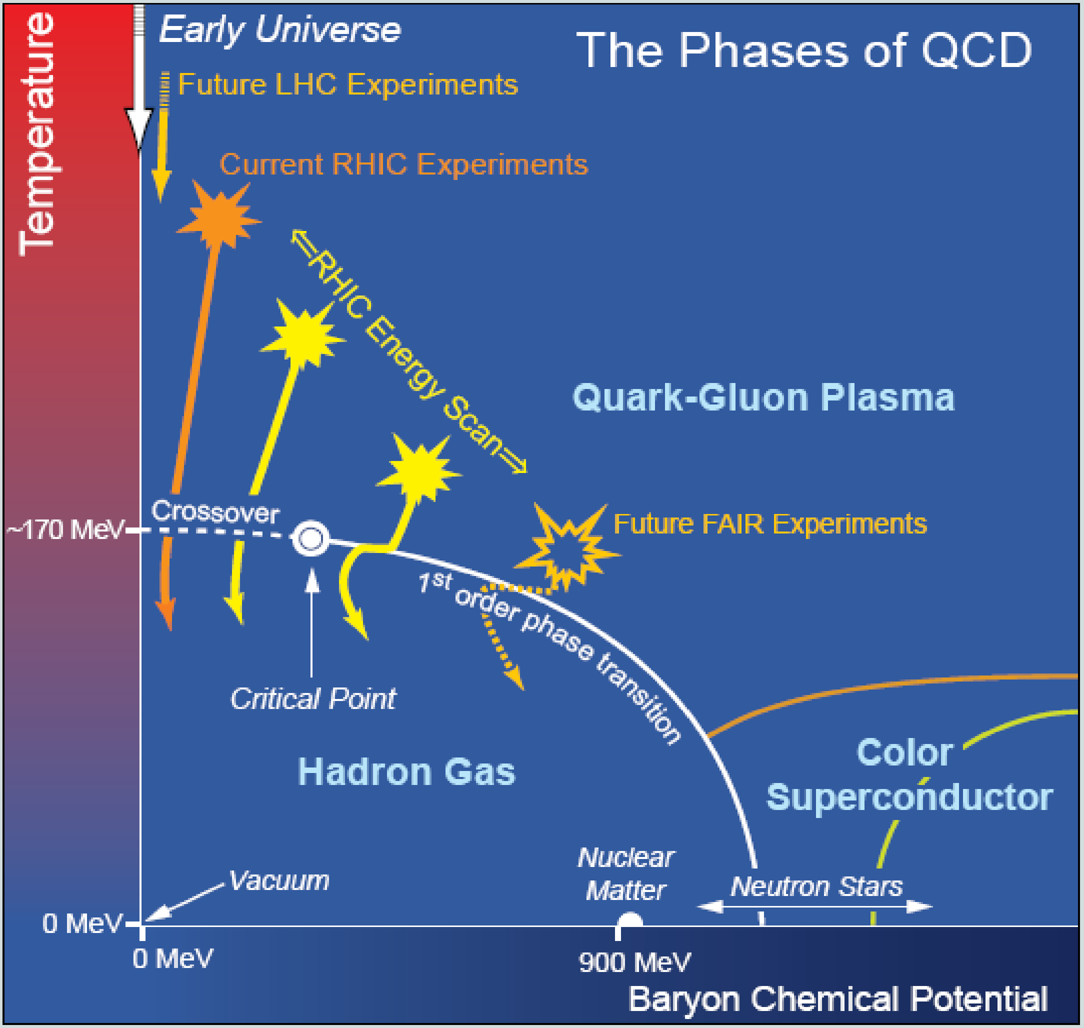
Outline

- Introduction
- Bethe-Salpeter Kernel
- Issue-1: rho-a1 mass splitting problem
- Issue-2: the dressed quark anomalous magnetic moments
- Summary

Introduction

- The Phases of QCD



Schematic QCD phase diagram for nuclear matter. The solid lines show the phase boundaries for the indicated phases. The solid circle depicts the critical point. Possible trajectories for systems created in the QGP phase at different accelerator facilities are also shown.

