

Chiral quark model for meson production in the resonance region

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Outline

- The "missing baryon resonances" problem
- Effective chiral Lagrangian for quarkpseudoscalar-meson interaction
- Baryon resonances in pseudoscalar meson photoproduction and meson-nucleon scatterings
 - **Prospects**

1. "Missing baryon resonances in πN scattering

- The non-relativistic constituent quark model (NRCQM) makes great success in the description of hadron spectroscopy: meson (q q), baryon (qqq).
- However, it also predicted a much richer baryon spectrum, where some of those have not been seen in πN scatterings.
 "Missing Resonances".



PDG2008: 22 nucleon resonances (uud, udd)

			Status as seen in —						
Particle	$L_{2I \cdot 2J}$	Overall status	$N\pi$	$N\eta$	ΛK	ΣK	$\Delta \pi$	$N\rho$	$N\gamma$
N(939)	P_{11}	****							
N(1440)	P_{11}	****	****	*			***	*	***
N(1520)	D_{13}	****	****	*			****	****	****
N(1535)	S_{11}	****	****	****			*	**	***
N(1650)	S_{11}	****	****	*	***	**	***	**	***
N(1675)	D_{15}	****	****	*	*		****	*	****
N(1680)	F_{15}	****	****				****	****	****
N(1700)	D_{13}	***	***	*	**	*	**	*	**
N(1710)	P_{11}	***	***	**	**	*	**	*	***
N(1720)	P_{13}	****	****	*	**	*	*	**	**
N(1900)	P_{13}	**	**					*	
N(1990)	F_{17}	**	**	*	*	*			*
N(2000)	F_{15}	**	**	*	*	*	*	**	
N(2080)	D_{13}	**	**	*	*				*
N(2090)	S_{11}	*	*						
N(2100)	P_{11}	*	*	*					
N(2190)	G_{17}	****	****	*	*	*		*	*
N(2200)	D_{15}	**	**	*	*				
N(2220)	H_{19}	****	****	*					
N(2250)	G_{19}	****	****	*					
N(2600)	I_{111}	***	***						
N(2700)	K_{113}	**	**						

(**) not wellestablished

Dilemma:

a) The NRCQM is WRONG:



quark-diquark configuration? ...



b) The NRCQM is CORRECT, but those missing states have only weak couplings to πN , i.e. small $g_{\pi N^*N}$. (Isgur, 1980)

Looking for "missing resonances" in N* $\rightarrow \eta N$, K Σ , K Λ , ρN , ωN , ϕN , γN ...

(Exotics ...)



Questions:

Should we take the naïve quark model seriously?

How far one can go with it?

What is the success and what is the failure?

....

□ The first orbital excitation states in the NRCQM

In the nonstrange sector, NRCQM allows the groundstate [56, 2 8] (*p* and *n*) to be excited to [70, 2 8] and [70, 4 8] octets, and [70, 2 10] decuplet via single photon absorption.



$ 70,\ ^{2}8,\ 1,1,J angle$	• $S_{11}(1535)$ (****), $D_{13}(1520)$ (****)
$ 70,\ ^{4}8,\ 1,1,J angle$	• $S_{11}(1650)$ (****), $D_{13}(1700)$ (***), $D_{15}(1675)$ (****)
$ 70, \ ^{2}10, \ 1, 1, J angle$	• $S_{31}(1620)$ (****), $D_{33}(1670)$ (****)
$ 70, \ ^{2}1, \ 1, 1, J angle$	• $\Lambda(1405) S_{01}$ (****), $\Lambda(1520) D_{03}$ (***)

Confirmed recently by JLab Lattice calculation. (Talk by D. Richards in MENU2010)

The SU(6) \otimes O(3) symmetry must be broken due to spindependent forces. Thus, state mixings are inevitable.

Several NRCQM selection rules are violated:

Moorhouse selection rule (Moorhouse, PRL16, 771 (1966))

$$\begin{split} \gamma + p(|\mathbf{56}, \mathbf{^28}; 0, 0, 1/2\rangle) \not\leftrightarrow & N^*(|\mathbf{70}, \mathbf{^48}\rangle) \\ \gamma + n(|\mathbf{56}, \mathbf{^28}; 0, 0, 1/2\rangle) \leftrightarrow N^*(|\mathbf{70}, \mathbf{^48}\rangle) \end{split}$$

• <u>Λ selection rule</u> (Zhao & Close, PRD74, 094014(2006)) in strong decays

$$N^*(|\mathbf{70}, \mathbf{^48}\rangle) \not\leftrightarrow \quad K(K^*) + \Lambda$$

• Faiman-Hendry selection rule (Faiman & Hendry, PR173, 1720 (1968)).

$$\Lambda^*(|\mathbf{70}, \mathbf{^48}\rangle) \not\leftrightarrow \quad N(|\mathbf{56}, \mathbf{^28}; 0, 0, 1/2\rangle) + \bar{K}$$

2. Effective chiral Lagrangian for quark-pseudoscalarmeson interactions

An effective chiral Lagrangian for quark-pseudoscalar-meson coupling to keep the meson-baryon interaction invariant under the chiral transformation:

$$\mathcal{L} = \overline{\psi} [\gamma_{\mu} (i\partial^{\mu} + V^{\mu} + \gamma_5 A^{\mu}) - m] \psi + \cdots,$$

where the vector and axial currents are

$$V_{\mu} = \frac{1}{2} (\xi^{\dagger} \partial_{\mu} \xi + \xi \partial_{\mu} \xi^{\dagger}),$$

$$A_{\mu} = i \frac{1}{2} (\xi^{\dagger} \partial_{\mu} \xi - \xi \partial_{\mu} \xi^{\dagger}),$$