- Quark and Gluon Confinement

No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon

- Dynamical Chiral symmetry breaking

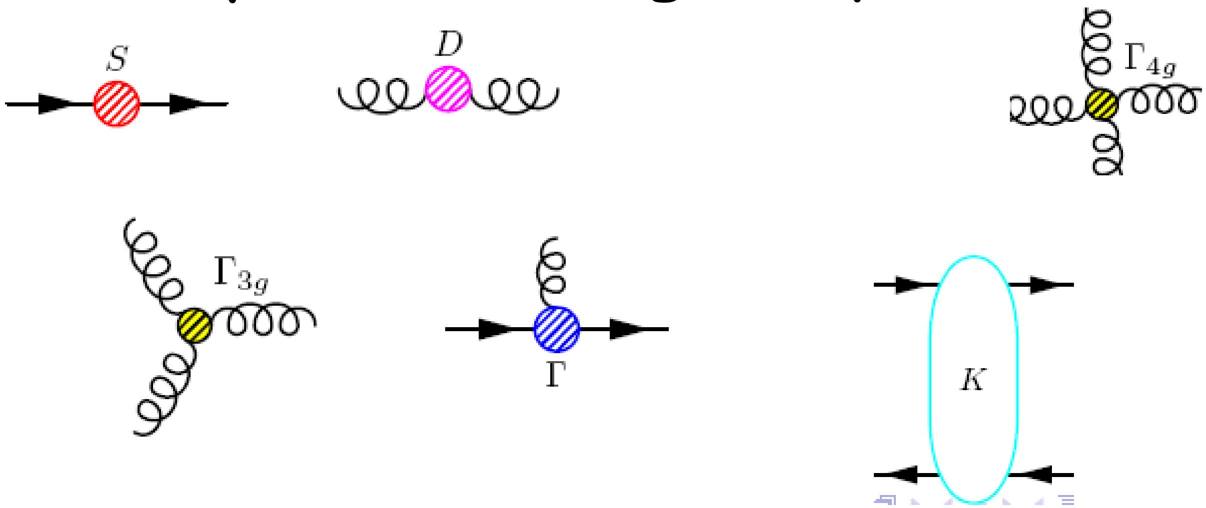
Lagrangian quark mass is small but no degeneracy between J^+ and J^-.

- Neither of these phenomena is apparent in QCD's Lagrangian yet they are the dominant determining characteristics of real-world QCD.

- Nonperturbative calculation of long-range piece of interaction

Introduction

- QCD and Dyson-Schwinger equations



- Each Greeen function satisfies integral equation involving other fuctions
- Infinite set of coupled integral equations
- Truncation scheme necessary

Generating tool for perturbation theory

Nonperturbative truncation scheme (respect symmetries, prove exact results, BS amplitudes->hadron observables)

- **Quark DS equation**

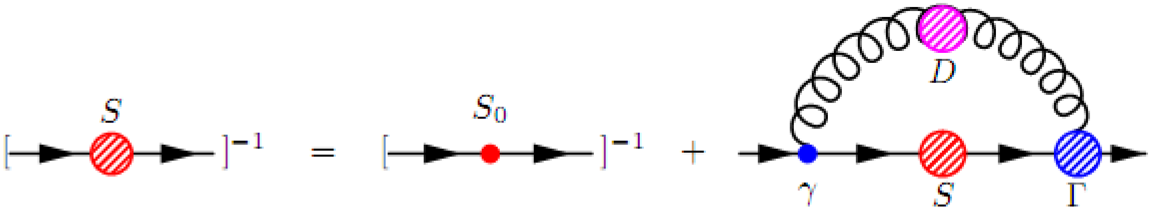
  - **dressed-quark-gluon vertex;**

    truncation (Bare, Ball-Chiu,…)