A. Manohar and H. Georgi, Nucl. Phys. B234, 189 (1984).

and the chiral transformation is,

$$\xi = e^{i\phi_m/f_m},\tag{77}$$

where f_m is the decay constant of the meson. The quark field ψ in the SU(3) symmetry is

$$\psi = \begin{pmatrix} \psi(u) \\ \psi(d) \\ \psi(s) \end{pmatrix}, \tag{78}$$

and the meson field ϕ_m is a 3 \otimes 3 matrix:

$$\phi_m = \begin{pmatrix} \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & K^0 \\ K^- & \overline{K}^0 & -\sqrt{\frac{2}{3}} \eta \end{pmatrix},$$
(79)

where the pseudoscalar mesons π , η and K are treated as Goldstone bosons. Thus, the Lagrangian in Eq. (121) is invariant under the chiral transformation. Expanding the nonlinear field ξ in Eq. (77) in terms of the Goldstone boson field ϕ_m , i.e. $\xi = 1 + i\phi_m/f_m + \cdots$, we obtain the standard quark-meson pseudovector coupling at tree level:

$$H_m = \sum_j \frac{1}{f_m} \overline{\psi}_j \gamma^j_\mu \gamma^j_5 \psi_j \partial^\mu \phi_m , \qquad (80)$$

where ψ_j ($\overline{\psi}_j$) represents the *j*th quark (anti-quark) field in the nucleon.

Test of Goldberger-Treiman relation:

The axial vector coupling, g_A , relates the hadronic operator σ to the quark operator σ_i for the *j*-th quark,

$$\langle N_f | \sum_j \hat{I}_j \sigma_j | N_i \rangle \equiv g_A \langle N_f | \sigma | N_i \rangle.$$

To equate the quark-level coupling to the hadronic level one for the π NN vertex, i.e. axial current conservation, one has



Baryon excitations in π - $p \rightarrow \eta n$

The process $\pi^- p \rightarrow \eta n$ can be expressed in term of the Mandelstam variables:

$$\mathcal{M} = \mathcal{M}_s + \mathcal{M}_u + \mathcal{M}_t.$$

The *s*- and *u*-channel transitions are given by

$$\mathcal{M}_{s} = \sum_{j} \langle N_{f} | H_{\eta} | N_{j} \rangle \langle N_{j} | \frac{1}{E_{i} + \omega_{\pi} - E_{j}} H_{\pi} | N_{i} \rangle$$
$$\mathcal{M}_{u} = \sum_{j} \langle N_{f} | H_{\pi} \frac{1}{E_{i} - \omega_{\eta} - E_{j}} | N_{j} \rangle \langle N_{j} | H_{\eta} | N_{i} \rangle.$$

Zhong, Zhao, He, and Saghai, PRC76, 065205 (2007); Zhong and Zhao, Phys. Rev. C 79, 045202 (2009)



for any operator \mathcal{O} , one has

$$(\hat{H} - E_i)\mathcal{O}|N_i\rangle = [\hat{H}, \mathcal{O}]|N_i\rangle$$

Refs.

Zhao, Li, & Bennhold, PLB436, 42(1998); PRC58, 2393(1998); Zhao, Didelez, Guidal, & Saghai, NPA660, 323(1999); Zhao, PRC63, 025203(2001); Zhao, Saghai, Al-Khalili, PLB509, 231(2001); Zhao, Al-Khalili, & Bennhold, PRC64, 052201(R)(2001); PRC65, 032201(R) (2002);





$$\mathcal{L}_{a_0\pi\eta} = g_{a_0\pi\eta} m_\pi \eta \vec{\pi} \, \vec{a}_0$$
$$H_{a_0} = \sum_j g_{a_0qq} m_\pi \bar{\psi}_j \psi_j \vec{a}_0$$

$$\mathcal{M}_{t} = g_{a_{0}\pi\eta}m_{\pi} \langle N_{f} | H_{a_{0}} | N_{i} \rangle \frac{1}{t^{2} - m_{a_{0}}^{2}}$$

S-channel transition amplitude with quark level operators

Non-relativistic expansion:

$$H_{\pi} = \sum_{j} \frac{I_{j}}{g_{A}^{\pi}} \sigma_{j} \cdot \left[\mathbf{A}_{\pi} e^{i\mathbf{k}\cdot\mathbf{r}_{j}} + \frac{\omega_{\pi}}{2m_{q}} \{\mathbf{p}_{j}, e^{i\mathbf{k}\cdot\mathbf{r}_{j}}\} \right],$$
$$H_{\eta} = \sum_{j} \frac{I_{j}}{g_{A}^{\eta}} \sigma_{j} \cdot \left[\mathbf{A}_{\eta} e^{-i\mathbf{q}\cdot\mathbf{r}_{j}} + \frac{\omega_{\eta}}{2m_{q}} \{\mathbf{p}_{j}, e^{-i\mathbf{q}\cdot\mathbf{r}_{j}}\} \right],$$

with

$$\mathbf{A}_{\pi} = -\left(\frac{\omega_{\pi}}{E_i + M_i} + 1\right)\mathbf{k},$$
$$\mathbf{A}_{\eta} = -\left(\frac{\omega_{\eta}}{E_f + M_f} + 1\right)\mathbf{q}.$$

 η, q

N', P_f

with

$$\mathcal{M}_{3}^{s} = \langle N_{f} | \frac{3I_{3}}{g_{A}^{\pi}} \left\{ \sigma_{3} \cdot \mathbf{A}_{\eta} \sigma_{3} \cdot \mathbf{A}_{\pi} \sum_{n=0}^{\infty} \frac{F_{s}(n)}{n!} \mathcal{X}^{n} \right. \\ \left. + \left[-\sigma_{3} \cdot \mathbf{A}_{\eta} \frac{\omega_{\pi}}{3m_{q}} \sigma_{3} \cdot \mathbf{q} - \frac{\omega_{\eta}}{3m_{q}} \sigma_{3} \cdot \mathbf{k} \sigma_{3} \cdot \mathbf{A}_{\pi} \right. \\ \left. + \frac{\omega_{\eta}}{m_{q}} \frac{\omega_{\pi}}{m_{q}} \frac{\alpha^{2}}{3} \right] \sum_{n=1}^{\infty} \frac{F_{s}(n)}{(n-1)!} \mathcal{X}^{n-1} \\ \left. + \frac{\omega_{\eta}}{3m_{q}} \frac{\omega_{\pi}}{3m_{q}} \sigma_{3} \cdot \mathbf{q} \sigma_{3} \cdot \mathbf{k} \sum_{n=2}^{\infty} \frac{F_{s}(n)}{(n-2)!} \mathcal{X}^{n-2} \right\} \left| N_{i} \right\rangle$$

where $\mathcal{X} \equiv \mathbf{k} \cdot \mathbf{q} / 3\alpha^2$.

with

♦ quark level \rightarrow hadron level

$$\mathcal{M}^{s} = \frac{1}{g_{A}^{\pi}} \Biggl\{ \mathbf{A}_{\eta} \cdot \mathbf{A}_{\pi} \sum_{n=0}^{\infty} [g_{s1} + (-2)^{-n} g_{s2}] \frac{F_{s}(n)}{n!} \mathcal{X}^{n} \\ + \left(-\frac{\omega_{\pi}}{3m_{q}} \mathbf{A}_{\eta} \cdot \mathbf{q} - \frac{\omega_{\eta}}{3m_{q}} \mathbf{A}_{\pi} \cdot \mathbf{k} + \frac{\omega_{\eta}}{m_{q}} \frac{\omega_{\pi}}{m_{q}} \frac{\alpha^{2}}{3} \right) \\ \times \sum_{n=1}^{\infty} [g_{s1} + (-2)^{-n} g_{s2}] \frac{F_{s}(n)}{(n-1)!} \mathcal{X}^{n-1} \\ + \frac{\omega_{\eta} \omega_{\pi}}{(3m_{q})^{2}} \mathbf{k} \cdot \mathbf{q} \sum_{n=2}^{\infty} \frac{F_{s}(n)}{(n-2)!} [g_{s1} + (-2)] F_{s}(n) = \frac{M_{n}}{P_{i} \cdot k - nM_{n} \omega_{h}} \\ + i \sigma \cdot (\mathbf{A}_{\eta} \times \mathbf{A}_{\pi}) \sum_{n=0}^{\infty} [g_{v1} + (-2)^{-n} g_{v2}] \frac{F_{s}(n)}{\cdot} \mathcal{X}^{n} \\ + \frac{\omega_{\eta} \omega_{\pi}}{(3m_{q})^{2}} i \sigma \cdot (\mathbf{q} \times \mathbf{k}) \qquad \rightarrow F_{s}(R) = \frac{2M_{R}}{s - M_{R}^{2} + iM_{R} \Gamma_{R}} \\ \times \sum_{n=2}^{\infty} [g_{v1} + (-2)^{-n} g_{v2}] \frac{F_{s}(n)}{(n-2)!} \mathcal{X}^{n-2} \Biggr\} e^{-(\mathbf{k}^{2} + \mathbf{q}^{2})/6\alpha^{2}}$$



- Compared with M^s₃, amplitude M^s₂ is relatively suppressed by a factor of (-1/2)ⁿ for each n.
- Higher excited states are relatively suppressed by $(k \cdot q/3\alpha^2)^n/n!$
- One can identify the quark motion correlations between the initial and final state baryon
- Similar treatment can be done for the u channel

Separate out individual resonances

A. n = 0 shell resonances

For n = 0, only the nucleon pole term contributes to the transition amplitude. Its *s*-channel amplitude is

$$\mathcal{M}_{N}^{s} = \mathcal{O}_{N} \frac{2M_{0}}{s - M_{0}^{2}} e^{-(\mathbf{k}^{2} + \mathbf{q}^{2})/6\alpha^{2}},$$

with

$$\mathcal{O}_N = [g_{s1} + g_{s2}] \mathbf{A}_{\eta} \cdot \mathbf{A}_{\pi} + [g_{v1} + g_{v2}] i\boldsymbol{\sigma} \cdot (\mathbf{A}_{\eta} \times \mathbf{A}_{\pi}),$$

where M_0 is the nucleon mass.

B. n = 1 shell resonances

For n = 1, only *S* and *D* waves contribute in the *s* channel. Note that the spin-independent amplitude for *D* waves is proportional to the Legendre function $P_2^0(\cos \theta)$ and the spin-dependent amplitude for *D* waves is in proportion to $\frac{\partial}{\partial \theta} P_2^0(\cos \theta)$. Moreover, the *S*-wave amplitude is independent of the scattering angle.

$$\mathcal{M}^{s}(S) = \mathcal{O}_{S}F_{s}(R)e^{-(\mathbf{k}^{2}+\mathbf{q}^{2})/6\alpha^{2}},$$
$$\mathcal{M}^{s}(D) = \mathcal{O}_{D}F_{s}(R)e^{-(\mathbf{k}^{2}+\mathbf{q}^{2})/6\alpha^{2}},$$

with

$$\mathcal{O}_{S} = \left(g_{s1} - \frac{1}{2}g_{s2}\right) \left(|\mathbf{A}_{\eta}||\mathbf{A}_{\pi}| \frac{|\mathbf{k}||\mathbf{q}|}{9\alpha^{2}} - \frac{\omega_{\pi}}{3m_{q}}\mathbf{A}_{\eta}' \cdot \mathbf{q}\right)$$
$$-\frac{\omega_{\eta}}{3m_{q}}\mathbf{A}_{\pi} \cdot \mathbf{k} + \frac{\omega_{\eta}}{m_{q}}\frac{\omega_{\pi}}{m_{q}}\frac{\alpha^{2}}{3}\right),$$
$$\mathcal{O}_{D} = \left(g_{s1} - \frac{1}{2}g_{s2}\right) |\mathbf{A}_{\eta}||\mathbf{A}_{\pi}|(3\cos^{2}\theta - 1)\frac{|\mathbf{k}||\mathbf{q}|}{9\alpha^{2}}$$
$$+ \left(g_{v1} - \frac{1}{2}g_{v2}\right)i\boldsymbol{\sigma} \cdot (\mathbf{A}_{\eta} \times \mathbf{A}_{\pi})\frac{\mathbf{k} \cdot \mathbf{q}}{3\alpha^{2}}.$$

$$\mathcal{M}^{s}(S) = [g_{S_{11}(1535)} + g_{S_{11}(1650)}]\mathcal{M}^{s}(S),$$

$$\mathcal{M}^{s}(D) = [g_{D_{13}(1520)} + g_{D_{13}(1700)} + g_{D_{15}(1675)}]\mathcal{M}^{s}(D),$$