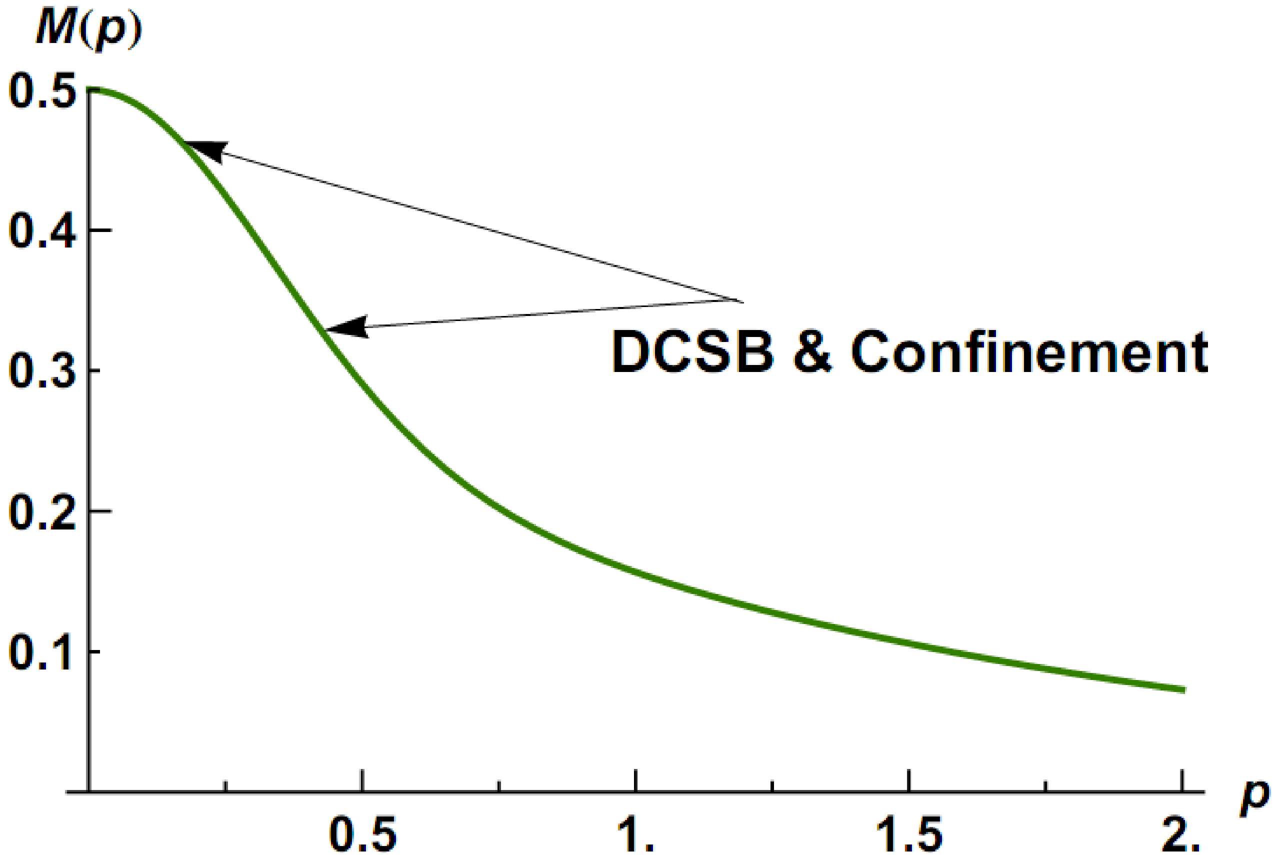
$$S^{-1}(p) = i\gamma \cdot p + m + \frac{4}{3}\int \frac{d^4q}{(2\pi)^4} D_{\mu\nu}(k)\gamma_\mu S(q)\Gamma_\nu^{BC}(q,p)$$

- **dressed-gluon propagator**

  (Maris-Tandy model, Gaussian model, MN model, Separable model,…)



Dressed–quark Mass Function



Dynamical chiral symmetry breaking is evident in the inhancement around p=0.5GeV

Confinement is signalled by the inflexion point at around p=0.25GeV

• Bethe-Salpeter equation

- Without bound states, Comparison with experiment is impossible;
- They appear as pole contributions to n>=3- point color-singlet Schwinger functions

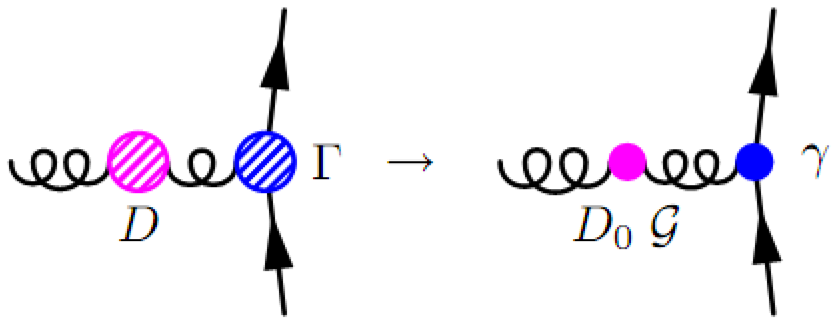
$$[\Gamma^a_{5\mu}(k;P)]_{tu} = [\gamma_5\gamma_\mu\mathcal{F}^a]_{tu} + \int_q [S(q_+)\Gamma^a_{5\mu}(q;P)S(q_-)]_{sr} \boxed{K^{rs}_{tu}(q,k;P)}$$

What is the Kernel?

1. Proof of Exact results in QCD
2. There is at least one systematic nonperturbative, symmetry-preserving truncation scheme
   Rainbow-Ladder truncation + effective gluon propagator
   Diagram expansion + MN model

Rainbow-Ladder truncation

$$Z_1\ g^2\ D_{\mu\nu}(p-q)\Gamma^a_\nu(q,p) \rightarrow \mathcal{G}((p-q)^2)\,D^{\text{free}}_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\nu$$



- Model of gluon propagator
- Bare quark-gluon vertex
- Problem of RL truncation

  too low mass splitting between vector and axial-vector mesons

  more attraction in the prediction of first radial excitation

  inconsistent truncation?

Bethe-Salpeter Kernel

- **Axial-vector Ward-Takahashi identity**

$$P_\mu \Gamma_{5\mu}^{us}(k;P) + i(m_u + m_s)\Gamma_5^{us}(k;P) = S_u^{-1}(k_+) i\gamma_5 + i\gamma_5 S_s^{-1}(k_-)$$

Satisfies BSE   Satisfies DSE

- Relation must be preserved by truncation
- Failure ⇨ Explicit violation of QCD's chiral symmetry

- **Vector Ward-Takahashi identity**

$$P_\mu i\Gamma_\mu^{us}(k;P) + (m_f - m_g)\Gamma_I^{us}(k;P) = S_u^{-1}(k_+) - S_s^{-1}(k_-)$$

Bethe-Salpeter equation

the general form

$$\Gamma^{us}_{5\mu}(k;P) = \gamma_5\gamma_\mu \quad - \quad \frac{4}{3}\int\frac{d^4q}{(2\pi)^4}D_{\alpha\beta}(k-q)\gamma_\alpha S_u(q_+)\Gamma^{us}_{5\mu}(q;P)S_s(q_-)\underline{\Gamma^s_\beta(q_-,k_-)}$$
$$+ \quad \frac{4}{3}\int\frac{d^4q}{(2\pi)^4}D_{\alpha\beta}(k-q)\gamma_\alpha S(q_+)\underline{\Lambda^{us}_{5\mu\beta}(k,q;P)}$$

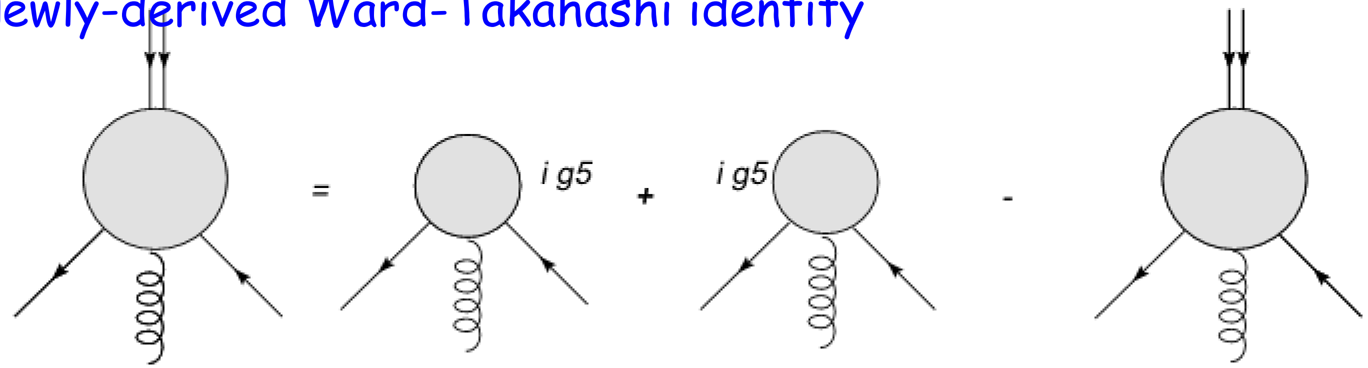
$$\Gamma^{us}_{5}(k;P) = \gamma_5 \quad - \quad \frac{4}{3}\int\frac{d^4q}{(2\pi)^4}D_{\alpha\beta}(k-q)\gamma_\alpha S_u(q_+)\Gamma^{us}_{5}(q;P)S_s(q_-)\underline{\Gamma^s_\beta(q_-,k_-)}$$
$$+ \quad \frac{4}{3}\int\frac{d^4q}{(2\pi)^4}D_{\alpha\beta}(k-q)\gamma_\alpha S(q_+)\underline{\Lambda^{us}_{5\beta}(k,q;P)}$$

Is completely defined via the dressed-quark self-energy

Bethe-Salpeter Kernel
60 year problem

Lei Chang and C. D. Roberts, Phys. Rev. Lett. 103 (2009) 081601

- Bethe-Salpeter equation introduced in 1951
- Newly-derived Ward-Takahashi identity



$$P_\mu \Lambda^{us}_{5\mu\beta}(k,q;P) + i(m_u+m_s)\Lambda^{us}_{5\beta} = \Gamma^u_\beta(q_+,k_+)i\gamma_5 + i\gamma_5\Gamma^s_\beta(q_-,k_-)$$

Necessary and sufficient to ensure axial-vector identity satisfied.