Factor	Value	Factor	Value	Factor	Value
<i>8s</i> 1	1	$g_{S_{11}(1535)}$	2	82	5/3
g_{s2}	2/3	$g_{S_{11}(1650)}$	-1	$g_{P_{11}(1710)}$	180/619
g_{v1}	5/3	$g_{D_{13}(1520)}$	2	$g_{P_{13}(1900)}$	18/619
g_{v2}	0	$g_{D_{13}(1700)}$	-1/10	$g_{P_{11}(2100)}$	-16/619
g^{π}_{A}	5/3	$g_{D_{15}(1675)}$	-9/10	$g_{F_{15}(1680)}$	5/3
g^{η}_A	1	$g_{P_{11}(1440)}$	225/619	$g_{F_{15}(2000)}$	-2/21
g 1	1	$g_{P_{13}(1720)}$	180/619	$g_{F_{17}(1990)}$	-4/7

In the SU(6) symmetry limit,

Model parameters

Goldberger-Treiman relation:

$$g_{mNN} = \frac{g_A^m M_N}{f_m}$$

 $g_{mNN} = 0.81$
 $g_{a_0NN} g_{a_0\pi\eta} = 100$
 $m_q = 330 \text{ MeV},$
 $\alpha^2 = 0.16 \text{ GeV}^2.$

TABLE II. Breit-Wigner masses M_R (in MeV) and widths Γ_R (in MeV) for the resonances. n = 1 and n = 2 stand for the degenerate states with quantum number n = 1 and n = 2 in the *u* channel.

Resonance	M_R	Γ_R	Resonance	M_R	Γ_R
<i>S</i> ₁₁ (1535)	1535	150	$P_{11}(1440)$	1440	300
$S_{11}(1650)$	1655	165	$P_{11}(1710)$	1710	100
$D_{13}(1520)$	1520	115	$P_{13}(1720)$	1720	200
$D_{13}(1700)$	1700	115	$P_{13}(1900)$	1900	500
$D_{15}(1675)$	1675	150	$P_{11}(2100)$	2100	150
n = 1	1650	230	$F_{15}(1680)$	1685	130
n = 2	1750	300	$F_{15}(2000)$	2000	200
_	_	_	$F_{17}(1990)$	1990	350

Differential cross sections



Left panel: • Solid: full calculation • Dot-dashed: without nucleon Born term

Right panel:

- Solid: full calculation
- Dotted lines: exclusive S11(1535)
- Dot-dashed: without S11(1650)
- Dashed: without t-channel



Left panel:

- Solid: full calculation
- Dot-dashed: without nucleon
 Born term
- Dashed: without D13(1520)

Right panel:

- Solid: full calculation
- Dotted lines: exclusive S11(1535)
- Dot-dashed: without S11(1650)
- Dashed: without t-channel

Total cross sections

- S11(1535) is dominant near threshold. The exclusive cross section is even larger than the data.
- S11(1650) has a destructive interference with the S11(1535), and appears to be a dip in the total cross section.
- States from n=2 shell account for the second enhancement around 1.7 GeV.

Zhong, Zhao, He, and Saghai, PRC76, 065205 (2007)



□ S-channel resonance excitations in $K^-p \rightarrow \Sigma^0 \pi^0$

$$\mathcal{O}_{S} = [g_{S_{01}(1405)} + g_{S_{01}(1670)}]\mathcal{O}_{S},$$

$$\mathcal{O}_{D} = [g_{D_{03}(1520)} + g_{D_{03}(1690)}]\mathcal{O}_{D},$$

 $\frac{g_{S_{01}(1405)}}{g_{S_{01}(1670)}} = \frac{\langle N_f | I_3^{\pi} \sigma_3 | S_{01}(1405) \rangle \langle S_{01}(1405) | I_3^K \sigma_3 | N_i \rangle}{\langle N_f | I^{\pi} \sigma_3 | S_{01}(1670) \rangle \langle S_{01}(1670) | I_3^K \sigma_3 | N_i \rangle}$

$$|S_{01}(1405)\rangle = \cos(\theta)|\mathbf{70},^2 \mathbf{1}\rangle - \sin(\theta)|\mathbf{70},^2 \mathbf{8}\rangle$$
$$|S_{01}(1670)\rangle = \sin(\theta)|\mathbf{70},^2 \mathbf{1}\rangle + \cos(\theta)|\mathbf{70},^2 \mathbf{8}\rangle$$

$g_{S_{01}(1405)}$	$[3\cos(\theta) - \sin(\theta)][\cos(\theta) + \sin(\theta)]$
$g_{S_{01}(1670)}$	$[3\sin(\theta) + \cos(\theta)][\sin(\theta) - \cos(\theta)]$

 $g_{S_{01}(1405)}/g_{S_{01}(1670)} = -3$ leads to $\theta = 0^{\circ}$, i.e., no configuration mixing between [70,² 1] and [70,² 8].