Kernel's Ward-Takahashi identity provides means by which to construct a symmetry preserving kernel of the Bethe-Salpeter equation that is matched to any reasonable Ansatz for the dressed-quark-gluon vertex

- Began with the quark-gluon vertex, whose diagrammatic content is unknown, but which expressed important additional nonperturbative effects that are difficult to capture in any finite sum of contributions
- The kernel-WTI provide symmetry-preserving closed system whose solution yields predictions for the properties of mesons
- This system and its predictions can smoothly be connected with those obtained, e.g., in a rainbow-ladder or kindred symmetry-preserving truncation of the DSEs
- This system can be used to anticipate, elucidate and understand the impact on hadron properties of the rich nonperturbative structure expected of the fully-dressed quark-gluon vertex in QCD

Model Calculation

first application (pion and sigma)

- gluon propagator

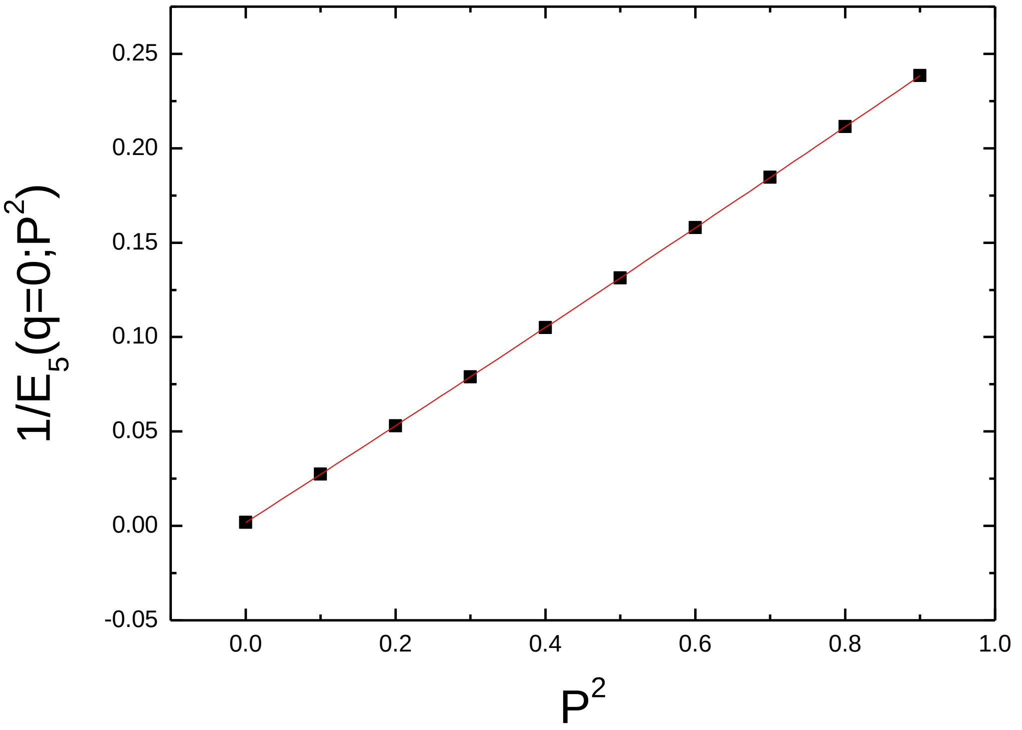
$$\frac{\mathcal{G}(\ell^2)}{\ell^2} = \frac{4\pi^2}{\omega^6} D\, \ell^2\, \mathrm{e}^{-\ell^2/\omega^2}$$

- quark-gluon vertex

$$i\Gamma_\mu(q,k) = i\Sigma_A(q^2,k^2)\,\gamma_\mu + 2\ell_\mu\left[i\gamma\cdot\ell\,\Delta_A(q^2,k^2) + \Delta_B(q^2,k^2)\right], \qquad (12)$$

$$\Gamma_\sigma(q,p) = \gamma_\sigma\,. \qquad (14)$$

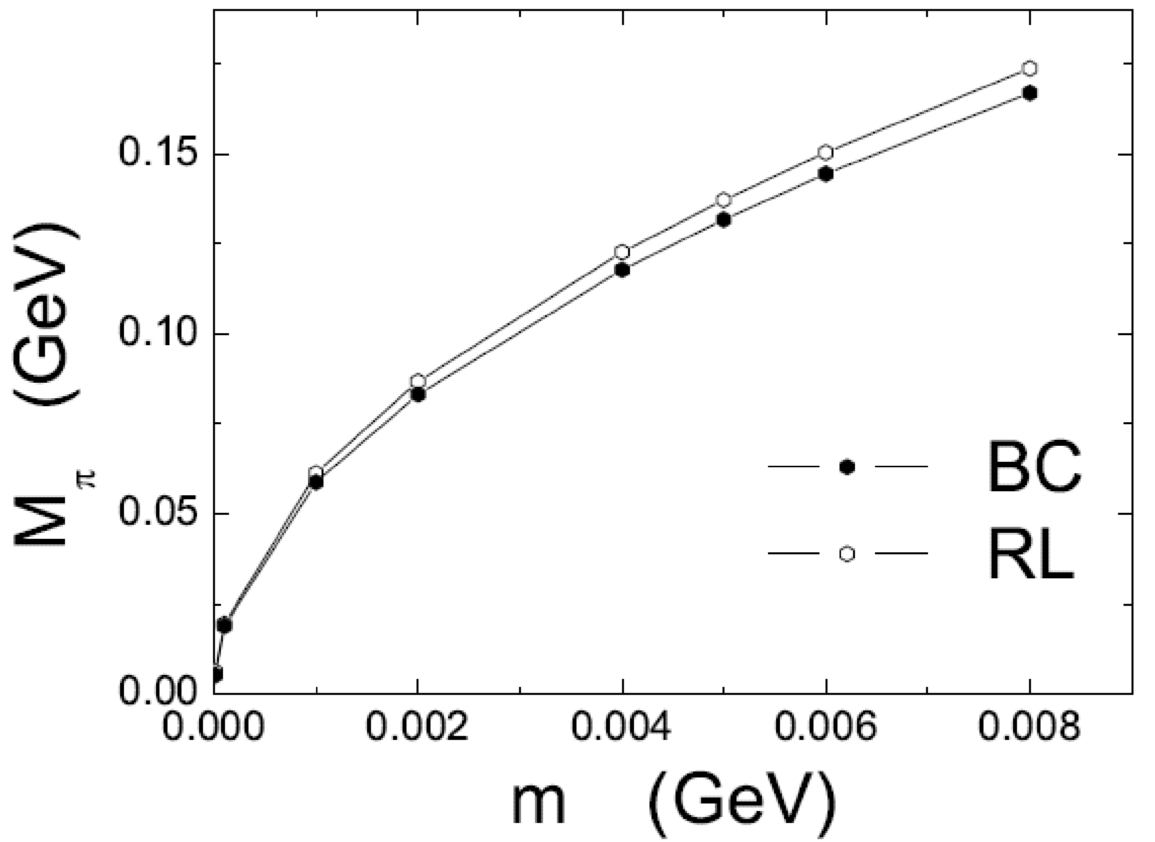
Finding Mass



$$f_N(x) = \frac{c_0 + c_1 x + \ldots + c_N x^N}{1 + c_{N+1} x + \ldots + c_{2N} x^N}, \quad x = P^2$$

f(P^2=-M^2)=0

Finding pion decay constant



$$(f_\pi^0)^2 = \frac{-\langle \bar{q}q \rangle_\zeta^0}{s_\pi^0(\zeta)}\,,\; s_\pi^0(\zeta) = m_\pi \frac{dm_\pi}{dm(\zeta)}\bigg|_{\hat{m}=0}$$

Vertex	D	$A(0)$	$M(0)$	$-(\langle\bar{q}q\rangle^0)^{1/3}$	f_π^0	m_π	m_σ
Eq. (14), RL	$\frac{1}{2}$	0.97	0.049	0.13	0.029	0.16	0.27
Eq. (12), BC		1.1	0.28	0.26	0.11	0.14	0.56
Eq. (14), RL	$\frac{2}{3}$	1.1	0.21	0.21	0.071	0.14	0.44
Eq. (12), BC		1.3	0.44	0.30	0.13	0.14	0.81
Eq. (14), RL	1	1.3	0.40	0.25	0.091	0.14	0.64
Eq. (12), BC		1.8	0.62	0.36	0.16	0.13	1.1

- Added attraction in pseudoscalar channel
- Added repulsion in scalar channel
- Clear sign that the BC-consistent truncation magnifies spin-orbit splitting. Effect owes to influence of quark's dynamically-enhanced scalar self-energy in the Bethe-Salpeter kernel. Impossible to demonstrate effect without our new procedure.