Zhong and Zhao, PRC79, 045202 (2009)

We thus determine the mixing angle by experimental data which requires $g_{S_{01}(1405)}/g_{S_{01}(1670)} \simeq -9$





Diff. Xsect. for K⁻ $p_{0.8} \rightarrow \Sigma^0 \pi^0$





 \mathcal{M}_2^s is the only s-channel amplitude

U-channel turns to be important



Baryon excitations in meson photoproduction

Quark-photon electromagnetic coupling:

$$H_e = -\sum_j e_j \gamma^j_\mu A^\mu(\mathbf{k}, \mathbf{r})$$

Transition amplitudes in terms of the Mandelstam variables:



The seagull term is composed of two parts,

$$M_{fi}^{sg} = \langle N_f | H_{m,e} | N_i \rangle + i \langle N_f | [g_e, H_m] | N_i \rangle, \tag{83}$$

where $|N_i\rangle$ and $|N_f\rangle$ are the initial and final state nucleon, respectively, and

$$H_{m,e} = \sum_{j} \frac{e_m}{f_m} \phi_m(\mathbf{q}, \mathbf{r}_j) \overline{\psi}_j \gamma_j^j \gamma_j^j \psi_j A^\mu(\mathbf{k}, \mathbf{r}_j)$$
(84)

is the contact term from the pseudovector coupling, and

$$g_e = \sum_j e_j \mathbf{r}_j \cdot \epsilon e^{i\mathbf{k}\cdot\mathbf{r}_j} \tag{85}$$

comes from the transformation of the electromagnetic interaction in the s- and u-channel [9, 6].

The s- and u-channel amplitudes have the following expression:

$$\begin{split} &M_{fi}^{s} + M_{fi}^{u} \\ = & i\omega_{\gamma} \sum_{j} \langle N_{f} | H_{m} | N_{j} \rangle \langle N_{j} | \frac{1}{E_{i} + \omega_{\gamma} - E_{j}} h_{e} | N_{i} \rangle \\ &+ & i\omega_{\gamma} \sum_{j} \langle N_{f} | h_{e} \frac{1}{E_{i} - \omega_{m} - E_{j}} | N_{j} \rangle \langle N_{j} | H_{m} | N_{i} \rangle, \end{split}$$

where

$$h_e = \sum_j e_j \mathbf{r}_j \cdot \boldsymbol{\epsilon} (1 - \boldsymbol{\alpha}_j \cdot \hat{\mathbf{k}}) e^{i\mathbf{k} \cdot \mathbf{r}_j},$$

and $\hat{\mathbf{k}} \equiv \mathbf{k}/\omega_{\gamma}$ is the unit vector in the direction of the photon momentum.

The nonrelativistic expansions of Eqs. (87) and (80) become [6]:

$$h_e = \sum_j \left[e_j \mathbf{r}_j \cdot \boldsymbol{\epsilon} - \frac{e_j}{2m_j} \boldsymbol{\sigma}_j \cdot (\boldsymbol{\epsilon}_\gamma \times \hat{\mathbf{k}}) \right] e^{i\mathbf{k} \cdot \mathbf{r}_j},\tag{88}$$

and

$$H_m^{nr} = \sum_j \left[\frac{\omega_m}{E_f + M_f} \boldsymbol{\sigma}_j \cdot \mathbf{P}_f + \frac{\omega_m}{E_i + M_i} \boldsymbol{\sigma}_j \cdot \mathbf{P}_i - \boldsymbol{\sigma}_j \cdot \mathbf{q} + \frac{\omega_m}{2\mu_q} \boldsymbol{\sigma}_j \cdot \mathbf{p}_j \right] \frac{\hat{I}_j}{g_A} e^{-i\mathbf{q}\cdot\mathbf{r}_j},$$
(89)

where $M_i(M_f)$, $E_i(E_f)$ and $\mathbf{P}_i(\mathbf{P}_f)$ are mass, energy and three-vector momentum for the initial (final) nucleon; \mathbf{r}_j and \mathbf{p}_j are the internal coordinate and momentum for the *j*th quark in the nucleon rest system.

Transition amplitudes in the harmonic oscillator basis

$$M_{fi}^{sg} = -e^{-(\mathbf{k}-\mathbf{q})^2/6\alpha^2} e_m \left[1 + \frac{\omega_m}{2} \left(\frac{1}{E_i + M_i} + \frac{1}{E_f + M_f} \right) \right] \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}_{\gamma},$$
$$M_{fi}^t = e^{-(\mathbf{k}-\mathbf{q})^2/6\alpha^2} \frac{e_m (M_f + M_i)}{q \cdot k} \left(\frac{\boldsymbol{\sigma} \cdot \mathbf{q}}{E_f + M_f} - \frac{\boldsymbol{\sigma} \cdot \mathbf{k}}{E_i + M_i} \right) \mathbf{q} \cdot \boldsymbol{\epsilon}_{\gamma},$$

$$M_{fi}^s = (M_2^s + M_3^s)e^{-(\mathbf{k}^2 + \mathbf{q}^2)/6\alpha^2},$$

$$M_{fi}^u = (M_2^u + M_3^u)e^{-(\mathbf{k}^2 + \mathbf{q}^2)/6\alpha^2},$$



$$\begin{split} \frac{M_3^s}{g_3^s} &= -\frac{1}{2m_q} \left[ig_v \mathbf{A}_s \cdot (\epsilon_\gamma \times \mathbf{k}) - \boldsymbol{\sigma} \cdot (\mathbf{A}_s \times (\epsilon_\gamma \times \mathbf{k})) \right] \\ &\times \frac{M_n}{n!(P_i \cdot k - nM\omega_h)} \left(\frac{\mathbf{k} \cdot \mathbf{q}}{3\alpha^2} \right)^n \\ &+ \frac{1}{6} \left[\frac{\omega_\gamma \omega_m}{\mu_q} (1 + \frac{\omega_\gamma}{2m_q}) \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}_\gamma + \frac{2\omega_\gamma}{\alpha^2} \boldsymbol{\sigma} \cdot \mathbf{A}_s \boldsymbol{\epsilon}_\gamma \cdot \mathbf{q} \right] \\ &\times \frac{M_n}{(n-1)!(P_i \cdot k - nM\omega_h)} \left(\frac{\mathbf{k} \cdot \mathbf{q}}{3\alpha^2} \right)^{n-1} \\ &+ \frac{\omega_\gamma \omega_m}{18\mu_q \alpha^2} \boldsymbol{\sigma} \cdot \mathbf{k} \boldsymbol{\epsilon}_\gamma \cdot \mathbf{q} \frac{M_n}{(n-2)!(P_i \cdot k - nM\omega_h)} \left(\frac{\mathbf{k} \cdot \mathbf{q}}{3\alpha^2} \right)^{n-2}, \end{split}$$