Rho-a1 problem

momentum dependence calculation

	exp.	rainbow-ladder	one-loop	Ball-Chiu consistent	Ball-Chiu plus anom. cm mom.
mass a_1	1230	759	885	1066	1230
mass ρ	775	644	764	924	745
mass-splitting	455	115	121	142	485

Ball-Chiu *Ansatz* for quark-gluon vertex

$$\Gamma_\mu^{\mathrm{BC}}(k,p) = \ldots + (k+p)_\mu \frac{B(k)-B(p)}{k^2-p^2}$$

Ball-Chiu augmented by *quark anomalous chromomagnetic moment* term: $\Gamma_\mu(k,p) = \Gamma_\mu^{\mathrm{BC}} + \sigma_{\mu\nu}(k-p)_\nu \frac{B(k)-B(p)}{k^2-p^2}$

A simple model

contact interaction calculation

H. L. L. Roberts, Lei Chang and C. D. Roberts, arXiv: 1007.4318 [nucl-th]

$$g^2 D_{\mu\nu}(p-q) = \delta_{\mu\nu} \frac{1}{m_G^2}$$

Model spin-orbit repulsion

	m_π	m_ρ	m_σ	m_{a_1}	m_{π^*}	m_{ρ^*}	m_{σ^*}	$m_{a_1^*}$
RL	0.141	0.798	0.825	1.073	1.434	1.434	1.479	1.461
RL + g_{SO}	0.141	0.798	1.079	1.243	1.434	1.434	1.487	1.485
experiment	0.140	0.777	0.4 - 1.2	1.243	1.3 ± 0.1	1.465	0.980	1.426

- Our results indicate that the impact of dress-quark-mass-driven vertex corrections can be large on ground-state masses but is much diminished for meson radial excitations;

- Notice that we calculate the first radial excitation via the method of

  M.K.Volkov and V.L.Yudichev, Phys. Part. Nucl. 31, 282 (2000)

Problem Solved?

- DCSB is the answer. Subtle interplay between competing effects, which can only be explicated
- Promise of first reliable prediction of light-quark meson spectrum, including the so-called hybrid and exotic states.

Quark Anomalous Magnetic Moments

Lei Chang, Yu-xin Liu and C. D. Roberts, in progress

- Massless fermion can not possess an anomalous magnetic moment

  Interaction term

  $$\int d^4x \, \frac{1}{2} g \, \bar{\psi}(x) \sigma_{\mu\nu} \psi(x) F_{\mu\nu}(x)$$

  explicit breaks chiral symmetry

- However, DCSB can generate a large anomalous chromomagnetic moment even in chiral limit (This explains the rho-a1 mass-splitting)

- New BSE formulation (Phys. Rev. Lett. 103 (2009) 081601) enalbes computation of dressed-quark electromagnetic moent given dressed-quark-gluon vertex with ACM-term

In the case of elastic scattering, the quark-photon vertex has the general form

$$\bar{u}(k)\Gamma_\mu^\gamma(k,p)u(p) = \bar{u}(p)\left[\gamma_\mu F_1(q^2) + \sigma_{\mu\nu}(k-p)_\nu F_2(q^2)\right]u(p)$$

The general form of vertex

$$\begin{aligned}\Gamma_\mu^{\mathrm{T}}(k,p;q) \;&=\; \gamma_\mu^{\mathrm{T}}\hat{F}_1 + i\gamma_\mu^{\mathrm{T}}\gamma\cdot q\hat{F}_2 + T_{\mu\rho}\sigma_{\rho\nu}\ell_\nu\,\ell\cdot q\,\hat{F}_3 \\ &\quad +[\ell_\mu^{\mathrm{T}}\gamma\cdot q + i\gamma_\mu^{\mathrm{T}}\sigma_{\nu\rho}\ell_\nu q_\rho]\hat{F}_4 - i\ell_\mu^{\mathrm{T}}\hat{F}_5 \\ &\quad +\ell_\mu^{\mathrm{T}}\gamma\cdot q\,\ell\cdot q\,\hat{F}_6 - \ell_\mu^{\mathrm{T}}\gamma\cdot\ell\,\hat{F}_7 + \ell_\mu^{\mathrm{T}}\sigma_{\nu\rho}\ell_\nu q_\rho\,\hat{F}_8\,.\end{aligned}$$