$$\begin{split} \frac{M_2^s(-2)^n}{g_2^s} &= -\frac{1}{2m_q} \left[ig_v' \mathbf{A}_s \cdot (\boldsymbol{\epsilon}_{\gamma} \times \mathbf{k}) - g_a' \boldsymbol{\sigma} \cdot (\mathbf{A}_s \times (\boldsymbol{\epsilon}_{\gamma} \times \mathbf{k})) \right] \\ &\times \frac{M_n}{n! (P_i \cdot k - nM\omega_h)} \left(\frac{\mathbf{k} \cdot \mathbf{q}}{3\alpha^2} \right)^n \\ &+ \frac{1}{6} \left[\frac{\omega_{\gamma} \omega_m}{\mu_q} (1 + g_a' \frac{\omega_{\gamma}}{2m_q}) \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}_{\gamma} + \frac{2\omega_{\gamma}}{\alpha^2} \boldsymbol{\sigma} \cdot \mathbf{A}_s \boldsymbol{\epsilon}_{\gamma} \cdot \mathbf{q} \right] \\ &\times \frac{M_n}{(n-1)! (P_i \cdot k - nM\omega_h)} \left(\frac{\mathbf{k} \cdot \mathbf{q}}{3\alpha^2} \right)^{n-1} \\ &+ \frac{\omega_{\gamma} \omega_m}{18\mu_q \alpha^2} \boldsymbol{\sigma} \cdot \mathbf{k} \boldsymbol{\epsilon}_{\gamma} \cdot \mathbf{q} \frac{M_n}{(n-2)! (P_i \cdot k - nM\omega_h)} \left(\frac{\mathbf{k} \cdot \mathbf{q}}{3\alpha^2} \right)^{n-2} \end{split}$$

- Compared with M^s₃, amplitude M^s₂ is relatively suppressed by a factor of (-1/2)ⁿ for each n.
- Higher excited states are relatively suppressed by $(k \cdot q/3\alpha^2)^n/n!$.
- One can identify the quark motion correlations between the initial and final state baryon.
- Similar treatment can be done for the u channel.
- In principle, all the s- and u-channel states have been included in the amplitudes, and the quark level operators have been related to the hadronic level ones through g-factors defined as follows.
- Then, one has to separate out the amplitudes for each single resonance (see *Ref. Zhao et al, PRC65, 065204 (2002)*).

$$\begin{split} g_3^u &= \langle N_f | \sum_j e_j \hat{I}_j \sigma_j^z | N_i \rangle / g_A, \\ g_2^u &= \langle N_f | \sum_{i \neq j} e_j \hat{I}_i \sigma_i^z | N_i \rangle / g_A, \\ g_v &= \frac{1}{g_3^u g_A} \langle N_f | \sum_j e_j \hat{I}_j | N_i \rangle, \\ &= \frac{1}{3g_2^u g_A} \langle N_f | \sum_{i \neq j} e_j \hat{I}_i \sigma_i \cdot \sigma_j | N_i \rangle, \end{split}$$

$$g'_a = \frac{1}{2g_2^u g_A} \langle N_f | \sum_{i \neq j} e_j \hat{I}_i (\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j)_z | N_i \rangle.$$

 g'_v

Some numerical results for pion photoproduction

 Δ magnetic dipole moment:

$$M_{1+}^{3/2} = -g_{\pi NN}g_R \frac{1}{2m_q} \left[\frac{\omega_m}{E_f + M_f} + 1 \right]$$

$$\times \frac{2M_{\Delta}}{s - M_{\Delta}^2 + iM_{\Delta}\Gamma_{\Delta}} e^{-(\mathbf{k}^2 + \mathbf{q}^2)/6\alpha^2}$$

$$g_R \equiv g_3^s g_v + g_2^u g_v' - \mu_i$$



Zhao et al, PRC65, 065204 (2002)

Differential cross sections for $\gamma p \rightarrow \pi^+ n$.



Polarized beam asymmetry for $\gamma p \rightarrow \pi^+ n$.



Polarized target asymmetry for $\gamma p \rightarrow \pi^+ n$.



Recoil polarization asymmetry for $\gamma p \rightarrow \pi^+ n$.



Simultaneous account for $\gamma \mathbf{p} \rightarrow \pi^0 \mathbf{p}$ and $\gamma \mathbf{n} \rightarrow \pi^- \mathbf{p}$ reaction and other relevant reactions.

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Zhao et al, PRC65, 065204 (2002)
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Number of states with the principle quantum number $n \le 2$:

$\gamma \mathbf{n} \rightarrow \mathbf{N}^* (\Delta^*) \rightarrow \pi \mathbf{N}$	27 states	Difference due to Moorhouse section rule
$\gamma \mathbf{p} \rightarrow \mathbf{N}^* (\Delta^*) \rightarrow \pi \mathbf{N}$	19 states	
$\gamma \textbf{n} \rightarrow \textbf{N}^* \rightarrow \eta \textbf{N}$	16 states	$P_{11}(1440)$ $D_{13}(1520)$
$\gamma \mathbf{p} \rightarrow \mathbf{N}^* \rightarrow \eta \mathbf{N}$	8 states	$S_{11}(1535)$ $E_{12}(1680)$
$\gamma \mathbf{n} \to \mathbf{N}^* \to \mathbf{K} \Lambda$	8 states	$P_{11}(1710)$
$\gamma \mathbf{p} \to \mathbf{N}^* \to \mathbf{K} \Lambda$	Due to Λ selection	rule $P_{13}(1720)$ $P_{13}(1900)$ $F_{15}(2000)$

 Λ Selection rule: Zhao & Close, PRD74, 094014(2006)

- 1. For the purpose of searching for individual resonance excitations, it is essential to have a quark model guidance for both known and "missing" states. And then allow the data to tell:
 - i) which state is favored;
 - ii) whether a state beyond the conventional quark model is needed;
 - iii) how quark model prescriptions for N*NM form factors complement with isobaric models.

2. Understanding the non-resonance background

A reliable estimate of the non-resonance background, such as the t- and u-channel. Their interferences with the resonances are essentially important.

3. Unitarity constraint

A coherent study of the pseudoscalar photoproduction and mesonbaryon scattering is needed. In particular, a coupled channel study will put a unitary constraint on the theory.

Photoproduction of pseudoscalar mesons (π , η , η' , **K); and** π **N** \rightarrow η **N**; **K**-**p** $\rightarrow \pi\Sigma$, and more are coming out soon...

Q. Z., **PRC 63**, 035205 (2001);

Q. Z., J.S. Al-Khalili, Z.P. Li, and R.L. Workman, PRC 65, 065204 (2002);
Q. Z., B. Saghai and Z.P. Li, JPG 28, 1293 (2002);
X.H. Zhong, Q. Z., J. He, and B. Saghai, PRC 76, 065205 (2007)
X.H. Zhong and Q. Z., arXiv:0811.4212, PRC79, 045202(2009)



A revisit to the S-wave state mixing

The mixing between pure [70, ²8] and [70, ⁴8] states is defined as

$$\begin{pmatrix} S_{11}(1535)\\S_{11}(1650) \end{pmatrix} = \begin{pmatrix} \cos\theta_S & -\sin\theta_S\\\sin\theta_S & \cos\theta_S \end{pmatrix} \begin{pmatrix} |[\mathbf{70}, \ ^2\mathbf{8}, 1, 1, 1/2^-]\rangle\\ |[\mathbf{70}, \ ^4\mathbf{8}, 1, 1, 1/2^-]\rangle \end{pmatrix}$$

Similarly, the *D*-wave mixing can be written as



The physical states should be orthogonal which means:



This expectation can be examined by the K-matrix propagator between [70, ²8] and [70, ⁴8] mixing states:

$$G = \frac{1}{D_a D_b - |D_{ab}|^2} \left(\begin{array}{cc} D_a & D_{ab} \\ D_{ab} & D_b \end{array} \right)$$

 $\begin{cases} D_a = s - m_a^2 + i\sqrt{s} \Gamma^a(s) \\ D_b = s - m_b^2 + i\sqrt{s} \Gamma^b(s) \end{cases} \qquad \begin{cases} \Gamma^a(s) = \Gamma^a_{\pi N} + \Gamma^a_{\eta N} + \dots , \\ \Gamma^b(s) = \Gamma^b_{\pi N} + \Gamma^b_{\eta N} + \dots . \end{cases}$

 $D_{ab} \simeq \frac{\imath}{16\pi} [\rho_{\pi N} g^a_{S_{11}N\pi} g^b_{S_{11}N\pi} + \rho_{\eta N} g^a_{S_{11}N\eta} g^b_{S_{11}N\eta}]$

$$\begin{cases} \mathcal{M}_{S_{11}\to NM} = \frac{1}{f_m} [C_1 \langle \hat{H}_1 \rangle \alpha(q) + C_2 \langle \hat{H}_2 \rangle (\gamma(q) - \sqrt{2}\beta(q))], \\ \\ \mathcal{M}_{D_{13}(D_{15})\to NM} = \frac{1}{f_m} \left[C_1 \langle \hat{H}_1 \rangle \alpha(q) + C_2 \langle \hat{H}_2 \rangle \left(\gamma(q) + \frac{\beta(q)}{\sqrt{2}} \right) \right] \end{cases}$$

with
$$C_1 \equiv -3\left(\frac{\omega_m}{E_f + M_f} + 1\right), \quad C_2 \equiv \frac{3\omega_m}{2\mu_q}$$

$\hat{H}_1(lpha), \hat{H}_2(\gamma - \sqrt{2}\beta)$	$S^+_{11} \to \Lambda K^+$	$S^+_{11} \to p\eta$	$S_{11}^+ \rightarrow n\pi^+$	$S_{11}^+ \rightarrow p \pi^0$	$S^+_{11} \to \Sigma^+ K^0$
$\langle N, J_z = \frac{1}{2} \hat{H}_1 S_{11}^+, J_z = \frac{1}{2} \rangle$	$-\frac{1}{6}$	$-\frac{\cos\theta}{3\sqrt{3}}$	$-\frac{2\sqrt{2}}{9\sqrt{3}}$	$\frac{2}{9\sqrt{3}}$	$-\frac{1}{9\sqrt{6}}$
$\langle N, J_z = \frac{1}{2} \hat{H}_2 S_{11}^+, J_z = \frac{1}{2} \rangle$	$-\frac{1}{6}$	$-\frac{\cos\theta}{3\sqrt{3}}$	$-\frac{2\sqrt{2}}{9\sqrt{3}}$	$\frac{2}{9\sqrt{3}}$	$-\frac{1}{9\sqrt{6}}$

We can then extract the N*NM form factors given by the chiral effective Lagrangian in the NRCQM, e.g.

$$\sum_{spin} |\mathcal{M}_{hadron}|^2 \equiv (E_i + M_i)(E_f + M_f) \sum_{spin} |\mathcal{M}_{quark}|^2,$$

where

$$\begin{split} N^*(S_{11} \to NM) : & \mathcal{M}_{hadron}^{S_{11}} = g_{S_{11}NM} \bar{u}_N u_R, \\ N^*(D_{13} \to NM) : & \mathcal{M}_{hadron}^{D_{13}} = g_{D_{13}NM} \bar{u}_N \gamma_5 \gamma_\mu u_{R\nu} p_M^\mu p_M^\nu, \\ N^*(D_{15} \to NM) : & \mathcal{M}_{hadron}^{D_{15}} = g_{D_{15}NM} \bar{u}_N u_{R\mu\nu} p_M^\mu p_M^\nu, \end{split}$$

$$\begin{cases} \mathcal{M}_{quark}^{S_{11}} = \frac{1}{f_m} [C_1 \alpha(q) + C_2(\gamma(q) - \sqrt{2}\beta(q))] \langle \hat{H} \rangle \\ = \frac{1}{f_m} \frac{i\alpha_h e^{-q^2/6\alpha^2}}{\sqrt{3}} \left[C_1 \frac{q^2}{\alpha_h^2} + C_2 \left(3 + \frac{q^2}{3\alpha_h^2} \right) \right] \langle \hat{H} \rangle, \\ \mathcal{M}_{quark}^{D_{13}/D_{15}} = \frac{1}{f_m} [C_1 \alpha(q) + C_2(\gamma(q) + \frac{\beta(q)}{\sqrt{2}})] \langle \hat{H} \rangle \\ = \frac{1}{f_m} \frac{iq^2 e^{-q^2/6\alpha^2}}{3\sqrt{3}\alpha_h} \langle \hat{H} \rangle, \end{cases}$$