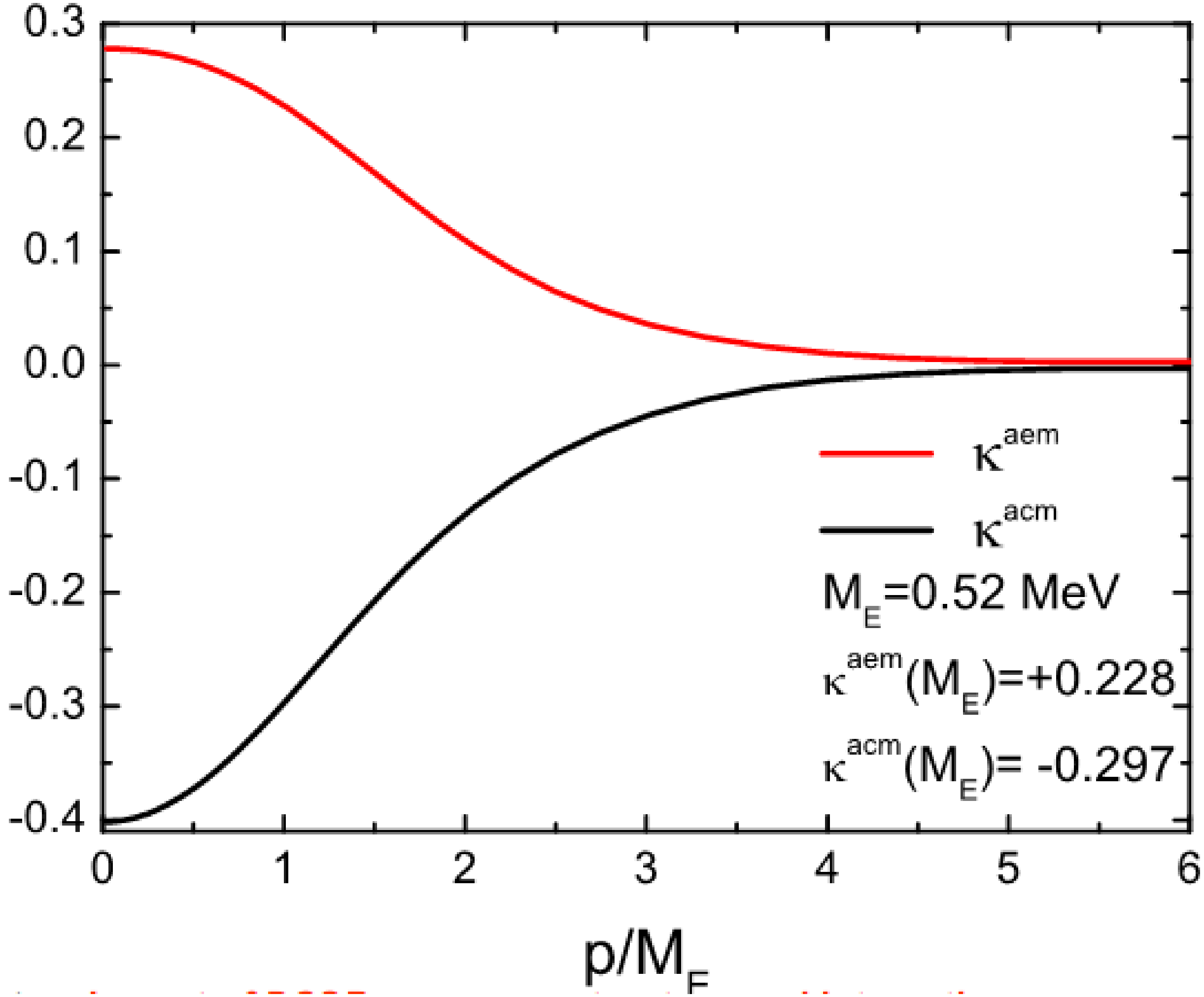
Based on the Gordon identity

$$2M\,\bar{u}(k)i\gamma_\mu u(p) \;=\; \bar{u}(k)\left[(k+p)_\mu + i\sigma_{\mu\nu}(k-p)_\nu\right]u(p)$$

One can obtain

$$\begin{aligned}F_2^\gamma \;&=\; 2M^2\delta_A - 2M\delta_B + 2M\hat{F}_2 + 2M^2\hat{F}_4 - M\hat{F}_5 - M^2\hat{F}_7 \\ F_1^\gamma \;&=\; \sigma_A - 2M^2\delta_A + 2M\delta_B + \hat{F}_1 + M\hat{F}_5 + M^2\hat{F}_7\end{aligned}$$

Mass dependence of Anomalous moments



- Preliminary result for mu distributions
- Effect on hadron form factors?

Summary

- DCSB impacts dramatically upon observables
  mass splitting
  anomalous magnetic moments

- DSEs provide such a framework: Tool enabling insight to be drawn from experiment into long-range piece of interaction between light-quarks

- Elucidate effects of confinement and DCSB in the light-quark meson spectrum, including so-called hybrids and exotics, using symmetry-preserving BS equation

- Elucidate signals of dressed mass in nucleon elastic and transition form factors

Thank you