With the data for $S_{11} \to N\pi$ and $N\eta$ [1], i.e.

$$Br(S_{11}(1535) \to N\pi) = 35 \sim 55\%$$

 $Br(S_{11}(1650) \to N\pi) = 60 \sim 95\%$

$$\begin{aligned} Br(S_{11}(1535) \to N\eta) &= 45 \sim 60\% \\ Br(S_{11}(1650) \to N\eta) &= 3 \sim 10\% , \end{aligned}$$

$$\theta_S \approx 24.6^\circ \sim 32.1^\circ$$

Similarly, with the data for $D_{13} \to N\pi$

$$Br(D_{13}(1520) \to N\pi) = 55 \sim 65\%$$

$$Br(D_{13}(1700) \to N\pi) = 5 \sim 15\% ,$$

 $Br(D_{13}(1520) \rightarrow N\eta) = 0.23 \pm 0.04\%$ $Br(D_{13}(1700) \rightarrow N\eta) = 0.0 \pm 1.0\%$,



$\theta_S^{OPE} = 25.5^{\circ}$
$\theta_S^{OGE} = -32^{\circ}$

$$\begin{aligned} \theta_D^{OPE} &= -52.7^\circ \\ \theta_D^{OGE} &= 6^\circ \end{aligned}$$

Relative signs for the N*NM couplings are given by the NRCQM

$\theta_S(24.6^{\circ} \sim 32.1^{\circ})$	$S_{11}^+ \rightarrow p\eta$	$S^+_{11} \to \Lambda K^+$	$S_{11}^+ \rightarrow n\pi^+$	$S_{11}^+ \rightarrow p\pi^0$	$S_{11}^+ \rightarrow \Sigma^+ K^0$
$\mathcal{M}_{N \star \rightarrow NM}$	$6.86\sim7.18$	$4.32 \sim 4.07$	$3.29\sim 2.68$	$-2.312\sim-1.92$	$3.32\sim3.88$
$g_{S_{11}NM}$	$7.03 \sim 7.35$	$4.42 \sim 4.16$	$3.37 \sim 2.74$	$-2.38\sim-1.94$	$3.41\sim3.99$
$g_{S_{11}NM}/g_{S_{11}p\eta}$	1	$0.63\sim 0.57$	$0.48\sim 0.37$	$-0.34\sim-0.27$	$0.49\sim 0.54$

TABLE VI: Strong coupling constants for $S_{11}(1535) \rightarrow NM$.

$\theta_S(24.6^{\circ} \sim 32.1^{\circ})$	$S_{11}^+ \to p\eta$	$S_{11}^+ \rightarrow \Lambda K^+$	$S_{11}^+ \rightarrow n\pi^+$	$S_{11}^+ \rightarrow p\pi^0$	$S_{11}^+ \to \Sigma^+ K^0$
$\mathcal{M}_{N^* \rightarrow NM}$	$-2.56\sim-1.67$	$2.0\sim 2.57$	$4.06\sim 4.44$	$-2.85\sim-3.15$	$-4.19\sim-3.75$
$g_{S_{11}NM}$	$-2.50\sim-1.63$	$1.96\sim 2.51$	$3.96\sim 4.34$	$-2.80\sim-3.07$	$-4.0\sim-3.58$
$g_{S_{11}NM}/g_{S_{11}p\eta}$	1	$-0.78\sim-1.54$	$-1.58\sim-2.66$	$1.12\sim 1.88$	$1.6\sim 2.2$

TABLE VII: Strong coupling constants for $S_{11}(1650) \rightarrow NM$.

Indication of a destructive sign between S11(1535) and S11(1650) amplitudes in $\gamma p \rightarrow \eta p$, and $\pi^- p \rightarrow \eta n$.

arXiv: 0810.0997[nucl-th] by Aznauryan, Burkert and Lee.

It is important to have a correct definition of the common sign of amplitudes and relative sign between helicity amplitudes, i.e. A1/2, A3/2, and S1/2.



$$A_{\frac{1}{2},\frac{3}{2}} = \zeta \mathcal{A}_{\frac{1}{2},\frac{3}{2}}, \quad S_{\frac{1}{2}} = \zeta \mathcal{S}_{\frac{1}{2}}. \qquad \qquad \zeta = -sign(g^*/g)$$





LF RQM:

Capstick, Keister, PR D51 (1995) 3598 —————————————————Pace, Simula et.al., PR D51 (1995) 3598 Combined with the difficulties in the description of large width of $S_{11}(1535) \rightarrow \eta N$ and large $S_{11}(1535) \rightarrow \phi N, \Lambda K$ couplings, this shows that <u>3q picture for</u> $S_{11}(1535)$ should be complemented

From I. Aznauryan, Electromagnetic N-N* Transition Form Factors Workshop, 2008



FIG. 1: Helicity amplitude for $\gamma^* p \to S_{11}(1535)$



FIG. 2: Helicity amplitude for $\gamma^* n \to S_{11}(1535)$



FIG. 3: Helicity amplitude for $\gamma^* p \to S_{11}(1650)$



FIG. 4: Helicity amplitude for $\gamma^* n \to S_{11}(1650)$